Energetic Efficiency of Quantum Annealers

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We study the thermodynamical efficiency of D-Wave quantum annealers subject to reverse annealing and pausing protocols. We show the trade-off between computational precision and its energy cost

I. INTRODUCTION

II. THEORY

The D-wave processor is an open quantum system that exchanges energy in the form of heat with the environment and work with the external time-dependent control fields. Accordingly, the D-wave processor is a thermal machine which was recently confirmed, where it was shown that the D-Wave 2000Q under the reverse-annealing protocol, operates as a quantum heat engine. Motivated by the recent results for high fidelity control of the ground state of the Ising model by minimising the adiabatic action, we are interested in analyzing the thermodynamical efficiency of D-Wave quantum annealers, via different quantum annealing schedules: reverse, and pausing.

The Hamiltonian governing the evolution of the D-Wave chip is:

$$H(s_t) = (1-s_t)\sum_i \sigma_i^x + s_t\left(\sum_i h_i\sigma_i^z + \sum_{\langle i,j\rangle} J_{i,j}\sigma_i^z\sigma_j^z\right). \tag{1}$$

We assume that initially the system+environment state is given by:

$$\rho = \frac{\exp\left(-\beta_1 H_S\right)}{Z_S} \otimes \frac{\exp\left(-\beta_2 H_E\right)}{Z_E}. \tag{2}$$

Accordingly, the quantum exchange fluctuation theorem gives:

$$\frac{p(\Delta E_1, \Delta E_2)}{p(-\Delta E_1, -\Delta E_2)} = e^{\beta_1\Delta E_1 + \beta_2\Delta E_2}, \tag{3}$$

Where $\Delta E_i, i = 1, 2$ are, respectively, the (stochastic) energy changes of the processor and its environment, occurring in the scheduled time τ, and $p(\Delta E_1, \Delta E_2)$ is the joint probability of their occurrence in a single run of the annealing schedule.

By identifiying the entropy production $\Sigma = \beta_1\Delta E_1 + \beta_2\Delta E_2$. Eq. (3) can be re-written as:

$$\frac{p(\Sigma, \Delta E_1)}{p(-\Sigma, -\Delta E_1)} = e^{\Sigma}. \tag{4}$$

Using the thermodynamic uncertainty relation, we can express lower bounds of the entropy production $\langle\Sigma\rangle$, the average work $\langle W\rangle$, and the average heat $\langle Q\rangle$ as a function of the energy change of the processor ΔE_1:

$$\langle\Sigma\rangle \geq 2g\left(\frac{\langle\Delta E_1\rangle}{\sqrt{\langle\Delta E_1^2\rangle}}\right), \tag{5}$$

$$-\langle Q\rangle \geq \frac{2}{\beta_2} g\left(\frac{\langle\Delta E_1\rangle}{\sqrt{\langle\Delta E_1^2\rangle}}\right) - \frac{\beta_1}{\beta_2}\langle\Delta E_1\rangle, \tag{6}$$

$$\langle W\rangle \geq \frac{2}{\beta_2} g\left(\frac{\langle\Delta E_1\rangle}{\sqrt{\langle\Delta E_1^2\rangle}}\right) + \left(1 - \frac{\beta_1}{\beta_2}\right)\langle\Delta E_1\rangle, \tag{7}$$

where $g(x) = x\tanh^{-1}(x)$.

The thermodynamic efficiency is given by

$$\eta_{\text{th}} \leq -\frac{\langle W\rangle}{\langle Q\rangle}, \tag{8}$$

while the computational efficiency can be written as

$$\eta_{\text{comp}} \leq \frac{\mathcal{P}_{\text{GS}}}{\langle W\rangle}. \tag{9}$$

Here $\mathcal{P}_{\text{GS}}$ is the probability to find the ground state of the Hamiltonian, Eq. (1).

III. EXPERIMENTS

IV. CONCLUSION

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