STA130 (Winter 2022) Midterm Examination

Professor Scott Lee Schwartz

Thursday October 28th, 2022

First name (please write as legibly as possible within the boxes)

Last name

UTROid (e.g., schwa265)

- **All answers you provide must be your own.**
- **A pencil rather than a pen is recommended for this examination.**
- **You may have ONE "any size font front+back"** $8\frac{1}{2} \times 11\frac{1}{2}$ **"cheat sheet".**
- ***You may not use any other resources during the duration of the exam.***
- **You may not use your phone or a calculator or a computer, etc.**
- **All such items (phone, etc.) must remain stored at all times.**
- **You may not take any items with you if you leave the room.**

0. To keep things fair, no questions about the exam will be answered during the duration of the exam. If you think there is a problem with a question, note the question and briefly describe the problem in the space below and your concern will be evaluated during marking.

1. *In the box below*, explain what the code below does, explaining explicitly what `%>%` and `object` are.

   `"file.csv" %>% read_csv() -> object`

2. *Use three of the four options below to fill in the blanks and complete the following analogy sentence.*

   "If functions in R are like light, then ______________ breaks a lot of lightbulbs, but ______________ screws in a new lightbulb, and ______________ turns on the light switch."

   A. cloud GUI IDE B. `install.packages()` C. `library()` D. restarting Jupyterhub

Broken Off On

3. *In the follow blank space*, put the value of `(TRUE | (TRUE | FALSE)) & (TURE & FALSE)`: ______________

4. What is the most important thing that Rstudio does? *Select ONE of the following.*

   A. Facilitates R analysis reproducibility

   B. Saves files and manages R packages

   C. Allows you to code and program in R

   D. Makes R easily accessible in the cloud

5. *Fill in the blanks below.*

   - A colour measured scientifically as a wave frequency number is
     ____________________ variable represented as a ______________ data type and visualized using
     `geom_`__________ in R.
   - A word describing a colour is
     ____________________ variable represented as a ______________ data type and visualized using
     `geom_`__________ in R.
   - A day of the week starting on Monday is
     ____________________ variable represented as a ______________ data type and visualized using
     `geom_`__________ in R.
   - An either/or variable is
     ____________________ variable represented as a ______________ data type and visualized using
     `geom_`__________ in R.
   - The number of questions on an exam is
     ____________________ variable represented as a ______________ data type and visualized using
     `geom_`__________ in R.

6. *Indicate in the box below* what important parameter you should consider when using `geom_histogram()` that you don't need to consider when using `geom_boxplot()` and `geom_bar()`.

7. For the *tibble* called `people` with the column `handedness`, with values that are either `"left"` and `"right"`, *use the three boxes below to indicate the three things that are wrong with the code below.*

```
people %>%
  ggplot(x=handedness, y="") %>%
  geom_boxplot()
```

   Problem # 1:

   Problem # 2:

   Problem # 3:

8. The encoding of the handedness variable in the people *tibble* above is not the only way this variable could be encoded. *Indicate in the boxes below* what the encoding of the handedness variable in the people *tibble* is expected by the mutate() function for each of the two code chunks.

   Hint: the code comments in the NEXT problem may be helpful for understanding the "TRUE ~" code.

```
# left code chunk
people %>% mutate(
  case_when(handedness ~ "right",
                    TRUE ~ "left"))
```

```
# right code chunk
people %>% mutate(
  case_when(handedness == "right" ~ 1,
                              TRUE ~ 0’))
```

   # left code chunk

   # right code chunk

9. Assume the people *tibble* doesn't have columns named row_id and even_odd before this code is run.

   Hint: the comments in the code truthfully below explain what each part of the code does.

```
people %>%
  mutate(row_id = row_number(), # column row_id now stores the row number
                     # if the row_id value divided by 2 has remainder 0
                     # then row_id %% 2 == 0 is TRUE
         even_odd = case_when(row_id %% 2 == 0 ~ "even", # if this line is TRUE
                                          TRUE ~ "odd")) # this case is not run
```

   What columns are added to the people *tibble* after this code is run? *Select ONE of the following.*

   A. row_id B. even_odd C. Both row_id and even_odd D. Neither row_id nor even_odd

10. *Fill in the blanks in the following sentences and in the code below.* "Possible ____________________ can be handled with either of the completed code constructions below, but the _____________ version is preferable because then the ____________________ functions operate on the same input."

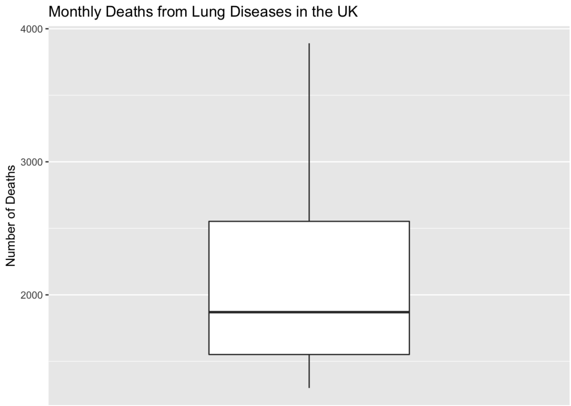
```
people %>% _______________________________  %>%
  summarise(n=n(), ‘% right handed‘ = paste(100*mean(handedness, __________), "%", sep=""))
                                    # paste() uses coercion to change the number to
                                    # a character string onto which it appends "%"
```

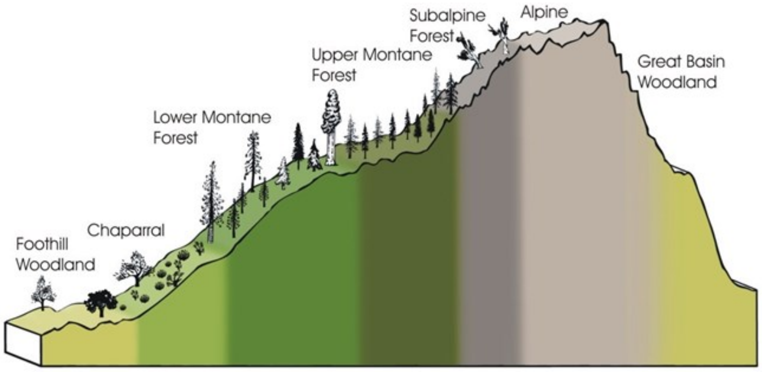
   "Still, this code above will only work if handedness is encoded as __________________________ or (due to _____________) if handedness is encoded as _________________________________."

11. What is the modality of this distribution given in the boxplot below (left figure)?

    A. Unimodal B. Bimodal C. Multimodal D. Uniform E. Can't tell

12. *Fill in the blanks to complete the following sentence.* "Describe the mountain below (right figure) as if it was a data distribution. This mountain is ______________ and _____________________ (which means that as a data distribution its *median* will be ______________ the *mean.*)"





13. *In the box below* write the *sample mean* $\bar{x}$ and *sample standard deviation* $s = \frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n-1}$ for the sample x which has the values `c(1, 2, 3)`?

14. For a *tibble* called `olympics` (which includes the `year`, `athlete`, `age`, and `event` variables), which of the following R code chunks results in a *tibble* containing only the `athlete`, `age`, and `event` information for the 10 youngest `athletes` for `year` $\geq$ 2000 (greater than or equal to 2000)?
Select ONE of the following choices.

A.
```
olympics %>% select(athlete, age, event) %>%
    filter(year >= 2000) %>% arrange(desc(age)) %>% head(10)
```

B.
```
olympics %>% filter(year >= 2000) %>%
    select(athlete, age, event) %>% arrange(age) %>% head(10)
```

C.
```
olympics %>% filter(year > 2000) %>% head(10) %>%
     select(athlete, age, event) %>% arrange(age)
```

D.
```
olympics %>% filter(year > 2000) %>% arrange(age) %>%
    head() %>% select(athlete, age, event)
```

15. For the `olympics` *tibble* noted above, which of the following R code chunks gives the average `age` of *athletes* for each `event` for `year` $<$ 2000 and `year` $\geq$ 2000 sorted by oldest to youngest average ages? *Select ONE of the following choices.*

    A.
    ```
    olympics %>% mutate(pre2000 = case_when(year<2000~TRUE, TRUE~FALSE)) %>%
      group_by(event, pre2000) %>% summarize(ave_age = mean(age, na.rm=TRUE)) %>%
      arrange(desc(ave_age))
    ```
    B.
    ```
    olympics %>% arrange(age, year<2000) %>%
      group_by(event, year<2000) %>% summarize(mean(age, na.rm=TRUE))
    ```
    C.
    ```
    olympics %>% mutate(case_when(pre2000 = year<2000~TRUE,
                                  post2000 = year>=2000~TRUE) %>%
      group_by(event, pre2000, post2000) %>% arrange(desc(ave_age)) %>%
      summarize(ave_age = mean(age, na.rm=TRUE))
    ```
    D.
    ```
    olympics %>% group_by(event & year<2000 | event & year>=2000) %>%
      summarize(ave_age = mean(age, na.rm=TRUE))) %>% arrange(desc(ave_age))
    ```

16. Which of the following best describes the *test statistic* of the *data generating mechanism* given in the following code? *Select ONE of the following choices.*

    ```
    phat <- mean(sample(c(0,1), size=100, prob=c(1/3,2/3), replace=TRUE))
    ```

    A. The proportion of heads for coin flips
    B. The proportion of heads for 100 coin flips where the chance of heads is 1/3
    C. The proportion of heads for 100 coin flips where the chance of heads is 2/3
    D. This code won't run as is with `replace=TRUE`

17. *In the two boxes below*, describe the values in the `test_stats` object when the following code is run for `n=2` and `n=3`.

    ```
    test_stats = 1:N; for(i in 1:N){set.seed(123)
      x <- sample(c(0,1), size=n, replace=FALSE)
      test_stats[i] <- mean(x, na.rm=TRUE)
    }
    ```

    n=2

    n=3

18. Which TWO of the following return a “shuffled” version of `x`? *Choose TWO that apply.*
    Hint: `rep(1,2)` returns `c(1,1)`.

    A. `sample(x, size=length(x), replace=FALSE)`
    B. `sample(x, size=length(x), prob=rep(1/length(x), length(x)), replace=TRUE)`
    C. `sample(x, size=length(x), prob=rep(1/length(x), length(x)), replace=FALSE)`
    D. `sample(x, size=length(x), prob=c(0.5,0.5), replace=TRUE)`

19. What assumption does mixing and shuffling two samples up implicitly make about the nature of the two samples? *In the box below, describe what you’re assuming about two populations generating two samples when you’re willing to shuffle the two samples, as in a two-sample permutation test.*

20. *Explain in the box below*, why the code below would no longer simulate a sampling distribution for a *two-sample permutation test* for `data` if `size=n1+n2` (in the first function in the `for` loop) was changed to `size=n1`? Address the *test statistic* compared to `permutation_test_stats` if `size=n1`.

```
# synthetic data created as an example data set
n1 <- 100; n2 <- 100; groups <- c(rep("one", n1), rep("two", n2))
x1 <- rnorm(mean=0, n=n1) # sample(c(0,1), size=n1, replace=TRUE)
x2 <- rnorm(mean=1, n=n1) # sample(c(0,1), size=n2, p=c(1/3,2/3), replace=TRUE)
data <- tibble(group = groups, outcome = c(x1, x2))
N <- 10000; permutation_test_stats <- 1:N; set.seed(130)
for(i in 1:N){
  shuffled_data <- data %>% mutate(group = sample(group, size=n1+n2, replace=FALSE))
  # shuffling the groups above, instead of shuffling the outcomes as done below
  # shuffled_data <- data %>% mutate(outcome = sample(outcome), size=n1+n2, replace=FALSE)
  # shuffled_xs <- sample(c(x1,x2), size=n1+n2, replace=FALSE) # sample(c(x1,x2))
  permutation_test_stats[i] <- shuffled_data %>% group_by(group) %>%
    summarise(means = mean(outcome), .groups="drop") %>%
    summarise(value = diff(means)) %>% as.numeric()
  # the above is equivalent to the following if we were instead using ‘shuffled_xs‘
  # permutation_test_stats[i] <- mean(shuffled_xs[1:n1])-mean(shuffled_xs[(n1+1):(n1+n2)])
}
```

21. Which code below is the most general statement of a *p-value*? *Select ONE of the following choices.*

   A. `mean(abs(sim_teststats)<=abs(obs_teststat))`

   B. `mean(abs(sim_teststats)>=abs(obs_teststat))`

   C. `mean(abs(sim_teststats-H0_parameter)<=abs(obs_teststat-H0_parameter))`

   D. `mean(abs(sim_teststats-H0_parameter)>=abs(obs_teststat-H0_parameter))`

22. *Write in the box below* the definition of a *p-value*.

23. Which of the following is a true description of the *p-value*? *Select ONE of the following choices.*

   A. The probability the *NULL hypothesis* H_0 is true

   B. The probability the parameter p of the *NULL hypothesis* $H_0 : p = p_0$ equals p_0

   C. The probability of a *Type-I* error.

   D. None of the above

24. For an $\alpha = 0.01$ *significance* (formal hypothesis) test, if a *p-value* of 0.02 has been calculated, what type of error might we make? *Select ONE of the following choices.*

   A. A Type I Error B. A Type II Error C. A Type III Error D. None of the above

25. *Explain in the box below* what happens to the *p-value* as `N` is increased in the following procedure.

```
for(i in 1:N){
  # sample data generating mechanism
  # calculate and save simulated test statistic
}
# compare observed test statistic to simulated test statistics
# to produce the estimated p-value of the observed test statistic
# with respect to the hypothesized data generating mechanism
```

26. From "Safety and Efficacy of the BNT162b2 mRNA Covid-19 Vaccine" published in the New England Journal of Medicine by Polack et al. of the C4591001 Clinical Trial Group:

> A total of 43,548 participants underwent randomization, of whom 43,448 received injections: 21,720 with BNT162b2 and 21,728 with placebo. There were 8 cases of [laboratory-confirmed] Covid-19 with onset at least 7 days after the second dose among participants assigned to receive BNT162b2 and 162 cases among those assigned to placebo. ... Among 10 cases of severe Covid-19 with onset after the first dose, 9 occurred in placebo recipients and 1 in a BNT162b2 recipient.

and the study found a vaccine efficacy of $\tilde{x} = 1 - (8/21720)/(162/21728) \approx 95\%$ with an associated *p-value* that resulted in the rejection of the *NULL hypothesis* at the $\alpha = 0.025$ level.

(a) What does the test statistic $\tilde{x} \approx 95\%$ measure? *Select ONE of the following choices.*

A. The rate of Covid-19 in the placebo group ($162/21728 \approx 0.0075$)

B. The rate of Covid-19 in the BNT162b2 group ($8/21720 \approx 0.0004$)

C. A ratio of Covid-19 rates between BNT162b2 and placebo groups

D. The chance of death from Covid-19 between BNT162b2 and placebo groups

(b) *State in the box below* the *NULL hypothesis* of the hypothesis in terms of p_1 and p_2, the chances of laboratory-confirmed Covid-19 cases in the BNT162b2 and placebo groups?

(c) *Fill in the following blanks.* "The *test statistic*'s *p-value* is _______ than or equal to $\alpha =$_______."

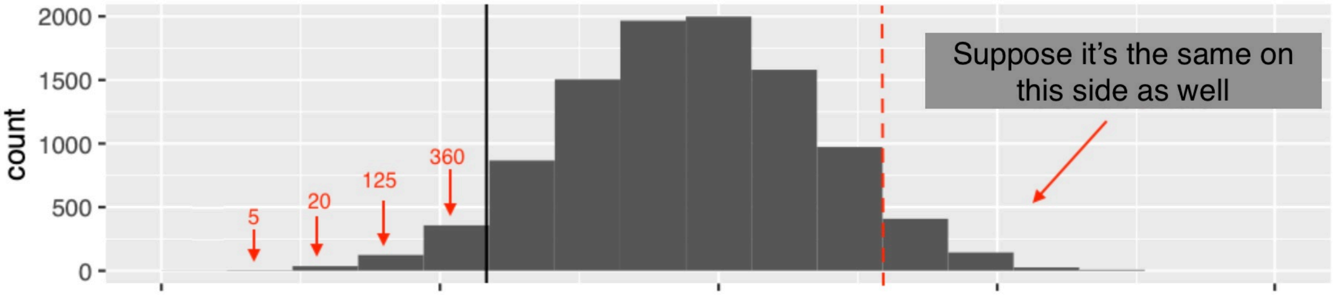
(d) Suppose the following *simulated sampling distribution* of the *test statistic* based on 10,000 *simulated test statistics* created assuming the *NULL hypothesis* is true was used to produce the *p-value* for this study. Assuming the *p-value* is based on a *two-sided hypothesis test* (where "as or more extreme" is symmetric), what is the "tallest" bin the *test statistic* could have fallen into while still definitely producing the result of this study? *Select ONE of the following choices.*

A. The bin with 5 counts

B. The bin 20 counts

C. The bin with 125 counts

D. The bin with 360 counts



27. *Fill in the blanks to complete the following sentence.* "A ____________ estimates a ____________; but, when bootstrapping a ____________ approximates a ____________."

28. *Fill in the blanks to complete the following bootstrapping code and, as asked by the prompts below, explain in the boxes below the code* why your choices for `size` and `replace` parameters respectively makes `bootstrap_stats` approximate a *sampling distribution* that is relevant for the *test statistic.*

```
bootstrap_statistics <- 1:B
for(b in 1:B){
  bootstrap_sample <- sample(x, size=_________, replace=_______)
  bootstrap_stats[b] <- statistic(bootstrap_sample) # a sample statistic
}
_________(bootstrap_stats, c(_________)) # 95% bootstrap confidence interval
```

Why would the opposite choice of `replace` *not work for your choice of* `size`?

For your choice of `replace` *why must you also use your choice of* `size`?

29. *Indicate in the box below* what, for the given *fixed confidence level* of 95% above, could be increased in the context of the *bootstrapping* code above to reduce the width of the *confidence interval*?

30. Which of the following correctly represents a 90% confidence interval $I = [\hat{\theta}_{\texttt{lower bound}}, \hat{\theta}_{\texttt{upper bound}}]$ for parameter θ? *Select ONE of the following choices.*

    A. A probability about θ; namely, $\Pr_\theta(\hat{\theta}_{\texttt{lower bound}} \leq \theta \leq \hat{\theta}_{\texttt{upper bound}}) = 0.90$

    B. A probability about the statistic I; namely, $\Pr_I(I \texttt{ bounds the true parameter value}) = 0.90$

    C. Exactly one out of every ten 90% *confidence intervals* will capture the true *parameter* value

    D. None of these correctly represent 90% *confidence intervals*