Internship Report

A Report on

Optimization and Implementation of Source Reconstruction Methods for Estimating Radiated Emissions in Electronic Devices

Submitted by

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Internship Period: 7 June,2024 - 22 July,2024

Submitted to

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July 16, 2024

Contents

1. Abstract

During my internship, I have comprehensively understood the methods of numerical and analytical integration and developed codes for computing the scattered field by a closed surface of an arbitrarily shaped body. The methodology involves discretizing the mesh into triangles, forming a simplex structure, and subsequently evaluating the vector and scalar potential due to one triangle over another. This process considers one triangle as the source triangle and the other as the observation triangle. The following steps outline the procedure and objectives achieved during the internship:

1.1 Discretization and Potential Evaluation

- The closed surface is discretized into triangular elements.
- For each pair of triangles (source and observation), the vector and scalar potentials are evaluated.
- For triangles in the neighborhood of the source triangle, an analytical method is used to increase accuracy. This is crucial as the $\frac{1}{R}$ factor increases drastically with decreasing distance, which can otherwise compromise accuracy.

1.2 Analytical and Numerical Integration

- Analytical integration is applied to neighboring triangles to handle the $\frac{1}{R}$ singularity smoothly across the triangle.
- Numerical integration is used for non-neighboring triangles to reduce computation time.

1.3 Computation of Moment Matrix and Forcing Vector

- Once the potentials due to all interactions are computed, the moment matrix Z_{mn} and the forcing vector V_m are calculated.
- These form the linear equation $ZI = V$, which is solved to find the current I for each triangle.

1.4 Surface Current and Scattered Field Calculation

- The surface current is expressed as $J = I_n \cdot f_n$, where f_n is the basis function.
- Using this surface current, the scattered electric field E can be calculated for any point near the arbitrarily shaped body.

This approach ensures a robust and efficient computation of the scattered field by leveraging both numerical and analytical methods, tailored to the specific requirements of the mesh configuration and the proximity of the interacting triangles. The combination of these methods provides a balance between accuracy and computational efficiency, enabling effective modeling of electromagnetic scattering for complex geometries.

State your internship objectives and expectations.

2. Tasks

2.1 Understanding Research Paper: Electromagnetic Scattering by Surfaces of Arbitrary Shape by IEEE

2.1.1 Introduction to the Paper

Electromagnetic scattering by surfaces of arbitrary shape is a critical problem in various fields, including radar, wireless communications, remote sensing, and electromagnetic compatibility. Traditional analytical methods often fall short in addressing the complexities of real-world structures, necessitating robust numerical techniques. This paper focuses on the application of the Method of Moments (MoM) to solve integral equations that describe the scattering of electromagnetic waves by these irregular surfaces.

The MoM is a powerful numerical method that transforms integral equations into a solvable matrix equation, making it suitable for dealing with complex geometries and boundary conditions. By discretizing the surface into smaller elements and using basis functions to approximate the current distribution, the MoM efficiently converts the continuous problem into a discrete one.

By implementing these techniques, the paper aims to enhance the accuracy and computational efficiency of electromagnetic scattering simulations.

2.1.2 Topics Covered in the Paper

Key Techniques

- **Method of Moments (MoM)**: Converts integral equations into a matrix form, facilitating the solution of complex scattering problems.

- **Discretization**: Breaks down the surface into smaller elements, approximating the current distribution using basis functions.

- **Planar Triangular Surface Patches**: Objects are modeled using planar triangular patches, applicable to both open and closed surfaces.

- **Special Basis Functions**: Develops subdomain-type basis functions defined on pairs of adjacent triangular patches to yield a current representation free of line or point charges at subdomain boundaries.

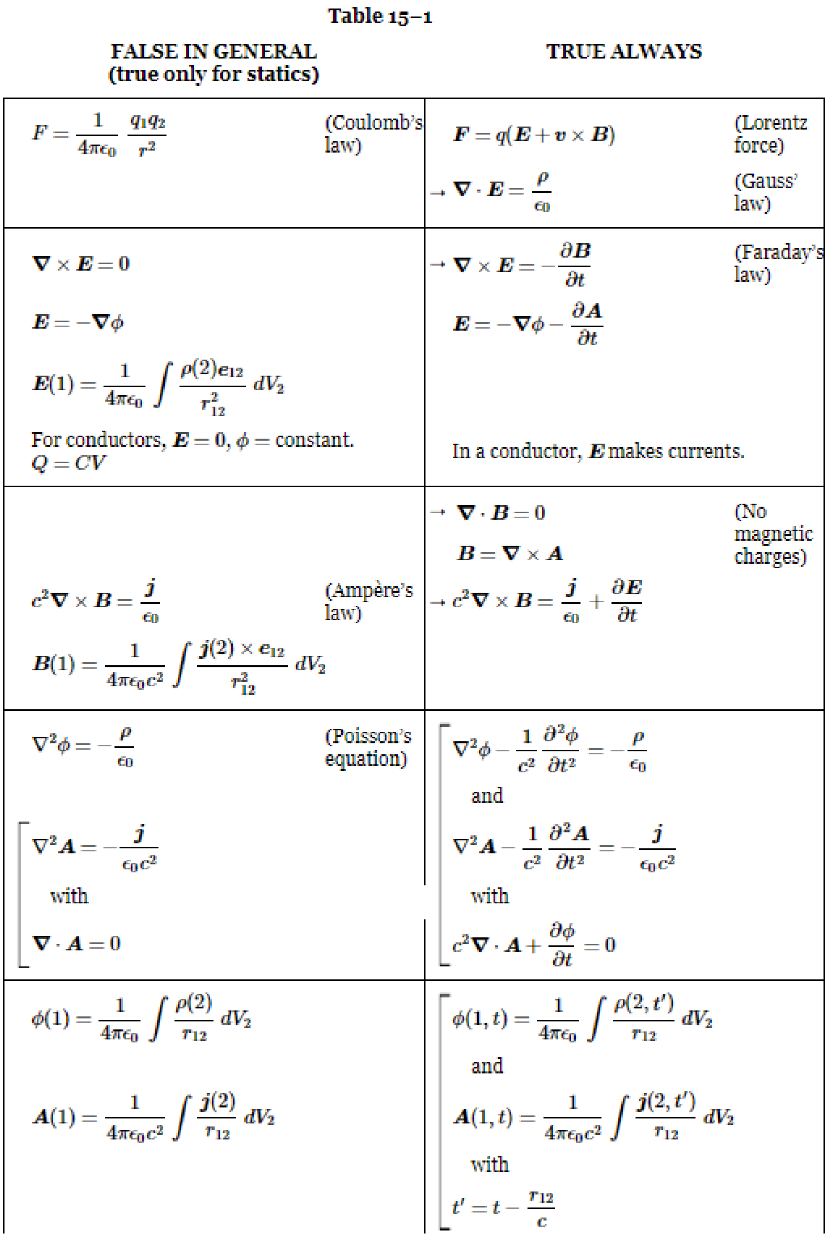

Table 15–1

FALSE IN GENERAL (true only for statics)		**TRUE ALWAYS**	
$\boldsymbol{F} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$	(Coulomb's law)	$\boldsymbol{F} = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$ $\boldsymbol{\nabla} \cdot \boldsymbol{E} = \frac{\rho}{\epsilon_0}$	(Lorentz force) (Gauss' law)
$\boldsymbol{\nabla} \times \boldsymbol{E} = 0$ $\boldsymbol{E} = -\boldsymbol{\nabla}\phi$ $\boldsymbol{E}(1) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(2)\boldsymbol{e}_{12}}{r_{12}^2}\,dV_2$ For conductors, $\boldsymbol{E} = 0$, $\phi =$ constant. $Q = CV$		$\boldsymbol{\nabla} \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$ $\boldsymbol{E} = -\boldsymbol{\nabla}\phi - \frac{\partial \boldsymbol{A}}{\partial t}$ In a conductor, $\boldsymbol{E}$ makes currents.	(Faraday's law)
$c^2\boldsymbol{\nabla} \times \boldsymbol{B} = \frac{\boldsymbol{j}}{\epsilon_0}$ $\boldsymbol{B}(1) = \frac{1}{4\pi\epsilon_0 c^2}\int \frac{\boldsymbol{j}(2) \times \boldsymbol{e}_{12}}{r_{12}^2}\,dV_2$	(Ampère's law)	$\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0$ $\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A}$ $c^2\boldsymbol{\nabla} \times \boldsymbol{B} = \frac{\boldsymbol{j}}{\epsilon_0} + \frac{\partial \boldsymbol{E}}{\partial t}$	(No magnetic charges)
$\nabla^2\phi = -\frac{\rho}{\epsilon_0}$ $\nabla^2\boldsymbol{A} = -\frac{\boldsymbol{j}}{\epsilon_0 c^2}$ with $\boldsymbol{\nabla} \cdot \boldsymbol{A} = 0$	(Poisson's equation)	$\nabla^2\phi - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = -\frac{\rho}{\epsilon_0}$ and $\nabla^2\boldsymbol{A} - \frac{1}{c^2}\frac{\partial^2\boldsymbol{A}}{\partial t^2} = -\frac{\boldsymbol{j}}{\epsilon_0 c^2}$ with $c^2\boldsymbol{\nabla} \cdot \boldsymbol{A} + \frac{\partial\phi}{\partial t} = 0$	
$\phi(1) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(2)}{r_{12}}\,dV_2$ $\boldsymbol{A}(1) = \frac{1}{4\pi\epsilon_0 c^2}\int \frac{\boldsymbol{j}(2)}{r_{12}}\,dV_2$		$\phi(1,t) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(2,t')}{r_{12}}\,dV_2$ and $\boldsymbol{A}(1,t) = \frac{1}{4\pi\epsilon_0 c^2}\int \frac{\boldsymbol{j}(2,t')}{r_{12}}\,dV_2$ with $t' = t - \frac{r_{12}}{c}$	

Figure 2.1: Maxwell's Equation

Main Points Covered

1. **Formulation of the Electric Field Integral Equation (EFIE)**

$$E_s = -j\omega A - \nabla\Phi \tag{2.1}$$

Here, A is the magnetic vector potential and Φ is the electric scalar potential, defined as:

$$A(r) = \frac{\mu}{4\pi}\int_S \frac{J(r')e^{-jk|r-r'|}}{|r-r'|}dS' \tag{2.2}$$

$$\Phi(r) = \frac{1}{4\pi\epsilon}\int_S \frac{\sigma(r')e^{-jk|r-r'|}}{|r-r'|}dS' \tag{2.3}$$

- μ is the permeability of the medium.
- r is the observation point.
- r' is the source point on the surface S.
- k is the wave number ($k = \frac{2\pi}{\lambda}$, where λ is the wavelength).
- $J(r')$ is the surface current density.
- ϵ is the permittivity of the medium.
- $\sigma(r')$ is the surface charge density.

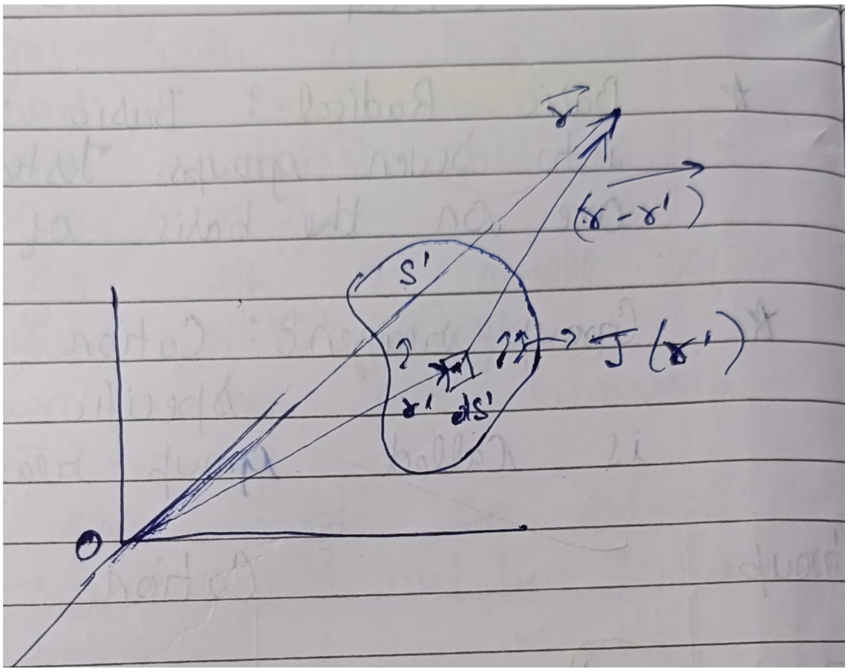

Figure 2.2: Vector Potential

Derives an integral equation for the surface current induced on a conducting scatterer from boundary conditions on the electric field. The surface charge density σ is related to the surface current by the continuity equation:

$$\nabla_s \cdot J = -j\omega\sigma \tag{2.4}$$

For a perfect conductor, the boundary condition at the surface S is that the tangential component of the total electric field must be zero:

$$(E_i + E_s)_{\tan} = 0 \tag{2.5}$$

This leads to the integral equation:

$$E^i_{\tan} = -(E_s)_{\tan} = j\omega A_{\tan} + (\nabla\Phi)_{\tan} \tag{2.6}$$

2. **Expansion of Current**

- Represents the unknown current J as a sum of weighted basis functions:

$$J = \sum_{n=1}^{N} I_n f_n$$

where I_n are the coefficients to be determined.

3. **RWG Basis Functions** The RWG basis functions are defined on pairs of adjacent triangular elements and are particularly useful for modeling arbitrary surfaces. Each RWG function is associated with an edge shared by two triangles, T^+ and T^-.

   Let's define an RWG basis function for an edge shared by two triangles, T^+ and T^-:

Definition

$$\mathbf{f}_n(\mathbf{r}) = \begin{cases} \frac{\mathbf{l}_n}{2A_n^+} p_n^+, & \mathbf{r} \in T^+ \\ \frac{\mathbf{l}_n}{2A_n^-} p_n^-, & \mathbf{r} \in T^- \end{cases} \tag{2.7}$$

where:

- $\mathbf{l}_n$ is the length of the edge shared by triangles T^+ and T^-.
- A_n^+ and A_n^- are the areas of triangles T^+ and T^-, respectively.
- $\mathbf{f}_n(\mathbf{r})$ is the RWG basis function.
- p_n^+ is the position vector with respect to the free vertex of T_n^+.
- p_n^- is the position vector with respect to the free vertex of T_n^-.

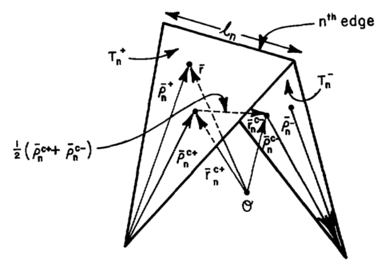

Fig. 2. Triangle pair and geometrical parameters associated with interior edge.

It is convenient to start our development by noting that each basis function is to be associated with an *interior edge*

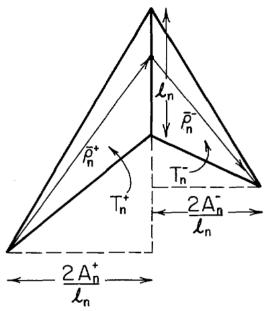

Fig. 3. Geometry for construction of component of basis function normal to edge.

Properties

- RWG functions are vector basis functions.
- They are defined piecewise on pairs of adjacent triangles.
- The basis function is linear within each triangle.
- RWG functions ensure the continuity of the tangential component of the current across the shared edge.

2.1.3 Moment Calculations Using Centroid Integration

To simplify the moment calculations, we approximate the integrals by evaluating the function at the centroid of each triangular element. The steps are as follows:

Steps

(a) **Approximate Integrals with Centroid Points**: Approximate the integral by evaluating the function at the centroid of the triangle.

(b) **Calculate Centroids**: Determine the centroid $\mathbf{r}_c$ of each triangular element T_i:

$$\mathbf{r}_c = \frac{\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3}{3}$$

(c) **Evaluate Magnetic Vector Potential at Centroids**: Compute the magnetic vector potential $\mathbf{A}(\mathbf{r}_c)$ at the centroid $\mathbf{r}_c$:

$$\mathbf{A}(\mathbf{r}_c) = \frac{\mu}{4\pi} \sum_j \frac{\mathbf{J}(\mathbf{r}_{c,j}) e^{-jk|\mathbf{r}_c - \mathbf{r}_{c,j}|}}{|\mathbf{r}_c - \mathbf{r}_{c,j}|} \Delta S_j$$

(d) **Evaluate Electric Scalar Potential at Centroids**: Compute the electric scalar potential $\Phi(\mathbf{r}_c)$ at the centroid $\mathbf{r}_c$:

$$\Phi(\mathbf{r}_c) = \frac{1}{4\pi\epsilon} \sum_j \frac{\sigma(\mathbf{r}_{c,j}) e^{-jk|\mathbf{r}_c - \mathbf{r}_{c,j}|}}{|\mathbf{r}_c - \mathbf{r}_{c,j}|} \Delta S_j$$

Example of Moment Method Matrix Entry

Each entry Z_{mn} in the MoM matrix can be approximated using centroid integration as follows:

$$Z_{mn} \approx j\omega \frac{\mu}{4\pi} \frac{\mathbf{f}_n(\mathbf{r}_{c,m}) \cdot \mathbf{f}_m(\mathbf{r}_{c,m}) e^{-jk|\mathbf{r}_{c,m} - \mathbf{r}_{c,n}|}}{|\mathbf{r}_{c,m} - \mathbf{r}_{c,n}|} \Delta S_n$$

Where:

- $\mathbf{f}_n(\mathbf{r}_{c,m})$ is the RWG basis function evaluated at the centroid $\mathbf{r}_{c,m}$.
- $\mathbf{f}_m(\mathbf{r}_{c,m})$ is the RWG basis function evaluated at the centroid $\mathbf{r}_{c,m}$.
- ΔS_n is the area of the n-th triangle.

Matrix Equation Formulation

Assembles the integral equation into a matrix form:

$$\mathbf{ZI} = \mathbf{V}$$

where $\mathbf{Z}$ is the impedance matrix, $\mathbf{I}$ is the vector of unknown coefficients, and $\mathbf{V}$ is the excitation vector.

- $[Z]_{mn}\,[I]_n = [V]_m$. This matrix has m rows because the test procedure has been performed m times on the electric field integral equation and has n columns because of the substitution of $J(r') = \sum I_n f_n$.

2.1.4 Coding for Selecting all pairs of triangular cells

Objective

The objective of this code is to read mesh data from a custom `.msh` file format, identify internal edges shared by triangles, and determine the third vertex from each of the two triangles connected by a common edge. Additionally, the code measures the time taken to perform these operations to analyze the relationship between the number of triangles and execution time.

Implementation

(a) **Reading the Mesh File**: The `read_custom_msh` function reads a custom `.msh` file and extracts vertex coordinates and element connectivity data. The function processes the file line by line to identify sections containing vertex and element data.

(b) **Finding Internal Edges and Third Vertices**: The`find_internal_edges_and_third_vert` function processes the mesh data to identify internal edges shared by two triangles and determines the third vertex for each of these triangles. It uses an edge-to-triangle mapping to identify shared edges.

(c) **Measuring Execution Time**: The execution time is measured using the `time` module. The time taken to read the mesh file and process the data is recorded and printed, allowing for analysis of the relationship between the number of triangles and the execution time.

2.1.5 Numerical Integration Technique in Triangular Domain

Numerical integration over a triangular domain is a critical technique in various fields such as finite element analysis, computational fluid dynamics, and structural engineering. This technique involves the approximation of an integral over a triangular region using a weighted sum of function values at specific points within the triangle. One common method for this is Gaussian quadrature, which provides an efficient and accurate way to perform numerical integration.

Gaussian Quadrature in Triangular Domain

Gaussian quadrature is a numerical integration method that approximates the integral of a function using a sum of weighted function values at specified points, called Gaussian points. In the context of a triangular domain, these points and weights are specifically chosen to maximize accuracy.

The integral of a function $f(x, y)$ over a triangular domain Δ can be approximated as:

$$\iint_{\Delta} f(x,y)\,dA \approx \sum_{i=1}^{n} w_i f(x_i, y_i)$$

where (x_i, y_i) are the Gaussian points within the triangle, and w_i are the corresponding weights.

This method is particularly useful for triangular domains because it can handle the irregular geometry of triangles efficiently. Gaussian quadrature provides an iterative way of solving integrals, making it easy to implement in computational codes.

Gauss Points and Weights

The following table lists the Gaussian points and weights for different orders of Gaussian quadrature in a triangular domain. For the unit triangle, which is an isosceles right triangle, the weights are multiplied by 0.5 to account for the triangle's area.

The Gaussian quadrature method is highly efficient for triangular domains due to its ability to handle the irregular shape of triangles. It provides an iterative and systematic way of solving integrals, making it straightforward to implement in computational codes. This method ensures accuracy and computational efficiency, which are crucial for applications in engineering and scientific simulations.

2.2 Developed Analytic Method for Solving Integrals on Triangular Domains

In my research, I focused on the numerical integration of linear shape functions multiplied by the 3-D Green's function or its gradient on triangular domains. This work builds on the methods discussed in two pivotal research papers:

(a) *On the Numerical Integration of the Linear Shape Functions Times the 3-D Green's Function or its Gradient on a Plane Triangle*

(b) *Potential Integrals for Uniform and Linear Source Distributions on Polygonal and Polyhedral Domains*

2.2.1 Key Objectives

The primary objective was to develop an analytic method for solving integrals on triangular domains that effectively removes singularities. This method is crucial for accurately solving electromagnetic scattering problems where integral equations often involve singular kernels.

2.2.2 Methodology

The method involves several key computational steps, which I implemented in a Python code. The approach includes the following:

(a) **Compute Triangle Properties**: Calculate side lengths, normal vector, area, and unit vectors for the triangle.

(b) **Compute Local Coordinates and Perpendicular Distances**: Determine local coordinates of points and distances from the projection of observation points to triangle sides.

(c) **Compute Distance Functions**: Calculate distances to nodes and sides from both the projection point and the observation point.

(d) **Compute Function Sets**: Compute logarithmic and arctangent functions that help in removing singularities.

(e) **Final Integration**: Integrate the derived functions to compute the integral values accurately.

2.2.3 Highlights of the Code

(a) **Triangle Properties Calculation**:
- **Side Lengths**: $l1$, $l2$, $l3$
- **Normal Vector**: $\mathbf{n}$
- **Area**: A
- **Unit Vectors**: $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$

(b) **Local Coordinates**:
- **Local Coordinates of P3**: $u3$, $v3$
- **Local Coordinates of Observation Point** r: $u0$, $v0$, $w0$

(c) **Perpendicular Distances to Nodes**:
- Distances $t1_0$, $t2_0$, $t3_0$

(d) **Distance Functions**:
- Compute $R1_0$, $R2_0$, $R3_0$ and their variations $R1_-$, $R1_+$, etc.

(e) **Logarithmic and Arctangent Functions**:
- Functions $f2i$ and $f3i$ to handle logarithmic calculations.
- Functions bi to handle arctangent calculations.

(f) **Integration**:
- Final integral values $I1$ and $I2$ are computed, which incorporate all the derived functions and distances.

Order	Points (x, y)	Weights
1	(1/3, 1/3)	1/2
2	(1/6, 1/6)	1/6
	(2/3, 1/6)	1/6
	(1/6, 2/3)	1/6
3	(1/3, 1/3)	-27/96
	(3/5, 1/5)	25/96
	(1/5, 3/5)	25/96
	(1/5, 1/5)	25/96
4	(0.108103018168070, 0.445948490915965)	0.223381589678011
	(0.445948490915965, 0.108103018168070)	0.223381589678011
	(0.445948490915965, 0.445948490915965)	0.223381589678011
	(0.816847572980459, 0.091576213509771)	0.109951743655322
	(0.091576213509771, 0.816847572980459)	0.109951743655322
	(0.091576213509771, 0.091576213509771)	0.109951743655322
5	(0.333333333333333, 0.333333333333333)	0.225000000000000
	(0.059715871789770, 0.470142064105115)	0.132394152788506
	(0.470142064105115, 0.059715871789770)	0.132394152788506
	(0.470142064105115, 0.470142064105115)	0.132394152788506
	(0.797426985353087, 0.101286507323456)	0.125939180544827
	(0.101286507323456, 0.797426985353087)	0.125939180544827
	(0.101286507323456, 0.101286507323456)	0.125939180544827
6	(0.501426509658179, 0.249286745170910)	0.0583931378631895
	(0.249286745170910, 0.501426509658179)	0.0583931378631895
	(0.249286745170910, 0.249286745170910)	0.0583931378631895
	(0.873821971016996, 0.063089014491502)	0.0254224531851035
	(0.063089014491502, 0.873821971016996)	0.0254224531851035
	(0.063089014491502, 0.063089014491502)	0.0254224531851035
	(0.053145049844817, 0.310352451033784)	0.041425537809187
	(0.053145049844817, 0.636502499121399)	0.041425537809187
	(0.310352451033784, 0.053145049844817)	0.041425537809187
	(0.310352451033784, 0.636502499121399)	0.041425537809187
	(0.636502499121399, 0.053145049844817)	0.041425537809187
	(0.636502499121399, 0.310352451033784)	0.041425537809187
7	(0.333333333333333, 0.333333333333333)	-0.074785022233841
	(0.479308067841920, 0.260345966079040)	0.087807628716604
	(0.260345966079040, 0.479308067841920)	0.087807628716604
	(0.260345966079040, 0.260345966079040)	0.087807628716604
	(0.869739794195568, 0.065130102902216)	0.026673617804419
	(0.065130102902216, 0.869739794195568)	0.026673617804419
	(0.065130102902216, 0.065130102902216)	0.026673617804419
	(0.048690315425316, 0.312865496004874)	0.038556880445129
	(0.048690315425316, 0.638444188569810)	0.038556880445129
	(0.312865496004874, 0.048690315425316)	0.038556880445129
	(0.312865496004874, 0.638444188569810)	0.038556880445129
	(0.638444188569810, 0.048690315425316)	0.038556880445129
	(0.638444188569810, 0.312865496004874)	0.038556880445129

Table 2.1: Gaussian points and weights for various orders in a triangular domain.

3. Conclusion

The developed analytic method and accompanying Python code provide an efficient and accurate way to solve integrals on triangular domains, crucial for electromagnetic scattering and other related problems. By removing singularities through careful computation of distances and function sets, this method enhances the precision and reliability of numerical solutions in practical applications.

A. Supporting Documents

This chapter lists the supporting documents and references that were instrumental during the internship for understanding and developing the methods and codes for electromagnetic scattering by surfaces of arbitrary shape.

A.1 References

(a) *Electromagnetic Scattering by Surfaces of Arbitrary Shape*

(b) *High Degree Efficient Symmetrical Gaussian Quadrature Rules for the Triangle*

(c) *On the Numerical Integration of the Linear Shape Functions Times the 3-D Green's Function or its Gradient on a Plane Triangle*

(d) *Potential Integrals for Uniform and Linear Source Distributions on Polygonal and Polyhedral Domains*

(e) *Dwight H.R. - Tables of Integrals and Other Mathematical Data (1957)*

These documents provided crucial theoretical background, numerical methods, and mathematical tables that facilitated the development of the analytical and numerical integration techniques used in this project.