Outline

- Relational Algebra
  - Unary Relational Operations
    - → Select, Project
  - Relational Algebra Operations From Set Theory
    - → Union, Intersection, Difference, Cartesian Product
  - Binary Relational Operations
    - → Join, Set Division
  - Additional Relational Operations
    - → Generalized project, Aggregates, Outer Join
  - Examples and exercise
- Relational Calculus
  - Tuple Relational Calculus
- Coming up
  - SQL

Additional Relational Operations – Generalized projection

- Allows functions of attributes to be included in the projection list

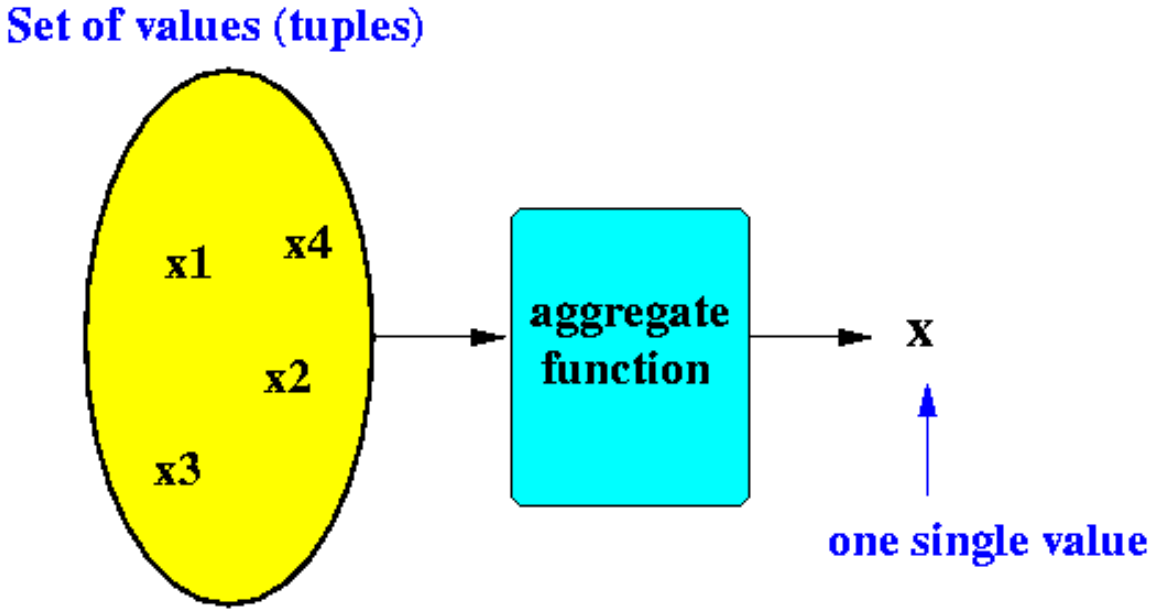
$$\pi_{F1,\, F2,\, \ldots,\, Fn}\,(R)$$

- Example

$$\pi_{\text{LNAME, FNAME, SALARY*1.03}}\,(\text{EMPLOYEE})$$

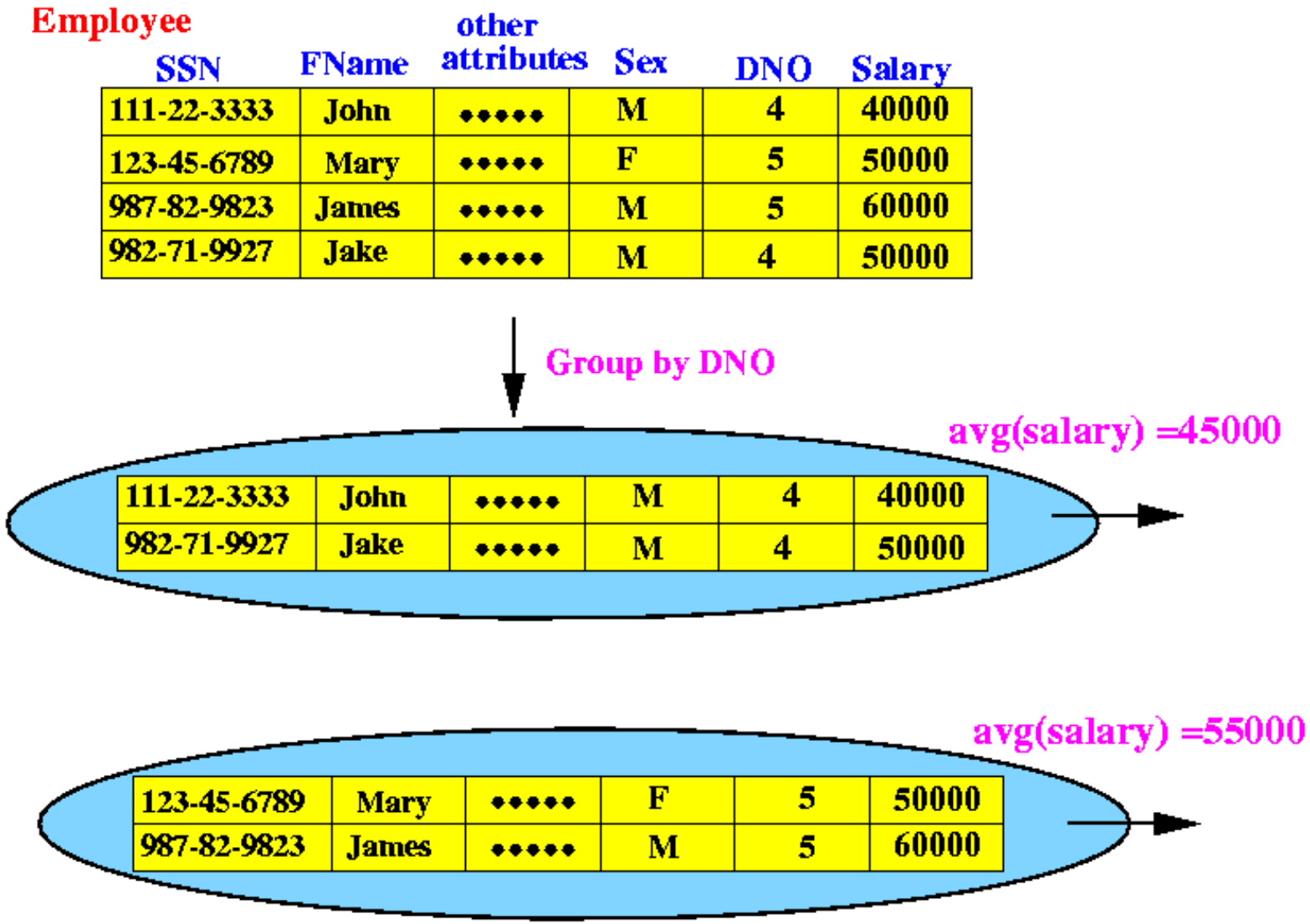
Additional Operations – Aggregate Functions

- An **aggregate functions** operates on a collection of values (tuples) from the database and computes one single value as output.
- Common functions include SUM, AVERAGE, MAXIMUM, and MINIMUM and COUNT.

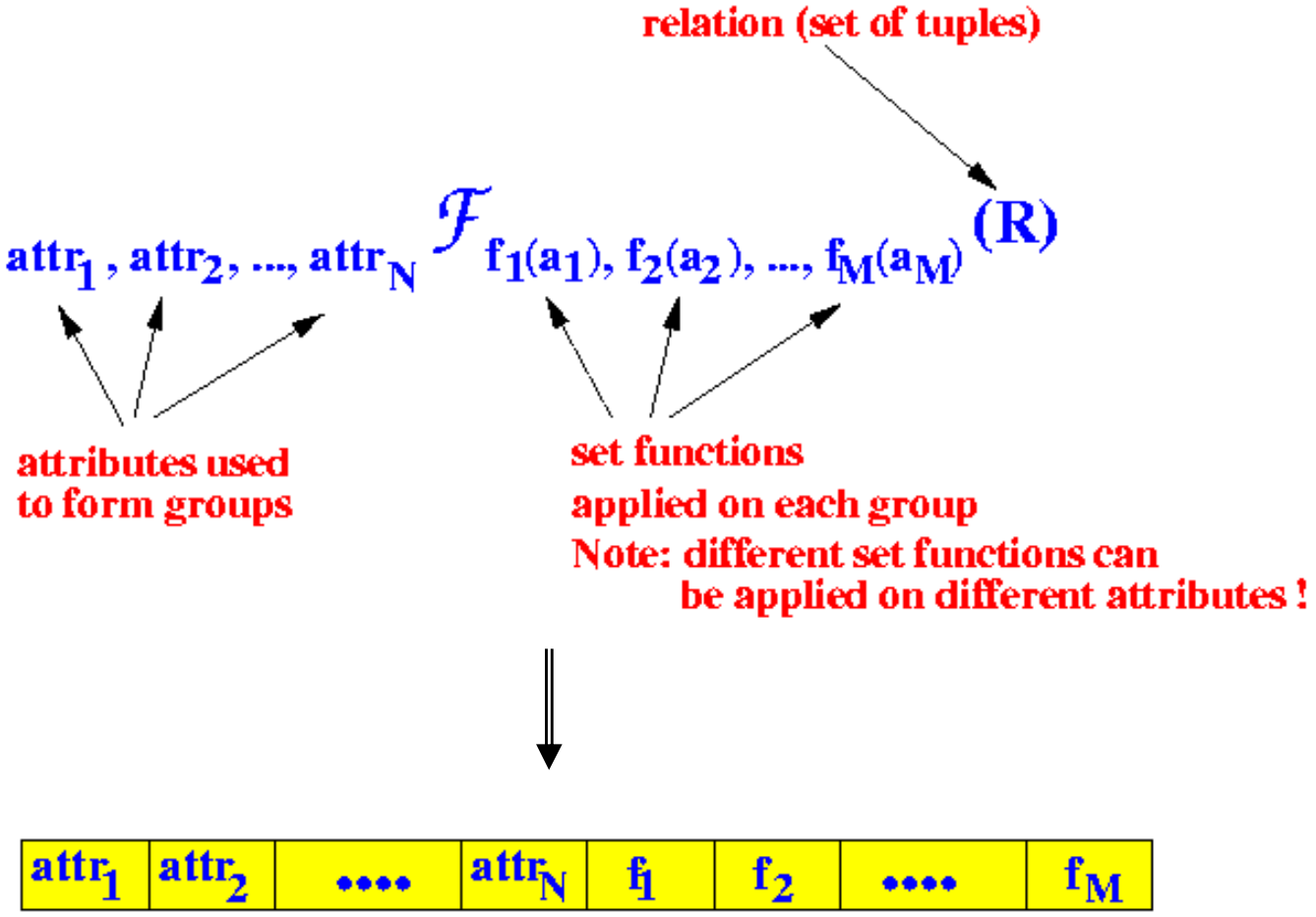


Aggregate Functions and Grouping

- Groups the tuples by the values of some attributes and then apply an aggregate function independently to each group



Aggregation and Grouping – Formal Notation



EMPLOYEE

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
John	B	Smith	123456789	1965-01-09	731 Fondren, Houston, TX	M	30000	333445555	5
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston, TX	M	40000	888665555	5
Alicia	J	Zelaya	999887777	1968-01-19	3321 Castle, Spring, TX	F	25000	987654321	4
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Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5
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Figure 6.10

The aggregate function operation.

a. $\rho_{R(\text{Dno, No\_of\_employees, Average\_sal})}(\text{Dno } \Im \text{ COUNT Ssn, AVERAGE Salary}(\text{EMPLOYEE}))$.

b. $\text{Dno } \Im \text{ COUNT Ssn, AVERAGE Salary}(\text{EMPLOYEE})$.

c. $\Im \text{ COUNT Ssn, AVERAGE Salary}(\text{EMPLOYEE})$.

R

(a)

Dno	No_of_employees	Average_sal
5	4	33250
4	3	31000
1	1	55000

(b)

Dno	Count_ssn	Average_salary
5	4	33250
4	3	31000
1	1	55000

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(b)

Dno	Count_ssn	Average_salary
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4	3	31000
1	1	55000

(c)

Count_ssn	Average_salary
8	35125

Operations of Relational Algebra

Table 6.1 Operations of Relational Algebra

OPERATION	PURPOSE	NOTATION
SELECT	Selects all tuples that satisfy the selection condition from a relation R.	$\sigma_{\text{<selection condition>}}(R)$
PROJECT	Produces a new relation with only some of the attributes of R, and removes duplicate tuples.	$\pi_{\text{<attribute list>}}(R)$
THETA JOIN	Produces all combinations of tuples from R_1 and R_2 that satisfy the join condition.	$R_1 \bowtie_{\text{<join condition>}} R_2$
EQUIJOIN	Produces all the combinations of tuples from R_1 and R_2 that satisfy a join condition with only equality comparisons.	$R_1 \bowtie_{\text{<join condition>}} R_2$, OR $R_1 \bowtie_{\text{(<join attributes 1>), (<join attributes 2>)}} R_2$
NATURAL JOIN	Same as EQUIJOIN except that the join attributes of R_2 are not included in the resulting relation; if the join attributes have the same names, they do not have to be specified at all.	$R_1 *_{\text{<join condition>}} R_2$, OR $R_1 *_{\text{(<join attributes 1>), (<join attributes 2>)}} R_2$ OR $R_1 * R_2$

Operations of Relational Algebra

UNION	Produces a relation that includes all the tuples in R_1 or R_2 or both R_1 and R_2; R_1 and R_2 must be union compatible.	$R_1 \cup R_2$
INTERSECTION	Produces a relation that includes all the tuples in both R_1 and R_2; R_1 and R_2 must be union compatible.	$R_1 \cap R_2$
DIFFERENCE	Produces a relation that includes all the tuples in R_1 that are not in R_2; R_1 and R_2 must be union compatible.	$R_1 - R_2$
CARTESIAN PRODUCT	Produces a relation that has the attributes of R_1 and R_2 and includes as tuples all possible combinations of tuples from R_1 and R_2.	$R_1 \times R_2$
DIVISION	Produces a relation $R(X)$ that includes all tuples $t[X]$ in $R_1(Z)$ that appear in R_1 in combination with every tuple from $R_2(Y)$, where $Z = X \cup Y$.	$R_1(Z) \div R_2(Y)$

Complete Set of Relational Operations

- The set of operations including **select σ, project π , union ∪, set difference - , and cartesian product X** is called a complete set because any other relational algebra expression can be expressed by a combination of these five operations.

- For example:

  $\mathbf{R \cap S = (R \cup S) - ((R - S) \cup (S - R))}$

  $R \bowtie_{\text{<join condition>}} S = \sigma_{\text{<join condition>}} (\mathbf{R \times S})$

Additional Operations

- Generalized Projections
- Aggregate and Groupings
- Outer Joins

Examples of Queries in Relational Algebra

Query 1. Retrieve the name and address of all employees who work for the 'Research' department.

$$\text{RESEARCH\_DEPT} \leftarrow \sigma_{\text{Dname='Research'}}(\text{DEPARTMENT})$$
$$\text{RESEARCH\_EMPS} \leftarrow (\text{RESEARCH\_DEPT} \bowtie_{\text{Dnumber=Dno}} \text{EMPLOYEE})$$
$$\text{RESULT} \leftarrow \pi_{\text{Fname, Lname, Address}}(\text{RESEARCH\_EMPS})$$

As a single in-line expression, this query becomes:

$$\pi_{\text{Fname, Lname, Address}} (\sigma_{\text{Dname='Research'}}(\text{DEPARTMENT} \bowtie_{\text{Dnumber=Dno}} (\text{EMPLOYEE}))$$

Query 3. Find the names of employees who work on *all* the projects controlled by department number 5.

$$\text{DEPT5\_PROJS} \leftarrow \rho_{(\text{Pno})}(\pi_{\text{Pnumber}}(\sigma_{\text{Dnum=5}}(\text{PROJECT})))$$
$$\text{EMP\_PROJ} \leftarrow \rho_{(\text{Ssn, Pno})}(\pi_{\text{Essn, Pno}}(\text{WORKS\_ON}))$$
$$\text{RESULT\_EMP\_SSNS} \leftarrow \text{EMP\_PROJ} \div \text{DEPT5\_PROJS}$$
$$\text{RESULT} \leftarrow \pi_{\text{Lname, Fname}}(\text{RESULT\_EMP\_SSNS} * \text{EMPLOYEE})$$

Examples of Queries in Relational Algebra

Query 6. Retrieve the names of employees who have no dependents.

This is an example of the type of query that uses the MINUS (SET DIFFERENCE) operation.

$$\text{ALL\_EMPS} \leftarrow \pi_{\text{Ssn}}(\text{EMPLOYEE})$$
$$\text{EMPS\_WITH\_DEPS(Ssn)} \leftarrow \pi_{\text{Essn}}(\text{DEPENDENT})$$
$$\text{EMPS\_WITHOUT\_DEPS} \leftarrow (\text{ALL\_EMPS} - \text{EMPS\_WITH\_DEPS})$$
$$\text{RESULT} \leftarrow \pi_{\text{Lname, Fname}}(\text{EMPS\_WITHOUT\_DEPS} * \text{EMPLOYEE})$$

Query 7. List the names of managers who have at least one dependent.

$$\text{MGRS(Ssn)} \leftarrow \pi_{\text{Mgr\_ssn}}(\text{DEPARTMENT})$$
$$\text{EMPS\_WITH\_DEPS(Ssn)} \leftarrow \pi_{\text{Essn}}(\text{DEPENDENT})$$
$$\text{MGRS\_WITH\_DEPS} \leftarrow (\text{MGRS} \cap \text{EMPS\_WITH\_DEPS})$$
$$\text{RESULT} \leftarrow \pi_{\text{Lname, Fname}}(\text{MGRS\_WITH\_DEPS} * \text{EMPLOYEE})$$

1. Find fname and lname of all employees who have 2 or more dependents

EMPLOYEE

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
John	B	Smith	123456789	1965-01-09	731 Fondren, Houston, TX	M	30000	333445555	5
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston, TX	M	40000	888665555	5
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James	E	Borg	888665555	1937-11-10	450 Stone, Houston, TX	M	55000	NULL	1

Essn	Dependent_name	Sex	Bdate	Relationship
333445555	Alice	F	1986-04-05	Daughter
333445555	Theodore	M	1983-10-25	Son
333445555	Joy	F	1958-05-03	Spouse
987654321	Abner	M	1942-02-28	Spouse
123456789	Michael	M	1988-01-04	Son
123456789	Alice	F	1988-12-30	Daughter
123456789	Elizabeth	F	1967-05-05	Spouse

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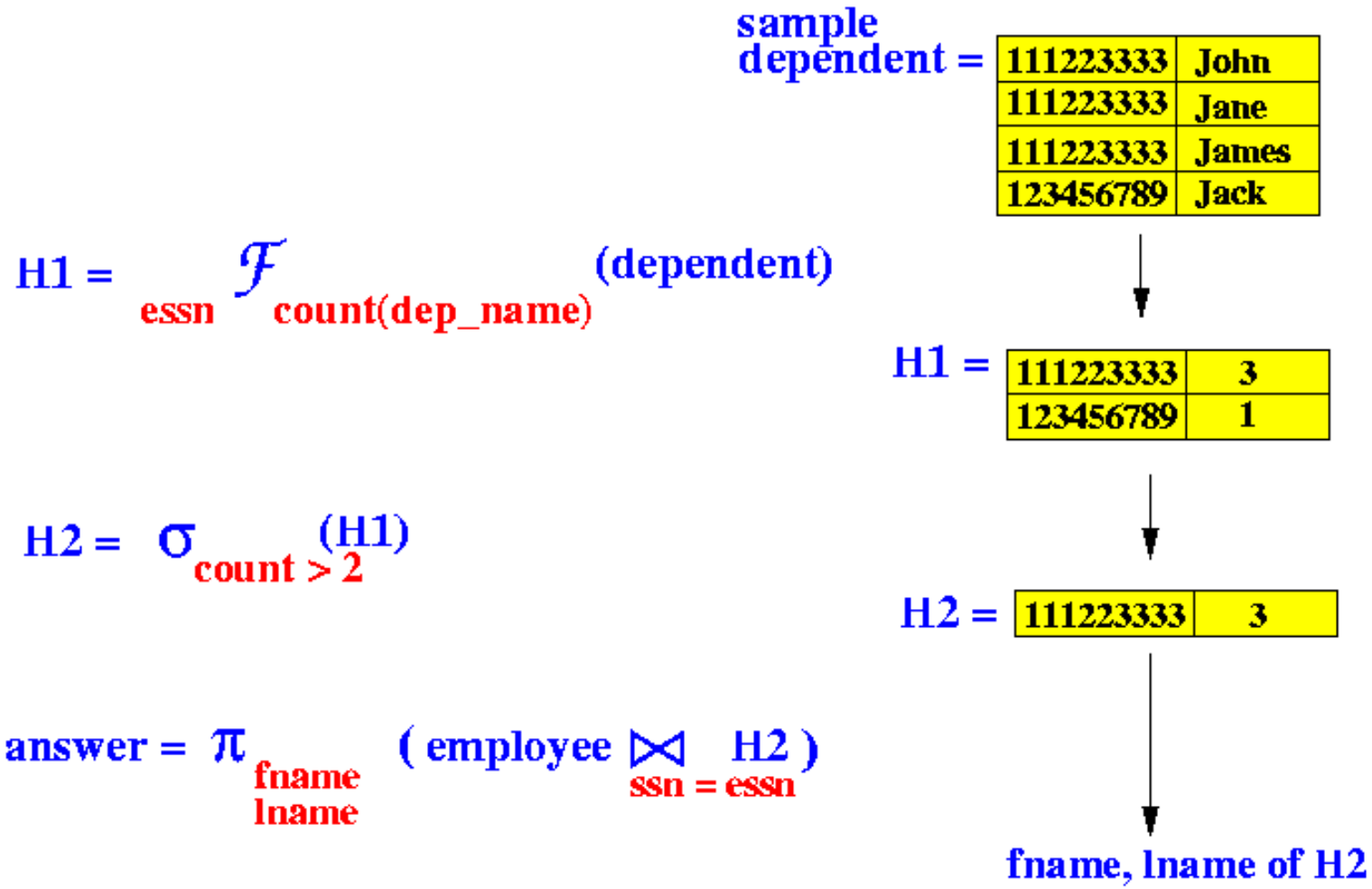

2. Find fname and lname of the employees who have the most number of dependents

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Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
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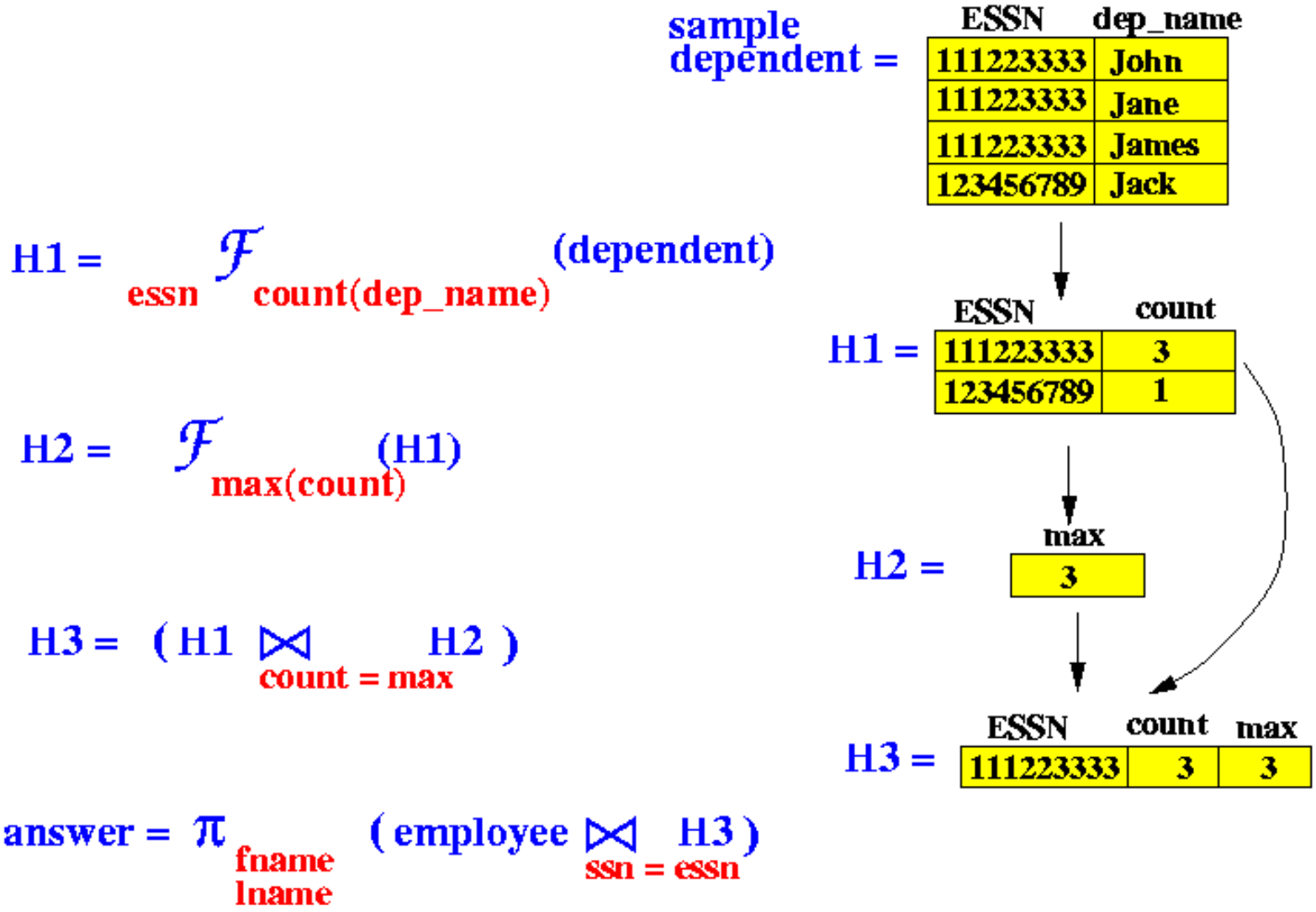

3. Find fname and lname of the employee(s) in the ‘Research’ department who earn the highest salary in the department

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Figure 3.6
One possible database state for the COMPANY relational database schema.

EMPLOYEE

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John	B	Smith	123456789	1965-01-09	731 Fondren, Houston, TX	M	30000	333445555	5
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DEPARTMENT

Dname	Dnumber	Mgr_ssn	Mgr_start_date
Research	5	333445555	1988-05-22
Administration	4	987654321	1995-01-01
Headquarters	1	888665555	1981-06-19

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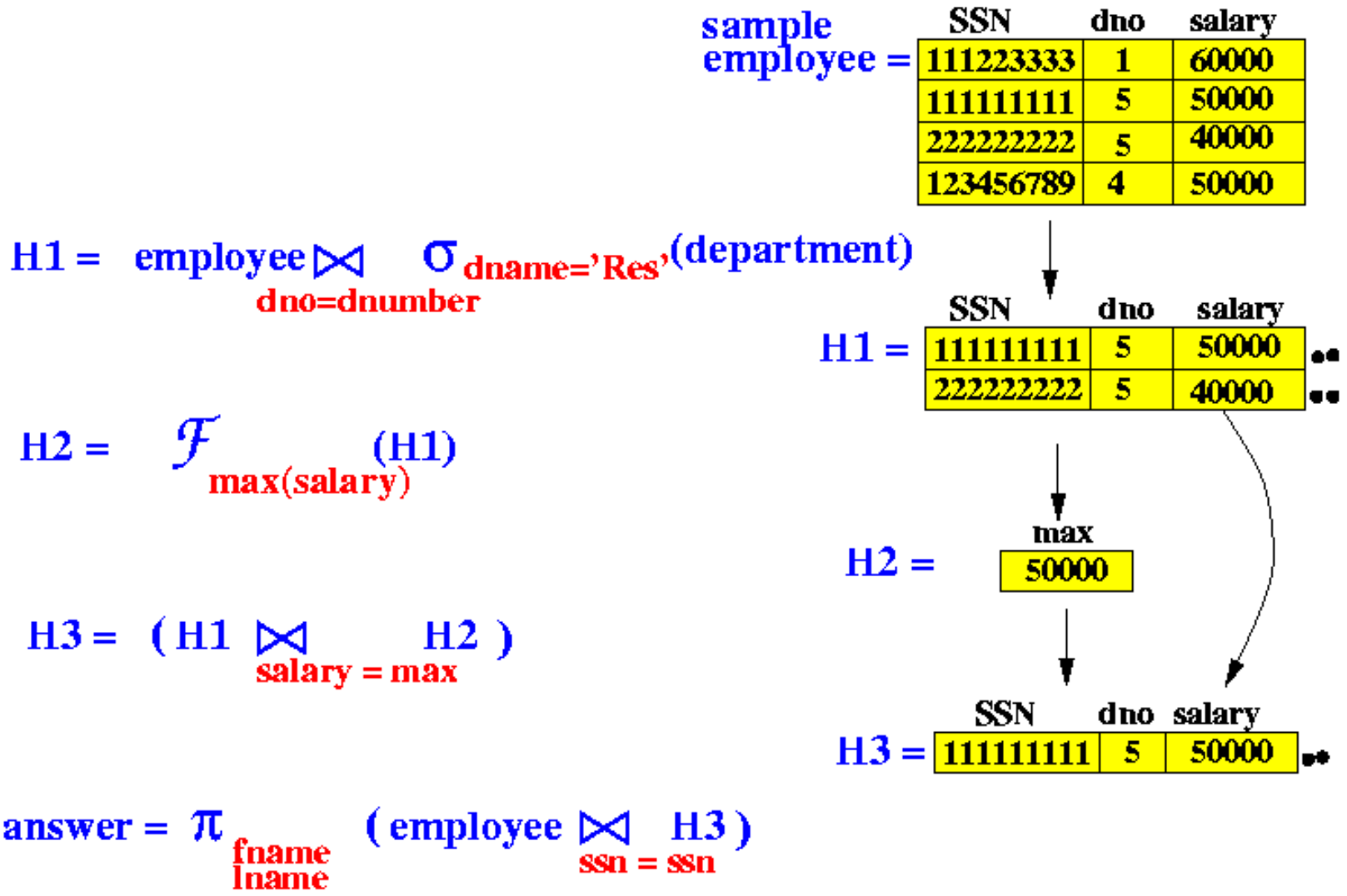

4. For each department, show the name, number of employees, minimum salary and maximum salary paid to the employees in the department

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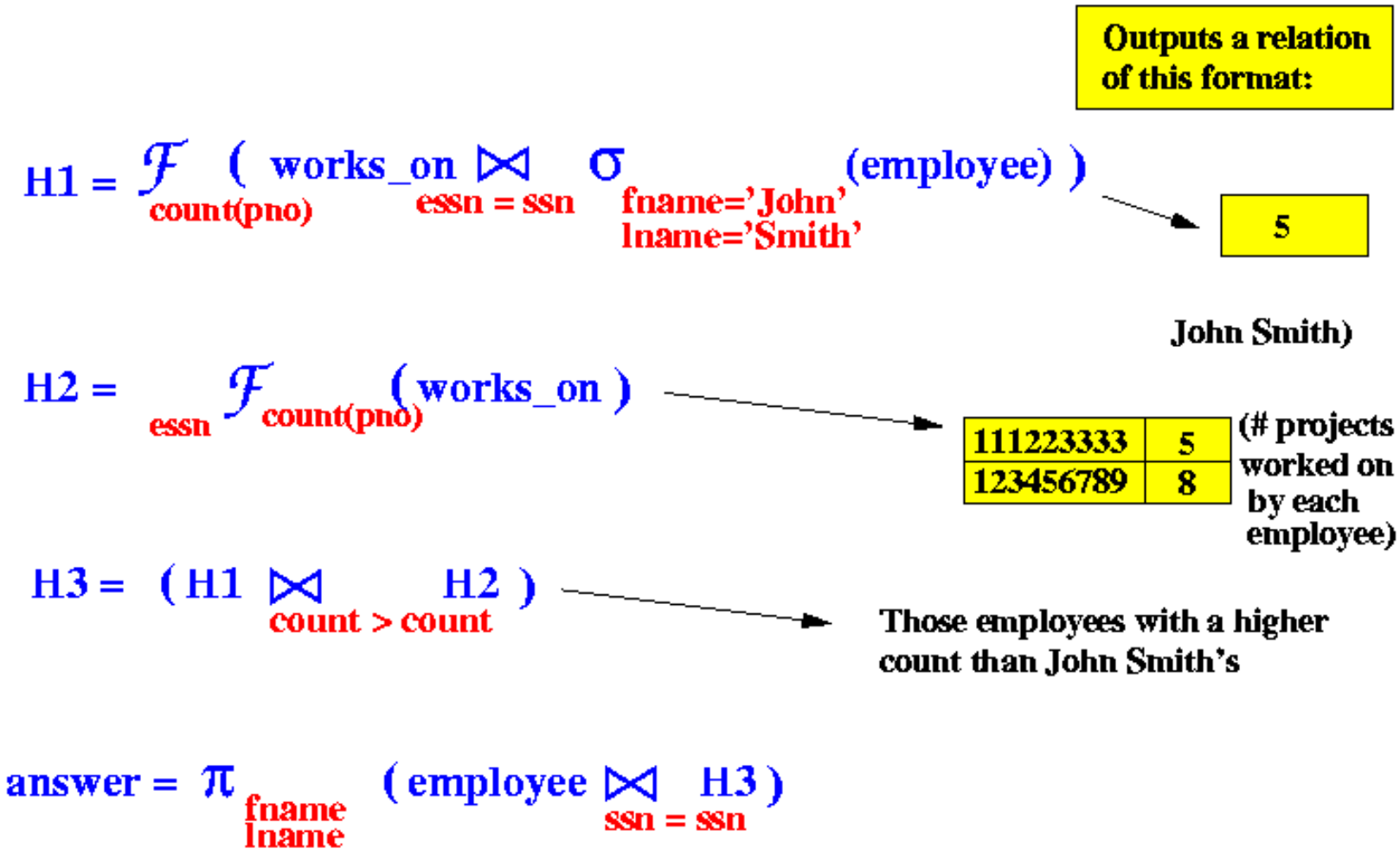
$$\text{dname}\ \mathcal{F}_{\substack{\text{count(ssn)}\\ \text{min(salary)}\\ \text{max(salary)}}}\ (\ \text{department} \bowtie_{\text{dnumber} = \text{dno}} \text{employee}\ )$$

Exercise

5. Find fname and lname of all employees who work on more projects than ‘John Smith’
6. Find fname and lname of all employees who work on 2 or more projects controlled by the ‘Research’ department
7. Find fname and lname of all employees who work on all projects controlled by the ‘Research’ department
8. Find fname and lname of all employeeds that do not work on any projects controlled by the ‘Research’ department

5. Find fname and lname of all employees who work on more projects than ‘John Smith’



6. Find fname and lname of all employees who work on 2 or more projects controlled by the 'Research' department

$$H1 = project \underset{dnum=dnumber}{\bowtie} \sigma_{dname='Research'}(department)$$

(all projects controlled by the research department)

$$H2 = {}_{essn}\mathcal{F}_{count(pno)}(works\_on \underset{pno=pnumber}{\bowtie} H1)$$

(employee ssn, #projects in H1 that emp. works on)

$$H3 = \sigma_{count>=2}(H3)$$

(employee ssn that work on >=2 proj controlled by 'Res' dept)

$$answer = \pi_{\substack{fname \\ lname}}(employee \underset{ssn = ssn}{\bowtie} H3)$$

7. Find fname and lname of all employees who work on all projects controlled by the 'Research' department

$$H1 = \pi_{pnumber}\ (\ project \bowtie_{dnum=dnumber}\ \sigma_{dname='Research'}\ (department)\ )$$

(proj# of all projects controlled by the research department)

$$H2 = \pi_{essn\ pno}\ (project)$$

(who works on which project)

$$H3 = H2 \div H1$$

(ssn of employees that works on all projects in H1)

$$answer = \pi_{fname\ lname}\ (employee \bowtie_{ssn = ssn}\ H3)$$

8. Find fname and lname of all employeeds that do not work on any projects controlled by the 'Research' department

$$H1 = \pi_{pnumber}\left(project \bowtie_{dnum=dnumber} \sigma_{dname='Research'}(department)\right)$$

(proj# of all projects controlled by the research department)

$$H2 = \pi_{essn}\left(works\_on \bowtie_{pno=pnumber} H1\right)$$

(essn of employees who DO works on a project in H1)

$$H3 = \pi_{ssn}(employee) - H2$$

(essn of employees who DO NOT works on any project in H1)

$$answer = \pi_{fname, lname}\left(employee \bowtie_{ssn=ssn} H3\right)$$