Package 'fitAutoReg'

September 25, 2023
Title Fit Autoregressive Models
Version 0.9.1
Description Fit autoregressive models using 'RcppArmadillo'.
<pre>URL https://github.com/ijapesigan/fitAutoReg,</pre>
https://ijapesigan.github.io/fitAutoReg/
BugReports https://github.com/ijapesigan/fitAutoReg/issues
License GPL (>= 3)
Encoding UTF-8
LazyData true
Roxygen list(markdown = TRUE)
VignetteBuilder knitr
Depends R (>= $3.5.0$)
LinkingTo Rcpp, RcppArmadillo
Imports Rcpp, dynr
Suggests knitr, rmarkdown, testthat, MASS
RoxygenNote 7.2.3
NeedsCompilation yes
Author Ivan Jacob Agaloos Pesigan [aut, cre, cph] (https://orcid.org/0000-0003-4818-8420)
Maintainer Ivan Jacob Agaloos Pesigan < r.ijapesigan@gmail.com>
R topics documented:
BootCI

2 BootCI

YXExo	
YX	
StdMat	
SimVARExo	
SimVAR	
SelectVARLasso	
SearchVARLasso	
RBootVAROLS	
RBootVARLasso	
RBootVARExoOLS	
RBootVARExoLasso	
print.dynr_model	
plot.dynr model	
PBootVAROLS	
PBootVARLasso	
PBootVARExoOLS	
PBootVARExoLasso	
OrigScale	
ModelVARP2Dynr	
ModelVARP1Dynr	
LambdaSeq	
FitVAROLS	
FitVARMplus	
FitVARLassoSearch	
FitVARLasso	
FitVARDynr	
FitMLVARMplus	
FitMLVARDynr	
dat_p2_yx	
dat_p2_exo_yx	
dat_p2	

 ${\tt BootCI}$

Bootstrap Percentile Confidence Intervals

Description

Bootstrap Percentile Confidence Intervals

Usage

BootCI(x, alpha = 0.05)

BootCI 3

Arguments

Value

A list with two elements, namely 11 for the lower limit and u1 for the upper limit.

Author(s)

Ivan Jacob Agaloos Pesigan

See Also

```
Other Simulation of Autoregressive Data Functions: BootSE(), SelectVARLasso(), SimVARExo(), SimVAR(), YXExo(), YX()
```

```
set.seed(42)
# Parametric bootstrap
system.time(
  pb <- PBootVAROLS(</pre>
    data = dat_p2,
    p = 2,
    B = 5,
    burn_in = 20
  )
)
pb$est
BootCI(pb)
system.time(
  pb <- PBootVARLasso(</pre>
    data = dat_p2,
    p = 2,
    B = 5,
    burn_in = 20,
    n_{\text{lambdas}} = 50,
    crit = "ebic",
    max_iter = 1000,
    tol = 1e-5
  )
)
pb$est
BootCI(pb)
# Residual bootstrap
system.time(
  rb <- RBootVAROLS(</pre>
    data = dat_p2,
```

4 BootSE

```
p = 2,
    B = 5
  )
)
rb$est
BootCI(rb)
system.time(
  rb <- RBootVARLasso(</pre>
    data = dat_p2,
    p = 2,
    B = 5,
    n_{\text{lambdas}} = 50,
    crit = "ebic",
    max_iter = 1000,
    tol = 1e-5
)
rb$est
BootCI(rb)
```

BootSE

Bootstrap Standard Errors

Description

Bootstrap Standard Errors

Usage

BootSE(x)

Arguments

Х

 $Numeric\ matrix.\ Output\ of\ PBootVAROLS(),\ PBootVARLasso(),\ RBootVAROLS(),\ or\ RBootVARLasso().$

Value

A matrix of standard errors.

Author(s)

Ivan Jacob Agaloos Pesigan

See Also

```
Other Simulation of Autoregressive Data Functions: BootCI(), SelectVARLasso(), SimVARExo(), SimVAR(), YXExo(), YX()
```

BootSE 5

```
set.seed(42)
# Parametric bootstrap
system.time(
  pb <- PBootVAROLS(</pre>
    data = dat_p2,
    p = 2,
    B = 5,
    burn_in = 20
  )
)
pb$est
BootSE(pb)
system.time(
  pb <- PBootVARLasso(</pre>
    data = dat_p2,
    p = 2,
    B = 5,
    burn_in = 20,
    n_{\text{lambdas}} = 50,
    crit = "ebic",
    max_iter = 1000,
    tol = 1e-5
  )
)
pb$est
BootSE(pb)
# Residual bootstrap
system.time(
  rb <- RBootVAROLS(</pre>
    data = dat_p2,
    p = 2,
    B = 5
  )
)
rb$est
BootSE(rb)
system.time(
  rb <- RBootVARLasso(</pre>
    data = dat_p2,
    p = 2,
    B = 5,
    n_{\text{lambdas}} = 50,
    crit = "ebic",
    max_iter = 1000,
    tol = 1e-5
  )
)
rb$est
BootSE(rb)
```

6 coef.dynr_model

coef.dynr_model

Auto and Cross Regression Coeffcients

Description

Auto and Cross Regression Coeffcients

Usage

```
## S3 method for class 'dynr_model'
coef(object, dynr_cook, ...)
```

Arguments

```
object Object of class dynr_model.dynr_cook Ouput of FitVARDynr().... additional arguments.
```

Value

A list with the following elements:

- constant Numeric vector. The constant term vector of length k, where k is the number of variables.
- coef Numeric matrix. Coefficient matrix with dimensions k by (k * p). Each k by k block corresponds to the coefficient matrix for a particular lag.
- cov Numeric matrix. The k by k covariance matrix of the noise.

Author(s)

Ivan Jacob Agaloos Pesigan

```
## Not run:
dynr_model <- ModelVARP2Dynr(data = dat_p2)
dynr_cook <- FitVARP2Dynr(model = dynr_model)
coef(object = dynr_model, dynr_cook = dynr_cook)
## End(Not run)</pre>
```

dat_ml_p1 7

dat_ml_p1

Data from the Multilevel Vector Autoregressive Model (p = 1)

Description

Data from the Multilevel Vector Autoregressive Model (p = 1)

Usage

dat_ml_p1

Format

A list of length n = 100 consisting of matrices with 1000 rows (time points) and k = 3 columns (variables) generated from the p = 1 multilevel vector autoregressive model given by

$$Y_{1_t} = 1 + \mathcal{N} \left(\mu = 0.4, \sigma^2 = 0.01 \right) Y_{1_{t-1}} + 0.0 Y_{2_{t-1}} + 0.0 Y_{3_{t-1}} + \varepsilon_{1_t},$$

$$Y_{2_t} = 1 + 0.0Y_{1_{t-1}} + \mathcal{N}\left(\mu = 0.5, \sigma^2 = 0.01\right)Y_{2_{t-1}} + 0.0Y_{3_{t-1}} + \varepsilon_{2_t},$$

and

$$Y_{3_t} = 1 + 0.0Y_{1_{t-1}} + 0.0Y_{2_{t-1}} + \mathcal{N}\left(\mu = 0.6, \sigma^2 = 0.01\right)Y_{3_{t-1}} + \varepsilon_{3_t}$$

which simplifies to

$$Y_{1_t} = 1 + \mathcal{N} \left(\mu = 0.4, \sigma^2 = 0.01 \right) Y_{1_{t-1}} + \varepsilon_{1_t},$$

$$Y_{2_t} = 1 + \mathcal{N} \left(\mu = 0.5, \sigma^2 = 0.01 \right) Y_{2_{t-1}} + \varepsilon_{2_t},$$

and

$$Y_{3_t} = 1 + \mathcal{N} \left(\mu = 0.6, \sigma^2 = 0.01 \right) Y_{3_{t-1}} + \varepsilon_{3_t}.$$

The covariance matrix of process noise is an identity matrix.

dat_ml_p2

Data from the Multilevel Vector Autoregressive Model (p = 2)

Description

Data from the Multilevel Vector Autoregressive Model (p = 2)

Usage

dat_ml_p2

8 dat_p1

Format

A list of length n = 100 consisting of matrices with 1000 rows (time points) and k = 3 columns (variables) generated from the p = 2 multilevel vector autoregressive model given by

$$Y_{3_t} = 1 + 0.0Y_{1_{t-1}} + 0.0Y_{2_{t-1}} + \mathcal{N}\left(\mu = 0.6, \sigma^2 = 0.01\right)Y_{3_{t-1}} + 0.0Y_{1_{t-2}} + 0.0Y_{2_{t-2}} + \mathcal{N}\left(\mu = 0.3, \sigma^2 = 0.01\right)Y_{3_{t-2}} + \varepsilon_{3_t}$$
 which simplifies to

$$Y_{1_t} = 1 + \mathcal{N} \left(\mu = 0.4, \sigma^2 = 0.01 \right) Y_{1_{t-1}} + \mathcal{N} \left(\mu = 0.1, \sigma^2 = 0.01 \right) Y_{1_{t-2}} + \varepsilon_{1_t},$$

$$Y_{2_t} = 1 + \mathcal{N} \left(\mu = 0.5, \sigma^2 = 0.01 \right) Y_{2_{t-1}} + \mathcal{N} \left(\mu = 0.2, \sigma^2 = 0.01 \right) Y_{2_{t-2}} + \varepsilon_{2_t},$$

and

$$Y_{3_t} = 1 + \mathcal{N} \left(\mu = 0.6, \sigma^2 = 0.01 \right) Y_{3_{t-1}} + \mathcal{N} \left(\mu = 0.3, \sigma^2 = 0.01 \right) Y_{3_{t-2}} + \varepsilon_{3_t}.$$

The covariance matrix of process noise is an identity matrix.

dat_p1

Data from the Vector Autoregressive Model (p = 1)

Description

Data from the Vector Autoregressive Model (p = 1)

Usage

dat_p1

Format

A matrix with 1000 rows (time points) and k = 3 columns (variables) generated from the p = 1 vector autoregressive model given by

$$Y_{1_t} = 1 + 0.4 Y_{1_{t-1}} + 0.0 Y_{2_{t-1}} + 0.0 Y_{3_{t-1}} + \varepsilon_{1_t},$$

$$Y_{2_t} = 1 + 0.0Y_{1_{t-1}} + 0.5Y_{2_{t-1}} + 0.0Y_{3_{t-1}} + \varepsilon_{2_t},$$

and

$$Y_{3_t} = 1 + 0.0Y_{1_{t-1}} + 0.0Y_{2_{t-1}} + 0.6Y_{3_{t-1}} + \varepsilon_{3_t}$$

which simplifies to

$$Y_{1_t} = 1 + 0.4Y_{1_{t-1}} + \varepsilon_{1_t},$$

$$Y_{2_t} = 1 + 0.5Y_{2_{t-1}} + \varepsilon_{2_t},$$

and

$$Y_{3_t} = 1 + 0.6Y_{3_{t-1}} + \varepsilon_{3_t}.$$

The covariance matrix of process noise is an identity matrix.

dat_p1_yx

dat_p1_yx

Data from the Vector Autoregressive Model (Y) and Lagged Predictors (X) (p = 1)

Description

Data from the Vector Autoregressive Model (Y) and Lagged Predictors (X) (p = 1)

Usage

Format

A list with elements Y and X where Y is equal to the dat_p1 data set minus p = 1 terminal rows and X is a matrix of ones for the first column and lagged values of Y for the rest of the columns.

dat_p2

Data from the Vector Autoregressive Model (p = 2)

Description

Data from the Vector Autoregressive Model (p = 2)

Usage

dat_p2

Format

A matrix with 1000 rows (time points) and k = 3 columns (variables) generated from the p = 2 vector autoregressive model given by

$$Y_{1_t} = 1 + 0.4Y_{1_{t-1}} + 0.0Y_{2_{t-1}} + 0.0Y_{3_{t-1}} + 0.1Y_{1_{t-2}} + 0.0Y_{2_{t-2}} + 0.0Y_{3_{t-2}} + \varepsilon_{1_t},$$

$$Y_{2_t} = 1 + 0.0Y_{1_{t-1}} + 0.5Y_{2_{t-1}} + 0.0Y_{3_{t-1}} + 0.0Y_{1_{t-2}} + 0.2Y_{2_{t-2}} + 0.0Y_{3_{t-2}} + \varepsilon_{2_t},$$

and

$$Y_{3_t} = 1 + 0.0Y_{1_{t-1}} + 0.0Y_{2_{t-1}} + 0.6Y_{3_{t-1}} + 0.0Y_{1_{t-2}} + 0.0Y_{2_{t-2}} + 0.3Y_{3_{t-2}} + \varepsilon_{3_t}$$

which simplifies to

$$Y_{1_t} = 1 + 0.4Y_{1_{t-1}} + 0.1Y_{1_{t-2}} + \varepsilon_{1_t},$$

$$Y_{2_t} = 1 + 0.5Y_{2_{t-1}} + 0.2Y_{2_{t-2}} + \varepsilon_{2_t},$$

and

$$Y_{3_t} = 1 + 0.6Y_{3_{t-1}} + 0.3Y_{3_{t-2}} + \varepsilon_{3_t}.$$

The covariance matrix of process noise is an identity matrix.

10 dat_p2_exo_yx

dat_p2_exo

Data from the Vector Autoregressive Model with Exogenous Variables (p = 2)

Description

Data from the Vector Autoregressive Model with Exogenous Variables (p = 2)

Usage

dat_p2_exo

Format

A matrix with 1000 rows (time points) and k = 3 (autoregressive variables) plus m = 3 columns (exogenous variables) generated from the p = 2 vector autoregressive model given by

$$Y_{1t} = 1 + 0.4Y_{1t-1} + 0.0Y_{2t-1} + 0.0Y_{3t-1} + 0.1Y_{1t-2} + 0.0Y_{2t-2} + 0.0Y_{3t-2} + 0.5X_1 + 0.0X_2 + 0.0X_3\varepsilon_{1t},$$

$$Y_{2_t} = 1 + 0.0Y_{1_{t-1}} + 0.5Y_{2_{t-1}} + 0.0Y_{3_{t-1}} + 0.0Y_{1_{t-2}} + 0.2Y_{2_{t-2}} + 0.0Y_{3_{t-2}} + 0.0X_1 + 0.5X_2 + 0.0X_3\varepsilon_{2_t},$$

and

$$Y_{3_t} = 1 + 0.0Y_{1_{t-1}} + 0.0Y_{2_{t-1}} + 0.6Y_{3_{t-1}} + 0.0Y_{1_{t-2}} + 0.0Y_{2_{t-2}} + 0.3Y_{3_{t-2}} + 0.0X_1 + 0.0X_2 + 0.5X_3\varepsilon_{3_t} + 0.0Y_{1_{t-1}} + 0.0Y_{1$$

which simplifies to

$$Y_{1_t} = 1 + 0.4Y_{1_{t-1}} + 0.1Y_{1_{t-2}} + 0.5X_1\varepsilon_{1_t},$$

$$Y_{2_t} = 1 + 0.5Y_{2_{t-1}} + 0.2Y_{2_{t-2}} + 0.5X_2\varepsilon_{2_t},$$

and

$$Y_{3_t} = 1 + 0.6Y_{3_{t-1}} + 0.3Y_{3_{t-2}} + 0.5X_3\varepsilon_{3_t}.$$

The covariance matrix of process noise is an identity matrix.

dat_p2_exo_yx

Data from the Vector Autoregressive Model (Y) and Lagged Predictors and Exogenous Variables (X) (p = 2)

Description

Data from the Vector Autoregressive Model (Y) and Lagged Predictors and Exogenous Variables (X) (p = 2)

Usage

dat_p2_exo_yx

dat_p2_yx

Format

A list with elements Y and X where Y is equal to the k = 3 autoregressive variables of the dat_p2_exo data set minus p = 2 terminal rows and X is a matrix of ones for the first column, lagged values of Y, and m = 3 exogenous variables.

dat_p2_yx

Data from the Vector Autoregressive Model (Y) and Lagged Predictors (X) (p = 2)

Description

Data from the Vector Autoregressive Model (Y) and Lagged Predictors (X) (p = 2)

Usage

```
dat_p2_yx
```

Format

A list with elements Y and X where Y is equal to the dat_p2 data set minus p = 2 terminal rows and X is a matrix of ones for the first column and lagged values of Y for the rest of the columns.

FitMLVARDynr

Fit Vector Autoregressive (VAR) Model using dynr on Each of the Data Matrix in a List

Description

This function estimates the parameters of a VAR model using the dynr package for each of the data matrix in a list.

```
FitMLVARDynr(
  data,
  p,
  ncores = 1,
  conf_level = 0.95,
  optimization_flag = TRUE,
  hessian_flag = FALSE,
  verbose = FALSE,
  weight_flag = FALSE,
  debug_flag = FALSE,
  perturb_flag = FALSE
)
```

12 FitMLVARMplus

Arguments

data	List. Each element is a numeric matrix of time series data with dimensions t by k, where t is the number of observations and k is the number of variables.
р	Positive integer. Number of lags. Only supports $p = 1$ and $p = 2$.
ncores	Positive integer. Number of cores to use. Not supported on Windows.
conf_level	a cumulative proportion indicating the level of desired confidence intervals for the final parameter estimates (default is .95)
optimization_f	lag
	a flag (TRUE/FALSE) indicating whether optimization is to be done.
hessian_flag	a flag (TRUE/FALSE) indicating whether the Hessian matrix is to be calculated.
verbose	a flag (TRUE/FALSE) indicating whether more detailed intermediate output during the estimation process should be printed
weight_flag	a flag (TRUE/FALSE) indicating whether the negative log likelihood function should be weighted by the length of the time series for each individual
debug_flag	a flag (TRUE/FALSE) indicating whether users want additional dynr output that can be used for diagnostic purposes
perturb_flag	a flag (TRUE/FLASE) indicating whether to perturb the latent states during estimation. Only useful for ensemble forecasting.

Value

A list each element of which is an object of class dynrCook.

Author(s)

Ivan Jacob Agaloos Pesigan

See Also

Other Fitting Autoregressive Model Functions: FitMLVARMplus(), FitVARDynr(), FitVARLassoSearch(), FitVARLasso(), FitVARMplus(), FitVAROLS(), LambdaSeq(), ModelVARP1Dynr(), ModelVARP2Dynr(), OrigScale(), PBootVARExoLasso(), PBootVARExoLs(), PBootVARLasso(), PBootVAROLS(), RBootVARExoLasso(), RBootVARCLS(), RBootVARCLS(), SearchVARLasso(), StdMat()

FitMLVARMplus	Fit Multilevel Vector Autoregressive Model using Mplus

Description

This function estimates the parameters of a VAR model using Mplus.

```
FitMLVARMplus(data, p = 1, mplus_bin, iter = 5000L)
```

FitVARDynr 13

Arguments

data Numeric matrix. The time series data with dimensions t by k, where t is the

number of observations and k is the number of variables.

p Positive integer. Number of lags.

mplus_bin Character string. Path to the Mplus binary.

iter Positive integer. Maximum number of iterations for each MCMC chain.

Author(s)

Ivan Jacob Agaloos Pesigan

See Also

```
Other Fitting Autoregressive Model Functions: FitMLVARDynr(), FitVARDynr(), FitVARLassoSearch(), FitVARLasso(), FitVARPulus(), FitVAROLS(), LambdaSeq(), ModelVARP1Dynr(), ModelVARP2Dynr(), OrigScale(), PBootVARExoLasso(), PBootVARExoOLS(), PBootVARLasso(), PBootVAROLS(), RBootVARExoLasso(), RBootVARExoLasso(), RBootVARDLS(), SearchVARLasso(), StdMat()
```

Examples

```
## Not run:
FitMLVARMplus(data = dat_ml_p1, p = 1, mplus_bin = "mplus")
FitMLVARMplus(data = dat_ml_p2, p = 2, mplus_bin = "mplus")
## End(Not run)
```

FitVARDynr

Fit Vector Autoregressive Model using dynr

Description

This function estimates the parameters of a VAR model using the dynr package.

```
FitVARDynr(
  model,
  conf_level = 0.95,
  optimization_flag = TRUE,
  hessian_flag = FALSE,
  verbose = FALSE,
  weight_flag = FALSE,
  debug_flag = FALSE,
  perturb_flag = FALSE
)
```

14 FitVARDynr

Arguments

model	Ouput of ModelVARP1Dynr() or ModelVARP2Dynr().
conf_level	a cumulative proportion indicating the level of desired confidence intervals for the final parameter estimates (default is .95)
optimization_f	lag
	a flag (TRUE/FALSE) indicating whether optimization is to be done.
hessian_flag	a flag (TRUE/FALSE) indicating whether the Hessian matrix is to be calculated.
verbose	a flag (TRUE/FALSE) indicating whether more detailed intermediate output during the estimation process should be printed
weight_flag	a flag (TRUE/FALSE) indicating whether the negative log likelihood function should be weighted by the length of the time series for each individual
debug_flag	a flag (TRUE/FALSE) indicating whether users want additional dynr output that can be used for diagnostic purposes
perturb_flag	a flag (TRUE/FLASE) indicating whether to perturb the latent states during estimation. Only useful for ensemble forecasting.

Value

Object of class dynrCook.

Author(s)

Ivan Jacob Agaloos Pesigan

See Also

```
Other Fitting Autoregressive Model Functions: FitMLVARDynr(), FitMLVARMplus(), FitVARLassoSearch(), FitVARLasso(), FitVARDus(), FitVAROLS(), LambdaSeq(), ModelVARP1Dynr(), ModelVARP2Dynr(), OrigScale(), PBootVARExoLasso(), PBootVARExoLasso(), PBootVARCus(), PBootVARCus(), RBootVARCus(), RBootVARCus(), RBootVARCus(), SearchVARLasso(), StdMat()
```

```
## Not run:
FitVARDynr(model = ModelVARP1Dynr(data = dat_p1))
FitVARDynr(model = ModelVARP2Dynr(data = dat_p2))
## End(Not run)
```

FitVARLasso 15

FitVARLasso	Fit Vector Autoregressive (VAR) Model Parameters using Lasso Regularization

Description

This function estimates the parameters of a VAR model using the Lasso regularization method with cyclical coordinate descent. The Lasso method is used to estimate the autoregressive and cross-regression coefficients with sparsity.

Usage

```
FitVARLasso(YStd, XStd, lambda, max_iter, tol)
```

Arguments

YStd	Numeric matrix. Matrix of standardized dependent variables (Y).
XStd	Numeric matrix. Matrix of standardized predictors (X). XStd should not include a vector of ones in column one.
lambda	Numeric. Lasso hyperparameter. The regularization strength controlling the sparsity.
max_iter	Integer. The maximum number of iterations for the coordinate descent algorithm (e.g., max_iter = 10000).
tol	Numeric. Convergence tolerance. The algorithm stops when the change in coefficients between iterations is below this tolerance (e.g., tol = 1e-5).

Details

The FitVARLasso() function estimates the parameters of a Vector Autoregressive (VAR) model using the Lasso regularization method. Given the input matrices YStd and XStd, where YStd is the matrix of standardized dependent variables, and XStd is the matrix of standardized predictors, the function computes the autoregressive and cross-regression coefficients of the VAR model with sparsity induced by the Lasso regularization.

The steps involved in estimating the VAR model parameters using Lasso are as follows:

- **Initialization**: The function initializes the coefficient matrix beta with OLS estimates. The beta matrix will store the estimated autoregressive and cross-regression coefficients.
- Coordinate Descent Loop: The function performs the cyclical coordinate descent algorithm to estimate the coefficients iteratively. The loop iterates max_iter times, or until convergence is achieved. The outer loop iterates over the predictor variables (columns of XStd), while the inner loop iterates over the outcome variables (columns of YStd).
- Coefficient Update: For each predictor variable (column of XStd), the function iteratively updates the corresponding column of beta using the coordinate descent algorithm with L1 norm regularization (Lasso). The update involves calculating the soft-thresholded value c, which encourages sparsity in the coefficients. The algorithm continues until the change in coefficients between iterations is below the specified tolerance tol or when the maximum number of iterations is reached.

16 FitVARLassoSearch

• Convergence Check: The function checks for convergence by comparing the current beta matrix with the previous iteration's beta_old. If the maximum absolute difference between beta and beta_old is below the tolerance tol, the algorithm is considered converged, and the loop exits.

Value

Matrix of estimated autoregressive and cross-regression coefficients.

Author(s)

Ivan Jacob Agaloos Pesigan

See Also

```
Other Fitting Autoregressive Model Functions: FitMLVARDynr(), FitMLVARMplus(), FitVARDynr(), FitVARDynr(), FitVARDynr(), FitVARLassoSearch(), FitVARPulus(), FitVAROLS(), LambdaSeq(), ModelVARP1Dynr(), ModelVARP2Dynr(), OrigScale(), PBootVARExoLasso(), PBootVARExoLsso(), PBootVARDLS(), RBootVARDLS(), RBootVARDLS(), RBootVARDLS(), SearchVARLasso(), StdMat()
```

Examples

```
YStd <- StdMat(dat_p2_yx$Y)
XStd <- StdMat(dat_p2_yx$X[, -1]) # remove the constant column
lambda <- 73.90722
FitVARLasso(
   YStd = YStd,
   XStd = XStd,
   lambda = lambda,
   max_iter = 10000,
   tol = 1e-5
)</pre>
```

FitVARLassoSearch

Fit Vector Autoregressive (VAR) Model Parameters using Lasso Regularization with Lambda Search

Description

Fit Vector Autoregressive (VAR) Model Parameters using Lasso Regularization with Lambda Search

```
FitVARLassoSearch(YStd, XStd, lambdas, crit, max_iter, tol)
```

FitVARLassoSearch 17

Arguments

YStd	Numeric matrix. Matrix of standardized dependent variables (Y).
XStd	Numeric matrix. Matrix of standardized predictors (X) . XStd should not include a vector of ones in column one.
lambdas	Numeric vector. Lasso hyperparameter. The regularization strength controlling the sparsity.
crit	Character string. Information criteria to use. Valid values include "aic", "bic", and "ebic".
max_iter	Integer. The maximum number of iterations for the coordinate descent algorithm (e.g., max_iter = 10000).
tol	Numeric. Convergence tolerance. The algorithm stops when the change in coefficients between iterations is below this tolerance (e.g., tol = 1e-5).

Value

Matrix of estimated autoregressive and cross-regression coefficients.

Author(s)

Ivan Jacob Agaloos Pesigan

See Also

```
Other Fitting Autoregressive Model Functions: FitMLVARDynr(), FitMLVARMplus(), FitVARDynr(), FitVARLasso(), FitVARDynr(), FitVARLasso(), FitVARDus(), FitVARDus(), LambdaSeq(), ModelVARP1Dynr(), ModelVARP2Dynr(), OrigScale(), PBootVARExoLasso(), PBootVARExoLasso(), PBootVARLasso(), PBootVARDus(), RBootVARExoLasso(), RBootVARDus(), RBootVARDus(), SearchVARLasso(), StdMat()
```

```
YStd <- StdMat(dat_p2_yx$Y)
XStd <- StdMat(dat_p2_yx$X[, -1]) # remove the constant column
lambdas <- LambdaSeq(
   YStd = YStd,
    XStd = XStd,
    n_lambdas = 100
)
FitVARLassoSearch(
   YStd = YStd,
   XStd = XStd,
   lambdas = lambdas,
   crit = "ebic",
   max_iter = 1000,
   tol = 1e-5
)</pre>
```

18 FitVARMplus

FitVARMplus	Fit Vector Autoregressive Model using Mplus
TEVAINIPLUS	Til vector Mitoregressive Model usung Impius

Description

This function estimates the parameters of a VAR model using Mplus.

Usage

```
FitVARMplus(data, p = 1, mplus_bin, iter = 5000L)
```

Arguments

data Numeric matrix. The time series data with dimensions t by k, where t is the

number of observations and k is the number of variables.

p Positive integer. Number of lags.

mplus_bin Character string. Path to the Mplus binary.

iter Positive integer. Maximum number of iterations for each MCMC chain.

Author(s)

Ivan Jacob Agaloos Pesigan

See Also

```
Other Fitting Autoregressive Model Functions: FitMLVARDynr(), FitMLVARMplus(), FitVARDynr(), FitVARLassoSearch(), FitVARLasso(), FitVAROLS(), LambdaSeq(), ModelVARP1Dynr(), ModelVARP2Dynr(), OrigScale(), PBootVARExoLasso(), PBootVARExoLsso(), PBootVARCLsso(), PBootVARCLsso(), RBootVARCLsso(), RBootVARCLsso(), RBootVARCLsso(), SearchVARLasso(), StdMat()
```

```
## Not run:
FitVARMplus(data = dat_p1, p = 1, mplus_bin = "mplus")
FitVARMplus(data = dat_p2, p = 2, mplus_bin = "mplus")
## End(Not run)
```

FitVAROLS 19

FitVAROLS	Fit Vector Autoregressive (VAR) Model Parameters using Ordinary Least Squares (OLS)

Description

This function estimates the parameters of a VAR model using the Ordinary Least Squares (OLS) method. The OLS method is used to estimate the autoregressive and cross-regression coefficients.

Usage

```
FitVAROLS(Y, X)
```

Arguments

Y Numeric matrix. Matrix of dependent variables (Y).

X Numeric matrix. Matrix of predictors (X).

Details

The FitVAROLS() function estimates the parameters of a Vector Autoregressive (VAR) model using the Ordinary Least Squares (OLS) method. Given the input matrices Y and X, where Y is the matrix of dependent variables, and X is the matrix of predictors, the function computes the autoregressive and cross-regression coefficients of the VAR model. Note that if the first column of X is a vector of ones, the constant vector is also estimated.

The steps involved in estimating the VAR model parameters using OLS are as follows:

- Compute the QR decomposition of the lagged predictor matrix X using the qr_econ function from the Armadillo library.
- Extract the Q and R matrices from the QR decomposition.
- Solve the linear system R * coef = Q.t() * Y to estimate the VAR model coefficients coef.
- The function returns a matrix containing the estimated autoregressive and cross-regression coefficients of the VAR model.

Value

Matrix of estimated autoregressive and cross-regression coefficients.

Author(s)

Ivan Jacob Agaloos Pesigan

20 LambdaSeq

See Also

```
Other Fitting Autoregressive Model Functions: FitMLVARDynr(), FitMLVARMplus(), FitVARDynr(), FitVARDynr(), FitVARLassoSearch(), FitVARLasso(), FitVARMplus(), LambdaSeq(), ModelVARP1Dynr(), ModelVARP2Dynr(), OrigScale(), PBootVARExoLasso(), PBootVARExoLasso(), PBootVARDLS(), RBootVARExoLasso(), RBootVARDLS(), RBootVARDLS(), SearchVARLasso(), StdMat()
```

Examples

```
Y <- dat_p2_yx$Y
X <- dat_p2_yx$X
FitVAROLS(Y = Y, X = X)
```

LambdaSeq

Function to generate the sequence of lambdas

Description

Function to generate the sequence of lambdas

Usage

```
LambdaSeq(YStd, XStd, n_lambdas)
```

Arguments

YStd Numeric matrix. Matrix of standardized dependent variables (Y).

Numeric matrix. Matrix of standardized predictors (X). XStd should not include

a vector of ones in column one.

n_lambdas Integer. Number of lambdas to generate.

Value

Returns a vector of lambdas.

Author(s)

Ivan Jacob Agaloos Pesigan

See Also

```
Other Fitting Autoregressive Model Functions: FitMLVARDynr(), FitMLVARMplus(), FitVARDynr(), FitVARDynr(), FitVARLassoSearch(), FitVARLasso(), FitVARDus(), FitVAROLS(), ModelVARP1Dynr(), ModelVARP2Dynr(), OrigScale(), PBootVARExoLasso(), PBootVARExoLsso(), PBootVARDus(), PBootVARDus(), RBootVARDus(), RBootVARDus(), RBootVARDus(), SearchVARLasso(), StdMat()
```

ModelVARP1Dynr 21

Examples

```
YStd <- StdMat(dat_p2_yx$Y)
XStd <- StdMat(dat_p2_yx$X[, -1]) # remove the constant column
LambdaSeq(YStd = YStd, XStd = XStd, n_lambdas = 100)
```

ModelVARP1Dynr

Specify Vector Autoregressive (VAR(p = 1)) Model using dynr

Description

This function specifies a VAR(p = 1) model using the dynr::dynr.model() package.

Usage

```
ModelVARP1Dynr(data, outfile = tempfile())
```

Arguments

data Numeric matrix. The time series data with dimensions t by k, where t is the

number of observations and k is the number of variables.

outfile a character string of the name of the output C script of model functions to be

compiled for parameter estimation. The default is the name for a potential tem-

porary file returned by tempfile().

Author(s)

Ivan Jacob Agaloos Pesigan

See Also

```
Other Fitting Autoregressive Model Functions: FitMLVARDynr(), FitMLVARMplus(), FitVARDynr(), FitVARLassoSearch(), FitVARLasso(), FitVARMplus(), FitVAROLS(), LambdaSeq(), ModelVARP2Dynr(), OrigScale(), PBootVARExoLasso(), PBootVARExoLasso(), PBootVARCLS(), RBootVARExoLasso(), RBootVARCLS(), RBootVARCLS(), SearchVARLasso(), StdMat()
```

```
ModelVARP1Dynr(data = dat_p1)
```

OrigScale OrigScale

ModelVARP2Dvnr

Specify Vector Autoregressive (VAR(p = 2)) Model using dynr

Description

This function specifies a VAR(p = 2) model using the dynr::dynr.model() package.

Usage

```
ModelVARP2Dynr(data, outfile = tempfile())
```

Arguments

data Numeric matrix. The time series data with dimensions t by k, where t is the

number of observations and k is the number of variables.

outfile a character string of the name of the output C script of model functions to be

compiled for parameter estimation. The default is the name for a potential tem-

porary file returned by tempfile().

Author(s)

Ivan Jacob Agaloos Pesigan

See Also

```
Other Fitting Autoregressive Model Functions: FitMLVARDynr(), FitMLVARMplus(), FitVARDynr(), FitVARLassoSearch(), FitVARLasso(), FitVARMplus(), FitVAROLS(), LambdaSeq(), ModelVARP1Dynr(), OrigScale(), PBootVARExoLasso(), PBootVARExoLasso(), PBootVARCLS(), RBootVARCLS(), RBootVARCLS(), RBootVARCLS(), SearchVARLasso(), StdMat()
```

Examples

```
ModelVARP2Dynr(data = dat_p2)
```

OrigScale

Return Standardized Estimates to the Original Scale

Description

Return Standardized Estimates to the Original Scale

```
OrigScale(coef_std, Y, X)
```

PBootVARExoLasso 23

Arguments

coef_std	Numeric matrix. Standardized estimates of the autoregression and cross regression coefficients.
Υ	Numeric matrix. Matrix of dependent variables (Y).
Χ	Numeric matrix. Matrix of predictors (X).

Author(s)

Ivan Jacob Agaloos Pesigan

See Also

```
Other Fitting Autoregressive Model Functions: FitMLVARDynr(), FitMLVARMplus(), FitVARDynr(),
FitVARLassoSearch(), FitVARLasso(), FitVARMplus(), FitVAROLS(), LambdaSeq(), ModelVARP1Dynr(),
ModelVARP2Dynr(), PBootVARExoLasso(), PBootVARExoUS(), PBootVARLasso(), PBootVAROLS(),
RBootVARExoLasso(), RBootVARExoOLS(), RBootVARLasso(), RBootVAROLS(), SearchVARLasso(),
StdMat()
```

Examples

```
Y \leftarrow dat_p2_yx$Y
X \leftarrow dat_p2_yx$X[, -1] \# remove the constant column
YStd <- StdMat(Y)
XStd <- StdMat(X)
coef_std <- FitVAROLS(Y = YStd, X = XStd)</pre>
FitVAROLS(Y = Y, X = X)
OrigScale(coef_std = coef_std, Y = Y, X = X)
```

Parametric Bootstrap for the Vector Autoregressive Model with Exoge-**PBootVARExoLasso** nous Variables Using Lasso Regularization

Description

Parametric Bootstrap for the Vector Autoregressive Model with Exogenous Variables Using Lasso Regularization

```
PBootVARExoLasso(data, exo_mat, p, B, burn_in, n_lambdas, crit, max_iter, tol)
```

24 PBootVARExoLasso

Arguments

data	Numeric matrix. The time series data with dimensions t by k, where t is the number of observations and k is the number of variables.
exo_mat	Numeric matrix. Matrix of exogenous variables with dimensions t + burn_in by m. If the number of rows is equal to t, set burn_in = 0.
р	Integer. The order of the VAR model (number of lags).
В	Integer. Number of bootstrap samples to generate.
burn_in	Integer. Number of burn-in observations to exclude before returning the results in the simulation step.
n_lambdas	Integer. Number of lambdas to generate.
crit	Character string. Information criteria to use. Valid values include "aic", "bic", and "ebic".
max_iter	Integer. The maximum number of iterations for the coordinate descent algorithm (e.g., max_iter = 10000).
tol	Numeric. Convergence tolerance. The algorithm stops when the change in coefficients between iterations is below this tolerance (e.g., tol = 1e-5).

Value

List with the following elements:

- est: Numeric matrix. Original OLS estimate of the coefficient matrix.
- boot: Numeric matrix. Matrix of vectorized bootstrap estimates of the coefficient matrix.

Author(s)

Ivan Jacob Agaloos Pesigan

See Also

```
Other Fitting Autoregressive Model Functions: FitMLVARDynr(), FitMLVARMplus(), FitVARDynr(), FitVARLassoSearch(), FitVARLasso(), FitVARMplus(), FitVAROLS(), LambdaSeq(), ModelVARP1Dynr(), ModelVARP2Dynr(), OrigScale(), PBootVARExoOLS(), PBootVARLasso(), PBootVAROLS(), RBootVARExoLasso(), RBootVARLasso(), RBootVARLasso(), StdMat()
```

```
data <- dat_p2_exo$data
exo_mat <- dat_p2_exo$exo_mat
pb <- PBootVARExoLasso(
   data = data,
   exo_mat = exo_mat,
   p = 2,
   B = 5,
   burn_in = 0,
   n_lambdas = 10,
   crit = "ebic",</pre>
```

PBootVARExoOLS 25

```
max_iter = 1000,
  tol = 1e-5
)
str(pb)
```

PBootVARExoOLS

Parametric Bootstrap for the Vector Autoregressive Model with Exogenous Variables Using Ordinary Least Squares

Description

Parametric Bootstrap for the Vector Autoregressive Model with Exogenous Variables Using Ordinary Least Squares

Usage

```
PBootVARExoOLS(data, exo_mat, p, B, burn_in)
```

Arguments

data	Numeric matrix. The time series data with dimensions t by k, where t is the number of observations and k is the number of variables.
exo_mat	Numeric matrix. Matrix of exogenous variables with dimensions t + burn_in by m. If the number of rows is equal to t, set burn_in = 0.
р	Integer. The order of the VAR model (number of lags).
В	Integer. Number of bootstrap samples to generate.
burn_in	Integer. Number of burn-in observations to exclude before returning the results in the simulation step.

Value

List with the following elements:

- est: Numeric matrix. Original OLS estimate of the coefficient matrix.
- boot: Numeric matrix. Matrix of vectorized bootstrap estimates of the coefficient matrix.

Author(s)

Ivan Jacob Agaloos Pesigan

See Also

```
Other Fitting Autoregressive Model Functions: FitMLVARDynr(), FitMLVARMplus(), FitVARDynr(), FitVARLassoSearch(), FitVARLasso(), FitVARMplus(), FitVAROLS(), LambdaSeq(), ModelVARP1Dynr(), ModelVARP2Dynr(), OrigScale(), PBootVARExoLasso(), PBootVARLasso(), PBootVAROLS(), RBootVARExoLasso(), RBootVARCLasso(), RBootVARCLasso(), SearchVARLasso(), StdMat()
```

26 PBootVARLasso

Examples

```
data <- dat_p2_exo$data
exo_mat <- dat_p2_exo$exo_mat
pb <- PBootVARExoOLS(
   data = data,
    exo_mat = exo_mat,
   p = 2,
   B = 5,
   burn_in = 0
)
str(pb)</pre>
```

PBootVARLasso

Parametric Bootstrap for the Vector Autoregressive Model Using Lasso Regularization

Description

Parametric Bootstrap for the Vector Autoregressive Model Using Lasso Regularization

Usage

```
PBootVARLasso(data, p, B, burn_in, n_lambdas, crit, max_iter, tol)
```

Arguments

data	Numeric matrix. The time series data with dimensions t by k, where t is the number of observations and k is the number of variables.
р	Integer. The order of the VAR model (number of lags).
В	Integer. Number of bootstrap samples to generate.
burn_in	Integer. Number of burn-in observations to exclude before returning the results in the simulation step.
n_lambdas	Integer. Number of lambdas to generate.
crit	Character string. Information criteria to use. Valid values include "aic", "bic", and "ebic".
max_iter	Integer. The maximum number of iterations for the coordinate descent algorithm (e.g., max_iter = 10000).
tol	Numeric. Convergence tolerance. The algorithm stops when the change in coefficients between iterations is below this tolerance (e.g., tol = 1e-5).

Value

List with the following elements:

- est: Numeric matrix. Original Lasso estimate of the coefficient matrix.
- boot: Numeric matrix. Matrix of vectorized bootstrap estimates of the coefficient matrix.

PBootVAROLS 27

Author(s)

Ivan Jacob Agaloos Pesigan

See Also

```
Other Fitting Autoregressive Model Functions: FitMLVARDynr(), FitMLVARMplus(), FitVARDynr(), FitVARLassoSearch(), FitVARLasso(), FitVARMplus(), FitVAROLS(), LambdaSeq(), ModelVARP1Dynr(), ModelVARP2Dynr(), OrigScale(), PBootVARExoLasso(), PBootVARExoUS(), PBootVAROLS(), RBootVARExoLasso(), RBootVARExoLasso(), RBootVAROLS(), SearchVARLasso(), StdMat()
```

Examples

```
pb <- PBootVARLasso(
   data = dat_p2,
   p = 2,
   B = 5,
   burn_in = 20,
   n_lambdas = 10,
   crit = "ebic",
   max_iter = 1000,
   tol = 1e-5
)
str(pb)</pre>
```

PBootVAROLS

Parametric Bootstrap for the Vector Autoregressive Model Using Ordinary Least Squares

Description

Parametric Bootstrap for the Vector Autoregressive Model Using Ordinary Least Squares

Usage

```
PBootVAROLS(data, p, B, burn_in)
```

data	Numeric matrix. The time series data with dimensions t by k, where t is the number of observations and k is the number of variables.
р	Integer. The order of the VAR model (number of lags).
В	Integer. Number of bootstrap samples to generate.
burn_in	Integer. Number of burn-in observations to exclude before returning the results in the simulation step.

28 plot.dynr_model

Value

List with the following elements:

- est: Numeric matrix. Original OLS estimate of the coefficient matrix.
- boot: Numeric matrix. Matrix of vectorized bootstrap estimates of the coefficient matrix.

Author(s)

Ivan Jacob Agaloos Pesigan

See Also

```
Other Fitting Autoregressive Model Functions: FitMLVARDynr(), FitMLVARMplus(), FitVARDynr(), FitVARLassoSearch(), FitVARLasso(), FitVARMplus(), FitVAROLS(), LambdaSeq(), ModelVARP1Dynr(), ModelVARP2Dynr(), OrigScale(), PBootVARExoLasso(), PBootVARExoLsso(), PBootVARLasso(), RBootVARExoLasso(), RBootVARCLasso(), RBootVARCLasso(), SearchVARLasso(), StdMat()
```

Examples

```
pb <- PBootVAROLS(data = dat_p2, p = 2, B = 5, burn_in = 20)
str(pb)</pre>
```

plot.dynr_model

Plot Method for an Object of Class dynr_model

Description

Plot Method for an Object of Class dynr_model

Usage

```
## S3 method for class 'dynr_model'
plot(x, ...)
```

Arguments

```
x Object of class dynr_model.
... additional arguments.
```

Author(s)

Ivan Jacob Agaloos Pesigan

```
model <- ModelVARP2Dynr(data = dat_p2)
plot(model)</pre>
```

print.dynr_model 29

print.dynr_model

Print Method for an Object of Class dynr_model

Description

Print Method for an Object of Class dynr_model

Usage

```
## S3 method for class 'dynr_model'
print(x, ...)
```

Arguments

X Object of class dynr_model.

... additional arguments.

Author(s)

Ivan Jacob Agaloos Pesigan

Examples

```
model <- ModelVARP2Dynr(data = dat_p2)
print(model)</pre>
```

RBootVARExoLasso

Residual Bootstrap for the Vector Autoregressive Model with Exogenous Variables Using Lasso Regularization

Description

Residual Bootstrap for the Vector Autoregressive Model with Exogenous Variables Using Lasso Regularization

```
RBootVARExoLasso(data, exo_mat, p, B, n_lambdas, crit, max_iter, tol)
```

30 RBootVARExoLasso

Arguments

data	Numeric matrix. The time series data with dimensions t by k, where t is the number of observations and k is the number of variables.
exo_mat	Numeric matrix. Matrix of exogenous variables with dimensions t by m.
p	Integer. The order of the VAR model (number of lags).
В	Integer. Number of bootstrap samples to generate.
n_lambdas	Integer. Number of lambdas to generate.
crit	Character string. Information criteria to use. Valid values include "aic", "bic", and "ebic".
max_iter	Integer. The maximum number of iterations for the coordinate descent algorithm (e.g., max_iter = 10000).
tol	Numeric. Convergence tolerance. The algorithm stops when the change in coefficients between iterations is below this tolerance (e.g., tol = 1e-5).

Value

List with the following elements:

- est: Numeric matrix. Original Lasso estimate of the coefficient matrix.
- boot: Numeric matrix. Matrix of vectorized bootstrap estimates of the coefficient matrix.
- X: Numeric matrix. Original X
- Y: List of numeric matrices. Bootstrapped Y

Author(s)

Ivan Jacob Agaloos Pesigan

See Also

```
Other Fitting Autoregressive Model Functions: FitMLVARDynr(), FitMLVARMplus(), FitVARDynr(), FitVARLassoSearch(), FitVARLasso(), FitVARMplus(), FitVAROLS(), LambdaSeq(), ModelVARP1Dynr(), ModelVARP2Dynr(), OrigScale(), PBootVARExoLasso(), PBootVARExoOLS(), PBootVARLasso(), PBootVAROLS(), RBootVARExoOLS(), RBootVARLasso(), SearchVARLasso(), StdMat()
```

```
data <- dat_p2_exo$data
exo_mat <- dat_p2_exo$exo_mat
rb <- RBootVARExoLasso(
   data = data,
   exo_mat = exo_mat,
   p = 2,
   B = 5,
   n_lambdas = 10,
   crit = "ebic",
   max_iter = 1000,</pre>
```

RBootVARExoOLS 31

```
tol = 1e-5
)
str(rb)
```

RBootVARExoOLS

Residual Bootstrap for the Vector Autoregressive Model with Exogenous Variables Using Ordinary Least Squares

Description

Residual Bootstrap for the Vector Autoregressive Model with Exogenous Variables Using Ordinary Least Squares

Usage

```
RBootVARExoOLS(data, exo_mat, p, B)
```

Arguments

data	Numeric matrix. The time series data with dimensions t by k, where t is the number of observations and k is the number of variables.
exo_mat	Numeric matrix. Matrix of exogenous variables with dimensions t by m.
р	Integer. The order of the VAR model (number of lags).
В	Integer. Number of bootstrap samples to generate.

Value

List with the following elements:

- est: Numeric matrix. Original OLS estimate of the coefficient matrix.
- boot: Numeric matrix. Matrix of vectorized bootstrap estimates of the coefficient matrix.
- X: Numeric matrix. Original X
- Y: List of numeric matrices. Bootstrapped Y

Author(s)

Ivan Jacob Agaloos Pesigan

See Also

```
Other Fitting Autoregressive Model Functions: FitMLVARDynr(), FitMLVARMplus(), FitVARDynr(), FitVARLassoSearch(), FitVARLasso(), FitVARMplus(), FitVAROLS(), LambdaSeq(), ModelVARP1Dynr(), ModelVARP2Dynr(), OrigScale(), PBootVARExoLasso(), PBootVARExoLS(), PBootVARLasso(), PBootVAROLS(), RBootVARCASSO(), RBootVAROLS(), SearchVARLasso(), StdMat()
```

32 RBootVARLasso

Examples

```
data <- dat_p2_exo$data
exo_mat <- dat_p2_exo$exo_mat
rb <- RBootVARExoOLS(data = data, exo_mat = exo_mat, p = 2, B = 5)
str(rb)</pre>
```

RBootVARLasso Residual Bootstrap for the Vector Autoregressive Model Using Lasso Regularization

Description

Residual Bootstrap for the Vector Autoregressive Model Using Lasso Regularization

Usage

```
RBootVARLasso(data, p, B, n_lambdas, crit, max_iter, tol)
```

Arguments

data	Numeric matrix. The time series data with dimensions t by k , where t is the number of observations and k is the number of variables.
р	Integer. The order of the VAR model (number of lags).
В	Integer. Number of bootstrap samples to generate.
n_lambdas	Integer. Number of lambdas to generate.
crit	Character string. Information criteria to use. Valid values include "aic", "bic", and "ebic".
max_iter	Integer. The maximum number of iterations for the coordinate descent algorithm (e.g., max_iter = 10000).
tol	Numeric. Convergence tolerance. The algorithm stops when the change in coefficients between iterations is below this tolerance (e.g., tol = 1e-5).

Value

List with the following elements:

- est: Numeric matrix. Original Lasso estimate of the coefficient matrix.
- boot: Numeric matrix. Matrix of vectorized bootstrap estimates of the coefficient matrix.
- X: Numeric matrix. Original X
- Y: List of numeric matrices. Bootstrapped Y

Author(s)

Ivan Jacob Agaloos Pesigan

RBootVAROLS 33

See Also

```
Other Fitting Autoregressive Model Functions: FitMLVARDynr(), FitMLVARMplus(), FitVARDynr(), FitVARLassoSearch(), FitVARLasso(), FitVARMplus(), FitVAROLS(), LambdaSeq(), ModelVARP1Dynr(), ModelVARP2Dynr(), OrigScale(), PBootVARExoLasso(), PBootVARExoLs(), PBootVARCLS(), RBootVARCLS(), RBootVARCLS(), SearchVARLasso(), StdMat()
```

Examples

```
rb <- RBootVARLasso(
   data = dat_p2,
   p = 2,
   B = 5,
   n_lambdas = 10,
   crit = "ebic",
   max_iter = 1000,
   tol = 1e-5
)
str(rb)</pre>
```

RBootVAROLS

Residual Bootstrap for the Vector Autoregressive Model Using Ordinary Least Squares

Description

Residual Bootstrap for the Vector Autoregressive Model Using Ordinary Least Squares

Usage

```
RBootVAROLS(data, p, B)
```

Arguments

data	Numeric matrix. The time series data with dimensions t by k, where t is the
	number of observations and k is the number of variables.
р	Integer. The order of the VAR model (number of lags).
В	Integer. Number of bootstrap samples to generate.

Value

List with the following elements:

- est: Numeric matrix. Original OLS estimate of the coefficient matrix.
- boot: Numeric matrix. Matrix of vectorized bootstrap estimates of the coefficient matrix.
- X: Numeric matrix. Original X
- Y: List of numeric matrices. Bootstrapped Y

34 Search VARLasso

Author(s)

Ivan Jacob Agaloos Pesigan

See Also

Other Fitting Autoregressive Model Functions: FitMLVARDynr(), FitMLVARMplus(), FitVARDynr(), FitVARLassoSearch(), FitVARLasso(), FitVARMplus(), FitVAROLS(), LambdaSeq(), ModelVARP1Dynr(), ModelVARP2Dynr(), OrigScale(), PBootVARExoLasso(), PBootVARExoOLS(), PBootVARLasso(), PBootVARCLS(), RBootVARCLS(), RBootVARLasso(), SearchVARLasso(), StdMat()

Examples

```
rb <- RBootVAROLS(data = dat_p2, p = 2, B = 5)
str(rb)</pre>
```

SearchVARLasso

Compute AIC, BIC, and EBIC for Lasso Regularization

Description

This function computes the Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and Extended Bayesian Information Criterion (EBIC) for a given matrix of predictors X, a matrix of outcomes Y, and a vector of lambda hyperparameters for Lasso regularization.

Usage

```
SearchVARLasso(YStd, XStd, lambdas, max_iter, tol)
```

YStd	Numeric matrix. Matrix of standardized dependent variables (Y).
XStd	Numeric matrix. Matrix of standardized predictors (X) . XStd should not include a vector of ones in column one.
lambdas	Numeric vector. Lasso hyperparameter. The regularization strength controlling the sparsity.
max_iter	Integer. The maximum number of iterations for the coordinate descent algorithm (e.g., \max_i ter = 10000).
tol	Numeric. Convergence tolerance. The algorithm stops when the change in coefficients between iterations is below this tolerance (e.g., tol = 1e-5).

SelectVARLasso 35

Value

List with the following elements:

- criteria: Matrix with columns for lambda, AIC, BIC, and EBIC values.
- fit: List of matrices containing the estimated autoregressive and cross-regression coefficients for each lambda.

Author(s)

Ivan Jacob Agaloos Pesigan

See Also

```
Other Fitting Autoregressive Model Functions: FitMLVARDynr(), FitMLVARMplus(), FitVARDynr(), FitVARLassoSearch(), FitVARLasso(), FitVARMplus(), FitVAROLS(), LambdaSeq(), ModelVARP1Dynr(), ModelVARP2Dynr(), OrigScale(), PBootVARExoLasso(), PBootVARExoLsso(), PBootVARCLS(), RBootVARCLS(), RBootVARCLS(), RBootVAROLS(), StdMat()
```

Examples

```
YStd <- StdMat(dat_p2_yx$Y)
XStd <- StdMat(dat_p2_yx$X[, -1])
lambdas <- 10^seq(-5, 5, length.out = 100)
search <- SearchVARLasso(YStd = YStd, XStd = XStd, lambdas = lambdas,
    max_iter = 10000, tol = 1e-5)
plot(x = 1:nrow(search$criteria), y = search$criteria[, 4],
    type = "b", xlab = "lambda", ylab = "EBIC")</pre>
```

SelectVARLasso

Select the Lasso Estimates from the Grid Search

Description

Select the Lasso Estimates from the Grid Search

Usage

```
SelectVARLasso(search, crit = "ebic")
```

```
search Object. Output of the SearchVARLasso() function.

crit Character string. Information criteria to use. Valid values include "aic", "bic", and "ebic".
```

36 SimVAR

Value

Returns the Lasso estimates of autoregression and cross regression coefficients.

Author(s)

Ivan Jacob Agaloos Pesigan

See Also

```
Other Simulation of Autoregressive Data Functions: BootCI(), BootSE(), SimVARExo(), SimVAR(), YXExo(), YX()
```

Examples

```
YStd <- StdMat(dat_p2_yx$Y)
XStd <- StdMat(dat_p2_yx$X[, -1])
lambdas <- 10^seq(-5, 5, length.out = 100)
search <- SearchVARLasso(
   YStd = YStd, XStd = XStd, lambdas = lambdas,
   max_iter = 10000, tol = 1e-5
)
SelectVARLasso(search, crit = "ebic")</pre>
```

 ${\tt SimVAR}$

Simulate Data from a Vector Autoregressive (VAR) Model

Description

This function generates synthetic time series data from a Vector Autoregressive (VAR) model.

Usage

```
SimVAR(time, burn_in, constant, coef, chol_cov)
```

time	Integer. Number of time points to simulate.
burn_in	Integer. Number of burn-in observations to exclude before returning the results.
constant	Numeric vector. The constant term vector of length k, where k is the number of variables.
coef	Numeric matrix. Coefficient matrix with dimensions k by $(k * p)$. Each k by k block corresponds to the coefficient matrix for a particular lag.
chol_cov	Numeric matrix. The Cholesky decomposition of the covariance matrix of the multivariate normal noise. It should have dimensions k by k.

SimVAR 37

Details

The SimVAR() function generates synthetic time series data from a Vector Autoregressive (VAR) model. The VAR model is defined by the constant term constant, the coefficient matrix coef, and the Cholesky decomposition of the covariance matrix of the multivariate normal process noise chol_cov. The generated time series data follows a VAR(p) process, where p is the number of lags specified by the size of coef. The generated data includes a burn-in period, which is excluded before returning the results.

The steps involved in generating the VAR time series data are as follows:

- Extract the number of variables k and the number of lags p from the input.
- Create a matrix data of size k by (time + burn_in) to store the generated VAR time series data.
- Set the initial values of the matrix data using the constant term constant.
- For each time point starting from the p-th time point to time + burn_in 1:
 - Generate a vector of random noise from a multivariate normal distribution with mean 0 and covariance matrix chol_cov.
 - Generate the VAR time series values for each variable j at time t using the formula:

$$Y_{tj} = \text{constant}_j + \sum_{l=1}^p \sum_{m=1}^k (\text{coef}_{jm} * Y_{im}) + \text{noise}_j$$

where Y_{tj} is the value of variable j at time t, constant_j is the constant term for variable j, coef_{jm} are the coefficients for variable j from lagged variables up to order p, Y_{tm} are the lagged values of variable m up to order p at time t, and noise_j is the element j from the generated vector of random process noise.

• Transpose the matrix data and return only the required time period after the burn-in period, which is from column burn_in to column time + burn_in - 1.

Value

Numeric matrix containing the simulated time series data with dimensions k by time, where k is the number of variables and time is the number of observations.

Author(s)

Ivan Jacob Agaloos Pesigan

See Also

Other Simulation of Autoregressive Data Functions: BootCI(), BootSE(), SelectVARLasso(), SimVARExo(), YXExo(), YX()

```
set.seed(42)
time <- 50L
burn_in <- 10L</pre>
```

38 SimVARExo

```
k <- 3
p <- 2
constant <- c(1, 1, 1)
coef <- matrix(</pre>
  data = c(
    0.4, 0.0, 0.0, 0.1, 0.0, 0.0,
    0.0, 0.5, 0.0, 0.0, 0.2, 0.0,
    0.0, 0.0, 0.6, 0.0, 0.0, 0.3
  ),
  nrow = k,
  byrow = TRUE
chol_cov <- chol(diag(3))</pre>
y <- SimVAR(
  time = time,
  burn_in = burn_in,
  constant = constant,
  coef = coef,
  chol_cov = chol_cov
head(y)
```

SimVARExo

Simulate Data from a Vector Autoregressive (VAR) Model with Exogenous Variables

Description

This function generates synthetic time series data from a Vector Autoregressive (VAR) model with exogenous variables.

Usage

```
SimVARExo(time, burn_in, constant, coef, chol_cov, exo_mat, exo_coef)
```

time	Integer. Number of time points to simulate.
burn_in	Integer. Number of burn-in observations to exclude before returning the results.
constant	Numeric vector. The constant term vector of length k, where k is the number of variables.
coef	Numeric matrix. Coefficient matrix with dimensions k by (k * p). Each k by k block corresponds to the coefficient matrix for a particular lag.
chol_cov	Numeric matrix. The Cholesky decomposition of the covariance matrix of the multivariate normal noise. It should have dimensions k by k.
exo_mat	Numeric matrix. Matrix of exogenous covariates with dimensions time + burn_in by x. Each column corresponds to a different exogenous variable.

SimVARExo 39

exo_coef

Numeric vector. Coefficient matrix with dimensions k by x associated with the exogenous covariates.

Value

Numeric matrix containing the simulated time series data with dimensions k by time, where k is the number of variables and time is the number of observations.

Author(s)

Ivan Jacob Agaloos Pesigan

See Also

```
Other Simulation of Autoregressive Data Functions: BootCI(), BootSE(), SelectVARLasso(), SimVAR(), YXExo(), YX()
```

```
set.seed(42)
time <- 1000L
burn_in <- 200
k <- 3
p <- 2
constant \leftarrow c(1, 1, 1)
coef <- matrix(</pre>
  data = c(
    0.4, 0.0, 0.0, 0.1, 0.0, 0.0,
    0.0, 0.5, 0.0, 0.0, 0.2, 0.0,
    0.0, 0.0, 0.6, 0.0, 0.0, 0.3
  ),
  nrow = k,
  byrow = TRUE
chol_cov <- chol(diag(3))</pre>
exo_mat <- MASS::mvrnorm(</pre>
  n = time + burn_in,
  mu = c(0, 0, 0),
  Sigma = diag(3)
exo_coef <- matrix(</pre>
  data = c(
    0.5, 0.0, 0.0,
    0.0, 0.5, 0.0,
    0.0, 0.0, 0.5
  ),
  nrow = 3
y <- SimVARExo(
  time = time,
  burn_in = burn_in,
  constant = constant,
```

40 StdMat

```
coef = coef,
  chol_cov = chol_cov,
  exo_mat = exo_mat,
  exo_coef = exo_coef
)
str(y)
```

StdMat

Standardize Matrix

Description

This function standardizes the given matrix by centering the columns and scaling them to have unit variance.

Usage

StdMat(X)

Arguments

Χ

Numeric matrix. The matrix to be standardized.

Value

Numeric matrix with standardized values.

Author(s)

Ivan Jacob Agaloos Pesigan

See Also

```
Other Fitting Autoregressive Model Functions: FitMLVARDynr(), FitMLVARMplus(), FitVARDynr(), FitVARLassoSearch(), FitVARLasso(), FitVARMplus(), FitVAROLS(), LambdaSeq(), ModelVARP1Dynr(), ModelVARP2Dynr(), OrigScale(), PBootVARExoLasso(), PBootVARExoOLS(), PBootVARLasso(), PBootVAROLS(), RBootVARExoLasso(), RBootVARCLS(), RBootVARCLS(), SearchVARLasso()
```

```
std <- StdMat(dat_p2)
colMeans(std)
var(std)</pre>
```

YX 41

ΥX

Create Y and X Matrices

Description

This function creates the dependent variable (Y) and predictor variable (X) matrices.

Usage

```
YX(data, p)
```

Arguments

data	Numeric matrix. The time series data with dimensions t by k, where t is the
	number of observations and k is the number of variables.
p	Integer. The order of the VAR model (number of lags).

Details

The YX() function creates the Y and X matrices required for fitting a Vector Autoregressive (VAR) model. Given the input data matrix with dimensions t by k, where t is the number of observations and k is the number of variables, and the order of the VAR model p (number of lags), the function constructs lagged predictor matrix X and the dependent variable matrix Y.

The steps involved in creating the Y and X matrices are as follows:

- Determine the number of observations t and the number of variables k from the input data matrix.
- Create matrices X and Y to store lagged variables and the dependent variable, respectively.
- Populate the matrices X and Y with the appropriate lagged data. The predictors matrix X contains a column of ones and the lagged values of the dependent variables, while the dependent variable matrix Y contains the original values of the dependent variables.
- The function returns a list containing the Y and X matrices, which can be used for further analysis and estimation of the VAR model parameters.

Value

List containing the dependent variable (Y) and predictor variable (X) matrices. Note that the resulting matrices will have t - p rows.

Author(s)

Ivan Jacob Agaloos Pesigan

See Also

```
The SimVAR() function for simulating time series data from a VAR model.
```

```
Other Simulation of Autoregressive Data Functions: BootCI(), BootSE(), SelectVARLasso(), SimVARExo(), SimVAR(), YXExo()
```

42 YXExo

Examples

```
set.seed(42)
time <- 50L
burn_in <- 10L
k <- 3
p <- 2
constant <- c(1, 1, 1)
coef <- matrix(</pre>
  data = c(
    0.4, 0.0, 0.0, 0.1, 0.0, 0.0,
    0.0, 0.5, 0.0, 0.0, 0.2, 0.0,
    0.0, 0.0, 0.6, 0.0, 0.0, 0.3
  ),
  nrow = k,
  byrow = TRUE
)
chol_cov <- chol(diag(3))</pre>
y <- SimVAR(
  time = time,
  burn_in = burn_in,
  constant = constant,
  coef = coef,
  chol\_cov = chol\_cov
)
yx \leftarrow YX(data = y, p = 2)
str(yx)
```

YXExo

Create Y and X Matrices with Exogenous Variables

Description

This function creates the dependent variable (Y) and predictor variable (X) matrices.

Usage

```
YXExo(data, p, exo_mat)
```

data	Numeric matrix. The time series data with dimensions t by k, where t is the number of observations and k is the number of variables.
p	Integer. The order of the VAR model (number of lags).
exo_mat	Numeric matrix. Matrix of exogenous variables with dimensions t by m.

YXExo 43

Details

The YX() function creates the Y and X matrices required for fitting a Vector Autoregressive (VAR) model. Given the input data matrix with dimensions t by k, where t is the number of observations and k is the number of variables, and the order of the VAR model p (number of lags), the function constructs lagged predictor matrix X and the dependent variable matrix Y.

The steps involved in creating the Y and X matrices are as follows:

- Determine the number of observations t and the number of variables k from the input data matrix
- Create matrices X and Y to store lagged variables and the dependent variable, respectively.
- Populate the matrices X and Y with the appropriate lagged data. The predictors matrix X contains a column of ones and the lagged values of the dependent variables, while the dependent variable matrix Y contains the original values of the dependent variables.
- The function returns a list containing the Y and X matrices, which can be used for further analysis and estimation of the VAR model parameters.

Value

List containing the dependent variable (Y) and predictor variable (X) matrices. Note that the resulting matrices will have t - p rows.

Author(s)

Ivan Jacob Agaloos Pesigan

See Also

The SimVAR() function for simulating time series data from a VAR model.

Other Simulation of Autoregressive Data Functions: BootCI(), BootSE(), SelectVARLasso(), SimVARExo(), SimVAR(), YX()

```
set.seed(42)
time <- 1000L
burn_in <- 200
k <- 3
p <- 2
constant <- c(1, 1, 1)
coef <- matrix(
   data = c(
      0.4, 0.0, 0.0, 0.1, 0.0, 0.0,
      0.0, 0.5, 0.0, 0.0, 0.2, 0.0,
      0.0, 0.0, 0.6, 0.0, 0.0, 0.3
),
   nrow = k,
   byrow = TRUE
)
chol_cov <- chol(diag(3))</pre>
```

YXExo

```
exo_mat <- MASS::mvrnorm(</pre>
 n = time + burn_in,
 mu = c(0, 0, 0),
 Sigma = diag(3)
exo_coef <- matrix(</pre>
 data = c(
   0.5, 0.0, 0.0,
   0.0, 0.5, 0.0,
   0.0, 0.0, 0.5
  ),
  nrow = 3
y <- SimVARExo(
  time = time,
  burn_in = burn_in,
  constant = constant,
  coef = coef,
  chol_cov = chol_cov,
  exo_mat = exo_mat,
  exo_coef = exo_coef
)
yx <- YXExo(
 data = y,
  p = 2,
  exo_mat = exo_mat[(burn_in + 1):(time + burn_in), ]
str(yx)
```

Index

* Fitting Autoregressive Model Functions	dat_p2_yx, 11
FitMLVARDynr, 11	SimVAR, 36
FitMLVARMplus, 12	SimVARExo, 38
FitVARDynr, 13	* fitAutoReg
FitVARLasso, 15	BootCI, 2
FitVARLassoSearch, 16	BootSE, 4
FitVARMplus, 18	$dat_ml_p1, 7$
FitVAROLS, 19	$dat_ml_p2, 7$
LambdaSeq, 20	dat_p1, 8
ModelVARP1Dynr, 21	dat_p1_yx, 9
ModelVARP2Dynr, 22	dat_p2, 9
OrigScale, 22	dat_p2_exo, 10
PBootVARExoLasso, 23	dat_p2_exo_yx, 10
PBootVARExoOLS, 25	dat_p2_yx, 11
PBootVARLasso, 26	FitMLVARDynr, 11
PBootVAROLS, 27	FitMLVARMplus, 12
RBootVARExoLasso, 29	FitVARDynr, 13
RBootVARExoOLS, 31	FitVARLasso, 15
RBootVARLasso, 32	FitVARLassoSearch, 16
RBootVAROLS, 33	FitVARMplus, 18
SearchVARLasso, 34	FitVAROLS, 19
StdMat, 40	LambdaSeq, 20
* Simulation of Autoregressive Data	ModelVARP1Dynr, 21
Functions	ModelVARP2Dynr, 22
BootCI, 2	OrigScale, 22
BootSE, 4	PBootVARExoLasso, 23
SelectVARLasso, 35	PBootVARExoOLS, 25
SimVAR, 36	PBootVARLasso, 26
SimVARExo, 38	PBootVAROLS, 27
YX, 41	RBootVARExoLasso, 29
YXExo, 42	RBootVARExoOLS, 31
* data	RBootVARLasso, 32
$dat_ml_p1, 7$	RBootVAROLS, 33
$dat_ml_p2, 7$	SearchVARLasso, 34
$dat_p1, 8$	SelectVARLasso, 35
dat_p1_yx, 9	StdMat, 40
dat_p2, 9	* fit
dat_p2_exo, 10	FitMLVARDynr, 11
dat n2 exp vx 10	FitMLVARMolus 12

46 INDEX

FitVARDynr, 13	$dat_ml_p1, 7$
FitVARLasso, 15	$dat_ml_p2, 7$
FitVARLassoSearch, 16	dat_p1, 8
FitVARMplus, 18	dat_p1_yx, 9
FitVAROLS, 19	dat_p2,9
LambdaSeg, 20	dat_p2_exo, 10
ModelVARP1Dynr, 21	dat_p2_exo_yx, 10
ModelVARP2Dynr, 22	dat_p2_yx, 10 dat_p2_yx, 11
· · · · · · · · · · · · · · · · · · ·	
SearchVARLasso, 34	dynr::dynr.model(), 21, 22
SelectVARLasso, 35	FitMLVARDynr, 11, <i>13</i> , <i>14</i> , <i>16–18</i> , <i>20–25</i> , <i>27</i> ,
* methods	
<pre>coef.dynr_model, 6</pre>	28, 30, 31, 33–35, 40
plot.dynr_model, 28	FitMLVARMplus, 12, 12, 14, 16–18, 20–25, 27,
print.dynr_model, 29	28, 30, 31, 33–35, 40
* pb	FitVARDynr, 12, 13, 13, 16–18, 20–25, 27, 28,
BootCI, 2	30, 31, 33–35, 40
BootSE, 4	FitVARDynr(), 6
PBootVARExoLasso, 23	FitVARLasso, <i>12–14</i> , 15, <i>17</i> , <i>18</i> , <i>20–25</i> , <i>27</i> ,
PBootVARExoOLS, 25	28, 30, 31, 33–35, 40
PBootVARLasso, 26	FitVARLasso(), 15
PBootVAROLS, 27	FitVARLassoSearch, 12-14, 16, 16, 18,
_	20–25, 27, 28, 30, 31, 33–35, 40
* rb	FitVARMplus, 12–14, 16, 17, 18, 20–25, 27,
BootCI, 2	28, 30, 31, 33–35, 40
BootSE, 4	FitVAROLS, 12–14, 16–18, 19, 20–25, 27, 28,
RBootVARExoLasso, 29	30, 31, 33–35, 40
RBootVARExoOLS, 31	
RBootVARLasso, 32	FitVAROLS(), 19
RBootVAROLS, 33	LambdaSeq, 12–14, 16–18, 20, 20, 21–25, 27,
* simAutoReg	28, 30, 31, 33–35, 40
SimVAR, 36	20, 30, 31, 33–33, 40
SimVARExo, 38	ModelVARP1Dynr, 12-14, 16-18, 20, 21,
YX, 41	22–25, 27, 28, 30, 31, 33–35, 40
YXExo, 42	ModelVARP1Dynr(), 14
* sim	
SimVAR, 36	ModelVARP2Dynr, 12–14, 16–18, 20, 21, 22,
SimVARExo, 38	23–25, 27, 28, 30, 31, 33–35, 40
* utils	ModelVARP2Dynr(), <i>14</i>
OrigScale, 22	OrigScale, 12–14, 16–18, 20–22, 22, 24, 25,
	_
StdMat, 40	27, 28, 30, 31, 33–35, 40
YX, 41	PBootVARExoLasso, 12–14, 16–18, 20–23, 23,
YXExo, 42	25, 27, 28, 30, 31, 33–35, 40
* var	
SimVAR, 36	PBootVARExoOLS, 12–14, 16–18, 20–24, 25,
SimVARExo, 38	27, 28, 30, 31, 33–35, 40
	PBootVARLasso, 12–14, 16–18, 20–25, 26, 28,
BootCI, 2, 4, 36, 37, 39, 41, 43	30, 31, 33–35, 40
BootSE, 3, 4, 36, 37, 39, 41, 43	PBootVARLasso(), 3 , 4
	PBootVAROLS, 12–14, 16–18, 20–25, 27, 27,
<pre>coef.dynr_model, 6</pre>	30, 31, 33–35, 40

INDEX 47

```
PBootVAROLS(), 3, 4
plot.dynr_model, 28
print.dynr_model, 29
RBootVARExoLasso, 12–14, 16–18, 20–25, 27,
         28, 29, 31, 33–35, 40
RBootVARExoOLS, 12–14, 16–18, 20–25, 27,
         28, 30, 31, 33–35, 40
RBootVARLasso, 12-14, 16-18, 20-25, 27, 28,
         30, 31, 32, 34, 35, 40
RBootVARLasso(), 3, 4
RBootVAROLS, 12-14, 16-18, 20-25, 27, 28,
         30, 31, 33, 33, 35, 40
RBootVAROLS(), 3, 4
SearchVARLasso, 12–14, 16–18, 20–25, 27,
         28, 30, 31, 33, 34, 34, 40
SearchVARLasso(), 35
SelectVARLasso, 3, 4, 35, 37, 39, 41, 43
SimVAR, 3, 4, 36, 36, 39, 41, 43
SimVAR(), 37, 41, 43
SimVARExo, 3, 4, 36, 37, 38, 41, 43
StdMat, 12–14, 16–18, 20–25, 27, 28, 30, 31,
         33–35, 40
YX, 3, 4, 36, 37, 39, 41, 43
YX(), 41, 43
YXExo, 3, 4, 36, 37, 39, 41, 42
```