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References

Aroian: The probability function of the product of two normally distributed variables

Aroian-1947

Leo A. Aroian. “The probability function of the product of two normally distributed variables”. In: *The Annals of Mathematical Statistics* 18.2 (June 1947), pp. 265–271. DOI: 10.1214/aoms/1177730442.

Abstract: Let x and y follow a normal bivariate probability function with means $\bar{X}, \bar{Y}$, standard deviations σ_1, σ_2, respectively, r the coefficient of correlation, and $\rho_1 = \bar{X}/\sigma_1, \rho_2 = \bar{Y}/\sigma_2$. Professor C. C. Craig [1] has found the probability function of $z = xy/\sigma_1\sigma_2$ in closed form as the difference of two integrals. For purposes of numerical computation he has expanded this result in an infinite series involving powers of z, ρ_1, ρ_2, and Bessel functions of a certain type; in addition, he has determined the moments, semin-variants, and the moment generating function of z. However, for ρ_1 and ρ_2 large, as Craig points out, the series expansion converges very slowly. Even for ρ_1 and ρ_2 as small as 2, the expansion is unwieldy. We shall show that as ρ_1 and $\rho_2 \to \infty$, the probability function of z approaches a normal curve and in case $r = 0$ the Type III function and the Gram-Charlier Type A series are excellent approximations to the z distribution in the proper region. Numerical integration provides a substitute for the infinite series wherever the exact values of the probability function of z are needed. Some extensions of the main theorem are given in section 5 and a practical problem involving the probability function of z is solved.