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## References

**Aroian: The probability function of the product of two normally distributed variables**

**Aroian-1947**

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Leo A. Aroian. “The probability function of the product of two normally distributed variables”. In: *The Annals of Mathematical Statistics* 18.2 (June 1947), pp. 265–271. DOI: [10.1214/aoms/1177730442](https://doi.org/10.1214/aoms/1177730442).

Abstract: Let  $x$  and  $y$  follow a normal bivariate probability function with means  $\bar{X}, \bar{Y}$ , standard deviations  $\sigma_1, \sigma_2$ , respectively,  $r$  the coefficient of correlation, and  $\rho_1 = \bar{X}/\sigma_1, \rho_2 = \bar{Y}/\sigma_2$ . Professor C. C. Craig [1] has found the probability function of  $z = xy/\sigma_1\sigma_2$  in closed form as the difference of two integrals. For purposes of numerical computation he has expanded this result in an infinite series involving powers of  $z, \rho_1, \rho_2$ , and Bessel functions of a certain type; in addition, he has determined the moments, semin-variants, and the moment generating function of  $z$ . However, for  $\rho_1$  and  $\rho_2$  large, as Craig points out, the series expansion converges very slowly. Even for  $\rho_1$  and  $\rho_2$  as small as 2, the expansion is unwieldy. We shall show that as  $\rho_1$  and  $\rho_2 \rightarrow \infty$ , the probability function of  $z$  approaches a normal curve and in case  $r = 0$  the Type III function and the Gram-Charlier Type A series are excellent approximations to the  $z$  distribution in the proper region. Numerical integration provides a substitute for the infinite series wherever the exact values of the probability function of  $z$  are needed. Some extensions of the main theorem are given in section 5 and a practical problem involving the probability function of  $z$  is solved.