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## References

**Goodman: On the exact variance of products**

**Goodman-1960**

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Leo A. Goodman. “On the exact variance of products”. In: *Journal of the American Statistical Association* 55.292 (Dec. 1960), pp. 708–713. DOI: [10.1080/01621459.1960.10483369](https://doi.org/10.1080/01621459.1960.10483369).

Abstract: A simple exact formula for the variance of the product of two random variables, say,  $x$  and  $y$ , is given as a function of the means and central product-moments of  $x$  and  $y$ . The usual approximate variance formula for  $xy$  is compared with this exact formula; e.g., we note, in the special case where  $x$  and  $y$  are independent, that the “variance” computed by the approximate formula is less than the exact variance, and that the accuracy of the approximation depends on the sum of the reciprocals of the squared coefficients of variation of  $x$  and  $y$ . The case where  $x$  and  $y$  need not be independent is also studied, and exact variance formulas are presented for several different “product estimates.” (The usefulness of exact formulas becomes apparent when the variances of these estimates are compared.) When  $x$  and  $y$  are independent, simple unbiased estimates of these exact variances are suggested; in the more general case, consistent estimates are presented.

**Granger: Investigating causal relations by econometric models and cross-spectral methods**

**Granger-1969**

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C. W. J. Granger. “Investigating causal relations by econometric models and cross-spectral methods”. In: *Econometrica* 37.3 (Aug. 1969), p. 424. ISSN: 0012-9682. DOI: [10.2307/1912791](https://doi.org/10.2307/1912791).

Abstract: There occurs on some occasions a difficulty in deciding the direction of causality between two related variables and also whether or not feedback is occurring. Testable definitions of causality and feedback are proposed and illustrated by use of simple two-variable models. The important

problem of apparent instantaneous causality is discussed and it is suggested that the problem often arises due to slowness in recording information or because a sufficiently wide class of possible causal variables has not been used. It can be shown that the cross spectrum between two variables can be decomposed into two parts, each relating to a single causal arm of a feedback situation. Measures of causal lag and causal strength can then be constructed. A generalisation of this result with the partial cross spectrum is suggested.

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**Kalman: A new approach to linear filtering and prediction problems**      **Kalman-1960**

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R. E. Kalman. “A new approach to linear filtering and prediction problems”. In: *Journal of Basic Engineering* 82.1 (Mar. 1960), pp. 35–45. DOI: [10.1115/1.3662552](https://doi.org/10.1115/1.3662552).

Abstract: The classical filtering and prediction problem is re-examined using the Bode-Shannon representation of random processes and the “state-transition” method of analysis of dynamic systems. New results are: (1) The formulation and methods of solution of the problem apply without modification to stationary and nonstationary statistics and to growing-memory and infinite-memory filters. (2) A nonlinear difference (or differential) equation is derived for the covariance matrix of the optimal estimation error. From the solution of this equation the co-efficients of the difference (or differential) equation of the optimal linear filter are obtained without further calculations. (3) The filtering problem is shown to be the dual of the noise-free regulator problem. The new method developed here is applied to two well-known problems, confirming and extending earlier results. The discussion is largely self-contained and proceeds from first principles; basic concepts of the theory of random processes are reviewed in the Appendix.