## longMI: Internal Tests

### Ivan Jacob Agaloos Pesigan

#### Tests

```
#> test
#> Call:
#> Comparison(configural = configural_fit, weak = weak_fit, strong = strong_fit,
#> strict = strict_fit)
                chisq df pvalue
                                   cfi
                                         tli rmsea srmr
                                                               aic
#> configural 25.9682 19 0.1311 0.9915 0.9875 0.0424 0.0306 11252.23 11335.18
#> weak 41.8973 22 0.0064 0.9757 0.9691 0.0666 0.0763 11262.16 11335.16
             53.7228 25 0.0007 0.9650 0.9608 0.0750 0.0872 11267.98 11331.03
#> strong
            134.5591 29 0.0000 0.8712 0.8757 0.1336 0.1690 11340.82 11390.59
#> strict
#> Comparison(configural = configural_fit, weak = weak_fit, strong = strong_fit,
      strict = strict_fit)
#>
#> CONFIGURAL INVARIANCE MODEL
#> lavaan 0.6.16 ended normally after 75 iterations
#>
#>
   Estimator
                                                     ML
#>
    Optimization method
                                                  NLMINB
   Number of model parameters
#>
                                                     27
#>
    Number of equality constraints
                                                      2
#>
                                                     204
#>
    Number of observations
    Number of missing patterns
                                                       1
#>
#> Model Test User Model:
#>
                                                  25.968
#>
    Test statistic
    Degrees of freedom
                                                      19
#>
    P-value (Chi-square)
                                                   0.131
#>
#> Parameter Estimates:
```

```
#>
   Standard errors
                                           Standard
#>
    Information
                                           Observed
#>
    Observed information based on
                                           Hessian
#>
#> Latent Variables:
#>
                   Estimate Std.Err z-value P(>|z|)
#>
   f1t1 =~
            (111)
                     4.451
                             0.400
                                    11.137
                                             0.000
#>
    y1_1
#>
     y1_2
                      6.850
                            0.637
                                    10.745
                                             0.000
     y1_3
#>
                      4.590
                            0.520
                                    8.821
                                           0.000
     y1_4
#>
                      5.039
                            0.396
                                    12.728
                                            0.000
   f1t6 =~
#>
            (111) 4.451
                            0.400 11.137
                                            0.000
#>
    y6_1
#>
                     4.006
                            0.489 8.194
                                           0.000
     y6_2
                     4.551
                             0.545
                                           0.000
#>
     y6_3
                                   8.346
#>
      y6_4
                      4.102
                              0.453
                                     9.057
                                            0.000
#>
#> Covariances:
                  Estimate Std.Err z-value P(>|z|)
#>
    f1t1 ~~
#>
#>
     f1t6
                    1.837 0.215 8.558
                                            0.000
#>
#> Intercepts:
#>
                  Estimate Std.Err z-value P(>|z|)
               (i1) 19.776
                            0.427
                                    46.273
                                            0.000
#>
    .y1_1
#>
    .y1_2
                    21.797
                            0.680
                                    32.036
                                            0.000
     .y1_3
                    14.903
                            0.528
                                     28.223
                                            0.000
#>
#>
                     20.396
                            0.439 46.416
                                             0.000
     .y1_4
               (i1) 19.776
                                           0.000
#>
     .y6_1
                            0.427 46.273
                    19.317
                            2.299 8.404
                                           0.000
#>
     .y6_2
                             2.516 4.738
                     11.922
#>
     .y6_3
                                            0.000
                     17.970
#>
     .y6_4
                            1.844 9.747
                                           0.000
#>
     f1t1
                     0.000
                            0.606 10.649
#>
     f1t6
                     6.455
                                            0.000
#>
#> Variances:
#>
                   Estimate Std.Err z-value P(>|z|)
#>
                    17.448
                           2.240
                                   7.789
                                            0.000
     .y1_1
#>
     .y1_2
                     47.511
                            5.754
                                   8.257
                                             0.000
    .y1_3
                     35.810
                            4.031 8.884
                                            0.000
#>
#>
     .y1_4
                    13.999
                            2.133 6.563
                                            0.000
#>
                     47.096
                            6.432 7.322
                                            0.000
     .y6_1
#>
     .y6_2
                     73.850
                            8.388
                                   8.805
                                             0.000
                            10.354 8.588
#>
                     88.920
                                           0.000
     .y6_3
#>
     .y6_4
                     23.267
                            4.182
                                     5.564
                                           0.000
     f1t1
                     1.000
#>
```

```
#>
   f1t6
                       5.834 1.167 4.997 0.000
#>
#>
#>
#> WEAK INVARIANCE MODEL
#> lavaan 0.6.16 ended normally after 54 iterations
#>
#>
    Estimator
                                                    ML
#>
    Optimization method
                                                NLMINB
#>
    Number of model parameters
                                                    27
    Number of equality constraints
                                                     5
#>
#>
                                                   204
#>
    Number of observations
#>
    Number of missing patterns
                                                     1
#>
#> Model Test User Model:
#>
                                                41.897
#>
   Test statistic
    Degrees of freedom
                                                    22
#>
                                                 0.006
#>
    P-value (Chi-square)
#>
#> Parameter Estimates:
#>
#>
   Standard errors
                                              Standard
#>
   Information
                                              Observed
#>
   Observed information based on
                                               Hessian
#>
#> Latent Variables:
#>
                    Estimate Std.Err z-value P(>|z|)
   f1t1 =~
#>
     y1_1 (111) 4.933 0.339 14.562 0.000
y1_2 (112) 5.172 0.429 12.052 0.000
#>
    y1_1
#>
                     5.072
                                       12.762 0.000
#>
      y1_3
               (113)
                              0.397
                                       14.492
#>
               (114)
                     4.865
                              0.336
                                               0.000
      y1_4
    f1t6 =~
#>
#>
     y6_1
               (111)
                     4.933 0.339
                                       14.562
                                               0.000
#>
      y6_2
               (112)
                     5.172
                                0.429
                                       12.052
                                                 0.000
#>
      y6_3
               (113)
                       5.072
                                0.397
                                        12.762
                                                 0.000
               (114)
                       4.865
                                0.336
                                       14.492
                                                 0.000
#>
      y6_4
#>
#> Covariances:
#>
                     Estimate Std.Err z-value P(>|z|)
    f1t1 ~~
#>
#>
     f1t6
                       1.558
                                0.136 11.439
                                                 0.000
#>
```

```
#> Intercepts:
                  Estimate Std.Err z-value P(>|z|)
#>
              (i1) 19.776 0.445 44.430 0.000
#>
    .y1_1
#>
                    21.797
                           0.629 34.633 0.000
     .y1_2
    .y1_3
                           0.544 27.411
#>
                    14.903
                                          0.000
#>
    .y1_4
                    (i1) 19.776 0.445 44.430 0.000
#>
     .y6_1
                    15.049 2.299 6.547 0.000
#>
     .y6_2
                           2.212 5.315 0.000
#>
     .y6_3
                    11.756
#>
    .y6_4
                    16.111 1.765 9.130 0.000
#>
     f1t1
                    0.000
#>
     f1t6
                    5.824
                           0.429 13.591
                                          0.000
#>
#> Variances:
                  Estimate Std.Err z-value P(>|z|)
#>
#>
    .y1_1
                   16.079
                           2.188
                                  7.349
                                          0.000
#>
    .y1_2
                    54.055
                           6.012 8.992
                                          0.000
                   34.578 3.941 8.775 0.000
#>
    .y1_3
                   15.075 2.109 7.149 0.000
#>
    .y1_4
                           6.494 7.661 0.000
                   49.748
#>
    .y6_1
                   72.254 8.353 8.650 0.000
#>
    .y6_2
#>
    .y6_3
                   91.610 10.487 8.736 0.000
                   22.022 4.032 5.462 0.000
#>
     .y6_4
                    1.000
#>
     f1t1
#>
     f1t6
                    4.240
                           0.546 7.759
                                          0.000
#>
#>
#>
#> STRONG INVARIANCE MODEL
#> lavaan 0.6.16 ended normally after 52 iterations
#>
#>
   Estimator
                                              ML
#>
    Optimization method
                                           NLMINB
    Number of model parameters
                                              27
#>
    Number of equality constraints
#>
                                              8
#>
#>
   Number of observations
                                              204
#>
    Number of missing patterns
                                               1
#>
#> Model Test User Model:
#>
#>
    Test statistic
                                           53.723
    Degrees of freedom
                                              25
#>
#>
    P-value (Chi-square)
                                            0.001
#>
```

```
#> Parameter Estimates:
#>
#>
    Standard errors
                                            Standard
#>
    Information
                                            Observed
#>
    Observed information based on
                                            Hessian
#>
#> Latent Variables:
                   Estimate Std.Err z-value P(>|z|)
#>
   f1t1 =~
#>
                            0.331 15.917
#>
    y1_1
            (111)
                    5.270
                                             0.000
#>
     y1_2 (112)
                    4.525
                            0.312 14.487
                                            0.000
           (113)
(114)
                    4.960
                            0.328
                                     15.112
                                            0.000
#>
      y1_3
                    4.547
                             0.291
                                     15.634
                                            0.000
#>
     y1_4
#>
   f1t6 =~
                    5.270
                                             0.000
#>
     y6_1
              (111)
                            0.331
                                     15.917
#>
      y6_2
              (112)
                     4.525
                              0.312
                                     14.487
                                              0.000
#>
     y6_3
              (113)
                     4.960
                              0.328
                                     15.112
                                            0.000
              (114)
                      4.547
                              0.291
                                     15.634
                                              0.000
#>
      y6_4
#>
#> Covariances:
#>
                   Estimate Std.Err z-value P(>|z|)
#>
    f1t1 ~~
#>
                     1.608
                              0.142 11.316
     f1t6
                                            0.000
#>
#> Intercepts:
#>
                   Estimate Std.Err z-value P(>|z|)
#>
               (i1) 19.929
                             0.457
                                     43.623
                                              0.000
     .y1_1
#>
               (i2)
                    21.459
                              0.598
                                     35.901
                                              0.000
     .y1_2
                    14.882
                            0.529 28.146
                                            0.000
#>
     .y1_3
               (i3)
               (i4)
                    20.311
                            0.421
                                     48.252
                                            0.000
#>
     .y1_4
#>
     .y6_1
               (i1)
                    19.929
                             0.457
                                     43.623
                                            0.000
#>
     .y6_2
               (i2)
                    21.459
                            0.598
                                     35.901
                                            0.000
#>
     .y6_3
               (i3)
                    14.882
                            0.529 28.146 0.000
#>
     .y6_4
               (i4)
                    20.311
                              0.421
                                     48.252
                                            0.000
                     0.000
#>
     f1t1
#>
     f1t6
                      5.337
                              0.354
                                     15.077
                                            0.000
#>
#> Variances:
#>
                   Estimate Std.Err z-value P(>|z|)
                    15.124 2.238 6.757
#>
                                             0.000
    .y1_1
#>
     .y1_2
                     57.819
                            6.150 9.402
                                             0.000
                     34.209
                             3.869
                                    8.841
                                             0.000
#>
     .y1_3
#>
     .y1_4
                     16.339
                             2.114
                                      7.729
                                              0.000
#>
                     45.360
                             6.338
                                    7.157
                                            0.000
     .y6_1
#>
     .y6_2
                     74.229
                              8.352
                                      8.887
                                            0.000
                     89.572 10.110 8.860 0.000
#>
     .y6_3
```

```
#>
   .y6_4
                     24.586 3.951 6.223 0.000
                      1.000
#>
     f1t1
     f1t6
                      4.557
#>
                            0.585 7.791
                                              0.000
#>
#>
#>
#> STRICT INVARIANCE MODEL
#> lavaan 0.6.16 ended normally after 57 iterations
#>
#>
   Estimator
                                                 ML
    Optimization method
                                             NLMINB
#>
    Number of model parameters
                                                 27
#>
#>
    Number of equality constraints
                                                 12
#>
#>
   Number of observations
                                                204
#>
    Number of missing patterns
                                                  1
#>
#> Model Test User Model:
#>
#>
   Test statistic
                                            134.559
#>
    Degrees of freedom
                                                 29
    P-value (Chi-square)
                                              0.000
#>
#> Parameter Estimates:
#>
#>
   Standard errors
                                            Standard
#>
    Information
                                            Observed
    Observed information based on
#>
                                            Hessian
#>
#> Latent Variables:
#>
                  Estimate Std.Err z-value P(>|z|)
   f1t1 =~
#>
                    5.083
#>
    y1_1
            (111)
                            0.364 13.978
                                            0.000
                    4.309
                            0.333 12.944
     y1_2
              (112)
                                            0.000
#>
                    4.785 0.356 13.447
#>
    y1_3
              (113)
                                            0.000
#>
     y1_4
              (114) 4.358 0.317
                                     13.765 0.000
#>
   f1t6 =~
     y6_1
y6_2
#>
    y6_1
              (111)
                    5.083
                              0.364
                                     13.978
                                            0.000
              (112)
                     4.309
                              0.333 12.944 0.000
#>
                      4.785
                              0.356 13.447 0.000
#>
     y6_3
              (113)
#>
              (114)
                      4.358
                              0.317 13.765
                                            0.000
     y6_4
#>
#> Covariances:
#>
                   Estimate Std.Err z-value P(>|z|)
#> f1t1 ~~
```

```
#> f1t6
               1.812 0.168 10.812 0.000
#>
#> Intercepts:
#>
                   Estimate Std.Err z-value P(>|z|)
#>
     .y1_1
               (i1)
                     20.019
                              0.509
                                    39.339
                                              0.000
#>
               (i2)
                     21.513
                              0.621
                                     34.648
                                              0.000
     .y1_2
#>
     .y1_3
               (i3) 14.805
                            0.617
                                     23.988
                                            0.000
               (i4) 20.313
                            0.433 46.915
                                            0.000
#>
     .y1_4
     .y6_1
#>
               (i1)
                    20.019
                             0.509 39.339
                                             0.000
#>
     .y6_2
               (i2)
                    21.513
                            0.621
                                     34.648 0.000
#>
     .y6_3
               (i3)
                   14.805
                            0.617 23.988 0.000
               (i4)
                     20.313
                            0.433
                                    46.915
                                           0.000
#>
     .y6_4
#>
                     0.000
     f1t1
#>
     f1t6
                      5.557
                            0.415 13.396
                                            0.000
#>
#> Variances:
#>
                   Estimate Std.Err z-value P(>|z|)
              (u1) 28.657 2.958 9.689
                                            0.000
#>
    .y1_1
#>
     .y1_2
              (u2)
                   68.013
                            5.253 12.946
                                             0.000
                     61.387
                            4.897 12.535
#>
     .y1_3
               (u3)
                                              0.000
                                            0.000
#>
     .y1_4
              (u4)
                    20.878
                            2.142 9.746
#>
     .y6_1
               (u1) 28.657 2.958 9.689 0.000
                            5.253 12.946 0.000
#>
               (u2) 68.013
     .y6_2
#>
     .y6_3
               (u3)
                   61.387
                             4.897 12.535
                                            0.000
#>
     .y6_4
               (u4)
                   20.878
                            2.142 9.746 0.000
#>
     f1t1
                     1.000
#>
     f1t6
                      5.056
                              0.692 7.304
                                            0.000
#>
#> Call:
#> Comparison(configural = configural_fit, weak = weak_fit, strong = strong_fit,
#>
      strict = strict_fit)
#>
#> Chi-Squared Difference Test
#>
#>
                 AIC BIC
                            Chisq Chisq diff RMSEA Df diff Pr(>Chisq)
              Df
#> 1.configural 19 11252 11335 25.968
#> 1.weak
             22 11262 11335 41.897
                                     15.929 0.14535
                                                        3 0.0011726 **
#> 2.configural 19 11252 11335 25.968
#> 2.strong
            25 11268 11331 53.723
                                     27.755 0.13332
                                                        6 0.0001045 ***
#> 3.configural 19 11252 11335 25.968
#> 3.strict 29 11341 11391 134.559
                                   108.591 0.21984
                                                       10 < 2.2e-16 ***
#> 4.weak
             22 11262 11335 41.897
#> 4.strong
             25 11268 11331 53.723
                                     11.826 0.12009
                                                        3 0.0080053 **
#> 5.weak
             22 11262 11335 41.897
#> 5.strict
            29 11341 11391 134.559
                                      92.662 0.24492
                                                        7 < 2.2e-16 ***
#> 6.strong 25 11268 11331 53.723
```

```
#> 6.strict 29 11341 11391 134.559 80.836 0.30686 4 < 2.2e-16 ***
#> ---
#> Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
#> Call:
#> Invariance(data = osbornesudick1972, time_points = time_points,
      factor_loadings = factor_loadings)
              chisq df pvalue cfi
                                       tli rmsea srmr
                                                           aic
#> configural 25.9682 19 0.1311 0.9915 0.9875 0.0424 0.0306 11252.23 11335.18
             41.8973 22 0.0064 0.9757 0.9691 0.0666 0.0720 11262.16 11335.16
            53.7228 25 0.0007 0.9650 0.9608 0.0750 0.0859 11267.98 11331.03
#> strong
#> strict
           134.5591 29 0.0000 0.8712 0.8757 0.1336 0.1328 11340.82 11390.59
#> Call:
#> Invariance(data = osbornesudick1972, time_points = time_points,
#> factor_loadings = factor_loadings)
#>
#>
#> CONFIGURAL INVARIANCE MODEL
#> lavaan 0.6.16 ended normally after 75 iterations
#>
#>
   Estimator
                                                   MT.
#> Optimization method
                                               NLMINB
#> Number of model parameters
                                                   27
#>
   Number of equality constraints
#>
#>
   Number of observations
                                                  204
#>
#> Model Test User Model:
#>
#> Test statistic
                                               25.968
#>
   Degrees of freedom
                                                   19
#>
   P-value (Chi-square)
                                                0.131
#>
#> Parameter Estimates:
#>
#>
   Standard errors
                                             Standard
#> Information
                                             Expected
#> Information saturated (h1) model
                                          Structured
#> Latent Variables:
                   Estimate Std.Err z-value P(>|z|)
#>
#> f1t1 =~
#>
    y1_1
              (111)
                       4.451
                               0.396 11.244
                                                0.000
                               0.637 10.750 0.000
                       6.850
#>
     y1_2
#>
     y1_3
                       4.590
                               0.515 8.918 0.000
                       5.039 0.393 12.809 0.000
#> y1_4
```

```
#>
    f1t6 =~
#>
      y6_1
               (111)
                       4.451
                                0.396
                                        11.244
                                                  0.000
                        4.006
                                0.485
                                         8.261
                                                  0.000
#>
      y6_2
                        4.551
                                 0.546
                                          8.342
                                                  0.000
#>
      y6_3
#>
      y6_4
                        4.102
                                 0.448
                                          9.150
                                                  0.000
#>
#> Covariances:
                     Estimate Std.Err z-value P(>|z|)
#>
    f1t1 ~~
#>
#>
      f1t6
                        1.837
                                 0.211
                                          8.703
                                                  0.000
#>
#> Intercepts:
                     Estimate Std.Err z-value P(>|z|)
#>
                                                  0.000
#>
                (i1) 19.776
                                0.427
                                        46.273
     .y1_1
                       21.797
                                 0.680
                                        32.036
                                                  0.000
#>
      .y1_2
#>
     .y1_3
                       14.903
                                 0.528
                                        28.223
                                                  0.000
#>
     .y1_4
                       20.396
                                0.439
                                       46.416
                                                  0.000
                (i1)
                      19.776
                               0.427
                                        46.273
                                                0.000
#>
     .y6_1
#>
                       19.317
                                2.281
                                        8.467
                                                0.000
     .y6_2
                       11.922
#>
     .y6_3
                                 2.538
                                         4.697
                                                  0.000
#>
     .y6_4
                       17.970
                               1.815 9.903
                                                  0.000
#>
      f1t1
                       0.000
                        6.455
                                 0.601
                                                  0.000
#>
      f1t6
                                        10.743
#>
#> Variances:
#>
                     Estimate Std.Err z-value P(>|z|)
#>
                       17.448
                                 2.186
                                         7.981
                                                  0.000
     .y1_1
#>
                       47.511
                                 5.748
                                         8.266
                                                  0.000
     .y1_2
                                3.969
                                         9.022
                                                  0.000
#>
     .y1_3
                       35.810
                      13.999
                                2.085
                                         6.712
                                                  0.000
#>
     .y1_4
#>
     .y6_1
                       47.096
                                 6.305
                                         7.470
                                                  0.000
#>
     .y6_2
                       73.850
                                8.395
                                         8.797
                                                0.000
#>
     .y6_3
                       88.920
                               10.222 8.699 0.000
#>
     .y6_4
                       23.267
                                4.076 5.709
                                                  0.000
      f1t1
                       1.000
#>
#>
      f1t6
                       5.834
                               1.159
                                         5.035
                                                  0.000
#>
#>
#>
#> WEAK INVARIANCE MODEL
#> lavaan 0.6.16 ended normally after 54 iterations
#>
#>
    Estimator
                                                     ML
    Optimization method
                                                 NLMINB
    Number of model parameters
#>
```

```
#>
    Number of equality constraints
#>
    Number of observations
                                                 204
#>
#>
#> Model Test User Model:
#>
#>
    Test statistic
                                              41.897
    Degrees of freedom
                                                  22
#>
#>
    P-value (Chi-square)
                                               0.006
#>
#> Parameter Estimates:
#>
#>
    Standard errors
                                            Standard
#>
   Information
                                            Expected
#>
   Information saturated (h1) model
                                          Structured
#>
#> Latent Variables:
#>
                   Estimate Std.Err z-value P(>|z|)
   f1t1 =~
#>
    y1_1
            (111)
                      4.933
                               0.342
                                      14.420
                                               0.000
#>
                    5.172
                                             0.000
#>
     y1_2
              (112)
                             0.413 12.534
#>
     y1_3
              (113)
                    5.072
                              0.398 12.733
                                             0.000
                      4.865
                              0.330
                                     14.733
                                             0.000
#>
      y1_4
              (114)
   f1t6 =~
#>
     y6_1 (111)
y6_2 (112)
                    4.933
                             0.342
                                     14.420
                                             0.000
#>
#>
                      5.172
                             0.413 12.534 0.000
#>
      y6_3
              (113)
                      5.072
                               0.398
                                     12.733 0.000
#>
      y6_4
              (114)
                      4.865
                              0.330
                                     14.733
                                               0.000
#>
#> Covariances:
#>
                   Estimate Std.Err z-value P(>|z|)
    f1t1 ~~
#>
      f1t6
                     1.558
                               0.136 11.482
#>
                                               0.000
#>
#> Intercepts:
                   Estimate Std.Err z-value P(>|z|)
#>
#>
     .y1_1
               (i1) 19.776 0.445 44.430
                                             0.000
#>
     .y1_2
                     21.797
                              0.629 34.633
                                               0.000
#>
     .y1_3
                     14.903
                              0.544 27.411
                                              0.000
                     20.396
                             0.436 46.803
                                             0.000
#>
     .y1_4
                             0.445 44.430
#>
     .y6_1
               (i1)
                     19.776
                                             0.000
#>
                     15.049
                              2.223
                                     6.770
                                             0.000
     .y6_2
#>
     .y6_3
                     11.756
                              2.238
                                      5.253
                                               0.000
                             1.699
#>
                     16.111
                                       9.484
                                               0.000
     .y6_4
#>
      f1t1
                      0.000
     f1t6
                      5.824 0.432 13.476 0.000
#>
```

```
#> Variances:
                   Estimate Std.Err z-value P(>|z|)
#>
#>
                    16.079 2.165
                                    7.427
                                              0.000
     .y1_1
    .y1_2
#>
                    54.055
                            5.831
                                     9.270
                                              0.000
#>
    .y1_3
                    34.578 3.922 8.817
                                            0.000
     .y1_4
#>
                    15.075 2.051 7.350 0.000
                            6.242 7.970 0.000
                    49.748
#>
     .y6_1
                             8.498 8.503 0.000
#>
     .y6_2
                    72.254
                    91.610 10.272 8.919 0.000
#>
     .y6_3
#>
     .y6_4
                    22.022
                            3.989 5.521 0.000
     f1t1
                     1.000
#>
#>
     f1t6
                     4.240
                            0.539 7.867 0.000
#>
#>
#>
#> STRONG INVARIANCE MODEL
#> lavaan 0.6.16 ended normally after 52 iterations
#>
#>
   Estimator
                                                 MT.
#>
   Optimization method
                                             NLMINB
    Number of model parameters
                                                 27
#>
#>
    Number of equality constraints
#>
#>
   Number of observations
                                                204
#>
#> Model Test User Model:
#>
#>
   Test statistic
                                             53.723
                                                 25
#>
    Degrees of freedom
    P-value (Chi-square)
#>
                                              0.001
#>
#> Parameter Estimates:
#>
    Standard errors
#>
                                           Standard
#>
   Information
                                           Expected
#>
   Information saturated (h1) model
                                        Structured
#> Latent Variables:
                  Estimate Std.Err z-value P(>|z|)
#>
#>
   f1t1 =~
#>
    y1_1
              (111)
                      5.270
                              0.333
                                     15.813
                                              0.000
                     4.525
                              0.308 14.681
                                              0.000
#>
     y1_2
              (112)
#>
     y1_3
              (113)
                      4.960
                              0.328
                                     15.120
                                              0.000
              (114) 4.547 0.289 15.707 0.000
#> y1_4
```

```
#>
    f1t6 =~
#>
      y6_1
               (111)
                      5.270
                                 0.333
                                        15.813
                                                  0.000
                        4.525
                                 0.308
                                                  0.000
#>
      y6_2
               (112)
                                         14.681
               (113)
                        4.960
                                 0.328
                                         15.120
                                                  0.000
#>
      y6_3
#>
      y6_4
               (114)
                        4.547
                                 0.289
                                         15.707
                                                  0.000
#>
#> Covariances:
                     Estimate Std.Err z-value P(>|z|)
#>
    f1t1 ~~
#>
#>
      f1t6
                        1.608
                                 0.143
                                        11.281
                                                  0.000
#>
#> Intercepts:
                     Estimate Std.Err z-value P(>|z|)
#>
                                                  0.000
                (i1)
                       19.929
                               0.456
                                        43.727
#>
     .y1_1
                       21.459
                                 0.598
                                         35.868
                                                  0.000
#>
      .y1_2
                (i2)
#>
     .y1_3
                (i3)
                       14.882
                                 0.529
                                         28.153
                                                  0.000
#>
     .y1_4
                (i4)
                       20.311
                                 0.421
                                         48.237
                                                  0.000
                (i1)
                      19.929
                               0.456
                                        43.727
                                                0.000
#>
     .y6_1
#>
     .y6_2
                (i2)
                       21.459
                               0.598
                                         35.868
                                                0.000
                                 0.529
#>
     .y6_3
                (i3)
                       14.882
                                         28.153
                                                  0.000
#>
     .y6_4
                (i4)
                       20.311
                               0.421
                                         48.237
                                                  0.000
#>
      f1t1
                        0.000
                        5.337
                                                  0.000
#>
      f1t6
                                 0.353
                                        15.109
#>
#> Variances:
#>
                     Estimate Std.Err z-value P(>|z|)
#>
                                 2.193
                                          6.895
                                                  0.000
     .y1_1
                       15.124
#>
                       57.819
                                 6.078
                                          9.512
                                                  0.000
     .y1_2
                                 3.850
                                         8.885
                                                  0.000
#>
     .y1_3
                       34.209
                       16.339
                                2.062
                                         7.925
                                                  0.000
#>
     .y1_4
#>
     .y6_1
                       45.360
                                 6.182
                                         7.337
                                                  0.000
#>
     .y6_2
                       74.229
                                8.350
                                       8.890
                                                0.000
#>
     .y6_3
                       89.572
                               10.064 8.900 0.000
#>
     .y6_4
                       24.586
                                3.878 6.339
                                                  0.000
      f1t1
                       1.000
#>
                                 0.584
#>
      f1t6
                        4.557
                                         7.806
                                                  0.000
#>
#>
#>
#> STRICT INVARIANCE MODEL
#> lavaan 0.6.16 ended normally after 57 iterations
#>
#>
    Estimator
                                                     ML
    Optimization method
                                                 NLMINB
    Number of model parameters
#>
```

```
#>
    Number of equality constraints
                                                 12
#>
    Number of observations
                                                 204
#>
#>
#> Model Test User Model:
#>
#>
    Test statistic
                                             134.559
    Degrees of freedom
                                                 29
#>
    P-value (Chi-square)
                                               0.000
#>
#> Parameter Estimates:
#>
#>
    Standard errors
                                            Standard
#>
   Information
                                            Expected
#>
   Information saturated (h1) model
                                          Structured
#>
#> Latent Variables:
#>
                   Estimate Std.Err z-value P(>|z|)
   f1t1 =~
#>
    y1_1
            (111)
                      5.083
                              0.364
                                     13.968
                                               0.000
#>
                    4.309
                                             0.000
#>
     y1_2 (112)
                            0.325 13.248
#>
     y1_3
              (113)
                    4.785
                              0.354 13.521 0.000
                      4.358
                              0.312
                                            0.000
#>
     y1_4
              (114)
                                     13.971
   f1t6 =~
#>
     y6_1 (111)
y6_2 (112)
                    5.083
                             0.364
                                     13.968
                                             0.000
#>
#>
                      4.309
                            0.325 13.248 0.000
#>
      y6_3
              (113)
                      4.785
                              0.354
                                     13.521 0.000
#>
      y6_4
              (114)
                      4.358
                              0.312
                                     13.971
                                               0.000
#>
#> Covariances:
#>
                   Estimate Std.Err z-value P(>|z|)
    f1t1 ~~
#>
     f1t6
                     1.812
                              0.168 10.790
#>
                                               0.000
#>
#> Intercepts:
                   Estimate Std.Err z-value P(>|z|)
#>
#>
     .y1_1
               (i1) 20.019 0.506 39.547
                                             0.000
#>
     .y1_2
               (i2) 21.513
                            0.621
                                      34.617
                                              0.000
#>
     .y1_3
               (i3)
                     14.805
                              0.617
                                     24.001
                                             0.000
               (i4)
                    20.313
                             0.433 46.903 0.000
#>
     .y1_4
                             0.506
#>
     .y6_1
               (i1)
                     20.019
                                     39.547 0.000
#>
               (i2)
                     21.513
                             0.621
                                      34.617
                                             0.000
     .y6_2
#>
     .y6_3
               (i3)
                     14.805
                              0.617
                                      24.001
                                               0.000
#>
               (i4)
                     20.313
                             0.433
                                     46.903
                                               0.000
     .y6_4
#>
     f1t1
                      0.000
    f1t6
                      5.557 0.411 13.526 0.000
#>
```

```
#> Variances:
                   Estimate Std.Err z-value P(>|z|)
#>
              (u1) 28.657 2.747 10.433
                                               0.000
#>
     .y1_1
#>
     .y1_2
               (u2)
                     68.013
                              5.170 13.155
                                               0.000
#>
     .y1_3
               (u3) 61.387 4.821 12.734
                                             0.000
#>
     .y1_4
               (u4) 20.878 2.008 10.398
                                             0.000
                             2.747 10.433
#>
     .y6_1
               (u1) 28.657
                                             0.000
     .y6_2
               (u2) 68.013
#>
                             5.170 13.155
                                             0.000
#>
     .y6_3
               (u3) 61.387 4.821 12.734 0.000
#>
     .y6_4
               (u4) 20.878
                             2.008 10.398 0.000
     f1t1
                      1.000
#>
      f1t6
                      5.056
                               0.685
                                      7.378
                                             0.000
#>
#>
#> Call:
#> Invariance(data = osbornesudick1972, time_points = time_points,
      factor_loadings = factor_loadings)
#>
#> Chi-Squared Difference Test
#>
#>
              Df
                 AIC BIC
                             Chisq Chisq diff RMSEA Df diff Pr(>Chisq)
#> 1.configural 19 11252 11335 25.968
              22 11262 11335 41.897
                                      15.929 0.14535
                                                          3 0.0011726 **
#> 1.weak
#> 2.configural 19 11252 11335 25.968
                                      27.755 0.13332
                                                          6 0.0001045 ***
#> 2.strong
            25 11268 11331 53.723
#> 3.configural 19 11252 11335 25.968
#> 3.strict
            29 11341 11391 134.559
                                    108.591 0.21984
                                                         10 < 2.2e-16 ***
#> 4.weak
              22 11262 11335 41.897
11.826 0.12009
                                                          3 0.0080053 **
#> 5.strict 29 11341 11391 134.559
#> 6.strong 25 11268 11331 53.723
                                                          7 < 2.2e-16 ***
                                     92.662 0.24492
#> 6.strict
             29 11341 11391 134.559
                                                          4 < 2.2e-16 ***
                                     80.836 0.30686
#> Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
#> Test passed
#> [[1]]
#> [[1]][[1]]
#> [[1]][[1]]$value
#> [[1]][[1]]$value[[1]]
#> lavaan 0.6.16 ended normally after 90 iterations
```

```
#>
#> Estimator
                                                   ML
#> Optimization method
                                               NLMINB
#> Number of model parameters
                                                   31
   Number of equality constraints
#>
                                                   12
#>
#>
   Number of observations
                                                  204
#>
   Number of missing patterns
                                                   1
#>
#> Model Test User Model:
#>
#>
   Test statistic
                                              133.220
#> Degrees of freedom
                                                   25
#> P-value (Chi-square)
                                                0.000
#>
#>
#> [[1]][[1]]$visible
#> [1] TRUE
```

# Environment

```
ls()
#> [1] "osbornesudick1972" "root" "tex_file"
```

## Class

```
#> [[1]]
#> [1] "data.frame"
#>
#> [[2]]
#> [1] "root_criterion"
#>
#> [[3]]
#> [1] "character"
```

### References

Pesigan, I. J. A., Sun, R. W., & Cheung, S. F. (2023). betaDelta and betaSandwich: Confidence intervals for standardized regression coefficients in R. *Multivariate Behavioral Research*, 1–4. https://doi.org/10.1080/00273171.2023.2201277

R Core Team. (2023). R: A language and environment for statistical computing. R Foundation for Statistical Computing. Vienna, Austria. https://www.R-project.org/