longMI: Internal Tests

Ivan Jacob Agaloos Pesigan

Tests

```
#> test
#> Call:
#> Comparison(configural = configural_fit, weak = weak_fit, strong = strong_fit,
       strict = strict_fit)
                 chisq df pvalue
                                    cfi
                                          tli rmsea
                                                        srmr
                                                                  aic
#> configural 25.9682 19 0.1311 0.9915 0.9875 0.0424 0.0306 11252.23 11335.18
              41.8973 22 0.0064 0.9757 0.9691 0.0666 0.0763 11262.16 11335.16
#> strong
             53.7228 25 0.0007 0.9650 0.9608 0.0750 0.0872 11267.98 11331.03
#> strict
            134.5591 29 0.0000 0.8712 0.8757 0.1336 0.1690 11340.82 11390.59
#> Call:
#> Comparison(configural = configural_fit, weak = weak_fit, strong = strong_fit,
      strict = strict_fit)
#>
#>
#> CONFIGURAL INVARIANCE MODEL
#> lavaan 0.6.16 ended normally after 75 iterations
#>
#>
    Estimator
                                                       ML
    Optimization method
#>
                                                   NLMINB
    Number of model parameters
#>
                                                       27
    Number of equality constraints
                                                        2
#>
#>
#>
    Number of observations
                                                      204
#>
    Number of missing patterns
                                                        1
#> Model Test User Model:
#>
#>
    Test statistic
                                                   25.968
#>
     Degrees of freedom
                                                       19
    P-value (Chi-square)
                                                    0.131
#>
#> Parameter Estimates:
```

```
#>
#>
    Standard errors
                                            Standard
#>
    Information
                                           Observed
#>
    Observed information based on
                                            Hessian
#>
#> Latent Variables:
#>
                   Estimate Std.Err z-value P(>|z|)
#>
   f1t1 =~
#>
    y1_1
            (111)
                     4.451
                              0.400
                                     11.137
                                              0.000
     y1_2
#>
                      6.850
                            0.637
                                     10.745
                                            0.000
#>
     y1_3
                      4.590
                            0.520
                                     8.821
                                            0.000
                      5.039
                            0.396
                                     12.728
                                            0.000
#>
      y1_4
#>
   f1t6 =~
             (111) 4.451
#>
   y6_1
                            0.400
                                    11.137
                                             0.000
                      4.006
                            0.489
                                            0.000
#>
     y6_2
                                    8.194
#>
      y6_3
                      4.551
                              0.545
                                     8.346
                                              0.000
#>
     y6_4
                      4.102
                              0.453
                                      9.057
                                              0.000
#>
#> Covariances:
                   Estimate Std.Err z-value P(>|z|)
#>
   f1t1 ~~
#>
#>
   f1t6
                     1.837
                              0.215
                                      8.558
                                              0.000
#>
#> Intercepts:
                   Estimate Std.Err z-value P(>|z|)
#>
   .y1_1
#>
               (i1) 19.776
                            0.427
                                     46.273
                                            0.000
#>
                     21.797
                             0.680
                                     32.036
                                            0.000
     .y1_2
#>
    .y1_3
                     14.903
                             0.528
                                     28.223
                                              0.000
                     20.396
                            0.439 46.416
                                            0.000
#>
     .y1_4
               (i1) 19.776
                            0.427
                                     46.273
                                            0.000
#>
     .y6_1
                             2.299
                                    8.404
                                            0.000
#>
     .y6_2
                     19.317
#>
     .y6_3
                     11.922
                            2.516 4.738
                                            0.000
#>
     .y6_4
                     17.970
                            1.844 9.747
                                            0.000
#>
     f1t1
                     0.000
                     6.455
#>
     f1t6
                             0.606 10.649
                                              0.000
#>
#> Variances:
#>
                   Estimate Std.Err z-value P(>|z|)
#>
    .y1_1
                     17.448
                             2.240
                                    7.789
                                             0.000
                     47.511
                            5.754 8.257
                                             0.000
#>
     .y1_2
                            4.031 8.884
#>
     .y1_3
                     35.810
                                            0.000
#>
                             2.133 6.563
                                            0.000
     .y1_4
                     13.999
#>
     .y6_1
                     47.096
                             6.432
                                      7.322
                                              0.000
                             8.388 8.805
#>
                     73.850
                                            0.000
     .y6_2
#>
     .y6_3
                     88.920
                            10.354
                                    8.588
                                            0.000
                     23.267 4.182
                                   5.564 0.000
#>
     .y6_4
```

```
#>
    f1t1
                       1.000
      f1t6
#>
                       5.834
                               1.167
                                       4.997
                                                0.000
#>
#>
#>
#> WEAK INVARIANCE MODEL
#> lavaan 0.6.16 ended normally after 54 iterations
#>
    Estimator
                                                   ML
#>
    Optimization method
                                               NLMINB
    Number of model parameters
                                                   27
#>
    Number of equality constraints
#>
                                                   5
#>
                                                  204
#>
    Number of observations
#>
    Number of missing patterns
                                                    1
#>
#> Model Test User Model:
#>
#>
    Test statistic
                                               41.897
#>
    Degrees of freedom
                                                   22
    P-value (Chi-square)
                                                0.006
#>
#> Parameter Estimates:
#>
#>
   Standard errors
                                              Standard
                                              Observed
#>
   Information
#>
    Observed information based on
                                              Hessian
#>
#> Latent Variables:
#>
                    Estimate Std.Err z-value P(>|z|)
#>
   f1t1 =~
                     4.933
                              0.339 14.562
                                              0.000
#>
    y1_1
             (111)
                             0.429 12.052
#>
     y1_2
              (112)
                    5.172
                                              0.000
                     5.072
                               0.397
                                       12.762
      y1_3
              (113)
                                                0.000
#>
                     4.865
#>
     y1_4
             (114)
                             0.336
                                       14.492
                                                0.000
#>
   f1t6 =~
#>
              (111)
                      4.933
                               0.339
                                       14.562
                                                0.000
      y6_1
#>
      y6_2
               (112)
                       5.172
                               0.429
                                       12.052
                                                0.000
               (113)
                      5.072
                               0.397
                                       12.762
                                                0.000
#>
      y6_3
                       4.865
#>
      y6_4
               (114)
                               0.336
                                      14.492
                                                0.000
#>
#> Covariances:
                    Estimate Std.Err z-value P(>|z|)
#>
   f1t1 ~~
#>
                       1.558 0.136 11.439 0.000
#> f1t6
```

```
#> Intercepts:
                  Estimate Std.Err z-value P(>|z|)
#>
#>
              (i1) 19.776 0.445 44.430
                                             0.000
     .y1_1
#>
    .y1_2
                    21.797
                             0.629 34.633
                                             0.000
#>
    .y1_3
                    14.903
                           0.544 27.411
                                           0.000
     .y1_4
#>
                    20.396   0.436   46.803   0.000
              (i1) 19.776
                           0.445 44.430
                                           0.000
#>
     .y6_1
                                   6.547
                                           0.000
#>
    .y6_2
                    15.049
                            2.299
#>
    .y6_3
                    11.756
                            2.212 5.315 0.000
#>
     .y6_4
                    16.111
                            1.765 9.130 0.000
#>
     f1t1
                     0.000
#>
     f1t6
                     5.824
                             0.429 13.591
                                           0.000
#>
#> Variances:
#>
                  Estimate Std.Err z-value P(>|z|)
#>
    .y1_1
                   16.079
                           2.188 7.349
                                            0.000
                           6.012 8.992
#>
                    54.055
                                           0.000
    .y1_2
                    34.578
                           3.941 8.775
#>
    .y1_3
                                           0.000
                            2.109 7.149
                    15.075
                                           0.000
#>
    .y1_4
#>
    .y6_1
                    49.748
                            6.494 7.661 0.000
#>
    .y6_2
                   72.254 8.353 8.650 0.000
                   91.610 10.487 8.736 0.000
#>
     .y6_3
                            4.032 5.462 0.000
#>
                    22.022
    .y6_4
                    1.000
#>
     f1t1
#>
    f1t6
                    4.240
                            0.546 7.759
                                           0.000
#>
#>
#> STRONG INVARIANCE MODEL
#> lavaan 0.6.16 ended normally after 52 iterations
#>
#>
   Estimator
                                                ML
                                            NLMINB
    Optimization method
#>
    Number of model parameters
#>
                                                27
#>
    Number of equality constraints
                                                8
#>
#>
    Number of observations
                                               204
#>
    Number of missing patterns
                                                 1
#>
#> Model Test User Model:
#>
    Test statistic
                                             53.723
#>
    Degrees of freedom
                                                25
    P-value (Chi-square)
                                             0.001
#>
```

```
#> Parameter Estimates:
#>
#>
    Standard errors
                                           Standard
#>
    Information
                                           Observed
#>
    Observed information based on
                                           Hessian
#>
#> Latent Variables:
#>
                  Estimate Std.Err z-value P(>|z|)
#>
   f1t1 =~
#>
    y1_1
            (111)
                    5.270
                            0.331
                                    15.917
                                            0.000
           (112)
     y1_2
                    4.525
                            0.312 14.487
                                           0.000
#>
           (113)
                    4.960
                            0.328
                                    15.112
                                           0.000
#>
    y1_3
#>
            (114)
                   4.547
                            0.291
                                    15.634
                                           0.000
     y1_4
   f1t6 =~
#>
#>
    y6_1
              (111)
                    5.270
                             0.331
                                    15.917
                                            0.000
#>
     y6_2
             (112)
                     4.525
                             0.312
                                    14.487
                                            0.000
     y6_3
             (113)
                   4.960
                            0.328 15.112
                                           0.000
#>
#>
     y6_4
              (114)
                     4.547
                             0.291
                                    15.634
                                             0.000
#>
#> Covariances:
#>
                  Estimate Std.Err z-value P(>|z|)
   f1t1 ~~
#>
#>
     f1t6
                     1.608
                             0.142
                                   11.316
                                             0.000
#>
#> Intercepts:
#>
                   Estimate Std.Err z-value P(>|z|)
#>
    .y1_1
              (i1)
                   19.929
                            0.457
                                    43.623
                                            0.000
                   21.459
                             0.598
                                    35.901
                                             0.000
#>
    .y1_2
              (i2)
              (i3)
                   14.882
                           0.529 28.146
                                           0.000
#>
     .y1_3
     .y1_4
#>
               (i4)
                   20.311
                            0.421
                                    48.252
                                            0.000
#>
     .y6_1
              (i1)
                    19.929
                            0.457
                                    43.623
                                           0.000
#>
    .y6_2
              (i2)
                    21.459
                            0.598 35.901
                                           0.000
                    14.882
#>
     .y6_3
              (i3)
                            0.529
                                    28.146 0.000
#>
               (i4)
                     20.311
                            0.421
                                    48.252
                                           0.000
     .y6_4
                     0.000
#>
     f1t1
#>
     f1t6
                     5.337
                            0.354
                                    15.077
                                            0.000
#>
#> Variances:
                  Estimate Std.Err z-value P(>|z|)
#>
#>
    .y1_1
                    15.124
                            2.238
                                   6.757
                                            0.000
                                   9.402
                                             0.000
#>
     .y1_2
                    57.819
                            6.150
#>
    .y1_3
                    34.209
                             3.869
                                     8.841
                                             0.000
#>
                    16.339
                            2.114
                                    7.729
                                           0.000
     .y1_4
#>
     .y6_1
                     45.360
                             6.338
                                     7.157
                                           0.000
                    74.229 8.352 8.887 0.000
#>
     .y6_2
```

```
#>
     .y6_3
                     89.572 10.110 8.860 0.000
                     24.586 3.951 6.223
#>
     .y6_4
                                               0.000
     f1t1
                      1.000
#>
#>
      f1t6
                      4.557 0.585 7.791 0.000
#>
#>
#>
#> STRICT INVARIANCE MODEL
#> lavaan 0.6.16 ended normally after 57 iterations
#>
   Estimator
                                                  ML
    Optimization method
                                               NLMINB
#>
    Number of model parameters
                                                  27
#>
    Number of equality constraints
#>
                                                  12
#>
#>
    Number of observations
                                                 204
#>
    Number of missing patterns
                                                   1
#>
#> Model Test User Model:
#>
#>
   Test statistic
                                              134.559
    Degrees of freedom
                                                  29
#>
#>
    P-value (Chi-square)
                                               0.000
#>
#> Parameter Estimates:
#>
#>
    Standard errors
                                             Standard
   Information
                                             Observed
#>
   Observed information based on
                                             Hessian
#>
#> Latent Variables:
                   Estimate Std.Err z-value P(>|z|)
#>
#>
   f1t1 =~
            (111) 5.083
    y1_1
                               0.364 13.978
                                               0.000
#>
     y1_2 (112) 4.309 0.333 12.944 0.000
#>
#>
     y1_3
              (113) 4.785 0.356 13.447 0.000
#>
     y1_4
             (114) 4.358 0.317 13.765 0.000
   f1t6 =~
#>
    y6_1 (111) 5.083 0.364 13.978 0.000
y6_2 (112) 4.309 0.333 12.944 0.000
#>
#>
     y6_3
#>
              (113) 4.785
                               0.356 13.447 0.000
#>
      y6_4
              (114)
                      4.358
                               0.317 13.765
                                               0.000
#>
#> Covariances:
                    Estimate Std.Err z-value P(>|z|)
```

```
#> f1t1 ~~
#>
     f1t6
                      1.812
                              0.168
                                     10.812
                                              0.000
#>
#> Intercepts:
#>
                   Estimate Std.Err z-value P(>|z|)
#>
               (i1)
                     20.019 0.509
                                    39.339
                                              0.000
     .y1_1
     .y1_2
               (i2) 21.513
                              0.621
                                     34.648
                                              0.000
#>
               (i3)
                    14.805
                             0.617
                                     23.988
                                              0.000
#>
     .y1_3
     .y1_4
#>
               (i4)
                    20.313
                             0.433 46.915
                                              0.000
#>
     .y6_1
               (i1)
                    20.019
                            0.509 39.339
                                            0.000
#>
     .y6_2
               (i2)
                    21.513
                            0.621 34.648 0.000
               (i3)
                    14.805
                            0.617 23.988 0.000
#>
     .y6_3
               (i4)
                     20.313
                            0.433 46.915
                                            0.000
#>
     .y6_4
#>
                     0.000
     f1t1
#>
     f1t6
                     5.557 0.415 13.396 0.000
#>
#> Variances:
                   Estimate Std.Err z-value P(>|z|)
#>
#>
              (u1)
                    28.657
                            2.958
                                    9.689
                                              0.000
     .y1_1
                              5.253 12.946
#>
     .y1_2
               (u2)
                     68.013
                                              0.000
#>
     .y1_3
               (u3)
                    61.387
                            4.897 12.535
                                              0.000
#>
     .y1_4
               (u4) 20.878 2.142 9.746
                                            0.000
                            2.958 9.689
#>
     .y6_1
               (u1) 28.657
                                            0.000
     .y6_2
#>
               (u2)
                    68.013
                            5.253 12.946
                                             0.000
                            4.897 12.535
#>
     .y6_3
               (u3)
                    61.387
                                            0.000
#>
     .y6_4
               (u4) 20.878
                            2.142 9.746 0.000
#>
     f1t1
                     1.000
#>
     f1t6
                      5.056
                              0.692
                                     7.304
                                              0.000
#>
#> Call:
#> Comparison(configural = configural_fit, weak = weak_fit, strong = strong_fit,
      strict = strict_fit)
#>
#> Chi-Squared Difference Test
#>
                            Chisq Chisq diff RMSEA Df diff Pr(>Chisq)
#>
              Df
                 AIC BIC
#> 1.configural 19 11252 11335 25.968
             22 11262 11335 41.897
                                     15.929 0.14535
                                                        3 0.0011726 **
#> 1.weak
#> 2.configural 19 11252 11335 25.968
             25 11268 11331 53.723
                                                        6 0.0001045 ***
#> 2.strong
                                     27.755 0.13332
#> 3.configural 19 11252 11335 25.968
#> 3.strict
             29 11341 11391 134.559
                                   108.591 0.21984
                                                        10 < 2.2e-16 ***
#> 4.weak
              22 11262 11335 41.897
                                                         3 0.0080053 **
#> 4.strong
              25 11268 11331 53.723
                                     11.826 0.12009
#> 5.weak
             22 11262 11335 41.897
#> 5.strict 29 11341 11391 134.559
                                   92.662 0.24492 7 < 2.2e-16 ***
```

```
#> 6.strong 25 11268 11331 53.723
#> 6.strict
             29 11341 11391 134.559
                                        80.836 0.30686
                                                          4 < 2.2e-16 ***
#> ---
#> Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
#> Call:
#> Invariance(data = osbornesudick1972, time_points = time_points,
#> factor_loadings = factor_loadings)
#>
#>
               chisq df pvalue
                                cfi tli rmsea srmr
                                                             aic bic
#> configural 25.9682 19 0.1311 0.9915 0.9875 0.0424 0.0306 11252.23 11335.18
             41.8973 22 0.0064 0.9757 0.9691 0.0666 0.0720 11262.16 11335.16
            53.7228 25 0.0007 0.9650 0.9608 0.0750 0.0859 11267.98 11331.03
#> strong
            134.5591 29 0.0000 0.8712 0.8757 0.1336 0.1328 11340.82 11390.59
#> strict
#> Call:
#> Invariance(data = osbornesudick1972, time_points = time_points,
#> factor_loadings = factor_loadings)
#>
#>
#> CONFIGURAL INVARIANCE MODEL
#> lavaan 0.6.16 ended normally after 75 iterations
#>
#> Estimator
                                                    ML
#>
   Optimization method
                                                NLMINB
   Number of model parameters
#>
                                                    27
#>
    Number of equality constraints
                                                    2
#>
#>
    Number of observations
                                                   204
#>
#> Model Test User Model:
#>
#>
   Test statistic
                                                25.968
#>
    Degrees of freedom
                                                    19
    P-value (Chi-square)
                                                 0.131
#>
#>
#> Parameter Estimates:
#>
#>
   Standard errors
                                              Standard
#>
    Information
                                              Expected
   Information saturated (h1) model
#>
                                           Structured
#>
#> Latent Variables:
#>
                    Estimate Std.Err z-value P(>|z|)
   f1t1 =~
#>
#>
      y1_1
               (111)
                       4.451
                                0.396 11.244
                                                 0.000
                       6.850 0.637 10.750 0.000
      y1_2
#>
```

```
#>
   y1_3
                      4.590 0.515 8.918 0.000
#>
      y1_4
                      5.039
                               0.393
                                      12.809
                                               0.000
   f1t6 =~
#>
              (111)
                      4.451
                              0.396
                                     11.244
                                             0.000
#>
      y6_1
#>
     y6_2
                      4.006
                               0.485
                                      8.261
                                               0.000
#>
                      4.551
                               0.546 8.342 0.000
     y6_3
      y6_4
#>
                      4.102
                               0.448
                                       9.150
                                             0.000
#>
#> Covariances:
#>
                   Estimate Std.Err z-value P(>|z|)
#>
    f1t1 ~~
#>
     f1t6
                               0.211
                                       8.703
                      1.837
                                               0.000
#>
#> Intercepts:
#>
                    Estimate Std.Err z-value P(>|z|)
#>
     .y1_1
               (i1) 19.776
                               0.427
                                     46.273
                                               0.000
#>
     .y1_2
                     21.797
                               0.680
                                      32.036
                                               0.000
                     14.903
                             0.528
                                      28.223
                                             0.000
#>
     .y1_3
#>
     .y1_4
                     20.396
                             0.439
                                      46.416
                                             0.000
                     19.776
                                      46.273
#>
     .y6_1
               (i1)
                              0.427
                                               0.000
#>
     .y6_2
                     19.317
                              2.281
                                    8.467
                                             0.000
#>
     .y6_3
                     11.922
                              2.538 4.697
                                             0.000
#>
                     17.970
                              1.815 9.903
                                             0.000
     .y6_4
#>
      f1t1
                      0.000
      f1t6
                      6.455
                              0.601 10.743
                                               0.000
#>
#>
#> Variances:
#>
                   Estimate Std.Err z-value P(>|z|)
                             2.186
                                     7.981
                                               0.000
#>
     .y1_1
                    17.448
                     47.511
                               5.748
                                       8.266
                                               0.000
#>
     .y1_2
     .y1_3
#>
                     35.810
                               3.969
                                       9.022
                                               0.000
#>
     .y1_4
                     13.999
                              2.085
                                       6.712
                                             0.000
#>
     .y6_1
                     47.096
                             6.305 7.470
                                             0.000
                             8.395 8.797
#>
     .y6_2
                     73.850
                                             0.000
                             10.222 8.699 0.000
                     88.920
#>
     .y6_3
#>
     .y6_4
                     23.267
                             4.076 5.709 0.000
#>
     f1t1
                     1.000
#>
     f1t6
                     5.834
                             1.159 5.035 0.000
#>
#>
#>
#> WEAK INVARIANCE MODEL
#> lavaan 0.6.16 ended normally after 54 iterations
#>
#>
                                                  ML
   Estimator
```

```
#>
    Optimization method
                                             NLMINB
    Number of model parameters
#>
                                                 27
    Number of equality constraints
#>
                                                 5
#>
#>
    Number of observations
                                                204
#>
#> Model Test User Model:
#>
#>
    Test statistic
                                             41.897
    Degrees of freedom
#>
                                                 22
    P-value (Chi-square)
                                              0.006
#>
#> Parameter Estimates:
#>
   Standard errors
#>
                                           Standard
#>
    Information
                                           Expected
#>
    Information saturated (h1) model
                                        Structured
#>
#> Latent Variables:
                   Estimate Std.Err z-value P(>|z|)
#>
   f1t1 =~
#>
#>
   y1_1
            (111)
                     4.933
                              0.342
                                     14.420
                                              0.000
     y1_2 (112) 5.172
                            0.413 12.534
                                            0.000
#>
    y1_3
                    5.072
#>
              (113)
                              0.398
                                     12.733
                                              0.000
              (114) 4.865
                            0.330
                                     14.733
                                            0.000
#>
     y1_4
   f1t6 =~
#>
            (111)
(112)
#>
     y6_1
                    4.933
                            0.342
                                     14.420
                                            0.000
#>
    y6_2
                     5.172
                              0.413
                                     12.534
                                              0.000
#>
              (113)
                     5.072
                              0.398
                                     12.733 0.000
     y6_3
#>
              (114)
                      4.865
                              0.330
                                     14.733
                                            0.000
      y6_4
#>
#> Covariances:
                   Estimate Std.Err z-value P(>|z|)
#>
    f1t1 ~~
#>
     f1t6
#>
                     1.558
                              0.136 11.482
                                              0.000
#>
#> Intercepts:
#>
                   Estimate Std.Err z-value P(>|z|)
    .y1_1
#>
               (i1) 19.776
                            0.445
                                    44.430
                                             0.000
                     21.797
                            0.629 34.633
                                            0.000
#>
     .y1_2
                     14.903
                            0.544
#>
     .y1_3
                                     27.411
                                            0.000
#>
                     20.396
                            0.436
                                     46.803
                                            0.000
     .y1_4
#>
     .y6_1
               (i1) 19.776
                              0.445 44.430
                                              0.000
#>
                     15.049
                              2.223 6.770 0.000
     .y6_2
#>
     .y6_3
                     11.756
                              2.238
                                      5.253
                                              0.000
                     16.111 1.699 9.484 0.000
#>
     .y6_4
```

```
#>
   f1t1
                      0.000
      f1t6
#>
                      5.824
                               0.432 13.476
                                               0.000
#>
#> Variances:
                   Estimate Std.Err z-value P(>|z|)
#>
#>
     .y1_1
                    16.079 2.165 7.427
                                               0.000
#>
     .y1_2
                    54.055
                             5.831
                                      9.270
                                               0.000
                    34.578
                             3.922 8.817
                                              0.000
#>
     .y1_3
#>
     .y1_4
                     15.075
                              2.051
                                      7.350
                                             0.000
                     49.748
                             6.242 7.970 0.000
#>
     .y6_1
#>
     .y6_2
                     72.254
                             8.498 8.503 0.000
                    91.610
                            10.272 8.919 0.000
#>
     .y6_3
#>
                     22.022
                             3.989 5.521 0.000
     .y6_4
                     1.000
#>
     f1t1
                            0.539 7.867 0.000
                     4.240
#>
    f1t6
#>
#>
#>
#> STRONG INVARIANCE MODEL
#> lavaan 0.6.16 ended normally after 52 iterations
#>
#>
   Estimator
                                                 ML
#>
    Optimization method
                                              NLMINB
    Number of model parameters
#>
                                                 27
#>
    Number of equality constraints
#>
#>
    Number of observations
                                                 204
#>
#> Model Test User Model:
#>
#>
   Test statistic
                                              53.723
    Degrees of freedom
#>
                                                  25
#>
    P-value (Chi-square)
                                               0.001
#>
#> Parameter Estimates:
#>
#>
   Standard errors
                                            Standard
#>
    Information
                                            Expected
#>
   Information saturated (h1) model
                                         Structured
#>
#> Latent Variables:
#>
                    Estimate Std.Err z-value P(>|z|)
    f1t1 =~
#>
#>
      y1_1
              (111)
                      5.270
                               0.333
                                    15.813
                                               0.000
      y1_2 (112)
                      4.525
                              0.308 14.681
#>
                                            0.000
```

```
#>
   y1_3 (113) 4.960 0.328 15.120 0.000
             (114)
                     4.547
                             0.289
                                   15.707
                                            0.000
#>
     y1_4
   f1t6 =~
#>
#>
     y6_1
             (111)
                   5.270
                             0.333
                                   15.813
                                          0.000
#>
    y6_2
             (112)
                    4.525
                           0.308 14.681 0.000
             (113)
                    4.960 0.328 15.120 0.000
#>
     y6_3
#>
             (114)
                     4.547
                            0.289
                                   15.707 0.000
     y6_4
#>
#> Covariances:
#>
                  Estimate Std.Err z-value P(>|z|)
#>
   f1t1 ~~
#>
    f1t6
                    1.608
                             0.143
                                  11.281
                                            0.000
#>
#> Intercepts:
#>
                  Estimate Std.Err z-value P(>|z|)
#>
    .y1_1
              (i1) 19.929
                           0.456
                                   43.727
                                            0.000
    .y1_2
              (i2) 21.459
                           0.598
                                   35.868
                                          0.000
#>
#>
    .y1_3
              (i3)
                   14.882
                           0.529
                                   28.153
                                          0.000
              (i4)
                    20.311
                           0.421
                                   48.237
                                          0.000
#>
    .y1_4
#>
    .y6_1
              (i1)
                    19.929
                           0.456 43.727
                                          0.000
#>
    .y6_2
              (i2) 21.459 0.598 35.868 0.000
              (i3)
#>
     .y6_3
                   14.882 0.529 28.153 0.000
              (i4) 20.311
                           0.421
                                   48.237
                                          0.000
#>
    .y6_4
#>
     f1t1
                     0.000
#>
     f1t6
                     5.337
                           0.353 15.109
                                          0.000
#>
#> Variances:
#>
                  Estimate Std.Err z-value P(>|z|)
#>
    .y1_1
                   15.124
                           2.193
                                  6.895
                                           0.000
                   57.819
                             6.078
                                    9.512
                                            0.000
#>
     .y1_2
#>
    .y1_3
                    34.209
                            3.850
                                    8.885
                                            0.000
#>
    .y1_4
                   16.339
                           2.062 7.925
                                          0.000
#>
    .y6_1
                    45.360
                           6.182 7.337
                                          0.000
                           8.350 8.890
                    74.229
                                          0.000
#>
    .y6_2
                           10.064 8.900 0.000
#>
                    89.572
    .y6_3
#>
    .y6_4
                   24.586 3.878 6.339 0.000
#>
                    1.000
     f1t1
                    4.557 0.584 7.806 0.000
#>
     f1t6
#>
#>
#>
#> STRICT INVARIANCE MODEL
#> lavaan 0.6.16 ended normally after 57 iterations
#>
```

```
#> Estimator
                                                        ML
#>
    Optimization method
                                                    NLMINB
    Number of model parameters
#>
                                                        27
#>
    Number of equality constraints
                                                        12
#>
#>
    Number of observations
                                                        204
#>
#> Model Test User Model:
#>
                                                   134.559
#>
    Test statistic
#>
    Degrees of freedom
                                                       29
    P-value (Chi-square)
                                                     0.000
#>
#>
#> Parameter Estimates:
#>
#>
   Standard errors
                                                   Standard
    Information
                                                   Expected
#>
    Information saturated (h1) model
                                                Structured
#>
#> Latent Variables:
#>
                      Estimate Std.Err z-value P(>|z|)
#>
    f1t1 =~

    y1_1
    (111)
    5.083
    0.364
    13.968
    0.000

    y1_2
    (112)
    4.309
    0.325
    13.248
    0.000

    y1_3
    (113)
    4.785
    0.354
    13.521
    0.000

    y1_4
    (114)
    4.358
    0.312
    13.971
    0.000

#>
#>
#>
#>
   f1t6 =~
#>
#>
    y6_1
              (111) 5.083
                                   0.364
                                           13.968
                                                   0.000
                (112) 4.309 0.325 13.248 0.000
#>
     y6_2
                (113) 4.785
                                   0.354 13.521 0.000
#>
     y6_3
#>
       y6_4
                (114)
                         4.358
                                   0.312 13.971
                                                   0.000
#>
#> Covariances:
#>
                      Estimate Std.Err z-value P(>|z|)
   f1t1 ~~
#>
                         1.812
                                   0.168 10.790
#>
    f1t6
                                                      0.000
#>
#> Intercepts:
                     Estimate Std.Err z-value P(>|z|)
#>
    .y1_1
.y1_2
                 (i1) 20.019 0.506 39.547
                                                   0.000
#>
                                                   0.000
#>
                 (i2) 21.513 0.621
                                           34.617
#>
     .y1_3
                 (i3) 14.805
                                 0.617 24.001
                                                   0.000
                                 0.433 46.903
#>
     .y1_4
                 (i4) 20.313
                                                   0.000
#>
                 (i1) 20.019 0.506 39.547 0.000
     .y6_1
#>
      .y6_2
                 (i2) 21.513 0.621
                                           34.617 0.000
                 (i3) 14.805 0.617 24.001 0.000
#>
     .y6_3
```

```
.y6_4 (i4) 20.313 0.433 46.903 0.000
      f1t1
                       0.000
#>
#>
      f1t6
                       5.557
                                                0.000
                             0.411
                                      13.526
#>
#> Variances:
                    Estimate Std.Err z-value P(>|z|)
#>
#>
     .y1_1
               (u1) 28.657 2.747 10.433
                                               0.000
     .y1_2
                             5.170 13.155
                                               0.000
#>
               (u2)
                      68.013
#>
     .y1_3
               (u3)
                    61.387
                             4.821 12.734
                                              0.000
#>
     .y1_4
               (u4)
                     20.878
                             2.008 10.398 0.000
#>
     .y6_1
               (u1)
                     28.657
                             2.747 10.433
                                             0.000
#>
     .y6_2
               (u2)
                     68.013
                             5.170 13.155
                                              0.000
               (u3) 61.387 4.821 12.734 0.000
#>
     .y6_3
#>
     .y6_4
               (u4) 20.878 2.008 10.398 0.000
#>
      f1t1
                      1.000
#>
      f1t6
                       5.056
                               0.685 7.378
                                             0.000
#>
#> Call:
#> Invariance(data = osbornesudick1972, time_points = time_points,
      factor_loadings = factor_loadings)
#>
#> Chi-Squared Difference Test
#>
              Df AIC BIC
                             Chisq Chisq diff RMSEA Df diff Pr(>Chisq)
#> 1.configural 19 11252 11335 25.968
              22 11262 11335 41.897
                                      15.929 0.14535
                                                          3 0.0011726 **
#> 1.weak
#> 2.configural 19 11252 11335 25.968
#> 2.strong
            25 11268 11331 53.723
                                    27.755 0.13332 6 0.0001045 ***
#> 3.configural 19 11252 11335 25.968
#> 3.strict 29 11341 11391 134.559
                                     108.591 0.21984
                                                        10 < 2.2e-16 ***
#> 4.weak
              22 11262 11335 41.897
#> 4.strong 25 11268 11331 53.723
                                      11.826 0.12009
                                                          3 0.0080053 **
#> 5.weak 22 11262 11335 41.897
#> 5.strict 29 11341 11391 134.559
#> 6.strong 25 11268 11331 53.723
                                    92.662 0.24492
                                                           7 < 2.2e-16 ***
#> 6.strict
             29 11341 11391 134.559 80.836 0.30686
                                                          4 < 2.2e-16 ***
#> ---
#> Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
#> Test passed
#> [[1]]
```

```
#> [[1]][[1]]
#> [[1]][[1]]$value
#> [[1]][[1]]$value[[1]]
#> lavaan 0.6.16 ended normally after 90 iterations
#>
#> Estimator
                                                    ML
#> Optimization method
                                                NLMINB
#> Number of model parameters
                                                    31
#> Number of equality constraints
                                                   12
#>
#>
   Number of observations
                                                   204
#>
   Number of missing patterns
                                                    1
#>
#> Model Test User Model:
#>
#> Test statistic
                                               133.220
#> Degrees of freedom
                                                    25
#> P-value (Chi-square)
                                                 0.000
#>
#>
#> [[1]][[1]]$visible
#> [1] TRUE
```

Environment

```
ls()
#> [1] "osbornesudick1972" "root" "tex_file"
```

Class

```
#> [[1]]
#> [1] "data.frame"
#>
#> [[2]]
#> [1] "root_criterion"
#>
#> [[3]]
#> [1] "character"
```

References

Pesigan, I. J. A., Sun, R. W., & Cheung, S. F. (2023). betaDelta and betaSandwich: Confidence intervals for standardized regression coefficients in R. *Multivariate Behavioral Research*, 1–4. https://doi.org/10.1080/00273171.2023.2201277

R Core Team. (2023). R: A language and environment for statistical computing. R Foundation for Statistical Computing. Vienna, Austria. https://www.R-project.org/