## longMI: Internal Tests

### Ivan Jacob Agaloos Pesigan

#### Tests

```
#> test
#> Call:
#> Comparison(configural = configural_fit, weak = weak_fit, strong = strong_fit,
      strict = strict_fit)
#>
                chisq df pvalue
                                    cfi
                                          tli rmsea
                                                        srmr
                                                                  aic
#> configural 25.9682 19 0.1311 0.9915 0.9875 0.0424 0.0306 11252.23 11335.18
              41.8973 22 0.0064 0.9757 0.9691 0.0666 0.0763 11262.16 11335.16
              53.7228 25 0.0007 0.9650 0.9608 0.0750 0.0872 11267.98 11331.03
#> strong
             134.5591 29 0.0000 0.8712 0.8757 0.1336 0.1690 11340.82 11390.59
#> strict
#> Call:
#> Comparison(configural = configural_fit, weak = weak_fit, strong = strong_fit,
      strict = strict_fit)
#>
#> Fit Measures
                chisq df pvalue
                                    cfi
                                          tli rmsea
                                                        srmr
                                                                  aic
                                                                           bic
#> configural 25.9682 19 0.1311 0.9915 0.9875 0.0424 0.0306 11252.23 11335.18
              41.8973 22 0.0064 0.9757 0.9691 0.0666 0.0763 11262.16 11335.16
#> strong
              53.7228 25 0.0007 0.9650 0.9608 0.0750 0.0872 11267.98 11331.03
            134.5591 29 0.0000 0.8712 0.8757 0.1336 0.1690 11340.82 11390.59
#> strict
#>
#>
#> CONFIGURAL INVARIANCE MODEL
#> lavaan 0.6.16 ended normally after 75 iterations
#>
    Estimator
#>
                                                       ML
#>
    Optimization method
                                                   NLMINB
    Number of model parameters
                                                       27
#>
    Number of equality constraints
                                                        2
#>
#>
    Number of observations
                                                      204
    Number of missing patterns
```

```
#> Model Test User Model:
#>
#> Test statistic
                                          25.968
#>
   Degrees of freedom
                                             19
#>
   P-value (Chi-square)
                                           0.131
#>
#> Parameter Estimates:
#>
   Standard errors
#>
                                        Standard
#> Information
                                        Observed
#>
   Observed information based on
                                         Hessian
#>
#> Latent Variables:
                 Estimate Std.Err z-value P(>|z|)
#>
#>
   f1t1 =~
#>
   y1_1 (111) 4.451 0.400 11.137
                                         0.000
    y1_2
                    6.850 0.637 10.745 0.000
#>
                    4.590 0.520 8.821 0.000
#>
     y1_3
                    5.039 0.396 12.728 0.000
#>
    y1_4
#>
   f1t6 =~
#>
   y6_1
           (111) 4.451 0.400 11.137 0.000
                    4.006 0.489 8.194 0.000
#>
     y6_2
                           0.545 8.346 0.000
#>
    y6_3
                    4.551
                    4.102 0.453 9.057 0.000
#>
     y6_4
#>
#> Covariances:
#>
                 Estimate Std.Err z-value P(>|z|)
   f1t1 ~~
#>
#>
    f1t6
                   1.837 0.215 8.558 0.000
#>
#> Intercepts:
                 Estimate Std.Err z-value P(>|z|)
#>
   .y1_1
#>
             (i1) 19.776 0.427 46.273 0.000
                   21.797
                          0.680 32.036 0.000
#>
    .y1_2
                   14.903 0.528 28.223 0.000
#>
   .y1_3
#>
    .y1_4
                   20.396  0.439  46.416  0.000
#>
             (i1) 19.776 0.427 46.273 0.000
    .y6_1
                                 8.404 0.000
#>
    .y6_2
                   19.317
                          2.299
                   11.922 2.516 4.738 0.000
#>
    .y6_3
#>
    .y6_4
                   17.970 1.844 9.747 0.000
     f1t1
#>
                    0.000
#>
     f1t6
                    6.455
                            0.606 10.649
                                         0.000
#>
#> Variances:
                  Estimate Std.Err z-value P(>|z|)
#>
```

```
#>
     .y1_1
                      17.448 2.240 7.789 0.000
                       47.511 5.754 8.257 0.000
#>
     .y1_2
                      35.810 4.031 8.884 0.000
#>
     .y1_3
                      13.999 2.133 6.563 0.000
#>
     .y1_4
                               6.432 7.322 0.000
#>
     .y6_1
                      47.096
                      73.850 8.388 8.805 0.000
#>
     .y6_2
#>
     .y6_3
                      88.920 10.354 8.588 0.000
                      23.267 4.182 5.564 0.000
#>
     .y6_4
                       1.000
#>
      f1t1
     f1t6
                       5.834 1.167 4.997 0.000
#>
#>
#>
#> WEAK INVARIANCE MODEL
#> lavaan 0.6.16 ended normally after 54 iterations
#>
#>
    Estimator
                                                      ML
    Optimization method
#>
                                                  NLMINB
    Number of model parameters
                                                      27
#>
#>
    Number of equality constraints
                                                      5
#>
#>
   Number of observations
                                                     204
#>
    Number of missing patterns
                                                       1
#>
#> Model Test User Model:
#>
#>
    Test statistic
                                                  41.897
   Degrees of freedom
                                                      22
#>
   P-value (Chi-square)
                                                   0.006
#>
#> Parameter Estimates:
#>
   Standard errors
#>
                                                Standard
#> Information
                                                Observed
    Observed information based on
#>
                                                Hessian
#>
#> Latent Variables:
#>
                    Estimate Std.Err z-value P(>|z|)
   f1t1 =~
#>

    y1_1
    (111)
    4.933
    0.339
    14.562
    0.000

    y1_2
    (112)
    5.172
    0.429
    12.052
    0.000

    y1_3
    (113)
    5.072
    0.397
    12.762
    0.000

#>
     y1_1
#>
#>
     y1_3
              (114) 4.865 0.336 14.492 0.000
#>
     y1_4
#>
   f1t6 =~
#> y6_1 (111) 4.933 0.339 14.562 0.000
```

```
#>
      y6_2
              (112) 5.172
                              0.429 12.052 0.000
              (113)
                      5.072
                                               0.000
      y6_3
                              0.397
                                      12.762
#>
#>
      y6_4
              (114)
                      4.865
                              0.336
                                      14.492
                                               0.000
#>
#> Covariances:
#>
                   Estimate Std.Err z-value P(>|z|)
    f1t1 ~~
#>
      f1t6
                                      11.439
                                               0.000
#>
                      1.558
                              0.136
#>
#> Intercepts:
#>
                   Estimate Std.Err z-value P(>|z|)
                              0.445
                                     44.430
#>
     .y1_1
               (i1)
                     19.776
                                               0.000
                     21.797
                              0.629
                                      34.633
                                               0.000
#>
     .y1_2
#>
     .y1_3
                     14.903
                             0.544
                                     27.411
                                             0.000
#>
     .y1_4
                     20.396
                              0.436 46.803
                                             0.000
#>
     .y6_1
               (i1) 19.776
                              0.445 44.430
                                             0.000
                     15.049
                             2.299 6.547 0.000
#>
     .y6_2
#>
     .y6_3
                     11.756
                             2.212 5.315 0.000
                     16.111
#>
                             1.765 9.130 0.000
     .y6_4
#>
     f1t1
                      0.000
#>
      f1t6
                      5.824
                              0.429
                                     13.591
                                              0.000
#>
#> Variances:
                   Estimate Std.Err z-value P(>|z|)
#>
#>
    .y1_1
                    16.079
                            2.188 7.349
                                              0.000
#>
     .y1_2
                     54.055
                             6.012 8.992
                                               0.000
#>
     .y1_3
                     34.578
                             3.941
                                      8.775
                                               0.000
     .y1_4
#>
                     15.075
                            2.109 7.149
                                             0.000
                            6.494 7.661
                                            0.000
#>
     .y6_1
                     49.748
                     72.254
                             8.353 8.650
                                            0.000
#>
     .y6_2
                            10.487 8.736 0.000
#>
     .y6_3
                     91.610
                     22.022
#>
     .y6_4
                             4.032 5.462 0.000
#>
     f1t1
                     1.000
                      4.240
#>
     f1t6
                             0.546 7.759
                                              0.000
#>
#>
#>
#> STRONG INVARIANCE MODEL
#> lavaan 0.6.16 ended normally after 52 iterations
#>
#>
    Estimator
                                                 ML
    Optimization method
                                              NLMINB
#>
#>
    Number of model parameters
                                                 27
#>
    Number of equality constraints
                                                  8
```

```
#>
    Number of observations
                                                204
#>
#>
    Number of missing patterns
                                                  1
#>
#> Model Test User Model:
#>
#>
    Test statistic
                                             53.723
                                                 25
#>
    Degrees of freedom
    P-value (Chi-square)
                                              0.001
#>
#> Parameter Estimates:
#>
#>
    Standard errors
                                           Standard
#>
   Information
                                           Observed
#>
    Observed information based on
                                            Hessian
#>
#> Latent Variables:
                  Estimate Std.Err z-value P(>|z|)
#>
   f1t1 =~
#>
                                              0.000
            (111)
                     5.270
                              0.331
                                     15.917
#>
    y1_1
     y1_2 (112)
#>
                    4.525
                            0.312 14.487
                                            0.000
#>
     y1_3
              (113)
                    4.960 0.328 15.112
                                            0.000
                    4.547 0.291
                                            0.000
#>
     y1_4
              (114)
                                     15.634
   f1t6 =~
#>
     y6_1 (111)
y6_2 (112)
                    5.270
                            0.331
                                     15.917 0.000
#>
#>
                    4.525
                            0.312 14.487 0.000
#>
     y6_3
              (113)
                      4.960
                              0.328 15.112
                                            0.000
#>
     y6_4
              (114)
                      4.547
                              0.291
                                     15.634
                                              0.000
#>
#> Covariances:
#>
                   Estimate Std.Err z-value P(>|z|)
#>
   f1t1 ~~
     f1t6
#>
                     1.608
                              0.142
                                    11.316
                                            0.000
#>
#> Intercepts:
                   Estimate Std.Err z-value P(>|z|)
#>
#>
     .y1_1
               (i1) 19.929 0.457 43.623
                                            0.000
#>
     .y1_2
               (i2) 21.459
                            0.598 35.901
                                            0.000
#>
     .y1_3
               (i3)
                    14.882
                             0.529 28.146
                                             0.000
               (i4)
                    20.311
                            0.421
                                     48.252 0.000
#>
     .y1_4
#>
     .y6_1
               (i1)
                    19.929
                            0.457
                                     43.623 0.000
#>
               (i2)
                    21.459
                            0.598
                                     35.901
                                            0.000
     .y6_2
#>
     .y6_3
               (i3)
                     14.882
                              0.529
                                     28.146
                                              0.000
#>
               (i4)
                     20.311 0.421
                                     48.252
                                              0.000
     .y6_4
#>
     f1t1
                      0.000
    f1t6
                      5.337 0.354 15.077 0.000
#>
```

```
#> Variances:
                   Estimate Std.Err z-value P(>|z|)
#>
#>
                     15.124 2.238
                                       6.757
                                              0.000
     .y1_1
    .y1_2
#>
                     57.819
                              6.150 9.402
                                               0.000
#>
                    34.209 3.869 8.841 0.000
     .y1_3
     .y1_4
#>
                    16.339 2.114 7.729 0.000
     .y6_1
                    45.360
                             6.338 7.157
                                            0.000
#>
                             8.352 8.887
                                            0.000
#>
     .y6_2
                     74.229
                    89.572 10.110 8.860 0.000
#>
     .y6_3
#>
     .y6_4
                    24.586
                             3.951 6.223 0.000
#>
     f1t1
                     1.000
#>
     f1t6
                     4.557 0.585 7.791 0.000
#>
#>
#>
#> STRICT INVARIANCE MODEL
#> lavaan 0.6.16 ended normally after 57 iterations
#>
#>
   Estimator
                                                 MT.
#>
   Optimization method
                                              NLMINB
    Number of model parameters
                                                  27
#>
#>
    Number of equality constraints
                                                 12
#>
#>
   Number of observations
                                                 204
#>
   Number of missing patterns
                                                  1
#>
#> Model Test User Model:
#>
   Test statistic
#>
                                             134.559
    Degrees of freedom
#>
                                                 29
    P-value (Chi-square)
                                              0.000
#>
#>
#> Parameter Estimates:
#>
#>
   Standard errors
                                            Standard
#>
   Information
                                            Observed
#>
    Observed information based on
                                             Hessian
#>
#> Latent Variables:
#>
                   Estimate Std.Err z-value P(>|z|)
   f1t1 =~
#>
                      5.083
                              0.364
                                      13.978
#>
    y1_1
              (111)
                                               0.000
#>
      y1_2
              (112)
                      4.309
                              0.333
                                     12.944
                                               0.000
              (113) 4.785
                              0.356 13.447 0.000
#>
      y1_3
```

```
#>
    y1_4 (114) 4.358
                              0.317 13.765
                                               0.000
    f1t6 =~
#>
               (111)
                       5.083
                                0.364
                                        13.978
                                                 0.000
#>
      y6_1
#>
      y6_2
               (112)
                       4.309
                                0.333
                                       12.944
                                                 0.000
#>
      y6_3
               (113)
                       4.785
                                0.356
                                       13.447
                                                 0.000
               (114)
                       4.358
                                0.317
                                       13.765
                                                 0.000
#>
      y6_4
#>
#> Covariances:
#>
                    Estimate Std.Err z-value P(>|z|)
    f1t1 ~~
#>
#>
                       1.812
                                0.168 10.812
                                                 0.000
      f1t6
#>
#> Intercepts:
#>
                    Estimate Std.Err z-value
                                              P(>|z|)
#>
     .y1_1
                (i1)
                      20.019
                                0.509
                                       39.339
                                                 0.000
#>
     .y1_2
                (i2)
                      21.513
                                0.621
                                        34.648
                                                 0.000
                      14.805
                                0.617
                                        23.988
                                                 0.000
#>
     .y1_3
                (i3)
#>
     .y1_4
                (i4)
                      20.313
                                0.433
                                       46.915
                                                 0.000
                (i1)
                      20.019
                                0.509
                                        39.339
                                                 0.000
#>
     .y6_1
#>
     .y6_2
                (i2)
                      21.513
                               0.621
                                        34.648
                                                 0.000
#>
     .y6_3
                (i3) 14.805 0.617 23.988 0.000
                      20.313
                              0.433 46.915 0.000
#>
     .y6_4
                (i4)
      f1t1
                       0.000
#>
#>
      f1t6
                       5.557
                                0.415
                                       13.396
                                                 0.000
#>
#> Variances:
#>
                    Estimate Std.Err z-value P(>|z|)
#>
                (u1)
                      28.657
                              2.958
                                        9.689
                                                 0.000
     .y1_1
                                5.253
                                                 0.000
#>
     .y1_2
                (u2)
                      68.013
                                       12.946
                      61.387
                                4.897
                                       12.535
                                                 0.000
#>
     .y1_3
                (u3)
#>
     .y1_4
                (u4)
                      20.878
                                2.142
                                        9.746
                                                 0.000
#>
     .y6_1
                (u1)
                      28.657
                              2.958
                                       9.689
                                               0.000
#>
     .y6_2
                (u2)
                      68.013
                              5.253 12.946
                                               0.000
                (u3)
                      61.387
                               4.897
                                       12.535
                                                 0.000
#>
     .y6_3
#>
                (u4)
                      20.878
                                2.142
                                       9.746
                                                 0.000
     .y6_4
#>
      f1t1
                       1.000
#>
      f1t6
                       5.056
                                0.692
                                        7.304
                                                 0.000
#>
#> Call:
#> Comparison(configural = configural_fit, weak = weak_fit, strong = strong_fit,
#>
      strict = strict_fit)
#>
#> Chi-Squared Difference Test
#>
#>
               Df
                   AIC
                         BIC
                               Chisq Chisq diff RMSEA Df diff Pr(>Chisq)
```

```
#> 1.configural 19 11252 11335 25.968
#> 1.weak 22 11262 11335 41.897
                                     15.929 0.14535
                                                        3 0.0011726 **
#> 2.configural 19 11252 11335 25.968
            25 11268 11331 53.723
                                    27.755 0.13332 6 0.0001045 ***
#> 2.strong
#> 3.configural 19 11252 11335 25.968
#> 3.strict 29 11341 11391 134.559
                                     #> 4.weak
             22 11262 11335 41.897
11.826 0.12009
                                                        3 0.0080053 **
                                    92.662 0.24492
                                                        7 < 2.2e-16 ***
#> 6.strict 29 11341 11391 134.559 80.836 0.30686 4 < 2.2e-16 ***
#> ---
#> Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
#> Invariance(data = osbornesudick1972, time_points = time_points,
   factor_loadings = factor_loadings)
              chisq df pvalue
                              cfi tli rmsea srmr
                                                         aic
#> configural 25.9682 19 0.1311 0.9915 0.9875 0.0424 0.0306 11252.23 11335.18
#> weak 41.8973 22 0.0064 0.9757 0.9691 0.0666 0.0720 11262.16 11335.16
#> strong
            53.7228 25 0.0007 0.9650 0.9608 0.0750 0.0859 11267.98 11331.03
           134.5591 29 0.0000 0.8712 0.8757 0.1336 0.1328 11340.82 11390.59
#> strict
#> Invariance(data = osbornesudick1972, time_points = time_points,
#> factor_loadings = factor_loadings)
#>
#> Fit Measures
              chisq df pvalue
                                cfi
                                     tli rmsea srmr
                                                           aic
#> configural 25.9682 19 0.1311 0.9915 0.9875 0.0424 0.0306 11252.23 11335.18
#> weak 41.8973 22 0.0064 0.9757 0.9691 0.0666 0.0720 11262.16 11335.16
#> strong
            53.7228 25 0.0007 0.9650 0.9608 0.0750 0.0859 11267.98 11331.03
#> strict 134.5591 29 0.0000 0.8712 0.8757 0.1336 0.1328 11340.82 11390.59
#>
#>
#> CONFIGURAL INVARIANCE MODEL
#> lavaan 0.6.16 ended normally after 75 iterations
#>
#>
   Estimator
                                                 ML
   Optimization method
                                             NLMINB
#>
#>
    Number of model parameters
                                                 27
    Number of equality constraints
#>
                                                  2
#>
#> Number of observations
                                                204
```

```
#> Model Test User Model:
#>
#>
   Test statistic
                                           25.968
#>
   Degrees of freedom
                                              19
#>
   P-value (Chi-square)
                                           0.131
#>
#> Parameter Estimates:
#>
#>
   Standard errors
                                         Standard
#> Information
                                         Expected
#>
   Information saturated (h1) model
                                     Structured
#>
#> Latent Variables:
                  Estimate Std.Err z-value P(>|z|)
#>
#>
   f1t1 =~
#>
   y1_1 (111) 4.451 0.396 11.244
                                          0.000
    y1_2
                    6.850 0.637 10.750 0.000
#>
                    4.590 0.515 8.918 0.000
#>
     y1_3
                    5.039 0.393 12.809 0.000
#>
    y1_4
#>
   f1t6 =~
#>
   y6_1
           (111) 4.451 0.396 11.244 0.000
                    4.006 0.485 8.261 0.000
#>
     y6_2
                            0.546 8.342 0.000
#>
    y6_3
                    4.551
                    4.102 0.448 9.150 0.000
#>
     y6_4
#>
#> Covariances:
#>
                 Estimate Std.Err z-value P(>|z|)
   f1t1 ~~
#>
#>
    f1t6
                    1.837 0.211 8.703 0.000
#>
#> Intercepts:
                 Estimate Std.Err z-value P(>|z|)
#>
   .y1_1
#>
              (i1) 19.776 0.427 46.273
                                          0.000
                   21.797
                           0.680 32.036
                                         0.000
#>
    .y1_2
                   14.903
#>
   .y1_3
                          0.528 28.223 0.000
#>
    .y1_4
                   20.396   0.439   46.416   0.000
#>
              (i1) 19.776 0.427 46.273 0.000
    .y6_1
                                  8.467 0.000
#>
    .y6_2
                   19.317
                           2.281
                   11.922 2.538 4.697 0.000
#>
    .y6_3
#>
    .y6_4
                   17.970 1.815 9.903 0.000
#>
     f1t1
                    0.000
#>
     f1t6
                    6.455
                            0.601 10.743
                                           0.000
#>
#> Variances:
                  Estimate Std.Err z-value P(>|z|)
#>
```

```
#>
    .y1_1
                    17.448 2.186 7.981 0.000
                    47.511 5.748 8.266 0.000
#>
     .y1_2
                    35.810 3.969 9.022 0.000
#>
    .y1_3
#>
     .y1_4
                    13.999 2.085 6.712 0.000
                            6.305 7.470 0.000
#>
    .y6_1
                    47.096
#>
    .y6_2
                    73.850 8.395 8.797 0.000
#>
     .y6_3
                   88.920 10.222 8.699 0.000
                    23.267 4.076 5.709 0.000
#>
     .y6_4
                     1.000
#>
     f1t1
#>
    f1t6
                    5.834 1.159 5.035 0.000
#>
#>
#> WEAK INVARIANCE MODEL
#> lavaan 0.6.16 ended normally after 54 iterations
#>
#>
   Estimator
                                                ML
   Optimization method
#>
                                             NLMINB
   Number of model parameters
                                                27
#>
#>
   Number of equality constraints
                                                5
#>
#>
   Number of observations
                                                204
#> Model Test User Model:
#>
#>
   Test statistic
                                             41.897
#>
    Degrees of freedom
                                                 22
                                              0.006
   P-value (Chi-square)
#>
#>
#> Parameter Estimates:
   Standard errors
                                           Standard
#>
#> Information
                                           Expected
#>
    Information saturated (h1) model
                                       Structured
#>
#> Latent Variables:
#>
                   Estimate Std.Err z-value P(>|z|)
   f1t1 =~
#>
    y1_1 (111) 4.933 0.342 14.420 0.000
y1_2 (112) 5.172 0.413 12.534 0.000
#>
#>
     y1_3
y1_4
#>
              (113) 5.072 0.398 12.733 0.000
#>
    y1_4
              (114)
                   4.865
                            0.330 14.733
                                            0.000
   f1t6 =~
#>
#>
     y6_1
              (111) 4.933
                              0.342
                                     14.420
                                              0.000
#> y6_2 (112) 5.172 0.413 12.534 0.000
```

```
y6_3
           (113) 5.072 0.398 12.733
                                            0.000
             (114)
#>
     y6_4
                     4.865
                             0.330
                                   14.733
                                            0.000
#>
#> Covariances:
#>
                  Estimate Std.Err z-value P(>|z|)
    f1t1 ~~
#>
                    1.558 0.136 11.482
#>
     f1t6
                                            0.000
#>
#> Intercepts:
#>
                  Estimate Std.Err z-value P(>|z|)
#>
              (i1) 19.776
                            0.445
                                  44.430
                                            0.000
    .y1_1
   .y1_2
                    21.797
                             0.629
                                   34.633
                                            0.000
#>
                                          0.000
                    14.903
                           0.544 27.411
#>
    .y1_3
#>
    .y1_4
                    20.396 0.436 46.803
                                          0.000
#>
    .y6_1
              (i1) 19.776
                           0.445 44.430
                                          0.000
#>
    .y6_2
                    15.049
                           2.223 6.770
                                          0.000
                    11.756
                           2.238 5.253 0.000
#>
    .y6_3
#>
    .y6_4
                    16.111
                           1.699 9.484 0.000
     f1t1
                    0.000
#>
#>
     f1t6
                    5.824
                           0.432 13.476 0.000
#>
#> Variances:
#>
                  Estimate Std.Err z-value P(>|z|)
                   16.079 2.165 7.427
#>
                                          0.000
   .y1_1
#>
   .y1_2
                   54.055
                           5.831 9.270
                                          0.000
#>
    .y1_3
                   34.578
                           3.922 8.817
                                          0.000
                           2.051
                                          0.000
#>
    .y1_4
                   15.075
                                    7.350
                           6.242 7.970 0.000
#>
                   49.748
    .y6_1
                   72.254 8.498 8.503 0.000
#>
    .y6_2
                   91.610 10.272 8.919 0.000
#>
     .y6_3
                           3.989 5.521 0.000
#>
    .y6_4
                   22.022
                    1.000
#>
    f1t1
#>
    f1t6
                    4.240
                           0.539 7.867 0.000
#>
#>
#> STRONG INVARIANCE MODEL
#> lavaan 0.6.16 ended normally after 52 iterations
#>
#>
   Estimator
                                              ML
    Optimization method
                                           NLMINB
#>
    Number of model parameters
                                               27
#>
#>
    Number of equality constraints
                                               8
#>
```

```
#>
   Number of observations
                                                204
#>
#> Model Test User Model:
#>
#>
    Test statistic
                                              53.723
#>
    Degrees of freedom
                                                 25
    P-value (Chi-square)
                                              0.001
#>
#> Parameter Estimates:
#>
#>
    Standard errors
                                            Standard
#>
   Information
                                            Expected
    Information saturated (h1) model
                                          Structured
#>
#>
#> Latent Variables:
                   Estimate Std.Err z-value P(>|z|)
#>
#>
   f1t1 =~
    y1_1
                    5.270
                             0.333 15.813
                                            0.000
#>
            (111)
                    4.525
                            0.308 14.681 0.000
#>
     y1_2 (112)
              (113)
                    4.960
                            0.328
                                     15.120
                                            0.000
#>
     y1_3
#>
     y1_4
              (114)
                     4.547
                             0.289
                                     15.707 0.000
#>
   f1t6 =~
              (111)
                    5.270
                              0.333
                                     15.813
                                            0.000
#>
      y6_1
#>
     y6_2
              (112)
                      4.525
                              0.308
                                     14.681
                                              0.000
              (113)
                      4.960
                              0.328
                                              0.000
#>
     y6_3
                                     15.120
#>
      y6_4
              (114)
                      4.547
                              0.289
                                     15.707
                                              0.000
#>
#> Covariances:
#>
                   Estimate Std.Err z-value P(>|z|)
#>
   f1t1 ~~
     f1t6
#>
                     1.608
                              0.143
                                    11.281
                                              0.000
#>
#> Intercepts:
#>
                   Estimate Std.Err z-value P(>|z|)
#>
               (i1) 19.929
                             0.456
                                    43.727
                                             0.000
     .y1_1
               (i2) 21.459
                              0.598
                                     35.868
#>
     .y1_2
                                              0.000
#>
     .y1_3
               (i3) 14.882 0.529 28.153
                                            0.000
#>
     .y1_4
               (i4)
                    20.311
                            0.421
                                     48.237
                                            0.000
#>
     .y6_1
               (i1)
                     19.929
                              0.456 43.727
                                              0.000
               (i2)
                    21.459
                             0.598
                                     35.868 0.000
#>
     .y6_2
#>
     .y6_3
               (i3)
                    14.882
                             0.529
                                     28.153 0.000
#>
               (i4)
                     20.311
                              0.421
                                     48.237
                                              0.000
     .y6_4
#>
     f1t1
                      0.000
                      5.337
                              0.353 15.109
                                              0.000
#>
      f1t6
#>
#> Variances:
```

```
Estimate Std.Err z-value P(>|z|)
                   15.124 2.193 6.895 0.000
#>
    .y1_1
#>
                    57.819 6.078 9.512 0.000
    .y1_2
                           3.850 8.885 0.000
#>
    .y1_3
                    34.209
#>
    .y1_4
                    16.339
                           2.062 7.925 0.000
                   45.360 6.182 7.337 0.000
#>
    .y6_1
                   74.229 8.350 8.890 0.000
#>
     .y6_2
                    89.572 10.064 8.900 0.000
#>
    .y6_3
#>
    .y6_4
                   24.586 3.878 6.339 0.000
#>
    f1t1
                    1.000
#>
    f1t6
                    4.557 0.584 7.806 0.000
#>
#>
#>
#> STRICT INVARIANCE MODEL
#> lavaan 0.6.16 ended normally after 57 iterations
#>
   Estimator
                                               ML
#>
    Optimization method
                                            NLMINB
#> Number of model parameters
                                               27
   Number of equality constraints
#>
                                               12
#>
#>
   Number of observations
                                              204
#>
#> Model Test User Model:
#>
                                           134.559
#>
   Test statistic
#>
    Degrees of freedom
                                               29
    P-value (Chi-square)
                                             0.000
#>
#> Parameter Estimates:
#>
                                          {\tt Standard}
#>
   Standard errors
#>
    Information
                                          Expected
#>
   Information saturated (h1) model
                                       Structured
#> Latent Variables:
                  Estimate Std.Err z-value P(>|z|)
#>
#>
   f1t1 =~
            (111) 5.083
#>
    y1_1
                           0.364 13.968
                                           0.000
     y1_2
             (112)
                     4.309
                           0.325 13.248
                                           0.000
#>
                     4.785 0.354 13.521 0.000
#>
    y1_3
             (113)
#>
     y1_4
             (114)
                     4.358 0.312 13.971 0.000
#>
   f1t6 =~
```

```
y6_1 (111) 5.083 0.364 13.968 0.000
      y6_2 (112) 4.309 0.325 13.248 0.000
#>
#>
      y6_3
                (113) 4.785 0.354 13.521 0.000
      y6_4
                (114) 4.358 0.312 13.971 0.000
#>
#>
#> Covariances:
                     Estimate Std.Err z-value P(>|z|)
#>
   f1t1 ~~
#>
#>
     f1t6
                        1.812 0.168 10.790
                                                    0.000
#>
#> Intercepts:
                      Estimate Std.Err z-value P(>|z|)
#>
                 (i1) 20.019 0.506 39.547 0.000
#>
     .y1_1
                                                    0.000
#>
     .y1_2
                 (i2) 21.513 0.621 34.617

    .y1_3
    (i3)
    14.805
    0.617
    24.001
    0.000

    .y1_4
    (i4)
    20.313
    0.433
    46.903
    0.000

    .y6_1
    (i1)
    20.019
    0.506
    39.547
    0.000

    .y6_2
    (i2)
    21.513
    0.621
    34.617
    0.000

#>
#>
#>
#>
    .y6_3 (i3) 14.805 0.617 24.001 0.000
.y6_4 (i4) 20.313 0.433 46.903 0.000
#>
#>
#>
     f1t1
                       0.000
     f1t6
                        5.557 0.411 13.526 0.000
#>
#>
#> Variances:
#>
                     Estimate Std.Err z-value P(>|z|)
   .y1_1 (u1) 28.657 2.747 10.433 0.000
.y1_2 (u2) 68.013 5.170 13.155 0.000
.y1_3 (u3) 61.387 4.821 12.734 0.000
#>
#>
#>
                 (u4) 20.878 2.008 10.398 0.000
#>
     .y1_4
     .y6_1
.y6_2
.y6_3
.y6_4
                 (u1) 28.657 2.747 10.433 0.000
#>
                 (u2) 68.013
                                 5.170 13.155 0.000
#>
#>
                 (u3) 61.387 4.821 12.734 0.000
#>
                 (u4) 20.878 2.008 10.398 0.000
#>
      f1t1
                         1.000
                                 0.685 7.378
#>
      f1t6
                         5.056
                                                    0.000
#>
#> Call:
#> Invariance(data = osbornesudick1972, time_points = time_points,
    factor_loadings = factor_loadings)
#> Chi-Squared Difference Test
#>
                                 Chisq Chisq diff RMSEA Df diff Pr(>Chisq)
                Df AIC BIC
#> 1.configural 19 11252 11335 25.968
#> 1.weak 22 11262 11335 41.897 15.929 0.14535 3 0.0011726 **
```

```
#> 2.configural 19 11252 11335 25.968
#> 2.strong 25 11268 11331 53.723
                                     27.755 0.13332 6 0.0001045 ***
#> 3.configural 19 11252 11335 25.968
#> ---
#> Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
#> Test passed
#> [[1]]
#> [[1]][[1]]
#> [[1]][[1]]$value
#> [[1]][[1]]$value[[1]]
#> lavaan 0.6.16 ended normally after 90 iterations
#>
#>
    Estimator
                                                ML
    Optimization method
                                            NLMINB
    Number of model parameters
#>
                                                31
#>
    Number of equality constraints
                                                12
#>
#>
   Number of observations
                                               204
#>
   Number of missing patterns
                                                1
#>
#> Model Test User Model:
#>
                                           133.220
#> Test statistic
#> Degrees of freedom
                                                25
#> P-value (Chi-square)
                                             0.000
#>
#>
#> [[1]][[1]]$visible
#> [1] TRUE
```

# Environment

```
ls()
#> [1] "osbornesudick1972" "root" "tex_file"
```

## Class

```
#> [[1]]
#> [1] "data.frame"
#>
#> [[2]]
#> [1] "root_criterion"
#>
#> [[3]]
#> [1] "character"
```

### References

Pesigan, I. J. A., Sun, R. W., & Cheung, S. F. (2023). betaDelta and betaSandwich: Confidence intervals for standardized regression coefficients in R. *Multivariate Behavioral Research*, 1–4. https://doi.org/10.1080/00273171.2023.2201277

R Core Team. (2023). R: A language and environment for statistical computing. R Foundation for Statistical Computing. Vienna, Austria. https://www.R-project.org/