

# Autoregressive Model as a State Space Model

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## 1 Measurement Model

The measurement model is given by

$$Y = \nu + \lambda\eta + \varepsilon \quad \text{with} \quad \varepsilon \sim \mathcal{N}(0, \theta^2) \quad (1)$$

where  $Y$ ,  $\eta$ , and  $\varepsilon$  are random variables and  $\nu$ ,  $\lambda$ , and  $\theta^2$  are model parameters.  $Y$  is the observed random variable,  $\eta$  is the latent random variable, and  $\varepsilon$  is the random measurement error while  $\nu$  is the intercept,  $\lambda$  is the factor loading, and  $\theta^2$  is the variance of  $\varepsilon$ . For the autoregressive model some constraints are applied specifically  $\nu = 0$ ,  $\lambda = 1$ ,  $\varepsilon = 0$ . Such that

$$Y = \eta. \quad (2)$$

### 1.1 Example

The measurement model is given by

$$\begin{aligned} Y &= \nu + \lambda\eta + \varepsilon \\ Y &= 0 + 1\eta + 0 \\ Y &= \eta. \end{aligned} \quad (3)$$

## 2 Dynamic Structure

The dynamic structure is given by

$$\eta_t = \alpha + \beta\eta_l + \zeta_t \quad \text{with} \quad \zeta_t \sim \mathcal{N}(0, \psi^2) \quad (4)$$

where  $\eta_t$ ,  $\eta_l$ , and  $\zeta_t$  are random variables and  $\alpha$ ,  $\beta$ , and  $\psi^2$  are model parameters.  $\eta_t$  is the latent variable at time  $t$ ,  $\eta_l$  is a vector of latent variables at lags  $l = \{1, \dots, p\}$ , and  $\zeta_t$  is the dynamic noise at time  $t$  while  $\alpha$  is the intercept,  $\beta$  is a row vector of autoregression coefficients, and  $\psi^2$  is the variance of  $\zeta_t$ .

## 2.1 Example

The random variables and the parameters are given below for  $p = 2$ .

$$\eta_t \tag{5}$$

$$\boldsymbol{\eta}_t = \begin{pmatrix} \eta_{t-1} \\ \eta_{t-2} \end{pmatrix} \tag{6}$$

$$\alpha \tag{7}$$

$$\beta = \begin{pmatrix} \beta_1 & \beta_2 \end{pmatrix} \tag{8}$$

$$\begin{matrix} \beta_1 \\ \beta_2 \end{matrix} \left| \begin{matrix} \eta_t & \text{regressed on} & \eta_{t-1} \\ \eta_t & \text{regressed on} & \eta_{t-2} \end{matrix} \right| \tag{9}$$

$$\psi^2 \tag{10}$$

## 3 Initial Condition

In the state space model, the initial value of the latent variable  $\eta$  given by  $\eta_{|0}$  needs to be specified. We can sample from a particular distribution, for example, the normal distribution as follows

$$\eta_{|0} \sim \mathcal{N}(\mu_0, \sigma_0^2). \tag{11}$$