

Autoregressive Model as a State Space Model

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1 Measurement Model

The measurement model is given by

$$Y = \nu + \lambda\eta + \varepsilon \quad \text{with} \quad \varepsilon \sim \mathcal{N}(0, \theta^2) \quad (1)$$

where Y , η , and ε are random variables and ν , λ , and θ^2 are model parameters. Y is the observed random variable, η is the latent random variable, and ε is the random measurement error while ν is the intercept, λ is the factor loading, and θ^2 is the variance of ε . For the autoregressive model some constraints are applied specifically $\nu = 0$, $\lambda = 1$, $\varepsilon = 0$. Such that

$$Y = \eta. \quad (2)$$

1.1 Example

The measurement model is given by

$$\begin{aligned} Y &= \nu + \lambda\eta + \varepsilon \\ Y &= 0 + 1\eta + 0 \\ Y &= \eta. \end{aligned} \quad (3)$$

2 Dynamic Structure

The dynamic structure is given by

$$\eta_t = \alpha + \beta\boldsymbol{\eta}_l + \zeta_t \quad \text{with} \quad \zeta_t \sim \mathcal{N}(0, \psi^2) \quad (4)$$

where η_t , $\boldsymbol{\eta}_l$, and ζ_t are random variables and α , β , and ψ^2 are model parameters. η_t is the latent variable at time t , $\boldsymbol{\eta}_l$ is a vector of latent variables at lags $l = \{1, \dots, p\}$, and ζ_t is the dynamic noise at time t while α is the intercept, β is a row vector of autoregression coefficients, and ψ^2 is the variance of ζ_t .

2.1 Example

The random variables and the parameters are given below for $p = 2$.

$$\eta_t \tag{5}$$

$$\boldsymbol{\eta}_t = \begin{pmatrix} \eta_{t-1} \\ \eta_{t-2} \end{pmatrix} \tag{6}$$

$$\alpha \tag{7}$$

$$\beta = \begin{pmatrix} \beta_1 & \beta_2 \end{pmatrix} \tag{8}$$

$$\begin{matrix} \beta_1 \\ \beta_2 \end{matrix} \left| \begin{matrix} \eta_t & \text{regressed on} & \eta_{t-1} \\ \eta_t & \text{regressed on} & \eta_{t-2} \end{matrix} \right| \tag{9}$$

$$\psi^2 \tag{10}$$

3 Initial Condition

In the state space model, the initial value of the latent variable η given by $\eta_{|0}$ needs to be specified. We can sample from a particular distribution, for example, the normal distribution as follows

$$\eta_{|0} \sim \mathcal{N}(\mu_0, \sigma_0^2). \tag{11}$$