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Assignment Title: Individual Report and Excel Modelling

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Table of Contents

1. Current business at the at the Students' Union Shop 3

1.1 Current situation described at the Students' Union Shop 3

2. Using Queuing theory for evaluating their current performance at the Students' Union Shop ..4

3. Case II: Suggestion improvements to their current strategies: 6

4. Multi-server queuing model using Excel: 8

5. Economic Analysis of the Waiting Lines: 12

References: 13

1. Current business at the at the Students' Union Shop

The current problem that needs to be addressed at the Students' Union Shop is to determine how many cashiers should be deployed at the cash counter, so that an optimized balance could be produced in terms of saving the time of the customers as the managers of the shop are concerned that some of the customers may be shunning in the shop because of high waiting times. Currently, there are 3 cashiers deployed at the Students' Union Shop. Another constraint that the shop also has to consider is that a that adding a new cashier leads to an additional costs of $20 an hour. So, an optimized solution has to be found balancing the time needs of the customers and the cost of adding new cashiers to the shop.

1.1 Current situation described at the Students' Union Shop

The objective with respect to the current problem is to achieve an optimized solution between the number of cashiers that must be used (number of queues that must be used), providing faster turnaround time for each customer standing in cash paying queue, so that no customer has to wait for service.

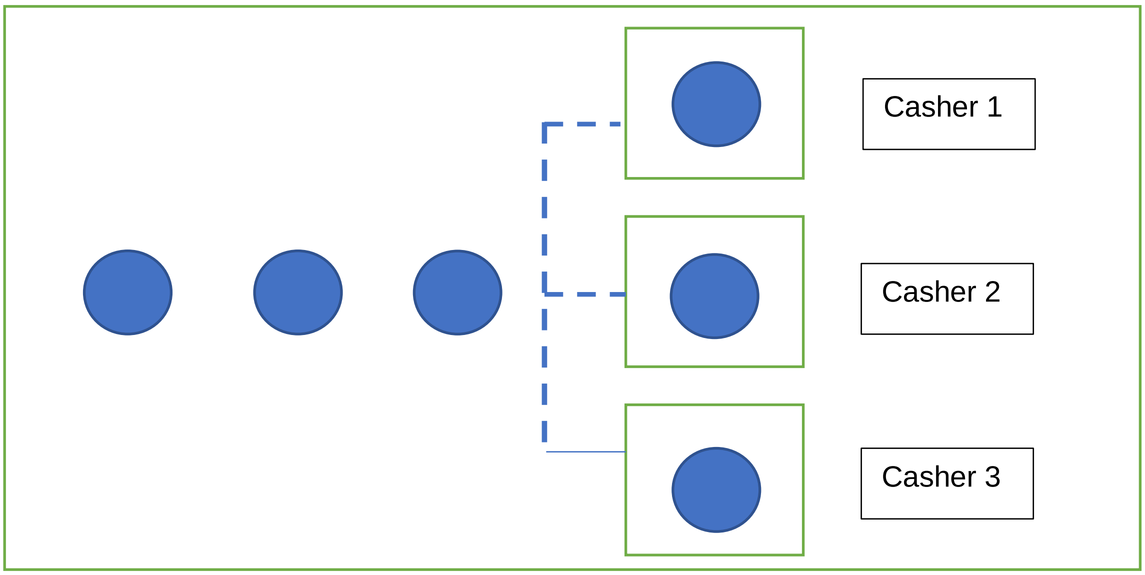
Objective: Determine how many cashiers must be placed at the cash counter to provide adequate service level

The operating characteristics of the current queue at the Students' Union Shop are:

- Currently there are 3 cashiers where the customers can pay for their order, with the maximum capacity of cashiers 6 and extra cashier costs $20 an hour
- There is a single queue where the customers line up
- The data about the number of customers at the till from 8 a.m. to 6.0 p.m., at 5 min intervals for 5 week period and all the data were collected during term time
- the time it takes for cashiers to take payment is also collected 1000 and the data comes from term time

2. Using Queuing theory for evaluating their current performance at the Students' Union Shop

The current situation highlighted at the Students' Union Shop falls under the queuing theory. Within the queuing theory, the current situation falls under the multiple server waiting model. As mentioned in the case study, there are total of 3 cashiers, but a single queuing line. The single queuing line forms the pooled queue for all the 3 servers.



The Student Union's Multi-server model with single shared queue and 3 servers

The blue dot showcases the customer, while there are 3 cashiers that are serving the customers. There is only 1 single queue, and the customers can go to any cashier which is free.

Operating characteristics of the current Students' Union Shop case study:

For understanding the current multi-server queuing problem, the first step is to determine the distribution of the customers arriving to be served. The customers are discrete values and are assumed to be arriving independently and randomly. So, such scenario can be modelled using Poisson distribution.

Poisson Distribution can only take discrete set of values, for example, 0,1,2 …. Etc, here which will mimic the number of customers. Further, the Poisson distribution describes the number of events that occur within fixed time interval. Further the services time can be understood to be following exponential, with each server being able to service μ customers per time.

Here within the given data, the time interval taken into consideration = 5 min

λ = the mean arrival rate for the system or the number of customers

μ = the service rate for each server

k = the number of servers = 3

Task 1: Finding the mean number of arrivals for the 5 min time interval:

The first task is to find out the λ, based on the given data about the customer arrival rate every 5 min. The first step as part of the queuing theory is to determine the probability distribution of the customer arrivals in a given time period.

There are total of 3000 unique values [Count = COUNT(A4:A3003)] of number of arrivals counter after each 5 min time interval:

So, Average = AVERAGE(C4:C3003) = 5.408333333 per 5 min = 1.08 customers/per

$$\lambda = 1.08$$

$$P(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

Here x is the probability of number of customers arriving at the shop within 1 min time interval, given λ = 1.08

Step 2: Service Time = μ

The service time is the time that a customer spends from entering the queue to existing the queue. For, this case study, the service time for 1000 customers is given, which will be used to calculate μ min.

The service times are assumed to follow an exponential distribution system. The service time in the current case for all the three cashiers would be equivalent, given the nature of the work.

P (service time <= t) = $1 - e^{-\mu t}$

The value of μ = based on the service time data, AVERAGE(B4:B1003) = 0.932 min for 1 single customer

Further, it also means that μ = 1.07 customers per minute combined

But, as there are 3 cashiers, so the value of = 0.357 customers/ min/ each cashier

Step 3: Measures of performance of the current Students' Union Shop case study:

For the queue system to be stable, $\lambda < k\mu$ (k = number of cashiers here)

But, in this case, it can be seen that λ (1.08) > $k\mu$ (1.07) therefore, in the current case the queue system is not stable. This means that the number of customers coming in per time are more that the number of customers that can be served, which could lead to the build up within the queue.

The value of $\lambda / \mu = 1.08/0.357 = 3.02$

More than 3 servers are required to maintain the steady state

So, for the current case , there will be a build-up in the queue, because the number of incoming customers are always greater than the number of outgoing customers, and the total servers can't reduce the time being spent by the customer, rather the average time spent by the customer is bound to increase.

3. Case II: Suggestion improvements to their current strategies:

An alternate case has to be setup, where the number of servers have to be increased if the serving time of each customer has to be decreased. So, the number of servers have to be increased.

A: the number of servers is increased from 3 to 4 :

$\lambda / \mu = 1.08/0.357 = 3.02$

a. L_q = Average length of the queue = 7.529 customer

b. W_q = Average waiting times at queues = $L_{q/}\lambda$ = 6.97 min

c. P_0 = The probability that there are no customers in the system = 0.1694

d. L = Average number of units in the system = $L_q + \lambda / \mu$ = 19.52 customers

e. Average time spent by the unit in the system = $W_q + 1 / \mu$ = 18.08 min

B. the number of servers is increased from 3 to 5 :

λ / μ = 1.08/0.357 = 3.02

a. L_q = Average length of the queue = 0.83

b. W_q = Average waiting times at queues = $L_{q/} \lambda$ = 0.77 min

c. P_0 = The probability that there are no customers in the system = 0.1025

d. L = Average number of units in the system = $L_q + \lambda / \mu$ = 2.17 customers

e. Average time spent by the unit in the system = $W_q + 1 / \mu$ = 2.01 min

C. the number of servers is increased from 3 to 6 :

λ / μ = 1.08/0.357 = 3.02

a. L_q = Average length of the queue = 0.11

b. W_q = Average waiting times at queues = $L_{q/} \lambda$ = 0.10 min

c. P_0 = The probability that there are no customers in the system = 0. 0516

d. L = Average number of units in the system = $L_q + \lambda / \mu$ = 0.29 customers

e. Average time spent by the unit in the system = $W_q + 1 / \mu$ = 0.27 min

So, the above three results clearly show that adding new servers will reduce the average length of the queue, reduce the waiting times of the customers, reduce the average number of customers in the system and also reduce the average time the customer spends in the system. Therefore, the greater the number of cashiers, the faster is the checkout time for the customer. The choice of number of servers that must be added has to be based on the trade-off between the cost of adding the new cashier and the additional number of customers that can be served, as a result of adding an extra cashier.

4. Multi-server queuing model using Excel:

Step 1: Input the values of the queuing model parameters, λ / μ / s

Queuing Parameters	Values
μ (Service Rate)	1.08
λ (Mean Arrival Rate)	0.357
s (Number of servers/cashiers)	3

The assumption is that the customers entering can be modelled using Poisson distribution while the serving of the customers can be modelled using exponential distribution

Step 2: Checking whether the system is the steady state or not:

		λ/kμ	Steady State
μ (Service Rate)	1.08		
λ (Mean Arrival Rate)	0.357		
s (k) (Number of servers/cashiers)	3	=$B$19/($B$21*B20)	=IF(C21<1,"YES","NO")
s (k) (Number of servers/cashiers)	4	=$B$19/($B$21*B21)	=IF(C22<1,"YES","NO")
s (k) (Number of servers/cashiers)	5	=$B$19/($B$21*B22)	=IF(C23<1,"YES","NO")
s (k) (Number of servers/cashiers)	6	=$B$19/($B$21*B23)	=IF(C24<1,"YES","NO")

		λ/kμ	Steady State
μ (Service Rate)	1.08		
λ (Mean Arrival Rate)	0.357		
s (k) (Number of servers/cashiers)	3	1.008403	NO

s (k) (Number of servers/cashiers)	4	0.12	YES
s (k) (Number of servers/cashiers)	5	0.09	YES
s (k) (Number of servers/cashiers)	6	0.072	YES

Step 3: Calculating Queuing values for Different Multi-server models:

Inputs	Column1	Column2
	Unit of Time	Min
	Arrival Time	1.08
	Service Time	0.357
	Number of Identical Servers	4
Outputs		
	Mean time between Arrivals	=1/C6
	Mean time per service	=1/C7
	Traffic intensity	=C6/C7/C8
Summary Measures		
	Average utilization rate of server	=C13*100
	Average number of customers waiting in line	=(C6/C7)^(C8+1)/C8/FACT(C8)/(1-C13)^2*C21
	Average number of customers in system	=C17/C6/C7
	Average time waiting in line	=C17/C6
	Average time in system	=C18/C6
	Probability of no customers in system	=POISSON.DIST(C8,C6/C7,FALSE)
	Probability that all servers are busy	=(C8*C13)^C8/FACT(C8)/(1-C13)*C21
	Probability that at least one server is idle	=1-C22

The above formulas when put in excel sheet will calculate the true values:

When testing with k =3, the model shows that the queuing system in not in steady stateL:

Inputs	Column1	Column2
	Unit of Time	Min
	Arrival Time	1.08
	Service Time	0.357
	Number of Identical Servers	3
Outputs		
	Mean time between Arrivals	0.9259259
	Mean time per service	2.8011204
	Traffic intensity	1.0084034
Summary Measures		
	Average utilization rate of server	100.84034
	Average number of customers waiting in line	14761.378
	Average number of customers in system	38285.553
	Average time waiting in line	13667.942
	Average time in system	35449.586
	Probability of no customers in system	0.2240182

For n = 4, the following are the solution values:

Inputs	Column1	Column2
	Unit of Time	Min
	Arrival Time	1.08
	Service Time	0.357
	Number of Identical Servers	4
Outputs		
	Mean time between Arrivals	0.9259259
	Mean time per service	2.8011204
	Traffic intensity	0.7563025
Summary Measures		
	Average utilization rate of server	75.630252
	Average number of customers waiting in line	7.5297969
	Average number of customers in system	19.529507
	Average time waiting in line	6.9720341
	Average time in system	18.082877
	Probability of no customers in system	0.1694255

For n = 5

Inputs	Column1	Column2
	Unit of Time	Min
	Arrival Time	1.08
	Service Time	0.357
	Number of Identical Servers	5
Outputs		
	Mean time between Arrivals	0.9259259
	Mean time per service	2.8011204
	Traffic intensity	0.605042
Summary Measures		
	Average utilization rate of server	60.504202
	Average number of customers waiting in line	0.8395462
	Average number of customers in system	2.1774722
	Average time waiting in line	0.7773576
	Average time in system	2.016178
	Probability of no customers in system	0.1025096

For n = 6,

Inputs	Column1	Column2
	Unit of Time	Min
	Arrival Time	1.08
	Service Time	0.357
	Number of Identical Servers	6
Outputs		
	Mean time between Arrivals	0.9259259
	Mean time per service	2.8011204
	Traffic intensity	0.5042017
Summary Measures		
	Average utilization rate of server	50.420168
	Average number of customers waiting in line	0.1128661
	Average number of customers in system	0.292733
	Average time waiting in line	0.1045057
	Average time in system	0.2710491
	Probability of no customers in system	0.0516855

5. Economic Analysis of the Waiting Lines:

Cost-benefit of adding an extra cashier in terms of staff utilisation and customer waiting time

a. The above results clearly show that there is a need to add an extra server to reduce the extra time that is required to serve the customers

There has to be choice made, whether 1 or 2 extra cashiers are to be added. This will be decided through taking a cost-benefit analysis. Adding a new cashier will cost $ 20 per hour. Every day there is 10 hrs shift. So, adding a new cashier, would cost $ 200 per day.

So, there has to be seen whether, 1 or 2 can be added. The preference should be to add 1 new cashier, as in that case, the average utilization of server is 75%, so 25% is the spare capacity. To improve deliver, 2 queues should be formed (Anderson et al., 2018).

References:

Anderson, D.R., Sweeney, D.J., Williams, T.A., Camm, J.D. and Cochran, J.J., 2018. *An introduction to management science: quantitative approach*. Cengage learning.