The full formalisation of the constraint optimisation problem

Formally, the decision variables of the constraint optimisation problem are: (1) $\{z^i_{p,t} \in \{0,1\}\}_{i\in I, p\in P, t\in T}$, indicating that the VM of task i is placed in MDC p ($z^i_{p,t} = 1$), or not ($z^i_{p,t} = 0$) at time step t. (2) $\{f^i_{l,p,j,k,t} \in \mathbb{R}^+\}_{i\in I, l\in L, p\in P, (j,k)\in\mathbb{E}, t\in T}$, indicating allocation of the bandwidth on each link for task i at time step t. (3) $\tilde{p}_i(\lambda_i, \theta^{\langle T^a_i\rangle}) \in \mathbb{R}^+$, denoting the payment of task i, which is a function of the allocation: λ_i and all tasks received by T^a_i: $\theta^{\langle T^a_i\rangle}$. So, for task i, its resource allocation scheme $\lambda_i = \{z^i_{p,t}\}_{i\in I, p\in P, t\in T} \cup \{f^i_{l,p,j,k,t}\}_{i\in I, l\in L, p\in P, (j,k)\in\mathbb{E}, t\in T}$ and its utility is $u_i = \boldsymbol{v}_i(\tilde{t}_i) - \tilde{p}_i(\lambda_i, \theta^{\langle T^a_i\rangle})$. The objective function maximises the total social welfare:

$$\underset{\lambda_i}{\text{maximise}} \sum_{i\in I} \boldsymbol{v}_i\Big(\sum_{p\in P, t\in T} z^i_{p,t}\Big) - o \tag{1}$$

where

$$o = \sum_{i\in I, r\in R, p\in P, t\in T} a_{i,r} z^i_{p,t} o_{p,r} + \sum_{i\in I, l\in L, p\in P, (j,k)\in\mathbb{E}, t\in T} 2 o_{j,k} f^i_{l,p,j,k,t}$$

Then, the following equations are the constraints.

$$\text{Subject to:} \quad \sum_{p\in P} z^i_{p,t} \le 1 \quad \forall i \in I, t\in T \tag{2a}$$

$$\sum_{i\in I} z^i_{p,t} a_{i,r} \le A_{p,r} \quad \forall p\in P, r\in R, t\in T \tag{2b}$$

$$z^i_{p,t} = 0 \quad \forall i\in I, p\in P, t < T^s_i \; or \; t > T^f_i \tag{2c}$$

$$\sum_{j:(j,p)\in\mathbb{E}} f^i_{l,p,j,p,t} = \Gamma^i_l z^i_{p,t} \quad \forall p\in P, i\in I, l\in L, t\in T \tag{2d}$$

$$\sum_{k:(l,k)\in\mathbb{E}} f^i_{l,p,l,k,t} = \Gamma^i_l z^i_{p,t} \quad \forall p\in P, i\in I, l\in L, t\in T \tag{2e}$$

$$\sum_{j:(j,k)\in\mathbb{E}} f^i_{l,p,j,k,t} = \sum_{j:(k,j)\in\mathbb{E}} f^i_{l,p,k,j,t} \forall p\in P, k\in P, i\in I, l\in L, t\in T \tag{2f}$$

$$\sum_{i\in I, l\in L, p\in P} f^i_{l,p,j,k,t} \le b_{j,k} \quad \forall (j,k)\in\mathbb{E}, t\in T \tag{2g}$$

$$f^i_{l,p,j,k,t} \ge 0 \quad \forall i\in I, l\in L, p\in P, (j,k)\in\mathbb{E}, t\in T \tag{2h}$$

To explain the above constraints in detail, constraint (2a) represents that every task only needs one VM. Constraint (2b) guarantees that at each time step the allocated resources at any MDC do not exceed its resource capacities. Constraint (2c) means that the VM is created within the start and finish time of the task. Constraint (2d) requires that the total inbound traffic to MDC p for the traffic from task i's location l equals its corresponding bandwidth demand at each time step if the VM for task i is placed in MDC p. Since the bandwidth demands and the bandwidth costs are both symmetrical. It is sufficient to just consider the traffic from L to P. Constraint (2e) indicates that the outbound traffic from task i's location l is equal to its corresponding bandwidth demand at each time step. Constraint (2f) represents that the inbound and outbound traffic of intermediate nodes for task i should be equal. Constraint (2g) guarantees that the aggregated traffic on each link does not exceed its bandwidth capacity at each time step. Finally, constraint (2h) ensures that the allocated bandwidth in each data link is not negative, which is impossible in practice. The above optimisation problem is a mixed integer linear programming problem.