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# XAI: Theoretically Unifying Conceptual Explanation and Generalization of DNNs

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Game-theoretic interactions to unify

1. attribution explanations
2. encoding of visual concepts
3. generalization power
4. adversarial transferability and robustness

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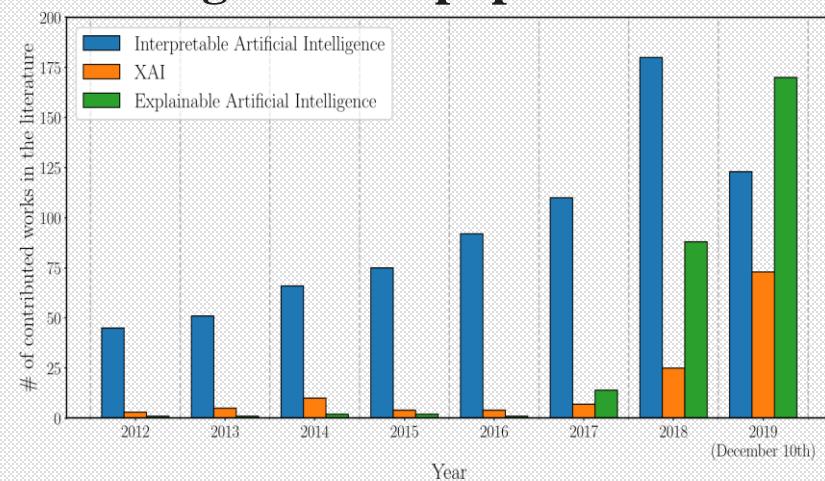
# Why XAI is important ?

## □ Key applications

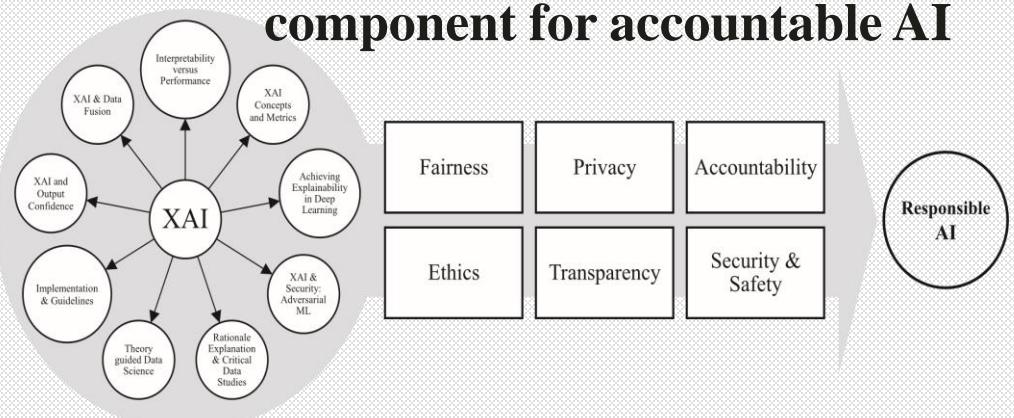
- Finance, autonomous driving, medical diagnosis, military

## □ Set standards for the AI safety and interpretability

### The growth of papers in XAI<sup>[1]</sup>



Interpretability is a necessary component for accountable AI



[1] Gonzalo Recio Domenech "Analysis of Explainability of Deep Learning Models for Medical Applicability" Minds Brains and Machines (MBM) — Master in Artificial Intelligence.





# Topics of explaining DNNs

## Semantic explanation

Which semantic concepts are modeled and used for prediction

How to quantify and improve the trustworthiness of a DNN

End-to-end learn interpretable features

Communicative learning at the semantic level

How to evaluate the explanation

Model and explain the representation capacity of a DNN

How to bridge the architecture with the knowledge representation

Explain classical deep-learning techniques (e.g., distillation, adversarial learning, compression)

How to debug DNNs using mathematical diagnosis of DNN features

## Mathematical explanation





# XAI topics

Which semantic concepts are modeled and used for prediction

How to quantify and improve the trustworthiness of a DNN

End-to-end learn interpretable features

Communicative learning at the semantic level

How to evaluate the explanation



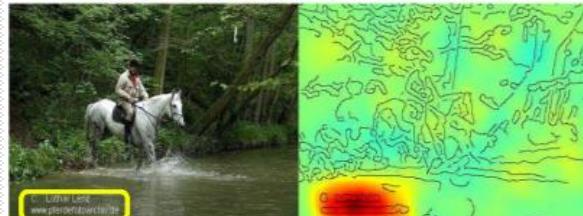
Make a surgery. Score=0.9

It is because

- 1) From Organ A. Score=0.2
- 2) From Organ B. Score=0.1

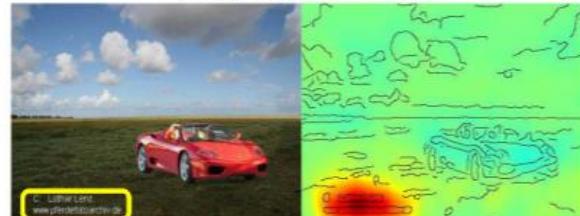
...

Horse-picture from Pascal VOC data set



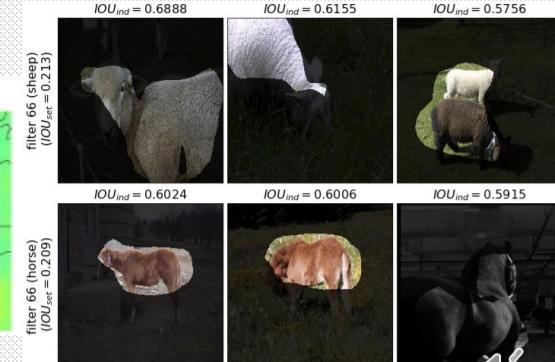
Source tag present  
↓  
Classified as horse

Artificial picture of a car



Original  
Pasted  
Masked

Score of lipstick



Lapuschkin et al. "unmasking clever hans predictors and assessing what machines really learn" in Nat Commun 10 1096, 2019  
Fong et al. "Net2Vec: Quantifying and Explaining how Concepts are encoded by filters in deep neural networks" in CVPR 2018  
Zhang et al. "Examining CNN Representations with respect to Dataset Bias" in AAAI 2018





# XAI topics

## Mathematical explanation

Model and explain  
the representation  
capacity of a DNN

How to bridge  
the architecture  
with the  
knowledge  
representation

Explain classical  
deep-learning  
techniques (e.g.,  
distillation,  
adversarial learning,  
compression)

How to debug  
DNNs using  
mathematical  
diagnosis of DNN  
features



- **How does an accident happen?**
- **What is the accident frequency if the car has run safely for a year?**
  - Once per year?
  - Once per ten years?
- **How to further boost the safety even without accident records?**



# XAI topics

## Mathematical explanation

Model and explain  
the representation  
capacity of a DNN

How to bridge  
the architecture  
with the  
knowledge  
representation

Explain classical  
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How to debug  
DNNs using  
mathematical  
diagnosis of DNN  
features

- **How to evaluate the generalization power of a DNN?**
- **Why does a specific DNN architecture outperform another architecture in a specific task?**
- **What is the relationship between the architecture and the knowledge.**
- **What is the common essence of existing DL methods? How to further improve these methods?**





# Problems with semantic explanations

Many semantic explanations are still heuristic technologies, rather than science

**Only self-consistency, no mutuality between XAI methods**

**Very few theoretic foundations**

**Difficult to improve DNNs**

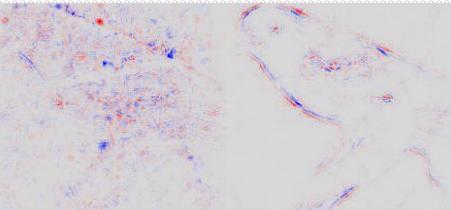
**Lack of convincing enough evaluation metrics**

Explanation results conflict with each other.

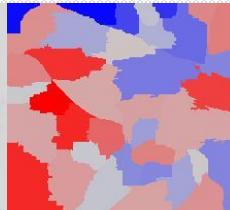
Input



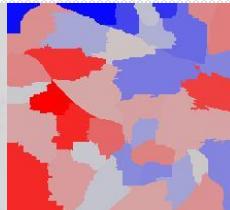
Gradient  
× Input



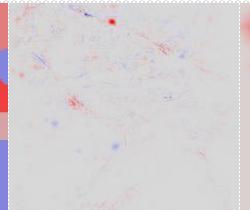
Guided Back-propagation



LIME



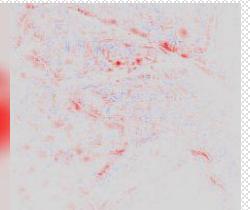
LRP



Perturbation



DeepSHAP





# Problems with explaining the representation power

Analysis of the representation capacity of a DNN

Limited to certain assumptions  
(shallow nets or infinite width)

Cannot provide semantic explanations

Cannot explain the emergence of semantics in deep layers.

“Mathematic proof” is not equivalent to “understanding.”

**Theorem 3 (Pitas et al. (2017))** Let  $B$  an upper bound on the  $\ell_2$  norm of any point in the input domain. For any  $B, \gamma, \delta > 0$ , the following bound holds with probability  $1 - \delta$  over the training set:

$$L \leq \hat{L}_\gamma + \sqrt{\frac{\left(84B \sum_{i=1}^d k_i \sqrt{c_i} + \sqrt{\ln(4n^2d)}\right)^2 \prod_{i=1}^d \|\mathbf{W}_i\|_2^2 \sum_{j=1}^d \frac{\|\mathbf{w}_j - \mathbf{w}_j^0\|_F^2}{\|\mathbf{w}_j\|_2^2} + \ln(\frac{m}{\delta})}{\gamma^2 m}} \quad (24)$$





# Vision for XAI science

Although still far from science

**Regional explanation  
with strict meanings**

- Strict meanings of visual concepts
- Accurate attributions

**XAI metrics for  
representation  
power of DNNs**

- Mutuality between different metrics
  - Feature transferability
  - Adversarial robustness/transferability
  - Transformation complexity
  - Generalization
  - Disentanglement
  - Feature information
  - Interactions
- Essence of existing deep-learning methods
  - Summarize effective factors
  - Improve existing methods
- Guide deep learning
  - Guide the design of network architecture
  - Guide the learning process

**Well-proved  
theoretic  
foundation**





# Game-theoretic interactions

Mathematical Foundations

Metric

Definitions and axioms

Theories of explaining DNNs

Game theory: Shapley Value

Game-theoretic interactions

Define multivariate interactions

Define multi-order interactions

Derive desirable axioms

Problem with baseline values of Shapley values

Explain the network's generalization ability

Construct interaction trees

Explain the DNN's semantic representation

Explain the adversarial transferability

Explain the aesthetic appreciation

Explain the adversarial robustness

- 1. The relationship between dropout and interactions
- 2. The relationship between the network's generalization ability and interactions

Extract hierarchical structures of interactions among words in a sentence

How the interactions reveal the semantics encoded in the DNN

- 1. The negative correlation between the adversarial transferability and the interaction
- 2. A unified explanation of transferability-boosting methods
- 3. Enhance the adversarial transferability by penalizing interactions

- 1. Aesthetic images make a neural network strengthen salient concepts and discard inessential concepts.
- 2. Operations to improve the aesthetic level of images automatically

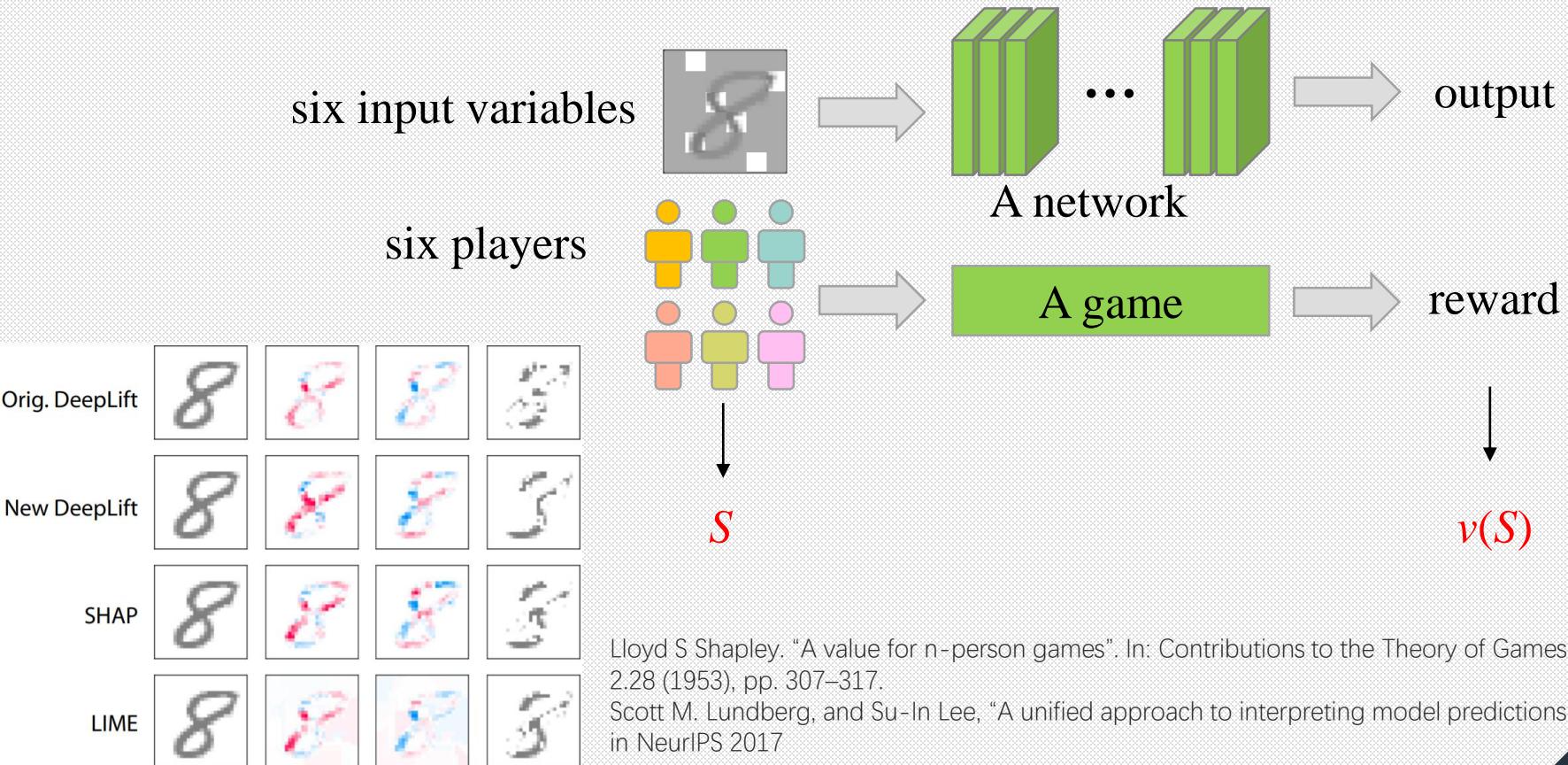
- 1. Adversarial perturbations mainly affect high-order interactions.
- 2. Adversarial training can boost the adversarial robustness by modeling more low-order interactions.



# Preliminaries: Shapley values

## □ Game

- Input variables → players
- Scalar network output/loss → total rewards of players in the game





## Preliminaries: Shapley values

- Given a game, how to fairly allocate contribution of each player?

The **Shapley value** is considered as a method that fairly allocates the reward to players.

$$\phi(i|N) = \sum_{S \subseteq N \setminus \{i\}} \frac{(n - |S| - 1)! |S|!}{n!} [v(S \cup \{i\}) - v(S)]$$

$$v(N) = v(\emptyset) + \sum_{i \in N} \phi(i|N)$$



# Preliminaries: Shapley values

□ **Question:** Given a game, how to fairly allocate contribution of each player?

Several **desirable axioms** ensure the fairness of allocation:

- **Linearity axiom**

If  $\forall S \subseteq N, u(S) = v(S) + w(S)$ , then  $\phi_u(i|N) = \phi_v(i|N) + \phi_w(i|N)$

- **Dummy axiom**

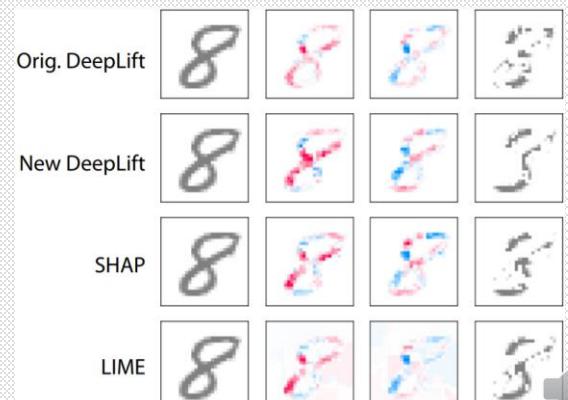
If  $\forall S \subseteq N \setminus \{i\}, v(S \cup \{i\}) = v(S) + v(\{i\})$ , then  $\phi(i|N) = v(\{i\}) - v(\emptyset)$

- **Symmetry axiom**

If  $\forall S \subseteq N \setminus \{i\}, v(S \cup \{i\}) = v(S \cup \{j\})$ , then  $\phi(i|N) = \phi(j|N)$

- **Efficiency axiom**

$\sum_{i \in N} \phi(i|N) = v(N) - v(\emptyset)$





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If two independent games  $v$  and  $w$  can be merged into one game, then the Shapley value of the player  $i$  in game  $v$  and game  $w$  also can be merged.

- **Dummy axiom**

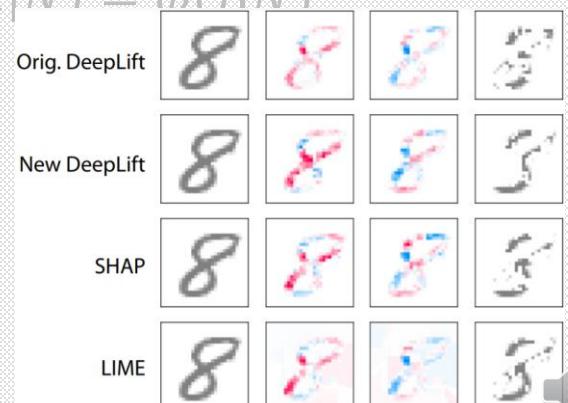
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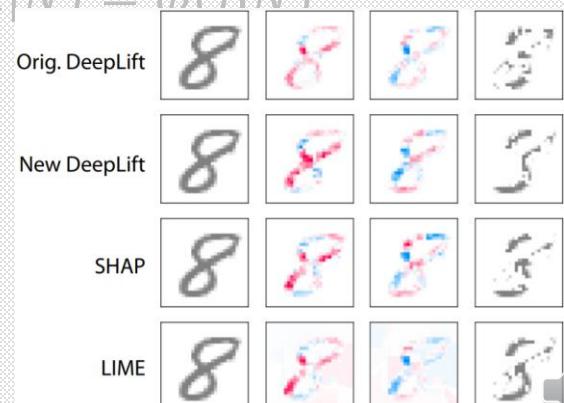
A dummy player  $i$  satisfies that the player  $i$  has no interaction with other players.

- **Symmetry axiom**

If  $\forall S \subseteq N \setminus \{i\}, v(S \cup \{i\}) = v(S \cup \{j\})$ , then  $\phi(i|N) = \phi(j|N)$

- **Efficiency axiom**

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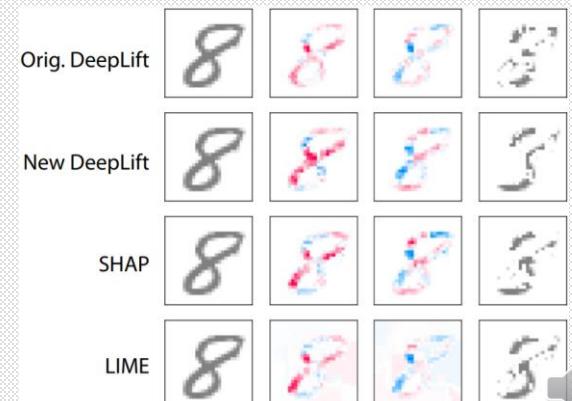
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If  $\forall S \subseteq N \setminus \{i\}, v(S \cup \{i\}) = v(S \cup \{j\})$ , then  $\phi(i|N) = \phi(j|N)$

If two players  $i, j$  have same collaborations with other players, then they have the same Shapley value.

- **Efficiency axiom**

$$\sum_{i \in N} \phi(i|N) = v(N) - v(\emptyset)$$





# Preliminaries: Shapley values

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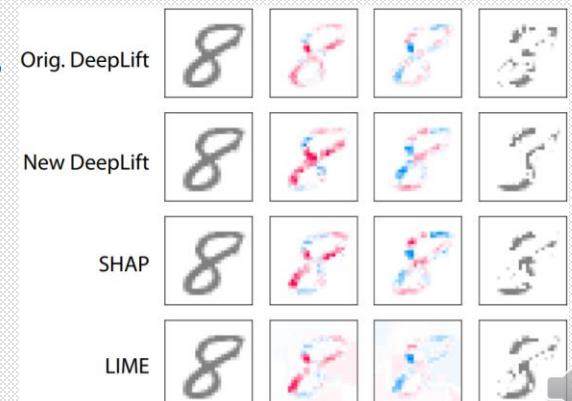
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$\sum_{i \in N} \phi(i|N) = v(N) - v(\emptyset)$

The overall reward can be allocated to all players in the game.





# Preliminaries: Shapley values

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## □ Remaining issues

- How to determine reasonable baseline values?
- How to determine the reasonable partition of players?



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# How to define interactions in game theory?

How to determine baseline values for the Shapley value?

What is the relationship between interactions and visual concepts?

What is the relationship between interactions and the aesthetic appreciation?

What is the relationship between interactions and the generalization?

What is the relationship between interactions and adversarial transferability?

What is the relationship between interactions and adversarial robustness?



# Game-theoretic interactions

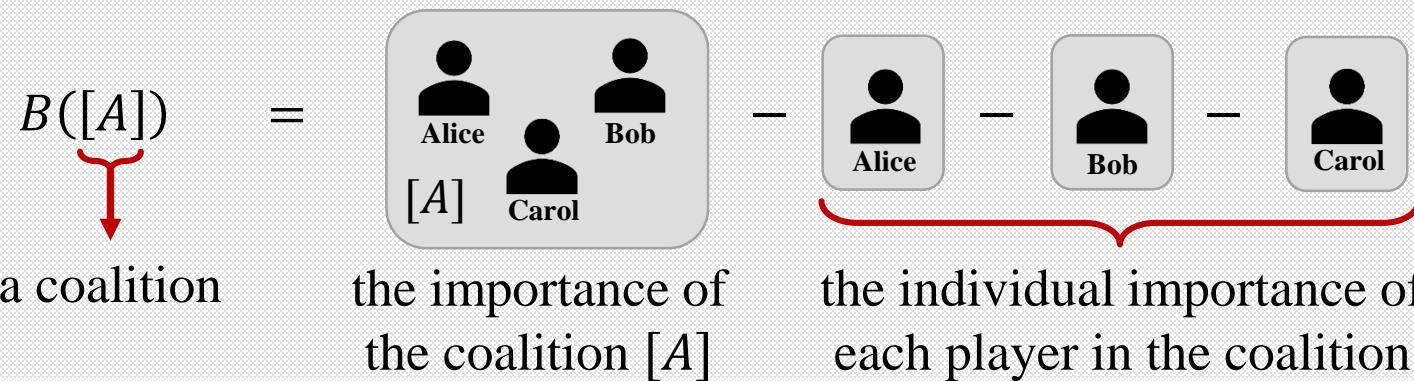
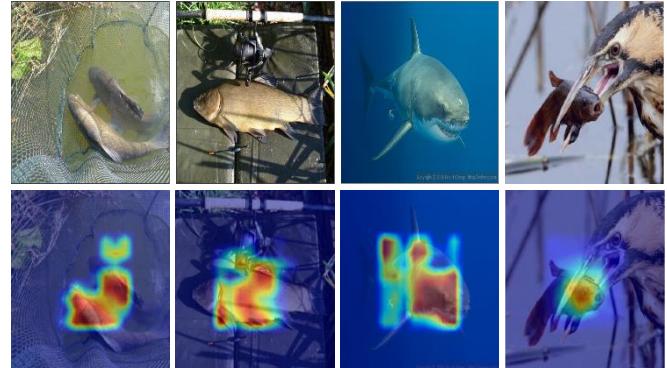
it's a remarkably solid and subtly satirical tour de force.

this is a good script, good dialogue, funny even for adults

dull, lifeless, and amateurishly assembled.

a warm but realistic meditation on friendship, family and affection.

no telegraphing is too obvious or simplistic for this movie.



$B([A]) > 0$  : Players in  $[A]$  mainly have **cooperative** relationships.

$B([A]) < 0$  : Players in  $[A]$  mainly have **adversarial** relationships.





# Game-theoretic interactions

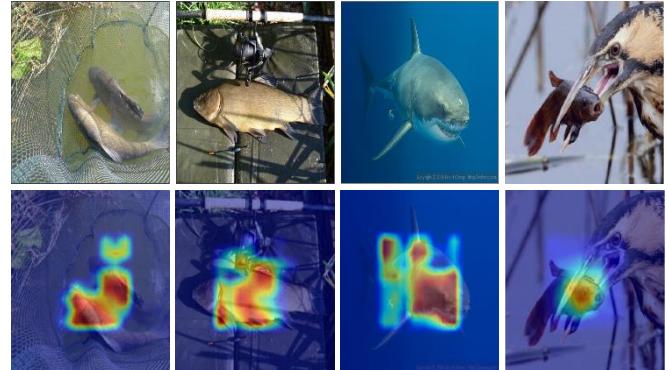
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- Input words of a sentence (or the pixels of an image) usually **cooperate** with each other, rather than **work individually** to make inferences.
- The cooperative input words (or pixels) have strong interactions.
- **Shapley interactions between two players ( $i,j$ ):** the change of the importance of  $i$  when  $j$  is present, w.r.t. the importance when  $j$  is absent.

$$I(i,j) = \phi_{w/j}(i|N) - \phi_{w/o\ j}(i|N)$$



# Multivariate Shapley interactions: properties

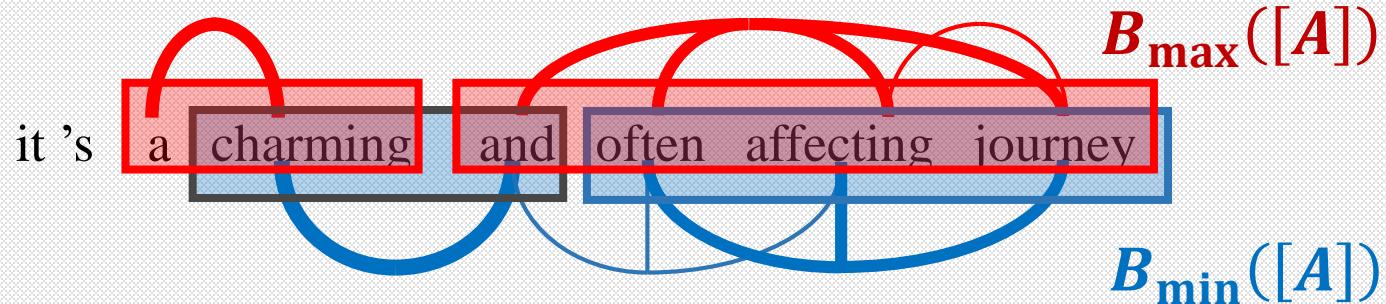
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**Properties** of multivariate Shapley interactions  $B([A])$ :

- **Linearity property :** If  $\forall S \subseteq N$ ,  $u(S) = v(S) + w(S)$ , then  $\forall A \subsetneq N$ ,  $B_u([A]) = B_v([A]) + B_w([A])$ .
- **Dummy property :** the dummy player has **no** interaction with other players. If  $\forall S \subseteq N \setminus \{i\}$ ,  $v(S \cup \{i\}) = v(S) + v(\{i\})$ , then  $\forall A \subsetneq N \setminus \{i\}$ ,  $B([A \cup \{i\}]) = B([A])$ .
- **Symmetry property :** symmetric players have **same** interaction with other players.  
If  $\forall S \subseteq N \setminus \{i, j\}$ ,  $v(S \cup \{i\}) = v(S \cup \{j\})$ , then  $\forall A \subsetneq N$ ,  $B([A \cup \{i\}]) = B([A \cup \{j\}])$



# Multivariate Shapley interactions

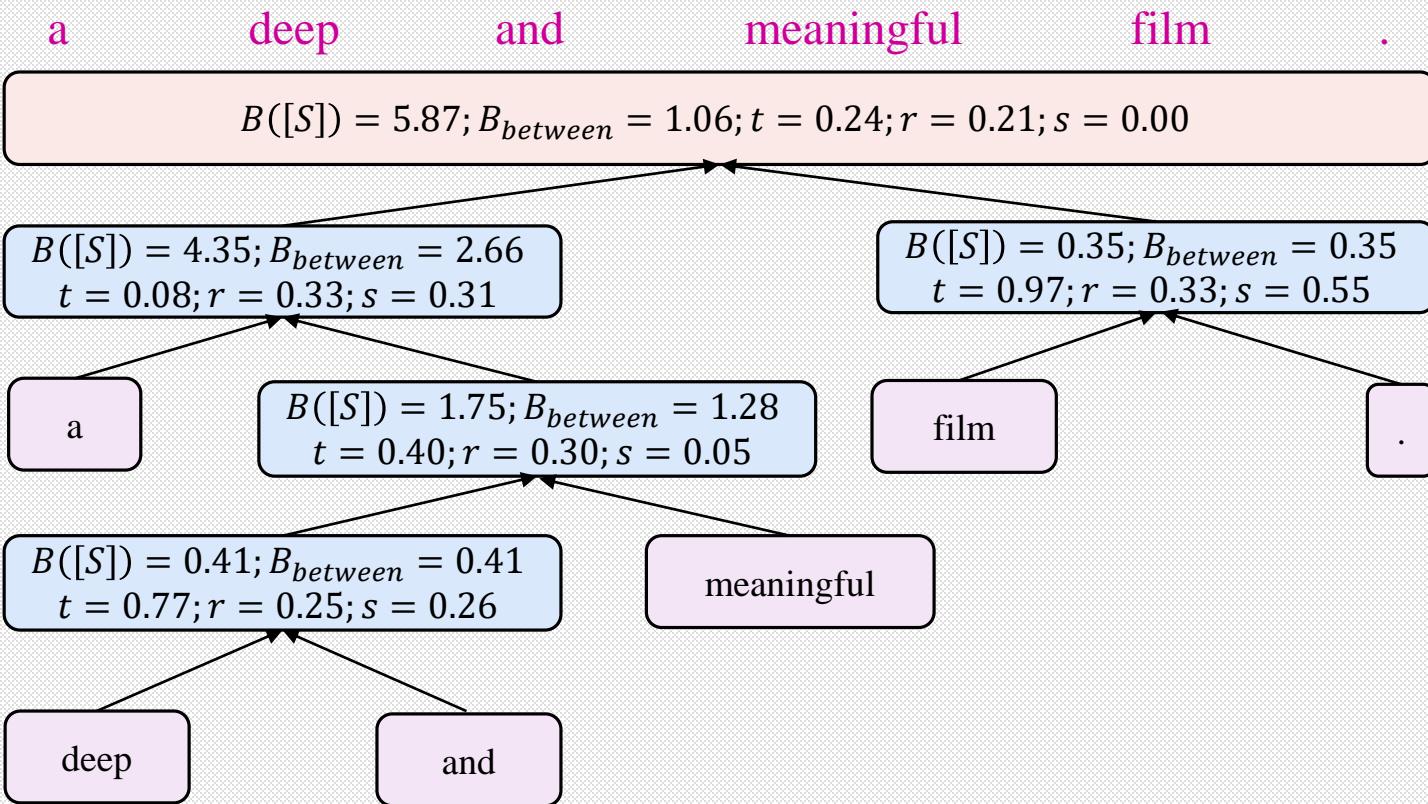


- $B_{\max}([A])$  reflects **positive interaction** inside  $[A]$ .
- $B_{\min}([A])$  reflects **negative interaction** inside  $[A]$ .
- $T([A]) = B_{\max}([A]) - B_{\min}([A])$
- $T([A])$  can measure both **positive** and **negative** interactions.
- We design an **effective** method to estimate the optimal partition and approximate  $T([A])$ .



## Explain the rationale of incorrect prediction

- **Multivariate interactions** can be used to extract tree structures that encoded interactions among words inside different DNNs.





# Explain the rationale of incorrect prediction

- **Multivariate interactions** show extract prototype features to help us understand the **incorrect predictions** of DNNs

**maximum (prototypes towards incorrect predictions):**

if steven soderbergh ' s ' solaris ' is a failure it is a glorious failure . predict: negative

**minimum (prototypes towards correct predictions):**

if steven soderbergh ' s ' solaris ' is a failure it is a glorious failure . label: positive

**maximum (prototypes towards incorrect predictions):**

the longer the movie goes , the worse it gets , but it ' s actually pretty good in the first few minutes. predict: positive

**minimum (prototypes towards correct predictions):**

the longer the movie goes , the worse it gets , but it ' s actually pretty good in the first few minutes. label: negative

**maximum (prototypes towards incorrect predictions):**

on the heels of the ring comes a similarly morose and humorless horror movie that , although flawed , predict: negative

is to be commended for its straight - ahead approach to creepiness .

**minimum (prototypes towards correct predictions):**

on the heels of the ring comes a similarly morose and humorless horror movie that , although flawed , label: positive

is to be commended for its straight - ahead approach to creepiness .



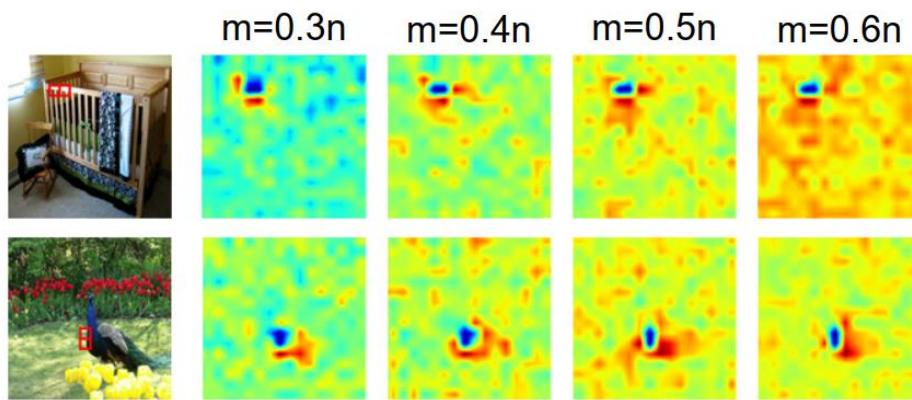
# Multi-order interactions to represent the complexity of representations

- We further define interactions of different orders as follows.

$$I^{(m)}(i, j) \stackrel{\text{def}}{=} \mathbb{E}_{S \subseteq N \setminus \{i, j\}, |S|=m} [\Delta v(S, i, j)]$$

$$I(i, j) = \frac{1}{n-1} \sum_{m=0}^{n-2} I^{(m)}(i, j)$$

$I^{(m)}(i, j)$  measures the average interaction between pixels  $(i, j)$  under all contexts consisting of  $m$  pixels.



**Low** order  $m$ : simple contextual collaborations with a few pixels → represents simple concepts;

**High** order  $m$ : complex contextual collaborations with massive pixels → represents complex concepts.



# Multi-order interactions: properties

## Properties of multi-order interactions

- **Marginal contribution property :**  $\forall i, j \in N, i \neq j, \phi^{(m+1)}(i|N) - \phi^{(m)}(i|N) = \mathbb{E}_{j \in N \setminus \{i\}} [I^{(m)}(i, j)]$
- **Accumulation property :**  $\phi^{(m)}(i|N) = \mathbb{E}_{j \in N \setminus \{i\}} \left[ \sum_{k=0}^{m-1} I^{(k)}(i, j) \right] + \phi^{(0)}(i|N)$
- **Efficiency property :**  $v(N) - v(\emptyset) = \sum_{i \in N} \phi^{(0)}(i|N) + \sum_{i \in N} \sum_{j \in N \setminus \{i\}} \left[ \sum_{k=0}^{n-2} \frac{n-1-k}{n(n-1)} I^{(k)}(i, j) \right]$
- **Linearity property :** If  $\forall S \subseteq N, u(S) = v(S) + w(S)$ , then  $I_u^{(m)}(i, j) = I_v^{(m)}(i, j) + I_w^{(m)}(i, j)$
- **Independency property :** If  $\forall S \subseteq N \setminus \{i\}, v(S \cup \{i\}) = v(S) + v(\{i\})$ , then  $\forall j \in N, I^{(m)}(i, j) = 0$
- **Symmetry property :** If  $\forall S \subseteq N, v(S \cup \{i\}) = v(S \cup \{j\})$ , then  $\forall k \in N \setminus \{i, j\}, I^{(m)}(i, k) = I^{(m)}(j, k)$
- **Summability property :**  $\phi^{(n-1)}(i|N) - \phi^{(0)}(i|N) = \mathbb{E}_{j \in N \setminus \{i\}} \left[ \sum_{m=0}^{n-2} I^{(m)}(i, j) \right] = I(N \setminus \{i\}, i) = \sum_{j \in N \setminus \{i\}} I(i, j)$



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How to define interactions in game theory?

# How to determine baseline values for the Shapley value?

What is the relationship between interactions and visual concepts?

What is the relationship between interactions and the aesthetic appreciation?

What is the relationship between interactions and the generalization?

What is the relationship between interactions and adversarial transferability?

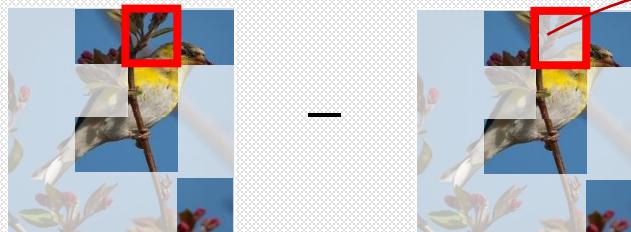
What is the relationship between interactions and adversarial robustness?



# Problem with baseline values

The marginal effects of the additional variable (red square)

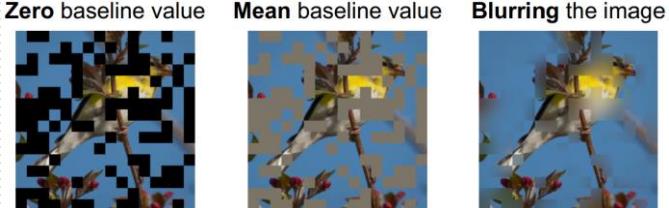
$$\phi_i = \sum_{S \subseteq N \setminus \{i\}} w_S [v(S \cup \{i\}) - v(S)]$$



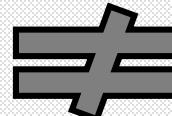
**Baseline values:** the value representing **the absence of the variables** (providing no signal to the model inference).

Previous settings of baseline values

- Zero
- Mean
- Blurring
- Depending on neighboring contexts  $S^{[1]}$ :



$$v(S) = E_{p(x'_{\bar{S}} | x_S)} [f(x_S \sqcup x'_{\bar{S}})]$$



Remove all information of variables w/o generating new edges/dots.

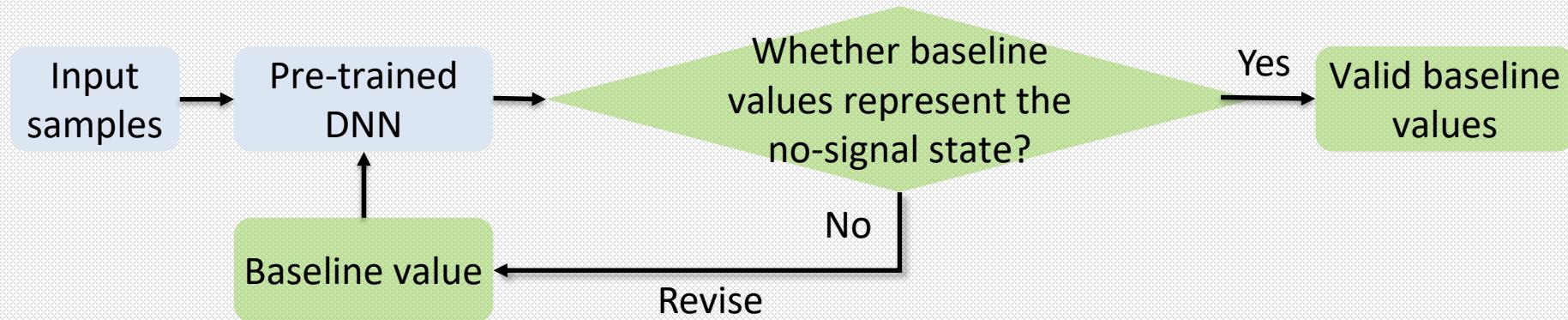


# Objective of learning baseline values

Input: A trained model and input samples

Output: Baseline values that satisfy the following two requirements:

- (1) retain the four axioms of Shapley values
- (2) push the baseline value towards representing **no-signal state** as much as possible.





## Multi-variate interaction

- The multi-variate interaction should ensure that

$$v(N) = v(\emptyset) + \sum_{S \subseteq N} I(S)$$

**Network output**

the benefit from all variables

a constant bias

the marginal benefit from the interaction of all variables in S

- Solution:

$$I(S) = \sum_{L \subseteq S} (-1)^{|S|-|L|} v(L)$$



## A new multi-variate interaction

- Transforming a DNN into an **AND-OR** representation.
- Decompose the overall utility of a DNN into utilities of different multi-variate interactions

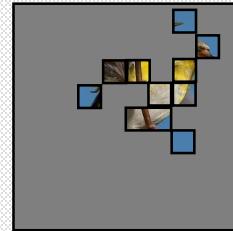
$$v(N) = v(\emptyset) + I(S_1) + I(S_2) + I(S_3) + \dots$$



Network output



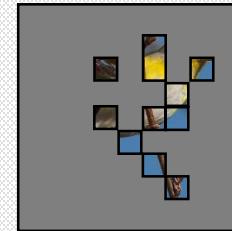
Constant bias



Elementary  
interaction  
component

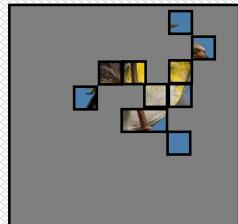


Elementary  
interaction  
component

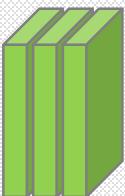


Elementary  
interaction  
component

$$\text{For } I(S_1) = \sum_{L \subseteq S_1} (-1)^{|S_1| - |L|} v(L)$$



...



Output  $v(S_1)$

A network



# Using interaction patterns to represent the no-signal state

---

- **Salient patterns  $I(S)$**  with significant influences,  $|I(S)|$  is large
- **Noisy patterns  $I(S)$ :** with little influences,  $|I(S)|$  is small

$$v(N) - v(\emptyset) = \sum_{S \subseteq N} I(S)$$

Learning baseline values that activate the least salient patterns → most likely to represent the no-signal state.



# Learning baseline values → to minimize the number of salient patterns

## Conclusion 1

The optimal baseline values



aim to

represent no-signal state

## Conclusion 2

How to represent signal state?



$$v(N) - v(\emptyset) = \sum_{S \subseteq N} I(S)$$

Using the number of salient patterns

Therefore, we can learn the baseline values that  
minimize the number of salient patterns

## Activation rate of different interaction patterns

Let  $\delta_i = 1$  denote the presence of the variable  $i$ , and let  $\delta_i = 0$  represent the absence of  $i$ . Let us consider a set of  $m$  variables. Let  $P(\delta_i = 1) = \frac{1}{2}$

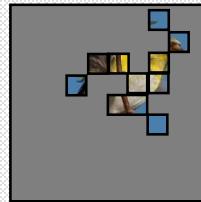
We can rewrite  $I(S)$  as

$$I(S) = \delta_1 \delta_2 \cdots \delta_m \cdot w_S \quad m = |S|$$

Then  $P(I(S) \neq 0) = P(\delta_1 = 1)P(\delta_2 = 1) \cdots P(\delta_m = 1) = 0.5^m$

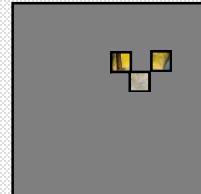
**We define  $m = |S|$  as the order of the interaction  $I(S)$ .**

- For high-order interactions, where  $m = |S|$  is large:



$$P(I(S) \neq 0) = \frac{1}{2} * \frac{1}{2} * \cdots = \frac{1}{2^{11}}$$

- For low-order interactions, where  $m = |S|$  is small:



$$P(I(S) \neq 0) = \frac{1}{2} * \frac{1}{2} * \frac{1}{2} = \frac{1}{2^3}$$



# Relationship between low-order interactions and high-order interactions

---

$$\nu(N) = \nu(\emptyset) + \underbrace{\sum_{S \in \Omega_{low}} I(S)}_{\text{Low-order interactions}} + \underbrace{\sum_{S \in \Omega_{high}} I(S)}_{\text{High-order interactions}}$$

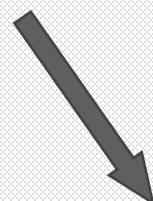
- $\Omega_{low} = \{S | |S| \leq threshold\}$ ,  $\Omega_{high} = \{S | |S| > threshold\}$



# Learning baseline values → to reduce low-order interaction patterns

$$v(N) = v(\emptyset) + \sum_{S \in \Omega_{low}} I(S) + \sum_{S \in \Omega_{high}} I(S)$$

High-order interactions  
→ Low activation rate  
→ Sparse activations



Reduce signals represented by low-order interactions

||

Strengthen signal represented by high-order interactions

||

Make most signals sparsely activated



## How to reduce low-order interactions

---

We prove that **low-order Shapley values only contain low-order interactions.**

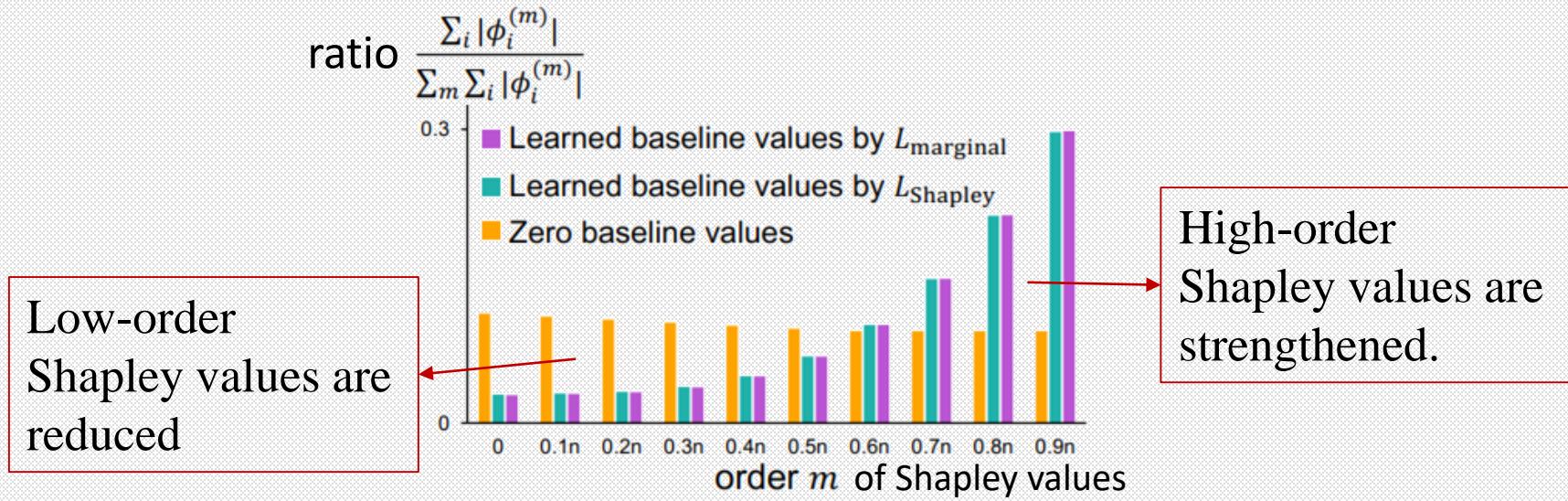
The m-order Shapley value  $\phi^{(m)}(i) = E_{S \subseteq N \setminus \{i\}, |S|=m} [\nu(S \cup \{i\}) - \nu(S)]$

The approximate-yet-efficient solution: penalizing low-order Shapley values

$$L_{Shapley} = \sum_{m \sim Unif(0; \lambda)} \sum_{x \in X} \sum_{i \in N} |\phi^{(m)}(i)|$$

# Verification

Objective: we aim to verify whether or not we can successfully reduce the ratio of low-order Shapley values and boost the influence of high-order Shapley values





# Connections between multi-variate interaction and other metrics

---

- Connecting the interaction  $I(S)$  to the Shapley value:

$$\text{The Shapley value } \phi(i) = \sum_{S:i \in S} \frac{1}{|S|} I(S)$$

- Connecting the interaction  $I(S)$  to the Shapley interaction index:

$$I_{shap}(S) = \sum_{T \subseteq N \setminus S} p(T) \sum_{L \subseteq S} (-1)^{|S|-|L|} v(L \cup T) = \sum_{T \subseteq N \setminus S} p(T) I(S|env(T))$$

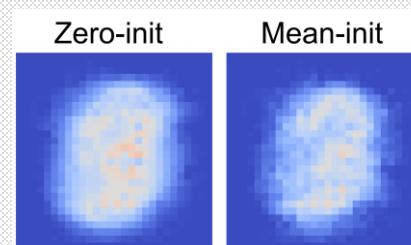
where  $I(S|env(T))$  denotes the specific interaction  $I(S)$  when variables in  $T$  are always presents.



# Experiments: learned baseline values and Shapley values

The baseline values learned by our method generated less noisy Shapley values than other methods

- On MNIST dataset
  - Learned baseline value: (shared by all MNIST images)



Focus on foreground

Shapley values based on different baseline values

| Input image | Zero Baseline value | Mean Baseline value | Blurring image | SHAP | SAGE magnitude $\times 2$ | Ours $L_{\text{shapley}}$ zero-init | Ours $L_{\text{shapley}}$ mean-init |
|-------------|---------------------|---------------------|----------------|------|---------------------------|-------------------------------------|-------------------------------------|
| 0           |                     |                     |                |      |                           |                                     |                                     |
| 1           |                     |                     |                |      |                           |                                     |                                     |
| 2           |                     |                     |                |      |                           |                                     |                                     |

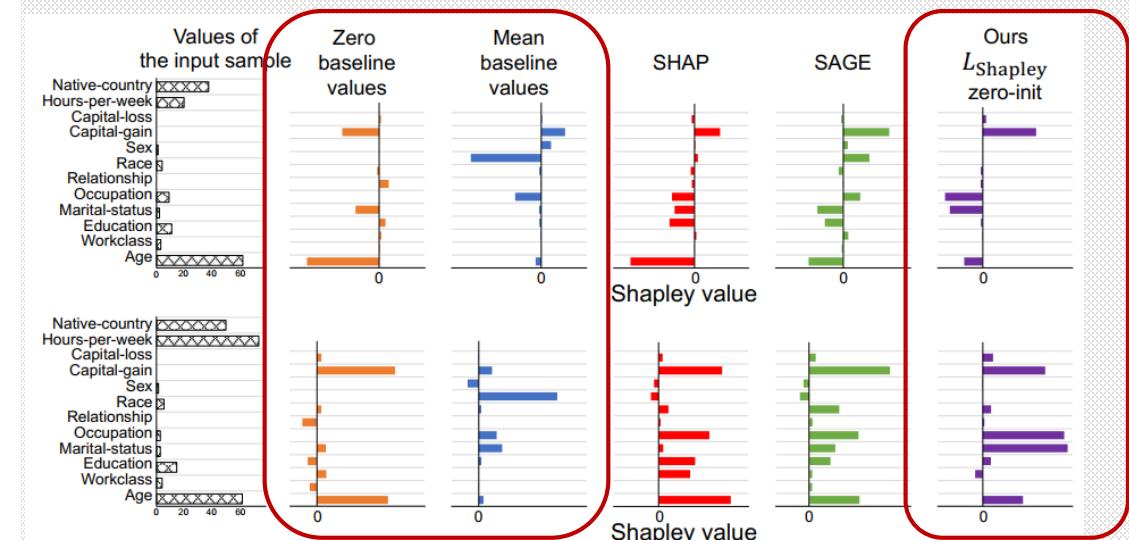
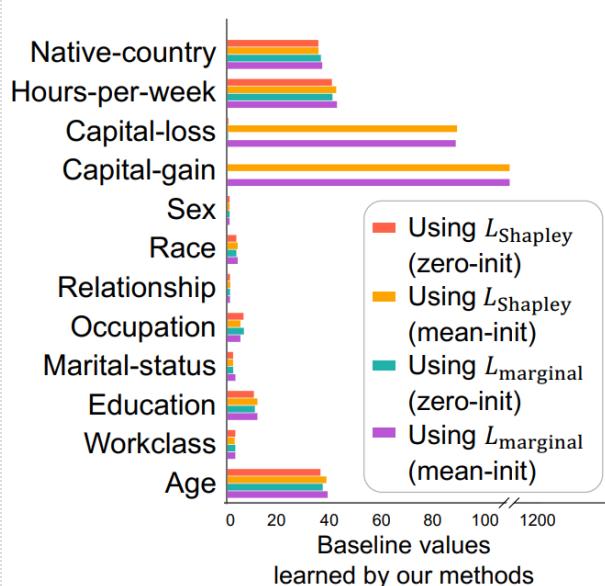
Less noise



# Experiments: learned baseline values and Shapley values

The learned baseline values generate Shapley values, which are consistent with SHAP and SAGE.

- On the UCI Census Income dataset **Shapley values**
  - Learned baseline value:



Contradict  
with other  
methods

Consistent  
with SHAP  
and SAGE



# Experiments: verification of the learned baseline values

## Verify the correctness of the learned baseline values

- On images, there are **no ground-truth baseline values** for verification.
- We generated functions, whose **ground truth of baseline values** could be easily determined.

| Functions ( $\forall i \in N, x_i \in \{0, 1\}$ )                                      |
|--|
| $-0.185x_1(x_2 + x_3)^{2.432} - x_4x_5x_6x_7$  |
| $-sigmoid(-4x_1 - 4x_2 - 4x_3 + 2.00) - 0.011x_4(x_5 + x_6 + x_7 + x_8 + x_9)^{2.341}$ |
| $-x_1x_2x_3 + sigmoid(-5x_4x_5x_6x_7 + 2.50) - x_8x_9$                                 |
| $-sigmoid(+4x_1 - 4x_2 + 4x_3 - 6.00) - x_4x_5x_6x_7 - x_8x_9x_{10}$                   |

| The ground truth of baseline values  |
|--|
| $b_i^* = 0$ for $i \in \{1, 2, 3, 4, 5, 6, 7\}$                                    |
| $b_i^* = 1$ for $i \in \{1, 2, 3\}$ , $b_i^* = 0$ for $i \in \{4, 5, 6, 7, 8, 9\}$ |
| $b_i^* = 1$ for $i \in \{4, 5, 6, 7\}$ , $b_i^* = 0$ for $i \in \{1, 2, 3, 8, 9\}$ |
| $b_i^* = 1$ for $i = 2$ , $b_i^* = 0$ for $i \in \{1, 3, 4, 5, 6, 7, 8, 9, 10\}$   |



# Experiments: verification of the learned baseline values

Verify the correctness of the learned baseline values ( $b_i^* \in \{0, 1\}$ )

- Metric: accuracy of the learned baseline values

$$\frac{1}{n} \sum_{i=1}^n [\mathbf{1}(b_i^* = 1 \& b_i > 0.5) + \mathbf{1}(b_i^* = 0 \& b_i < 0.5)]$$

Table 3: Accuracy of learned baseline values.

|                     | $L_{\text{Shapley}}$ |                     |                   | $L_{\text{marginal}}$ |                     |                   |
|---------------------|----------------------|---------------------|-------------------|-----------------------|---------------------|-------------------|
|                     | initialize with 0    | initialize with 0.5 | initialize with 1 | initialize with 0     | initialize with 0.5 | initialize with 1 |
| Synthetic functions | 98.06%               | 98.70%              | 98.70%            | 98.06%                | 98.14%              | 98.14%            |
| Functions in [47]   | 88.52%               | 91.80%              | 90.16%            | 86.89%                | 91.80%              | 90.16%            |

In most cases, the accuracy was above 90%, showing that **our method could effectively learn correct baseline values.**

# Can we unify all attribution methods using game-theoretic interactions?

Huiqi Deng  
Sun Yat-sen University





---

How to define interactions in game theory?

How to determine baseline values for the Shapley value?

# What is the relationship between interactions and visual concepts?

What is the relationship between interactions and the aesthetic appreciation?

What is the relationship between interactions and the generalization?

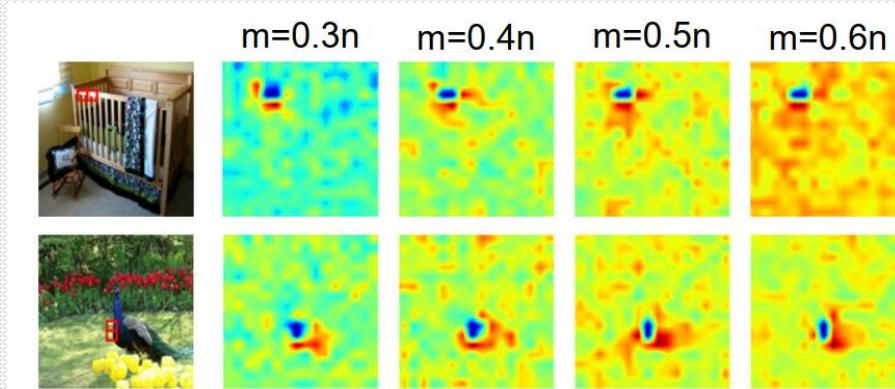
What is the relationship between interactions and adversarial transferability?

What is the relationship between interactions and adversarial robustness?



## Explaining textures, shapes, and beyond

- **Multi-order interaction:** measures the average interaction between pixels  $(i,j)$  under all contexts consisting of  $m$  pixels.



$$\text{M-order interaction} \quad I^{(m)}(i, j) \stackrel{\text{def}}{=} \mathbb{E}_{S \subseteq N \setminus \{i, j\}, |S|=m} [\Delta v(S, i, j)]$$

- Low-order interactions mainly reflect **simple and common** concepts.
- Middle-order interactions mainly represent **middle complex** concepts.
- High-order interactions mainly represent the **memory** of specific large-scale concepts.



# Multi-order interactions: properties

## Properties of multi-order interactions

- **Marginal contribution property :**  $\forall i, j \in N, i \neq j, \phi^{(m+1)}(i|N) - \phi^{(m)}(i|N) = \mathbb{E}_{j \in N \setminus \{i\}} [I^{(m)}(i, j)]$
- **Accumulation property :**  $\phi^{(m)}(i|N) = \mathbb{E}_{j \in N \setminus \{i\}} \left[ \sum_{k=0}^{m-1} I^{(k)}(i, j) \right] + \phi^{(0)}(i|N)$
- **Efficiency property :**  $v(N) - v(\emptyset) = \sum_{i \in N} \phi^{(0)}(i|N) + \sum_{i \in N} \sum_{j \in N \setminus \{i\}} \left[ \sum_{k=0}^{n-2} \frac{n-1-k}{n(n-1)} I^{(k)}(i, j) \right]$
- **Linearity property :** If  $\forall S \subseteq N, u(S) = v(S) + w(S)$ , then  $I_u^{(m)}(i, j) = I_v^{(m)}(i, j) + I_w^{(m)}(i, j)$
- **Independency property :** If  $\forall S \subseteq N \setminus \{i\}, v(S \cup \{i\}) = v(S) + v(\{i\})$ , then  $\forall j \in N, I^{(m)}(i, j) = 0$
- **Symmetry property :** If  $\forall S \subseteq N, v(S \cup \{i\}) = v(S \cup \{j\})$ , then  $\forall k \in N \setminus \{i, j\}, I^{(m)}(i, k) = I^{(m)}(j, k)$
- **Summability property :**  $\phi^{(n-1)}(i|N) - \phi^{(0)}(i|N) = \mathbb{E}_{j \in N \setminus \{i\}} \left[ \sum_{m=0}^{n-2} I^{(m)}(i, j) \right] = I(N \setminus \{i\}, i) = \sum_{j \in N \setminus \{i\}} I(i, j)$



## **What is the relationship between interactions and visual concepts?**

- **Understanding the encoding of textures**
- Understanding the difference between textures & shapes
- Understanding large-scale visual concepts
- Understanding outliers

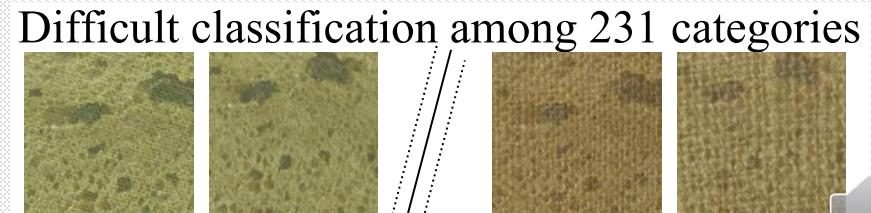
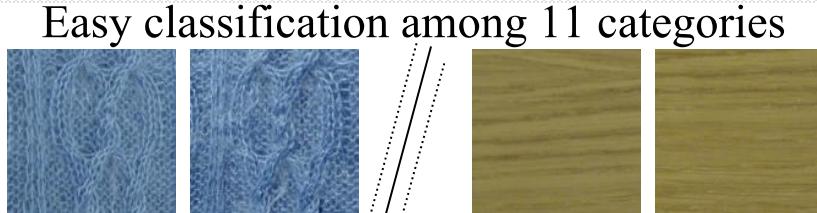




# Understanding the encoding of textures

- How does a DNN encodes textures?
  - **Low-order** interactions usually represent **common and widely-shared local** textures.
  - **Middle-order** interactions usually represent more **complex** textures.
- Hypothesis:

Compared to classify a few textures using low-order (simple) interactions, the classification of massive **fine-grained textures** usually forced a DNN to encode **fewer middle-order** interactions, which **subtly distinguish** fine-grained textures.





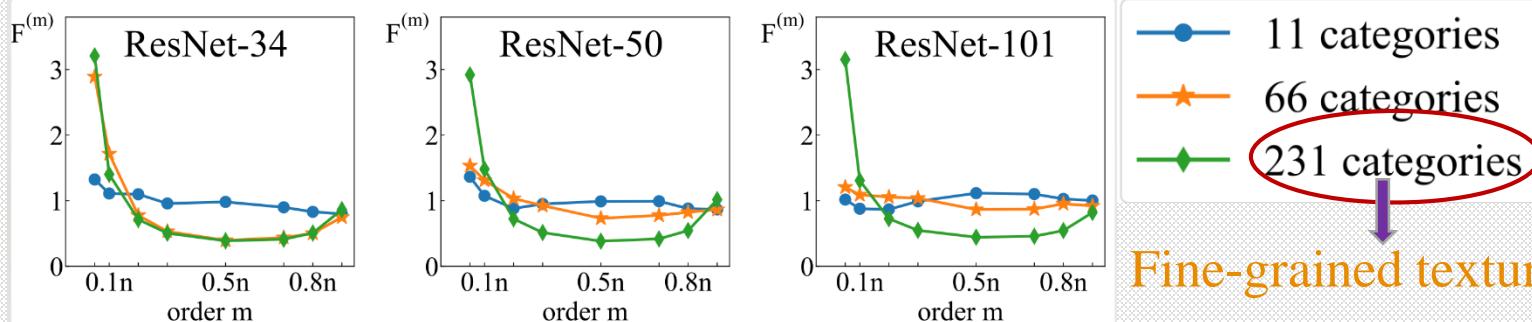
# Understanding the encoding of textures

- In order to verify hypothesis that **fine-grained texture classification** made the DNN encode **fewer but more complex middle-order** interactions.
  - The metric  $F^{(m)}$  → the relative strength of the m-th order

$$F^{(m)} = I_{\text{strength}}^{(m)} / \mathbb{E}_{m'}[I_{\text{strength}}^{(m')}],$$

$$I_{\text{strength}}^{(m)} = \mathbb{E}_{x \in \Omega} \left[ \mathbb{E}_{i,j} [|I^{(m)}(i, j|x)|] \right]$$

- Verification:



**Conclusion: The stricter encoding of fine-grained textures usually leads to fewer middle-order interactions.**



## • What is the relationship between interactions and visual concepts?

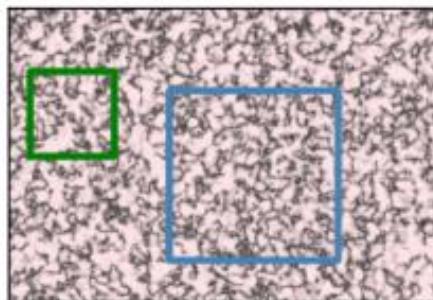
- Understanding the encoding of textures
- **Understanding difference between textures & shapes**
- Understanding large-scale visual concepts
- Understanding outliers



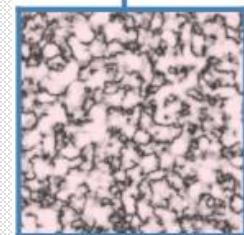
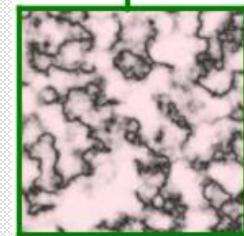
## ➤ Difference between textures & shapes

- Encoding textures is more flexible than encoding shapes.

### ➤ A large-scale texture



Can be modeled either as the ensemble of massive local textures.  
Or as the ensemble of a few middle-complex textures.



### ➤ A large-scale shape is usually encoded as the ensemble of middle-complex shapes.





# Difference between textures & shapes

---

- **Hypothesis:**  
If DNNs learned under **different noisy** conditions have **similar distributions** of the interaction orders, we consider the encoding of concepts is **not flexible**; otherwise, it is flexible.
- **Metric to verify hypothesis:**  
 $\Delta F^{(m)} = |F^{(m,noise)} - F^{(m)}|$  measures the difference of multi-order interaction strength between the DNN learned with noise and the DNN learned without noise.  
 A **large**  $\Delta F^{(m)}$  indicates the encoding of concepts is **flexible**.

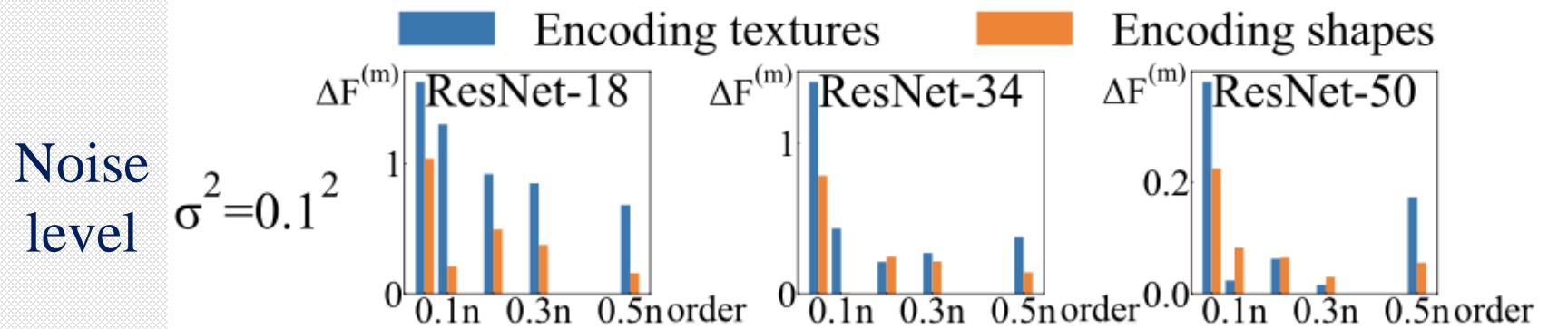
## ➤ Difference between textures & shapes

- Verification:

Compared with encoding shapes, encoding textures usually had **large  $\Delta F^{(m)}$**  values.



**Conclusion: Compared with encoding shapes,  
a DNN encodes textures with more flexibility.**





## **What is the relationship between interactions and visual concepts?**

- Understanding the encoding of textures
- Understanding the difference between textures & shapes
- **Understanding large-scale visual concepts**
- Understanding outliers





# Understanding large-scale visual concepts

- Concepts encoded as high-order interactions usually satisfy two requirements:
  1. Frequently appear in images, such as *sky or ocean*;
  2. The interaction between the background and the foreground is used for inference, such as *the interaction between the ocean and the red-breasted merganser*.

red-breasted  
merganser



(a)



(b)



(c)



(d)

Either only the foreground or only the background is not discriminative enough for inference.

# • Understanding large-scale visual concepts

- **Hypothesis:**

If a DNN **memorizes large-scale** concepts for inference, then this DNN is supposed to encode **more high-order interactions**.

- In order to verify this hypothesis, we construct two datasets.

- One dataset of classifying entire bird heads and partial bird heads forces the DNN to **hard memorize** the **entire large-scale concepts** for inference.
- The other dataset for the estimation of whether or not an image contains bird heads.

Entire  
bird heads



Partial  
bird heads



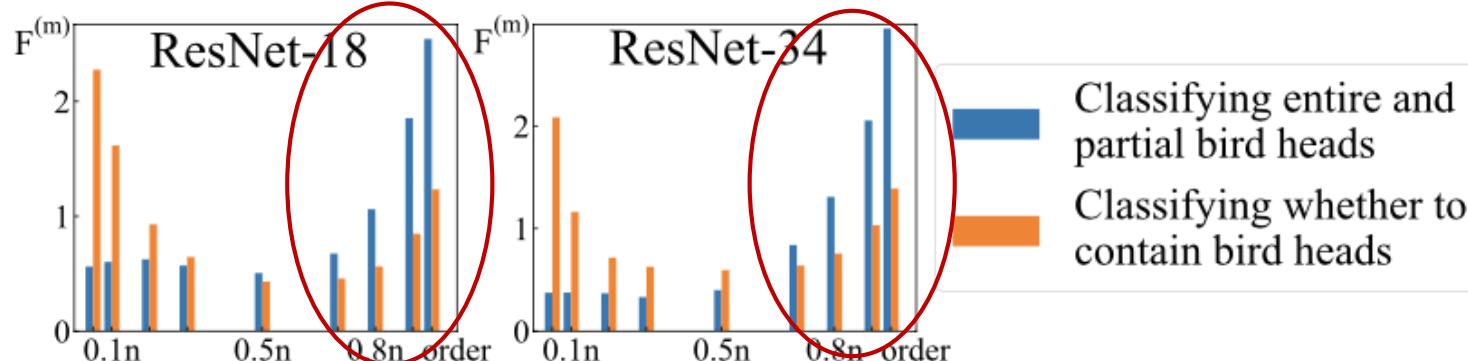
# • Understanding large-scale visual concepts

- **Metrics for verification:** Multi-order interaction strength  $F^{(m)}$ .

- **Verification:**

The classification of entire and partial bird heads encoded more high-order interactions.

→ **Conclusion: The DNN memorized large-scale concepts for inference usually encode more high-order interactions**





# • What is the relationship between interactions and visual concepts?

- Understanding the encoding of textures
- Understanding the difference between textures & shapes
- Understanding large-scale visual concepts
- **Understanding outliers**





## Understanding outliers

---

- Hypothesis:  
**The classification of outliers mainly depends on high-order interactions.**
- In order to verify this hypothesis, we construct **synthetic outliers**.
  - ✓ We add **negligible noises** to 50/100/200/300 randomly chosen training samples from Tiny ImageNet dataset, and assigned these noisy images with **random labels** to generate outliers.



## Understanding outliers

- Two **metrics** to verify the above hypothesis:

➤  $I_{avg}^{(m)}$ : measures the **average** m-order interaction.

$$I_{avg}^{(m)} = \mathbb{E}_{x \in \Omega} [\mathbb{E}_{i,j \in N} [I^{(m)}(i, j|x)]],$$

A **large**  $I_{avg}^{(m)}$  value indicates that the m-order interaction made a **significant contribution** to the classification.

➤  $P^{(m)}$ : measures the ratio of m-order interactions having **positive** effects among all m-order interactions.

$$P^{(m)} = \frac{\mathbb{E}_{x \in \Omega} \mathbb{E}_{i,j \in N} [\max(I^{(m)}(i, j|x), 0)]}{\mathbb{E}_{x \in \Omega} [\mathbb{E}_{i,j} [|I^{(m)}(i, j|x)|]]}.$$

A **large**  $P^{(m)}$  value indicates **more** m-order interactions contribute to the classification **positively**, *i.e.* being **more useful**.



## Understanding outliers

---

- In order to verify the hypothesis:

- ✓ We compare the difference of metrics  $I_{avg}^{(m)}$  and  $P^{(m)}$  between **outliers and normal samples**, i.e.

$$\Delta I_{avg}^{(m)} = I_{avg}^{(m,outlier)} - I_{avg}^{(m,normal)},$$
$$\Delta P^{(m)} = P^{(m,outlier)} - P^{(m,normal)}$$

- ✓ If  $\Delta I_{avg}^{(m)} > 0$  and  $\Delta P^{(m)} > 0$  for high order  $m$ , then the classification of outliers mainly depends on high-order interactions.

# • Understanding outliers

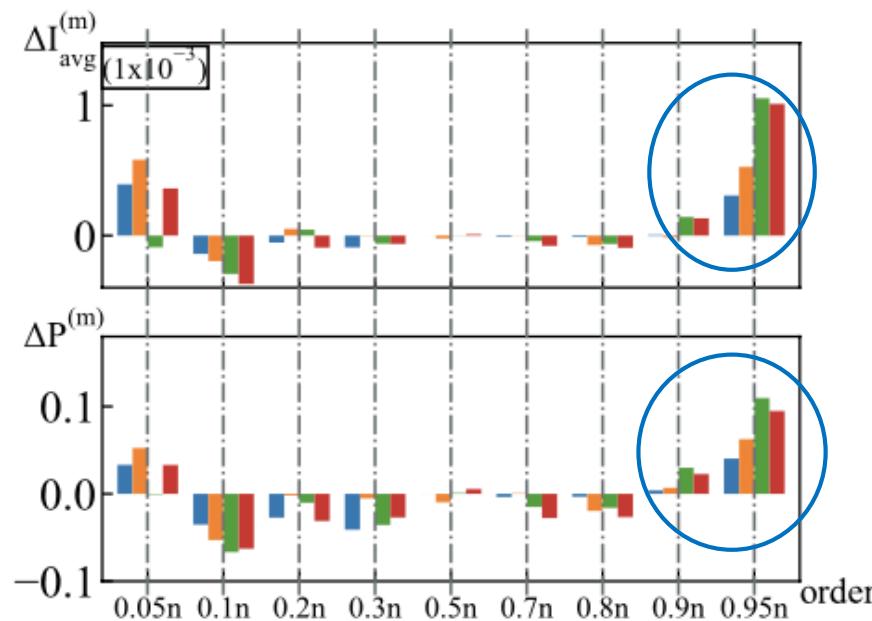
- Verification:

For DNNs trained using datasets contained 50/100/200/300 outliers,  $\Delta I_{avg}^{(m)} > 0$  and  $\Delta P^{(m)} > 0$ , when the order  $m > 0.8n$ .

**Conclusion: Compared to normal samples, the classification of outliers mainly depends on high-order interactions.**

(a) Trained on  
Tiny ImageNet  
dataset

- 50 outliers
- 100 outliers
- 200 outliers
- 300 outliers



# Can we learn meaningful features based on interactions?

Wen Shen  
Tongji University





---

How to define interactions in game theory?

How to determine baseline values for the Shapley value?

What is the relationship between interactions and visual concepts?

What is the relationship between interactions and the aesthetic appreciation?

# What is the relationship between interactions and the generalization?

What is the relationship between interactions and adversarial transferability?

What is the relationship between interactions and adversarial robustness?



# The Link between Interactions and the Network's Generalization Ability

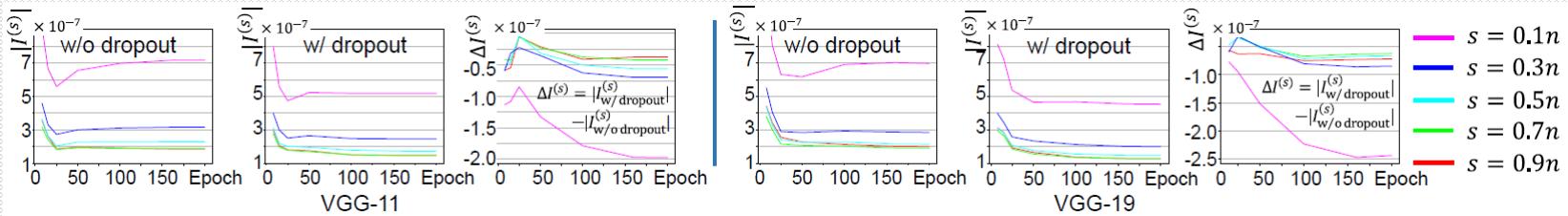
---

- Theoretically prove that Dropout can decrease the strength of interactions modeled by DNNs
- There is a negative correlation between the strength of interactions and the generalization ability of the network
- The generalization ability of the network can be enhanced by directly controlling the strength of interactions



# Overfitting → Strong Interactions

Dropout can **decrease** the strength of interactions modeled by DNNs



The relationship between  
interactions and the generalization  
ability:  
over-fitting → more interactions

| Dataset       | Model | Ordinary              | Over-fitted           |
|---------------|-------|-----------------------|-----------------------|
| MNIST         | RN-44 | $2.17 \times 10^{-3}$ | $3.64 \times 10^{-3}$ |
| Tiny-ImageNet | RN-34 | $2.57 \times 10^{-3}$ | $2.89 \times 10^{-3}$ |
| CelebA        | RN-34 | $6.46 \times 10^{-3}$ | $1.17 \times 10^{-2}$ |



# Suppressing Interactions → Boosting the generalization power

---

Enhance the generalization ability of the network by directly suppressing the interactions modeled by the network:

$$\text{Loss} = \text{Loss}_{\text{classification}} + \lambda \text{Loss}_{\text{interaction}}$$

$$\text{Loss}_{\text{interaction}} = \mathbb{E}_{i,j \in N, i \neq j} [|I(i,j)|]$$

$$= \mathbb{E}_{i,j \in N, i \neq j} \left[ \left| \sum_{S \subseteq N \setminus \{i,j\}} P_{\text{Shapley}}(S | N \setminus \{i,j\}) [\Delta f(S, i, j)] \right| \right]$$

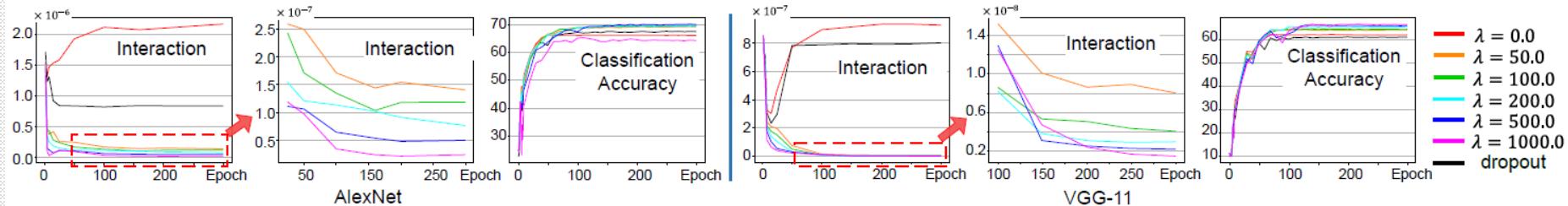
Based on the interactions, we improve the utility of dropout

- Explicitly control the DNN between over-fitting and under-fitting.
- Solve the issue that dropout is not compatible with batch normalization

Two advantages



# Suppressing Interactions → Boosting the generalization power



| $\lambda$ | AlexNet <sup>2</sup> | VGG-11 <sup>2</sup> | VGG-13 <sup>2</sup> | VGG-16 <sup>2</sup> |
|-----------|----------------------|---------------------|---------------------|---------------------|
| 0.0       | 66.2                 | 61.9                | 60.8                | 62.0                |
| 50.0      | 69.2                 | 63.9                | 64.0                | 63.8                |
| 100.0     | 69.6                 | 64.3                | 65.4                | 64.5                |
| 200.0     | 69.6                 | 65.3                | 65.9                | 64.7                |
| 500.0     | <b>70.0</b>          | 65.9                | <b>66.2</b>         | <b>64.9</b>         |
| 1000.0    | 64.3                 | <b>66.3</b>         | 66.0                | 64.5                |
| Dropout   | 67.5                 | 60.9                | 60.9                | 63.0                |

| $\lambda$ | RN-18 <sup>2</sup> | RN-34 <sup>2</sup> | $\lambda$ | VGG-16      | VGG-19      |
|-----------|--------------------|--------------------|-----------|-------------|-------------|
| 0.0       | 48.8               | 45.6               | 0.0       | 33.4        | 37.6        |
| 0.001     | 50.0               | 48.4               | 50.0      | 38.4        | 38.2        |
| 0.003     | 49.6               | 49.0               | 100.0     | 38.0        | 38.6        |
| 0.01      | <b>52.2</b>        | <b>49.6</b>        | 200.0     | 38.2        | 39.0        |
| 0.03      | 50.4               | 48.8               | 500.0     | <b>42.8</b> | 41.8        |
|           |                    |                    | 1000.0    | 40.8        | <b>45.2</b> |
| Dropout   | 47.4               | 46.0               | Dropout   | 36.8        | 32.6        |

| $\lambda$ | VGG-13      | VGG-16      | $\lambda$ | RN-18       |
|-----------|-------------|-------------|-----------|-------------|
| 0.0       | 94.6        | 93.7        | 0.0       | 92.7        |
| 5.0       | 94.8        | 93.8        | 0.001     | 93.0        |
| 10.0      | 94.7        | <b>94.6</b> | 0.003     | <b>93.1</b> |
| 20.0      | <b>94.9</b> | 94.1        | 0.01      | 93.0        |
| 50.0      | 94.7        | 94.08       | 0.03      | 92.9        |
| 100.0     | 94.7        | 94.3        |           |             |
| Dropout   | 94.6        | 92.4        | Dropout   | 92.1        |

Under-fitting      Over-fitting ↑



---

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What is the relationship between interactions and the aesthetic appreciation?

What is the relationship between interactions and the generalization?

# What is the relationship between interactions and adversarial transferability?

What is the relationship between interactions and adversarial robustness?

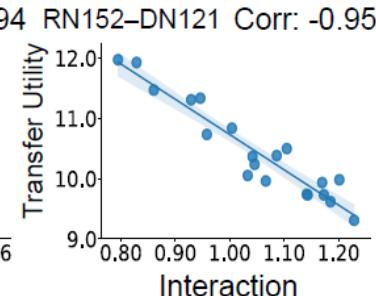
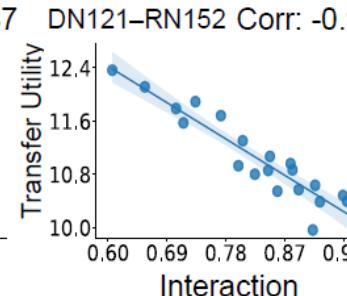
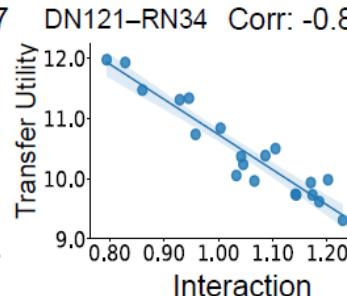
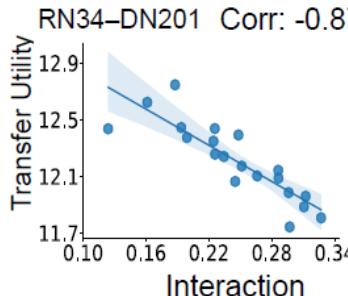
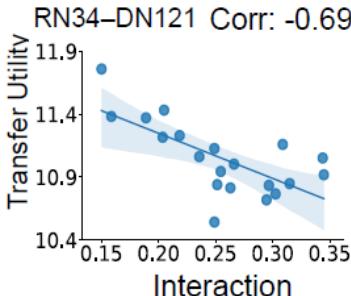




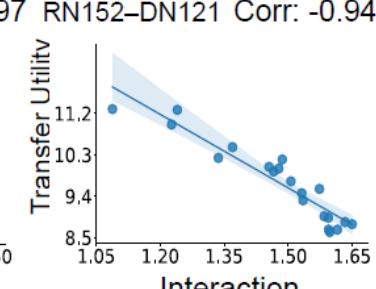
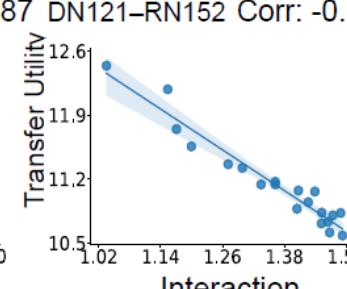
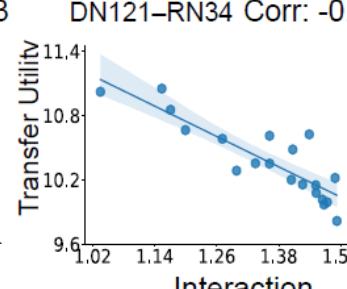
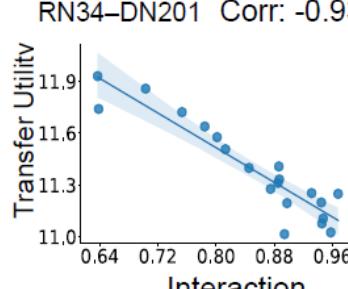
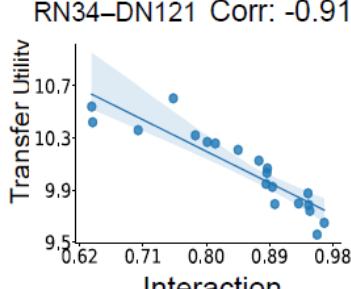
# The negative correlation between the interaction and the adversarial transferability

- Theoretical foundations : Multi-step attacks vs. Single-step attacks
  - Interaction : Multi-step attacks > Single-step attacks
  - Overfitting : Multi-step attacks > Single-step attacks<sup>[1]</sup>
- Empirical verification :

Setting 1



Setting 2



[1] Cihang Xie, Zhishuai Zhang, Yuyin Zhou, Song Bai, Jianyu Wang, Zhou Ren, and Alan L Yuille. Improving transferability of adversarial examples with input diversity. In Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition, pp. 2730–2739, 2019.

# 去芜存菁 Common essence: the reduction of interactions is the common mechanism of previous transferability-boosting methods

- Existing transferability-boosting methods can be approximately explained as the reduction of interactions.
  - Theoretically prove the attack based on momentum (MI Attack)<sup>[2]</sup>
  - Theoretically prove the attack based on smooth of gradients (VR Attack)<sup>[3]</sup>
  - Theoretically prove the attack based on skip connections (SGM Attack)<sup>[4]</sup>
  - Empirically verify the attack based on Translation-invariant (TI Attack)<sup>[5]</sup>
  - Empirically verify the attack based on Input diversity (DI Attack)<sup>[6]</sup>

## Proposition 1

The adversarial perturbation generated by the multi-step attack is given as  $\delta_{multi}^m = \alpha \sum_{t=0}^{m-1} \nabla_x l(h(x + \delta_{multi}^t), y)$ , where  $\delta_{multi}^t$  denotes the perturbation after the t-th step of updating, and m is referred to as the total number of steps. The adversarial perturbation generated by the single-step attack is given as  $\delta_{single} = \alpha m \nabla_x l(h(x), y)$ . Then, the expectation of interactions between perturbation units in  $\delta_{multi}^m$ ,  $\mathbb{E}_{a,b}[I_{ab}(\delta_{multi}^m)]$ , is larger than  $\mathbb{E}_{a,b}[I_{ab}(\delta_{single})]$ , i.e.  $\mathbb{E}_{a,b}[I_{ab}(\delta_{multi}^m)] \geq \mathbb{E}_{a,b}[I_{ab}(\delta_{single})]$ .

## Proposition 2

The adversarial perturbation generated by the multi-step attack is given as  $\delta_{multi}^m = \alpha \sum_{t=0}^{m-1} \nabla_x l(h(x + \delta_{multi}^t), y)$ . The adversarial perturbation generated by the VR Attack is computed as  $\delta_{vr}^m = \alpha \sum_{t=0}^{m-1} \nabla_x \hat{l}(h(x + \delta_{vr}^t), y)$ , where  $\hat{l}(h(x +$

## Proposition 3

The adversarial perturbation generated by the multi-step attack is given as  $\delta_{multi}^m = \alpha \sum_{t=0}^{m-1} \nabla_x l(h(x + \delta_{multi}^t), y)$ . The adversarial perturbation generated by the multi-step attack incorporating the momentum is computed as  $\delta_{mi}^m = \alpha \sum_{t=0}^{m-1} g_{mi}^t$ . Perturbation units of  $\delta_{mi}^m$  tend to exhibit smaller interactions than  $\delta_{multi}^m$ , i.e.  $\mathbb{E}_x \mathbb{E}_{a,b}[I_{ab}(\delta_{mi}^m)] < \mathbb{E}_x \mathbb{E}_{a,b}[I_{ab}(\delta_{multi}^m)]$ .

[2] Yinpeng Dong, Fangzhou Liao, and et al. Boosting adversarial attacks with momentum. In CVPR, 2018.

[3] Lei Wu, Zhanxing Zhu, and Cheng Tai. Understanding and enhancing the transferability of adversarial examples. arXiv preprint arXiv:1802.09707, 2018.

[4] Dongxian Wu, Yisen Wang, and et al. Skip connections matter: On the transferability of adversarial examples generated with resnets. In ICLR, 2020.

[5] Yinpeng Dong, Tianyu Pang, and et al. Evading defenses to transferable adversarial examples by translation-invariant attacks. In CVPR, 2019.

[6] Cihang Xie, Zhishuai Zhang, and et al. Improving transferability of adversarial examples with input diversity. In CVPR, 2019.



# Application: Penalizing interactions to improve adversarial transferability

- With the additional interaction-reduction loss, the PGD attack improves **more than 10%** adversarial transferability.
- Combining existing methods with the interaction-reduction loss, the adversarial transferability is improved from **54.6%-98.8%** to **70.2%-99.1%**

| Source | Method   | VGG-16          | RN152           | DN-201          | SE-154          | IncV3           | IncV4           | IncResV2        |
|--------|----------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| RN-34  | MI       | 80.1±0.5        | 73.0±2.3        | 77.7±0.5        | 48.9±0.8        | 46.2±1.2        | 39.9±0.5        | 34.8±2.5        |
|        | VR       | 88.8±0.2        | 86.4±1.6        | 87.9±2.4        | 62.1±1.5        | 58.4±3.0        | 56.3±2.3        | 49.7±0.9        |
|        | SGM      | 91.8±0.6        | 89.0±0.9        | 90.0±0.4        | 68.0±1.4        | 63.9±0.3        | 58.2±1.1        | 54.6±1.2        |
|        | SGM+IR   | 94.7±0.6        | 91.7±0.6        | 93.4±0.8        | 72.7±0.4        | 68.9±0.9        | 64.1±1.3        | 61.3±1.0        |
|        | HybridIR | <b>96.5±0.1</b> | <b>94.9±0.3</b> | <b>95.6±0.6</b> | <b>79.7±1.0</b> | <b>77.1±0.8</b> | <b>73.8±0.1</b> | <b>70.2±0.5</b> |
| RN-152 | MI       | 70.3±0.6        | —               | 74.8±1.4        | 51.7±0.8        | 47.1±0.9        | 40.5±1.6        | 36.8±2.7        |
|        | VR       | 83.9±3.4        | —               | 91.1±0.9        | 70.0±3.7        | 63.1±0.9        | 58.8±0.1        | 56.2±1.3        |
|        | SGM      | 88.2±0.5        | —               | 90.2±0.3        | 72.7±1.4        | 63.2±0.7        | 59.1±1.5        | 58.1±1.2        |
|        | SGM+IR   | 92.0±1.0        | —               | 92.5±0.4        | 79.3±0.1        | 69.6±0.8        | 66.2±1.0        | 63.6±0.9        |
|        | HybridIR | <b>95.3±0.4</b> | —               | <b>96.9±0.2</b> | <b>84.7±0.7</b> | <b>80.0±1.2</b> | <b>77.5±0.8</b> | <b>75.6±0.6</b> |
| DN-121 | MI       | 83.0±4.9        | 72.0±0.7        | 91.5±0.2        | 58.4±2.6        | 54.6±1.6        | 49.2±2.4        | 43.9±1.5        |
|        | VR       | 91.5±0.5        | 88.7±0.5        | 98.8±0.2        | 75.1±1.3        | 74.3±1.7        | 75.6±3.0        | 69.8±1.3        |
|        | SGM      | 88.7±0.9        | 88.1±1.0        | 98.0±0.4        | 78.0±0.9        | 64.7±2.5        | 65.4±2.3        | 59.7±1.7        |
|        | SGM+IR   | 91.7±0.2        | 90.4±0.4        | 94.3±0.1        | 87.0±0.4        | 78.8±1.3        | 79.5±0.2        | 75.8±2.7        |
|        | HybridIR | <b>96.9±0.4</b> | <b>96.8±0.4</b> | <b>99.1±0.4</b> | <b>90.9±0.5</b> | <b>88.4±0.8</b> | <b>87.8±0.8</b> | <b>87.1±0.4</b> |
| DN-201 | MI       | 77.3±0.8        | 74.8±1.4        | —               | 64.6±1.0        | 56.5±2.5        | 51.1±2.1        | 47.8±1.9        |
|        | VR       | 87.3±1.1        | 90.4±1.2        | —               | 78.0±1.5        | 75.8±2.1        | 75.8±1.3        | 71.3±1.2        |
|        | SGM      | 87.3±0.3        | 92.4±1.0        | —               | 82.9±0.2        | 72.3±0.3        | 71.3±0.6        | 68.8±0.5        |
|        | SGM+IR   | 89.5±0.9        | 91.8±0.7        | —               | 87.3±1.2        | 82.5±0.8        | 80.3±0.3        | 81.5±0.5        |
|        | HybridIR | <b>94.4±0.1</b> | <b>96.9±0.5</b> | —               | <b>91.7±0.2</b> | <b>89.6±0.6</b> | <b>88.3±0.3</b> | <b>87.3±0.7</b> |



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between interactions  
and adversarial robustness?**



# Previous explanations of adversarial robustness

□ Previous explanations lack an essential and unified explanation.

What is the essence  
of adversarial attacks  
and defense?

How to explain  
adversarial robustness  
from the perspective of  
feature representations?

- Explaining **adversarial examples**
  - Linearity of feature representations
  - Non-robust but discriminative features
- Explaining **adversarial training**
  - Learning general shapes of objects
  - Enumeration of all possible adversarial examples

- Explaining **adversarial robustness**
  - Proving the theoretical bound

[1] Yanpei Liu, Xinyun Chen, Chang Liu, and Dawn Song. Delving into transferable adversarial examples and black-box attacks. ICLR, 2016.

[2] Lei Wu, Zhanxing Zhu, and Cheng Tai. Understanding and enhancing the transferability of adversarial examples. arXiv preprint arXiv:1802.09707, 2018.

[3] Ambra Demontis, Marco Melis, Maura Pintor, Matthew Jagielski, Battista Biggio, Alina Oprea, Cristina Nita-Rotaru, and Fabio Roli. Why do adversarial attacks transfer? explaining transferability of evasion and poisoning attacks. In 28th USENIX Security Symposium USENIX Security, pp. 321–338, 2019.



## Contributions of this paper

---

- We discover that **adversarial attacks** mainly affect high-order interactions between input variables.
- The adversarial **training** boosts the robustness of DNNs by learning more discriminative low-order interactions.
- We propose a **unified explanation** for several adversarial defense methods.





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# Adversarial attacks mainly affect high-order interactions

Given a normal sample  $x$ , let  $\tilde{x} = x + \delta$  denotes its adversarial example.

Decompose the total adversarial utility of perturbations into attacking utilities on different interactions of different orders:

$$\Delta v(N|x) = v(N|x) - v(N|\tilde{x}) = \sum_{i \in N} \Delta \phi^{(0)}(i|N, x) + \sum_{i,j \in N, i \neq j} \sum_{m=0}^{n-2} \Delta J_{ij}^{(m)},$$
$$\Delta J_{ij}^{(m)} = \frac{n-1-m}{n(n-1)} \Delta I_{ij}^{(m)}$$


Small and can be ignored

$$\Delta I_{ij}^{(m)} = I_{ij}^{(m)}(x) - I_{ij}^{(m)}(\tilde{x})$$



# Adversarial attacks mainly affect high-order interactions

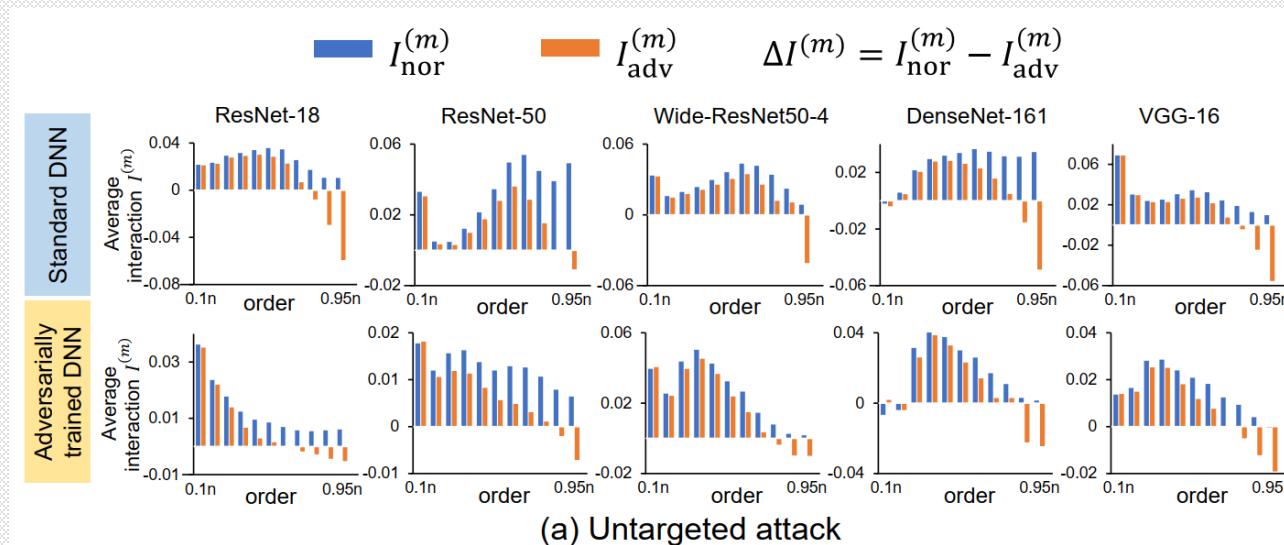


Figure: The multi-order interaction in normal samples and that in adversarial examples of standard DNNs and adversarially trained DNNs.

We discover that **adversarial attacks** mainly affect high-order interactions between input variables.



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# Adversarial training boosts the robustness of high-order interactions

Attacking utility of  $m$ -order interactions:  $\Delta J_{ij}^{(m)} = \frac{n-1-m}{n(n-1)} \Delta I_{ij}^{(m)}$

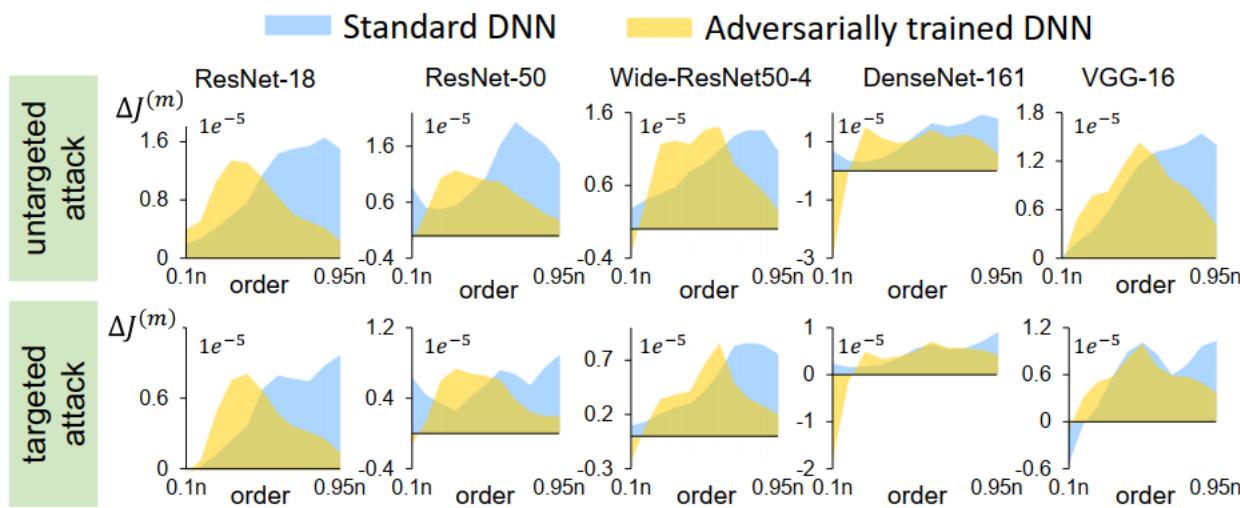


Figure: Distribution of compositional attacking utilities caused by interactions of different orders in standard DNNs and adversarially trained DNNs.

In adversarially learned DNNs, attacking utilities of high-order interactions significantly decreased.

# Adversarial training learns more reliable low-order interactions to boost the robustness of high-order interactions



Disentanglement: whether  $m$ -order interactions represent the information of a specific category.

In adversarially learned DNNs, low-order interactions exhibited higher disentanglement  $\rightarrow$  more category-specific  $\rightarrow$  strengthen the robustness of high-order interactions.

$$D^{(m)} = \mathbb{E}_{x \in \Omega} \mathbb{E}_{\substack{i,j \in N \\ i \neq j}} \frac{|I_{ij}^{(m)}(x)|}{\sum_{S \subseteq N \setminus \{i,j\}, |S|=m} |\Delta v(i, j, S|x)|}$$
$$= \mathbb{E}_{x \in \Omega} \mathbb{E}_{\substack{i,j \in N \\ i \neq j}} \frac{\left| \sum_{S \subseteq N \setminus \{i,j\}, |S|=m} \Delta v(i, j, S|x) \right|}{\sum_{S \subseteq N \setminus \{i,j\}, |S|=m} |\Delta v(i, j, S|x)|}$$

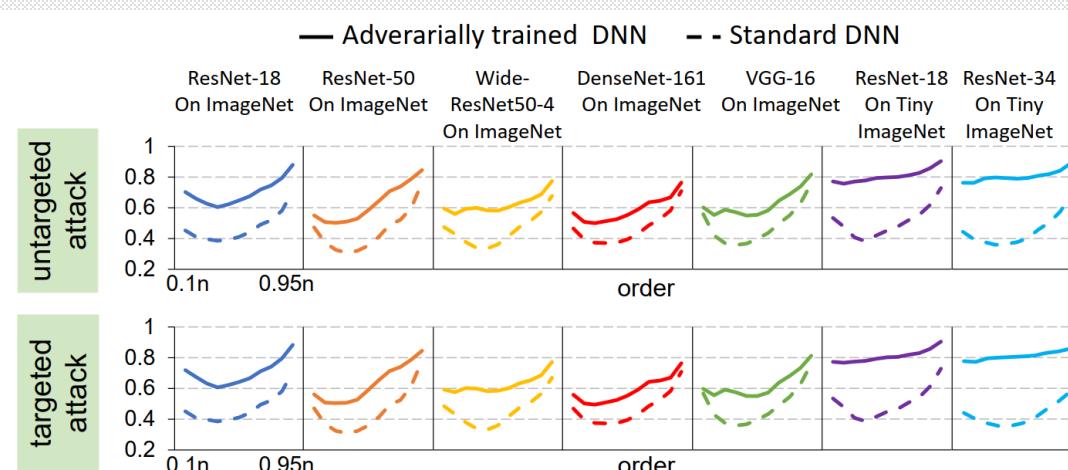


Figure: The interaction disentanglement.



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# 去芜存菁 The unified explanation for previous adversarial defenses

- Attribution-based method for detecting adversarial examples: ML-LOO [1]
- Rank-based method for detecting adversarial examples [2]



Detecting the **highest-order interaction** (the most sensitive component).

- Cutout method [3]
- High recoverability of adversarial examples in adversarially trained DNNs



Utilizing discriminative low-order interactions and **removing sensitive high-order interactions** boost the robustness.

[1] Puyudi Yang, Jianbo Chen, Cho-Jui Hsieh, Jane-Ling Wang, and Michael I. Jordan. ML-LOO: detecting adversarial examples with feature attribution. CoRR, abs/1906.03499, 2019.

[2] Malhar Jere, Maghav Kumar, and Farinaz Koushanfar. A singular value perspective on model robustness. arXiv preprint arXiv:2012.03516, 2020.

[3] Terrance DeVries and Graham W Taylor. Improved regularization of convolutional neural networks with cutout. arXiv preprint arXiv:1708.04552, 2017.

# Thank you

