## Learning Distance Structure for Ordinal Data Clustering

## Experimental Results and Space Complexity Analysis –

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1. Experimental Results with Significance Test: The symbol • indicates that the performance of the proposed DSLC is significantly different from the second best one according to Wilcoxon signed-rank test at 95% confidence interval (please note that Wilcoxon signed-rank test at 95% confidence interval is commonly adopted by existing works for testing the significance of clustering performance). All the compared methods are run 10 times, and all the demonstrated results are the mean of the 10 runs. The best and second best results are indicated by boldface and underline, respectively.

It can be observed that, DSLC significantly outperforms its counterparts on all the data sets in terms of ARI and NMI.

Table 1: ARI performance of the Type-1 approaches and DSLC.

Data	KMDH	KMDC	KMDJ	KMDE	WKMDH	WKMDC	WKMDE	CWO	DSLC
IQ	$-0.01\pm0.02$	$-0.01\pm0.01$	$0.007\pm0.01$	$0.044 \pm 0.04$	$-0.01\pm0.01$	$-0.02\pm0.00$	$0.072\pm0.09$	$-0.02\pm0.02$	0.171±0.12•
PE	$0.105 \pm 0.05$	$0.135 {\pm} 0.09$	$0.069 \pm 0.03$	$0.222 \pm 0.09$	$0.091 \pm 0.08$	$0.131 {\pm} 0.08$	$0.166{\pm}0.13$	$0.132 \pm 0.10$	$0.285{\pm}0.03$
AE	$0.140 \pm 0.07$	$0.147{\pm}0.08$	$0.115 \pm 0.02$	$0.270 \pm 0.06$	$0.144 {\pm} 0.07$	$0.139 \pm 0.07$	$0.279 \pm 0.07$	$0.118 {\pm} 0.05$	0.311±0.10•
PT	$0.073 \pm 0.01$	$0.068 {\pm} 0.01$	$0.074 \pm 0.01$	$0.078 \pm 0.01$	$0.055 {\pm} 0.02$	$0.063 \pm 0.02$	$0.045 {\pm} 0.02$	$0.084 \pm 0.00$	$0.086{\pm}0.00{ extstyle }$
BC	$0.015 \pm 0.04$	$0.103 \pm 0.07$	$0.095 {\pm} 0.07$	$0.042 {\pm} 0.06$	$0.043 \pm 0.06$	$0.103 \pm 0.07$	$0.057 \pm 0.07$	$0.127 \pm 0.07$	$0.149 \pm 0.07$
CE	$-0.01\pm0.01$	-	$0.037 \pm 0.03$	$0.031 \pm 0.03$	$0.010 \pm 0.01$	-	$0.029 \pm 0.02$	$0.013 \pm 0.00$	$0.072 \pm 0.06$
NS	$0.054 \pm 0.02$	-	$0.074\pm0.03$	$0.075 \pm 0.03$	$0.084 \pm 0.11$	-	$0.082 {\pm} 0.08$	$0.003 \pm 0.00$	0.150±0.10•
LE	$0.039 \pm 0.02$	$0.034 \pm 0.02$	$0.040 \pm 0.02$	$0.069 \pm 0.02$	$0.038\pm0.02$	$0.031 \pm 0.01$	$\underline{0.072{\pm}0.03}$	$0.050 \pm 0.03$	0.081±0.01•

Table 2: NMI performance of the Type-1 approaches and DSLC.

Data	KMDH	KMDC	KMDJ	KMDE	WKMDH	WKMDC	WKMDE	CWO	DSLC
IQ	$0.012\pm0.01$	$0.004\pm0.00$	$0.014\pm0.01$	$0.046\pm0.04$	$0.018\pm0.02$	$0.003\pm0.00$	$0.069\pm0.07$	$0.023\pm0.01$	0.115±0.08•
PE	$0.138 \pm 0.07$	$0.166{\pm}0.08$	$0.095 {\pm} 0.04$	$0.263 \pm 0.09$	$0.133 \pm 0.10$	$0.176 \pm 0.09$	$0.211 \pm 0.12$	$0.194{\pm}0.12$	0.344±0.03•
AE	$0.173 \pm 0.08$	$0.176 {\pm} 0.08$	$0.124{\pm}0.02$	$0.304 \pm 0.05$	$0.175 {\pm} 0.09$	$0.171 \pm 0.09$	$0.310 \pm 0.06$	$0.161 \pm 0.07$	0.371±0.07•
PT	$0.193 \pm 0.02$	$0.178 \pm 0.03$	$0.184{\pm}0.03$	$0.192 {\pm} 0.02$	$0.147{\pm}0.04$	$0.159 \pm 0.03$	$0.124 \pm 0.05$	$0.211 \pm 0.01$	0.216±0.01•
BC	$0.014 \pm 0.01$	$0.051 \pm 0.03$	$0.051 \pm 0.03$	$0.032 {\pm} 0.03$	$0.025 {\pm} 0.03$	$0.050 \pm 0.03$	$0.037 {\pm} 0.03$	$0.061 \pm 0.03$	$0.080 {\pm} 0.02 {ullet}$
CE	$0.043 \pm 0.02$	-	$0.075 \pm 0.04$	$0.078 \pm 0.04$	$0.023 \pm 0.02$	-	$0.067 {\pm} 0.04$	$0.051 \pm 0.01$	0.121±0.07•
NS	$0.057 \pm 0.02$	-	$0.077 \pm 0.03$	$0.080 \pm 0.03$	$0.115 \pm 0.16$	-	$0.106 \pm 0.09$	$0.006 \pm 0.00$	0.196±0.12•
LE	$0.065 \pm 0.02$	$0.064 {\pm} 0.02$	$0.075 {\pm} 0.02$	$0.096 {\pm} 0.02$	$0.066 \pm 0.03$	$0.058 {\pm} 0.02$	$\underline{0.099{\pm}0.03}$	$0.073 \pm 0.04$	0.137±0.02•

Table 3: ARI performance of the Type-2 approaches and DSLC.

Table 4: NMI performance of the Type-2 approaches and DSLC.

Data	KMS	WKMS	NWO	DSLC	Data	KMS	WKMS	NWO	DSLC
IQ	$0.090\pm0.10$	$0.102 \pm 0.11$	$0.102 \pm 0.11$	0.171±0.12•	IQ	$0.077 \pm 0.08$	$0.081 \pm 0.08$	$0.081 \pm 0.08$	0.115±0.08•
PE	$0.248 {\pm} 0.05$	$0.225{\pm}0.06$	$0.248 \pm 0.05$	0.285±0.03•	PE	$0.334 {\pm} 0.04$	$0.310 \pm 0.06$	$0.336 \pm 0.06$	$0.344 {\pm} 0.03 {\bullet}$
AE	$0.229 {\pm} 0.05$	$0.234{\pm}0.07$	$0.251 \pm 0.07$	0.311±0.10•	ΑE	$0.309 \pm 0.03$	$0.316 {\pm} 0.05$	$0.329 \pm 0.05$	$0.371 \pm 0.07 \bullet$
PT	$0.084 \pm 0.01$	$0.046 {\pm} 0.02$	$0.084 \pm 0.01$	$0.086{\pm}0.00{ullet}$	PT	$0.212 \pm 0.01$	$0.155{\pm}0.04$	$0.209 \pm 0.02$	$0.216\pm0.01$ $\bullet$
BC	$0.118 \pm 0.04$	$0.133 \pm 0.01$	$0.136 \pm 0.01$	$0.149 {\pm} 0.07 {\bullet}$	BC	$0.069 \pm 0.01$	$0.072 \pm 0.01$	$0.069 \pm 0.01$	$0.080 \pm 0.02 \bullet$
CE	$0.030 \pm 0.02$	$0.024 {\pm} 0.02$	$0.019 \pm 0.02$	$0.072 \pm 0.06 \bullet$	CE	$0.085 {\pm} 0.03$	$0.083 \pm 0.04$	$0.099 \pm 0.06$	$0.121\pm0.07 \bullet$
NS	$0.110\pm0.08$	$0.111 \pm 0.11$	$0.127 \pm 0.15$	$0.150 {\pm} 0.10 {\bullet}$	NS	$0.133 \pm 0.09$	$0.138 \pm 0.13$	$0.159 \pm 0.16$	$\textbf{0.196} \!\pm\! \textbf{0.12} \bullet$
LE	$\underline{0.077{\pm}0.02}$	$0.074 \pm 0.02$	$0.067 \pm 0.02$	$0.081 {\pm} 0.01 {\bullet}$	LE	$0.132 \pm 0.02$	$0.127{\pm}0.03$	$0.119 \pm 0.03$	$\textbf{0.137} {\pm} \textbf{0.02} \bullet$

**<sup>2.</sup> Space Complexity Analysis of DSLC:** Suppose  $X_{ord}$  is an ordinal data set with n data objects and m attributes. The m attributes have  $v_1, v_2, ..., v_m$  categories, respectively, and  $V = \max(v_1, v_2, ..., v_m)$ . When the number of clusters is set at k, space complexity of the proposed DSLC clustering algorithm is analyzed as follows:

An  $n \times m$  matrix  $X_{ord}$ , an  $n \times k$  matrix Q, a  $k \times m \times V$  matrix U, a  $1 \times m$  vector W, a  $V \times m$  matrix L, and m matrices  $M_1, M_2, ...M_m$ , each of which with size  $V \times V$ , should be maintained during DSLC clustering. Since  $V = \max(v_1, v_2, ..., v_m)$  is a small constant in real data sets, overall space complexity of DSLC is O(nm + nk + km), which is not high.