Лабораторная работа №1

Ларин Егор. 4 группа 3 курс

23 ноября 2022 г.

Теория

Задача

$$u'' = 2x + 2 - 2xu' + 4u$$

$$\begin{cases} u'(0) = -1 \\ u'(1) = 1 - 2u(1) \end{cases}$$

Разностная схема

$$u_{\overline{x}x} = \frac{u(x+h) - 2u(x) + u(x-h)}{h^2}$$

$$u_{\mathring{x}} = \frac{u(x+h) - u(x-h)}{2h}$$

$$y_{\overline{x}x} = 2x + 2 - 2xu_{\mathring{x}} + 4u$$

$$y(x-h)\left(\frac{1}{h^2} - \frac{x}{h}\right) + y(x)\left(\frac{-2}{h^2} - 4\right) + y(x+h)\left(\frac{1}{h^2} + \frac{x}{h}\right) = 2x + 2$$

$$y_{i-1}\left(\frac{1}{h^2} - \frac{x_i}{h}\right) + y_i\left(\frac{-2}{h^2} - 4\right) + y_{i+1}\left(\frac{1}{h^2} + \frac{x_i}{h}\right) = 2x + 2, i = \overline{1, N-1}$$

$$y_1 + y_0\left(-1 + 2h^2\right) = -h + h^2$$

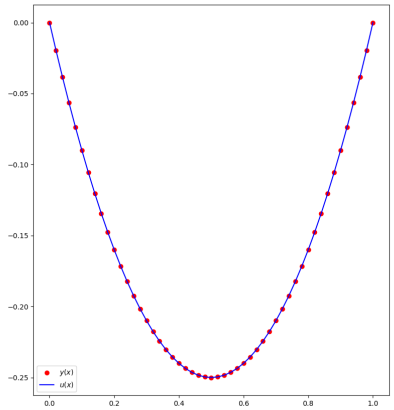
$$y_N\left(1 + 2h + 4h^2\right) - y_{N-1} = h - h^2$$

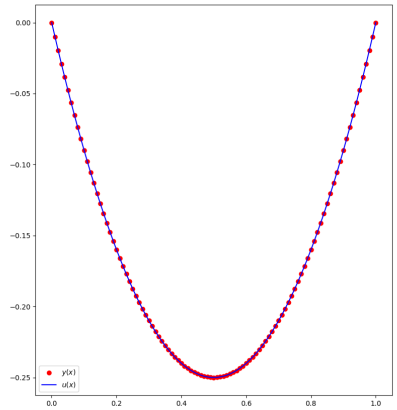
Листинг кода

```
from math import ceil
import numpy as np
from matplotlib import pyplot as plt

def TDMA(a, b, c, d):
    n = len(d)
    w= np.zeros(n-1,float)
    g= np.zeros(n, float)
    p = np.zeros(n,float)

    w[0] = c[0]/b[0]
    g[0] = d[0]/b[0]

    for i in range(1,n-1):
        w[i] = c[i]/(b[i] - a[i-1]*w[i-1])
    for i in range(1,n):
        g[i] = (d[i] - a[i-1]*g[i-1])/(b[i] - a[i-1]*w[i-1])
    p[n-1] = g[n-1]
    for i in range(n-1,0,-1):
        p[i-1] = g[i-1] - w[i-1]*p[i]
    return p

def u(x):
    return x**2 - x

def solve(h):
    # c lower
    # a middle
    # b upper

    N = ceil(1 / h)

    b_1 = [1] ##
    b_i = [1 + (i*h) * h for i in range(1,N)] #

    a_1 = [2*h**2 - 1] ##
    a_i = [-2 - 4 * h**2 for i in range(1, N)] #
    a_N = [1 + 2 *h +4*h**2]

    c_i = [1 - (i*h)*h for i in range(1, N)] #
    c_N = [-1]

    f_1 = [-h+h**2] ##
```

```
    f_i = [(2*(i*h) + 2) * h**2 for i in range(1, N)] #
    f_N = [h-h**2]

    ys = TDMA(
        c_i + c_N,
        a_1 + a_i + a_N,
        b_1 + b_i,
        f_1 + f_i + f_N,
    )

    xs = [i * h for i in range(N+1)]
    plt.plot(xs, ys, "ro", xs, [u(x) for x in xs], "b")
    plt.legend(["$y(x)$", "$u(x)$"])
    plt.show()

    print(ys)
    print(max([u(x) - y for (x,y) in zip(xs, ys)]))
    return ys


y1 = solve(0.02)
y2 = solve(0.01)
print(max([abs(y2[2*i] - y1[i]) for i in range(ceil(1 / 0.02))]))
```

Графики





Результаты вычислительного эксперимента

Величина	Погрешность
y_h	1.6681100944992977e-14
$y_{\frac{h}{2}}$	1.4016565685892601e-14
$\frac{1}{3}\max\left\|y^h - y^{\frac{h}{2}}\right\|_{\omega_h}$	2.8727020762175925e-15