RESONANCE DYNAMICS IN THE KUIPER BELT

BY BRETT GLADMAN AND J.J. KAVELAARS

Celestial mechanics is the queen of physics, being the longest-studied quantitative subject. The desire to predict the orbital motion of the planets drove both mathematics and physics to develop varied and precise techniques. For example, Newton's crowning achievment was the derivation of Kepler's laws of orbital motion from the three laws of motion and the law of universal gravitation. Bessel derived his famous functions to attempt to solve Kepler's nonlinear equation for the angular position of a planet along its orbit.

Kepler discovered (and stated in his First Law) that the planetary orbits are well described by ellipses in a fixed plane around the Sun, and that around the eccentric orbit the speed varies. Beginning students all learn Kepler's Third law in its simple form

$$P_{yr}^2 = a_{AU}^3 \tag{1}$$

for objects in orbit around the Sun, where the unit of distance is scaled to the Earth's orbital semimajor axis of 1 AU $\simeq 1.5 \times 10^8$ km. The semimajor axis a is one of 6 parameters (called 'orbital elements'), which describe the shape of the orbit (through a and the eccentricity e), its orientation (the inclination i, the longitude of ascending node Ω, and the position of the perihelion ω relative to the ascending node), and finally the angular position f along the orbit from the perihelion point. In the perturbation equations of celestial mechanics, the orbits of each of the planets slowly change due to the weak tugs from the other planets. In the classical secular perturbation theory approach to compute the evolution of the orbital elements, orbital resonances have an important role. The location of a so-called *mean-motion* resonance is easy to compute; if a planet has period P_1, then an external resonance with an second object whose orbital period $P_2 > P_1$ will occur when P_2/P_1 is a ratio of two integers m/n, whereupon Eq. (1) provides the resonant semimajor axis. For example, the external 5:2 mean-motion resonance of Neptune (with $a_1 \simeq 30$ AU) occurs where $a_2/a_1 = (P_2/P_1)^{2/3} = (5/2)^{2/3}$, corresponding to $a_2 \simeq 55$ AU. A transneptunian object (abbreviated TNO) with $\simeq 55$ AU will circle the Sun twice for every 5 orbits of Neptune. In analogy with the way amplitude can be built up by correctly timing the pushes of a child on a swing, the repetitive geometrical configuration caused by the resonant configuration results in the resonance affecting the orbital evolution of the TNO more than other (non-resonant) semimajor axes nearby.

SUMMARY

Over the last decade it has become clear that the Solar System's Kuiper Belt has a rich dynamical structure. Mean-motion resonances (which are depleted in the asteroid belt and appear as the Kirkwood gaps in the asteroidal semimajor axis distribution) are preferentially populated in the Kuiper Belt, confering orbital stability to objects which would otherwise have short lifetimes against gravitational encounters with Neptune. The basics of the orbital mechanics of these resonances is presented. How planet migration may be at the origin of the observed resonant structure is outlined, along with some of the observational complications involved.

The importance of mean-motion resonances in the Kuiper Belt is easily seen in the semimajor axis/eccentricity distribution shown in Figure 1. The most prominent group inhabits the 3:2 mean-motion resonance with $a \simeq 39.4$ AU; these objects are known as 'plutinos' because Pluto was the first object known to occupy this resonance. One can see that most of the resonant objects have higher orbital eccentricities than the main part of the Kuiper Belt (the 'classical' objects which range mostly from $a = 38 - 48$ AU); there are complex observational selection effects at play here, and this figure must be interpreted with care. What is clear is that resonant objects exist, and one naturally proposes the question as to how these objects entered

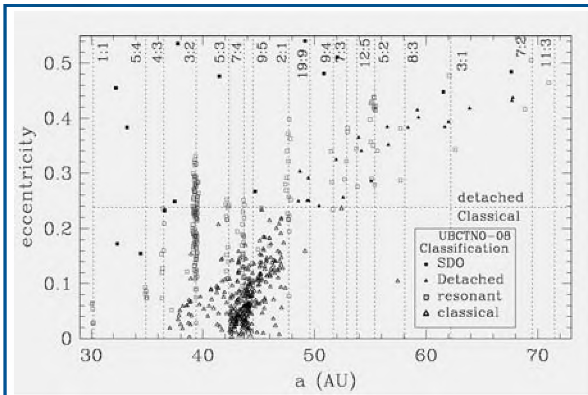

Fig. 1 The inner portions of the trans-neptunian region in semimajor axis/eccentricity space, for TNOs with high-quality orbits. Vertical lines show mean-motion resonance locations, along which many of the known TNOs sit; open squares denote TNOs whose resonant arguments are known to oscillate. This figure is an update of an orbital classification presented in Ref. [13].

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this dynamical state. We must first understand a little bit better what the resonant state actually is.

THE PENDULUM ANALOGY.

The language of orbital resonances share some terminology with the more familiar problem of the rigid-rod pendulum, where a mass m pivots on a rod of length L in a vertical plane, with ϕ measuring the angular displacement from "straight down". The dimensional Hamiltonian of the problem is written

$$H_{dim} = \frac{p^2}{2m} + mgL(1-\cos\phi) \tag{2}$$

where g is the local acceleration with respect to gravity. The single coordinate ϕ and its corresponding (generalized) momentum p describe the motion. One can choose units of mass, length, and time so that $m = g = L = 1$, giving the non-dimensional form:

$$H_{nd} = \frac{p^2}{2} + (1-\cos\phi) \tag{3}$$

with corresponding Hamiltonian equations of motion

$$\dot{\phi} = \frac{\partial H_{nd}}{\partial p} = p\,, \tag{4}$$

$$\dot{p} = -\frac{\partial H_{nd}}{\partial \phi} = -\sin\phi\,, \tag{5}$$

where the over-dot indicates differentiation with respect to time. The reader will note that differentiating the first equation and substituting for $\dot{p}$ from the second yields the second-order differential equation $\ddot{\phi} + \sin\phi = 0$ which, for small-amplitude oscillations which keep $\sin\phi \approx \phi$, produces the familiar simple harmonic oscillator for small amplitude oscillations around the straight-down point.

For what follows it is simpler to use an angular coordinate $\theta = \pi - \phi$ which is the angular position in the plane of oscilation, but measured from the 'straight-up' point, so that $\theta = \pi$ is the equilibrium point at the bottom of the swing. The Hamiltonian is

$$\mathcal{H} = \frac{p^2}{2} + \cos\theta \tag{6}$$

The full solutions for $\theta(t)$ and $p(t)$ of the simple-looking equations of motion analogous to (4) and (5) require knowledge of the Jacobian elliptic functions. However, if one only wishes to understand the *qualitative* nature of the trajectories one can simply plot level sets of the Hamiltonian $\mathcal{H}$ in the (θ,p) plane (Fig. 2). This is because the value of $\mathcal{H}$ (the system's mechanical energy) is conserved during the evolution, a familiar fact that is verifiable by computing $\dot{\mathcal{H}}$. Therefore, a curve of constant $\mathcal{H}$ is a trajectory. The figure shows the small-amplitude trajectories surrounding the vertically-down ($\theta = \pi$) point as circles, which become elongated as motions that approach $\theta = 0$ or π (the 'straight-up' point). The thick curve connecting $(\theta, p) = (0,0)$ with $(\pi,0)$ is called the *separatrix* because in separates the oscillatory regime, centered on $(\pi,0)$, from the rotor regime at higher momenta (where the pendulum circulates 'over the top' each period).

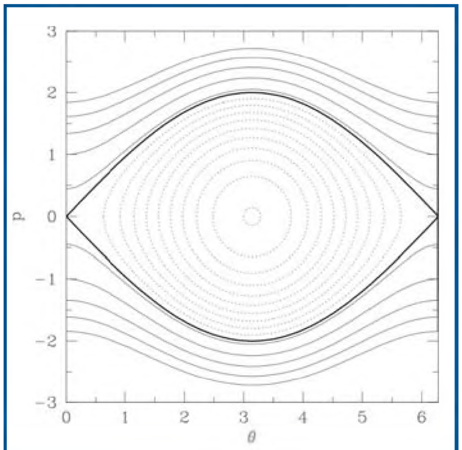

Fig. 2 The phase space of a planar rigid-rod pendulum, viewed in cylindrical coordinates. The angle θ measures the angle down from the vertically straight-up position, while p is the (angular) momentum in the intertial frame where the pivot is at rest. Trajectories (lines of constant $\mathcal{H}$) are shown. The oscillatory (also called librating) trajectories are dashed, while the 'rotor' solutions (which visit all values of θ) are solid. The thick trajectory is the separatrix.

The terminology as it relates to celestial mechanics is closer if one now moves to a new 'action' I which is just a linear offset of p (a new inertial reference frame) and plots the trajectories in polar coordinates (Fig. 3). Here the 'resonant' regime (of oscillatory motion) is positioned at left within the thick separatrix and surrounding the low amplitude oscillations. The 'resonant angle' θ is the polar position of a vector starting from the origin that points to the trajectory in question (determined by value of $\mathcal{H}$ determined by the initial values of I and θ). This vector moves along a surface of constant $\mathcal{H}$, controlling the value of the polar angle. For oscillatory motions inside the separatrix, θ does not explore all values, but has a restricted range centered on 180°. Recall that we have not determined the time behaviour along the trajectories (which is much

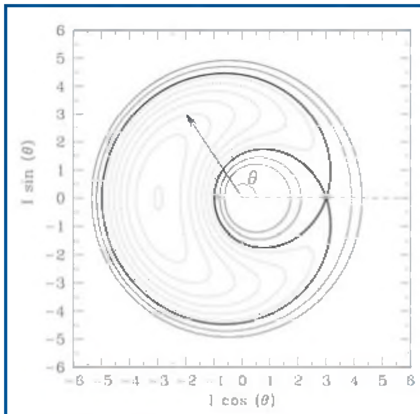

Fig. 3 The dynamics viewed in polar coordinates. The state of the system is expressed by the polar angle θ (measured counterclockwise from the dashed reference direction) and a generalized momentum I which determines the length of the state vector. Curves are still the level sets of the Hamiltonian and the thick curve is the separatrix of the previous figure. Rotor motions (solid trajectories) are both exterior to the separatrix and also near the origin (corresponding to 'above' and 'below' the separatrix in the previous figure). Librating trajectories are again dashed. As the system evolves along one of the trajectories the angle θ changes. In the case shown θ will not take on all values but rather will oscillate about θ = 180°.

more complicated), only the tracks in (p,θ) space; in fact the separatrix motion requires a formally-infinite time.

ORBITAL RESONANCE DYNAMICS

In celestial mechanics the 'resonant angle' θ is formed from a combination of angles which describe the orbit and positions along the orbit of the two particles. For example, for describing Pluto's motion in the 3:2 resonance, the resonant angle

$$\theta_{32} = 3\lambda - 2\lambda_N - \varpi \tag{7}$$

where

$$\lambda = \varpi + M \tag{8}$$

is the mean longitude of the particle, made up of the longitude of pericenter $\varpi = \Omega + \omega$ of the orbit and the mean anomaly M of the particle. The mean anomaly M of celestial mechanics is the angular position of the particle past the pericenter point, but in the 'average' sense rather the strict geometrical angle; the interested reader should refer to an orbital dynamics text for a deeper understanding, but here it is sufficient to think of λ as the angle that starts from the arbitrary reference direction from which the angular position of the ascending node Ω is measured (where the particle crosses the reference plane going north) around to the particle (this is precisely true for circular orbits in the reference plane).

The resonant angle θ_{32} is not a simple angle to interpret; one cannot draw it in coordinate space as an angle terminating at the plutino particle. This angle appears in the expansion of the 'disturbing function' of celestial mechanics, where it causes a singularity in the theory if the particle is precisely at the resonant semimajor axis. However, analytical treatments (beyond the scope of this article) can be developed. Some geometrical insight is obtained if one asks the question of the value of θ_{32} at an instant when the plutino is at perihelion. In such a case $M = 0$ for the particle (since M is measured from pericenter) and Eq. 8 indicates $\lambda = \varpi$ (that is, the angle around the particle from the reference direction is the same as the angle to perihelion, which must be true of course), so that

$$\theta_{32} = 3\varpi - 2\lambda_N - \varpi = 2(\varpi - \lambda_N)\,. \tag{9}$$

If the resonant angle has the value of 180°, then this equation demands that the perihelion longitude of the TNO is $\varpi = \lambda_N + 90°$, meaning that the perihelion point is 90° ahead of Neptune's position. Because of the 360° degeneracy of angles, another valid possibility is that $\varpi = \lambda_N - 90°$, corresponding to perihelion 90° behind Neptune. Thus, even a plutino with an orbit so eccentric that its closest solar approach is nearer the Sun than Neptune's distance of 30 AU (Pluto and many other plutinos have such large eccentricities) is protected from close encounters with Neptune by the resonance condition. If the resonant angle is near, but not exactly, 180°, a significant offset is still induced between the perihelion longitude and Neptune's location.

Because of the gravitational tugs which Neptune exerts on the plutino, the heliocentric orbit of the small body precesses and the perihelion direction relative to Neptune would drift. One would expect that eventually the perihelion direction of the plutino could align with Neptune. The reader should confirm that although both the longitudes of the plutino and Neptune advance rapidly, because of the fact that Neptune's period is 2/3 of the plutino's, Eq. 7 then results in $d\theta_{32}/dt$ being much smaller than either $d\lambda/dt$ or $d\lambda_N/dt$. Looking at Fig. 3, the resonant particle's trajectory follows one of the dashed curves inside the separatrix, and it is easy to see that as one follows such a trajectory away from $\theta = 180°$ there will be an extremal value of θ at which point the angle begins to return toward $\theta = 180°$; the angluar deviation of this extremal value is called the 'libration amplitude' of the trajectory. This libration of the resonant argument θ is the effect of the resonance, and it manifests itself in configuration space as a correlation between the plutino's perihelion direction and that of Neptune. (It is also true that at conjunction, where $\lambda = \lambda_N$ and thus Neptune and the plutino 'line up', the plutino is forced to be near its maximum distance from the Sun.) Fig. 4 shows a numerically-integrated time history of θ for a plutino with a libration amplitude of about 40°. The θ_{32} resonant argument completes one libration in only about 20 thousand years, but this libration is stable for time scales of billions of years. The small irregularities in Fig. 4 are caused by beating of the sampling interval and the small effects of the other planets.

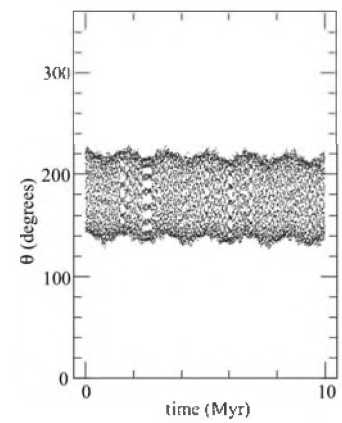

Fig. 4 The history of the resonant argument θ_{32} over 10 million years for a so-called 'plutino' in the 3:2 mean-motion resonance with Neptune. On time scales of 10^4 years the resonant argument librates around 180° with an amplitude of about 40°.

The libration's effect in space is illustrated in Fig. 5, which requires some explanation. This is for the same orbit numerically integrated in the previous figure. Imagine looking down on the Solar System from the north ecliptic pole (that is, directly above the plane of the Solar System), with the Sun at the coordinate-system origin. Fig. 5 shows the location of Neptune (N) this year, at a distance of 30 AU from the Sun (and nearly circular orbit); Neptune would trace out a counter-clockwise circle of this radius in an intertial frame, with a circle of that radius shown for reference. However, this figure is drawn in a reference frame that *co-rotates with Neptune* so that the planet's location remains fixed at its current position. In this co-rotating reference frame plutinos execute a complex motion like a child's spirograph, going clockwise around the Sun except for a small 'loop' near each pericenter which, as predicted above, occur near the points 90° ahead and behind Neptune.

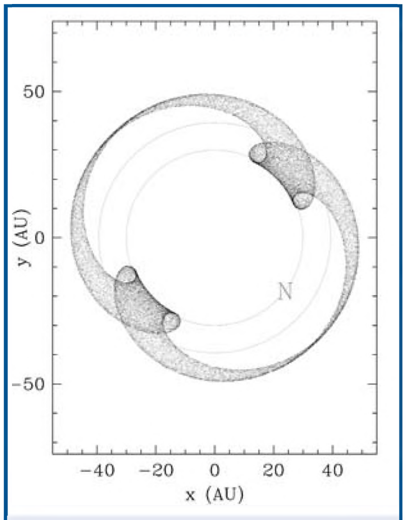

Fig. 5 The motion of a plutino in the reference frame co-rotating with Neptune. The two circles mark heliocentric distances of 30 and 40 AU. Neptune's position in indicated by the large "N". See text for discussion.

Two orbits of the plutino around the Sun correspond to one complete clockwise rotation around this figure (thus with two perihelion passages of course). The orbit's pericenter location then wobbles slowly back and forth, with the wobble direction reversing at the extremal points of the resonant argument.

REAL PLUTINOS

The phase-space structure of resonances presented in the previous section makes clear the special dynamics of objects in these resonances, and how the resonant configuration can influence the dynamics. However, except for Trojan asteroids, there were few real-world examples to provide case studies to compare to.

The 1930 discovery of Pluto created a puzzle regarding the origin of the orbit of this unusual object. The orbit is highly-inclined to the plane of the Solar System ($i \sim 17^o$), a semi-major axis of ~ 39.7 AU, and with a large-enough eccentricity ($e \sim 0.25$) that Pluto crosses the orbit of Neptune! These unusual orbital characteristics appeared to indicate that Pluto's orbit might not be stable.

Determining that Pluto and Neptune have orbital periods that are near the ratio of 3:2 merely requires a simple application of Kepler's Third Law (Eqn. 1). Simply having commensurate orbital periods, however, is not a sufficient condition to ensuring that Pluto and Neptune are in resonant orbits because the combination of the angular variables must be in the right range and the resonant argument must librate. The answer regarding the stability of Pluto's orbit needed to wait until 1965, at which time the computational power became available to conduct a numerical integration of Pluto's orbital evolution under the influence of the Sun and the four giant planets.

Cohen and Hubbard [1] demonstrated, via orbital integration, that the 3:2 resonant angle between of Pluto with respect to Neptune (θ_{32} as expressed in Eqn. 7) *librates* around a mean value of 180^o with an amplitude of $\simeq 80^o$ and period of ~ 19670 years. This libration of the resonant angle proved that the orbits of Pluto and Neptune are in resonance, but better observational data was required to secure the exact value of the libration amplitude. This yin-yang between observational discovery and computational modelling is a key feature of Kuiper belt research which continues to this day. At the time of writing, analytical/numerical understanding of resonance dynamics and migrational capture into resonances (see below) is slightly ahead of the observational data available.

A number of investigators have pursued ever-more detailed investigations of Pluto's orbital evolution, revealing that Pluto is simultaneously trapped in a number of other types of resonances inside the 3:2 mean-motion resonance. Discussion of the full complexity of Pluto's orbit is beyond the scope of this article and the interested reader is encourage to examine the review by Malhotra and Williams [2].

Soon after the discovery of other Kuiper Belt objects in the 1990s, other trans-neptunian objects were realized to be trapped in the 3:2 resonance. These objects were coined plutinos in analogy with Pluto. Other mean-motion resonances were then shown to be inhabited as well; Ref. [3] reviews the early development of the knowledge of the Kuiper Belt's resonant structure.

How do objects with an orbit like Pluto end up in this and other resonances? Formation of Pluto in a Neptune-crossing orbit seems unlikely. To understand the complexity of the problem requires a consideration of the process of planet formation and the general reversibility of Newtonian dynamics.

RESONANCE CAPTURE

Models of planetary accretion suggest that planets grow from a smooth disk of material initially via coagulation of dust particles which form into cm-sized dust balls and then coalesce to form larger like Pluto (see Ref. [4] for a review of the core accretion model of planet formation). Of critical importance for the current discussion is the finding that encounters between growing planetesimals must have very low relative velocities, or else the encounters are disruptive. For nearby orbits of small eccentricity and mutual inclination, encounter velocities between objects scale as $v_k\sqrt{e^2+i^2}$, where v_k is the keplerian orbital speed of one of the objects at the encounter. Thus, although an object with a low-inclination, low-eccentricity orbit could perhaps form in resonance with Neptune it seems highly unlikely that an object on a highly inclined or eccentric orbit, like Pluto, could have done so.

Based on expectations surrounding the planet formation process, Pluto most likely formed in a nearly circular orbit that was tightly confined to the ecliptic plane. After formation, some event(s) or action(s) must have provided the gravitational excitation required to leave Pluto in a high-inclination/high-eccentricity orbit. How can such a process have occurred and left Pluto in resonance with Neptune? Explaining the coupling between Pluto's orbital resonance in the context of the formation of the outer Solar System is where orbital dynamics provides its great constraint on models of planet formation.

During the final stages of the giant planet formation, and Neptune in particular, there was likely a surviving disk of at least a few Earth-masses of planetesimals orbiting in the ecliptic. When these particles are gravitationally scattered by the giant planets, an exchange of angular momentum occurs. If the planetesimal scatters inward of the giant planet then the giant planet's orbit will grow slightly or if the planetesimal scatters outward then the giant planet's orbit will shrink. The exact outcome, for the giant planet, depends on the net effect of all these scattering events. For each outward scattering the change of semi-major axis is approximately:

$$\frac{\delta a}{a} \simeq \frac{m}{M_N}$$

where m is the mass of the scattered planetesimals and M_N is that of Neptune [5].

In the case of the outer Solar System, outward scattering off Neptune rarely results in the planetesimal's ejection out of the Solar System while inward scatter usually leads to encounters which hand the planetesimals, via Uranus and Neptune, down to Jupiter. Mighty Jupiter easily ejects the planetesimals out of the system, and thus moves in. In essence this makes Neptune's total number of inward scattering events more numerous than the outward scattering ones and so Neptune's orbit slowly drifts outward, growing in semi-major axis. The slow expansion of the outer Solar System via this planetesimal scattering appears to be an inescapable result during the late stages of giant planet formation.

Imagine now that Pluto has formed and is far beyond pre-migration Neptune, say just outside the location of Neptune's 3:2 resonance at the time. As Neptune migrates outward and its semi-major axis increases, the heliocentric distance corresponding to the 3:2 resonance also slowly moves outward, eventually reaching the point where it sweeps past Pluto's location.

The dynamics of resonant capture are rather complex, although there are well-known examples. Likely the most familiar is that of the Moon's tidal locking to the same period as its revolution around the Earth. In that case a slowly-acting force (the tidal dissipation acting on the Moon's surface) de-spun the Moon's rotation until it was caputred into resonance and found an equilibrium.

In the study of capture into Neptune's 3:2 resonance, the transition depends on the initial conditions (namely the eccentricity of Pluto's non-resonant orbit), the rate of evolution and eccentricity of Neptune's orbit and the relative importance of other non-resonant gravitational perturbations (such as the graviational forces from the other planets). In essence, the timescale for migration of Neptune must be long compared to the timescale of the resonant perturbations (~ 10^4 yr). One can heuristically think about resonance capture by examining Fig. 3 and imagining that the pre-capture trajectory of the plutino is circulating on a trajectory near the origin. These curves are drawn in the case of a fixed orbit for Neptune. As Neptune migrates, the plutino orbit's *action* (to use the hamiltionian terminology), which here is the radius I, increases and the particle will approach the separatrix. One can productively think of this as the migratory motion actually 'breaking open' the separatrix near the cusp point along the $\theta = 0$ axis, and when the particle's orbit reaches this point it might end up escaping into the outer circulatory region (in which case the particle has passed through the resonance without capture), or becoming trapped into the librational region and thus captured into the resonance. In the language of nonlinear dynamics, one says that some of the initial conditions are part of the 'basin of attraction' of the librational fixed point at $\theta = 180^o$ (this analogy is only good if one imagines the planet migrating forever). When the planetary migration ceases, the structure of the resonance locks onto the phase space diagram like Fig. 3 and particles remain on the trajectory that they find themselves.

A phenomenon not obvious from the above discussion is that after capture, continued outward migration causes the expansion of the orbital eccentricity. Via angular-momentum conservations one can determine the growth of the eccentricity as a function of the change in semimajor axis of the migrating Neptune. Malhotra [5] found that

$$e_{final}^2 \simeq e_{capt}^2 + \frac{1}{3} \ln \frac{a_{N,final}}{a_{N,capt}} \tag{10}$$

where *capt* subscripts refers to the values at the instant of the resonant capture, and N subscripts refer to Neptune rather than Pluto. Thus, assuming that the pre-migration Pluto had $e_{initial} \approx 0$ and given the current eccentricity of Pluto, ($e_{final} \sim 0.25$) Eq. 10 provides an estimate of the migration distance:

$$\Delta a = (a_{N,final} - a_{N,capt}) = a_{N,final} \times \left[1 - \exp(-3 * e_{final}^2)\right] \simeq 5\ AU\ .$$

This assumes that Pluto was 'just exterior to' the 3:2 resonance when Neptune started to migrate (minimizing the migration distance) and that Pluto's pre-capture eccentricity was 0 (maximizing the migration). Since the publication of this theory [5], a large number of plutinos have been discovered and the median eccentricity of this population appears to be ~ 0.18 [6], so explaining the plutinos eccentricities with migration would require about half of the plutinos to be captured during the last 3 AU of Neptune's migration. Neptune is acting somewhat like a 'snowplow' (a good canadian analogy), with the plutinos first captured finishing 'highest up' in eccentricity. If one posits that all plutinos had initial $e \approx 0$ then the largest-observed $e = 0.33$ requires Neptune's total migration to have been 8 AU. However, some pre-migration stirring of the Kuiper Belt may have occured, in which case the total outward movement of Neptune would have been less.

Capture Efficiency

The dynamical process of resonance capture depends on a number of physical parameters, including the migration rate and eccentricity of Neptune and the particle at the time the resonance is crossed. Thus, we might hope to use the current number and orbital distribution of objects trapped in the 3:2 resonance, versus some estimate of the pre-migration distribution, as an archaeological indicator of Neptune's migration rate. Since Neptune's ancient migration rate is intimately tied to the primordial density of planetesimals in the outer Solar System we have, in effect, a measure of the density of material in the solar system at the epoch of planet formation.

A number of authors have examined the various interplays between the initial conditions of a disk of material beyond Neptune and the rate of capture into resonance (see Refs. [7-11] for examples). The dependencies between capture efficiency and migration rate (Ref. [9], in particular) and the initial eccentricity of captured material (e.g., Ref. [10]) have been examined as well as the long-term stability of captured objects [8].

The underlying driver for these studies has been an attempt to reconcile the observed orbital distribution with the migration theory. If the planetesimal disk beyond Neptune (the Kuiper belt) was completely quiescent ($< e > \sim < \sin(i) > \ll 0.01$), as is required for planetesimal accretion to be effective, then migration capture would have been nearly 100% effective and the majority of Kuiper belt objects should now be members of the 3:2 resonance. In addition, the currently-observed inclination distribution among the plutinos extends to higher inclination than the inclination produced via resonance migration.

Levison *et al.* [11] have proposed that instead of migration into a pre-existing low-eccentricity Kuiper Belt, Neptune, Uranus and a large planetesimal population were all flung outwards from initial locations interior to 30 AU. Migration still occurs, but during an epoch of large eccentricity for Neptune which greatly enhances resonance capture. (Neptune's eccentricity subsequently drops to the present value of nearly zero). This model better explains some of the observed features of the Kuiper Belt's orbital distribution.

TWOTINOS

The resonant dynamics in the outer Solar System are very rich. Many of the resonances are significantly more complicated than the simple picture developed above. Fig. 6 shows the same polar diagram as before, but now for the 2:1 mean-motion resonance with Neptune; trans-neptunian objects at this distance can be seen in Fig. 1 with $a \simeq 47.4$ AU. The resonant argument is $\theta_{21} = 2\lambda - \lambda_N - \varpi$, and if this argument librates the object is resonant and (whimsically) called a 'twotino'. The separatrix between the outer and inner circulation regions and the resonant libration region looks similar to before, but now the resonant region is broken up into 3 regions by an additional separatrix, called the symmetric and asymmetric islands as defined in Fig. 6's caption. Two types of resonant motion now exist: (1) *symmetric libration* where θ_{21} oscillates around 180°, as before, but as the figure shows this must occur with large amplitude, or (2) *asymmetric libration* where the libration center is not 180°, but rather one of two other possible values (the libration center depends on the eccentricity) and occuring with a smaller possible libration amplitude.

Twotinos librating around all three libration centers are known (Fig. 7). In fact, due to the gravitational forces from the other planets, a given twotino can switch between symmetric and

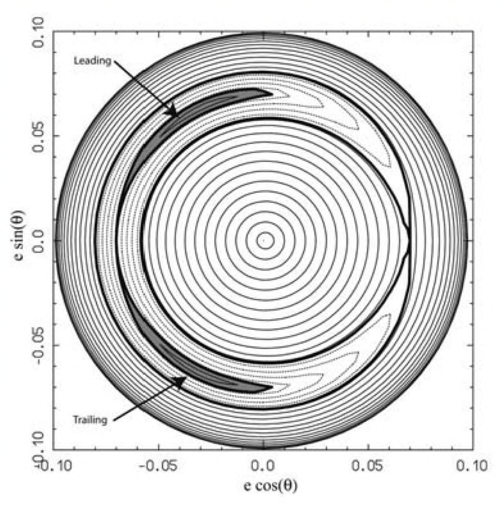

Fig. 6 The θ_{21} resonant argument trajectories for the 2:1 mean-motion resonance. Here the particle eccentricity is the radial coordinate. Separatrices are shown as heavy curves. There are now 3 resonance regions. The larger symmetric region encloses $\theta = 180°$ and allows large libration amplitudes. There are also the two small asymmetric libration islands, so named because a trajectory following one of them will never pass through 180°, but will instead oscillate with small amplitude around another average value (of roughly θ_{21} =120 or 240 degrees here). Note that there are no small-amplitude symmetric librators. For particles of different eccentricities the angle defining the libration center of the asymmetric islands changes. Figure provided by A. Morbidelli.

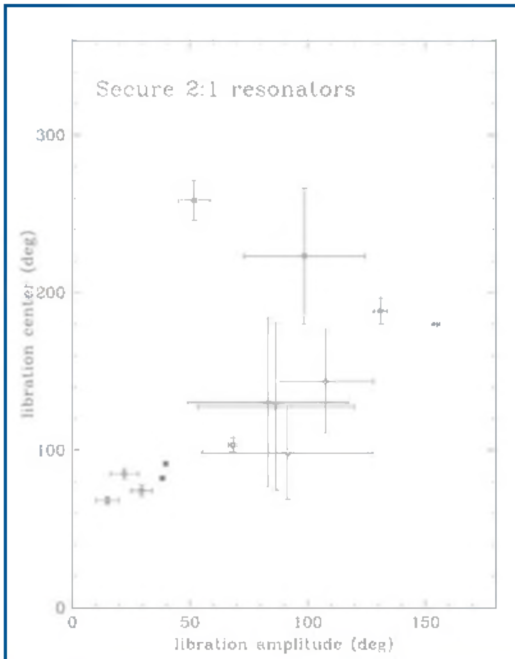

Fig. 7 The distribution of libration centers and resonance amplitudes for objects securely known to be in the 2:1 mean-motion resonance with Neptune. The error bars for each object represent the current range of allowable solutions. As expected, all the symmetric librators (which will have libration centers at 180°) have large libration amplitudes, and the smaller-amplitude symmetric librators have libration centers at depend on their eccentricity. The greater number of Kuiper Belt objects known with libration centers near 90° compared to near 270° is likely an observational bias due to objects in the 270° asymmetric island spending much of their time in the direction of the galactic center.

asymmetric libration over time. This more complex resonance structure opens new windows into the past, because assuming capture into the 2:1 resonance occured due to migration, the multiple libration islands may not be equally populated.

Asymmetric Capture

The unusual orbit of Pluto and the explanation of that orbit as the result of resonance capture during Neptune migration leads naturally to the question of what other signatures of migration might exist among the resonant Kuiper belt objects.

When a low-eccentricity object is initially captured into the 2:1 resonance, the effect of continued migration will be to cause the newly-captured twotino's eccentricity to grow, slowly leading it towards the separatrix entirely inside the resonant region (shown in Fig. 6), forcing a choice between one of the two asymmetric islands and the large libration-amplitude symmetric island. Which island will be selected?

As in the previous discussion, Fig. 6 is only true for the instantaneous, *i.e.* non-migrating, case. Ref. [12] numerically investigated the effects of migration-induced resonance capture on the distribution of libration amplitudes and libration centers of the twotino population. Remarkably, they found that the likelihood of capture into the so-called leading ($\theta < 180$) and trailing ($\theta > 180$) asymmetric libration islands is dependent on the rate of Neptune's ancient migration.

Murray-Clay and Chiang [12] further investigated the dynamics and provide an explanation of the resultant asymmetry. In the simplest case, a migrating Neptune causes the center of symmetric libration to shift slightly such that the average value of θ is greater than 180^o. Thus, as an object librates about the shifted 'symmetric' resonance center, the object will spend slightly more time on the   side of the libration potential than on the other side (see Fig. 8). This shift in libration center causes the asymmetry in the capture. Murray-Clay and Chiang further recognized that the libration amplitude is a function of the object's e prior to its capture into the 2:1 resonance. As the libration amplitude increases, the amount of time an object spends visiting phases space accessible to both asymmetric islands grows. In fact, for very small libration only the 'trailing' island is explored while for slightly larger eccentricities both islands can be explored.

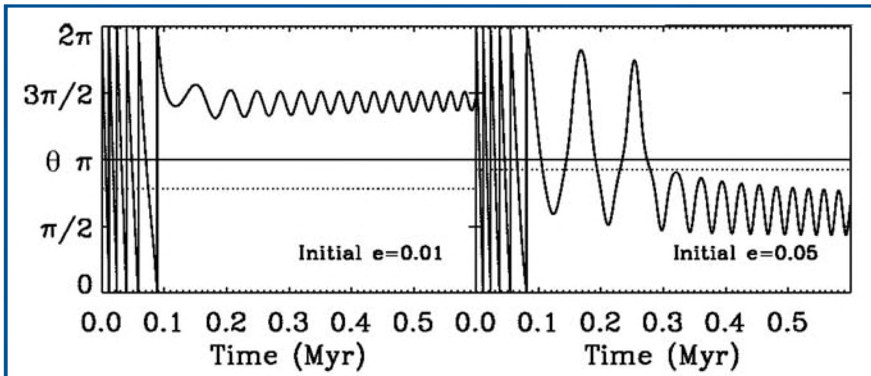

Fig. 8 The evolution of the libration angle ($\theta = \theta_{21}$) with a migrating Neptune for two transneptunian objects which become captured in the 2:1 resonance. At $t = 0$ the object is circulating (not in resonance) and is captured into resonance near t ~ 0.09 Myr. The left panel shows the low-eccentricity case where the libration amplitude is small and the separatrix (dashed line) between the two asymmetric islands is offset to $\theta < \pi$; clearly the object spends no time with small θ and so can not be captured into the island with $\theta < \pi$. For objects with larger initial eccentricities (right panel) the libration amplitudes are larger and the offset of the separatrix is smaller. In this case the object spends some time on the small- side of the separatrix point and so capture into that island becomes possible. (Figure provided by R. Murray-Clay and E. Chiang).

The strength of preference for the 'trailing' versus the 'leading' asymmetric islands is caused by the size of offset in the center of the symmetric libration. The size of this offset is, itself, a function of the migration rate and initial eccentricty of the captured twotino (see equation 26 of Ref. [12]). Thus, measuring the current ratio of 'trailing' to 'leading' twotinos could, when coupled to some estimate of the initial eccentricity of the twotino population, provide another measure of the migration rate of Neptune. The density of the primordial outer Solar System could then be inferred.

Although both the 3:2 capture efficiency and the 'trailing' versus 'leading' 2:1 asymmetry provide only indirect, and somewhat complicated, information on the rate of migration of Neptune, they both provide independent inferences. Coupling of the observational census of the Kuiper Belt's resonant populations with our evolving knowledge of the complex dynamical evolution of these bodies provides a path to unlocking the ancient history of our Solar System.

OBSERVATIONS

Since the discovery of the second trans-Neptunian object in 1992 there has been a veritable explosion of discovery. At the close of 2007, just 15 years after that transformative discovery, approximately 807 trans-Neptunian objects were known, of which some 120 might be plutinos and another dozen or so appear to be Twotinos (see Ref. [13] for a review of the nomenclature of the trans-Neptunian region and a census of its members). But wait, why 'approximately' 807 and 'might be plutinos' or 'appear to be twotinos'? The endeavour of wide-field search surveys for transneptunian bodies has been an exciting race, with multiple groups attempting to establish strong records of discovering new objects. Orbital determination of the discoveries, however, tends to be less exciting and requires many times the observational resources as compared to the initial discovery. Determining a transneptunian's precise orbit, particularly one in resonance, requires many dozen positional measurements spread over a number of years. Without a well-sampled and lengthy observational history, it is impossible to constrain quantities like the libration center and amplitudes (see Fig. 7), or even answering the question: "Is this object in resonance?"

The difficulty in determining the precise orbits leads to the problem of object loss. Without secure estimates of the orbital elements, the future positional uncertainty on the sky grows rapidly and soon (within a few months to a year) observers can no longer locate the object and it is lost. The constant leaking away of these discoveries contributes to a biased view of the known populations.

Determining the migration rate of Neptune from the relative strengths of the population of resonant objects (such as the relative size of the population of the 3:2 resonance as compared to say, the 2:1 resonant objects) requires that we either have unbiased estimates of those populations or, if the estimate is biased that we be able to account for that bias (see Ref. [14] for a discussion of observational biases that must be accounted for). Unfortunately, accounting for objects that have been lost due to insufficient observations is not a bias that we can corrected for.

One observational bias that can be understood in a straightforward way is the flux bias: brighter objects are easier to detect. Fig. 5 shows the positions, relative to the Sun in a frame rotating with Neptune, that a plutino explores during one libration cycle; this object makes closer approaches to the Sun when roughly 90 degrees away from Neptune on the sky. Since one only detects trans-Neptunians via reflected sunlight, their brightness L is given by $L \propto 1/r^4$, where r is their heliocentric distance. Thus, since plutinos are closest when 90 degrees from Neptune, a survey that looks in these directions is more likely to find plutinos than would a survey which looks towards Neptune. Because of this flux bias, we must know the pointing history of a survey before we determine the fraction of objects in the 3:2 resonance compared to other populations.

Fig. 7 presents the libration centers and amplitudes for the current sample of twotinos. We see that most of the currently-observed asymmetric population is in the 'leading' asymmetric island (libration center, $\theta \sim 90^o$, see Fig. 6). Does this then indicate that Neptune's migration was extremely slow or rather that the pre-capture orbits of the twotinos had large eccentricities; both effects would reduce the dynamical preference for 'leading' capture (see Ref. [12]) or is some other bias perhaps at work? For twotinos, as for plutinos, there is a bias towards discovery when the objects are near perihelion, for small-libration amplitude objects this occurs when they are near their libration centers relative to Neptune. Currently the trailing libration center, which is 60^o 'behind' Neptune, is aligned on the sky with the direction of the galactic center. Thus, although the trailing twotinos are brightest when on this part of their obit, discovery is hampered by observational confusion with the vast number of galactic plane stars in the background. Determining the intrinsic population from the observed one requires a knowledge of the pointing history of the discovery survey and an accurate estimate of the fraction of the search fields that are actually discoverable.

The authors of this manuscript are involved in a project (visit http://www.cfeps.net) that is striving to address the major problems with the observational constraints of Kuiper belt populations. This project carefully measures the internal biases for discovery and tracking, so that the sample of objects can be used to 'back out' the true populations.

CONCLUSION

Although this article has just scratched the surface of a very complex dynamical problem, we hope the reader will have garnered some appreciation for the dynamics of resonant orbital motion and how it can be used to diagnose the ancient history of the Solar System.

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