Lemma 1
Let X, Y be arbitrary spaces which are dense in cX and cY, repectively. Let f be a continuous function from X onto Y, $\bar{f}$ be the continuous extension of f. Let A be a closed subset of X. Then we have:
(i) If $f[A]$ *is not closed in* Y *then there exists an element* $x \in cl_{cX}A \backslash A$ *such that* $\bar{f}(x) \in cl_Y f[A] \backslash f[A]$.
(ii) If $f[A]$ *is closed in* Y *then for any element* $x \in cl_{cX}A \backslash A$ *for which* $\bar{f}(x) \in Y$ *holds, we have* $\bar{f}(x) \in f[A]$ *holds.*

Proof of (i):
We'll prove this fact first: $cl_{cY} f[A] \subseteq \bar{f}[cl_{cX}A]$.
Since $\bar{f}$ is a closed map from cX to cY, $\bar{f}[cl_{cX}A]$ is closed in cY. Moreover, since $\bar{f}[cl_{cX}A]$ is a closed set containing the set $\bar{f}[A]$ as well as the set $f[A]$, $cl_{cY} f[A]$ must be a subset of $\bar{f}[cl_{cX}A]$ by the definition of closures.
Now, back to the proof of (i). From $cl_{cY} f[A] \subseteq \bar{f}[cl_{cX}A]$, we get $cl_{cY} f[A] \cap Y \backslash f[A] \subseteq \bar{f}[cl_{cX}A] \cap Y \backslash f[A]$. By assymption, $f[A]$ is not closed in Y. So, $cl_{cY} f[A] \cap Y \backslash f[A] \neq \emptyset$. Then, we get $\bar{f}[cl_{cX}A] \cap Y \backslash f[A] \neq \emptyset$ as well, which is equivalent of saying that there exists an element $x \in cl_{cX}A$ such that $\bar{f}(x) \in \bar{f}[cl_{cX}A] \cap Y \backslash f[A] = cl_Y \bar{f}[A] \backslash f[A]$.

Proof of (ii):
We'll prove this fact first: $\bar{f}[cl_{cX}A] \subseteq cl_{cY} f[A]$.
Let $y \in \bar{f}(cl_{cX}A)$. We will show y is also in $cl_{cY} f(A)$. Pick x an element of $cl_{cX}(A)$ such that $\bar{f}(x) = y$. Now let V be any open subset of cY that contains y. Since $\bar{f}$ is continuous, there exist $U \in \tau(cX)$ such that $\bar{f}[U] \subseteq V$. Since $x \in U$ and $x \in cl_{cX}(A)$, U must meet A. So $\bar{f}[U]$ meets $\bar{f}[A]$ as well. So $\bar{f}[U] \subseteq V$ gives $V \cap \bar{f}[A] = \emptyset$.
Now back to the proof of (ii), we then have $\bar{f}[cl_{cX}A] \subseteq cl_{cY} f[A] \Rightarrow \bar{f}[cl_{cX}A] \cap Y \subseteq cl_{cY} f[A] \cap Y$. By assumption, $f[A]$ is closed in Y, so $cl_{cY} f[A] \cap Y = f[A]$. So now we have $\bar{f}[cl_{cX}A] \cap Y \subseteq f[A]$, and it gives us that if $x \in cl_{cX}A \backslash A$ and if $\bar{f}(x) \in Y$, then $\bar{f}(x) \in f[A]$, as desired.