Lemma 2 *If the Cartesian product $X \times Y$ of Tychonoff spaces X and Y is pseudocompact, then the projection $\pi_X : X \times Y \to X$ transforms functionally closed subsets of $X \times Y$ to closed subsets of X.*

Proof. Let X, Y be Tychonoff and $X \times Y$ be pseudocompact. By way of contradiction, suppose that there exists a functionally closed subset Z of $X \times Y$ such that $\pi_X[Z]$ is not closed in X. Let $x_0 \in cl_X \pi_X[Z] \setminus \pi_X[Z] \neq \emptyset$. Let $g : X \times Y \to [0,1]$ such that $Z = g^{\leftarrow}(0)$.

Define $f : X \times Y \to [0,1]$ such that $f(x,y) = min\{\frac{g(x,y)}{g(x_0,y)}, 1\}$. By our definition, f is a continuous function. Moreover, $f(x_0, y) = 1$ for all $y \in Y$ and $Z = f^{\leftarrow}(0)$.

Since X is Tychonoff, it is first-countable. Let $\mathcal{B}_{x_0} = \{B_k \in \tau(X) : B_0 \supset B_1 \supset B_2 \supset \cdots$ and $x_0 \in B_k, k = 1, 2, 3, ... \}$ be a neighborhood basis of x_0. We shall define two sequences, (x_0, y_i) and(x_i, y_i), and their open neighborhoods(open in $X \times Y$), V_i's and W_i's by induction:

Step 1: Pick a point in Z and label it (x_1, y_1). Since $X \times Y$ is Hausdorff and $\mathcal{B}$ is the neighborhood basis of x_0, there exist $W_1 \in \tau(X \times Y)$ and $V_1 \in \mathcal{B}$ such that $(x_1, y_1) \in W_1$ and $(x_0, y_1) \in V_1$, and $W_1 \cap V_1 = \emptyset$.

Since $g(x_1, y_1) = 0, f(x_1, y_1) = 0$ as well.

$\square$