Lemma 3 *Let X be a Tychonoff space. If X is pseudocompact, then every locally finite family of non-empty open subsets of X is finite.*

Proof. (By way of contradiction.)Suppose that there exists a locally finite family $\mathcal{F} = \{U_i \in \tau(X) : U_i \neq \emptyset, 1 \leq i \leq \infty\}$. Since each U_i is non-empty, choose a point $x_i \in U_i$ for $i = 1, 2, ...$ Since X is a Tychonoff space, there exists continuous functions $f_i : X \to [0, i]$ such that $f_i(x_i) = i$ and $f_i(X \backslash U_i) \subseteq \{0\}$, for each $i \in \mathbb{N}$. Define the fuction $f : X \to \mathbb{R}$ as $f(x) = \Sigma_{i=1}^{\infty}|f_i(x)|$. To show f is continuous, pick $x_0 \in X$ and open set V' of $\mathbb{R}$) containing $f(x_0)$. Let $V = V' \cap (f(x_0) - \frac{1}{10}, f(x_0) + \frac{1}{10})$. Since $\mathcal{F}$ is locally finite, there exists an open set $U_0 \in \tau(X)$ containing x_0 such that U_0 meets $\mathcal{F}$ only finitely many times. So there exists $\{a_i\{_{i=1}^{n} \subset \mathbb{N}$ such that $U_0 \cap U_{a_i} \neq \emptyset$ for $i = 1...n$. Define $\delta : \mathcal{P}(\mathbb{R}) \to \mathbb{R}$ to be $\delta(S) = sup(S) - inf(S)$. For each $i = 1...n$, since f_{a_i} is continuous, there exists $W_i \in \tau(X)$ such that $\delta(f[W_i]) < \frac{1}{10n}$. Let $W = W_1 \cap W_2 \cap \cdots \cap W_n$. Then $\delta(f_i[W]) < \frac{1}{10n}$ for each $i = 1...n$. So $\delta(f[W]) < \frac{1}{10}$. Hence f is a continuous function. However, as $f(x_i) \leq i$ for all $i \in \mathbb{N}$, f is clearly not bounded. This contradicts the pseudocompactness of X.

$\square$