Lemma 2

Let X, Y be Tychonoff spaces and cX, cY be T_2 compactifications of X, Y. Let f be a continuous function from X onto Y, $\bar{f}$ be the continuous extension of f. Let A be a closed subset of X. Then we have:
(i) If $f[A]$ is not closed in Y then there exists an element $x \in cl_{cX}A \backslash A$ such that $\bar{f}(x) \in cl_Y f[A] \backslash f[A]$.
(ii) If $f[A]$ is closed in Y then for any element $x \in cl_{cX}A \backslash A$ for which $\bar{f}(x) \in Y$ holds, we have $\bar{f}(x) \in f[A]$ holds.

Fact: $cl_{cY} f[A] = \bar{f}[cl_{cX}A]$.
Since $\bar{f}$ is a closed map from cX to cY, $\bar{f}[cl_{cX}A]$ is closed in cY. Moreover, since $\bar{f}[cl_{cX}A]$ is a closed set containing the set $\bar{f}[A]$ as well as the set $f[A]$, $cl_{cY} f[A]$ must be a subset of $\bar{f}[cl_{cX}A]$ by the definition of closures. Hence $cl_{cY} f[A] \subseteq \bar{f}[cl_{cX}A]$. By continuity, $\bar{f}[cl_{cX}A] \subseteq cl_{cY} \bar{f}[A] = cl_{cY} f[A]$.

Proof of (i):
By the Fact, $cl_{cY} f[A] \subseteq \bar{f}[cl_{cX}A]$, we get $cl_{cY} f[A] \cap Y \backslash f[A] \subseteq \bar{f}[cl_{cX}A] \cap Y \backslash f[A]$. By assymption, $f[A]$ is not closed in Y. So, $cl_{cY} f[A] \cap Y \backslash f[A] \neq \emptyset$. Then, we get $\bar{f}[cl_{cX}A] \cap Y \backslash f[A] \neq \emptyset$ as well, which is equivalent of saying that there exists an element $x \in cl_{cX}A$ such that $\bar{f}(x) \in \bar{f}[cl_{cX}A] \cap Y \backslash f[A] = cl_Y \bar{f}[A] \backslash f[A]$.

Proof of (ii):
From the Fact, We have $\bar{f}[cl_{cX}A] \subseteq cl_{cY} f[A] \Rightarrow \bar{f}[cl_{cX}A] \cap Y \subseteq cl_{cY} f[A] \cap Y$. By assumption, $f[A]$ is closed in Y, so $cl_{cY} f[A] \cap Y = f[A]$. So now we have $\bar{f}[cl_{cX}A] \cap Y \subseteq f[A]$, and it gives us that if $x \in cl_{cX}A \backslash A$ and if $\bar{f}(x) \in Y$, then $\bar{f}(x) \in f[A]$, as desired.