Lemma 2.4 Let X and Y be Tychonoff spaces, and $\pi_X : X \times Y \to X$ be the projection map. If π_X is z-closed, Z is a zero-set in $X \times Y$, and $(x,p) \in \overline{Z}^{X\times\beta Y}$, then $(x,p) \in \overline{Z \cap (\{x\} \times Y)}^{X\times\beta Y}$.

Proof: Assume that $(x,p) \notin \overline{Z \cap (\{x\} \times Y)}^{X\times\beta Y}$. Since $X \times \beta Y$ is Tychonoff, there exists a continuous function $f : X \times \beta Y \to [0,1]$ such that $f\left[\overline{Z \cap (\{x\} \times Y)}^{X\times\beta Y}\right] \subseteq \{1\}$ and $f(x,p) = 0$.

Let $Z_f = f^{\leftarrow}(0)$. So Z_f contains (x,p). Since $(x,p) \in \overline{Z}^{X\times\beta Y}$, we have $(x,p) \in \overline{Z}^{X\times\beta Y} \cap Z_f$. Thus,

$$x \in \pi_X\left[\overline{Z}^{X\times\beta Y} \cap Z_f\right] \subseteq \pi_X\left[\overline{Z \cap Z_f}^{X\times\beta Y}\right] \subseteq \overline{\pi_X\left[Z \cap Z_f\right]}^{X}.$$

On the other hand, since $\overline{Z \cap (\{x\} \times Y)}^{X\times\beta Y} \cap Z_f = \emptyset$, we have $Z \cap (\{x\} \times Y) \cap Z_f = \emptyset$. Now, if $x \in \pi_X\left[Z \cap Z_f\right]$, then $Z \cap Z_f \neq \emptyset$ and hence $Z \cap Z_f \cap (\{x\} \times Y) \neq \emptyset$, contradiction. So, $x \notin \pi_X\left[Z \cap Z_f\right]$.

Hence,

$$x \in \overline{\pi_X\left[Z \cap Z_f\right]}^{X} \setminus \pi_X\left[Z \cap Z_f\right].$$

As $Z \cap Z_f$ is a zero-set in $X \times Y$, $\pi_X\left[Z \cap Z_f\right]$ is closed in X, we have $\overline{\pi_X\left[Z \cap Z_f\right]}^{X} = \pi_X\left[Z \cap Z_f\right]$, contradiction.