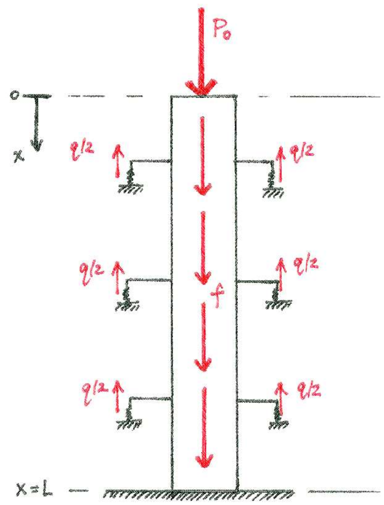
1. The figure below shows a steel pile of length $L = 4\,\mathrm{m}$, with constants $E = 210\,\mathrm{GPa}$ and $A = \pi(0.2)^2\,\mathrm{m}^2$. The pile is fixed at the bottom, and is considered to be a Winkler foundation across its length. In other words, we assume that the soil exerts a force that is opposite and linearly proportional to the displacement of the pile, i.e. $q = q(x) = ku(x)$ for some constant k (this is called foundation stiffness). Note, q has the unit of N/m.



We want to know how the pile deforms due to the load $P_0 = 10^5\,\mathrm{N}$ at its head, the distributed load $f = f(x)$ N/m, and the lateral force q due to the soil. For simplicity, we consider $k = EA/L^2$ and $f(x) = P_0/L$ below.

(a) By considering the equilibrium of a differential element, derive the governing equation and the BCs that the displacement of the pile $u = u(x)$ satisfies.

(b) Find the corresponding weak form. Given an element with $(p+1)$ basis functions, write down the element stiffness matrix and the element load vector.

(c) From the weak form, obtain the governing equation and the BCs that u satisfies.

(d) Consider 1 element with a linear polynomial basis ($p = 1$). Find the FE solution u_h and calculate the internal force $F_h(0) = EA\,u_h'(0)$ at the pile head. What do you see?

(e) This time, consider 2 elements with a linear polynomial basis ($p = 1$) in each element.

(f) Lastly, consider 1 and 2 elements with a quadratic polynomial basis ($p = 2$) in each. Assume that the middle node is placed at the center of the element.