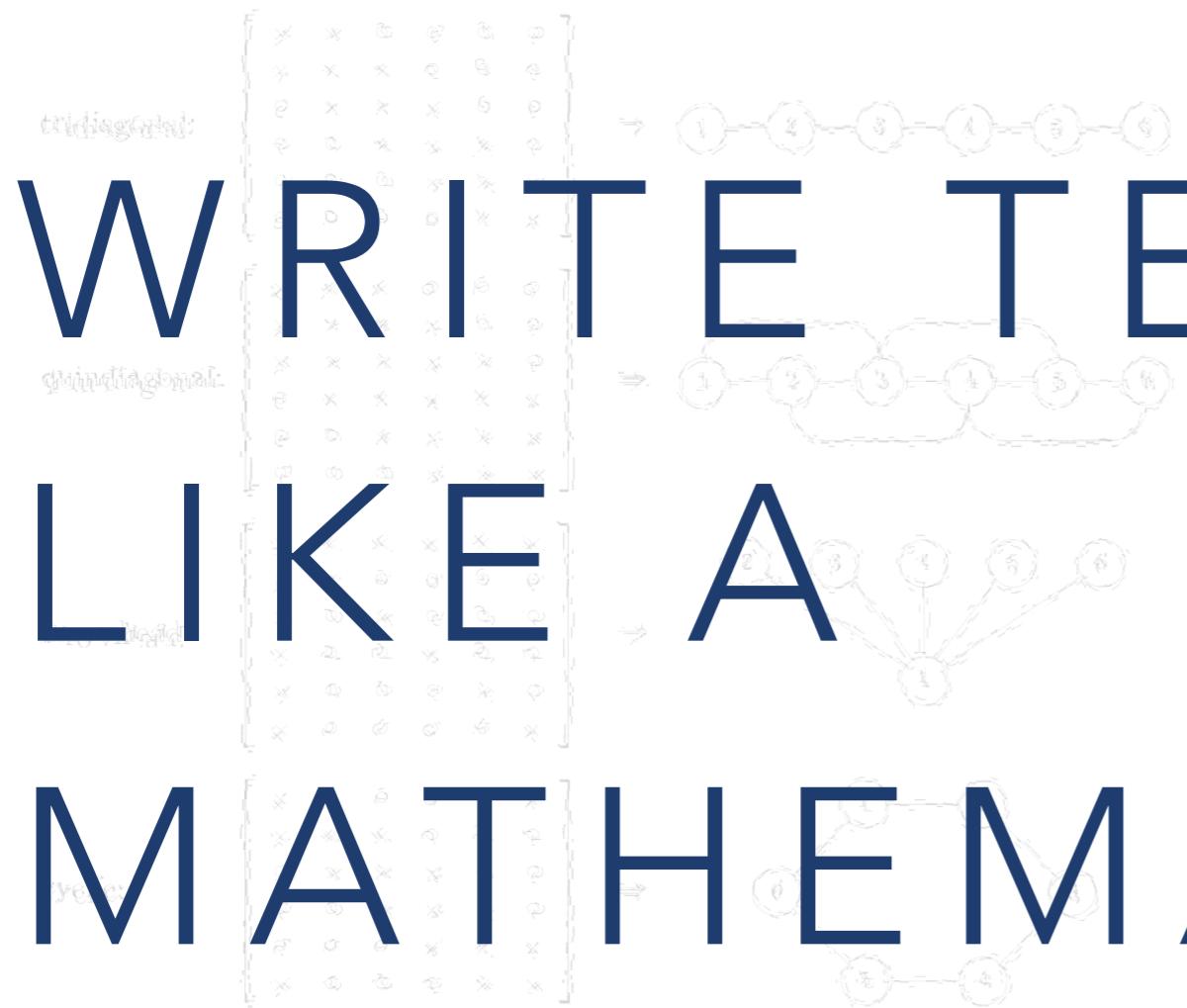


Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

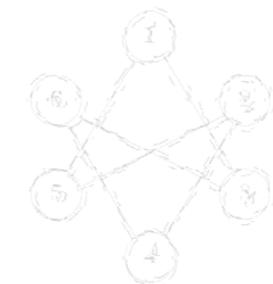


# WRITE TESTS LIKE A MATHEMATICIAN

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

ISAAC J. LEE

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, let us detail the relations as follows:

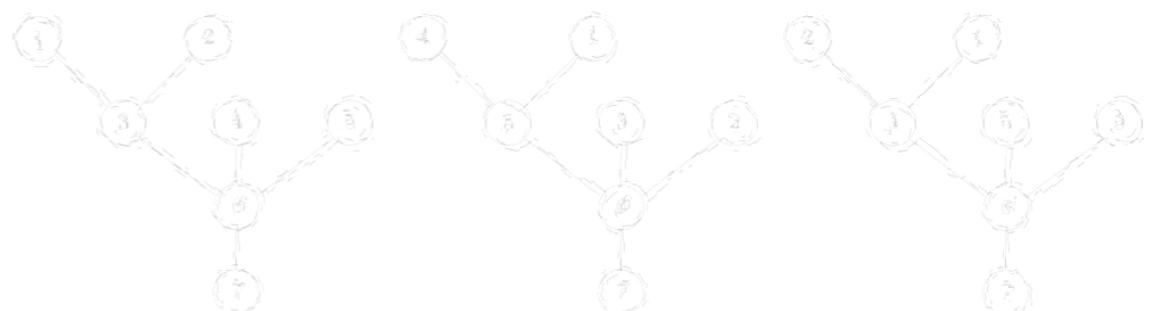
$$1 \rightarrow 2, 2 \rightarrow 3, 3 \rightarrow 2, 4 \rightarrow 5, 5 \rightarrow 6, 6 \rightarrow 3.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

Let  $\alpha$  be an ordered set of vertices  $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_n) \subseteq \mathbb{S}$  is called a *path* joining the vertices  $\alpha$  and  $\beta$  if  $\alpha \in \{1, 2, \dots, n\}$ ,  $\beta \in \{1, 2, \dots, n\}$  and for every  $k = 1, 2, \dots, n-1$  the set  $\{\alpha_k, \alpha_{k+1}\} \cap \{\beta_k, \beta_{k+1}\}$  contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that  $\mathbb{G}$  is a *tree* if each two members of  $\mathbb{V}$  are joined by a unique simple path. Both tridiagonal and pentadiagonal matrices correspond to trees, but this is not the case with either quadiagonal or cyclic matrices when  $n \geq 3$ .

Given a tree  $\mathbb{G}$  and an arbitrary vertex  $r \in \mathbb{V}$ , the pair  $\mathbb{T} = (\mathbb{G}, r)$  is called a *rooted tree*, while  $r$  is said to be the *root*. Unlike an ordinary graph,  $\mathbb{T}$  admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root  $r$  is the predecessor of all the vertices in  $\mathbb{V} \setminus \{r\}$  and these vertices are successors of  $r$ . Moreover, every  $\alpha \in \mathbb{V} \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex along this path, except for  $r$  and  $\alpha$ , as a predecessor of  $\alpha$  and a successor of  $r$ . We say that the rooted tree  $\mathbb{T}$  is *monotonically ordered* if each vertex is labelled before all its predecessors (or, more precisely, we have the relation from the top of the tree to the root. If we have already said it, relabeling a graph is tantamount to relabeling the rows and the columns of the underlying matrix).

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three monotone orderings of the sample rooted tree:



**Theorem III.1.** Let  $\mathbb{A}$  be a symmetric matrix whose graph  $\mathbb{G}$  is a tree. Choose a root  $r \in \{1, 2, \dots, n\}$  and assume that the rows and columns of  $\mathbb{A}$  have been arranged so that  $\mathbb{T} = (\mathbb{G}, r)$  is monotonically ordered. Given that  $\mathbb{A} = \mathbb{L}\mathbb{U}^T$  is a Cholesky factorization, it is true that

$$a_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

# 5 Rules of Writing Tests

Filter:

Module: Acceptance | Outline



EmberFest 18.10.2019

Running:

asymmetric:

$$\begin{bmatrix} 0 & 2 & 3 & 2 & 2 \\ 2 & 0 & 2 & 2 & 2 \\ 3 & 2 & 0 & 2 & 2 \\ 2 & 2 & 2 & 0 & 2 \\ 2 & 2 & 2 & 2 & 0 \end{bmatrix} \Rightarrow$$



cyclic:

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \Rightarrow$$



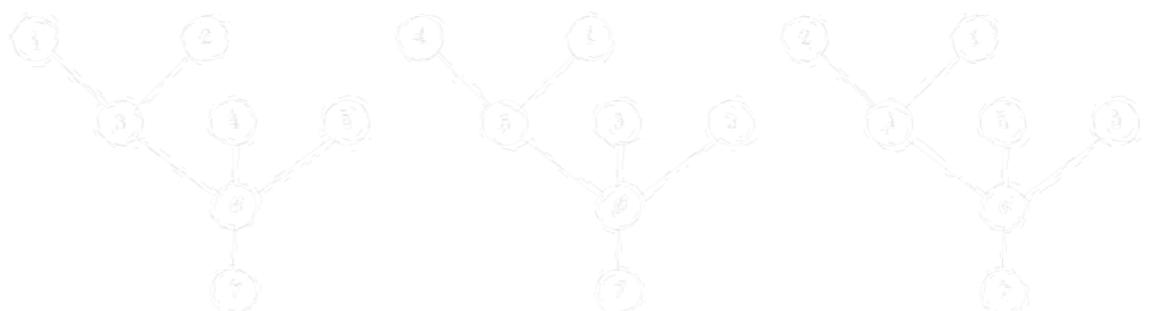
The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} 0 & 2 & * & 0 & * & 0 \\ 2 & 0 & 0 & * & x & 0 \\ * & 0 & 0 & 0 & 0 & * \\ 0 & * & 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

At a first glance, there is nothing to think in the case of the four matrices that we have just displayed, but its graph,

which can best be explained by an analogy with a family tree. Thus, the root  $r$  is the predecessor of all the vertices in  $T \setminus \{r\}$  and these vertices are successors of  $r$ . Moreover, every  $\alpha \in V \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex along this path, except for  $r$  and  $\alpha$ , as a predecessor of  $\alpha$  and a successor of  $r$ . We say that the rooted tree  $T$  is *monotonically ordered* if each vertex is labelled before all its predecessors; in other words, we label the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three consecutive orderings of the same rooted tree.



**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  are then arranged so that  $T = (G, r)$  is monotonically ordered. Then, that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ki}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

# 5 Rules of Writing Tests

Filter:

Module: Acceptance | Outline

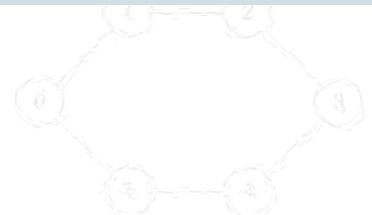
EmberFest 18.10.2019

1 assertion of 1 passed, 0 failed.

## If, if, if

cyclic:

$$\begin{bmatrix} * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \end{bmatrix} \Rightarrow$$



before all its predecessors in other words: we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three monotone orderings of the same rooted tree:



**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  are then arranged so, that  $T = (G, r)$  is monotonically ordered. Then, that  $A = L U$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ki}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

At a first glance, there is nothing to think in the case of the four matrices that we have just displayed, but its graph.

# 5 Rules of Writing Tests

Filter:

Module: Acceptance | Outline

# EmberFest 18.10.2019

2 assertions of 2 passed, 0 failed.

If, if, if

Use common, everyday words

# 5 Rules of Writing Tests

Filter:

Module:

Acceptance | Outline



EmberFest 18.10.2019

3 assertions of 3 passed, 0 failed.

! If, if, if

! Use common, everyday words

! Write less with theorems and new terms

$$\begin{bmatrix} * & * & * & 0 & * & 0 \\ 0 & * & 0 & * & * & 0 \\ * & 0 & * & * & 0 & * \\ 0 & * & 0 & * & * & 0 \\ * & * & 0 & * & * & 0 \\ 0 & * & * & 0 & * & * \end{bmatrix}.$$

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph:



**Theorem 11.1** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  are then arranged so, that  $\mathbf{T} = (G, r)$  is monotonically ordered. Then, that  $A = \mathbf{L}\mathbf{U}^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ii}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

# 5 Rules of Writing Tests

Filter:

Module:

Acceptance | Outline



EmberFest 18.10.2019

4 assertions of 4 passed, 0 failed.

! If, if, if

! Use common, everyday words

! Write less with theorems and new terms

! All your basis are belong to us

$$\begin{bmatrix} 0 & 3 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph.

**Theorem 11.1** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  are then arranged so, that  $\tilde{T} = (G, r)$  is monotonically ordered. Then, that  $A = \tilde{L}\tilde{U}$  is a Cholesky factorization, it is true that

$$t_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ki}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

# 5 Rules of Writing Tests

Filter:

Module: [Acceptance](#) | [Outline](#) ▾

EmberFest 18.10.2019

5 assertions of 5 passed, 0 failed.

! If, if, if

! Use common, everyday words

! Write less with theorems and new terms

! All your basis are belong to us

! 1 picture = 1000 words

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} * & * & * & 0 & * & * \\ 0 & * & 0 & * & * & * \\ * & 0 & * & 0 & * & * \\ 0 & * & 0 & * & 0 & * \\ * & * & 0 & 0 & * & * \\ 0 & 0 & * & * & * & * \end{bmatrix}$$

At a first glance, there is nothing to think in the case just displayed, but its graph,



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

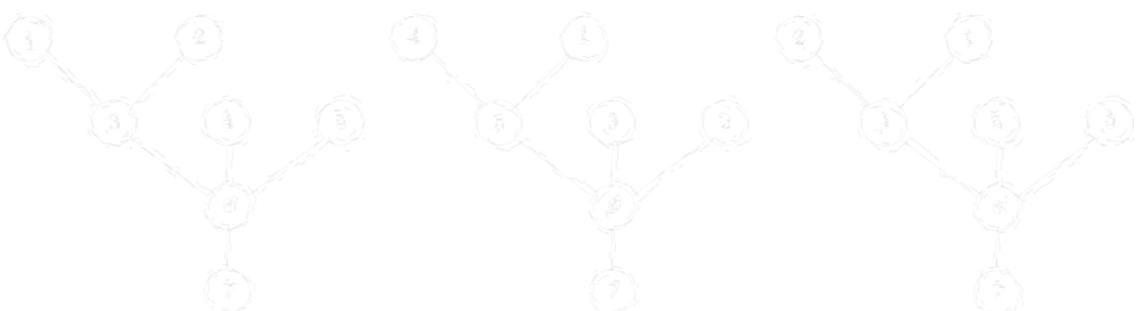
$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 3, \quad 5 \rightarrow 6, \quad 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges  $\{(i_0, j_0)\}_{i_0, j_0 \in \mathbb{N}_0}$  is called a path joining the vertices  $i_0$  and  $j_0$  if  $i_0 \in \{i_0, j_0\}$ ,  $j_0 \in \{i_0, j_0\}$  and for every  $k = 1, 2, \dots, n-1$  the set  $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$  contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that  $G$  is a tree if each two members of  $V$  are joined by a unique simple path. Both tridiagonal and quidiagonal matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when  $n \geq 3$ .

Given a tree  $T$  and an arbitrary vertex  $r \in V$ , the pair  $T = (G, r)$  is called a rooted tree, while  $r$  is said to be the root. Unlike in ordinary graph,  $T$  admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root  $r$  is the predecessor of all the vertices in  $T \setminus \{r\}$  and these vertices are successors of  $r$ . Moreover, every  $a \in V \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex along this path, except for  $r$  and  $a$ , as a predecessor of  $a$  and a successor of  $r$ . We say that the rooted tree  $T$  is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we lay out the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three consecutive orderings of the same rooted tree.



**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, n\}$  and assume that the rows and columns of  $A$  are then arranged so that  $T = (G, r)$  is monotonically ordered. Given that  $A = L U$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$



to tell a different story - it is nothing other than a cyclic matrix in disguised. To see this, first we look the variables as follows:

1 → 2, 2 → 3, 3 → 2, 4 → 1, 5 → 6, 6 → 3,

This, of course, is equivalent to reordering the columns and variables. An  $n \times n$  matrix  $G$  is a set of edges  $\{(i_1, j_1)\} \cup \{(i_2, j_2)\} \cup \dots \cup \{(i_n, j_n)\}$ , where  $i_1, i_2, \dots, i_n \in \{1, 2, \dots, n\}$  and for each  $i \in \{1, 2, \dots, n\}$  the set  $\{j_1, j_2, \dots, j_n\}$  contains exactly one member. We say that  $G$  is a path if for any two vertices  $v$  and  $w$  in  $V$  we can find a path from  $v$  to  $w$ . Both tridiagonal and banded matrices correspond to paths, the case with either quasi-diagonal or the matrix when  $n > 3$ .

Given a tree, while ordering a root  $r$  is the root of  $r$ . More each vertex of  $r$ . We say before all is true to the permutation

Every  $\mathcal{L}$  can be monotonically enlarged and, in general,  $\mathcal{L}$  is not unique.

The diagram consists of three separate network graphs arranged horizontally. Each graph has nodes numbered 1 through 5. In the first graph, nodes 1, 2, 3, 4, and 5 are connected in a sequence: 1-2, 2-3, 3-4, and 4-5. In the second graph, nodes 4, 3, 2, 1, and 5 are connected in a sequence: 4-3, 3-2, 2-1, and 1-5. In the third graph, nodes 2, 3, 4, 5, and 1 are connected in a sequence: 2-3, 3-4, 4-5, and 1-2. The word 'THEN' is centered in large, bold, dark blue capital letters. The background is white with a light gray grid.

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & 0 \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & * \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & * \end{bmatrix}.$$

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,



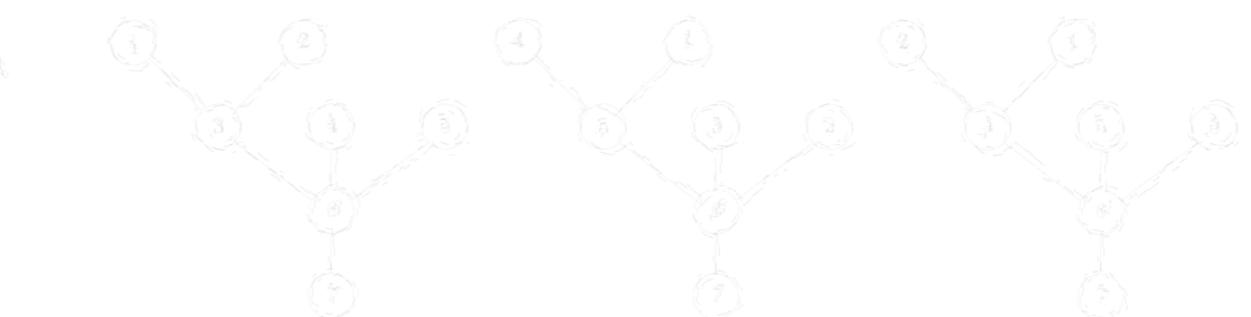
tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges  $\{(i_0, j_0)\}_{i_0, j_0 \in \mathbb{N}_0}$  in  $\mathbb{G}$  is called a path joining the vertices  $i_0$  and  $j_0$  if  $i_0 \in \{i_0, j_0\}$ ,  $j_0 \in \{i_0, j_0\}$  and for every  $k = 1, 2, \dots, n-1$  the set  $\{i_{k-1}, j_k\} \cap \{i_k, j_{k+1}\}$  contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that  $\mathbb{G}$  is a tree if each two members  $i_0, j_0$  are joined by a unique simple path. Both tridiagonal and quindiagonal matrices correspond to trees; this is not the case with either quidiagonal or cyclic matrices.

Let  $\mathbb{G}$  be a tree and an arbitrary vertex  $r \in V$ . The pair  $T = (\mathbb{G}, r)$  is called a rooted tree, where  $r$  is said to be the root. Unlike in a cyclic graph, there is no unique path from a vertex to the root. This can best be explained by an example. Let  $\mathbb{G}$  be a tree with  $r$  as the root, which is the predecessor of all the vertices in  $V \setminus \{r\}$ . Then  $r$  is called the root of  $\mathbb{G}$ . Moreover, every  $v \in V \setminus \{r\}$  is joined to  $r$  by a unique path, which consists of vertices along this path. The vertex  $r$  is called a parent of  $v$  and  $v$  is called a child of  $r$ . We say that the rooted tree  $T$  is monotonically ordered if every vertex  $v$  has all its predecessors in  $\mathbb{G}$  as its ancestors. In other words,  $v$  is layered above its root. (As we have already said, it is not possible to have two children with the same parent.) Every rooted tree will be monotonically ordered and, in general, it is not unique. Every rooted tree will give three consecutive properties of the underlying matrix.



**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $\mathbb{G}$  is a tree. Choose a root  $r \in \{1, 2, \dots, n\}$  and assume that the rows and columns of  $A$  are then arranged so that  $T = (\mathbb{G}, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{j,j}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their analysis.

1

```
assert.speaker().getsPersonal();  
  
await sing('Happy Birthday');  
  
assert.audience().isHappy();
```

just displayed, but its graph:

$$t_{k,j} = \frac{q_{kj}}{q_j}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

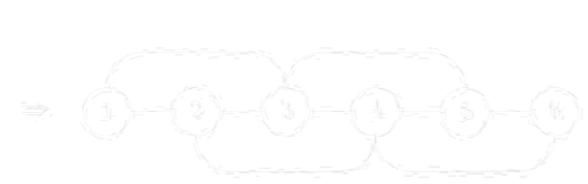
Therefore we will now give a few examples of matrices (represented by their sparsity)

and their graphs.

# SUSPECT 1

quindiagonal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$



symmetric:

$$\begin{bmatrix} * & * & * & * & * & * \\ * & * & 0 & 0 & 0 & 0 \\ * & 0 & * & 0 & 0 & 0 \\ * & 0 & 0 & * & 0 & 0 \\ * & 0 & 0 & 0 & * & 0 \\ * & 0 & 0 & 0 & 0 & * \end{bmatrix}$$



cyclic:

$$\begin{bmatrix} * & 0 & 0 & 0 & * & * \\ * & * & 0 & 2 & * & * \\ * & 0 & * & * & * & * \\ * & * & * & * & * & * \\ * & 0 & * & * & * & * \\ 0 & * & * & * & * & * \end{bmatrix}$$



# OBSERVER /

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

# COMPUTED

At a first glance, the matrix  $A$  and its structure of the form of a symmetric matrix just displayed, but its graph,



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 3, \quad 5 \rightarrow 6, \quad 6 \rightarrow 8.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges  $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$  is called a path joining the vertices  $i_0$  and  $j_0$  if  $i_0 \in \{i_0, j_0\}$ ,  $j_0 \in \{i_0, j_0\}$  and for every  $k = 1, 2, \dots, n-1$  the set  $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$  contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that  $G$  is a tree if each two members of  $V$  are joined by a unique simple path. Both tri-diagonal and any other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when  $n \geq 3$ .

Given a tree  $T$  and an arbitrary vertex  $r \in V$ , the pair  $T = (G, r)$  is called a rooted tree, while  $r$  is said to be the root. Unlike in ordinary graph,  $T$  admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root  $r$  is the predecessor of all the vertices in  $T \setminus \{r\}$  and these vertices are successors of  $r$ . Moreover, every  $\alpha \in V \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex along this path, except for  $r$  and  $\alpha$ , as a predecessor of  $\alpha$  and a successor of  $r$ . We say that the rooted tree  $T$  is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree.



**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  were then arranged so that  $T = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

and their graphs

# SUSPECT 2

quindiagonal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$



superdiagonal:

$$\begin{bmatrix} * & * & * & * & * & * \\ * & * & 0 & 0 & 0 & 0 \\ * & 0 & * & 0 & 0 & 0 \\ 0 & 0 & * & 0 & 0 & 0 \\ 0 & 0 & 0 & * & 0 & 0 \\ 0 & 0 & 0 & 0 & * & 0 \end{bmatrix}$$



cyclic:

$$\begin{bmatrix} * & 0 & 0 & 0 & * & 0 \\ * & * & 0 & 2 & * & 0 \\ * & 0 & * & * & 0 & 0 \\ 0 & 2 & * & * & * & 0 \\ 0 & * & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$



# EMBER DATA / FORM BUILDER

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & * & 0 \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$

At a first glance, this is nothing but the structure of the form builder matrix

just displayed, but its graph



tells a different story - it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

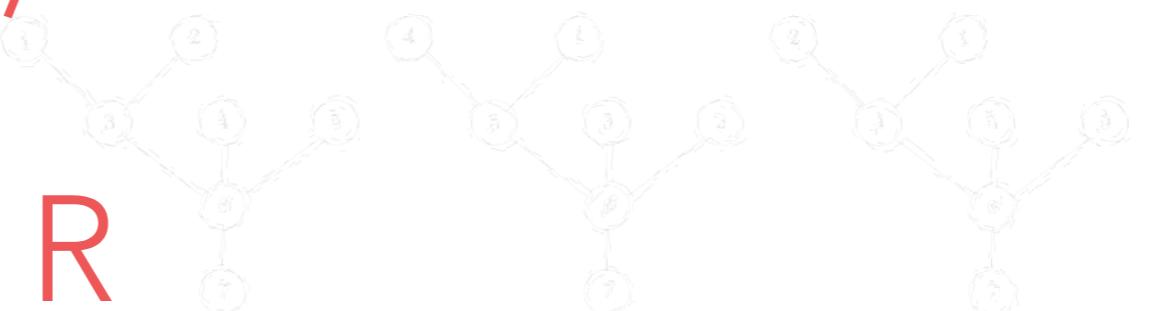
$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 8.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges  $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$  is called a path joining the vertices  $i_0$  and  $j_0$  if  $i_0 \in \{1, \dots, n\}$ ,  $j_0 \in \{1, \dots, n\}$  and for every  $k = 1, 2, \dots, n-1$  the set  $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$  contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that  $G$  is a tree if each two members of  $V$  are joined by a unique simple path. Both tri-diagonal and any other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when  $n \geq 3$ .

Given a tree  $T$  and an arbitrary vertex  $r \in V$ , the pair  $T = (G, r)$  is called a rooted tree, while  $r$  is said to be the root. Unlike in ordinary graph,  $T$  admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root  $r$  is the predecessor of all the vertices in  $T \setminus \{r\}$  and these vertices are successors of  $r$ . Moreover, every  $\alpha \in V \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex along this path, except for  $r$  and  $\alpha$ , as a predecessor of  $\alpha$  and a successor of  $r$ . We say that the rooted tree  $T$  is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree.



**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, n\}$  and assume that the rows and columns of  $A$  are then arranged so, that  $T = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

and their graphs.

# SUSPECT 3



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix



At a first glance, this is nothing but a collection of little dots on a white background.

just displayed, but its graph,



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges  $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$   $\subseteq \mathbb{E}$  is called a path joining the vertices  $i_0$  and  $j_0$  if  $i_0 \in \{i_0, j_0\}$ ,  $j_0 \in \{i_0, j_0\}$  and for every  $k = 1, 2, \dots, n-1$  the set  $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$  contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that  $G$  is a tree if each two members of  $V$  are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when  $n \geq 3$ .

Given a tree  $T$  and an arbitrary vertex  $r \in V$ , the pair  $T = (G, r)$  is called a rooted tree, while  $r$  is said to be the root. Unlike in a ordinary graph,  $T$  admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root  $r$  is the predecessor of all the vertices in  $T \setminus \{r\}$  and these vertices are successors of  $r$ . Moreover, every  $\alpha \in V \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex along this path, except for  $r$  and  $\alpha$ , as a predecessor of  $\alpha$  and a successor of  $r$ . We say that the rooted tree  $T$  is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three consecutive drawings of the same rooted tree.



**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  were been arranged so, that  $T = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

and their graphs.

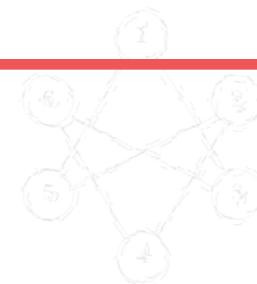
# SUSPECT 4



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

# LEAKAGE

At a first glance, this is nothing but a collection of the four matrices above, but just displayed, but its graph:



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

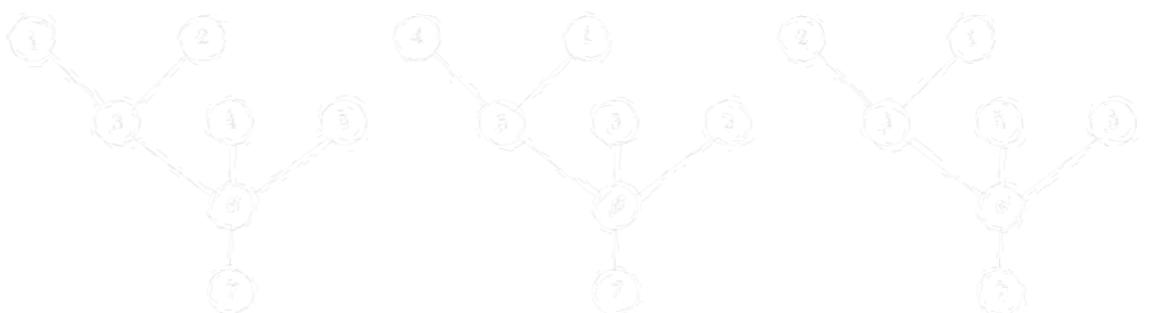
$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 1.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges  $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$   $\subseteq \mathbb{E}$  is called a path joining the vertices  $i_0$  and  $j_0$  if  $i_0 \in \{i_0, j_0\}$ ,  $j_0 \in \{i_0, j_0\}$  and for every  $k = 1, 2, \dots, n-1$  the set  $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$  contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that  $G$  is a tree if each two members of  $V$  are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when  $n \geq 3$ .

Given a tree  $T$  and an arbitrary vertex  $r \in V$ , the pair  $T = (G, r)$  is called a rooted tree, while  $r$  is said to be the root. Unlike in ordinary graph,  $T$  admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root  $r$  is the predecessor of all the vertices in  $T \setminus \{r\}$  and these vertices are successors of  $r$ . Moreover, every  $a \in V \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex along this path, except for  $r$  and  $a$ , as a predecessor of  $a$  and a successor of  $r$ . We say that the rooted tree  $T$  is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three consecutive orderings of the same rooted tree:



**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  were been arranged so, that  $T = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

and their graphs.

# SUSPECT 5

quindiagonal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ 0 & * & * & 0 & 0 & 0 \\ 0 & 0 & * & * & 0 & 0 \\ 0 & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & * & * \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$



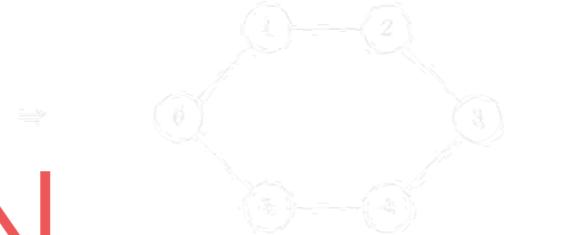
superdiagonal:

$$\begin{bmatrix} * & * & * & * & * & * \\ 0 & * & 0 & 0 & 0 & 0 \\ 0 & 0 & * & 0 & 0 & 0 \\ 0 & 0 & 0 & * & 0 & 0 \\ 0 & 0 & 0 & 0 & * & 0 \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$



cyclic:

$$\begin{bmatrix} * & 0 & 0 & 0 & * & * \\ 0 & * & 0 & 0 & 2 & 1 \\ 0 & 0 & * & 0 & 3 & 2 \\ 0 & 0 & 0 & * & 4 & 3 \\ 0 & 0 & 0 & 0 & * & 5 \\ 0 & 0 & 0 & 0 & 5 & * \end{bmatrix}$$

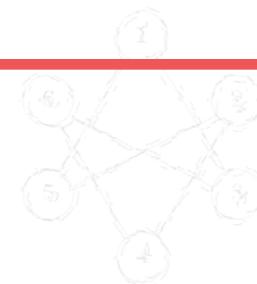


# ADMIN

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

# PRIVILEGE

At a first glance, this is nothing but the structure of the famous triangular matrix just displayed, but its graph:



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 3, \quad 5 \rightarrow 6, \quad 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges  $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$  is called a path joining the vertices  $i_0$  and  $j_0$  if  $i_0 \in \{1, \dots, n\}$ ,  $j_0 \in \{1, \dots, n\}$  and for every  $k = 1, 2, \dots, n-1$  the set  $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$  contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that  $G$  is a tree if each two members of  $V$  are joined by a unique simple path. Both tri-diagonal and any other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when  $n \geq 3$ .

Given a tree  $T$  and an arbitrary vertex  $r \in V$ , the pair  $T = (G, r)$  is called a rooted tree, while  $r$  is said to be the root. Unlike in ordinary graph,  $T$  admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root  $r$  is the predecessor of all the vertices in  $T \setminus \{r\}$  and these vertices are successors of  $r$ . Moreover, every  $\alpha \in V \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex along this path, except for  $r$  and  $\alpha$ , as a predecessor of  $\alpha$  and a successor of  $r$ . We say that the rooted tree  $T$  is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

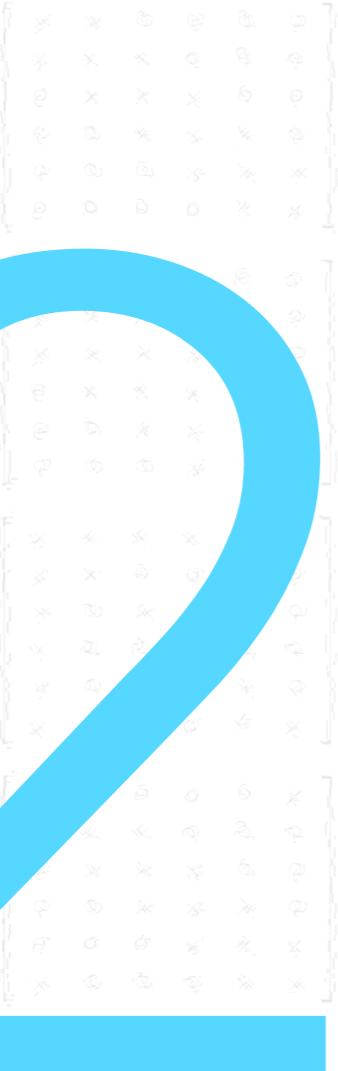
Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three consecutive orderings of the same rooted tree:

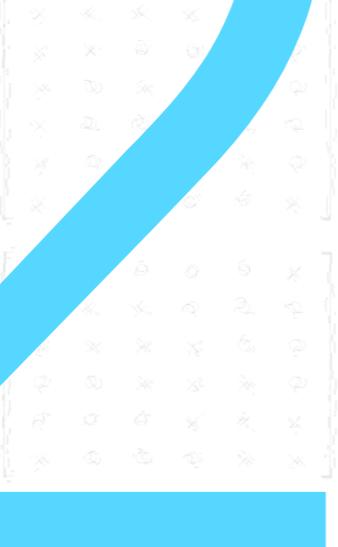


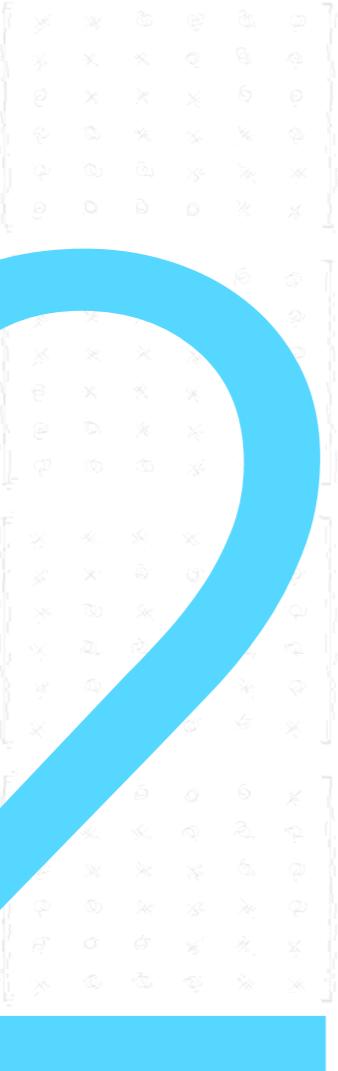
**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  are then arranged so that  $T = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

tridiagonal:  


symmetric:  


cyclic:  


2

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix



At a first glance, there is nothing to link it to any of the three structures just displayed, but its graph,

# USE COMMON EVERYDAY WORDS



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 1.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges  $\{v_1, v_2, \dots, v_p\} \subseteq E$  is called a path joining the vertices  $v_1$  and  $v_p$  ( $v_i \in V$ ). If  $v_1 = v_p$  and for every  $i = 1, \dots, p-1$  the set  $\{v_i, v_{i+1}\} \cap E$  contains exactly one member, it is a simple path if it does not visit any vertex more than once. We say that  $G$  is tree if each two members of  $V$  are joined by a unique simple path. (In this case, all other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when  $p \geq 3$ .)

Given a tree  $G$  and an arbitrary vertex  $r \in V$ , the pair  $T = (G, r)$  is called a rooted tree, while  $r$  is said to be the root. Unlike in ordinary graph,  $T$  admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root  $r$  is the predecessor of all the vertices in  $V \setminus \{r\}$  and these vertices are successors of  $r$ . Moreover, every  $v \in V \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex along this path, except for  $r$  and  $v$ , as a predecessor of  $v$  and a successor of  $r$ . We say that the rooted tree  $T$  is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we label the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is unique. To give the precise definition of the unique rooted tree,



**Theorem 1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  are then arranged so, that  $T = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{k,j}}{l_{j,j}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

and their graphs

triangular:  $\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ 0 & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & * & 0 \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix} \Rightarrow \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \xrightarrow{1 \rightarrow 1, 2 \rightarrow 3, 3 \rightarrow 5, 4 \rightarrow 6, 5 \rightarrow 6, 6 \rightarrow 6}$

quidiagonal:  $\begin{bmatrix} * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ 0 & * & * & * & * & * \\ 0 & 0 & * & * & * & * \\ 0 & 0 & 0 & * & * & * \end{bmatrix} \Rightarrow \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \xrightarrow{1 \rightarrow 1, 2 \rightarrow 3, 3 \rightarrow 4, 4 \rightarrow 5, 5 \rightarrow 6, 6 \rightarrow 6}$

superdiagonal:  $\begin{bmatrix} * & * & * & * & * & * \\ * & 0 & 0 & 0 & 0 & 0 \\ * & 2 & 0 & 0 & 0 & 0 \\ * & 2 & 2 & 0 & 0 & 0 \\ * & 2 & 2 & 2 & 0 & 0 \\ * & 2 & 2 & 2 & 2 & 0 \end{bmatrix} \Rightarrow \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \xrightarrow{1 \rightarrow 1, 2 \rightarrow 2, 3 \rightarrow 3, 4 \rightarrow 4, 5 \rightarrow 5, 6 \rightarrow 6}$

cyclic:  $\begin{bmatrix} * & 0 & 0 & 0 & * & * \\ * & * & 0 & 0 & 2 & 2 \\ * & * & * & 0 & 2 & 2 \\ * & * & * & * & * & 2 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix} \Rightarrow \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \xrightarrow{1 \rightarrow 1, 2 \rightarrow 2, 3 \rightarrow 3, 4 \rightarrow 4, 5 \rightarrow 5, 6 \rightarrow 6}$

# CONVENTION;

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

# AGREED ON BY MANY

At a first glance, this is not a triangular matrix, but it is a tree structure, as the one just displayed, but its graph:



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 3, 3 \rightarrow 2, 4 \rightarrow 4, 5 \rightarrow 6, 6 \rightarrow 8.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges  $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$   $\subseteq \mathbb{E}$  is called a path joining the vertices  $i_0$  and  $j_0$  if  $i_0, j_0 \in \{1, \dots, n\}$ ,  $i_0 \neq j_0$ ,  $\{i_0, j_0\} \subseteq \{i_1, \dots, i_{n+1}\}$  and for every  $k = 1, 2, \dots, n-1$  the set  $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$  contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that  $G$  is a tree if each two members of  $V$  are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when  $n \geq 3$ .

Given a tree  $T$  and an arbitrary vertex  $r \in V$ , the pair  $T = (G, r)$  is called a rooted tree, while  $r$  is said to be the root. Unlike in a ordinary graph,  $T$  admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root  $r$  is the predecessor of all the vertices in  $T \setminus \{r\}$  and these vertices are successors of  $r$ . Moreover, every  $\alpha \in V \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex along this path, except for  $r$  and  $\alpha$ , as a predecessor of  $\alpha$  and a successor of  $r$ . We say that the rooted tree  $T$  is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree:



**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, n\}$  and assume that the rows and columns of  $A$  are then arranged so, that  $T = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

tridiagonal:

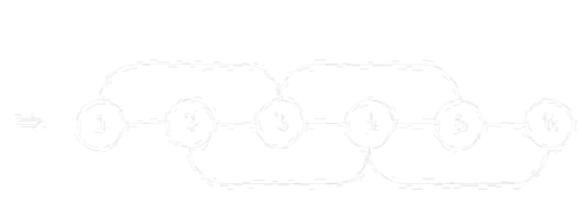
$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & 0 & * & * & 0 & 0 \\ 0 & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & * & * \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$

EVERYDAY



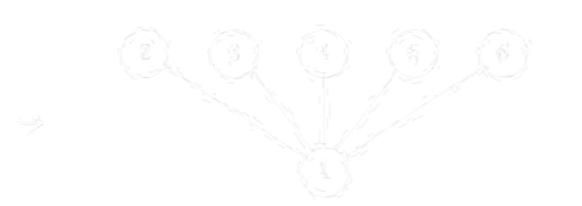
quindiagonal:

$$\begin{bmatrix} * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



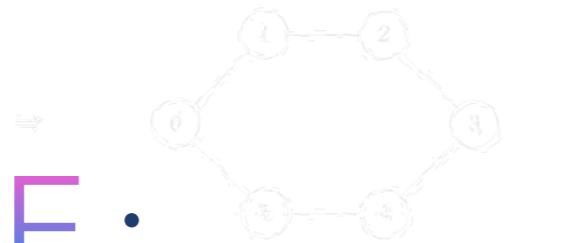
superdiagonal:

$$\begin{bmatrix} * & * & * & * & * & * \\ * & * & 0 & 0 & 0 & 0 \\ 0 & 0 & * & * & * & * \\ 0 & 0 & * & * & * & * \\ 0 & 0 & * & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



cyclic:

$$\begin{bmatrix} * & 0 & 0 & 0 & * & * \\ * & * & 0 & 2 & * & * \\ 0 & 0 & * & * & * & * \\ 0 & 2 & * & * & * & * \\ 0 & 0 & * & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



SIMPLE;

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

FAMILIAR TO MANY

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & * & * & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & * & * & * & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & * & * & * & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & * & * & * & * & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & * & * & * & * & 0 & 0 \\ 0 & 0 & 0 & 0 & * & * & * & * & * & 0 \\ 0 & 0 & 0 & 0 & 0 & * & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & * & * & * & * & * \end{bmatrix}$$

At a first glance this is not a matrix that looks like the ones we have been discussing so far. In fact, it is not the matrix that is just displayed, but its graph.



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

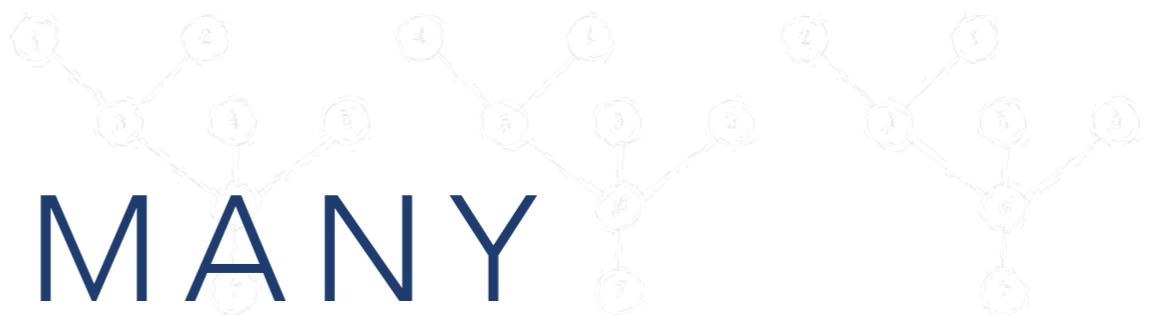
$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 3, \quad 5 \rightarrow 6, \quad 6 \rightarrow 8.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges  $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$  is called a path joining the vertices  $i_0$  and  $j_0$  if  $i_0 \in \{i_0, j_0\}$ ,  $j_0 \in \{i_0, j_0\}$  and for every  $k = 1, 2, \dots, n-1$  the set  $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$  contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that  $G$  is a tree if each two members of  $V$  are joined by a unique simple path. Both tridiagonal and quidiagonal matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when  $n \geq 3$ .

Given a tree  $T$  and an arbitrary vertex  $r \in V$ , the pair  $T = (G, r)$  is called a rooted tree, while  $r$  is said to be the root. Unlike in a ordinary graph,  $T$  admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root  $r$  is the predecessor of all the vertices in  $T \setminus \{r\}$  and these vertices are successors of  $r$ . Moreover, every  $\alpha \in V \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex along this path, except for  $r$  and  $\alpha$ , as a predecessor of  $\alpha$  and a successor of  $r$ . We say that the rooted tree  $T$  is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree.



**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, n\}$  and assume that the rows and columns of  $A$  are then arranged so that  $T = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

triangular:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$

quadrangular:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$

cyclic:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$

At a first glance, the last matrix is not symmetric, but it is a triangular matrix, as we have just displayed, but its graph,



**IF  $f$  IS CONTINUOUS IN  $[a, b]$ , AND IF  $f(a)f(b) < 0$ ,**

**THEN  $f$  MUST HAVE A ZERO IN  $(a, b)$ .**



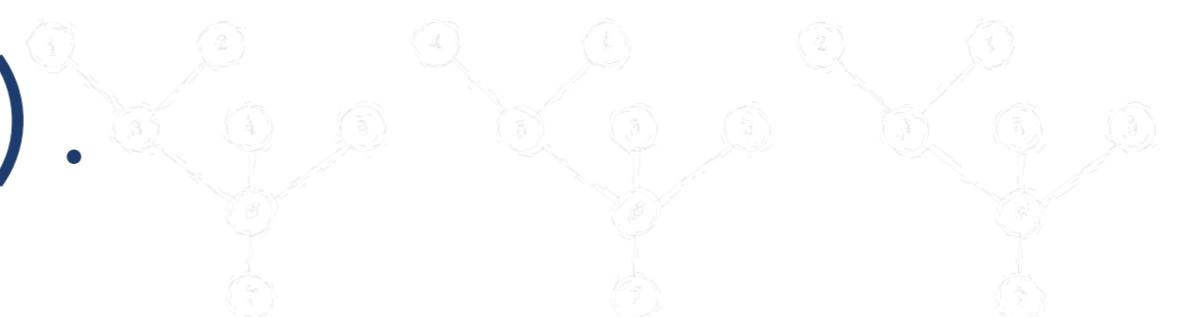
tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 3 \rightarrow 5, \quad 2 \rightarrow 4 \rightarrow 6, \quad 3 \rightarrow 1, \quad 4 \rightarrow 2.$$

Of course, this is equivalent to reordering (or relabeling) the equations and variables. As a result, the set  $\{(i, j) \in \mathbb{N} \times \mathbb{N} \mid 1 \leq i, j \leq n\}$  is called a *tree* if visiting the vertices  $i$  and  $j$  ( $i \in \{1, \dots, n\}$ ,  $j \in \{1, \dots, n\}$ ) and for every  $k = 1, 2, \dots, n-1$  the set  $\{(i_k, j_k) \in \{1, \dots, n\} \times \{1, \dots, n\} \mid i_k < i_{k+1}\}$  contains exactly one member. It is a *simple path* if it does not visit any vertex more than once. We say that  $G$  is a *tree* if each two members of  $V$  are joined by a unique simple path. Both triangular and quadrangular matrices correspond to trees, but this is not the case with either of quadrangular or cyclic matrices when  $n \geq 3$ .

Of course, a tree  $T = (G, r)$  is called a *rooted tree* if  $r \in V$  is a vertex in  $T$ . Unlike in a binary tree, there is a natural partial order on  $V$  which can best be explained by an analogy with a family tree. Thus, the root  $r$  is the *ancestor* of all the vertices in  $V \setminus \{r\}$  and these vertices are *successors* of  $r$ . Moreover, every  $v \in V \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex along this path, except for  $r$  and  $v$ , as a *predecessor* of  $v$  and a *successor* of  $r$ . We say that the rooted tree  $T$  is *monotonically ordered* if each vertex is labelled by an integer  $i$  in such a way that the vertices from the top of the tree to the bottom are in increasing order (in other words, we say the *height* from the top of the tree to the bottom is  $n-1$ ). (In words, we have already said, labelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree:



**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  are then arranged so that  $T = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

```
assert.dom('[data-test-message]')  
  .hasText(  
    'Thanks for signing up!',  
    'The user sees a welcome message.'  
  );
```

Therefore we will now give a few examples of matrices (represented by their sparsity)

triangular:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



quadrangular:

supersingular:

$$\begin{bmatrix} * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & * & * & * \end{bmatrix}$$



cyclic:

$$\begin{bmatrix} * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \end{bmatrix}$$



ZERO IN  $(a, b)$ .

$$\begin{bmatrix} * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \end{bmatrix}$$

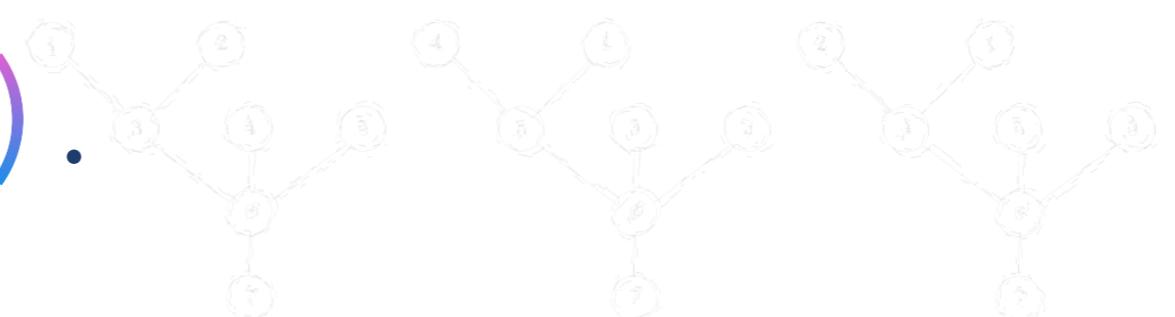
It tells a different story - it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:



It is of course, as equivalent to requiring (equivalently) the equations and variables. As you can see, the set  $\{v_i, v_{i+1}\}$  is called a *pair* (spelling the vertices  $v_i$  and  $v_{i+1}$   $\in \{v_1, v_2, \dots, v_n\}$ ,  $i \in \{1, 2, \dots, n-1\}$ ) and for every  $i = 1, 2, \dots, n-1$  the set  $\{v_i, v_{i+1}\} \cap \{v_{i+1}, v_{i+2}\}$  contains exactly one member. It is a *simple path* if it does not visit any vertex more than once. We say that  $G$  is a *tree* if each two members of  $V$  are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees, but this is not the case with either of bidiagonal or cyclic matrices when  $n \geq 3$ .

It is of course, as equivalent to requiring (equivalently) the equations and variables. As you can see, the set  $\{v_i, v_{i+1}\}$  is called a *pair* (spelling the vertices  $v_i$  and  $v_{i+1}$   $\in \{v_1, v_2, \dots, v_n\}$ ,  $i \in \{1, 2, \dots, n-1\}$ ) and for every  $i = 1, 2, \dots, n-1$  the set  $\{v_i, v_{i+1}\} \cap \{v_{i+1}, v_{i+2}\}$  contains exactly one member. It is a *simple path* if it does not visit any vertex more than once. We say that  $G$  is a *tree* if each two members of  $V$  are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees, but this is not the case with either of bidiagonal or cyclic matrices when  $n \geq 3$ .

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three consecutive orderings of the same rooted tree:



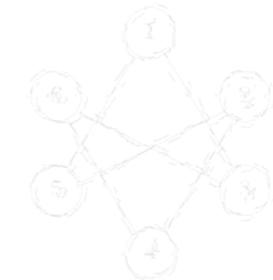
**Theorem 11.1** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  are then arranged so that  $T = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

At a first glance, this is not a very useful formula, but it is the basis for the following theorem:

just displayed, but its graph:

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, first relabel the vertices as follows:



Dashboard

Explore

Settings



vertices more than once. We say that  $G$  is a *tree* if each two members of  $V$  are joined by a unique simple path. Both tridiagonal and super-diagonal matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when  $p \geq 3$ .

Given a tree  $T$  and an arbitrary vertex  $r \in V$ , the pair  $T = (G, r)$  is called a *rooted tree*, while  $r$  is said to be the *root*. Unlike in ordinary graph,  $T$  admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root  $r$  is the *predecessor* of all the vertices in  $T \setminus \{r\}$  and these vertices are *successors* of  $r$ . Moreover, every  $\alpha \in V \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex along this path, except for  $r$  and  $\alpha$ , as a *predecessor* of  $\alpha$  and a *successor* of  $r$ . We say that the rooted tree  $T$  is *monotonically ordered* if each vertex is labelled before all its predecessors. In other words, if label the vertices from the root of the

'[data-test-link="Dashboard"]'

'[data-test-link="Explore"]'

'[data-test-link="Settings"]'

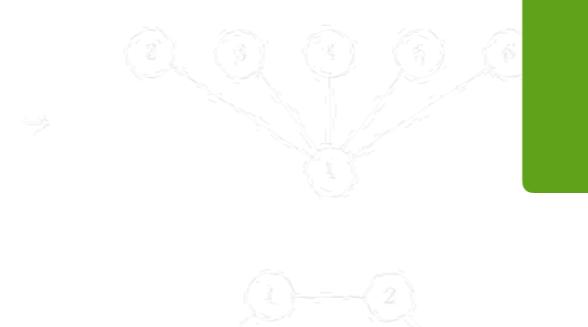
just displayed, but its graph.

$$t_{k,j} = \frac{a_{kj}}{a_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

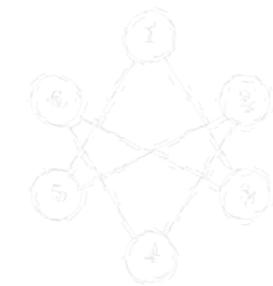
Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

Save

tridiagonal:	$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$
quindiagonal:	$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ 0 & * & * & * & * & 0 \\ 0 & 0 & * & * & * & * \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$
superdiagonal:	$\begin{bmatrix} * & * & * & * & * & * \\ * & * & 0 & 0 & 0 & 0 \\ 0 & * & * & 0 & 0 & 0 \\ 0 & 0 & * & * & 0 & 0 \\ 0 & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$



Cancel



tells a different story - it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$1 \rightarrow 5, 2 \rightarrow 3, 3 \rightarrow 4, 4 \rightarrow 6, 6 \rightarrow 5.$

or reordering (simultaneously) the equations and variables.

$\{(v_i, j)\}_{i,j=1}^n \subseteq \mathbb{S}$  is called a path joining the vertices  $v_i$  and  $v_j$ , and for every  $k = 1, 2, \dots, n-1$  the set  $\{v_i, j\} \cap$

contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that  $G$  is a tree if each two members of  $V$  are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees. This is not the case with either quindiagonal or cyclic matrices when  $n \geq 3$ .

Let a tree  $T$  and an arbitrary vertex  $r \in V$ , the pair  $T = (G, r)$  is called a rooted tree, while  $r$  is said to be the root. Unlike in ordinary graph,  $T$  admits a natural partial order, which can best be explained by an analogy with a family tree. Thus, the root  $r$  is the predecessor of all the vertices in  $T \setminus \{r\}$  and these vertices are successors of  $r$ . Moreover, every  $\alpha \in V \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex along this path, except for  $r$  and  $\alpha$ , as a predecessor of  $\alpha$  and a successor of  $r$ . We say that the rooted tree  $T$  is monotonically ordered if each vertex is labelled before all its predecessors. In other words, if travel the vertices from the root of the

'[data-test-button="Save"]'

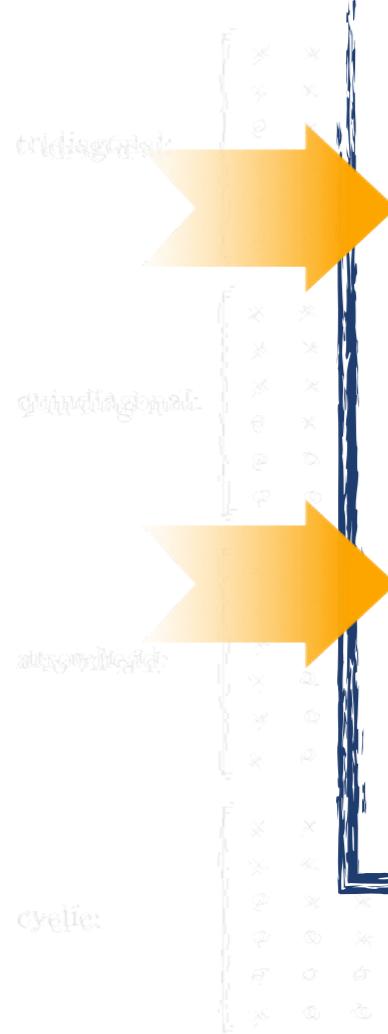
'[data-test-button="Cancel"]'

'[data-test-button="Add item"]'

just displayed, but its graph.

$$t_{k,j} = \frac{a_{k,j}}{a_{jj}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



Name\*

Description

permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three consecutive orderings of the sample rooted tree.

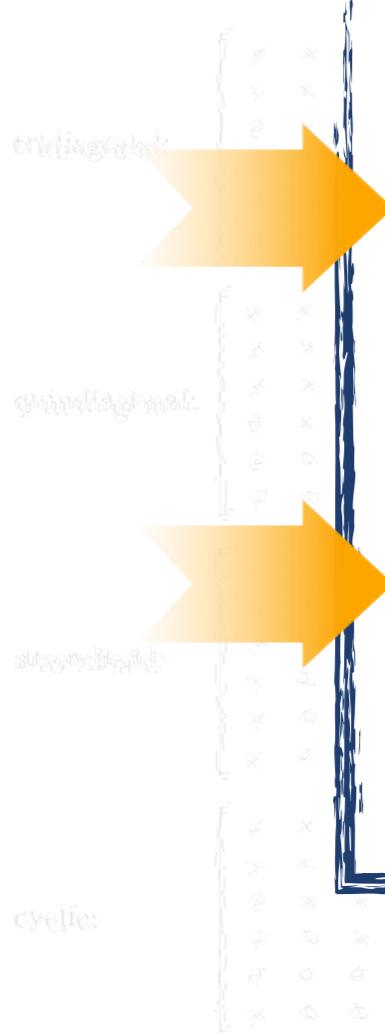
'[data-test-field="Name"]'

'[data-test-field="Description"]'

just displayed, but its graph.

$$t_{k,j} = \frac{a_{kj}}{a_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1, \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



Name

Little Bobby Tables

Description

Better not drop me!

matrix in disguised. To see  
 $6 \Rightarrow 3$ .  
equations and vegetables.  
th joining the vertices of  
 $\{v_1, v_2, \dots, v_p\} \cap V$  in  
ch if it does not visit any  
numbers of  $V$  are joined by  
vertices correspond to trees.  
atries when  $p \geq 3$ .  
-  $(G, \alpha)$  is called a rooted  
"admits a natural partial  
a family tree. Thus, the  
se vertices are *successors*  
e path and we designate  
son of  $\alpha$  and a *successor*  
if each vertex is labelled  
ices from the top of the  
graph is tantamount to  
permuting the rows and the columns of the underlying matrix.)  
Every rooted tree will be monotonically ordered and, in general, such an ordering  
is not unique. We now give three monotonic orderings of the sample rooted tree.

'[data-test-field="Name"]'

'[data-test-field="Description"]'

just displayed, but its graph.

$$t_{k,j} = \frac{a_{kj}}{a_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix



At a first glance, there is nothing to think in the case just displayed, but its graph,

# WRITE LESS WITH THEOREMS AND NEW TERMS

**Theorem 11.1.** Let  $A$  be a  $n \times n$  matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, n\}$  and assume that the rows and columns of  $A$  are then arranged so, that  $T = (G, r)$  is monotonically ordered. Then, that  $A = L U$  is a Cholesky factorization, it is true that

$$l_{kj} = \frac{a_{kj}}{L_{jj}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

triangular:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



quadrangular:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



supernodal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



cyclic:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



IF  $f$  IS CONTINUOUS IN  $[a, b]$ , AND IF  $f(a)f(b) < 0$ ,

THEN  $f$  MUST HAVE A

ZERO IN  $(a, b)$ .



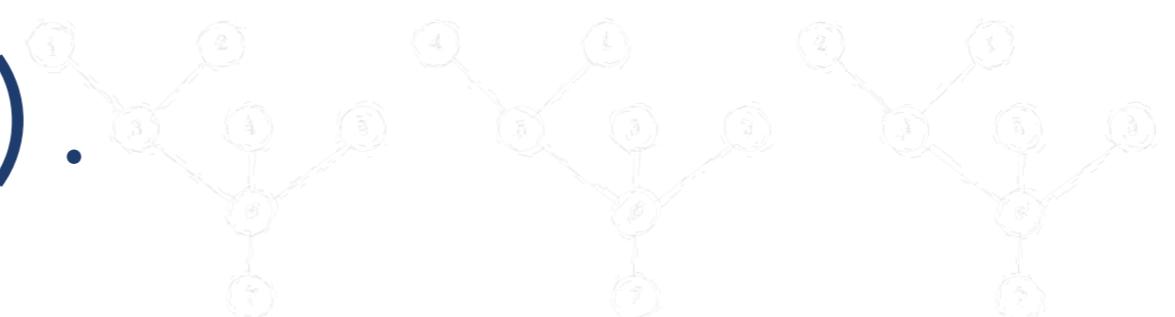
tells a different story - it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 2, 3 \rightarrow 3, 4 \rightarrow 4, 5 \rightarrow 5, 6 \rightarrow 3.$$

Of course, it is equivalent to replacing (arbitrarily) the equations and variables. As you can see from the figure, the graph  $G$  is called a tree, since the vertices  $a$  and  $b$  ( $a \in V \setminus \{b\}$ ,  $b \in V \setminus \{a\}$ ) and for every  $k = 1, 2, \dots, n-1$  the set  $\{v_1, v_2, \dots, v_k\} \cap \{v_{k+1}, v_{k+2}\}$  contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that  $G$  is a tree if each two members of  $V$  are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees, but this is not the case with either of banded or cyclic matrices when  $n \geq 3$ .

Of course, a tree is equivalent to a rooted tree. A vertex  $r \in V$  is called a root of the tree if it is not a predecessor of any other vertex. Unlike in a binary tree, there is a natural partial order in which each vertex is the predecessor of its children. Thus, the vertex  $r$  is the predecessor of all the vertices in  $V \setminus \{r\}$  and these vertices are successors of  $r$ . Moreover, every  $v \in V \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex along this path, except for  $r$  and  $v$ , as a predecessor of  $v$  and a successor of  $r$ . We say that the rooted tree  $T$  is monotonically ordered if each vertex is labelled by an integer from 1 to  $n$  in such a way that the vertices from the top of the tree to the bottom are in increasing order. (In other words, we say the vertices from the top of the tree to the bottom are in increasing order, because the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree:



**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, n\}$  and assume that the rows and columns of  $A$  are then arranged so that  $T = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

At a first glance, this is not a very useful formula, but it is the basis for the following algorithm.

just displayed, but its graph:

Therefore we will now give a few examples of matrices (represented by their sparsity)

# PROOF. USE THE INTERMEDIATE VALUE THEOREM.

cyclic:

$$\begin{bmatrix} * & * & 0 & 0 & 0 \\ 0 & * & * & 0 & 0 \\ * & 0 & * & 0 & 0 \\ 0 & 0 & 0 & * & 0 \\ 0 & 0 & 0 & 0 & * \end{bmatrix}$$



$$\begin{bmatrix} * & * & 0 & 0 & 0 \\ 0 & * & * & 0 & 0 \\ * & 0 & * & 0 & 0 \\ 0 & 0 & 0 & * & 0 \\ 0 & 0 & 0 & 0 & * \end{bmatrix}$$

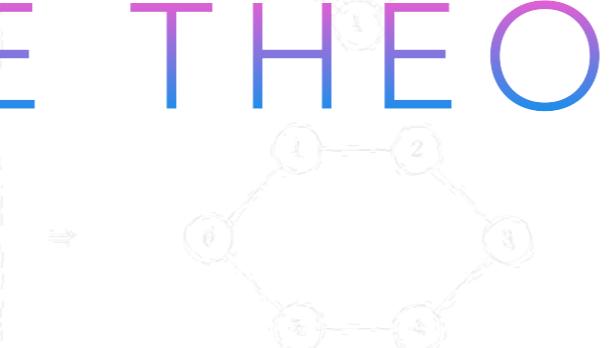


$$\begin{bmatrix} * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ * & 0 & * & * & 0 & 0 \\ 0 & 0 & 0 & * & 0 & 0 \\ 0 & 0 & 0 & 0 & * & 0 \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$



quidiagonal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 & 0 \\ 0 & * & * & 0 & 0 & 0 & 0 \\ * & 0 & * & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & * & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & * & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & * & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & 0 \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & * \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & * \end{bmatrix}$$

At a first glance this is not a cyclic matrix, but it is a bit. It has a triangular structure just displayed, but its graph,



tells a different story - it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

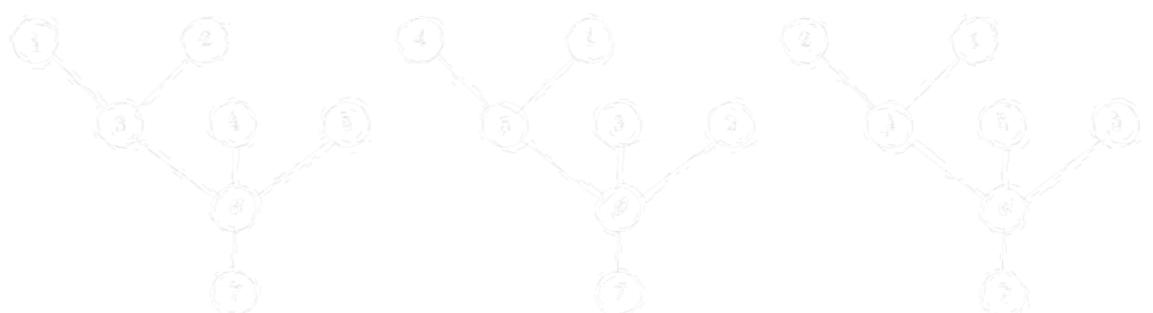
$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 3, \quad 5 \rightarrow 6, \quad 6 \rightarrow 3.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges  $\{(i_0, j_0)\}_{i_0, j_0 \in \mathbb{N}_0, i_0 \neq j_0}$  in  $\mathbb{G}$  is called a path joining the vertices  $i_0$  and  $j_0$  if  $i_0 \in \{0, 1, \dots, n-1\}$ ,  $j_0 \in \{0, 1, \dots, n-1\}$  and for every  $p = 1, 2, \dots, n-1$  the set  $\{i_p, j_p\} \cap \{i_{p-1}, j_{p-1}\}$  contains exactly one member. It is a simple path if it does not visit any vertices more than once. We say that  $\mathbb{G}$  is a tree if in two members of  $\mathbb{V}$  are joined by a unique simple path. Known triangular and quidiagonal matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when  $n \geq 3$ .

Given a tree  $\mathbb{T}$  and an arbitrary vertex  $r \in \mathbb{V}$ , the pair  $\mathbb{T} = (\mathbb{G}, r)$  is called a rooted tree, while  $r$  is said to be the root. Unlike in ordinary graph,  $\mathbb{T}$  admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root is the ancestor of all the vertices in  $\mathbb{V} \setminus \{r\}$  and these vertices are successors of  $r$ . Moreover, every  $\alpha \in \mathbb{V} \setminus \{r\}$  is joined to  $r$  by a simple path and we designate any vertex along this path, except for  $r$  and  $\alpha$ , as a predecessor of  $\alpha$  and a successor of  $r$ . We say that the rooted tree  $\mathbb{T}$  is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree:



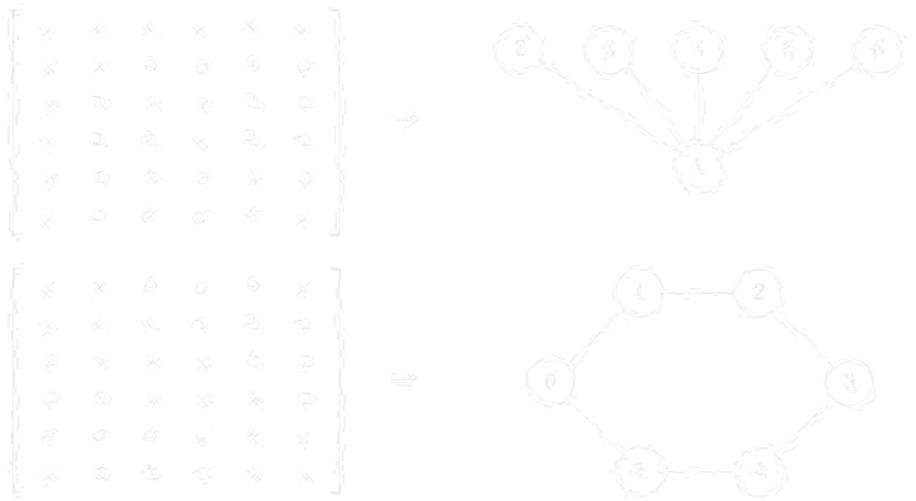
**Theorem 11.1.** Let  $\mathbb{A}$  be a symmetric matrix whose graph  $\mathbb{G}$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $\mathbb{A}$  were then arranged so that  $\mathbb{T} = (\mathbb{G}, r)$  is monotonically ordered. Given that  $\mathbb{A} = \mathbb{L}\mathbb{U}^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

PROOF.  
USE IVT.

triangular:  
quadratic:  
cyclic:



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & x & 0 \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & 0 & * & 0 \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & * & 0 \end{bmatrix}.$$

At a first glance this is not immediately clear what the structure of the matrix is. The graph, however, is just displayed, but its graph:



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 3, \quad 5 \rightarrow 6, \quad 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges  $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$  is called a path joining the vertices  $i_0$  and  $j_0$  if  $i_0 \in \{i_0, j_0\}$ ,  $j_0 \in \{i_0, j_0\}$  and for every  $k = 1, 2, \dots, n-1$  the set  $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$  contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that  $G$  is a tree if each two members of  $V$  are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when  $n \geq 3$ .

Given a tree  $T$  and an arbitrary vertex  $r \in V$ , the pair  $T = (G, r)$  is called a rooted tree, while  $r$  is said to be the root. Unlike in ordinary graph,  $T$  adapts a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root  $r$  is the predecessor of all the vertices in  $T \setminus \{r\}$  and these vertices are successors of  $r$ . Moreover, every  $a \in V \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex along this path, except for  $r$  and  $a$ , as a predecessor of  $a$  and a successor of  $r$ . We say that the rooted tree  $T$  is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we lay out the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three consecutive orderings of the same rooted tree:

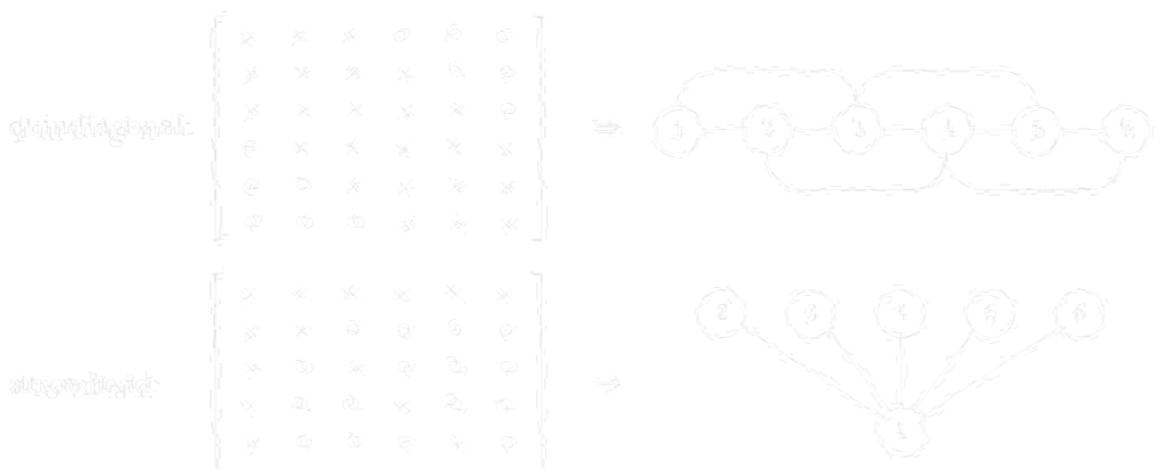


**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  are then arranged so that  $T = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

# THEOREM



# PROVEN

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

# TO BE TRUE

At a first glance this is not a symmetric matrix, but it is. It has a triangular structure just displayed, but its graph



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

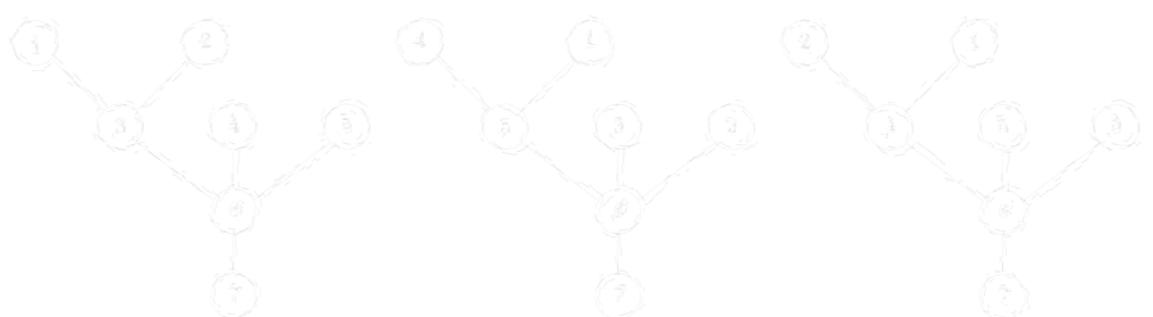
$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges  $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$  is called a path joining the vertices  $i_0$  and  $j_0$  if  $i_0 \in \{i_0, j_0\}$ ,  $j_0 \in \{i_0, j_0\}$  and for every  $k = 1, 2, \dots, n-1$  the set  $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$  contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that  $G$  is a tree if each two members of  $V$  are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when  $n \geq 3$ .

Given a tree  $T$  and an arbitrary vertex  $r \in V$ , the pair  $T = (G, r)$  is called a rooted tree, while  $r$  is said to be the root. Unlike in a ordinary graph,  $T$  admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root  $r$  is the predecessor of all the vertices in  $T \setminus \{r\}$  and these vertices are successors of  $r$ . Moreover, every  $a \in V \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex along this path, except for  $r$  and  $a$ , as a predecessor of  $a$  and a successor of  $r$ . We say that the rooted tree  $T$  is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three consecutive orderings of the same rooted tree.



**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  were been arranged so, that  $T = (G, r)$  is monotonically ordered. Given that  $A = L U$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{k,j}}{l_{j,j}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

```
import { fillIn } from '@ember/test-helpers';

export async function fillForm(fields) {
  for (const { label, value } of fields) {
    // input, textarea
    await fillIn(`[data-test-field="${label}"`, value);
  }
};
```

```
import { fillForm } from '../helpers/my-test-helpers';

...
test('User can create account', async function(assert) {
  await visit('/signup');
  await fillForm([
    { label: 'Name', value: 'Little Bobby Tables' },
    { label: 'Email', value: 'little.bobby@gmail.com' }
  ]);
  ...
});
```

Therefore we will now give a few examples of matrices (represented by their sparsity)

# NEW TERM

quidiagonal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ 0 & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$



superdiagonal:

$$\begin{bmatrix} * & * & * & * & * & * \\ * & 0 & 0 & 0 & 0 & 0 \\ 0 & * & 0 & 0 & 0 & 0 \\ 0 & 0 & * & 0 & 0 & 0 \\ 0 & 0 & 0 & * & 0 & 0 \\ 0 & 0 & 0 & 0 & * & 0 \end{bmatrix}$$



cyclic:

$$\begin{bmatrix} * & 0 & 0 & 0 & * & * \\ * & * & 0 & 0 & 0 & 0 \\ 0 & * & * & 0 & 0 & 0 \\ 0 & 0 & * & * & 0 & 0 \\ 0 & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



# UBIQUITOUS

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

# IDEA

$$\begin{bmatrix} * & * & 0 & 0 & * & * \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$

At a first glance, this is not a matrix that looks like the three structures just displayed, but its graph



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 3, \quad 5 \rightarrow 6, \quad 6 \rightarrow 8.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges  $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$  is called a path joining the vertices  $i_0$  and  $j_0$  if  $i_0 \in \{i_0, j_0\}$ ,  $j_0 \in \{i_0, j_0\}$  and for every  $k = 1, 2, \dots, n-1$  the set  $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$  contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that  $G$  is a tree if each two members of  $V$  are joined by a unique simple path. Both tri-diagonal and any other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when  $n \geq 3$ .

Given a tree  $T$  and an arbitrary vertex  $r \in V$ , the pair  $T = (G, r)$  is called a rooted tree, while  $r$  is said to be the root. Unlike in a ordinary graph,  $T$  admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root  $r$  is the predecessor of all the vertices in  $T \setminus \{r\}$  and these vertices are successors of  $r$ . Moreover, every  $\alpha \in V \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex along this path, except for  $r$  and  $\alpha$ , as a predecessor of  $\alpha$  and a successor of  $r$ . We say that the rooted tree  $T$  is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree:



**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  are then arranged so, that  $T = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$



Therefore we will now give a few examples of matrices (represented by their sparsity)

triangular:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



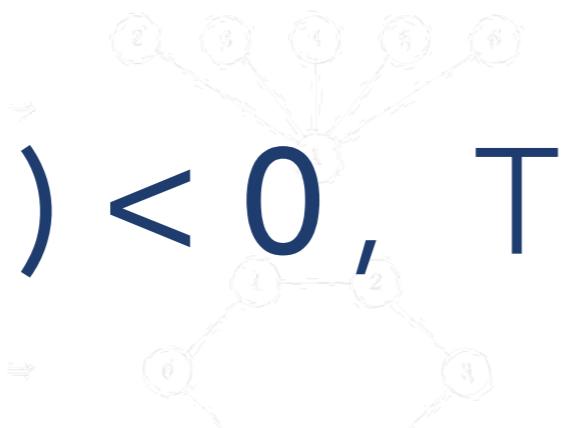
quadratic:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



symmetric:

$$\begin{bmatrix} * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ 0 & * & * & * & * & * \\ 0 & 0 & * & * & * & * \\ 0 & 0 & 0 & * & * & * \end{bmatrix}$$



cyclic:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} * & * & * & 0 & * & 0 \\ 0 & * & 0 & * & * & 0 \\ * & * & * & 0 & 0 & * \\ 0 & 0 & 0 & * & * & * \\ * & * & 0 & 0 & * & * \\ 0 & * & * & * & * & * \end{bmatrix}$$

At a first glance, this is not a triangular, quadratic, or cyclic matrix. But it is a matrix that has just been displayed, but its graph,



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges  $\{(i_1, j_1)\}_{i_1, j_1 \in \mathbb{N}, i_1 \neq j_1}$  in  $\mathbb{G}$  is called a path joining the vertices  $i_1$  and  $j_1$  if  $i_1, j_1 \in \{1, 2, \dots, n\}$ ,  $i_1 \neq j_1$ ,  $j_1 \in \{i_1, i_1+1, \dots, n\}$  and for every  $p = 1, 2, \dots, n-1$  the set  $\{i_p, j_p\} \cap \{i_{p+1}, j_{p+1}\}$  contains only one member. It is a simple path if it does not visit any vertex more than once. We say that  $\mathbb{G}$  is a tree if each two members of  $\mathbb{V}$  are joined by a unique simple path. Both quadiagonal and symmetric matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when  $n \geq 3$ .

Given a tree  $\mathbb{G}$  and an arbitrary vertex  $r \in \mathbb{V}$ , the pair  $T = (G, r)$  is called a rooted tree, while  $r$  is said to be the root. Unlike in a digraph,  $T$  admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the predecessor of the vertex  $v$  is the vertex  $u$  if  $v$  is a successor of  $u$  in the tree  $T$ . Moreover, every  $u \in \mathbb{V} \setminus \{r\}$  is the predecessor of  $v$  in the path and we designate  $u$  as a parent along this path, except for  $r$ , which is a rootless vertex of  $\mathbb{G}$  and a successor of  $r$ . We say that the rooted tree  $T$  is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we lay out the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We will give three examples consisting of the same rooted tree:

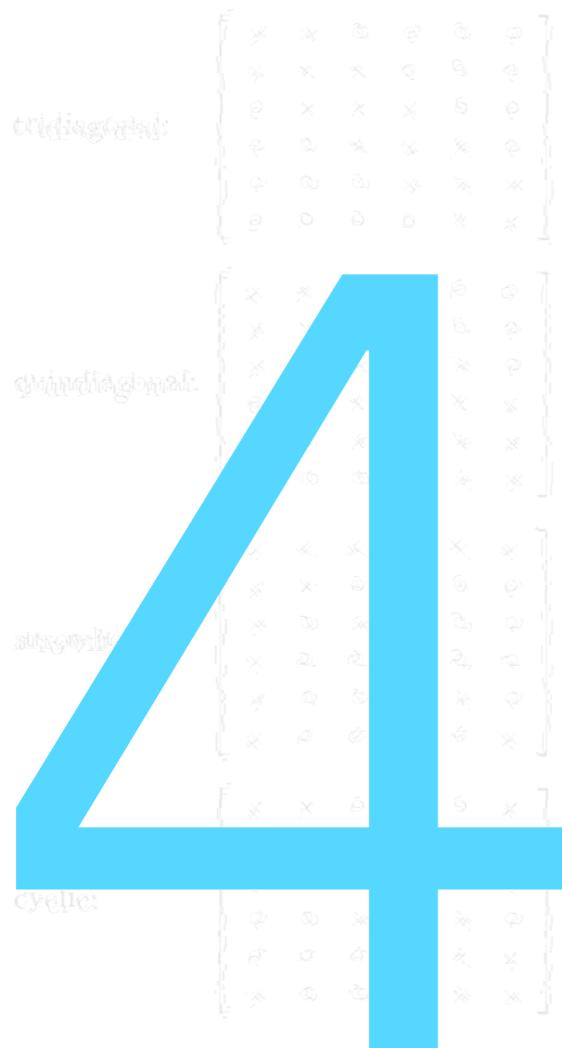


**Theorem 11.1.** Let  $\mathbb{A}$  be a symmetric matrix whose graph  $\mathbb{G}$  is a tree. Choose a root  $r \in \{1, 2, \dots, n\}$  and assume that the rows and columns of  $\mathbb{A}$  are now arranged so that  $T = (G, r)$  is monotonically ordered. Given that  $\mathbb{A} = \mathbb{L}\mathbb{U}^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{a_{jj}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

```
hooks.beforeEach(function(assert) {  
  ...  
  // Example: assert.isEnabled('Submit', 'Woot!');  
  assert.isEnabled = (label, message) => {  
    assert.dom(`[data-test-button="${label}"]`)  
      .doesNotHaveAttribute('disabled', message);  
  };  
  ...  
});
```

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & * \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & * \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & * \end{bmatrix}$$

At a first glance, there is nothing to link it to any of the four structures just displayed, but its graph,

# ALL YOUR BASIS ARE BELONG TO US



Let us now let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  are then arranged so that  $T = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{kj} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

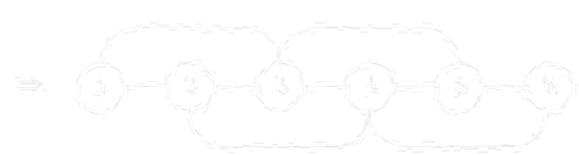
triangular:  

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ 0 & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



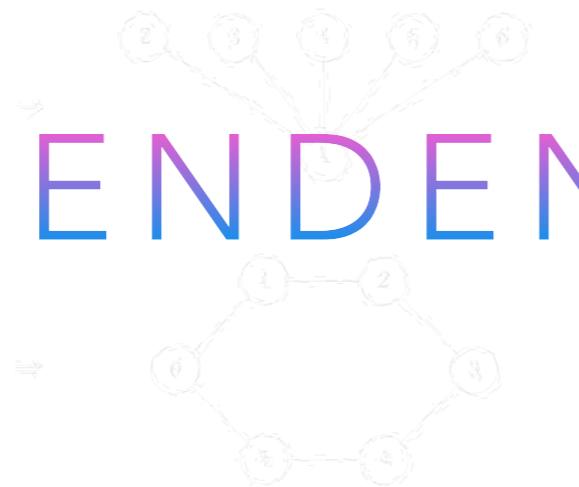
quidiagonal:  

$$\begin{bmatrix} * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ 0 & * & * & * & * & * \\ 0 & 0 & * & * & * & * \\ 0 & 0 & 0 & * & * & * \end{bmatrix}$$



symmetric:  

$$\begin{bmatrix} * & * & * & * & * & * \\ * & * & 0 & 0 & 0 & 0 \\ * & 0 & * & 0 & 0 & 0 \\ * & 0 & 0 & * & 0 & 0 \\ * & 0 & 0 & 0 & * & 0 \\ * & 0 & 0 & 0 & 0 & * \end{bmatrix}$$



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

SPAN THE ENTIRE SPACE

At a first glance, the matrix in the following looks like the matrix in the one just displayed, but its graph



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 1.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges  $\{(v_i, j)\}_{i=1}^n$  in  $\mathbb{G}$  is called a path joining the vertices  $v_i$  and  $v_j$  if  $i \in \{1, \dots, n\}$ ,  $j \in \{1, \dots, n\}$  and for every  $k = 1, 2, \dots, n-1$  the set  $\{v_i, j\} \cap \{v_{i+k}, v_{i+k+1}\}$  contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that  $\mathbb{G}$  is a tree if each two members of  $V$  are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when  $n \geq 3$ .

Given a tree  $\mathbb{G}$  and an arbitrary vertex  $r \in V$ , the pair  $T = (G, r)$  is called a rooted tree, while  $r$  is said to be the root. Unlike in ordinary graph,  $T$  admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root  $r$  is the root of all the edges and every vertex is an ancestor of  $r$ . Moreover, every  $v \in V \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex alone, throughout, except for  $r$  and  $v$ , as a predecessor of  $v$  and a successor of  $r$ . We say that the rooted tree  $T$  is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we lay out the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree:



**Theorem 11.1** Let  $A$  be a symmetric matrix whose graph  $\mathbb{G}$  is a tree. Choose a root  $r \in \{1, 2, \dots, n\}$  and assume that the rows and columns of  $A$  are then arranged so that  $T = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph.

tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 3, 3 \rightarrow 2, 4 \rightarrow 4, 5 \rightarrow 6, 6 \rightarrow 3.$$

Of course, it is equivalent to renumbering (cyclic shift) the vertices and to consider a different set of edges  $\{ (i, j) \mid i, j \in \{1, 2, \dots, 6\} \}$ . It is called a *cycle* if the vertices  $i, j \in \{1, 2, \dots, n\}$  and for every  $\ell = 1, 2, \dots, n-1$  the set  $\{i + \ell, j + \ell\}$  contains exactly one member. It is a *simple path* if it does not visit any vertex more than once. We say that  $G$  is a *tree* if each two members of  $V$  are joined by a unique path. Both tridiagonal and antidiagonal matrices correspond to the case with either quidiagonal or cyclic matrices when  $\ell = 1, 2, \dots, n-1$  and no arbitrary vertex  $v \in V \setminus \{r\}$  is called a *rooted tree* and  $r$  is called the *root*. Unlike in a ordinary graph,  $T$  admits a natural partial order that must be explained by an analogy with a family tree. Thus, the ancestor of all the vertices in  $T \setminus \{r\}$  and these vertices are *successors*. Every  $a \in V \setminus \{r\}$  is joined to  $r$  by a simple path and we designate this path, except for  $r$  and  $a$ , as a *predecessor* of  $a$  and a *successor* of  $r$ . The rooted tree  $T$  is *monotonically ordered* if each vertex is labelled with a natural number in other words, we label the vertices from the top of the tree to the bottom (we have already done it, relabeling a graph to a tree). (This is another reason to call  $T$  a *tree* and not a *forest*.) In general, such a ordering does not give three examples of the same sparse tree.



**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  are then arranged so, that  $T = (G, r)$  is monotonically ordered. Then, that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 3, 3 \rightarrow 2, 4 \rightarrow 4, 5 \rightarrow 6, 6 \rightarrow 8.$$

Of course, it is equivalent to renumbering (cyclic shift) the vertices and the edges. Consider a set of sites  $\{1, 2, \dots, n\}$ . It is called a *tree* if joining the vertices  $v_1, v_2, \dots, v_n$  ( $v_i \in \{1, 2, \dots, n\}$ ) and for every  $i = 1, 2, \dots, n$  the set  $\{v_i, v_{i+1}\}$  contains exactly one member. It is a *simple path* if it does not visit any vertex more than once. We say that  $G$  is a *tree* if each two members of  $V$  are joined by a path. Both tridiagonal and cyclic matrices correspond to trees. There are two types of trees with either quidiagonal or cyclic matrices:  $P_3$  and  $P_4$ . The first one is a path and an arbitrary vertex  $v \in V$  in the path  $P_3$  is called a *rooted* and to be the *root*. Unlike in a tridiagonal graph,  $T$  admits a natural partial order that must be explained by an analogy with a family tree. Thus, the ancestor of all the vertices in  $T \setminus \{r\}$  and these vertices are *successors*: every  $a \in V \setminus \{r\}$  is joined to  $r$  by a simple path and we designate this path, except for  $r$  and  $a$ , as a *predecessor* of  $a$  and a *successor*. The rooted tree  $T$  is *monotonically ordered* if each vertex is labelled with its *depth* in other words, we label the vertices from the top of the tree to the bottom (we have already done it, relabeling a graph to a tree). We will give three examples of the simple rooted trees.

**LATITUDE**

$30.267^\circ$

**LONGITUDE**

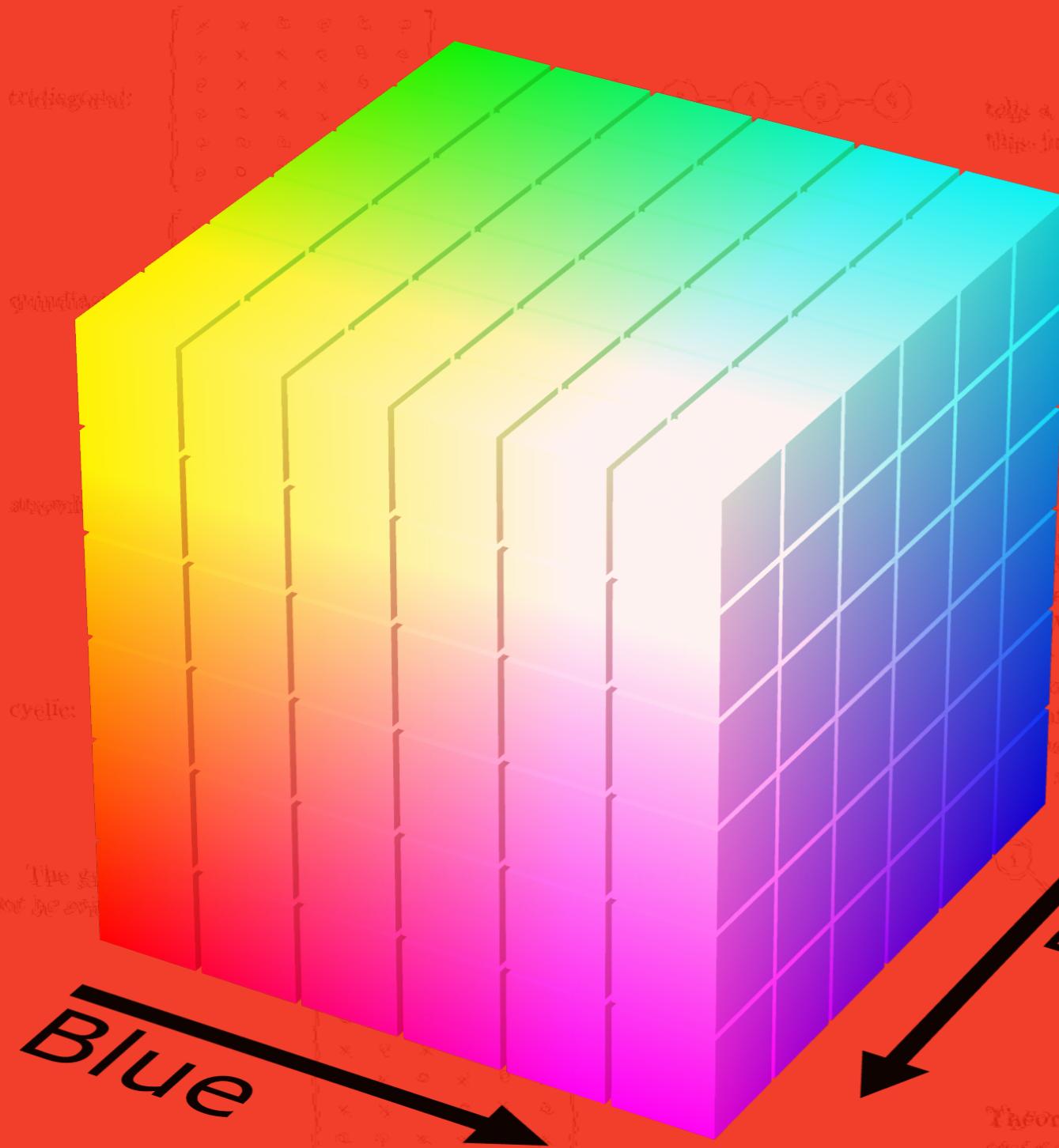
$-97.743^\circ$



**Theorem 11.1** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  are now arranged so that  $T = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



At a first glance, there is nothing to link in to any of the four matrices that we have just displayed, but its graph.



RED

224

tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, first relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 3, 3 \rightarrow 2, 4 \rightarrow 4, 5 \rightarrow 6, 6 \rightarrow 5$$

Of course, it is equivalent to reordering (simultaneously) the columns and rows. A *partial set of steps*  $\{(v_i, j_i)\}_{i=1}^n \subseteq \mathbb{E}$  is called a *path* joining the vertices  $v_i$  and  $v_j$  if  $(v_i, j_i), j_i \in \{1, \dots, n\}$  and for every  $k = 1, 2, \dots, n-1$  the set  $\{v_{k+1}, j_{k+1}\} \cap \{v_i, j_i\}$  contains exactly one member. It is a *simple path* if it does not visit any node more than once. We say that  $G$  is a *tree* if each two members of  $V$  are joined by a simple path. Both tri-diagonal and quasi-diagonal matrices correspond to trees. (This is not the case with either quidiagonal or cyclic matrices when  $n \geq 3$ .)

Let's start with an arbitrary vertex  $r \in V$ . The set  $T = \{v_i \in V \mid v_i \text{ is joined to } r \text{ by a simple path}\}$  is called a *rooted tree* with  $r$  added to be the root. Unlike a ordinary tree,  $T$  does not have a unique root, which can best be explained by an example. A family tree. This is the set of all the vertices in  $V \setminus \{r\}$  and these vertices are *successors* of  $r$ . If  $\alpha \in T$  and  $\alpha \neq r$ , then  $\alpha \in V \setminus \{r\}$  is joined to  $r$  by a simple path and we designate  $\alpha$  as an *ancestor* of  $r$ , except for  $r$  and  $\alpha$ , as a *predecessor* of  $\alpha$  and a *successor* of  $r$ . We say that a rooted tree  $T$  is *monotonically ordered* if each vertex is labelled with a unique integer  $\ell(v_i)$  in other words we label the vertices from the top of  $T$  to the bottom. (We have already said it, relabelling a graph is the same as relabelling the rows and the columns of the underlying matrix.)

Every vertex then will be monotonically ordered and, in general, such an ordering is unique. We can give three consecutive orderings of the same rooted tree.

Red

Green

Blue

Blue

Therefore we will now give a few examples of matrices (represented by their sparsity)

and their graphs

tridiagonal: 
$$\begin{bmatrix} * & * & 0 & 0 & 0 \\ * & * & * & 0 & 0 \\ 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & * & 0 \\ 0 & 0 & 0 & 0 & * \end{bmatrix}$$



BASIS

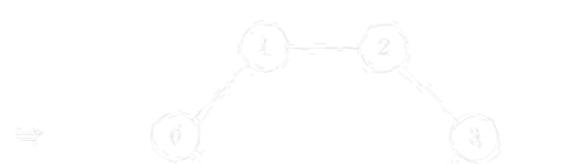
quindiagonal:

$$\begin{bmatrix} * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & * & 0 \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$


superdiagonal:

$$\begin{bmatrix} * & * & * & * & * & * \\ * & * & 0 & 0 & 0 & 0 \\ 0 & 0 & * & 0 & 0 & 0 \\ 0 & 0 & 0 & * & 0 & 0 \\ 0 & 0 & 0 & 0 & * & 0 \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$


cyclic:

$$\begin{bmatrix} * & 0 & 0 & 0 & * & * \\ * & * & 0 & 0 & 0 & 0 \\ 0 & 0 & * & 0 & 0 & 0 \\ 0 & 0 & 0 & * & 0 & 0 \\ 0 & 0 & 0 & 0 & * & 0 \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$


BUILDING BLOCKS

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

OF TESTS

At a first glance, this is not a matrix, but a graph of 10 nodes. It is, however, a matrix, just displayed, but its graph:



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges  $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$  is called a path joining the vertices  $i_0$  and  $j_0$  if  $i_0 \in \{1, \dots, n\}$ ,  $j_0 \in \{1, \dots, n\}$  and for every  $k = 1, 2, \dots, n-1$  the set  $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$  contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that  $G$  is a tree if each two members of  $V$  are joined by a unique simple path. Both tridiagonal and superdiagonal matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when  $n \geq 3$ .

Given a tree  $T$  and an arbitrary vertex  $r \in V$ , the pair  $T = (G, r)$  is called a rooted tree, while  $r$  is said to be the root. Unlike in a ordinary graph,  $T$  admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root  $r$  is the predecessor of all the vertices in  $T \setminus \{r\}$  and these vertices are successors of  $r$ . Moreover, every  $\alpha \in V \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex along this path, except for  $r$  and  $\alpha$ , as a predecessor of  $\alpha$  and a successor of  $r$ . We say that the rooted tree  $T$  is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is unique. The following diagram shows three consecutive orderings of the same rooted tree.



**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  were been arranged so, that  $T = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

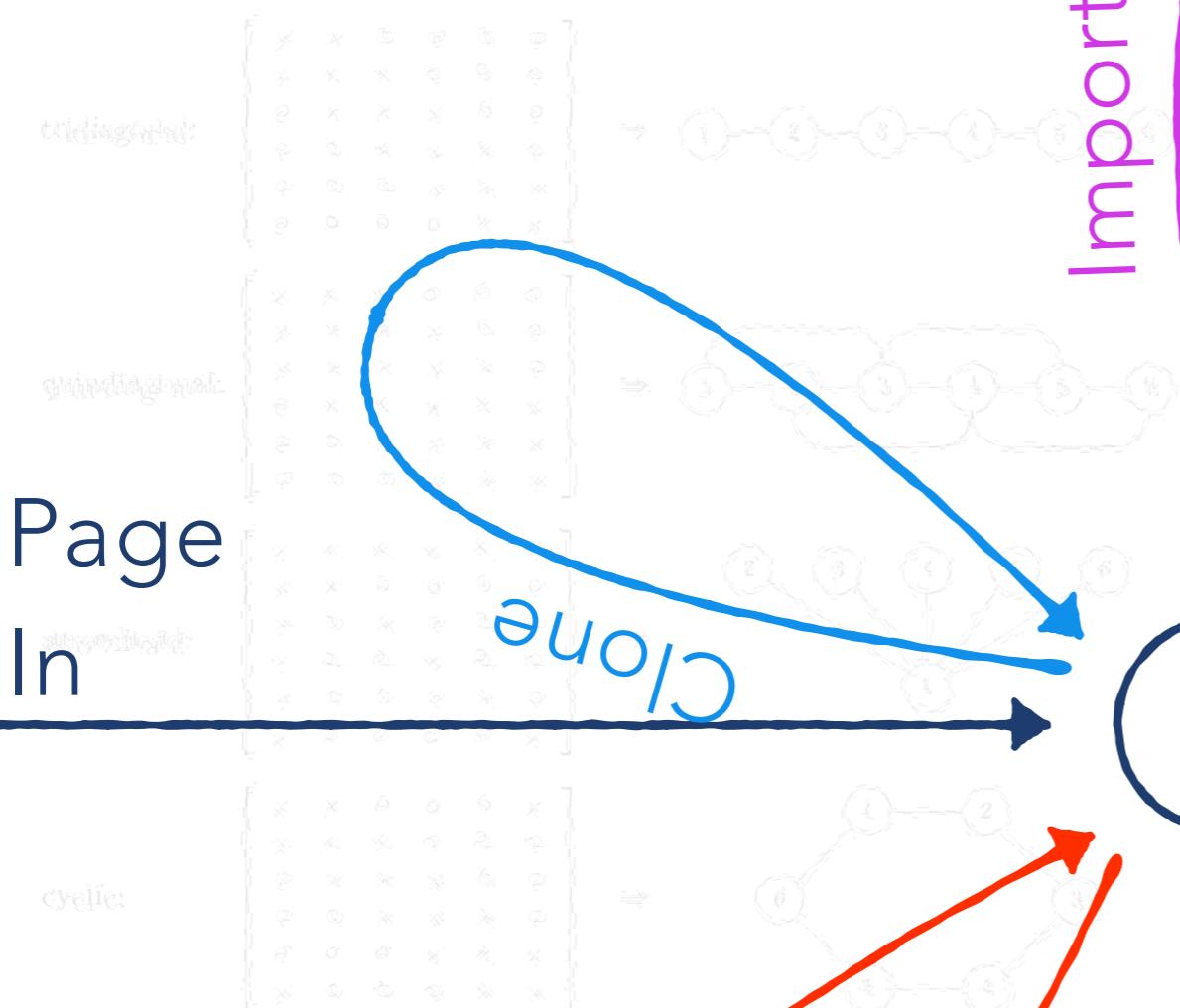
Create Edit Delete Clone Import

## 卷之三

a multiple simple path. Both tridiagonal and anti-pyramidal matrices composed for trees. The latter is more the case with the  $3 \times 3$  matrix in Fig. 10, which is a 10th-order polynomial.

	Name	Description
<input checked="" type="checkbox"/>	Little Bobby Tables	Better not drop me!
<input type="checkbox"/>	Big Bobby Tables	
<input type="checkbox"/>	Foo Bar	Making up names is hard.

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix



At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,

tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just read the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 1.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables. An ordered set of edges  $\{(i_0, j_0), \dots, (i_{\ell-1}, j_{\ell-1})\}$  in  $\mathbb{G}$  is called a path joining the vertices  $i_0$  and  $i_{\ell-1}$  ( $i_0, i_1, \dots, i_{\ell-1}, i_0 \in V$ ,  $j_0, j_1, \dots, j_{\ell-1} \in V$ ) and for any  $k = 1, 2, \dots, \ell - 1$  the set  $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$  contains exactly one vertex. It is a simple path if it does not visit any vertex more than once. We say that  $\mathbb{G}$  is a tree if none of members of  $V$  are joined by a single simple path. But tridiagonal and quadiagonal matrices contain cycles. Note this is not the case with either quadiagonal or cyclic matrices when

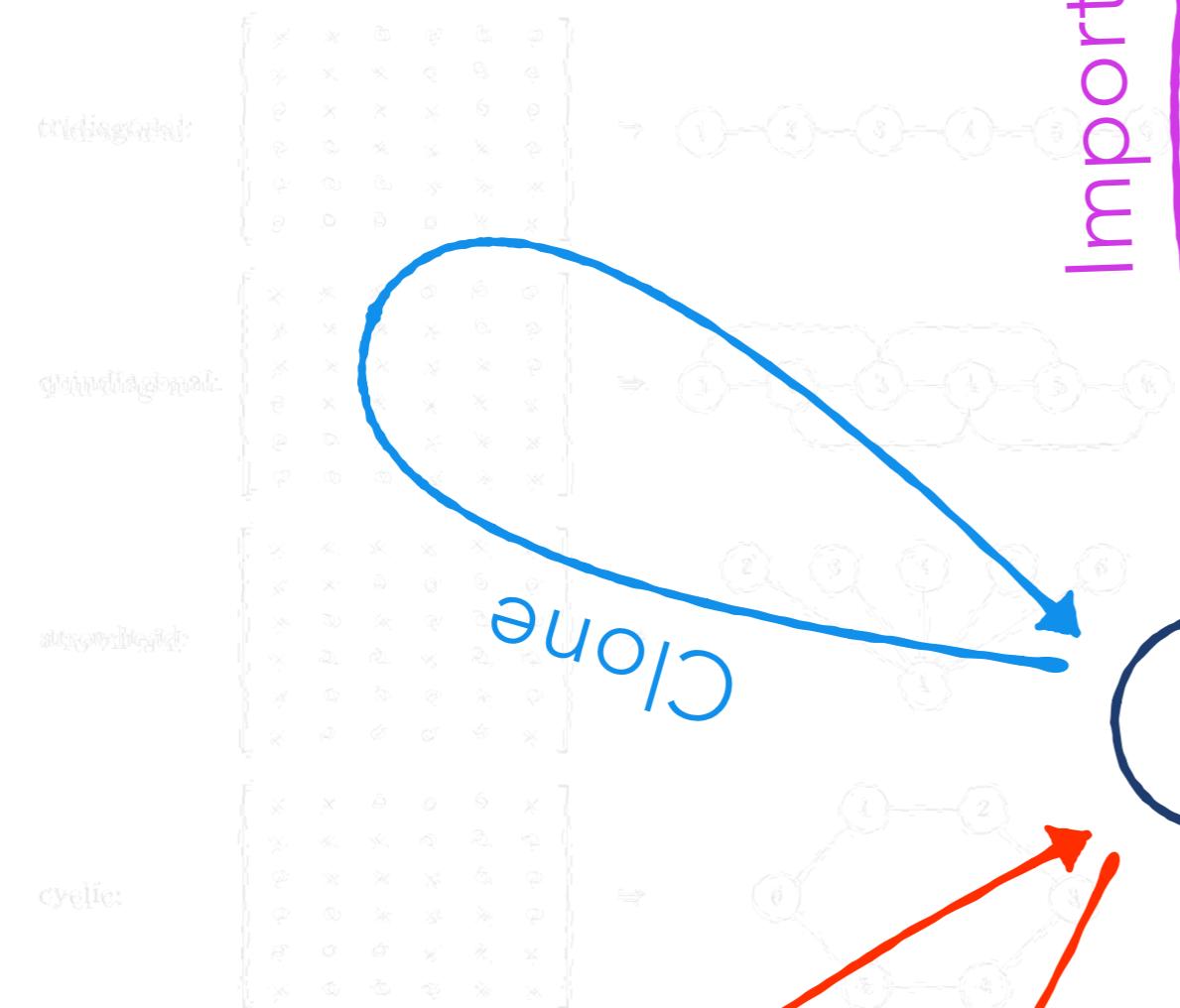
Create a tree  $T$  and a arbitrary vertex  $r \in V$ , the pair  $T - \{r\}$  is called a rooted tree, while  $r$  is said to be the root. Unlike in a ordinary graph,  $T$  has its natural partial order, which can best be explained by an analogy with a family tree. The root  $r$  is the predecessor of all the vertices in  $T \setminus \{r\}$  and these vertices are the successors of  $r$ . Moreover,  $r$  is the only vertex in  $T$  that has no predecessor. We say that a vertex  $\alpha$  is a predecessor of a vertex  $\beta$  if  $\beta$  is the only vertex in  $T$  that has  $\alpha$  as a predecessor along this path, except  $\alpha$  itself, as a predecessor of  $\alpha$  and a successor of  $\alpha$ . We say that the rooted tree  $T$  is monotonically ordered if each vertex is labelled with all its predecessors in other words we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to relabelling the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We will give three consecutive drawings of the same rooted tree.

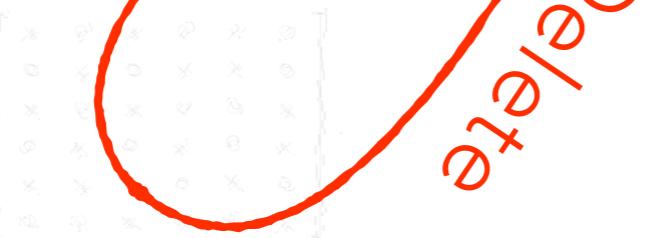
**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $\mathbb{G}$  is a tree. Choose a root  $r \in \{1, 2, \dots, n\}$  and assume that the rows and columns of  $A$  are then arranged so, that  $\mathbb{T} = (i_0, r)$  is monotonically ordered. Then, that  $A = LL^T$  is a Cholesky factorization it is true that

$$l_{k,j} = \frac{a_{k,j}}{l_{j,j}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

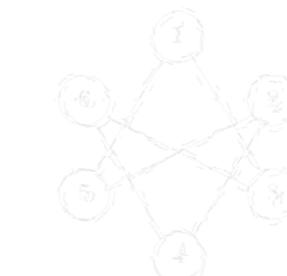


The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix



At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,

Import



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just read the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 3, 3 \rightarrow 2, 4 \rightarrow 5, 5 \rightarrow 6, 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges  $\{(i_0, j_0), (i_1, j_1), \dots, (i_{n-1}, j_{n-1})\}$  is called a path joining the vertices  $i_0$  and  $i_{n-1}$  ( $i_0, i_1, \dots, i_{n-1}, i_0 \in V$ ,  $j_0, j_1, \dots, j_{n-1} \in V$ ) and for any  $k = 1, 2, \dots, n-1$  the set  $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$  contains exactly one vertex. It is a simple path if it does not visit any vertex more than once. We say that  $G$  is a tree if no two members of  $V$  are joined by a single simple path. Both quadiagonal and pentadiagonal matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when  $p \geq 3$ .

Create a tree  $T$  and a arbitrary vertex  $r \in V$ , the pair  $T - \{r\}$  is called a rooted tree, while  $r$  is said to be the root. Unlike in a ordinary graph,  $T$  has a natural partial order, which can best be explained by an analogy with a family tree. Thus, the vertex  $v$  is a predecessor of all the vertices in  $T - \{v\}$  and these vertices are successors of  $v$ . Moreover, every  $a \in V \setminus \{r\}$  is joined to  $r$  by a simple path and we designate  $a$  as a predecessor of  $r$  and  $r$  as a predecessor of  $a$  and a successor of  $a$  along this path, except in  $r$  and  $a$ , as a predecessor of  $a$  and a successor of  $a$ . We say that the rooted tree  $T$  is monotonically ordered if each vertex is labelled with all its predecessors in other words, we label the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to relabelling the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We will give three consecutive drawings of the same rooted tree.

Edit



**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, n\}$  and assume that the rows and columns of  $A$  are now arranged so, that  $T = (i, j)$  is monotonically ordered. Then, that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{k,j}}{l_{j,j}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

Page  
In

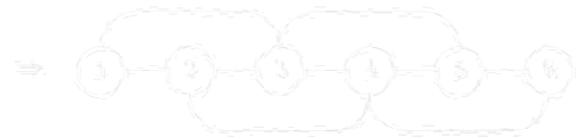
quidiagonal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 & 0 \\ 0 & * & * & 0 & 0 & 0 \\ 0 & 0 & * & * & 0 & 0 \\ 0 & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



quidiagonal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ 0 & 0 & * & * & * & * \\ 0 & 0 & 0 & * & * & * \end{bmatrix}$$

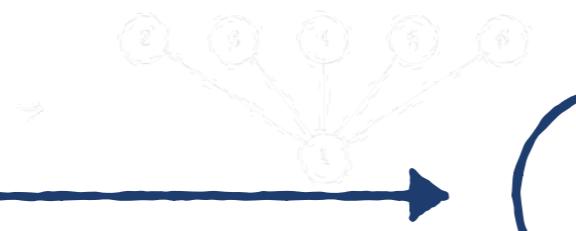


Page

Out

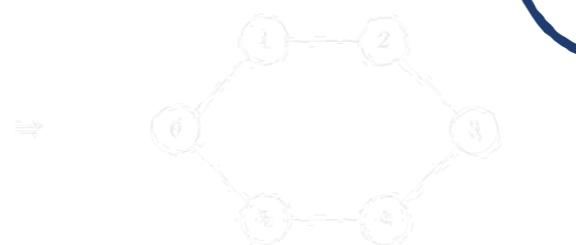
triangular:

$$\begin{bmatrix} * & * & * & * & * & * \\ * & * & 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$



cyclic:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & * \\ * & * & 0 & 0 & 0 & 0 \\ 0 & * & * & 0 & 0 & 0 \\ 0 & 0 & * & * & 0 & 0 \\ 0 & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & 0 \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & * \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & * \end{bmatrix}$$

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 4.$$

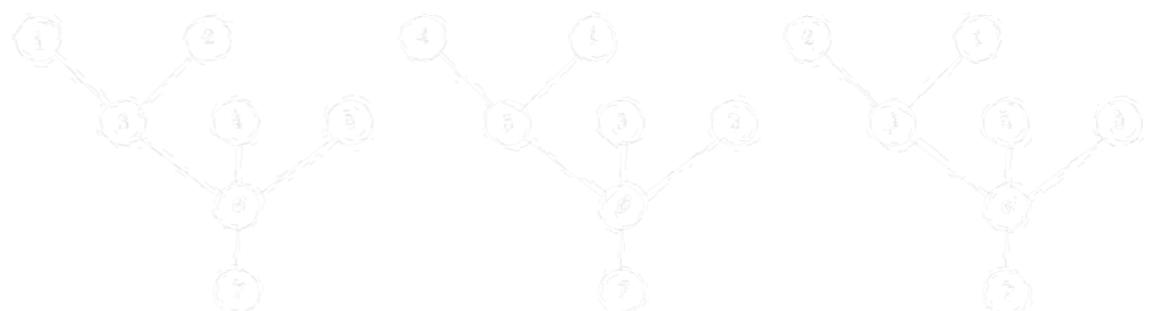
This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges  $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$  is called a path joining the vertices  $i_0$  and  $j_0$  if  $i_0 \in \{i_0, j_0\}$ ,  $j_0 \in \{i_0, j_0\}$  and for every  $k = 1, 2, \dots, n-1$  the set  $\{i_{k+1}, j_{k+1}\} \cap \{i_0, j_0\}$  contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that  $G$  is a tree if each two members of  $V$  are joined by a unique simple path. Both tri-diagonal and any other matrices  $A$  are not trees, but this is not the case with either quidiagonal or cyclic matrices.

Given a tree  $T$  and an arbitrary vertex  $r \in V$ , the pair  $T = (G, r)$  is called a rooted tree, while  $r$  is said to be the root. Unlike in ordinary graph,  $T$  admits a natural partial order, which can best be explained by an analogy with a family tree. The vertex  $r$  is the predecessor of all the vertices in  $T \setminus \{r\}$  and these vertices are successors. Moreover, every  $\alpha \in V \setminus \{r\}$  has a unique predecessor  $\alpha'$  and an immediate vertex along this path, except for  $r$  and  $\alpha$ , as a predecessor of  $\alpha$  and a successor.

We say that the rooted tree  $T$  is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree.

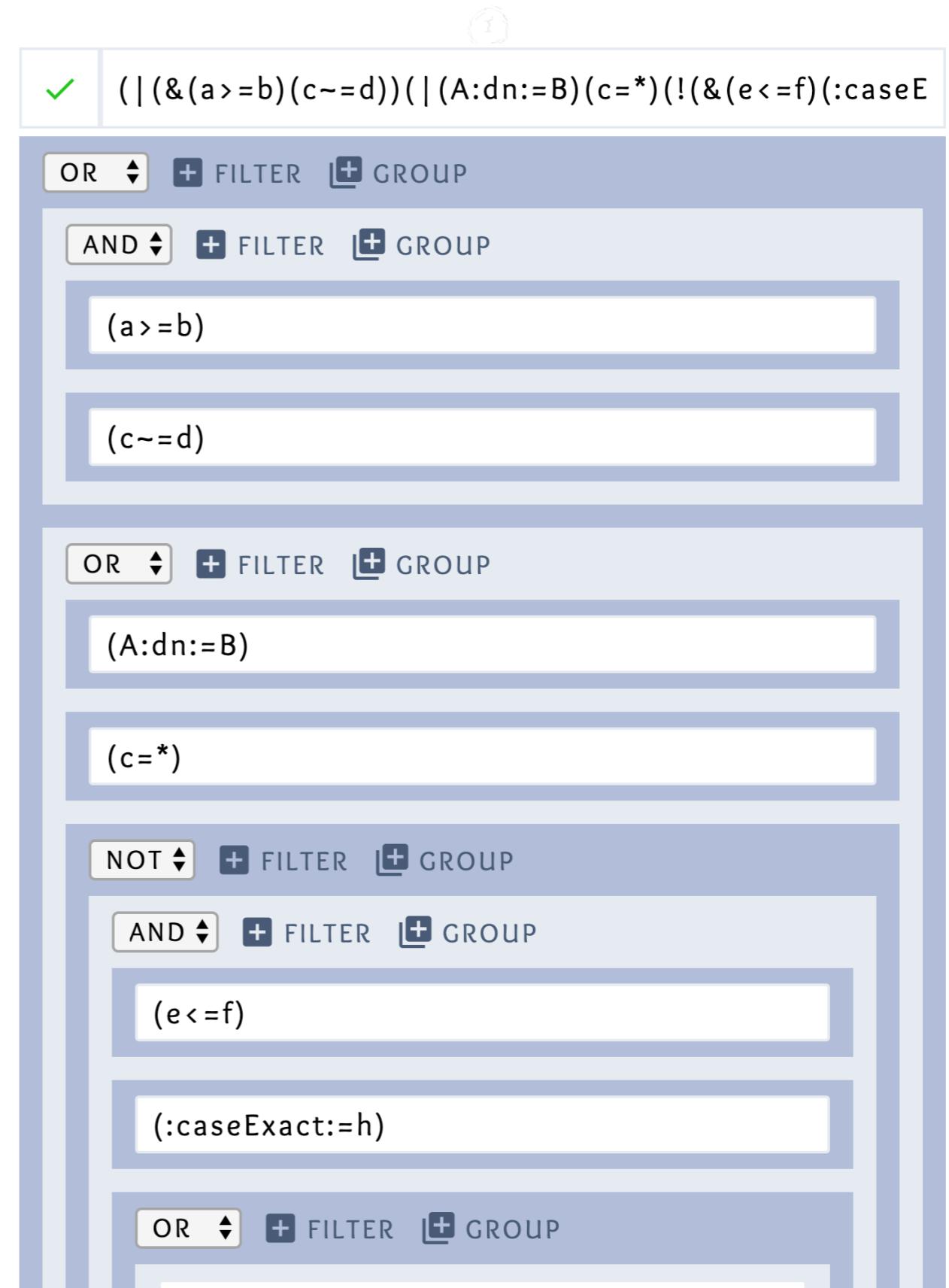


**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, n\}$  and assume that the rows and columns of  $A$  are then arranged so that  $T = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ki}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their analysis.

$(|(&(a>=b)(c\sim=d))$   
 $(| (A:dn:=B)(c=^*)$   
 $(!(&(e<=f)$   
 $(:caseExact:=h)$   
 $(| (i=j)(!(k<=l))))))))$



just displayed, but its graph.

$$t_{k,j} = \frac{a_{k,j}}{b_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1, \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

( | (! (a>=b) (c~d)))  
( | (A:dn := B) (c=\*))  
(!(&(e<=f))  
(:caseExact:=h))  
( | (i=j) (! (k<=l)))))))

! ( | (! (a>=b) (c~d))) ( | (A:dn := B) (c=\*)) (!(&(e<=f)) (:caseE>))

OR FILTER GROUP

NOT FILTER GROUP

You can negate only 1 filter in a group.

(a>=b)

(c~d)

You need to use ~=.

OR FILTER GROUP

(A:dn := B)

You need to trim the attribute and filter type.

(c=\*)

NOT FILTER GROUP

AND FILTER GROUP

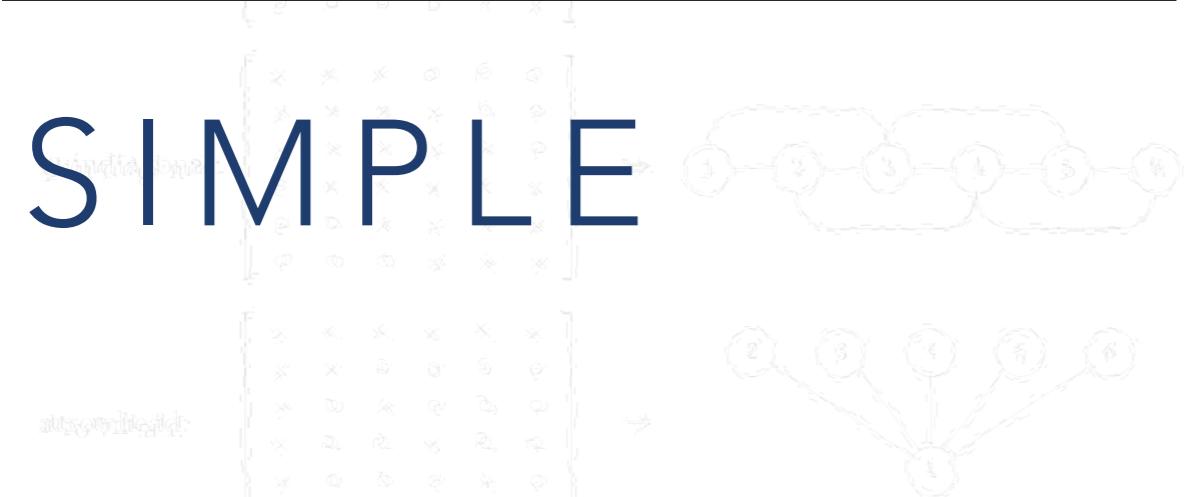
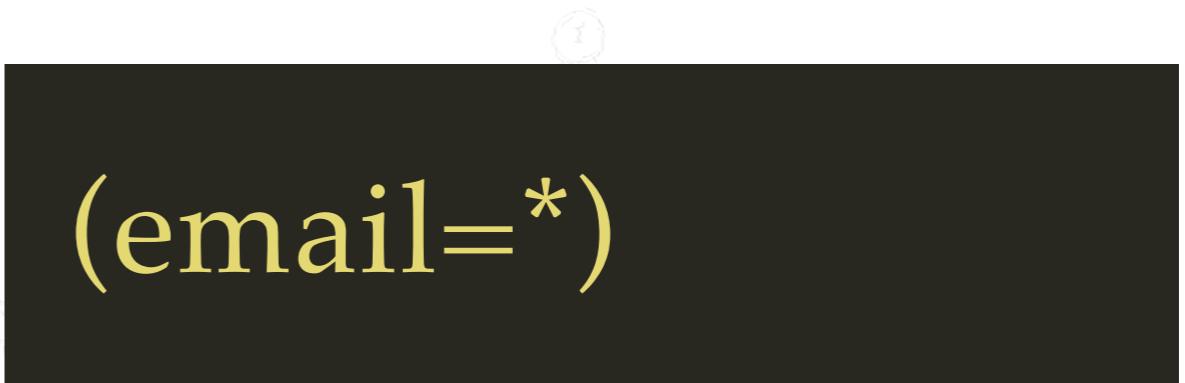
(e<=f)

(:caseExact:=h)

just displayed, but its graph.

$$t_{k,j} = \frac{a_{k,j}}{b_{j,j}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1, \quad (11.5)$$

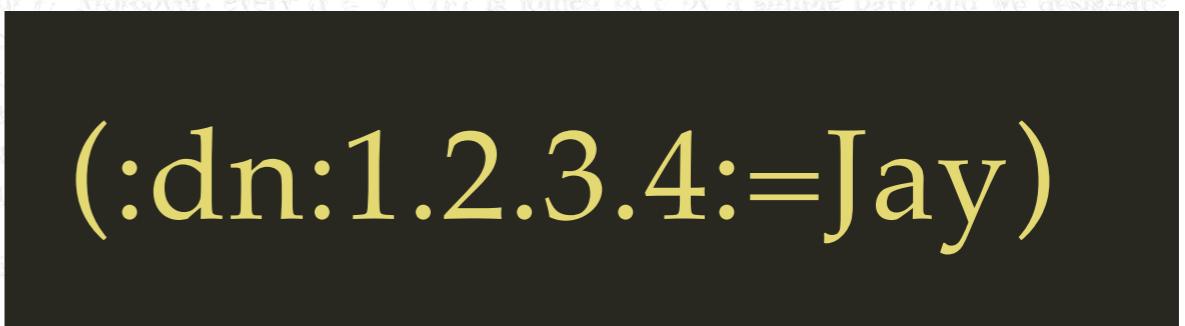
Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



**PRESENT**

This, of course, is equivalent to requiring (simultaneously) the equations and variables. An ordered set of edges  $\{v_i, w_i\}$  ( $i \in \{1, 2, \dots, n\}$ ) is called a path joining the vertices  $v_i$  and  $w_i$  ( $i \in \{1, 2, \dots, n\}$ ) and, for every  $i = 1, 2, \dots, n - 1$  the set  $\{v_i, w_i\} \cap \{v_{i+1}, w_{i+1}\}$  contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that  $G$  is a tree if each two members of  $V$  are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when  $p \geq 3$ .

Given a tree  $G$  and an arbitrary vertex  $r \in V$ , the pair  $T = (G, r)$  is called a rooted tree, while  $r$  is said to be the root. Unlike in a ordinary graph,  $T$  admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root  $r$  is the predecessor of all the vertices in  $T \setminus \{r\}$  and these vertices are successors of  $r$ . Moreover, every  $v \in V \setminus \{r\}$  has a unique path to  $r$  by a simple path and we designate



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparser pattern. Thus, consider the matrix

**SUBSTRING**



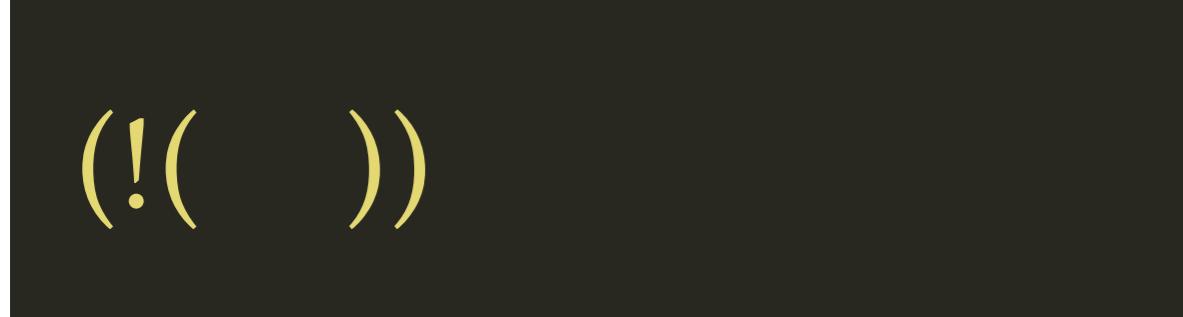
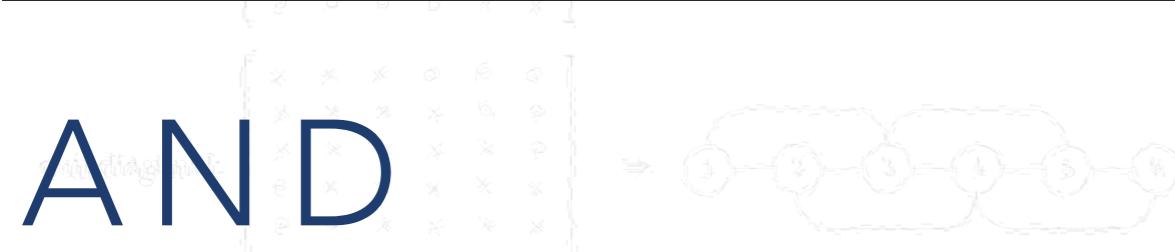
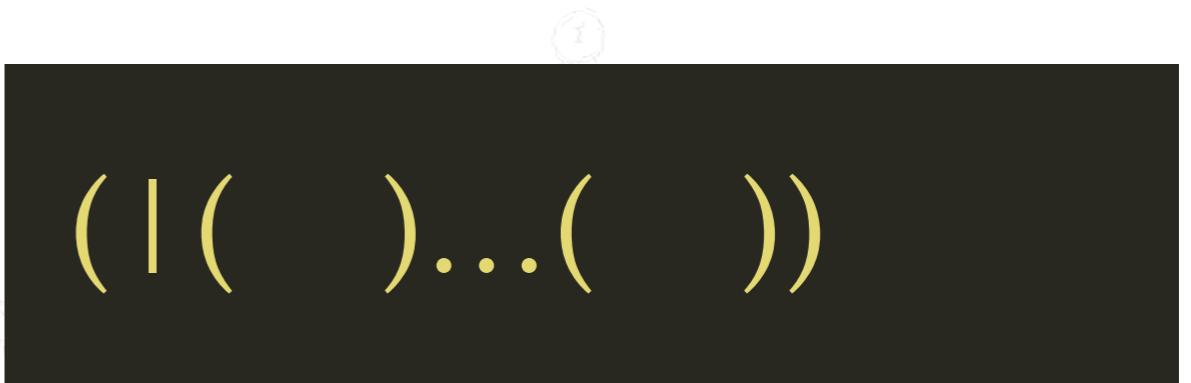
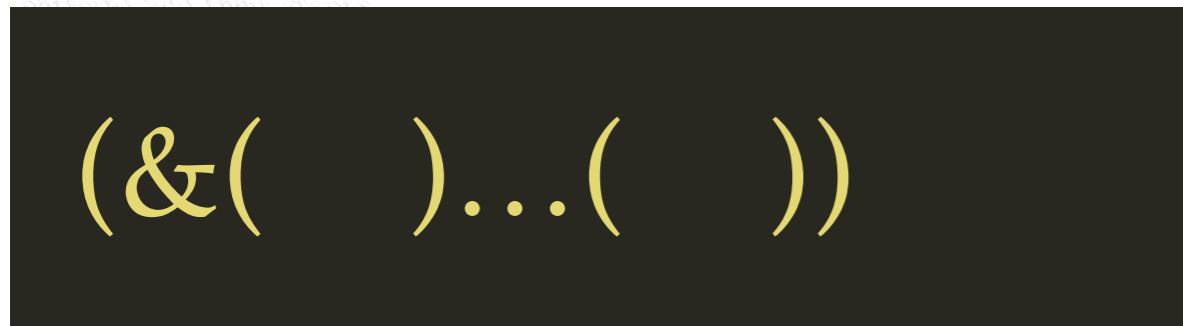
At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,



**Theorem 11.1** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  are then arranged so that  $T = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ki}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix



At a first glance, there is nothing to think in the case of the four matrices that we have just displayed, but its graph.

$1 \rightarrow 2, 2 \rightarrow 3, 3 \rightarrow 2, 1 \rightarrow 4, 5 \rightarrow 6, 6 \rightarrow 3$ .

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges  $\{(v_i, j_i)\}_{i=1}^n$  in  $\mathbb{G}$  is called a path joining the vertices  $v_i$  and  $v_j$  ( $i, j \in \{1, 2, \dots, n\}$ ,  $i \neq j$ ) if  $\{v_i, j_i\} \cap \{v_k, j_k\}$  contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that  $\mathbb{G}$  is a tree if each two members of  $\mathbb{V}$  are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when  $p \geq 3$ .

Given a tree  $\mathbb{G}$  and an arbitrary vertex  $r \in \mathbb{V}$ , the pair  $T = (G, r)$  is called a rooted tree, while  $r$  is said to be the root. Unlike in ordinary graph,  $T$  admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root  $r$  is the predecessor of all the vertices in  $\mathbb{V} \setminus \{r\}$  and these vertices are successors of  $r$ . Moreover, every  $v \in \mathbb{V} \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex along this path, except for  $r$  and  $v$ , as a predecessor of  $v$  and a successor of  $r$ . We say that the rooted tree  $T$  is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we label the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

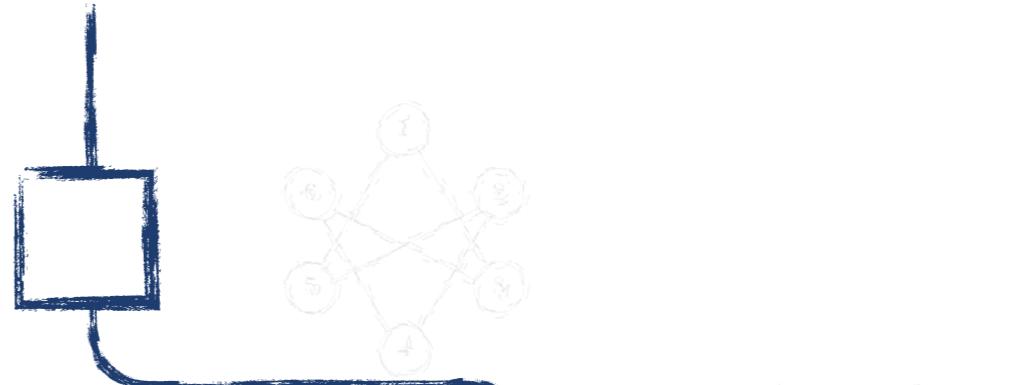
Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three consecutive orderings of the same rooted tree.



Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

```
function addOne(x) {
  if (Number.isFinite(x)) {
    x = x + 1;
  } else {
    console.log('error');
  }
  return x;
}
```

just displayed, but its graph:



tells a different story –  $\mathbf{g}$  is not a tree. In a cyclic matrix in discussed. To see this, just re-label the vertices as follows:

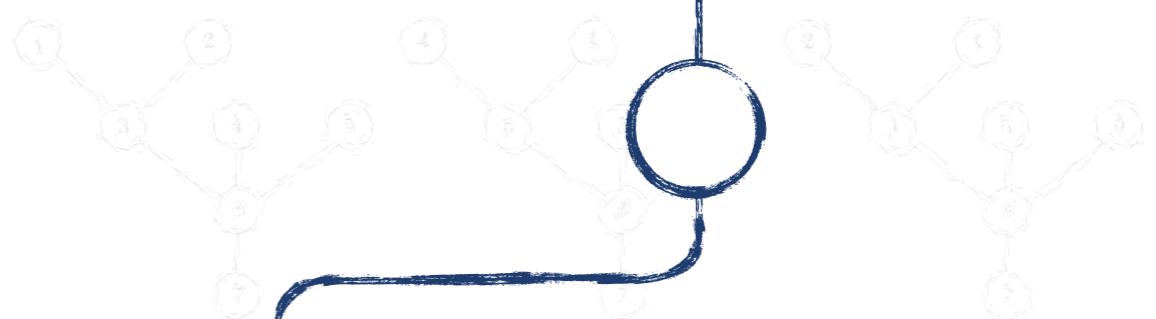
$$1 \rightarrow 1, \quad 2 \rightarrow 3, \quad 3 \rightarrow 2, \quad 4 \rightarrow 5, \quad 5 \rightarrow 6, \quad 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (swapping) the equations and variables.

An ordered set of edges  $\{(i_1, j_1)\}_{i_1, j_1 \in \{1, \dots, n\}}$  is a path joining the vertices  $i_1$  and  $j_1$  if  $i_1 \in \{1, \dots, n\}$ ,  $j_1 \in \{1, \dots, n\}$  and for every  $k = 1, 2, \dots, n-1$  the set  $\{i_{k+1}, j_{k+1}\} \cap \{i_k, j_k\}$  contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that  $\mathbf{G}$  is a tree if each two members of  $\mathbf{V}$  are joined by a unique simple path. Both tridiagonal and banded matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when  $n > 3$ .

Given a tree  $\mathbf{G}$  and an arbitrary vertex  $r \in \mathbf{V}$ , the path  $T = (Q, \pi)$  is called a rooted tree, while  $r$  is said to be the root. Unlike in an ordinary tree, the root  $r$  is not necessarily unique, which can best be explained by an analogy with a family tree. The root  $r$  is the predecessor of all the vertices in  $\mathbf{V} \setminus \{r\}$  and these vertices are successors of  $r$ . Moreover, every  $a \in \mathbf{V} \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex along this path, except for  $r$  and  $a$ , as a predecessor of  $a$  and a successor of  $r$ . We say that the rooted tree  $T$  is monotonically ordered if each vertex is labelled before all its predecessors; in other words, a vertex  $a$  is to the right of  $r$  if it is further from the root to the left. (As we have already said, we are permuting the rows and the columns of the matrix, so the order of the columns in the matrix is not unique.)

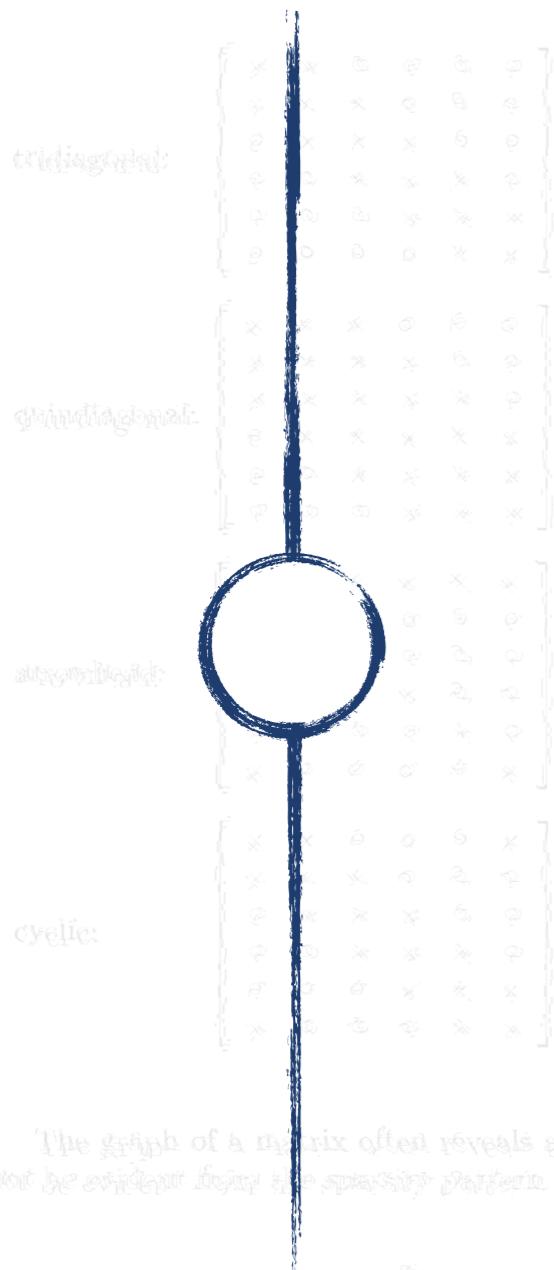
Every rooted tree will be monotonically ordered, but in general such an ordering is not unique. We now give three consecutive stages of the same rooted tree:



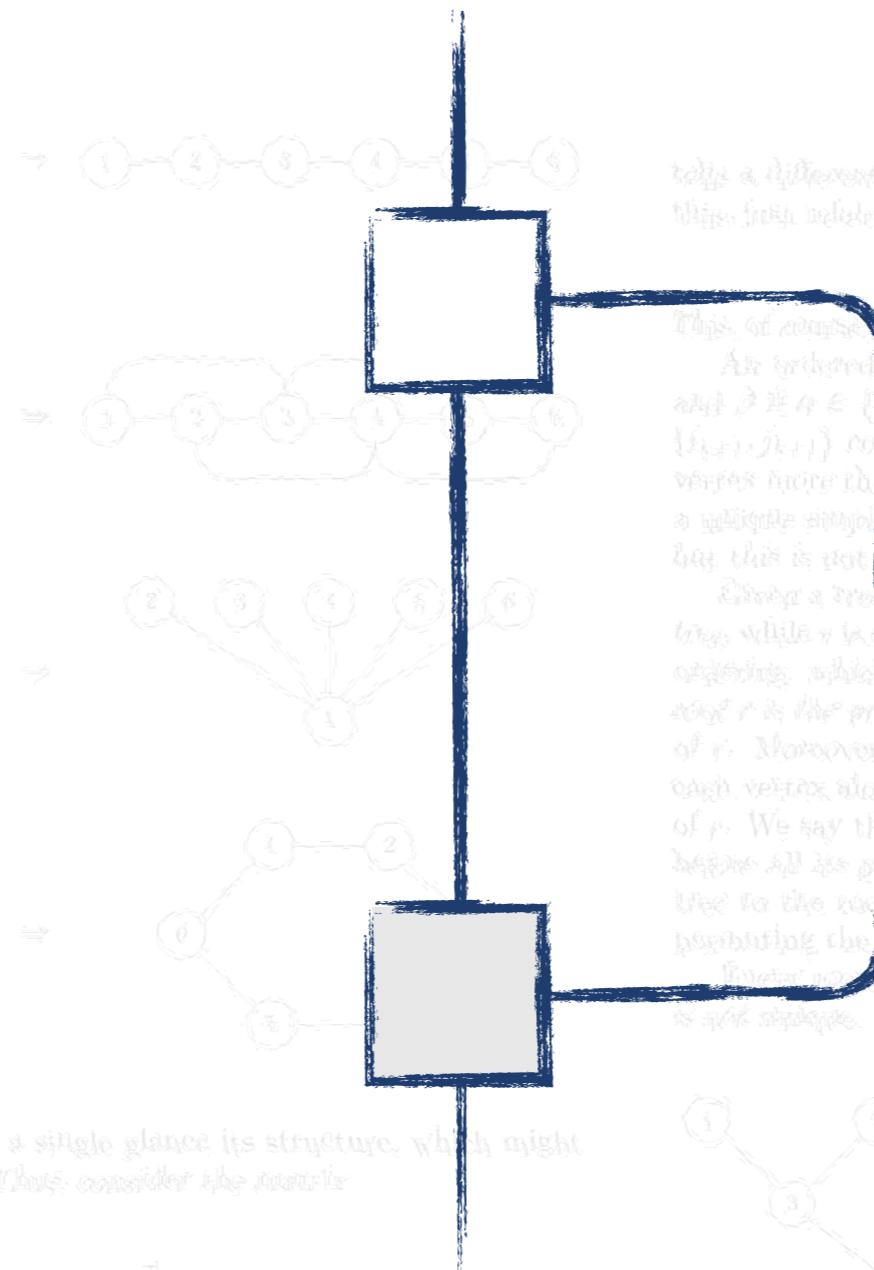
**Theorem 2.1** Let  $\mathbf{A}$  be a symmetric matrix whose graph  $\mathbf{G}$  is a tree. Choose a root  $r \in \{1, 2, \dots, n\}$ . Assume that the rows and columns of  $\mathbf{A}$  have been arranged so that  $T = (Q, \pi)$  is monotonically ordered. Then the Cholesky factorization of  $\mathbf{A}$  is

$$L_{k,j} = \frac{a_{k,j}}{a_{kk}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (1.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



cyclic:

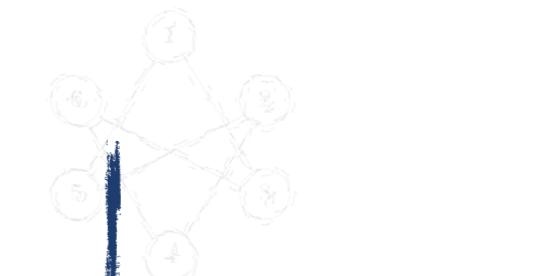


The graph of a matrix often reveals at a single glance its structure, which might not be evident from its sparsity pattern. Thus, consider the matrix

$$\text{LOG} \quad \begin{bmatrix} * & 0 & * & 0 & * & 0 \\ 0 & * & x & 0 & * & 0 \\ * & x & 0 & * & x & 0 \\ 0 & * & x & 0 & * & x \\ * & x & 0 & * & x & 0 \\ 0 & * & x & 0 & * & x \end{bmatrix}$$

| F

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, first recall that the vertices are

$$1 \rightarrow 1, \quad 2 \rightarrow 3, \quad \dots, \quad 5 \rightarrow 6, \quad 6 \rightarrow 8.$$

This, of course, is equivalent to row permuting (and simultaneously) the equations and variables.

An ordered set of edges  $\{(v_i, w_i)\}_{i=1}^n$  in  $G$  is called a path joining the vertices  $v_1$  and  $v_n$  if  $v_i \in V$ ,  $w_i \in V$ , for every  $i = 1, 2, \dots, n-1$  the set  $\{v_i, w_i\} \cap \{v_{i+1}, w_{i+1}\}$  contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that  $G$  is a tree if each two members of  $V$  are joined by a unique simple path. Both tridiagonal and pentadiagonal matrices correspond to trees, but this is not the case with either quadiagonal or cyclic matrices when  $n \geq 3$ .

Given a tree  $T$  and an arbitrary vertex  $r \in V$ , the pair  $T = (G, r)$  is called a rooted tree, while  $r$  is said to be the root. Unlike in a ordinary graph,  $T$  admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root  $r$  is the predecessor of all the vertices in  $T \setminus \{r\}$  and these vertices are successors of  $r$ . Moreover, every  $v \in V \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex along this path, except for  $r$  itself, as a predecessor of  $v$  and a successor of  $r$ . We say that the rooted tree  $T$  is monotonically ordered if each vertex is labelled before all its predecessors in other words, we layed the vertices from the top of the tree to the root. (As we have mentioned it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Even a binary tree will be monotonically ordered and, in general, such an ordering is not unique. We will give the



**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  are then arranged so that  $T = (G, r)$  is monotonically ordered. Then, that  $A = LL^T$  is a Cholesky factorization, it is true that

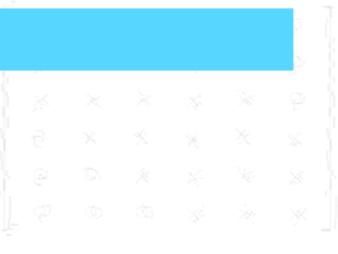
$$l_{k,j} = \frac{a_{kj}}{a_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

triangular:



quadratic:



symmetric:



cyclic:



The graph of a matrix often reveals at a single glance its structure, which may not be evident from the sparsity pattern. Thus, consider the matrix



At a first glance, there is nothing to think in the case just displayed, but its graph,

1

PICTURE

1000

WORDS



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges  $\{(i_1, j_1), (i_2, j_2), \dots, (i_v, j_v)\}$  in  $\mathbb{G}$  is called a *path* joining the vertices  $i_1, i_2, \dots, i_v$  ( $j_1, j_2, \dots, j_v$ ) and, for every  $k = 1, 2, \dots, v-1$  the set  $\{(i_k, j_k), (i_{k+1}, j_{k+1})\}$  contains exactly one edge. It is a *simple path* if it does not visit any vertex more than once. We say that  $\mathbb{G}$  is a *tree* if each two members of  $\mathbb{V}$  are joined by a single simple path. Back to graphs, the quadiagonal matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when  $p \geq 3$ .

Given a tree  $\mathbb{G}$  and an arbitrary vertex  $r \in \mathbb{V}$ , the pair  $T = (G, r)$  is called a *rooted tree*, while  $r$  is said to be the *root*. Unlike in ordinary graph,  $T$  admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root  $r$  is the *predecessor* of all the vertices in  $\mathbb{V} \setminus \{r\}$  and these vertices are *successors* of  $r$ . Moreover, every  $\alpha \in \mathbb{V} \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex along this path, except for  $r$  and  $\alpha$ , as a *predecessor* of  $\alpha$  and a *successor* of  $r$ . We say that the rooted tree  $T$  is *monotonically ordered* if each vertex is labelled before all its predecessors; in other words, we *layer* the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is unique. We now give three consecutive endings of the same rooted tree:



**Theorem 1.** Let  $A$  be a symmetric matrix whose graph  $\mathbb{G}$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  are then arranged so that  $T = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

```
// TODO: Write tests later

test('it renders', async function(assert) {
  await render(hbs`<ComplexComponent />`);
  assert.ok(true);
});
```

```
import { percySnapshot } from 'ember-percy';

...
// TODO: Write tests later

test('complex workflow', async function(assert) {
  await visit('/complex-page');
  await percySnapshot(assert);
});
```

Therefore we will now give a few examples of matrices (represented by their sparsity)

triangular:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



quadrangular:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



supernodal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



cyclic:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



IF  $f$  IS CONTINUOUS IN  $[a, b]$ , AND IF  $f(a)f(b) < 0$ ,

THEN  $f$  MUST HAVE A

ZERO IN  $(a, b)$ .



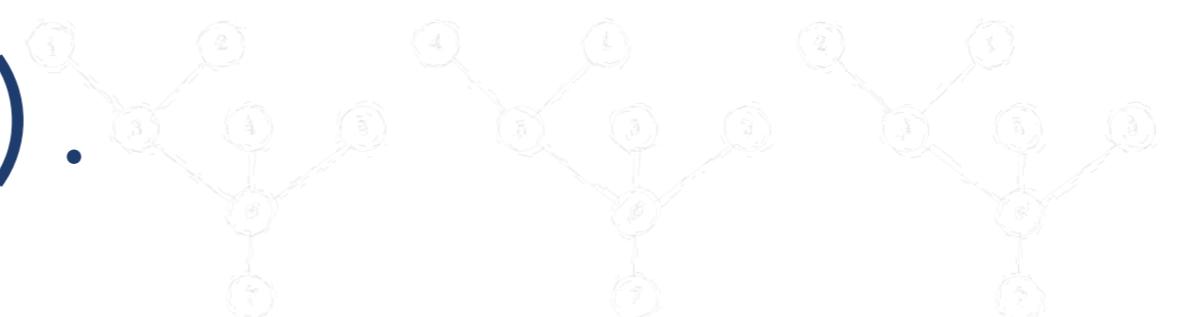
tells a different story - it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 2, 3 \rightarrow 3, 4 \rightarrow 4, 5 \rightarrow 5, 6 \rightarrow 3.$$

Of course, it is equivalent to replacing (arbitrarily) the equations and variables. As you can see from the figure, the graph  $G$  is called a tree, since the vertices  $a$  and  $b$  ( $a \in V \setminus \{b\}$ ,  $b \in V \setminus \{a\}$ ) and for every  $k = 1, 2, \dots, n-1$  the set  $\{v_1, v_2, \dots, v_k\} \cap \{v_{k+1}, v_{k+2}, \dots, v_{n-1}\}$  contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that  $G$  is a tree if each two members of  $V$  are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees, but this is not the case with either of banded or cyclic matrices when  $p \geq 3$ .

Of course, a tree is equivalent to a rooted tree. A vertex  $r \in V$  is called a root of the tree if it is not a predecessor of any other vertex. Unlike in a binary tree, there is a natural partial ordering which can best be explained by an analogy with a family tree. Thus, the root  $r$  is the predecessor of all the vertices in  $V \setminus \{r\}$  and these vertices are successors of  $r$ . Moreover, every  $v \in V \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex along this path, except for  $r$  and  $v$ , as a predecessor of  $v$  and a successor of  $r$ . We say that the rooted tree  $T$  is monotonically ordered if each vertex is labelled by an integer  $r$  ( $r \in \{1, 2, \dots, n\}$ ) in such a way that the vertices from the top of the tree to the bottom are in increasing order. (In other words, we say the vertices from the top of the tree to the bottom are in increasing order. Labeling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree:

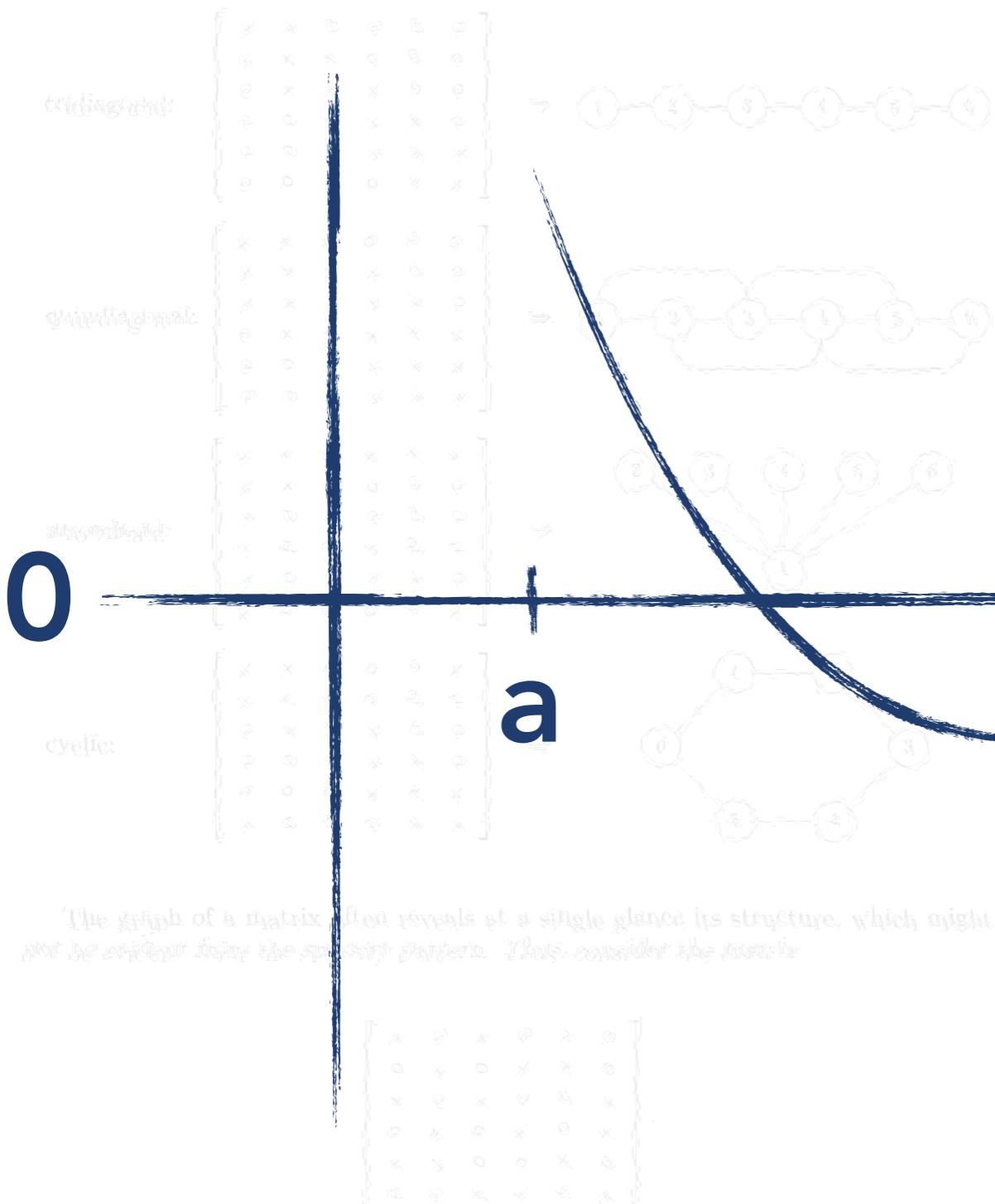


**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, n\}$  and assume that the rows and columns of  $A$  are then arranged so that  $T = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

At a first glance, this is not a very useful formula, but it is the basis for the algorithm just displayed, but its graph.

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



At a first glance, there is nothing to link in the case of the four matrices that we have just displayed, but its graph.



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 3, \quad 5 \rightarrow 6, \quad 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges  $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$  is called a path joining the vertices  $i_0$  and  $j_0$  if  $i_0 \in \{i_0, j_0\}$ ,  $j_0 \in \{i_0, j_0\}$  and for every  $k = 1, 2, \dots, n-1$  the set  $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$  contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that  $G$  is a tree if each two members of  $V$  are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when  $n \geq 3$ .

Given a tree  $T$  and an arbitrary vertex  $r \in V$ , the pair  $T = (G, r)$  is called a rooted tree, while  $r$  is said to be the root. Unlike in ordinary graph,  $T$  admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root  $r$  is the predecessor of all the vertices in  $T \setminus \{r\}$  and these vertices are successors of  $r$ . The set  $\{v \in V \mid v \neq r\text{ and }v \text{ is a successor of }r\}$  is called an  $r$ -successor. We designate each vertex along this path, except for  $r$  and  $\alpha$ , as a predecessor of  $\alpha$  and a successor of  $r$ . We say that the rooted tree  $T$  is monotonically ordered if each vertex is labelled before all its descendants. In other words, we layed the tree down the top at the tree's root. (As we have seen, it is said it, relabelling  $\alpha$  is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree:



**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, n\}$  and assume that the rows and columns of  $A$  are then arranged so that  $T = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

# YOU CAN FIND EQUALLY MANY NUMBERS BETWEEN 0 AND 1 AS YOU CAN

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

## BETWEEN $-\infty$ AND $\infty$ .

At a first glance this is not a matrix, but it is a little known matrix that is not

just displayed, but its graph.



It is a different story – it is nothing other than the cyclic matrix in disguised. To see this, let us relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 8.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges  $\{(u_i, v_i)\}_{i=1}^r$  in  $\mathbb{G}$  is called a path joining the vertices  $u_1$  and  $u_r$  if  $u_i \in \{u_1, u_2\}$ ,  $v_i \in \{v_1, v_2\}$  and for every  $i = 1, 2, \dots, r-1$  the set  $\{u_i, v_i\} \cap \{u_{i+1}, v_{i+1}\} = \emptyset$ . If a path contains exactly one segment, then it is a simple path if it does not visit any vertex more than once. We say that a path of length  $r$  is a  $r$ -cycle if it starts and ends at the same vertex. (Note that a path of length  $r$  corresponds to a  $(r+1) \times (r+1)$  matrix, but this is not the case with either quidiagonal or cyclic matrices when  $r \geq 3$ .)

Given a tree  $\mathbb{G}$  and an arbitrary vertex  $r \in \mathbb{V}$ , the pair  $T = (G, r)$  is called a rooted tree, while  $r$  is said to be the root. Unlike in a ordinary graph,  $T$  admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root is the ancestor of the vertices in  $T \setminus \{r\}$  and these vertices are successors of  $r$ . Moreover, every  $a \in \mathbb{V} \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex along this path, except for  $r$  and  $a$ , as a predecessor of  $a$  and a successor of  $r$ . We say that the rooted tree  $T$  is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree:



**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $\mathbb{G}$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  were been arranged so, that  $T = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

tridiagonal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix} \rightarrow \text{Graph: } 1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 6$$



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, first, re-label the weights as follows:

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99 100

$$\begin{bmatrix} * & 0 & * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & * & 0 & 0 \\ * & 0 & * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & * & 0 & * \\ * & 0 & * & 0 & * & 0 & * & 0 \\ 0 & * & * & 0 & * & 0 & * & 0 \\ * & 0 & * & * & 0 & * & 0 & 0 \\ 0 & * & * & 0 & * & * & 0 & 0 \\ * & 0 & * & * & 0 & * & * & 0 \\ 0 & * & * & 0 & * & * & 0 & * \end{bmatrix}.$$

At a first glance, there is nothing to think in the case of the first matrices that we have just displayed, but its graph:



**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  are then arranged so, that  $\mathbb{T} = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

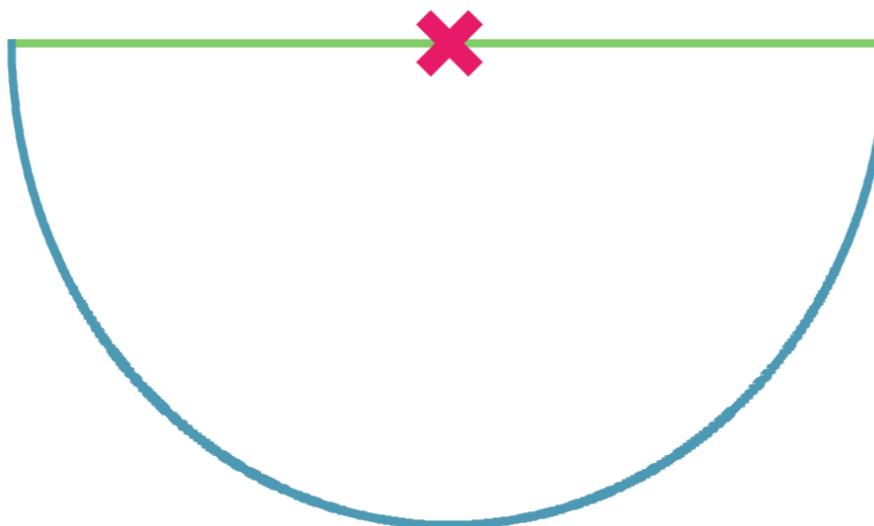
$$l_{k,j} = \frac{a_{kj}}{\sum_{i \sim j} a_{ki}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

tridiagonal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix} \rightarrow \text{Graph: } 1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 6$$

tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, first, re-label the vertices as follows:



$$\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & * & 0 \\ * & 0 & * & 0 & 0 & * \\ 0 & * & 0 & * & 0 & * \\ * & 0 & * & 0 & 0 & * \\ 0 & * & * & * & * & * \end{bmatrix}$$

At a first glance, there is nothing to think in the case of the first matrices that we have just displayed, but its graph:



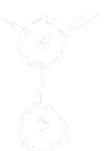
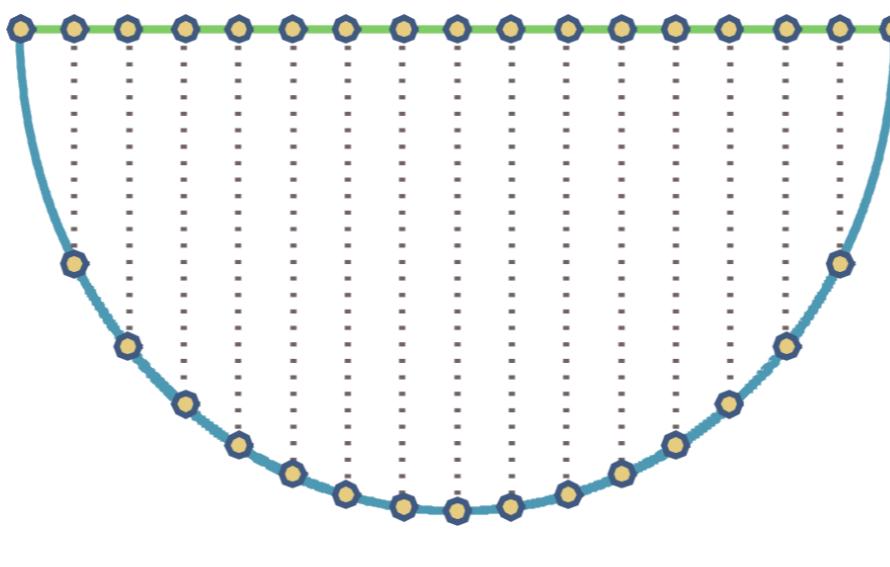
**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  are then arranged so, that  $\mathbb{T} = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ii}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, first recall the weights as follows:



**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, n\}$  and assume that the rows and columns of  $A$  are then arranged so that  $T = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

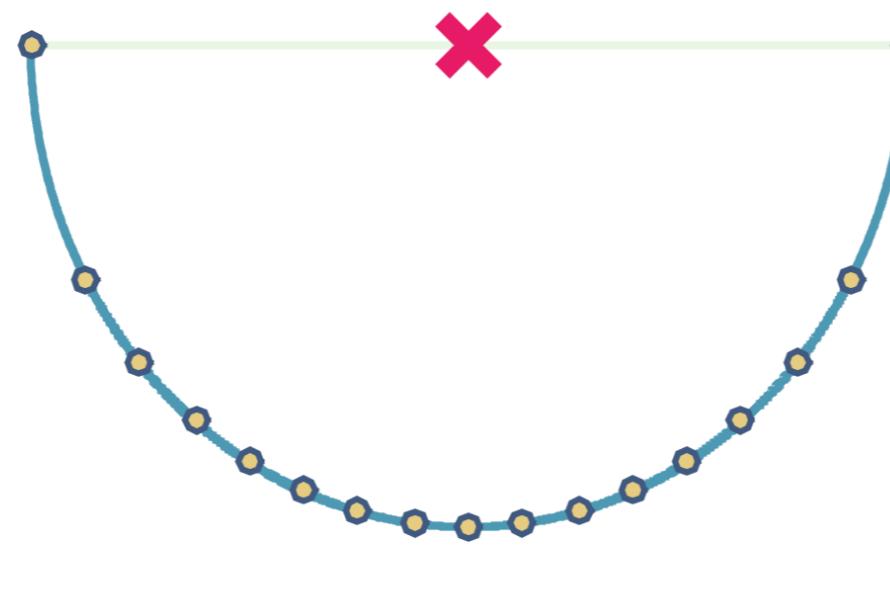
At a first glance, there is nothing to think in the case of the four matrices that we have just displayed, but its graph.

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

tridiagonal:

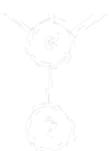
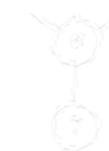
$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix} \rightarrow \text{Graph: } 1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 6$$

tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, first recall the weights as follows:



$$\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & * & 0 \\ * & 0 & * & 0 & 0 & * \\ 0 & * & 0 & * & 0 & * \\ * & 0 & * & 0 & 0 & * \\ 0 & * & * & * & * & * \end{bmatrix}$$

At a first glance, there is nothing to think in the case of the four matrices that we have just displayed, but its graph:



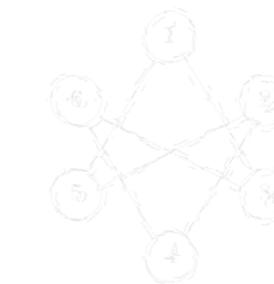
**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  are then arranged so, that  $\mathbb{T} = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ii}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

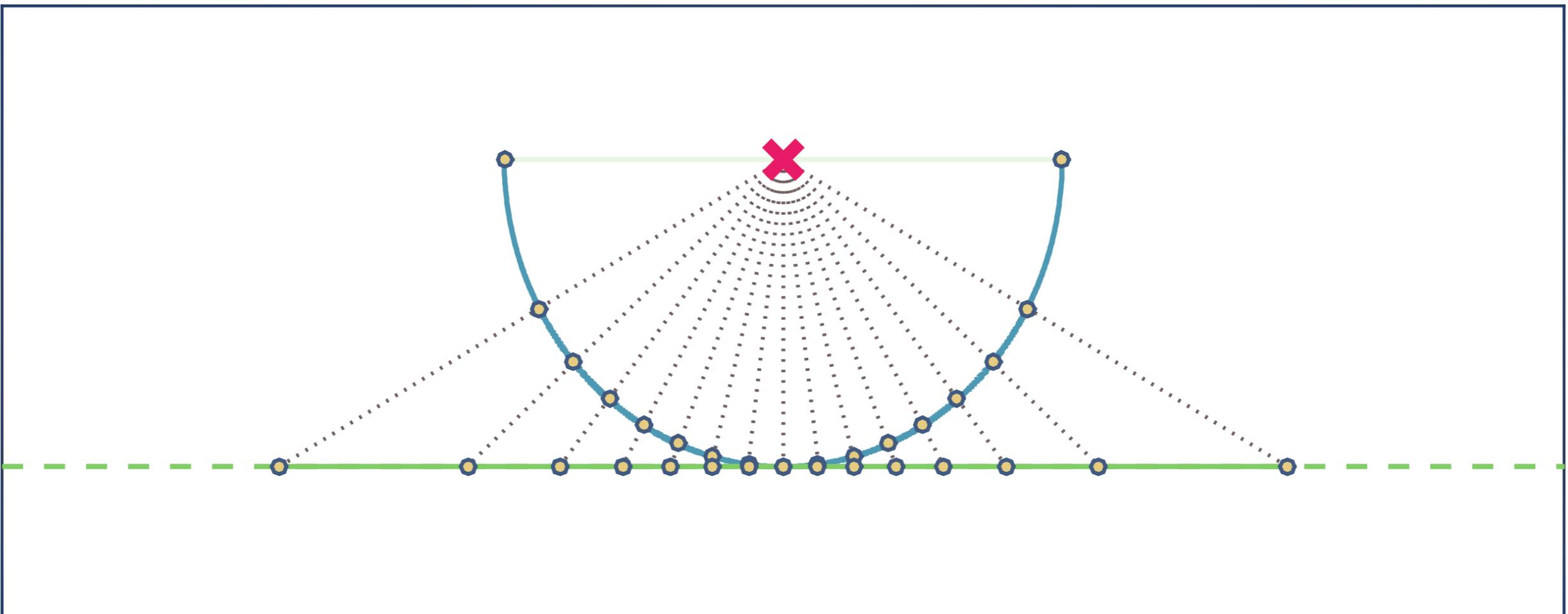
Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

tridiagonal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix} \rightarrow \text{Graph: } 1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 6$$



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just re-label the vertices as follows:



$$\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & 0 \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & 0 \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & * \end{bmatrix}.$$

At a first glance, there is nothing to think in the case of the four matrices that we have just displayed, but its graph:



**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  are then arranged so, that  $\mathbb{T} = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ii}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$



# 5 Rules of Writing Tests

Filter:

Module: Acceptance | Outline



EmberFest 18.10.2019

Running:

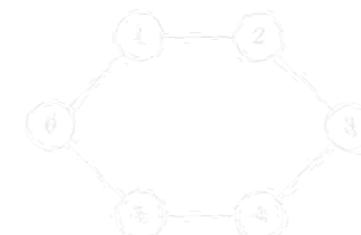
asymmetric:

$$\begin{bmatrix} 0 & 2 & 3 & 2 & 2 \\ 2 & 0 & 2 & 2 & 2 \\ 3 & 2 & 0 & 2 & 2 \\ 2 & 2 & 2 & 0 & 2 \\ 2 & 2 & 2 & 2 & 0 \end{bmatrix} \Rightarrow$$



cyclic:

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \Rightarrow$$



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} 0 & 2 & * & 0 & * & 0 \\ 2 & 0 & 0 & * & x & 0 \\ * & 0 & 0 & 0 & 0 & x \\ 0 & * & 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & x & 0 & 0 & 0 \end{bmatrix}.$$

At a first glance, there is nothing to think in the case of the four matrices that we have just displayed, but its graph,

which can best be explained by an analogy with a family tree. Thus, the root  $r$  is the predecessor of all the vertices in  $T \setminus \{r\}$  and these vertices are successors of  $r$ . Moreover, every  $a \in V \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex along this path, except for  $r$  and  $a$ , as a predecessor of  $a$  and a successor of  $r$ . We say that the rooted tree  $T$  is *monotonically ordered* if each vertex is labelled before all its predecessors; in other words, we label the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three consecutive orderings of the same rooted tree.



**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  are then arranged so that  $T = (G, r)$  is monotonically ordered. Then, that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ii}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

# 5 Rules of Writing Tests

Filter:

Module: Acceptance | Outline

EmberFest 18.10.2019

1 assertion of 1 passed, 0 failed.

## If, if, if

cyclic:

$$\begin{bmatrix} * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \end{bmatrix} \Rightarrow$$



before all its predecessors in other words: we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three monotone orderings of the same rooted tree:



**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  are then arranged so, that  $T = (G, r)$  is monotonically ordered. Then, that  $A = L U$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ki}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

At a first glance, there is nothing to think in the case of the four matrices that we have just displayed, but its graph.

# 5 Rules of Writing Tests

Filter:

Module: Acceptance | Outline

EmberFest 18.10.2019

2 assertions of 2 passed, 0 failed.

If, if, if

Use common, everyday words

# 5 Rules of Writing Tests

Filter:

Module:

Acceptance | Outline



EmberFest 18.10.2019

3 assertions of 3 passed, 0 failed.

! If, if, if

! Use common, everyday words

! Write less with theorems and new terms

$$\begin{bmatrix} * & * & * & 0 & * & 0 \\ 0 & * & 0 & * & * & 0 \\ * & 0 & * & * & 0 & * \\ 0 & * & 0 & * & * & 0 \\ * & * & 0 & * & * & 0 \\ 0 & * & * & 0 & * & * \end{bmatrix}.$$

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph:



**Theorem 11.1** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  are then arranged so, that  $\mathbf{T} = (G, r)$  is monotonically ordered. Then, that  $A = L U$  is a Cholesky factorization, it is true that

$$t_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j t_{i,j}^2}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

# 5 Rules of Writing Tests

Filter:

Module:

Acceptance | Outline



EmberFest 18.10.2019

4 assertions of 4 passed, 0 failed.

! If, if, if

! Use common, everyday words

! Write less with theorems and new terms

! All your basis are belong to us

$$\begin{bmatrix} 0 & 3 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph:

**Theorem 11.1** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  are then arranged so, that  $\tilde{T} = (G, r)$  is monotonically ordered. Then, that  $A = \tilde{L}\tilde{U}$  is a Cholesky factorization, it is true that

$$t_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ki}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

# 5 Rules of Writing Tests

Filter:

Module: [Acceptance](#) | [Outline](#) ▾

EmberFest 18.10.2019

5 assertions of 5 passed, 0 failed.

! If, if, if

! Use common, everyday words

! Write less with theorems and new terms

! All your basis are belong to us

! 1 picture = 1000 words

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

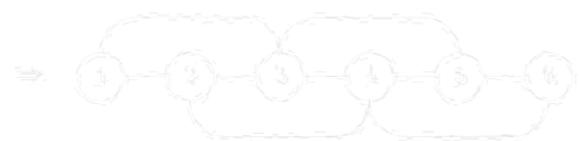
tridiagonal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



quadiagonal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ * & * & * & * & * & * \\ * & * & * & * & * & * \end{bmatrix}$$



# crunchingnumbers.live

@ijlee2

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & 0 \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & * \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & * \end{bmatrix}.$$

At a first glance, there is nothing to think in the case of the four matrices that we have just displayed, but its graph,



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

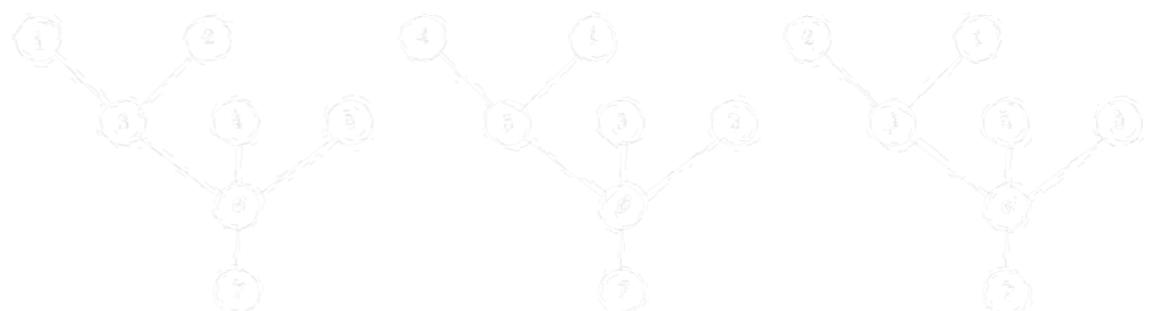
$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 3, \quad 5 \rightarrow 6, \quad 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges  $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$  is called a path joining the vertices  $i_0$  and  $j_0$  if  $i_0 \in \{i_0, j_0\}$ ,  $j_0 \in \{i_0, j_0\}$  and for every  $k = 1, 2, \dots, n-1$  the set  $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$  contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that  $G$  is a tree if each two members of  $V$  are joined by a unique simple path. Both tridiagonal and quadiagonal matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when  $n \geq 3$ .

Given a tree  $T$  and an arbitrary vertex  $r \in V$ , the pair  $T = (G, r)$  is called a rooted tree, where the vertex  $r$  is the root. Unlike in ordinary graph,  $T$  adapts a natural partial order which should be explained by an analogy with a family tree. Thus, the vertex  $r$  is the predecessor of all the vertices in  $T \setminus \{r\}$  and these vertices are successors of  $r$ . Moreover, every  $\alpha \in V \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex along this path, except for  $r$  and  $\alpha$ , as a predecessor of  $\alpha$  and a successor of  $r$ . We say that the rooted tree  $T$  is monotonically ordered if each vertex is labelled before all its predecessors in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three consecutive orderings of the same rooted tree.



**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Consider a vector  $r \in \{1, 2, \dots, n\}$  and assume that the rows and columns of  $A$  are now arranged so, that  $T = (G, r)$  is monotonically ordered. Given that  $A = PLU$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{p_{jj}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

**Q.E.D.**