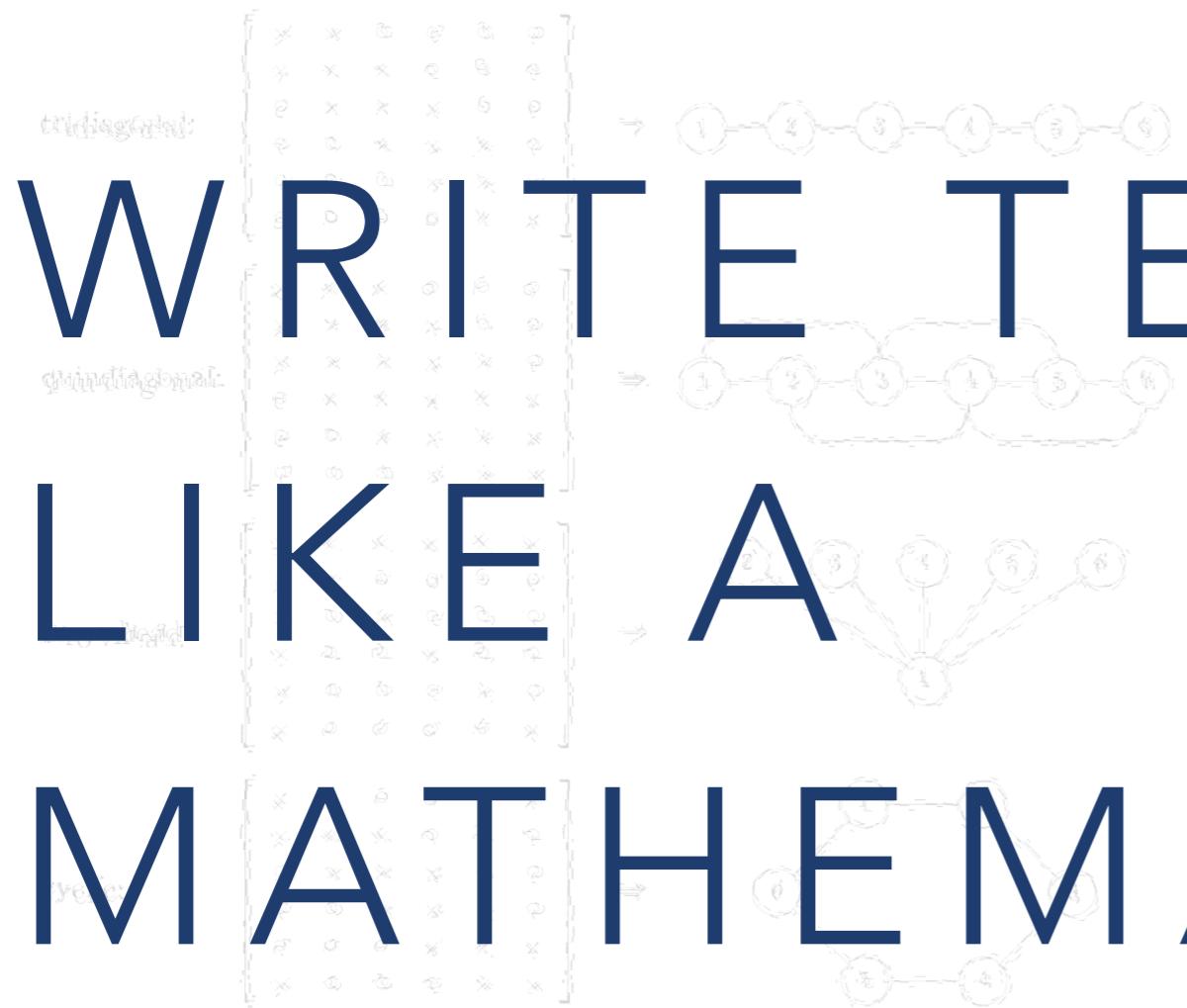


Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

ISAAC J. LEE

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, first recall the relation as follows

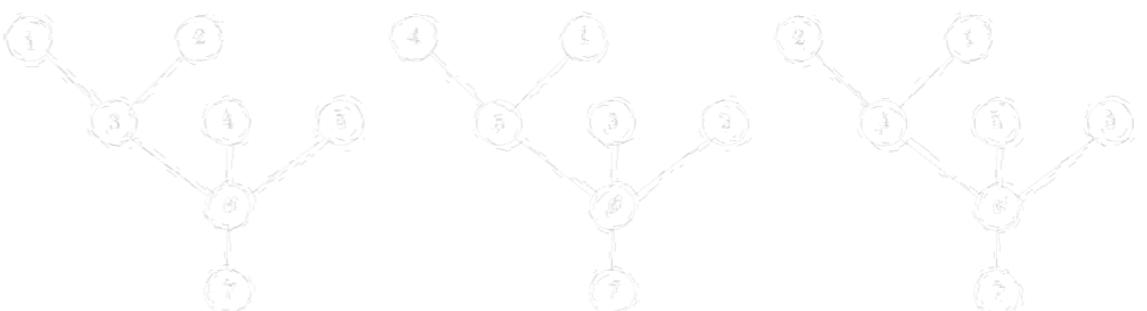
$$1 \rightarrow 2, 2 \rightarrow 3, 3 \rightarrow 2, 4 \rightarrow 5, 5 \rightarrow 6, 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

Let  $\alpha$  be an ordered set of vertices  $\{\alpha_i\}_{i=1}^n$ ,  $\beta \in \mathbb{S}$  is called a path joining the vertices  $\alpha$  and  $\beta$  if  $\alpha \in \{\alpha_i, \alpha_{i+1}\}_{i=1}^n$ ,  $\beta \in \{\alpha_i, \alpha_{i+1}\}_{i=1}^n$  and for every  $k = 1, 2, \dots, n-1$  the set  $\{\alpha_k, \alpha_{k+1}\} \cap \{\alpha_{k+1}, \alpha_{k+2}\}$  contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that  $\mathbb{G}$  is a tree if each two members of  $\mathbb{V}$  are joined by a unique simple path. Both tridiagonal and pentadiagonal matrices correspond to trees, but this is not the case with either quadiagonal or cyclic matrices when  $n \geq 3$ .

Given a tree  $\mathbb{G}$  and an arbitrary vertex  $r \in \mathbb{V}$ , the pair  $\mathbb{T} = (\mathbb{G}, r)$  is called a rooted tree, while  $r$  is said to be the root. Unlike an ordinary graph,  $\mathbb{T}$  admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root  $r$  is the predecessor of all the vertices in  $\mathbb{V} \setminus \{r\}$  and these vertices are successors of  $r$ . Moreover, every  $\alpha \in \mathbb{V} \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex along this path, except for  $r$  and  $\alpha$ , as a predecessor of  $\alpha$  and a successor of  $r$ . We say that the rooted tree  $\mathbb{T}$  is monotonically ordered if each vertex is labelled before all its predecessors (or, more precisely, we have the relation from the top of the tree to the root. If we have already said it, relabeling a graph is tantamount to relabeling the rows and the columns of the underlying matrix).

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three monotone orderings of the sample rooted tree:



**Theorem III.1.** Let  $\mathbb{A}$  be a symmetric matrix whose graph  $\mathbb{G}$  is a tree. Choose a root  $r \in \{1, 2, \dots, n\}$  and assume that the rows and columns of  $\mathbb{A}$  have been arranged so that  $\mathbb{T} = (\mathbb{G}, r)$  is monotonically ordered. Given that  $\mathbb{A} = \mathbb{L}\mathbb{U}^T$  is a Cholesky factorization, it is true that

$$a_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ii}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

# 5 Rules of Writing Tests

Filter:

Module: Acceptance | Outline



EmberFest 18.10.2019

Running:

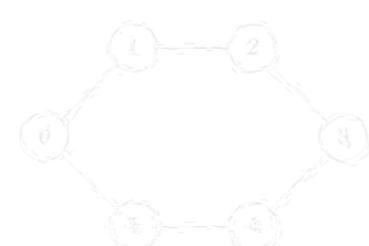
asymmetric:

$$\begin{bmatrix} 0 & 2 & 3 & 2 & 2 \\ 2 & 0 & 2 & 2 & 2 \\ 3 & 2 & 0 & 2 & 2 \\ 2 & 2 & 2 & 0 & 2 \\ 2 & 2 & 2 & 2 & 0 \end{bmatrix} \Rightarrow$$



cyclic:

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \Rightarrow$$



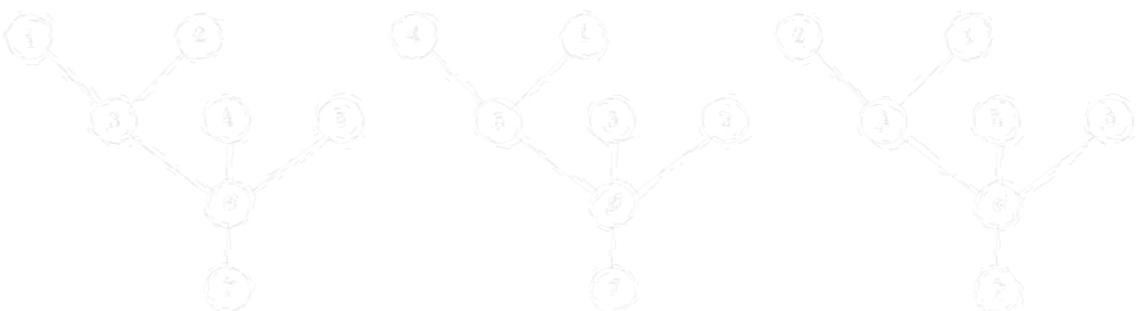
The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} 0 & 2 & * & 0 & * & 0 \\ 2 & 0 & 0 & * & x & 0 \\ * & 0 & 2 & 0 & 0 & x \\ 0 & * & 0 & 0 & x & 0 \\ * & x & 0 & 0 & 0 & 0 \\ 0 & 0 & x & 0 & 0 & 0 \end{bmatrix}.$$

At a first glance, there is nothing to think in the case of the four matrices that we have just displayed, but its graph,

which can best be explained by an analogy with a family tree. Thus, the root  $r$  is the predecessor of all the vertices in  $V \setminus \{r\}$  and these vertices are successors of  $r$ . Moreover, every  $v \in V \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex along this path, except for  $r$  and  $v$ , as a predecessor of  $v$  and a successor of  $r$ . We say that the rooted tree  $T$  is *monotonically ordered* if each vertex is labelled before all its predecessors; in other words, we label the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three monotonic orderings of the same rooted tree:



**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  are then arranged so that  $T = (G, r)$  is monotonically ordered. Then, that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ki}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

# 5 Rules of Writing Tests

Filter:

Module: Acceptance | Outline

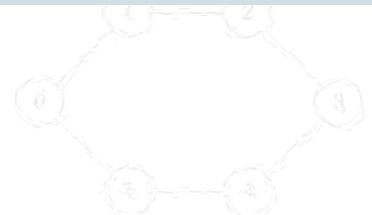
EmberFest 18.10.2019

1 assertion of 1 passed, 0 failed.

## If, if, if

cyclic:

$$\begin{bmatrix} * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \end{bmatrix} \Rightarrow$$



before all its predecessors in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three monotone orderings of the same rooted tree.

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \end{bmatrix}.$$

At a first glance, there is nothing to think in the case of the four matrices that we have just displayed, but its graph,



**Theorem 11.1** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  are then arranged so, that  $T = (G, r)$  is monotonically ordered. Then, that  $A = L U$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ki}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

# 5 Rules of Writing Tests

Filter:

Module: Acceptance | Outline

EmberFest 18.10.2019

2 assertions of 2 passed, 0 failed.

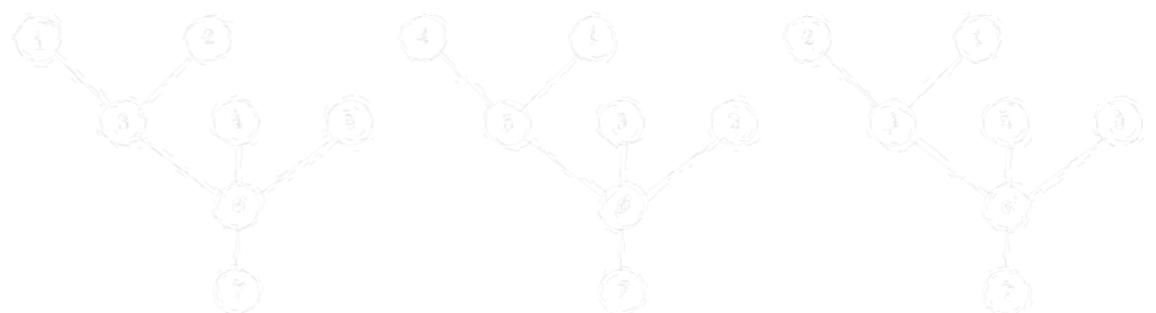
! If, if, if

! Use common, everyday words

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} * & * & * & 0 & * & 0 \\ 0 & * & 0 & * & * & 0 \\ * & 0 & * & 0 & * & * \\ 0 & * & 0 & * & 0 & * \\ * & * & 0 & 0 & * & * \\ 0 & 0 & * & * & * & * \end{bmatrix}.$$

At a first glance, there is nothing to think in to any of the four matrices that we have just displayed, but its graph.



**Theorem 11.1** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  are then arranged so, that  $\mathbb{T} = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ii}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

# 5 Rules of Writing Tests

Filter:

Module:

Acceptance | Outline



EmberFest 18.10.2019

3 assertions of 3 passed, 0 failed.

! If, if, if

! Use common, everyday words

! Write less with theorems and new terms

$$\begin{bmatrix} * & * & * & 0 & * & 0 \\ 0 & * & 0 & * & * & 0 \\ * & 0 & * & * & 0 & * \\ 0 & * & 0 & * & * & 0 \\ * & * & 0 & * & * & 0 \\ 0 & * & * & 0 & * & * \end{bmatrix}.$$

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph:



**Theorem 11.1** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  are then arranged so, that  $\mathbf{T} = (G, r)$  is monotonically ordered. Then, that  $A = \mathbf{L}\mathbf{U}^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ii}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

# 5 Rules of Writing Tests

Filter:

Module:

Acceptance | Outline



EmberFest 18.10.2019

4 assertions of 4 passed, 0 failed.

! If, if, if

! Use common, everyday words

! Write less with theorems and new terms

! All your basis are belong to us

$$\begin{bmatrix} 0 & 3 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,

**Theorem 11.1** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  are then arranged so, that  $\tilde{T} = (G, r)$  is monotonically ordered. Then, that  $A = \tilde{L}\tilde{U}$  is a Cholesky factorization, it is true that

$$t_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ki}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

# 5 Rules of Writing Tests

Filter:

Module: [Acceptance](#) | [Outline](#) ▾

EmberFest 18.10.2019

5 assertions of 5 passed, 0 failed.

! If, if, if

! Use common, everyday words

! Write less with theorems and new terms

! All your basis are belong to us

! 1 picture = 1000 words

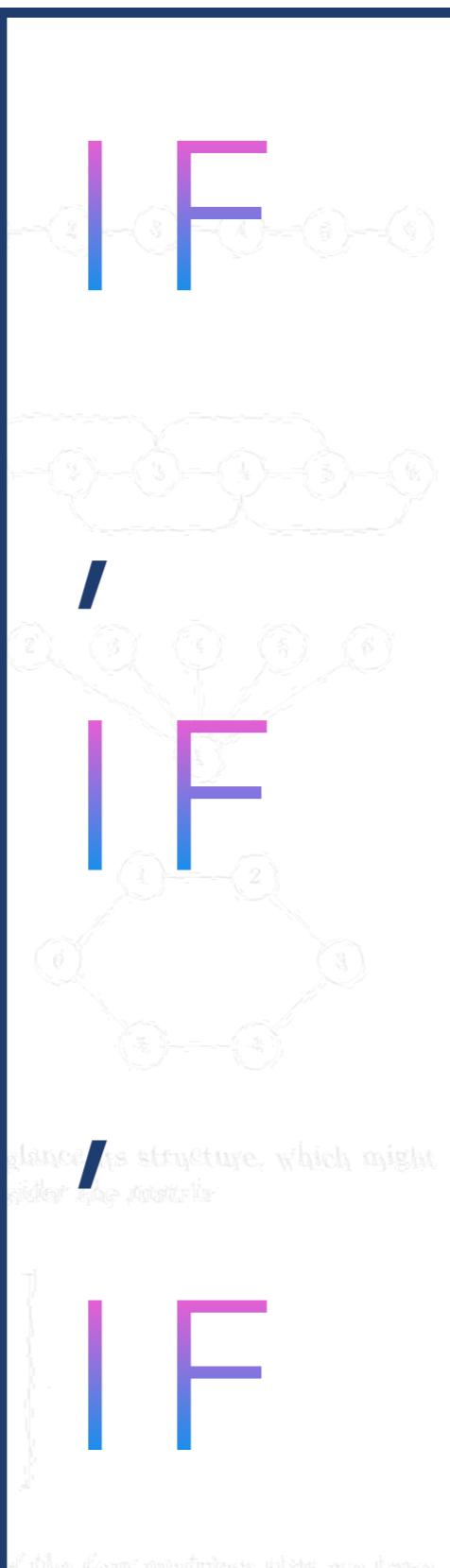
Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} * & * & * & 0 & * & * \\ 0 & * & 0 & * & * & * \\ * & 0 & * & 0 & * & * \\ 0 & * & 0 & * & 0 & * \\ * & * & 0 & 0 & * & * \\ 0 & 0 & * & * & * & * \end{bmatrix}$$

At a first glance, there is nothing to think in the case just displayed, but its graph,



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

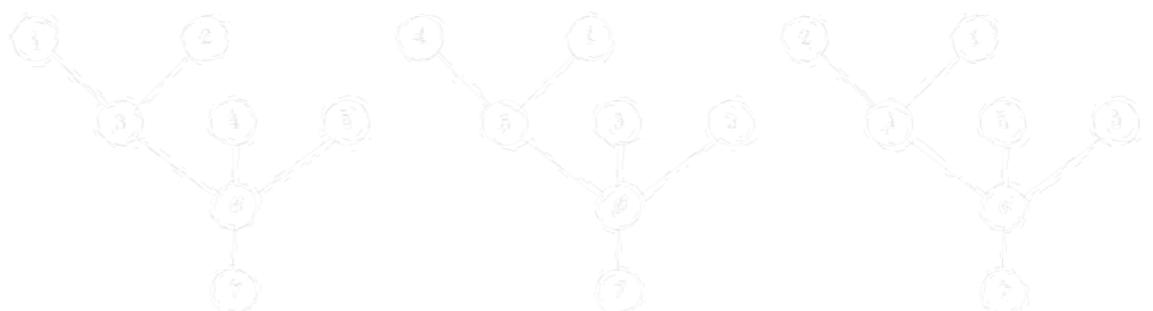
$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 3, \quad 5 \rightarrow 6, \quad 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges  $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$  is called a path joining the vertices  $i_0$  and  $j_0$  if  $i_0 \in \{i_0, j_0\}$ ,  $j_0 \in \{i_0, j_0\}$  and for every  $k = 1, 2, \dots, n-1$  the set  $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$  contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that  $G$  is a tree if each two members of  $V$  are joined by a unique simple path. Both tridiagonal and quidiagonal matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when  $n \geq 3$ .

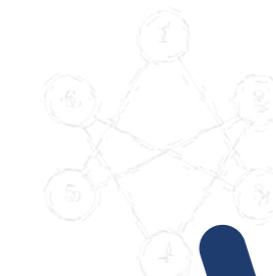
Given a tree  $T$  and an arbitrary vertex  $r \in V$ , the pair  $T = (G, r)$  is called a rooted tree, while  $r$  is said to be the root. Unlike in ordinary graph,  $T$  admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root  $r$  is the predecessor of all the vertices in  $T \setminus \{r\}$  and these vertices are successors of  $r$ . Moreover, every  $\alpha \in V \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex along this path, except for  $r$  and  $\alpha$ , as a predecessor of  $\alpha$  and a successor of  $r$ . We say that the rooted tree  $T$  is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we label the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three consecutive orderings of the same rooted tree.



**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, n\}$  and assume that the rows and columns of  $A$  are then arranged so that  $T = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$



tells a little story – it is nothing other than a cyclic matrix in disguised. To see this, first we look the entries as follows:

1 → 2 → 3 → 4 → 5 → 6 → 7 → 8 → 9 → 10 → 11 → 12 → 13 → 14 → 15 → 16 → 17 → 18 → 19 → 20 → 21 → 22 → 23 → 24 → 25 → 26 → 27 → 28 → 29 → 30 → 31 → 32 → 33 → 34 → 35 → 36 → 37 → 38 → 39 → 40 → 41 → 42 → 43 → 44 → 45 → 46 → 47 → 48 → 49 → 50 → 51 → 52 → 53 → 54 → 55 → 56 → 57 → 58 → 59 → 60 → 61 → 62 → 63 → 64 → 65 → 66 → 67 → 68 → 69 → 70 → 71 → 72 → 73 → 74 → 75 → 76 → 77 → 78 → 79 → 80 → 81 → 82 → 83 → 84 → 85 → 86 → 87 → 88 → 89 → 90 → 91 → 92 → 93 → 94 → 95 → 96 → 97 → 98 → 99 → 100

This is equivalent to considering a relation between the vertices of  $G$ , i.e.  $\{v_i, v_j\}$  and for  $i \neq j$  the set  $\{v_i, v_j\}$  contains exactly one member if and only if it does not visit any node once. We say that  $G$  is *acyclic* if two members of  $V$  are joined by the path. Both tridiagonal and pentadiagonal matrices correspond to trees, the case with either quaternary or  $p$ -ary matrices when  $p > 3$ .

Given a tree, while it can best be explained by an analogy with a family tree. Thus, the predecessor of all the vertices in  $V \setminus \{r\}$  are successors of  $r$ . Moreover, every  $\alpha \in V \setminus \{r\}$  is joined to  $r$  by a single path and we designate this path, except for  $r$  and  $\alpha$ , as a predecessor of  $\alpha$  and a successor of  $r$ . We say that the rooted tree  $T$  is *monotonic* if each vertex is labelled with its predecessors. In other words, we label the vertices from the top of the tree to the bottom. (As we have already said, relabelling a vertex is tantamount to rows and columns of the underlying matrix.)

Every individual in the monotonically ordered and, in general, non-convex ordering is not supplied

THEN

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & 0 \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & * \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & * \end{bmatrix}.$$

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,



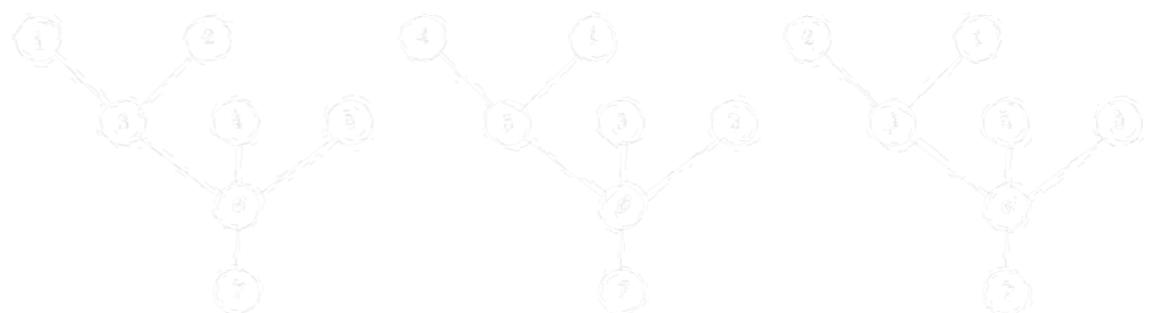
tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges  $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$  is called a path joining the vertices  $i_0$  and  $j_0$  if  $i_0 \in \{i_0, j_0\}$ ,  $j_0 \in \{i_0, j_0\}$  and for every  $k = 1, 2, \dots, n-1$  the set  $\{i_{k+1}, j_{k+1}\} \cap \{i_k, j_k\}$  contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that  $G$  is a tree if each two members  $i, j$  are joined by a unique simple path. Both tridiagonal and quidiagonal matrices correspond to trees; this is not the case with either quidiagonal or cyclic matrices.

Let  $T = (G, r)$  and an arbitrary vertex  $r \in V$ . The pair  $T = (G, r)$  is called a rooted tree and  $r$  is said to be the root. Unlike in a cyclic graph, there is no unique root, which can best be explained by an example. Let  $T = (G, r)$  be a tree. Then  $r$  is the predecessor of all the vertices in  $V \setminus \{r\}$ . Moreover, every  $v \in V \setminus \{r\}$  is joined to  $r$  along a path, which can best be explained by an example. Let  $T = (G, r)$  be a tree. Then  $r$  is the predecessor of all the vertices in  $V \setminus \{r\}$ . We say that the rooted tree  $T$  is monotonically ordered if every vertex  $v$  has all its predecessors in  $V \setminus \{r\}$  in the same order as they appear in  $T$ . In other words,  $v$  is layered above its predecessors. We call  $r$  the root. (As we have already said, it is not possible to choose a root in a cyclic graph.) Relabelling a graph in this way is called a topological ordering. Every rooted tree will be monotonically ordered and, in general, it is not possible to give three consecutive vertices of the same layer. This is not the case with a tree, which give three consecutive vertices of the same layer.



**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, n\}$  and assume that the rows and columns of  $A$  are then arranged so that  $T = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their analysis.

1

```
assert.speaker().getsPersonal();  
  
await sing('Happy Birthday');  
  
assert.audience().isHappy();
```

just displayed, but its graph:

$$t_{k,j} = \frac{q_{kj}}{q_j}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

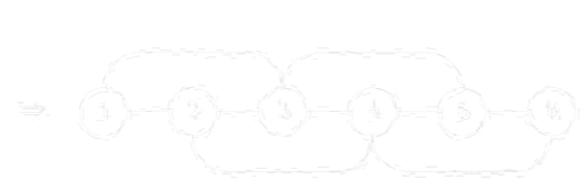
Therefore we will now give a few examples of matrices (represented by their sparsity)

and their graphs.

# SUSPECT 1

quindiagonal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$



symmetric:

$$\begin{bmatrix} * & * & * & * & * & * \\ * & * & 0 & 0 & 0 & 0 \\ * & 0 & * & 0 & 0 & 0 \\ * & 0 & 0 & * & 0 & 0 \\ * & 0 & 0 & 0 & * & 0 \\ * & 0 & 0 & 0 & 0 & * \end{bmatrix}$$



cyclic:

$$\begin{bmatrix} * & 0 & 0 & 0 & * & * \\ * & * & 0 & 2 & * & * \\ * & 0 & * & * & * & * \\ * & * & * & * & * & * \\ * & 0 & * & * & * & * \\ 0 & * & * & * & * & * \end{bmatrix}$$

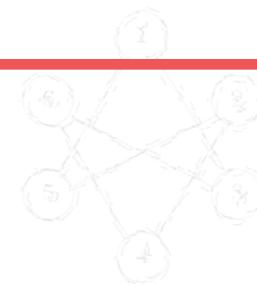


# OBSERVER /

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

# COMPUTED

At a first glance, the matrix  $A$  and its structure of the form of a triangular matrix just displayed, but its graph,



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

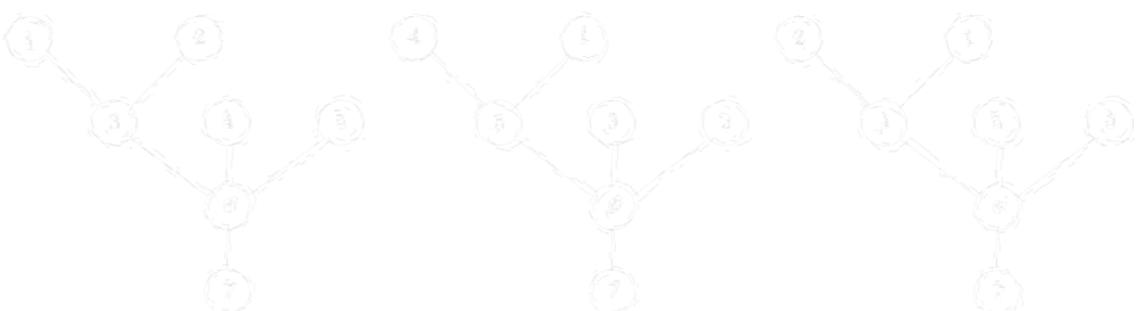
$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 3, \quad 5 \rightarrow 6, \quad 6 \rightarrow 8.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges  $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$  is called a path joining the vertices  $i_0$  and  $j_0$  if  $i_0 \in \{i_0, j_0\}$ ,  $j_0 \in \{i_0, j_0\}$  and for every  $k = 1, 2, \dots, n-1$  the set  $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$  contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that  $G$  is a tree if each two members of  $V$  are joined by a unique simple path. Both tri-diagonal and any other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when  $n \geq 3$ .

Given a tree  $T$  and an arbitrary vertex  $r \in V$ , the pair  $T = (G, r)$  is called a rooted tree, while  $r$  is said to be the root. Unlike in ordinary graph,  $T$  admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root  $r$  is the predecessor of all the vertices in  $T \setminus \{r\}$  and these vertices are successors of  $r$ . Moreover, every  $\alpha \in V \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex along this path, except for  $r$  and  $\alpha$ , as a predecessor of  $\alpha$  and a successor of  $r$ . We say that the rooted tree  $T$  is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree.



**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  were then arranged so that  $T = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

and their graphs.

# SUSPECT 2

quindiagonal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$



superdiagonal:

$$\begin{bmatrix} * & * & * & * & * & * \\ * & * & 0 & 0 & 0 & 0 \\ * & 0 & * & 0 & 0 & 0 \\ 0 & 0 & * & 0 & 0 & 0 \\ 0 & 0 & 0 & * & 0 & 0 \\ 0 & 0 & 0 & 0 & * & 0 \end{bmatrix}$$



cyclic:

$$\begin{bmatrix} * & 0 & 0 & 0 & * & 0 \\ * & * & 0 & 2 & * & 0 \\ * & 0 & * & * & 0 & 0 \\ 0 & 2 & * & * & * & 0 \\ 0 & * & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$



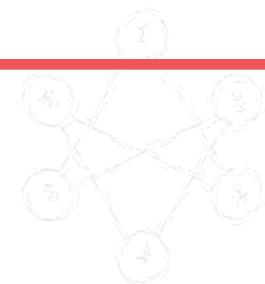
# EMBER DATA / FORM BUILDER

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & * & 0 \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$

At a first glance, this is nothing but the structure of the form builder matrix

just displayed, but its graph:



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 8.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges  $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$  is called a path joining the vertices  $i_0$  and  $j_0$  if  $i_0 \in \{1, \dots, n\}$ ,  $j_0 \in \{1, \dots, n\}$  and for every  $k = 1, 2, \dots, n-1$  the set  $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$  contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that  $G$  is a tree if each two members of  $V$  are joined by a unique simple path. Both tri-diagonal and any other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when  $n \geq 3$ .

Given a tree  $T$  and an arbitrary vertex  $r \in V$ , the pair  $T = (G, r)$  is called a rooted tree, while  $r$  is said to be the root. Unlike in ordinary graph,  $T$  admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root  $r$  is the predecessor of all the vertices in  $T \setminus \{r\}$  and these vertices are successors of  $r$ . Moreover, every  $\alpha \in V \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex along this path, except for  $r$  and  $\alpha$ , as a predecessor of  $\alpha$  and a successor of  $r$ . We say that the rooted tree  $T$  is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree:



**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  were then arranged so that  $T = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{k,j}}{l_{j,j}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

and their graphs.

# SUSPECT 3



# UNSETTLED

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix



At a first glance, the non-zero entries of this matrix are not at all clear. Let's look at its graph, which is



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

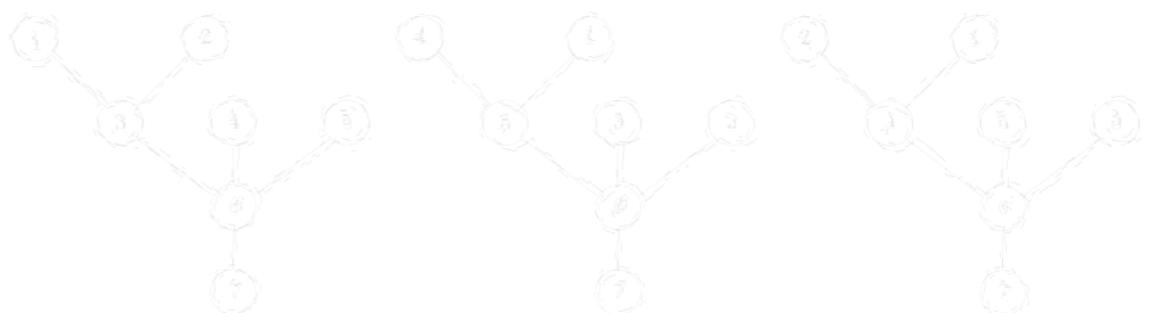
$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 1.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges  $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$  is called a path joining the vertices  $i_0$  and  $j_0$  if  $i_0 \in \{i_0, j_0\}$ ,  $j_0 \in \{i_0, j_0\}$  and for every  $k = 1, 2, \dots, n-1$  the set  $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$  contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that  $G$  is a tree if each two members of  $V$  are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when  $n \geq 3$ .

Given a tree  $T$  and an arbitrary vertex  $r \in V$ , the pair  $T = (G, r)$  is called a rooted tree, while  $r$  is said to be the root. Unlike in a ordinary graph,  $T$  admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root  $r$  is the predecessor of all the vertices in  $T \setminus \{r\}$  and these vertices are successors of  $r$ . Moreover, every  $\alpha \in V \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex along this path, except for  $r$  and  $\alpha$ , as a predecessor of  $\alpha$  and a successor of  $r$ . We say that the rooted tree  $T$  is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three consecutive orderings of the same rooted tree.



**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  were been arranged so, that  $T = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

and their graphs.

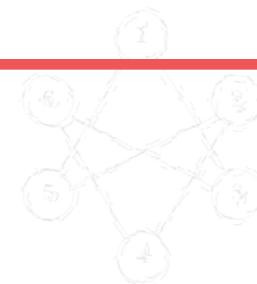
# SUSPECT 4



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

# LEAKAGE

At a first glance, this is nothing but a collection of the four matrices above, just displayed, but its graph,



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

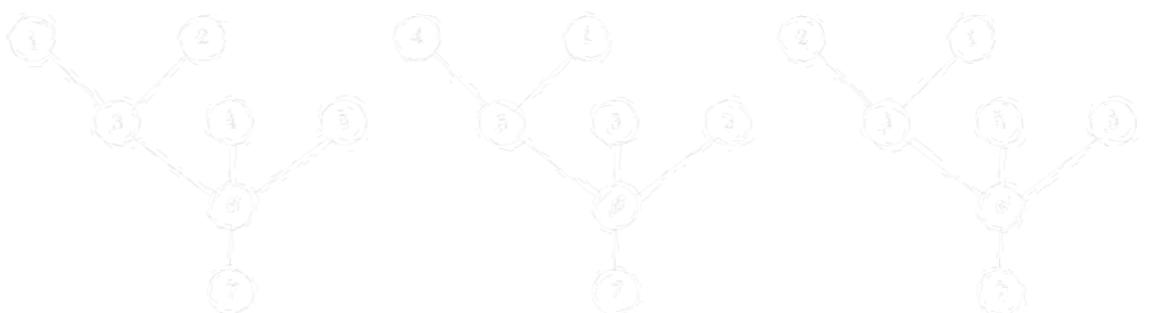
$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges  $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$   $\subseteq \mathbb{E}$  is called a path joining the vertices  $i_0$  and  $j_0$  if  $i_0 \in \{i_0, j_0\}$ ,  $j_0 \in \{i_0, j_0\}$  and for every  $k = 1, 2, \dots, n-1$  the set  $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$  contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that  $\mathbb{G}$  is a tree if each two members of  $\mathbb{V}$  are joined by a unique simple path. Both tri-diagonal and any other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when  $n \geq 3$ .

Given a tree  $\mathbb{T}$  and an arbitrary vertex  $r \in \mathbb{V}$ , the pair  $T = (G, r)$  is called a rooted tree, while  $r$  is said to be the root. Unlike in ordinary graph,  $T$  adapts a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root  $r$  is the predecessor of all the vertices in  $\mathbb{V} \setminus \{r\}$  and these vertices are successors of  $r$ . Moreover, every  $\alpha \in \mathbb{V} \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex along this path, except for  $r$  and  $\alpha$ , as a predecessor of  $\alpha$  and a successor of  $r$ . We say that the rooted tree  $T$  is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three consecutive orderings of the same rooted tree.



**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $\mathbb{G}$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  were been arranged so, that  $T = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

and their graphs.

# SUSPECT 5



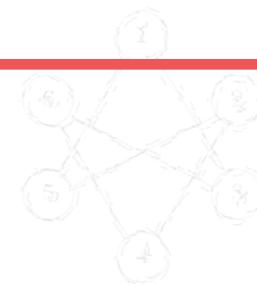
# ADMIN

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

# PRIVILEGE

At a first glance, this is nothing but a list of the four columns of the matrix

just displayed, but its graph,



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

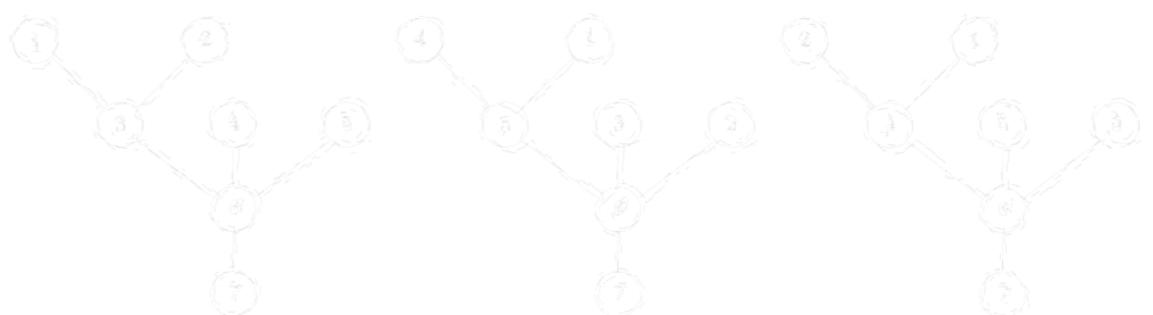
$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 8.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges  $\{(i_0, j_0)\}_{i_0, j_0 \in \mathbb{N}_0, i_0 \neq j_0}$  is called a path joining the vertices  $i_0$  and  $j_0$  if  $i_0 \in \{i_0, j_0\}$ ,  $j_0 \in \{i_0, j_0\}$  and for every  $k = 1, 2, \dots, n-1$  the set  $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$  contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that  $G$  is a tree if each two members of  $V$  are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when  $n \geq 3$ .

Given a tree  $T$  and an arbitrary vertex  $r \in V$ , the pair  $T = (G, r)$  is called a rooted tree, while  $r$  is said to be the root. Unlike in a ordinary graph,  $T$  admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root  $r$  is the predecessor of all the vertices in  $T \setminus \{r\}$  and these vertices are successors of  $r$ . Moreover, every  $a \in V \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex along this path, except for  $r$  and  $a$ , as a predecessor of  $a$  and a successor of  $r$ . We say that the rooted tree  $T$  is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

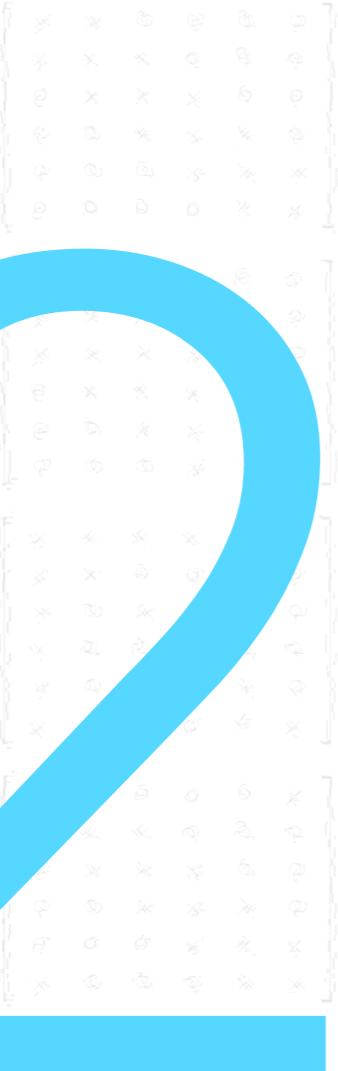
Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three consecutive orderings of the same rooted tree.

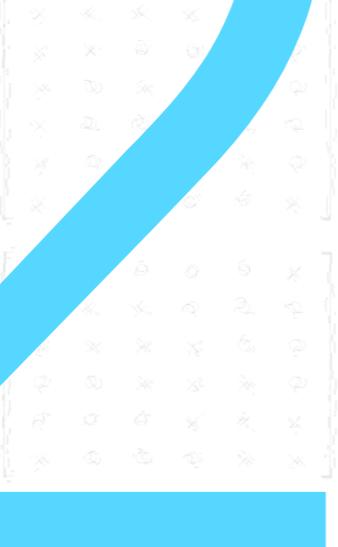


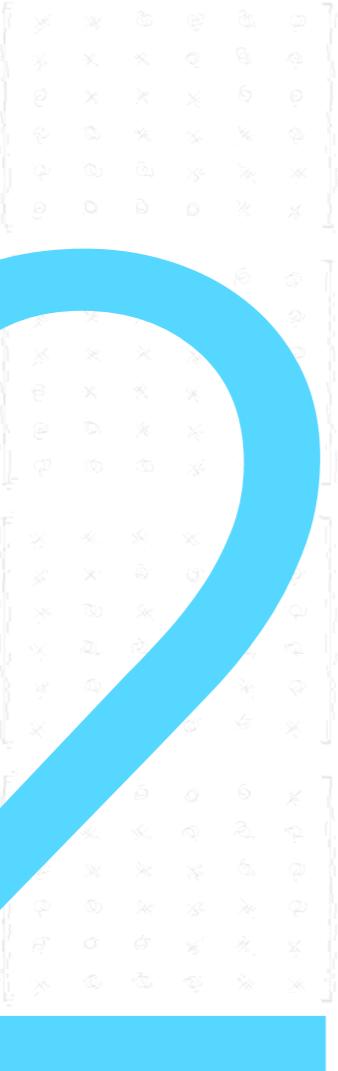
**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  were been arranged so, that  $T = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

tridiagonal:  


symmetric:  


cyclic:  


2

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix



At a first glance, there is nothing to link it to any of the three structures just displayed, but its graph,

# USE COMMON EVERYDAY WORDS



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges  $\{v_i, v_j\}$  in  $\mathbb{G}$  is called a path joining the vertices  $v_i$  and  $v_j$  ( $i, j \in \mathbb{N}$ ). If  $v_i = v_j$  and the edge  $\{v_i, v_i\} = \{v_i\}$  ( $i = 1, \dots, n-1$ ) contains only one member, it is a simple path if it does not visit any vertex more than once. We say that  $\mathbb{G}$  is free if each two members of  $\mathbb{V}$  are joined by a unique simple path. Both tridiagonal and cyclic matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when  $p \geq 3$ .

Given a tree  $\mathbb{T}$  and an arbitrary vertex  $r \in \mathbb{V}$ , the pair  $T = (G, r)$  is called a rooted tree, while  $r$  is said to be the root. Unlike in ordinary graph,  $T$  admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root  $r$  is the predecessor of all the vertices in  $T \setminus \{r\}$  and these vertices are successors of  $r$ . Moreover, every  $v \in \mathbb{V} \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex along this path, except for  $r$  and  $v$ , as a predecessor of  $v$  and a successor of  $r$ . We say that the rooted tree  $T$  is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we label the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is unique. To give the precise definition of the unique rooted tree,



**Theorem 1.** Let  $A$  be a symmetric matrix whose graph  $\mathbb{G}$  is a tree. Choose a root  $r \in \{1, 2, \dots, n\}$  and assume that the rows and columns of  $A$  are then arranged so, that  $T = (G, r)$  is monotonically ordered. Given that  $A = L U$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{k,j}}{l_{j,j}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

and their graphs

triangular:  $\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ 0 & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & * & 0 \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix} \Rightarrow \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \xrightarrow{1 \rightarrow 1, 2 \rightarrow 3, 3 \rightarrow 5, 4 \rightarrow 6, 5 \rightarrow 6, 6 \rightarrow 6}$

quidiagonal:  $\begin{bmatrix} * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ 0 & * & * & * & * & * \\ 0 & 0 & * & * & * & * \\ 0 & 0 & 0 & * & * & * \end{bmatrix} \Rightarrow \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \xrightarrow{1 \rightarrow 1, 2 \rightarrow 3, 3 \rightarrow 4, 4 \rightarrow 5, 5 \rightarrow 6, 6 \rightarrow 6}$

superdiagonal:  $\begin{bmatrix} * & * & * & * & * & * \\ * & 0 & 0 & 0 & 0 & 0 \\ * & 2 & 0 & 0 & 0 & 0 \\ * & 2 & 2 & 0 & 0 & 0 \\ * & 2 & 2 & 2 & 0 & 0 \\ * & 2 & 2 & 2 & 2 & 0 \end{bmatrix} \Rightarrow \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \xrightarrow{1 \rightarrow 1, 2 \rightarrow 2, 3 \rightarrow 3, 4 \rightarrow 4, 5 \rightarrow 5, 6 \rightarrow 6}$

cyclic:  $\begin{bmatrix} * & 0 & 0 & 0 & * & * \\ * & * & 0 & 0 & 2 & 2 \\ * & * & * & 0 & 2 & 2 \\ * & * & * & * & * & 2 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix} \Rightarrow \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \xrightarrow{1 \rightarrow 1, 2 \rightarrow 2, 3 \rightarrow 3, 4 \rightarrow 4, 5 \rightarrow 5, 6 \rightarrow 6}$

# CONVENTION;

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

# AGREED ON BY MANY

At a first glance, this is not a triangular matrix, but it is a tree structure, as the one just displayed, but its graph:



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 3, 3 \rightarrow 2, 4 \rightarrow 4, 5 \rightarrow 6, 6 \rightarrow 3.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges  $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$   $\subseteq \mathbb{E}$  is called a path joining the vertices  $i_0$  and  $j_0$  if  $i_0, j_0 \in \{1, \dots, n\}$ ,  $i_0 \neq j_0$ ,  $\{i_0, j_0\} \subseteq \{i_1, \dots, i_{n+1}\}$  and for every  $k = 1, 2, \dots, n-1$  the set  $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$  contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that  $G$  is a tree if each two members of  $V$  are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when  $n \geq 3$ .

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Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree:



**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, n\}$  and assume that the rows and columns of  $A$  are then arranged so, that  $T = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

tridiagonal:

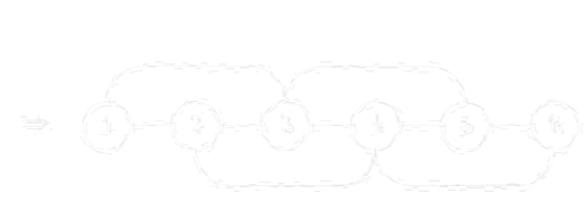
$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & 0 & * & * & 0 & 0 \\ 0 & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & * & * \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$

EVERYDAY



quindiagonal:

$$\begin{bmatrix} * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



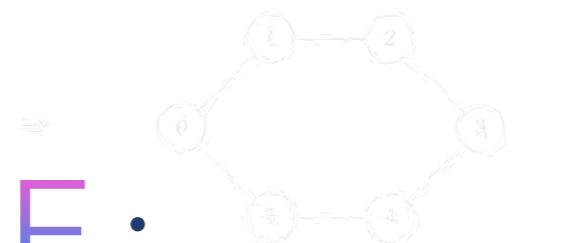
superdiagonal:

$$\begin{bmatrix} * & * & * & * & * & * \\ * & * & 0 & 0 & 0 & 0 \\ 0 & 0 & * & * & * & * \\ 0 & 0 & * & * & * & * \\ 0 & 0 & * & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



cyclic:

$$\begin{bmatrix} * & 0 & 0 & 0 & * & * \\ * & * & 0 & 2 & * & * \\ 0 & 0 & * & * & * & * \\ 0 & 2 & * & * & * & * \\ 0 & 0 & * & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



SIMPLE;

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

FAMILIAR TO MANY

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & * & * & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & * & * & * & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & * & * & * & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & * & * & * & * & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & * & * & * & * & 0 & 0 \\ 0 & 0 & 0 & 0 & * & * & * & * & * & 0 \\ 0 & 0 & 0 & 0 & 0 & * & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & * & * & * & * & * \end{bmatrix}$$

At a first glance this is not a matrix that looks like the ones we have been discussing, but it is. It is the matrix

just displayed, but its graph.



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

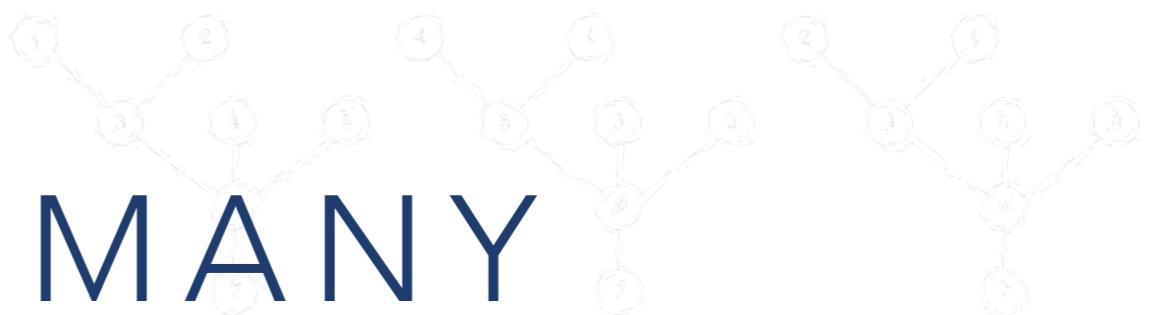
$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 3, \quad 5 \rightarrow 6, \quad 6 \rightarrow 8.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges  $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$   $\subseteq \mathbb{E}$  is called a path joining the vertices  $i_0$  and  $j_0$  if  $i_0 \in \{i_0, j_0\}$ ,  $j_0 \in \{i_0, j_0\}$  and for every  $k = 1, 2, \dots, n-1$  the set  $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$  contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that  $G$  is a tree if each two members of  $V$  are joined by a unique simple path. Both tridiagonal and quidiagonal matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when  $n \geq 3$ .

Given a tree  $T$  and an arbitrary vertex  $r \in V$ , the pair  $T = (G, r)$  is called a rooted tree, while  $r$  is said to be the root. Unlike in a ordinary graph,  $T$  adapts a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root  $r$  is the predecessor of all the vertices in  $T \setminus \{r\}$  and these vertices are successors of  $r$ . Moreover, every  $\alpha \in V \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex along this path, except for  $r$  and  $\alpha$ , as a predecessor of  $\alpha$  and a successor of  $r$ . We say that the rooted tree  $T$  is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree.



**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  are then arranged so that  $T = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

triangular:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$

quadrangular:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$

cyclic:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$

At a first glance, the last matrix is not symmetric, but it is a triangular matrix, as we have just displayed, but its graph,



IF  $f$  IS CONTINUOUS IN  $[a, b]$ , AND IF  $f(a)f(b) < 0$ ,

THEN  $f$  MUST HAVE A ZERO IN  $(a, b)$ .



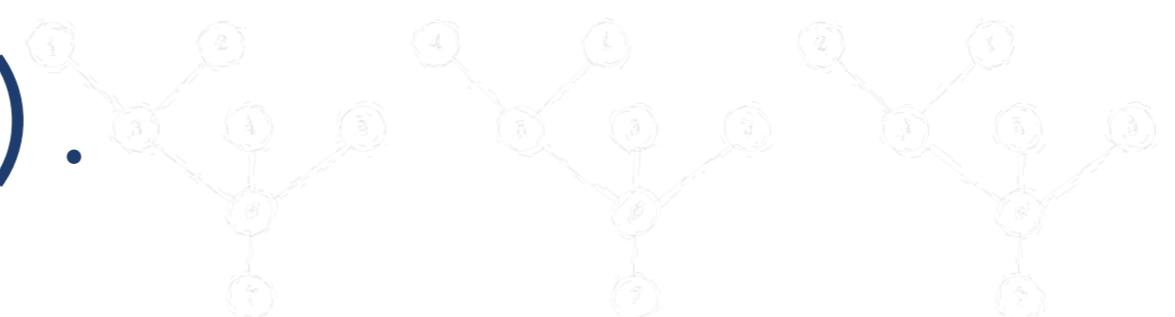
tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 3 \rightarrow 5, \quad 2 \rightarrow 4 \rightarrow 6, \quad 3 \rightarrow 1, \quad 4 \rightarrow 2.$$

Of course, it is equivalent to reordering simultaneously the equations and variables. As a result, we get the following graph  $G$ . It is called a tree spanning the vertices  $a$  and  $b$  ( $a \in \{1, 2, \dots, n\}$ ,  $b \in \{1, 2, \dots, n\}$ ) and for every  $k = 1, 2, \dots, n-1$  the set  $\{v_k, w_k\} \cap \{v_{k+1}, w_{k+1}\}$  contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that  $G$  is a tree if each two members of  $V$  are joined by a unique simple path. Both triangular and quadrangular matrices correspond to trees, but this is not the case with either quadrangular or cyclic matrices when  $n \geq 3$ .

It is natural to call a tree  $T = (G, r)$  a rooted tree, where  $r$  is the root. Unlike in a binary tree, there is a natural partial order, which can best be explained by an analogy with a family tree. Thus, the root  $r$  is the predecessor of all the vertices in  $V \setminus \{r\}$  and these vertices are successors of  $r$ . Moreover, every  $v \in V \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex along this path, except for  $r$  and  $v$ , as a predecessor of  $v$  and a successor of  $r$ . We say that the rooted tree  $T$  is monotonically ordered if each vertex is labelled by an integer from  $1$  to  $n$  in such a way that the vertices from the top of the tree down to the bottom are arranged in increasing order. (In other words, we say the vertices from the top of the tree down to the bottom are arranged in increasing order, because the top of the tree corresponds to the top of the permutation, the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree:



**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  are then arranged so that  $T = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

```
assert.dom('[data-test-message]')  
  .hasText(  
    'Thanks for signing up!',  
    'The user sees a welcome message.'  
  );
```

Therefore we will now give a few examples of matrices (represented by their sparsity)

triangular:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



quadrangular:



supernodal:



cyclic:



ZERO IN  $(a, b)$ .

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$

At a first glance this is not a triangular matrix, but it is a triangular matrix just displayed, but its graph,



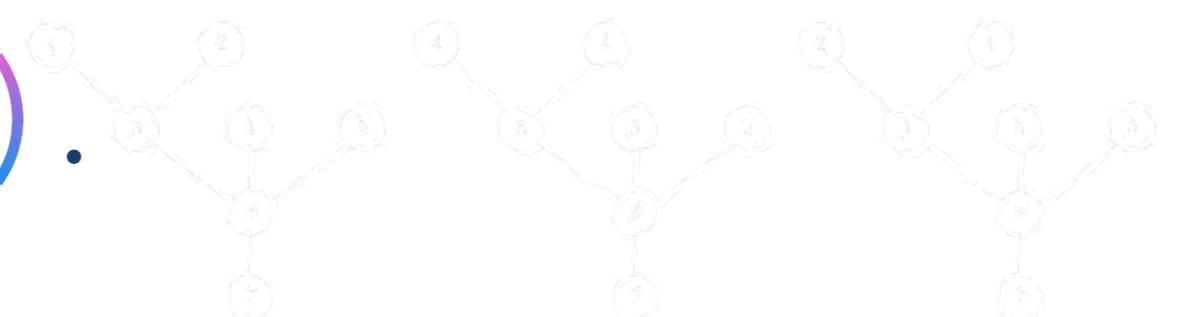
tells a different story - it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 2, 3 \rightarrow 3, 4 \rightarrow 4, 5 \rightarrow 5, 6 \rightarrow 6.$$

Of course, it is equivalent to relabeling the equations and variables. As you can see from the graph,  $G$  is a tree spanning the vertices  $a$  and  $b$  ( $a, b \in \{1, 2, \dots, 6\}$ ,  $a \neq b$ ) and for every  $i = 1, 2, \dots, n-1$  the set  $\{v_i, a, b\} \cap \{v_{i+1}, v_{i+2}\}$  contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that  $G$  is a tree if each two members of  $V$  are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees, but this is not the case with either quadiagonal or cyclic matrices when  $n \geq 3$ .

treating  $T$  as a binary tree, the root  $r$  of  $T$  is called a *rooted tree*. Unlike in ordinary graph,  $T$  admits a natural partial ordering which can best be explained by an analogy with a family tree. Thus, the root  $r$  is the predecessor of all the vertices in  $T \setminus \{r\}$  and these vertices are successors of  $r$ . Moreover, every  $v \in V \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex along this path, except for  $r$  and  $v$ , as a predecessor of  $v$  and a successor of  $r$ . We say that the rooted tree  $T$  is *monotonically ordered* if each vertex is labelled by an integer from  $1$  to  $n$  (in other words, we say the vertices from the top of the tree to the bottom). As we have already said, labelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

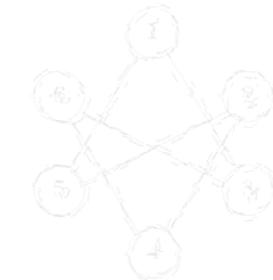
Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree:



**Theorem 11.1** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  are then arranged so that  $T = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, first relabel the vertices as follows:



Dashboard

Explore

Settings



vertices more than once. We say that  $G$  is a *tree* if each two members of  $V$  are joined by a unique simple path. Both tridiagonal and super-diagonal matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when  $p \geq 3$ .

Given a tree  $T$  and an arbitrary vertex  $r \in V$ , the pair  $T = (G, r)$  is called a *rooted tree*, while  $r$  is said to be the *root*. Unlike in ordinary graph,  $T$  admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root  $r$  is the *predecessor* of all the vertices in  $T \setminus \{r\}$  and these vertices are *successors* of  $r$ . Moreover, every  $\alpha \in V \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex along this path, except for  $r$  and  $\alpha$ , as a *predecessor* of  $\alpha$  and a *successor* of  $r$ . We say that the rooted tree  $T$  is *monotonically ordered* if each vertex is labelled before all its predecessors. In other words, if label the vertices from the root of the

'[data-test-link="Dashboard"]'

'[data-test-link="Explore"]'

'[data-test-link="Settings"]'

just displayed, but its graph.

$$t_{k,j} = \frac{a_{k,j}}{a_{j,j}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

Save



tridiagonal:	$\begin{bmatrix} x & * & 0 & 0 & 0 & 0 \\ * & x & * & 0 & 0 & 0 \\ 0 & * & x & * & 0 & 0 \\ 0 & 0 & * & x & * & 0 \\ 0 & 0 & 0 & * & x & 0 \\ 0 & 0 & 0 & 0 & * & x \end{bmatrix}$
quidiagonal:	$\begin{bmatrix} x & * & 0 & 0 & 0 & 0 \\ * & x & * & 0 & 0 & 0 \\ 0 & * & x & * & 0 & 0 \\ 0 & 0 & * & x & * & 0 \\ 0 & 0 & 0 & * & x & 0 \\ 0 & 0 & 0 & 0 & * & x \end{bmatrix}$
quintidiagonal:	$\begin{bmatrix} x & * & 0 & 0 & 0 & 0 \\ * & x & * & 0 & 0 & 0 \\ 0 & * & x & * & 0 & 0 \\ 0 & 0 & * & x & * & 0 \\ 0 & 0 & 0 & * & x & 0 \\ 0 & 0 & 0 & 0 & * & x \end{bmatrix}$



Cancel



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, first relabel the vertices as follows:

$1 \rightarrow 5, 2 \rightarrow 3, 3 \rightarrow 4, 4 \rightarrow 6, 6 \rightarrow 5.$

reordering (simultaneously) the equations and variables.

$\{(v_i, j)\}_{i,j=1}^n \subseteq \mathbb{S}$  is called a path joining the vertices  $v_i$  and  $v_j$ , and for every  $k = 1, 2, \dots, n-1$  the set  $\{v_i, j_k\} \cap$

contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that  $\mathbb{G}$  is a tree if each two members of  $\mathbb{V}$  are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees. This is not the case with either quidiagonal or cyclic matrices when  $n \geq 3$ .

Let a tree  $\mathbb{T}$  and an arbitrary vertex  $r \in \mathbb{V}$ , the pair  $\mathbb{T} = (Q, r)$  is called a rooted tree, while  $r$  is said to be the root. Unlike in ordinary graph,  $\mathbb{T}$  admits a natural partial order, which can best be explained by an analogy with a family tree. Thus, the root  $r$  is the predecessor of all the vertices in  $\mathbb{V} \setminus \{r\}$  and these vertices are successors of  $r$ . Moreover, every  $\alpha \in \mathbb{V} \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex along this path, except for  $r$  and  $\alpha$ , as a predecessor of  $\alpha$  and a successor of  $r$ . We say that the rooted tree  $\mathbb{T}$  is monotonically ordered if each vertex is labelled before all its predecessors. In other words, if label the vertices from the root of the

'[data-test-button="Save"]'

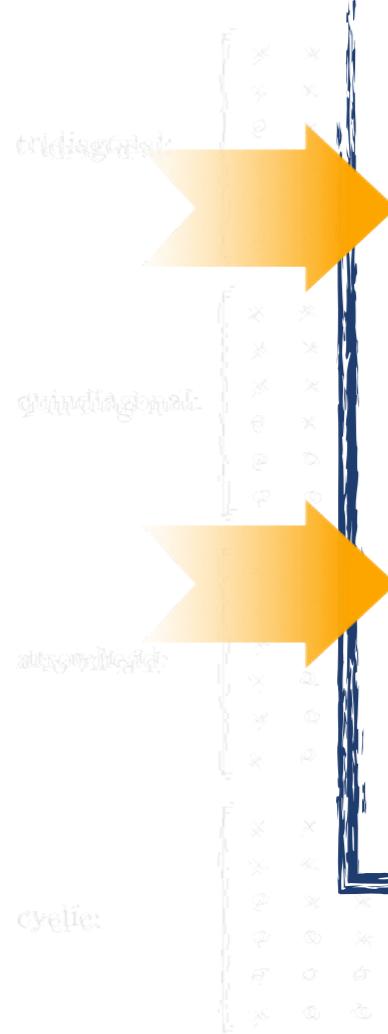
'[data-test-button="Cancel"]'

'[data-test-button="Add item"]'

just displayed, but its graph.

$$t_{k,j} = \frac{a_{k,j}}{a_{jj}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



## Name\*

## Description

permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three consecutive orderings of the sample rooted tree.

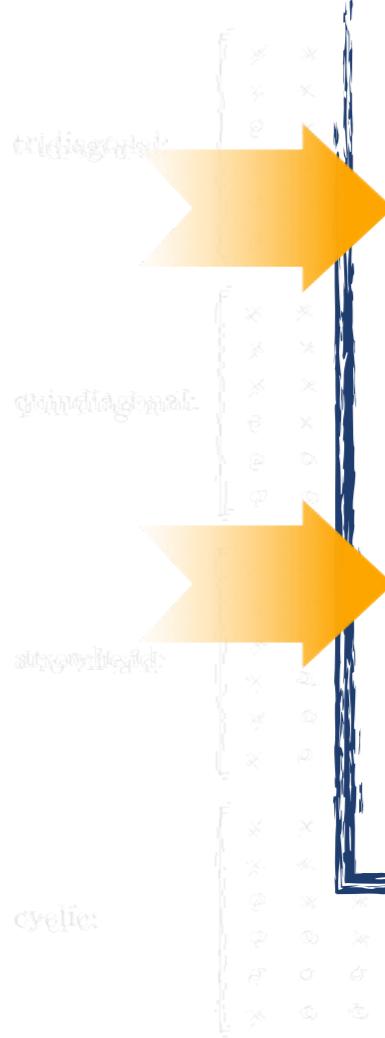
```
[data-test-field="Name"]
```

```
[data-test-field="Description"]
```

just displayed, but its graph.

$$t_{k,j} = \frac{a_{kj}}{a_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1, \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



Name

Little Bobby Tables

Description

Better not drop me!

matrix in disguised. To see  
 $6 \Rightarrow 3$ .  
equations and vegetables.  
th joining the vertices of  
 $\{v_1, v_2, \dots, v_p\} \cap V$  in  
ch if it does not visit any  
numbers of  $V$  are joined by  
vertices correspond to trees.  
atries when  $p \geq 3$ .  
-  $(G, \alpha)$  is called a rooted  
"admits a natural partial  
a family tree. Thus, the  
se vertices are *successors*  
e path and we designate  
son of  $\alpha$  and a *successor*  
if each vertex is labelled  
ices from the top of the  
graph is tantamount to  
permuting the rows and the columns of the underlying matrix.)  
Every rooted tree will be monotonically ordered and, in general, such an ordering  
is not unique. We now give three monotonic orderings of the sample rooted tree.

'[data-test-field="Name"]'

'[data-test-field="Description"]'

just displayed, but its graph.

$$t_{k,j} = \frac{a_{kj}}{a_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix



At a first glance, there is nothing to link it to any of the four structures just displayed, but its graph,

# WRITE LESS WITH THEOREMS AND NEW TERMS

**Theorem 11.1.** Let  $A$  be a  $n \times n$  matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, n\}$  and assume that the rows and columns of  $A$  are then arranged so, that  $T = (G, r)$  is monotonically ordered. Then, that  $A = L U$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{k,j}}{l_{j,j}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

triangular:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



quadrangular:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



supernodal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



cyclic:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



IF  $f$  IS CONTINUOUS IN  $[a, b]$ , AND IF  $f(a)f(b) < 0$ ,

THEN  $f$  MUST HAVE A

ZERO IN  $(a, b)$ .



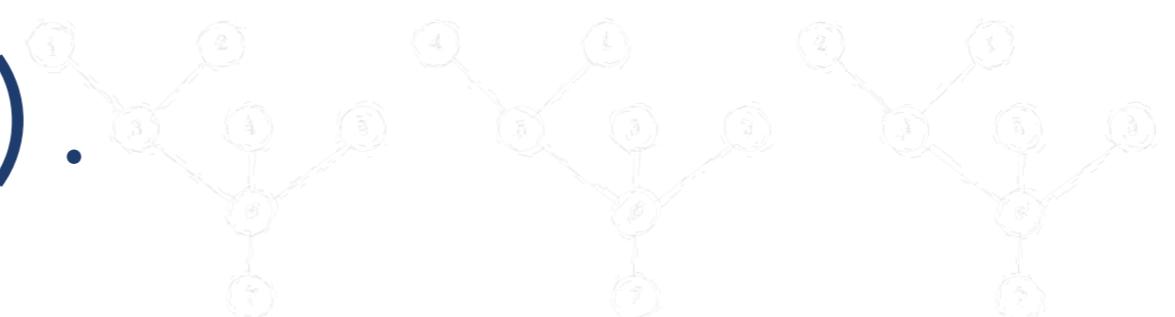
tells a different story - it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 2, 3 \rightarrow 3, 4 \rightarrow 4, 5 \rightarrow 5, 6 \rightarrow 3.$$

Of course, it is equivalent to replacing (arbitrarily) the equations and variables. As you can see from the figure, the graph  $G$  is called a tree, since the vertices  $a$  and  $b$  ( $a \in V \setminus \{b\}$ ,  $b \in V \setminus \{a\}$ ) and for every  $k = 1, 2, \dots, n-1$  the set  $\{v_1, v_2, \dots, v_k\} \cap \{v_{k+1}, v_{k+2}, \dots, v_{n-1}\}$  contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that  $G$  is a tree if each two members of  $V$  are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees, but this is not the case with either of banded or cyclic matrices when  $n \geq 3$ .

Of course, a rooted tree is a tree with a root vertex  $r$ . Unlike in a binary tree, there is a natural partial order which can best be explained by an analogy with a family tree. Thus, the root  $r$  is the predecessor of all the vertices in  $V \setminus \{r\}$  and these vertices are successors of  $r$ . Moreover, every  $v \in V \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex along this path, except for  $r$  and  $v$ , as a predecessor of  $v$  and a successor of  $r$ . We say that the rooted tree  $T$  is monotonically ordered if each vertex is labelled by an integer from  $1$  to  $n$  in such a way that the vertices from the top of the tree to the bottom are in increasing order. (In other words, we say the vertices from the top of the tree to the bottom are in increasing order. Labeling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree:



**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, n\}$  and assume that the rows and columns of  $A$  are then arranged so that  $T = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

At a first glance, this is not a very useful formula, but it is the basis for the following algorithm.

just displayed, but its graph:

Therefore we will now give a few examples of matrices (represented by their sparsity)

# PROOF. USE THE INTERMEDIATE VALUE THEOREM.

cyclic:

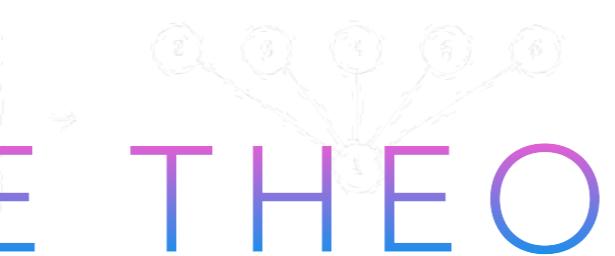
$$\begin{bmatrix} * & * & 0 & 0 & 0 \\ * & * & * & 0 & 0 \\ 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * \\ 0 & 0 & 0 & * & * \end{bmatrix}$$



$$\begin{bmatrix} * & * & 0 & 0 & 0 \\ * & * & * & 0 & 0 \\ 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * \\ 0 & 0 & 0 & * & * \end{bmatrix}$$



$$\begin{bmatrix} * & * & * & 0 & 0 \\ * & * & * & * & 0 \\ 0 & * & * & * & 0 \\ 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & * & * \end{bmatrix}$$



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & x & 0 \\ * & 0 & * & 0 & 0 & x \\ 0 & x & 0 & 0 & * & 0 \\ * & 0 & * & 0 & * & 0 \\ 0 & 0 & * & 0 & 0 & * \end{bmatrix}$$

At a first glance this is not a matrix, but it is a little known tree structure, just displayed, but its graph:



tells a different story - it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

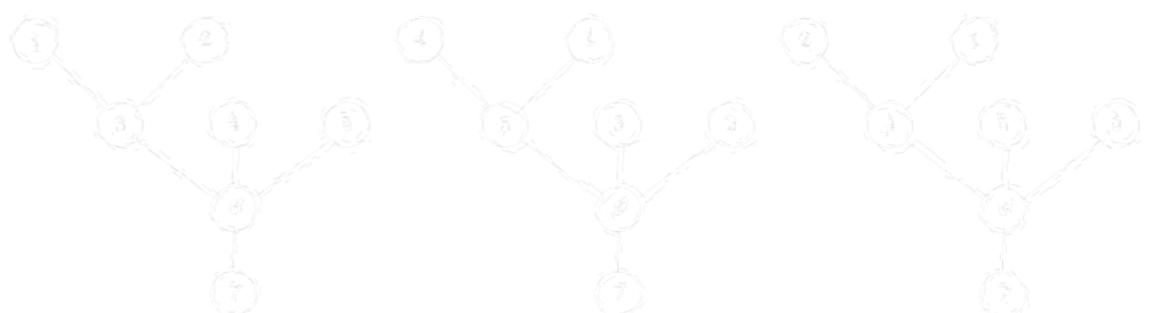
$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges  $\{(i_0, j_0)\}_{i_0, j_0 \in \mathbb{N}_0, i_0 \neq j_0}$  in  $\mathbb{G}$  is called a path joining the vertices  $i_0$  and  $j_0$  if  $i_0 \in \{i_0, j_0\}$ ,  $j_0 \in \{i_0, j_0\}$  and for every  $k = 1, 2, \dots, n-1$  the set  $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$  contains exactly one member. It is a simple path if it does not visit any vertices more than once. We say that  $\mathbb{G}$  is a tree if in two members of  $\mathbb{V}$  are joined by a unique simple path. Known tridiagonal and cyclic matrices correspond to trees, but this is not the case with either quadiagonal or cyclic matrices when  $n \geq 3$ .

Given a tree  $\mathbb{T}$  and an arbitrary vertex  $r \in \mathbb{V}$ , the pair  $\mathbb{T} = (\mathbb{G}, r)$  is called a rooted tree, while  $r$  is said to be the root. Unlike in ordinary graph,  $\mathbb{T}$  admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root is the ancestor of all the vertices in  $\mathbb{V} \setminus \{r\}$  and these vertices are successors of  $r$ . Moreover, every  $a \in \mathbb{V} \setminus \{r\}$  is joined to  $r$  by a simple path and we designate any vertex along this path, except for  $r$  and  $a$ , as a predecessor of  $a$  and a successor of  $r$ . We say that the rooted tree  $\mathbb{T}$  is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree:



**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $\mathbb{G}$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  are then arranged so, that  $\mathbb{T} = (\mathbb{G}, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

tridiagonal:  $\begin{bmatrix} * & * & 0 & 0 & 0 \\ * & * & * & 0 & 0 \\ 0 & 0 & * & * & 0 \\ 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & * \end{bmatrix}$

PROOF.

quasidiagonal:  $\begin{bmatrix} * & * & 0 & 0 & 0 \\ * & * & * & 0 & 0 \\ 0 & * & * & * & 0 \\ 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & * \end{bmatrix}$

graph:

USE IVT.

quasidiagonal:  $\begin{bmatrix} * & * & * & * & * \\ * & * & 0 & 0 & 0 \\ 0 & 0 & * & * & * \\ 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * \end{bmatrix}$

graph:

cyclic:  $\begin{bmatrix} * & 0 & 0 & 0 & * \\ * & * & 0 & 2 & * \\ 0 & * & * & * & 0 \\ 0 & 2 & * & * & 0 \\ 0 & 0 & 0 & * & * \end{bmatrix}$

graph:

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & 0 \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & * \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & * \end{bmatrix}$

At a first glance this is not a tridiagonal matrix, but it is a tridiagonal matrix, just displayed, but its graph:



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 3, \quad 5 \rightarrow 6, \quad 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges  $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$  is called a path joining the vertices  $i_0$  and  $j_0$  if  $i_0 \in \{i_0, j_0\}$ ,  $j_0 \in \{i_0, j_0\}$  and for every  $k = 1, 2, \dots, n-1$  the set  $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$  contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that  $G$  is a tree if each two members of  $V$  are joined by a unique simple path. Both tridiagonal and quasidiagonal matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when  $n \geq 3$ .

Given a tree  $T$  and an arbitrary vertex  $r \in V$ , the pair  $T = (G, r)$  is called a rooted tree, while  $r$  is said to be the root. Unlike in ordinary graph,  $T$  adapts a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root  $r$  is the predecessor of all the vertices in  $T \setminus \{r\}$  and these vertices are successors of  $r$ . Moreover, every  $\alpha \in V \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex along this path, except for  $r$  and  $\alpha$ , as a predecessor of  $\alpha$  and a successor of  $r$ . We say that the rooted tree  $T$  is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we lay out the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three consecutive orderings of the same rooted tree:

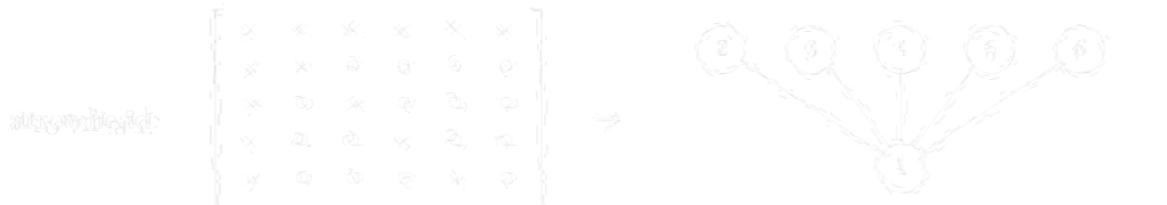
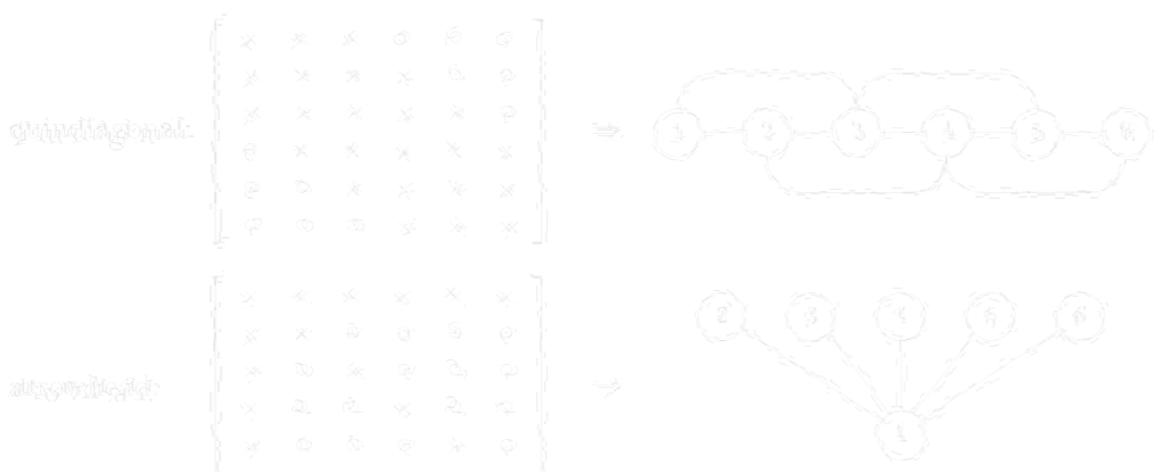


**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, n\}$  and assume that the rows and columns of  $A$  are then arranged so, that  $T = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

# THEOREM



# PROVEN

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

# TO BE TRUE

At a first glance this is not a symmetric matrix, but it is. It is a tridiagonal matrix, just displayed, but its graph,



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

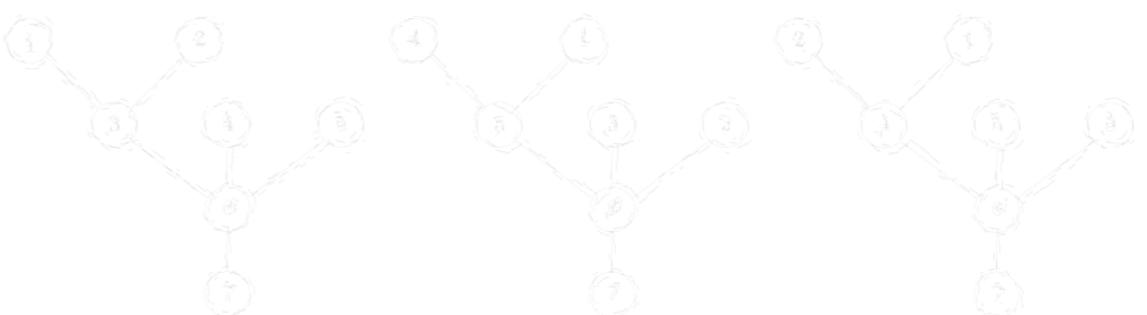
$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 3, \quad 5 \rightarrow 6, \quad 6 \rightarrow 3.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges  $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$  is called a path joining the vertices  $i_0$  and  $j_0$  if  $i_0 \in \{1, \dots, n\}$ ,  $j_0 \in \{1, \dots, n\}$  and for every  $k = 1, 2, \dots, n-1$  the set  $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$  contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that  $G$  is a tree if each two members of  $V$  are joined by a unique simple path. Both tridiagonal and quadiagonal matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when  $n \geq 3$ .

Given a tree  $T$  and an arbitrary vertex  $r \in V$ , the pair  $T = (G, r)$  is called a rooted tree, while  $r$  is said to be the root. Unlike in a ordinary graph,  $T$  adapts a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root  $r$  is the predecessor of all the vertices in  $T \setminus \{r\}$  and these vertices are successors of  $r$ . Moreover, every  $a \in V \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex along this path, except for  $r$  and  $a$ , as a predecessor of  $a$  and a successor of  $r$ . We say that the rooted tree  $T$  is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

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**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  are then arranged so, that  $T = (G, r)$  is monotonically ordered. Given that  $A = L U$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

```
import { fillIn } from '@ember/test-helpers';

export async function fillForm(fields) {
  for (const { label, value } of fields) {
    // input, textarea
    await fillIn(`[data-test-field="${label}"`, value);
  }
};
```

```
import { fillForm } from '../helpers/my-test-helper';

...
test('User can create account', async function(assert) {
  await visit('/signup');
  await fillForm([
    { label: 'Name', value: 'Little Bobby Tables' },
    { label: 'Email', value: 'little.bobby@gmail.com' }
  ]);
  ...
});
```

Therefore we will now give a few examples of matrices (represented by their sparsity)

tri-diagonal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & 0 & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix} \Rightarrow \text{graph: } \begin{array}{c} 1 \\ \text{---} \\ 2 \text{---} 3 \text{---} 4 \text{---} 5 \text{---} 6 \end{array}$$

quindiagonal:

$$\begin{bmatrix} * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ 0 & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix} \Rightarrow \text{graph: } \begin{array}{c} 1 \\ \text{---} \\ 2 \text{---} 3 \text{---} 4 \text{---} 5 \text{---} 6 \end{array}$$

super-diagonal:

$$\begin{bmatrix} * & * & * & * & * & * \\ * & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \end{bmatrix} \Rightarrow \text{graph: } \begin{array}{c} 1 \\ \text{---} \\ 2 \text{---} 3 \text{---} 4 \text{---} 5 \text{---} 6 \end{array}$$

cyclic:

$$\begin{bmatrix} * & 0 & 0 & 0 & * & * \\ * & * & 0 & 0 & 0 & 0 \\ 0 & 0 & * & 0 & 0 & 0 \\ 0 & 0 & 0 & * & 0 & 0 \\ 0 & 0 & 0 & 0 & * & 0 \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix} \Rightarrow \text{graph: } \begin{array}{c} 1 \text{---} 2 \text{---} 3 \text{---} 4 \text{---} 5 \text{---} 6 \text{---} 1 \end{array}$$

**UBIQUITOUS**

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

**IDEA**

At a first glance this is not a matrix, but a graph. But it is a matrix, as we have just displayed, but its graph:



tells a different story - it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 8.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges  $\{(v_i, j_i)\}_{i=1}^n$  in  $\mathbb{G}$  is called a path joining the vertices  $v_i$  and  $j_i$  if  $i \in \{1, \dots, n\}$ ,  $j_i \in \{1, \dots, n\}$  and for every  $k = 1, 2, \dots, n-1$  the set  $\{v_i, j_i\} \cap \{v_{i+1}, j_{i+1}\}$  contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that  $\mathbb{G}$  is a tree if each two members of  $\mathbb{V}$  are joined by a unique simple path. Both tri-diagonal and super-diagonal matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when  $n \geq 3$ .

Given a tree  $\mathbb{T}$  and an arbitrary vertex  $r \in \mathbb{V}$ , the pair  $T = (G, r)$  is called a rooted tree, while  $r$  is said to be the root. Unlike in ordinary graph,  $T$  adapts a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root  $r$  is the predecessor of all the vertices in  $\mathbb{V} \setminus \{r\}$  and these vertices are successors of  $r$ . Moreover, every  $v \in \mathbb{V} \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex along this path, except for  $r$  and  $v$ , as a predecessor of  $v$  and a successor of  $r$ . We say that the rooted tree  $T$  is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three consecutive orderings of the same rooted tree:



**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $\mathbb{G}$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  are then arranged so, that  $T = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

triangular:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$

quadrangular:

supernodal:

cyclic:



IF  $f$  IS CONTINUOUS IN  $[a, b]$ , AND IF  $f(a)f(b) < 0$ ,

THEN  $f$  MUST HAVE A

ZERO IN  $(a, b)$ .



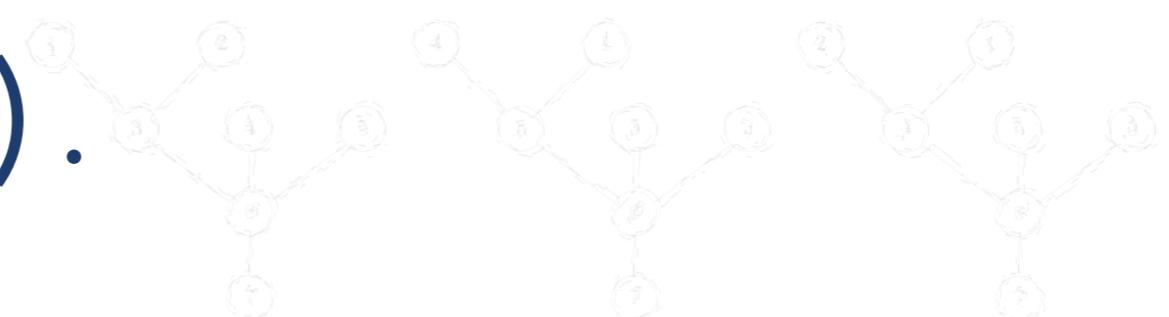
tells a different story - it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 3, 2 \rightarrow 5, 3 \rightarrow 6, 4 \rightarrow 3, 5 \rightarrow 4, 6 \rightarrow 3.$$

Of course, it is equivalent to reordering simultaneously the equations and variables. As a result, we get the following:  $G$  is called a tree if visiting the vertices  $\alpha$  and  $\beta$  ( $\alpha, \beta \in V \cup \{r\}$ ,  $\beta \in V \cup \{r\}$ ) and for every  $k = 1, 2, \dots, n-1$  the set  $\{v_k, \alpha\} \cap \{v_{k+1}, \beta\}$  contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that  $G$  is a tree if each two members of  $V$  are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees, but this is not the case with either of banded or cyclic matrices when  $n \geq 3$ .

As a result, a tree  $T = (G, r)$  is called a rooted tree, which is to be understood in the following sense. To date, a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root  $r$  is the predecessor of all the vertices in  $V \setminus \{r\}$  and these vertices are successors of  $r$ . Moreover, every  $v \in V \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex along this path, except for  $r$  and  $v$ , as a predecessor of  $v$  and a successor of  $r$ . We say that the rooted tree  $T$  is monotonically ordered if each vertex is labelled by an integer from  $1$  to  $n$  (in other words, we say the vertices from the top of the tree to the bottom, as we have already said). Labelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree:



**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  are then arranged so that  $T = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

At a first glance, this is nothing but the formula of the lower triangular matrix that was just displayed, but its graph.

Therefore we will now give a few examples of matrices (represented by their sparsity)

triangular:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



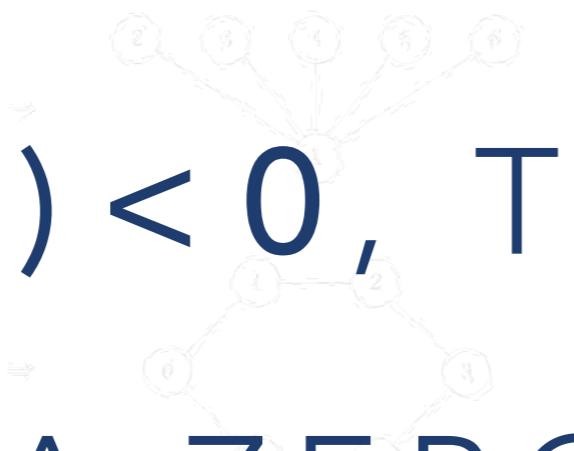
quadratic:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



symmetric:

$$\begin{bmatrix} * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ 0 & * & * & * & * & * \\ 0 & 0 & * & * & * & * \\ 0 & 0 & 0 & * & * & * \end{bmatrix}$$



cyclic:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



**IF  $f \in C([a, b])$ , AND  $f(a)f(b) < 0$ , THEN  $f$  MUST HAVE A ZERO IN  $(a, b)$ .**

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} * & * & * & 0 & * & 0 \\ 0 & * & 0 & * & * & 0 \\ * & * & * & 0 & 0 & * \\ 0 & 0 & 0 & * & * & * \\ * & * & 0 & 0 & * & * \\ 0 & * & * & * & * & * \end{bmatrix}$$



At a first glance, this is not a cyclic matrix, but it is. Because this matrix is not just displayed, but its graph.



tells a different story - it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges  $\{(i_0, j_0)\}_{i_0, j_0 \in \mathbb{N}_0, i_0 \neq j_0}$  in  $\mathbb{G}$  is called a path joining the vertices  $i_0$  and  $j_0$  if  $i_0, j_0 \in \{1, 2, \dots, n\}$ ,  $i_0 \neq j_0$ , and for every  $p = 1, 2, \dots, v-1$  the set  $\{i_p, j_p\} \cap \{i_{p+1}, j_{p+1}\}$  contains only one member. It is a simple path if it does not visit any vertex more than once. We say that  $\mathbb{G}$  is a tree if each two members of  $\mathbb{V}$  are joined by a unique simple path. Both quadiagonal and cyclic matrices correspond to trees, but this is not the case with either quadiagonal or cyclic matrices when  $n \geq 3$ .

Given a tree  $\mathbb{G}$  and an arbitrary vertex  $r \in \mathbb{V}$ , the pair  $T = (G, r)$  is called a rooted tree, while  $r$  is said to be the root. Unlike in a digraph,  $T$  admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the predecessor of the vertex  $v$  is the vertex  $u$  if  $v$  is an ancestor of  $u$  in the tree  $T$ . Note that every  $u \in \mathbb{V}$  is the ancestor of  $v$  in the path and we designate  $u$  as a parent along this path, except for  $r$ , which is a root, and  $v$  as a successor of  $r$ . We say that the rooted tree  $T$  is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we lay out the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

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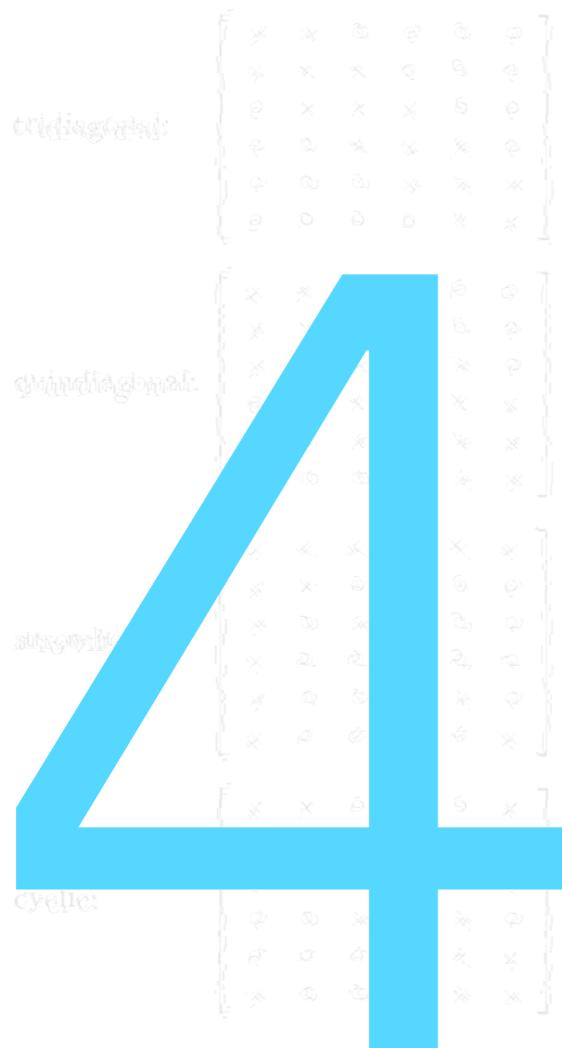


**Theorem 11.1** Let  $A$  be a symmetric matrix whose graph  $\mathbb{G}$  is a tree. Choose a root  $r \in \{1, 2, \dots, n\}$  and assume that the rows and columns of  $A$  are then arranged so that  $T = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

```
hooks.beforeEach(function(assert) {  
  ...  
  // Example: assert.isEnabled('Submit', 'Woot!');  
  assert.isEnabled = (label, message) => {  
    assert.dom(`[data-test-button="${label}"]`)  
      .doesNotHaveAttribute('disabled', message);  
  };  
  ...  
});
```

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & * \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & * \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & * \end{bmatrix}$$

At a first glance, there is nothing to link it to any of the four structures just displayed, but its graph,

# ALL YOUR BASIS ARE BELONG TO US



Let us now let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  are then arranged so that  $T = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{kj} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

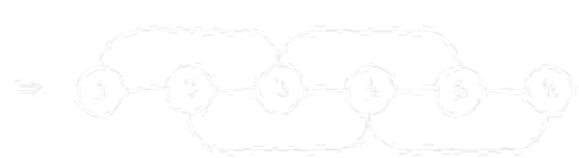
triangular:  

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$



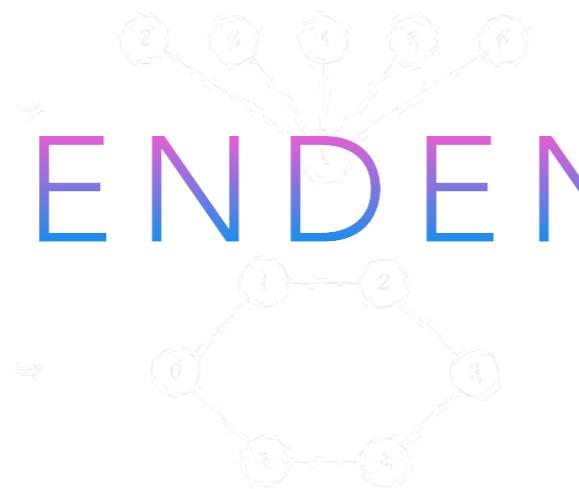
quidiagonal:  

$$\begin{bmatrix} * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$



acyclic:  

$$\begin{bmatrix} * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$



THAT

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

SPAN THE ENTIRE SPACE

At a first glance, this is not a matrix that looks like the basis matrices that we have just displayed, but its graph:



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges  $\{(v_i, j_i)\}_{i=1}^n$  in  $\mathbb{G}$  is called a path joining the vertices  $v_i$  and  $v_j$  if  $i \in \{1, \dots, n\}$ ,  $j \in \{1, \dots, n\}$  and for every  $k = 1, 2, \dots, n-1$  the set  $\{v_i, j_k\} \cap \{v_{k+1}, j_{k+1}\}$  contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that  $\mathbb{G}$  is a tree if each two members of  $V$  are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when  $n \geq 3$ .

Given a tree  $\mathbb{G}$  and an arbitrary vertex  $r \in V$ , the pair  $T = (G, r)$  is called a rooted tree, while  $r$  is said to be the root. Unlike in ordinary graph,  $T$  admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root  $r$  is the only vertex that has no predecessors and all other vertices are successors of  $r$ . Moreover, every  $v \in V \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex alone, throughout, except for  $r$  and  $v$ , as a predecessor of  $v$  and a successor of  $r$ . We say that the rooted tree  $T$  is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree:



**Theorem 11.1** Let  $A$  be a symmetric matrix whose graph  $\mathbb{G}$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  were then arranged so that  $T = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph.

tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 3, 3 \rightarrow 2, 4 \rightarrow 4, 5 \rightarrow 6, 6 \rightarrow 3.$$

Of course, it is equivalent to renumbering (cyclic shift) the vertices and to consider a different set of edges  $\{ (i, j) \mid i, j \in \{1, 2, \dots, 6\} \}$ . It is called a *cycle* if the vertices  $i, j \in \{1, 2, \dots, n\}$  and for every  $\ell = 1, 2, \dots, n-1$  the set  $\{i + \ell, j + \ell\}$  contains exactly one member. It is a *simple path* if it does not visit any vertex more than once. We say that  $G$  is a *tree* if each two members of  $V$  are joined by a unique path. Both tridiagonal and antidiagonal matrices correspond to the case with either quidiagonal or cyclic matrices when  $\ell = 1, 2, \dots, n-1$  and no arbitrary vertex  $v \in V \setminus \{r\}$  is called a *root* and  $r$  is called the *root*. Unlike in a ordinary graph,  $T$  admits a natural partial order that must be explained by an analogy with a family tree. Thus, the ancestor of all the vertices in  $T \setminus \{r\}$  and these vertices are successive. Every  $a \in V \setminus \{r\}$  is joined to  $r$  by a simple path and we designate this path, except for  $r$  and  $a$ , as a *predecessor* of  $a$  and a *successor* of  $r$ . The rooted tree  $T$  is *monotonically ordered* if each vertex is labelled with a natural number in other words, we label the vertices from the top of the tree to the bottom (we have already done it, relabeling a graph to obtain a tree). This is another way to give three examples of the same concept.



**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  are then arranged so, that  $T = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{j,j}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 3, 3 \rightarrow 2, 4 \rightarrow 4, 5 \rightarrow 6, 6 \rightarrow 8.$$

Of course, it is equivalent to renumbering (cyclic shift) the vertices and the corresponding set of edges  $\{ (i, i+1) \mid i \in \{1, 2, \dots, 6\} \}$ . It is called a *cycle* joining the vertices  $i \in \{1, 2, \dots, 6\}$ ,  $i \neq i+1$ , and for every  $i = 1, 2, \dots, 6$  the set  $\{i, i+1\}$  contains exactly one member. It is a *simple path* if it does not visit any vertex more than once. We say that  $G$  is a *tree* if each two members of  $V$  are joined by a *simple path*. Both tridiagonal and cyclic matrices correspond to trees. There are two types of trees: with either quidditonic or cyclic number of vertices  $n \geq 3$ . A *quidditonic tree* is a tree with an arbitrary vertex  $r \in V$  (the root). It is called a *rooted tree* and  $r$  is said to be the *root*. Unlike in a quidditonic graph,  $T$  admits a natural partial order that must be explained by an analogy with a family tree. Thus, the *ancestor* of all the vertices in  $T \setminus \{r\}$  and these vertices are *successors*. Every  $a \in V \setminus \{r\}$  is joined to  $r$  by a *simple path* and we designate this path, except for  $r$  and  $a$ , as a *predecessor* of  $a$  and a *successor*. The rooted tree  $T$  is *monotonically ordered* if each vertex is labelled with its *depth* in other words, we label the vertices from the top of the tree to the bottom (we have already done it, relabeling a graph to a quidditonic one). This is the only difference between a quidditonic and a cyclic tree. We will give three examples of the simple rooted trees.

**LATITUDE**

$30.267^\circ$

**LONGITUDE**

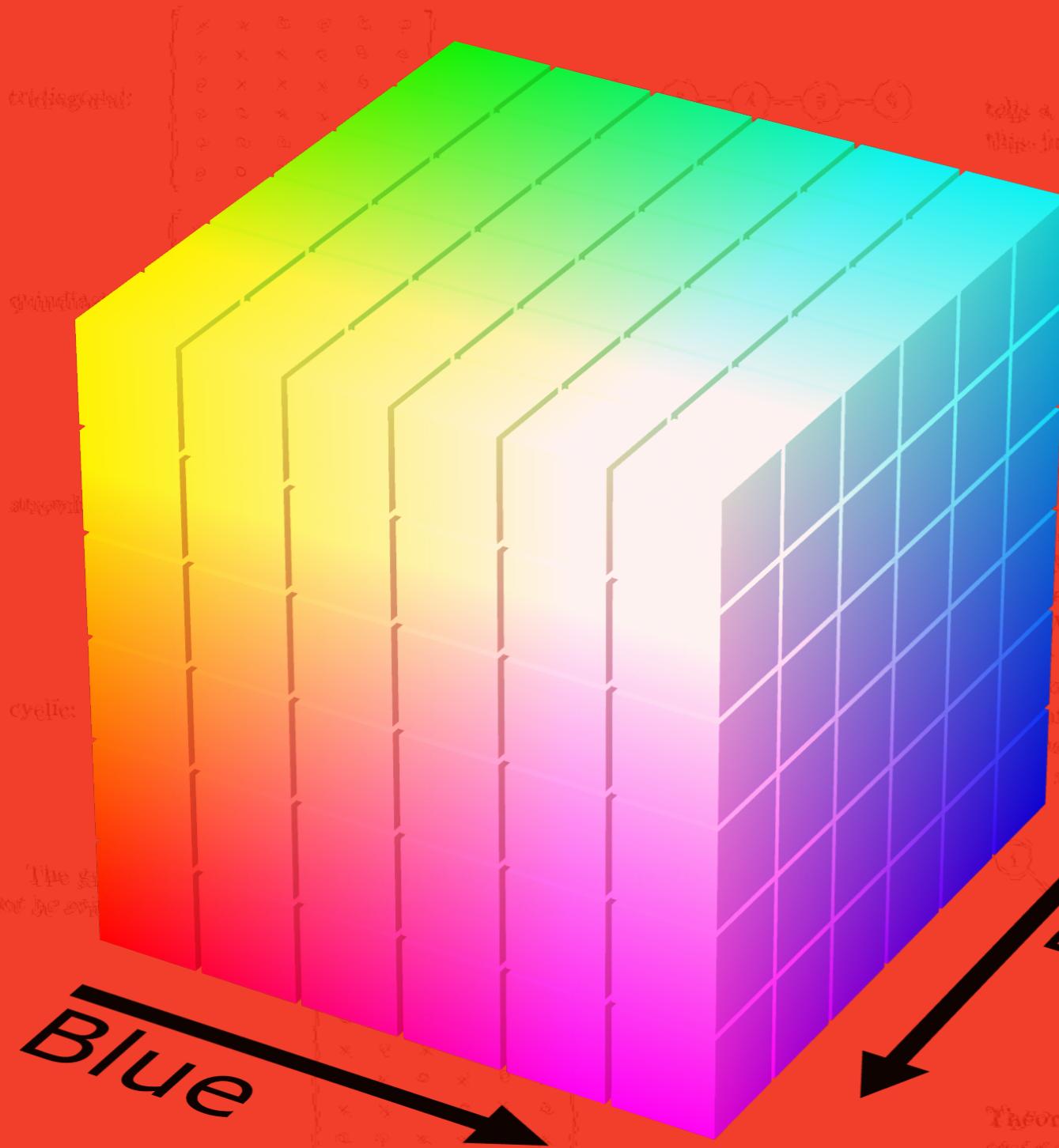
$-97.743^\circ$



**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  are then arranged so that  $T = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



At a first glance, there is nothing to link in to any of the four matrices that we have just displayed, but its graph.



RED

224

tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, first relabel the vertices as follows:

$$0 \rightarrow 1, 2 \rightarrow 3, 3 \rightarrow 2, 4 \rightarrow 5, 5 \rightarrow 6, 6 \rightarrow 4$$

course, is equivalent to reordering (simultaneously) the columns and rows. A *partial set of steps*  $\{(v_i, j_i)\}_{i=1}^n \subseteq \mathbb{E}$  is called a *path* joining the vertices  $v_i$  and  $v_j$  if  $(v_i, j_i) \in \mathbb{E}$ ,  $j_i \in \{i, j\}$ ,  $i \in \{1, n\}$  and for every  $k = 1, 2, \dots, n-1$  the set  $\{v_k, j_k\} \cap \{(v_i, j_i)\}$  contains exactly one member. It is a *simple path* if it does not visit any vertex more than once. We say that  $G$  is a *tree* if each two members of  $V$  are joined by a simple path. Both tri-diagonal and quasi-diagonal matrices correspond to trees. This is not the case with either quidiagonal or cyclic matrices when  $n \geq 3$ .

Let's start with an arbitrary vertex  $r \in V$ . The set  $T = \{v \in V \mid v \text{ is joined to } r \text{ by a simple path}\}$  is called a *rooted tree* with  $r$  added to be the root. Unlike a ordinary tree,  $T$  does not have a unique root, which can best be explained by an example of a family tree. This is the set of all the vertices in  $V \setminus \{r\}$  and these vertices are *successors* of  $r$ . Vertex  $\alpha \in V \setminus \{r\}$  is joined to  $r$  by a simple path and we designate  $\alpha$  as an *ancestor* of  $r$  and  $r$  as a *descendant* of  $\alpha$  and a *successor* of  $r$ . We say that a rooted tree  $T$  is *monotonically ordered* if each vertex is labelled with a unique integer  $\ell(v)$  in other words we label the vertices from the root  $r$  to the bottom. (We have already said it, relabelling a graph is the same as relabelling the rows and the columns of the underlying matrix.)

every vertex then will be monotonically ordered and, in general, such an ordering is called a *topological ordering*. We will give three consecutive orderings of the same rooted tree.

Red  
Green  
Blue



GREEN

78

BLUE

57

**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, n\}$  and assume that the rows and columns of  $A$  are renumbered so, that  $T = (G, r)$  is monotonically ordered. Then, that  $A = L U$  is a Cholesky factorization. It is true that

$$a_{k,j} = \frac{a_{k,j}}{a_{j,j}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

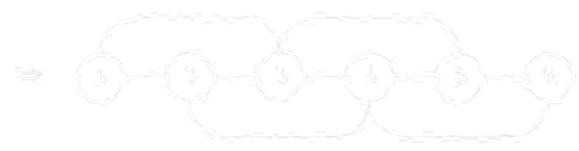
and their graphs

tridiagonal: 
$$\begin{bmatrix} * & * & 0 & 0 & 0 \\ * & * & * & 0 & 0 \\ 0 & * & * & * & 0 \\ 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & * & * \end{bmatrix}$$



BASIS

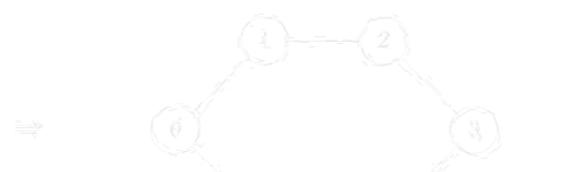
quindiagonal:

$$\begin{bmatrix} * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ 0 & * & * & * & * & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$


superdiagonal:

$$\begin{bmatrix} * & * & * & * & * & * \\ * & * & 0 & 0 & 0 & 0 \\ 0 & * & * & 0 & 0 & 0 \\ 0 & 0 & * & * & 0 & 0 \\ 0 & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$


cyclic:

$$\begin{bmatrix} * & 0 & 0 & 0 & * & * \\ * & * & 0 & 2 & * & * \\ 0 & * & * & * & * & 0 \\ 0 & 2 & * & * & * & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$


BUILDING BLOCKS

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

OF TESTS

At a first glance, this is not a matrix, but a collection of little boxes containing numbers just displayed, but its graph:



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges  $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$  is called a path joining the vertices  $i_0$  and  $j_0$  if  $i_0 \in \{1, \dots, n\}$ ,  $j_0 \in \{1, \dots, n\}$  and for every  $k = 1, 2, \dots, n-1$  the set  $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$  contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that  $G$  is a tree if each two members of  $V$  are joined by a unique simple path. Both tridiagonal and superdiagonal matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when  $n \geq 3$ .

Given a tree  $T$  and an arbitrary vertex  $r \in V$ , the pair  $T = (G, r)$  is called a rooted tree, while  $r$  is said to be the root. Unlike in a ordinary graph,  $T$  admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root  $r$  is the predecessor of all the vertices in  $T \setminus \{r\}$  and these vertices are successors of  $r$ . Moreover, every  $\alpha \in V \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex along this path, except for  $r$  and  $\alpha$ , as a predecessor of  $\alpha$  and a successor of  $r$ . We say that the rooted tree  $T$  is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is unique. The following diagram shows three consecutive orderings of the same rooted tree.



**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  were been arranged so, that  $T = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



tells a different story - it is nothing other than the cyclic matrix in disguised. To see this, just replace the weights as follows:



Create Edit Delete Clone Import

[[\*, 0, 0, 0, 0, 0], [0, \*, 0, 0, 0, 0], [0, 0, \*, 0, 0, 0], [0, 0, 0, \*, 0, 0], [0, 0, 0, 0, \*, 0], [0, 0, 0, 0, 0, \*]]

A single simple path. Both tridiagonal and any banded matrices correspond to trees, but this is not the case with either anti-diagonal or cyclic matrices when  $n > 3$ .

	Name	Description
<input checked="" type="checkbox"/>	Little Bobby Tables	Better not drop me!
<input type="checkbox"/>	Big Bobby Tables	
<input type="checkbox"/>	Foo Bar	Making up names is hard.



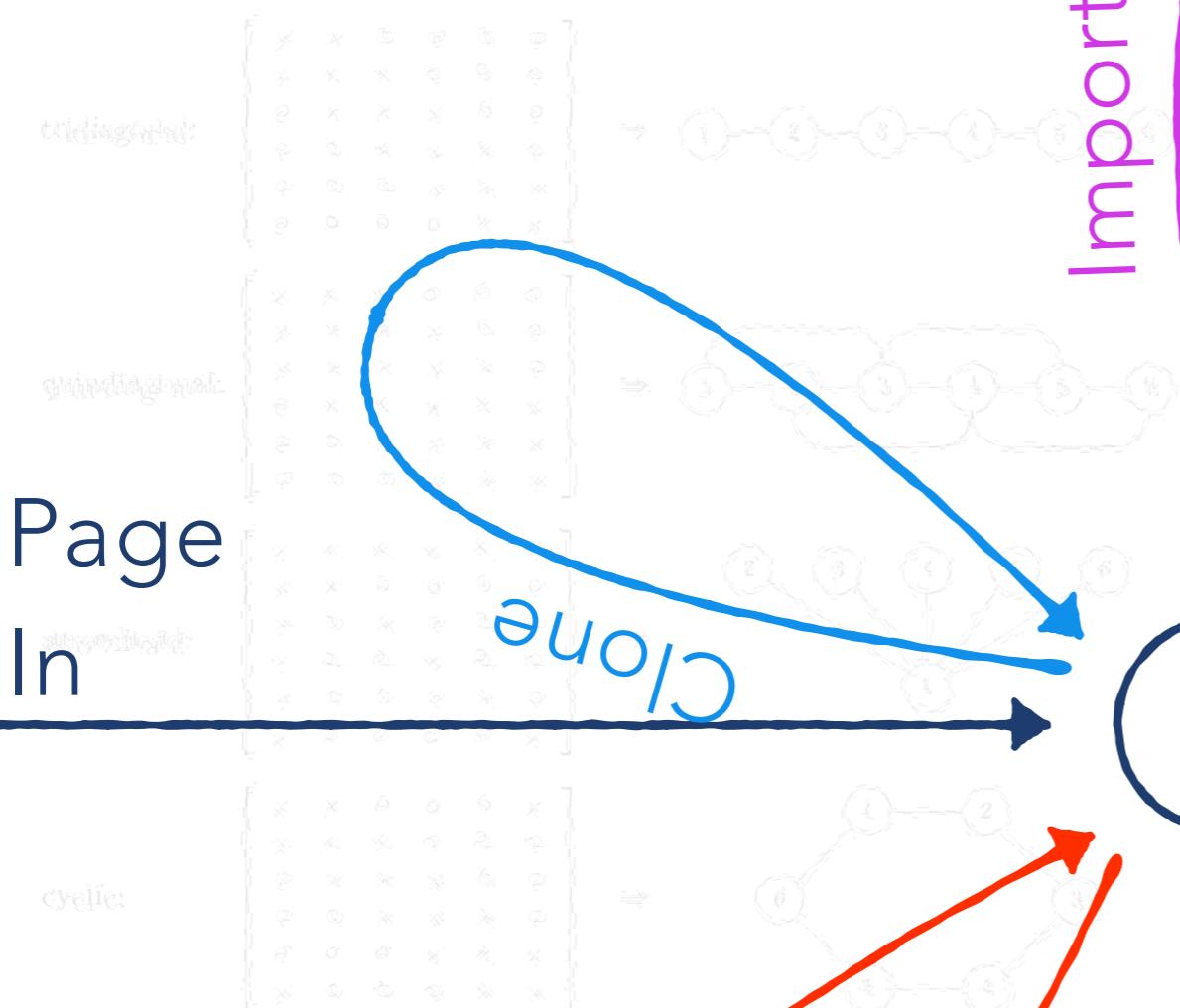
At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph:



**Theorem 11.1** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  are then arranged so, that  $\mathbb{T} = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,

tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just read the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 1.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges  $\{(v_i, j)\}_{i,j}$ ,  $\mathbb{S} \subseteq \mathbb{V} \times \mathbb{V}$  is called a path joining the vertices  $v_i$  and  $v_j$  if  $i, j \in \{1, \dots, n\}$ ,  $\mathbb{S} \subseteq \mathbb{V} \times \mathbb{V}$  and for any  $k = 1, 2, \dots, n-1$  the set  $\{v_i, j\} \cap \{v_{i+k}, j+k\}$  contains exactly one vertex. It is a simple path if it does not visit any vertex more than once. We say that  $G$  is a tree if none of members of  $\mathbb{V}$  are joined by a single simple path. But tridiagonal and quadiagonal matrices contain cycles. Note this is not the case with either quadiagonal or cyclic matrices when

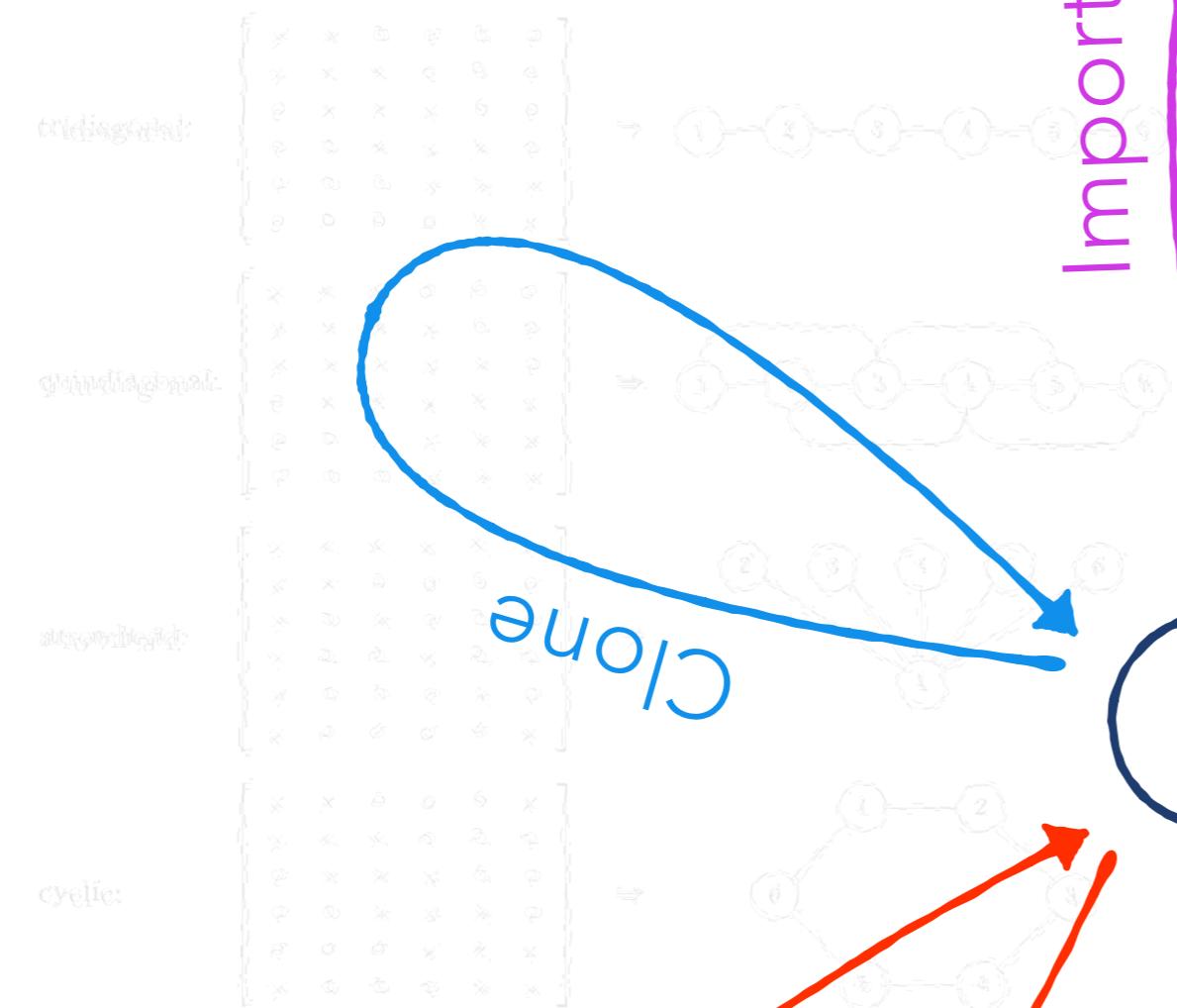
Create a tree  $T$  and a root vertex  $r \in \mathbb{V}$ , the pair  $T \sim (r, \mathbb{V})$  is called a rooted tree, while  $r$  is said to be the root. Unlike in a ordinary graph,  $T$  has its own partial ordering which can best be explained by an analogy with a family tree. The root  $r$  is the predecessor of all the vertices in  $T \setminus \{r\}$  and these vertices are the successors of  $r$ . The vertex  $v$  is the predecessor of  $u$  if  $v$  is an ancestor of  $u$  and no other vertex along this path, except  $v$  and  $u$ , is a predecessor of  $v$  and a successor of  $u$ . We say that the rooted tree  $T$  is monotonically ordered if each vertex is labelled before all its predecessors in other words we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to relabelling the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We will give three consecutive orderings of the same rooted tree.

Theorem 11.1. Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, n\}$  and assume that the rows and columns of  $A$  are then arranged so, that  $T = (r, \mathbb{V})$  is monotonically ordered. Then, that  $A = LL^T$  is a Cholesky factorization it is true that

$$l_{k,j} = \frac{a_{k,j}}{l_{j,j}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,

tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just read the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 3, 3 \rightarrow 2, 4 \rightarrow 5, 5 \rightarrow 6, 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges  $\{(i_0, j_0), (i_1, j_1), \dots, (i_{n-1}, j_{n-1})\}$  is called a path joining the vertices  $i_0$  and  $i_{n-1}$  ( $i_0, i_1, \dots, i_{n-1}, i_0 \in V$ ),  $j_0, j_1, \dots, j_{n-1} \in \{1, 2, \dots, n\}$  and for any  $k = 1, 2, \dots, n-1$  the set  $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$  contains exactly one vertex. It is a simple path if it does not visit any vertex more than once. We say that  $G$  is a tree if no two members of  $V$  are joined by a single simple path. Both triangular and pentangular matrices correspond to trees, but this is not the case with either quadrangular or cyclic matrices when  $n \geq 3$ .

Create a tree  $T$  and a arbitrary vertex  $r \in V$ , the pair  $T \sim (r, r)$  is called a rooted tree, while  $r$  is said to be the root. Unlike in a ordinary graph,  $T$  has a natural partial order, which can best be explained by an analogy with a family tree. Thus, the vertex  $r$  is the predecessor of all the vertices in  $T \setminus \{r\}$  and these vertices are successors of  $r$ . Moreover, every  $a \in V \setminus \{r\}$  is joined to  $r$  by a simple path and we designate  $a$  as a predecessor of  $r$  and  $r$  as a predecessor of  $a$  and a successor of  $a$  along this path, except in  $r$  and  $a$ , as a predecessor of  $a$  and a successor of  $a$ . We say that the rooted tree  $T$  is monotonically ordered if each vertex is labelled with all its predecessors in other words, we label the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to relabelling the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We will give three consecutive drawings of the same rooted tree.



**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, n\}$  and assume that the rows and columns of  $A$  are now arranged so, that  $T = (r, r)$  is monotonically ordered. Then, that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{k,j}}{l_{j,j}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

Page  
In

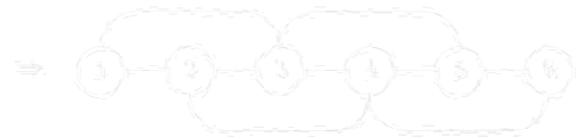
quidiagonal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 & 0 \\ 0 & * & * & 0 & 0 & 0 \\ 0 & 0 & * & * & 0 & 0 \\ 0 & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



quidiagonal:

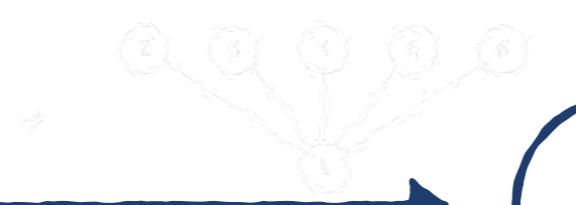
$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$



Page  
Out

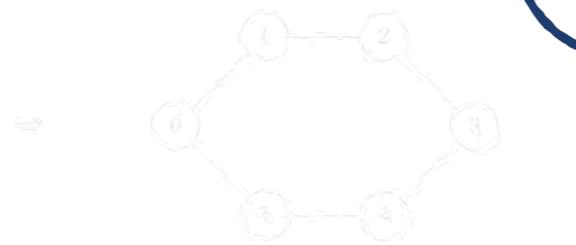
triangular:

$$\begin{bmatrix} * & * & * & * & * & * \\ * & * & 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$



cyclic:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & * \\ * & * & 0 & 0 & 0 & 0 \\ 0 & * & * & 0 & 0 & 0 \\ 0 & 0 & * & * & 0 & 0 \\ 0 & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & 0 \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & 0 \\ * & 0 & * & 0 & * & 0 \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 4.$$

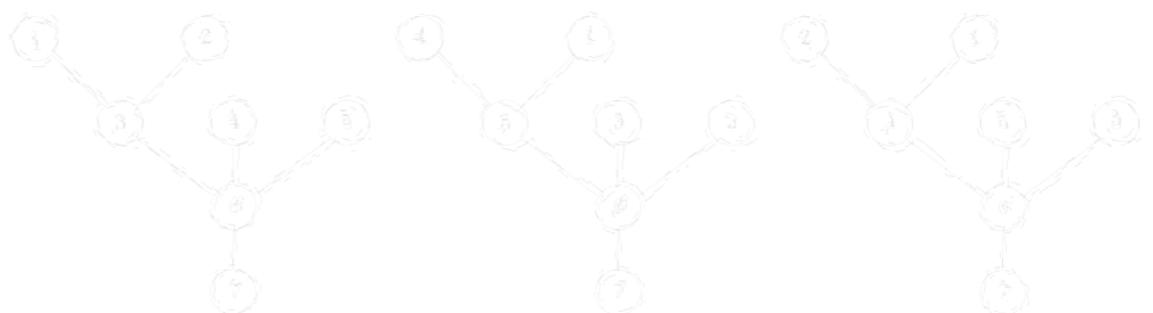
This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges  $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$  is called a path joining the vertices  $i_0$  and  $j_0$  if  $i_0 \in \{i_0, j_0\}$ ,  $j_0 \in \{i_0, j_0\}$  and for every  $k = 1, 2, \dots, n-1$  the set  $\{i_{k+1}, j_{k+1}\} \cap \{i_k, j_k\}$  contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that  $G$  is a tree if each two members of  $V$  are joined by a unique simple path. Both tri-diagonal and any other matrices  $A$  are not trees, but this is not the case with either quidiagonal or cyclic matrices.

Given a tree  $T$  and an arbitrary vertex  $r \in V$ , the pair  $T = (G, r)$  is called a rooted tree, while  $r$  is said to be the root. Unlike in ordinary graph,  $T$  admits a natural partial order, which can best be explained by an analogy with a family tree. The vertex  $r$  is the predecessor of all the vertices in  $T \setminus \{r\}$  and these vertices are successors. Moreover, every  $\alpha \in V \setminus \{r\}$  has a unique predecessor  $\alpha'$  and an immediate vertex along this path, except for  $r$  and  $\alpha$ , as a predecessor of  $\alpha$  and a successor.

We say that the rooted tree  $T$  is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three consecutive orderings of the same rooted tree.

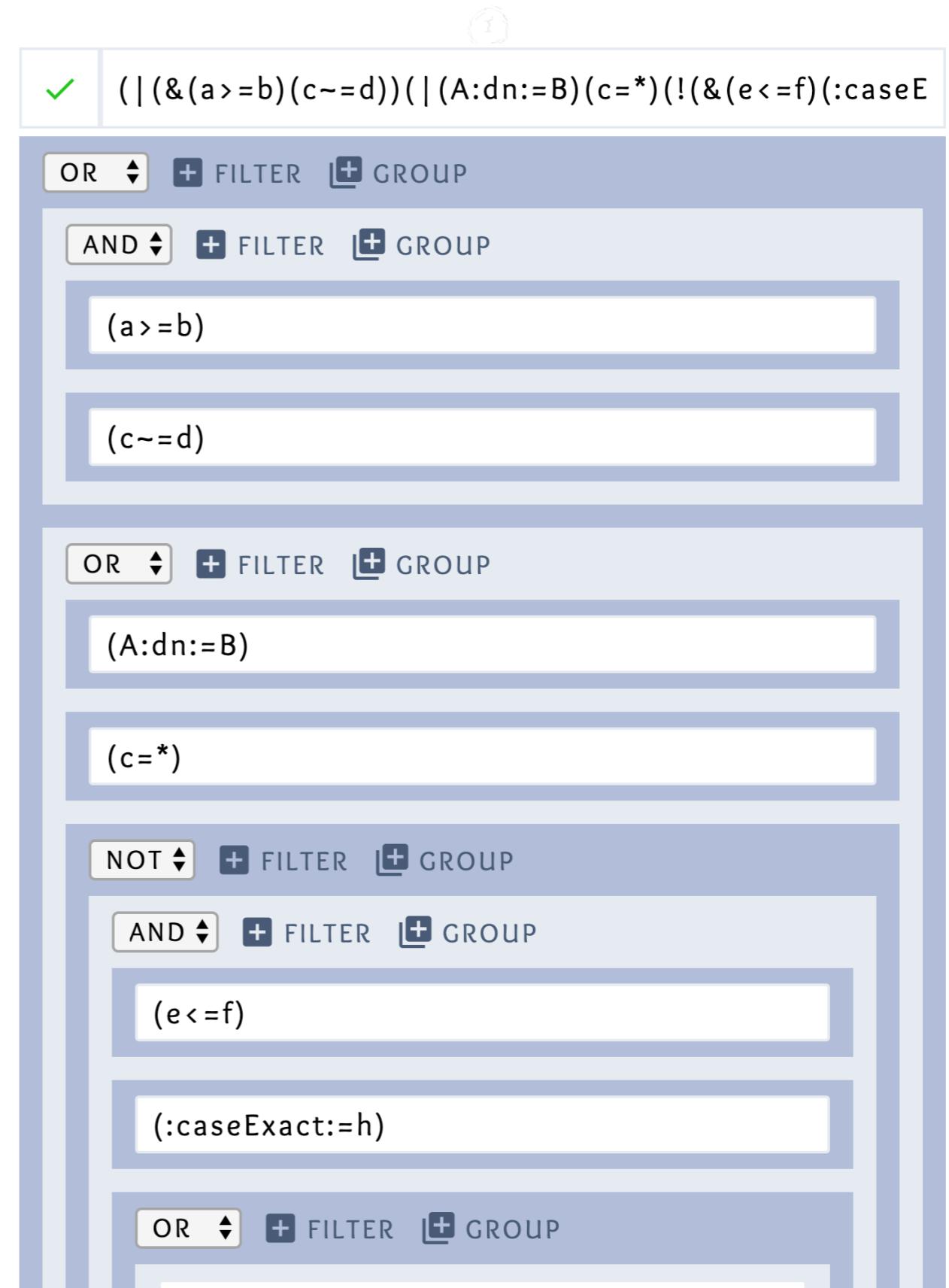


**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, n\}$  and assume that the rows and columns of  $A$  are then arranged so that  $T = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ki}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their ranks.

$(|(&(a>=b)(c\sim=d))$   
 $(| (A:dn:=B)(c=^*)$   
 $(!(&(e<=f)$   
 $(:caseExact:=h)$   
 $(| (i=j)(!(k<=l))))))))$



just displayed, but its graph.

$$t_{k,j} = \frac{a_{k,j}}{b_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1, \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

( | (! (a>=b) (c~d)))  
( | (A:dn := B) (c=\*))  
(!(&(e<=f))  
(:caseExact:=h))  
( | (i=j) (! (k<=l)))))))

! ( | (! (a>=b) (c~d))) ( | (A:dn := B) (c=\*)) (!(&(e<=f)) (:caseE>))

OR FILTER GROUP

NOT FILTER GROUP

You can negate only 1 filter in a group.

(a>=b)

(c~d)

You need to use ~=.

OR FILTER GROUP

(A:dn := B)

You need to trim the attribute and filter type.

(c=\*)

NOT FILTER GROUP

AND FILTER GROUP

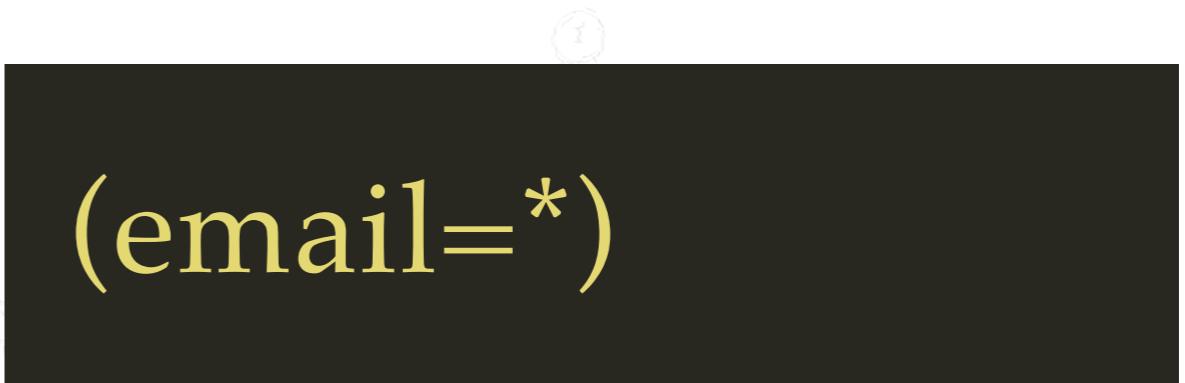
(e<=f)

(:caseExact:=h)

just displayed, but its graph.

$$t_{k,j} = \frac{a_{k,j}}{b_{j,j}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1, \quad (11.5)$$

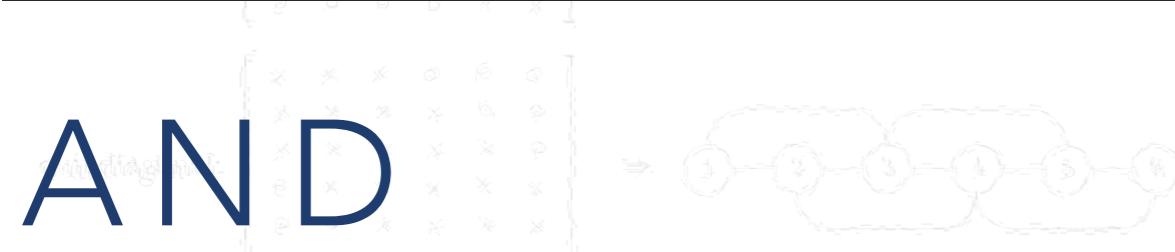
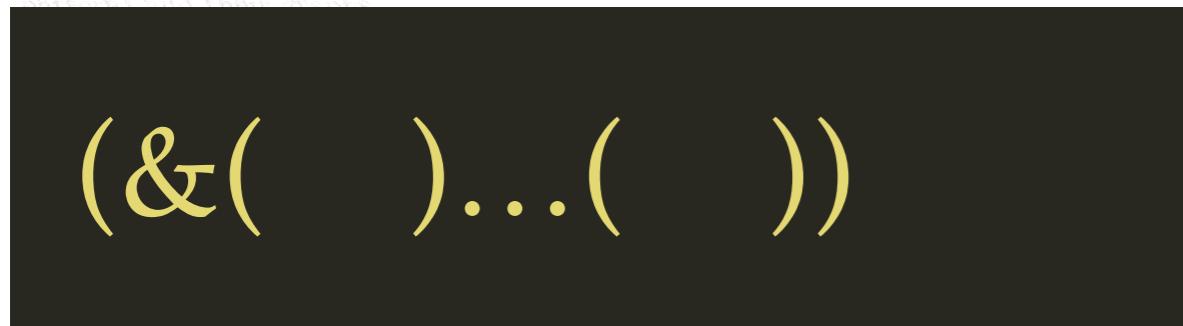
Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



**Theorem 11.1** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  are then arranged so that  $T = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ii}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

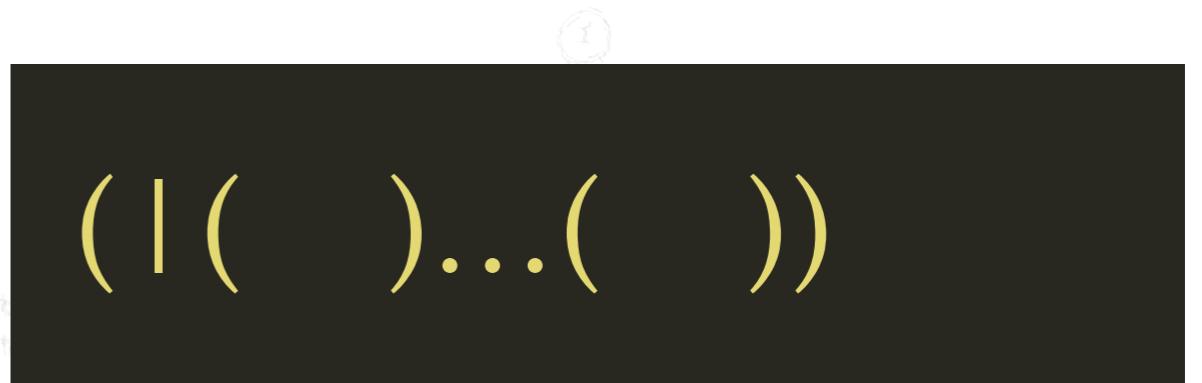
Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix



At a first glance, there is nothing to think in the case of the four matrices that we have just displayed, but its graph.



$1 \rightarrow 1, 2 \rightarrow 3, 3 \rightarrow 2, 4 \rightarrow 5, 5 \rightarrow 6, 6 \rightarrow 3$ .

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges  $\{(v_i, j_i)\}_{i=1}^n$  in  $\mathbb{G}$  is called a path joining the vertices  $v_i$  and  $v_j$  ( $i, j \in \{1, 2, \dots, n\}$ ,  $i \neq j$ ) if  $\{v_i, j_i\} \cap \{v_k, k_i\}$  contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that  $\mathbb{G}$  is a tree if each two members of  $\mathbb{V}$  are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when  $p \geq 3$ .

Given a tree  $\mathbb{G}$  and an arbitrary vertex  $r \in \mathbb{V}$ , the pair  $T = (G, r)$  is called a rooted tree, while  $r$  is said to be the root. Unlike in ordinary graph,  $T$  admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root  $r$  is the predecessor of all the vertices in  $\mathbb{V} \setminus \{r\}$  and these vertices are successors of  $r$ . Moreover, every  $v \in \mathbb{V} \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex along this path, except for  $r$  and  $v$ , as a predecessor of  $v$  and a successor of  $r$ . We say that the rooted tree  $T$  is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three consecutive orderings of the same rooted tree.



**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $\mathbb{G}$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  are then arranged so that  $T = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ki}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

```
function addOne(x) {  
  if (Number.isFinite(x)) {  
    x = x + 1;  
  } else {  
    console.log('error');  
  }  
  return x;  
}
```



tells a different story – it is mapping values from one cyclic matrix in disguise. To see this, first relate the variables as follows:

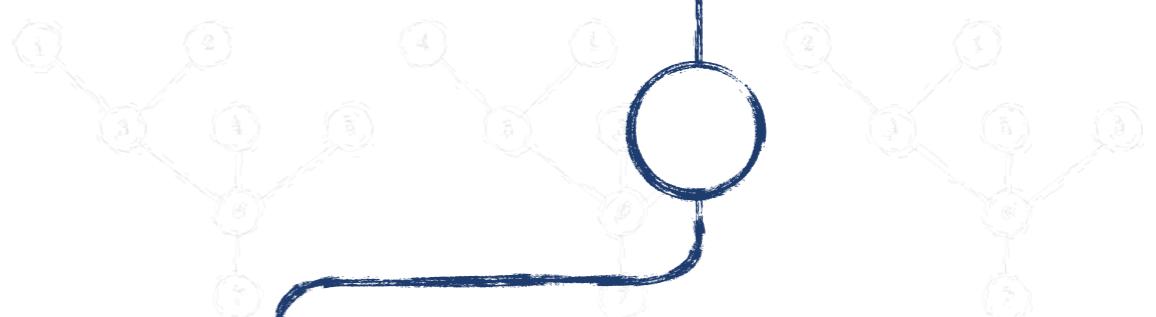
→  $3 \rightarrow 3$ ,  $3 \rightarrow 3$

This, of course, is equivalent to requiring (since  $\mathbf{A}$  is a  $n \times n$  matrix) that the equations and variables

An ordered set of edges  $\{(i_k, j_k)\}_{k=1}^n$  is a path joining the vertices  $i_1$  and  $j_n$  if  $i_k \in \{v, w\}$ ,  $j_k \in \{v, w\}$  and for every  $k = 1, 2, \dots, n-1$  the set  $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$  contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that  $G$  is a tree if each two members of  $V$  are joined by a unique simple path. Both tridiagonal and approach matrices correspond to trees, but this is not the case with either quindiajou or matrices when  $p > 3$ .

Given a tree  $T$  and an arbitrary vertex  $r$  in  $T$ , we say that  $T$  is *rooted* at  $r$ , while  $r$  is said to be the *root*. Unlike an *arbitrary* tree, a *rooted* tree  $T$  admits a natural ordering, which can best be explained by an example. Consider a family tree. The root  $r$  is the *predecessor* of all the vertices in  $V \setminus \{r\}$  and these vertices are *successors* of  $r$ . Moreover, every  $\alpha \in V \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex along this path, except for  $r$  and  $\alpha$ , as a *predecessor* of  $\alpha$  and a *successor* of  $r$ . We say that the rooted tree  $T$  is *monotonically ordered* if each vertex is labellable before all its predecessors; in other words, if we can label the vertices from the top of the tree to the root. (As we have already said, this is equivalent to permuting the rows and the columns of the incidence matrix.)

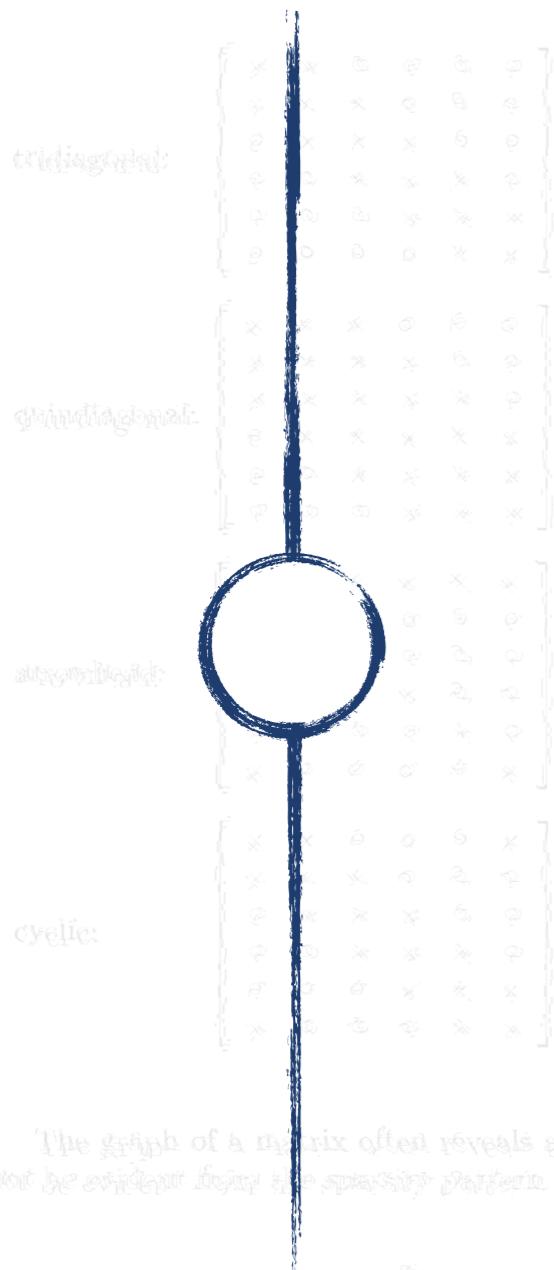
Every rooster that can be monogomically paired, in general, sticks to one mate. We note with these computer analyses of the same scored data,



Theorem 11.1. Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, n\}$ , that is,  $T = (V, E)$  is a spanning tree of  $G$  with root  $r$ . Then  $A = LL^T$  is a Cholesky factorization of  $A$ .

$$e_{k,j} = \frac{e_{k,j}}{e_{k,j}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

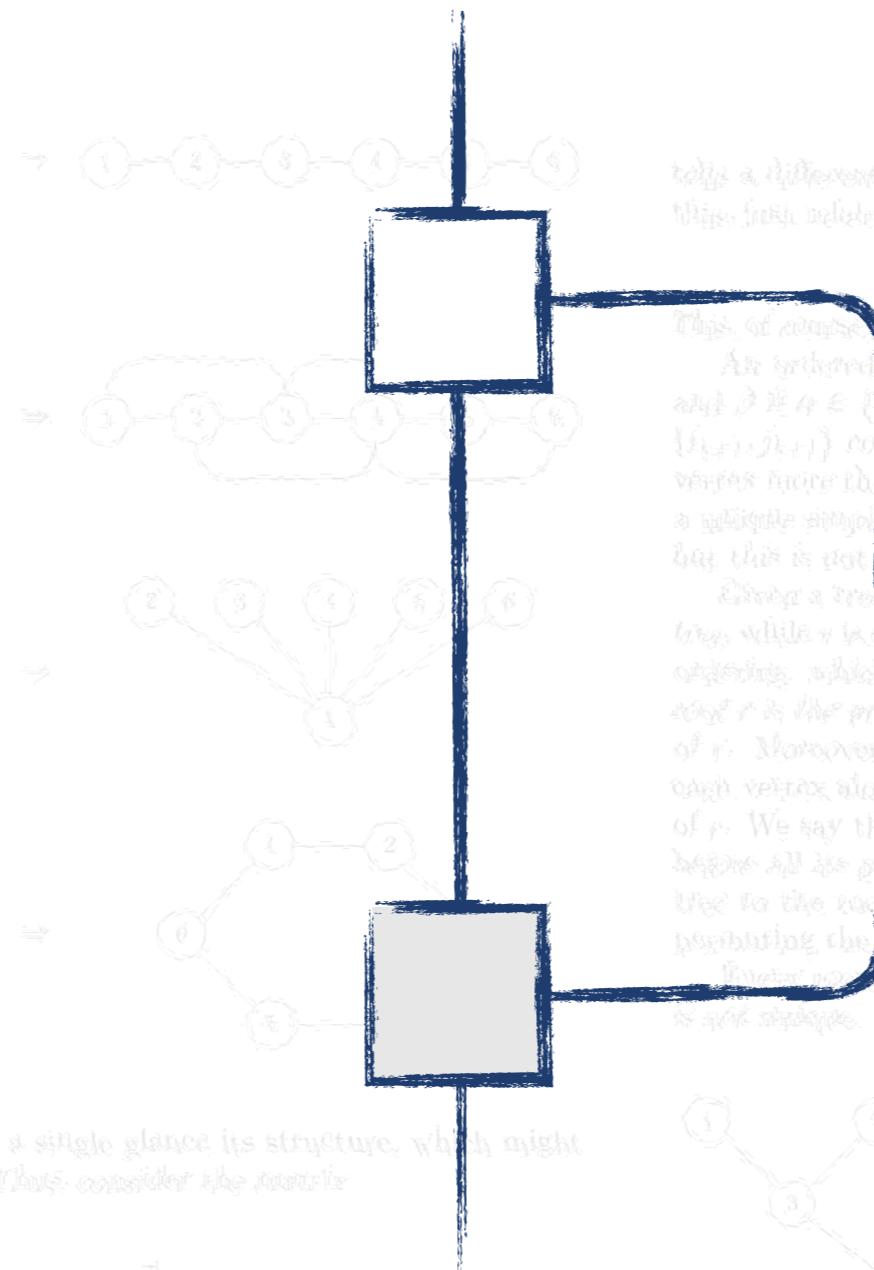


The graph of a matrix often reveals at a single glance its structure, which might not be evident from its sparsity pattern. Thus, consider the matrix

$$\text{LOG} \quad \begin{bmatrix} * & 0 & * & 0 & * & 0 \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & * \\ 0 & * & 0 & * & 0 & * \\ * & 0 & * & 0 & * & 0 \\ * & 0 & * & 0 & * & 0 \end{bmatrix}$$

IF

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, first recall that the vertices are

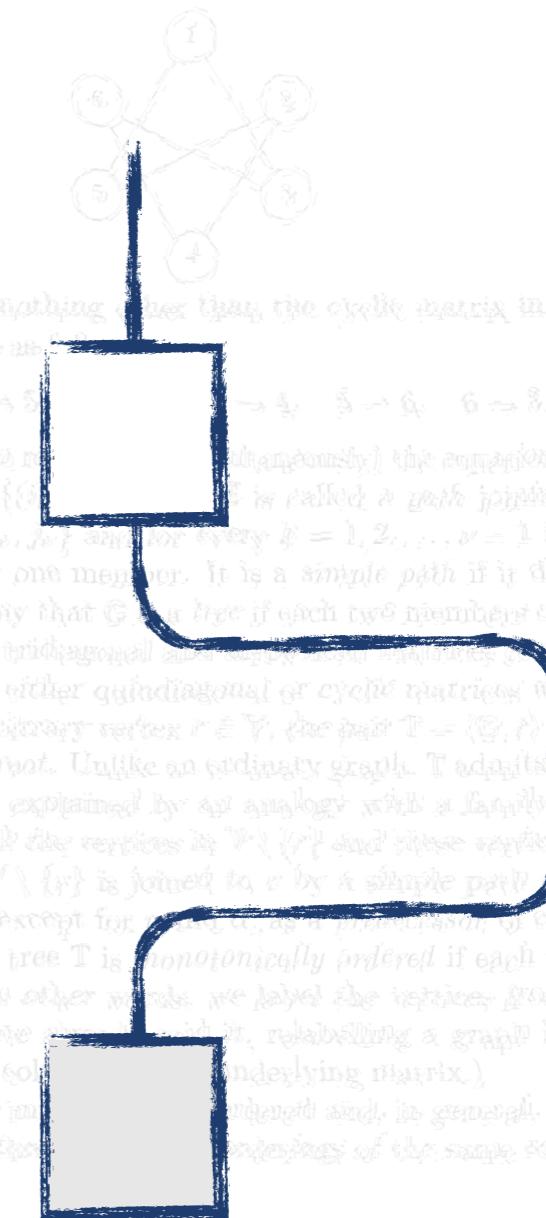
$$1 \rightarrow 1, \quad 2 \rightarrow 2, \quad \dots, \quad 5 \rightarrow 6, \quad 6 \rightarrow 1.$$

This, of course, is equivalent to renumbering (simultaneously) the equations and variables.

An ordered set of edges  $\{(v_i, w_i)\}_{i=1}^n$  in  $G = (V, E)$  is called a path joining the vertices  $v_1$  and  $v_n$  if  $i \in \{1, \dots, n\}$ ,  $v_i \in V$ ,  $w_i \in V$ , and for every  $k = 1, 2, \dots, n-1$  the set  $\{v_k, w_k\} \cap \{v_{k+1}, w_{k+1}\}$  contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that  $G$  is a tree if each two members of  $V$  are joined by a unique simple path. Both tridiagonal and pentadiagonal matrices correspond to trees, but this is not the case with either quadiagonal or cyclic matrices when  $n \geq 3$ .

Given a tree  $T$  and an arbitrary vertex  $r \in V$ , the pair  $T = (G, r)$  is called a rooted tree, while  $r$  is said to be the root. Unlike in a ordinary graph,  $T$  admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root  $r$  is the predecessor of all the vertices in  $T \setminus \{r\}$  and these vertices are successors of  $r$ . Moreover, every  $v \in V \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex along this path, except for  $r$  itself, as a predecessor of  $v$  and a successor of  $r$ . We say that the rooted tree  $T$  is monotonically ordered if each vertex is labelled before all its predecessors in other words, we layed the vertices from the top of the tree to the root. (As we have mentioned it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Even a binary tree will be monotonically ordered and, in general, such an ordering is not unique. We will give the



**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  are then arranged so that  $T = (G, r)$  is monotonically ordered. Then, that  $A = LL^T$  is a Cholesky factorization, it is true that

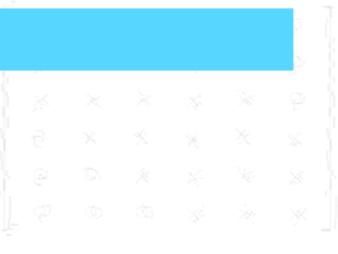
$$l_{k,j} = \frac{a_{kj}}{a_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

triangular:



quadratic:



symmetric:



cyclic:



The graph of a matrix often reveals at a single glance its structure, which may not be evident from the sparsity pattern. Thus, consider the matrix



At a first glance, there is nothing to think in the case just displayed, but its graph,

1

PICTURE

1000

WORDS



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges  $\{(i_1, j_1), (i_2, j_2), \dots, (i_v, j_v)\}$  in  $\mathbb{G}$  is called a *path* joining the vertices  $i_1, i_2, \dots, i_v$  ( $j_1, j_2, \dots, j_v$ ) and, for every  $k = 1, 2, \dots, v-1$  the set  $\{(i_k, j_k), (i_{k+1}, j_{k+1})\}$  contains exactly one edge. It is a *simple path* if it does not visit any vertex more than once. We say that  $\mathbb{G}$  is a *tree* if each two members of  $\mathbb{V}$  are joined by a single simple path. Back to graphs, the quadiagonal matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when  $p \geq 3$ .

Given a tree  $\mathbb{G}$  and an arbitrary vertex  $r \in \mathbb{V}$ , the pair  $T = (G, r)$  is called a *rooted tree*, while  $r$  is said to be the *root*. Unlike in ordinary graph,  $T$  admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root  $r$  is the *predecessor* of all the vertices in  $\mathbb{V} \setminus \{r\}$  and these vertices are *successors* of  $r$ . Moreover, every  $\alpha \in \mathbb{V} \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex along this path, except for  $r$  and  $\alpha$ , as a *predecessor* of  $\alpha$  and a *successor* of  $r$ . We say that the rooted tree  $T$  is *monotonically ordered* if each vertex is labelled before all its predecessors; in other words, we *layer* the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is unique. We now give three consecutive endings of the same rooted tree:



**Theorem 1.** Let  $A$  be a symmetric matrix whose graph  $\mathbb{G}$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  are then arranged so that  $T = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

```
// TODO: Write tests later

test('it renders', async function(assert) {
  await render(hbs`<ComplexComponent />`);
  assert.ok(true);
});
```

```
import { percySnapshot } from 'ember-percy';

...
// TODO: Write tests later

test('complex workflow', async function(assert) {
  await visit('/complex-page');
  await percySnapshot(assert);
});
```

Therefore we will now give a few examples of matrices (represented by their sparsity)

triangular:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



quadrangular:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



supernodal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



cyclic:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



IF  $f$  IS CONTINUOUS IN  $[a, b]$ , AND IF  $f(a)f(b) < 0$ ,

THEN  $f$  MUST HAVE A

ZERO IN  $(a, b)$ .



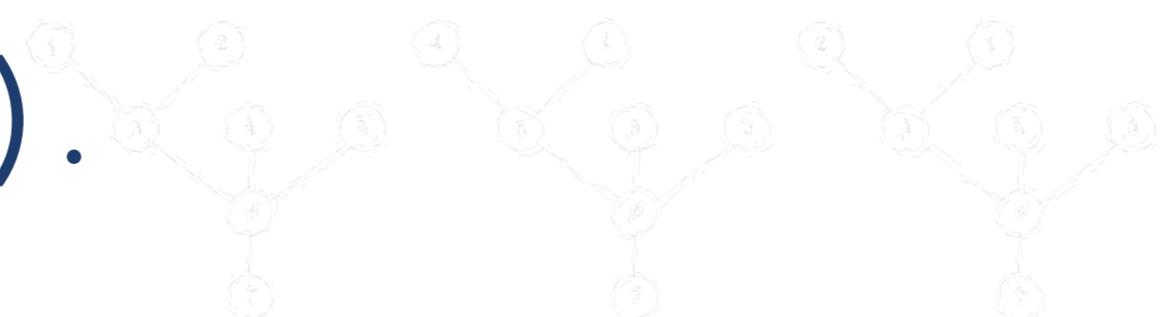
tells a different story - it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 2, 3 \rightarrow 3, 4 \rightarrow 4, 5 \rightarrow 5, 6 \rightarrow 6.$$

Of course, it is equivalent to relabeling the equations and variables. As you can see from the figure,  $G$  is called a tree, since the vertices  $v$  and  $u$  ( $v \in V \setminus \{u\}$ ,  $u \in V \setminus \{v\}$ ) and for every  $k = 1, 2, \dots, n-1$  the set  $\{v, u\} \cap \{v_k, u_k\}$  contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that  $G$  is a tree if each two members of  $V$  are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees, but this is not the case with either of banded or cyclic matrices when  $n \geq 3$ .

Of course, a tree is equivalent to a rooted tree. A vertex  $r \in V$  is called a root of the tree if it is not a predecessor of any other vertex. Unlike in a binary tree, there is a natural partial ordering which can best be explained by an analogy with a family tree. Thus, the root  $r$  is the predecessor of all the vertices in  $V \setminus \{r\}$  and these vertices are successors of  $r$ . Moreover, every  $v \in V \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex along this path, except for  $r$  and  $v$ , as a predecessor of  $v$  and a successor of  $r$ . We say that the rooted tree  $T$  is monotonically ordered if each vertex is labelled by an integer  $r$  in such a way that the vertices from the top of the tree to the bottom are in increasing order (in other words, we say the vertices from the top of the tree to the bottom are in increasing order). Labelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three consecutive orderings of the same rooted tree:



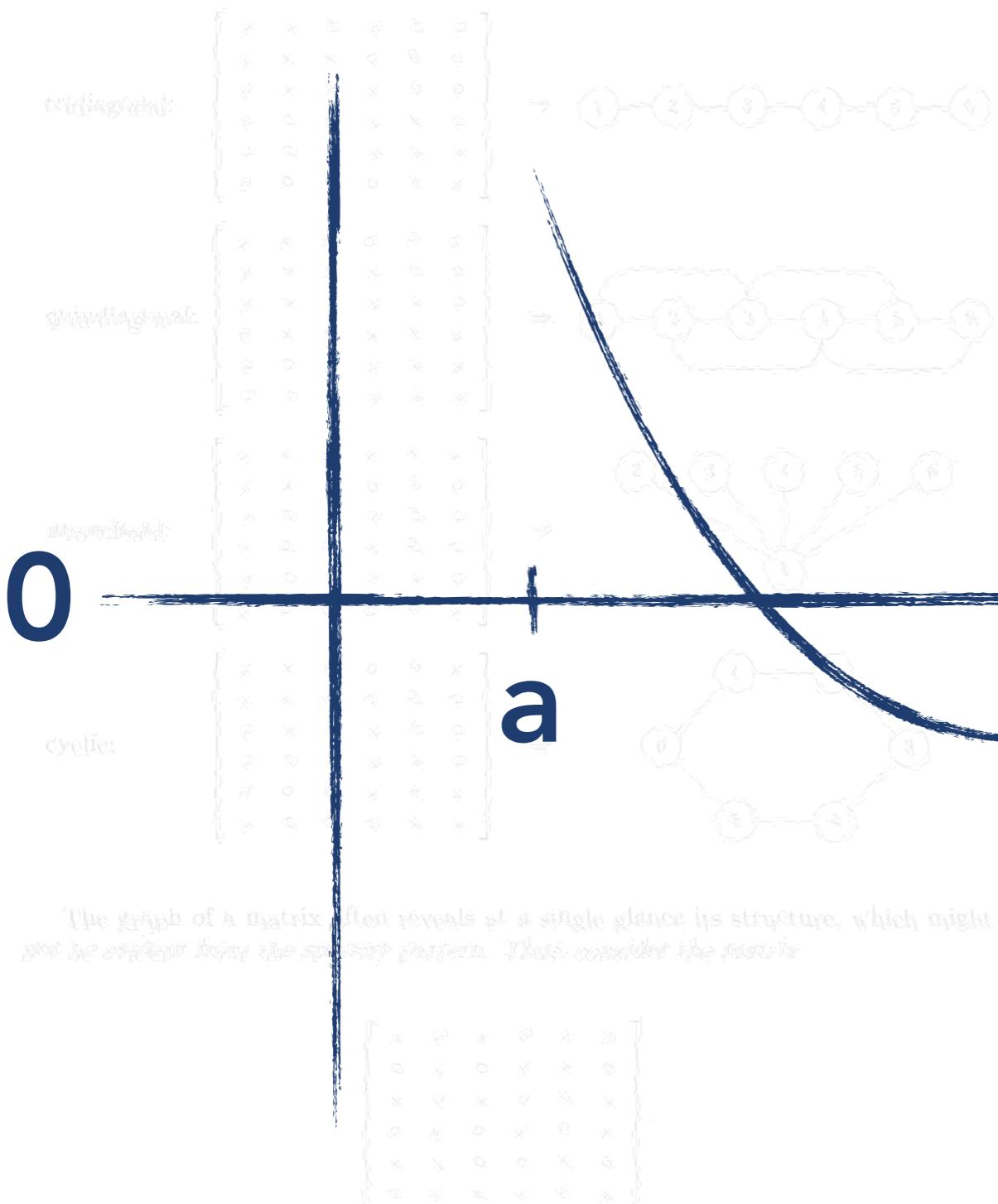
**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  are then arranged so that  $T = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

At a first glance, this is not a very useful formula, but it is the basis for the following algorithm.

just displayed, but its graph:

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



At a first glance, there is nothing to link in the case of the four matrices that we have just displayed, but its graph.



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 3, \quad 5 \rightarrow 6, \quad 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges  $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$  is called a path joining the vertices  $i_0$  and  $j_0$  if  $i_0 \in \{i_0, j_0\}$ ,  $j_0 \in \{i_0, j_0\}$  and for every  $k = 1, 2, \dots, n-1$  the set  $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$  contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that  $G$  is a tree if each two members of  $V$  are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when  $n \geq 3$ .

Given a tree  $T$  and an arbitrary vertex  $r \in V$ , the pair  $T = (G, r)$  is called a rooted tree, while  $r$  is said to be the root. Unlike in ordinary graph,  $T$  admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root  $r$  is the predecessor of all the vertices in  $T \setminus \{r\}$  and these vertices are successors of  $r$ . The set  $\{v \in V \mid v \neq r\text{ and }v \text{ is a successor of }r\}$  is called an  $r$ -successor. We designate each vertex along this path, except for  $r$  and  $\alpha$ , as a predecessor of  $\alpha$  and a successor of  $r$ . We say that the rooted tree  $T$  is monotonically ordered if each vertex is labelled before all its descendants. In other words, we layed the tree down the top at the tree's root. (As we have seen, it is said it, relabelling  $\alpha$  is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree:



Therefore we will now give a few examples of matrices (represented by their sparsity)

# YOU CAN FIND EQUALLY MANY NUMBERS BETWEEN 0 AND 1 AS YOU CAN

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

## BETWEEN $-\infty$ AND $\infty$ .

At a first glance this is not a matrix, but it is a little known triangular matrix, just displayed, but its graph,



It is a different story – it is nothing other than the cyclic matrix in disguised. To see this, let us relabel the vertices as follows:

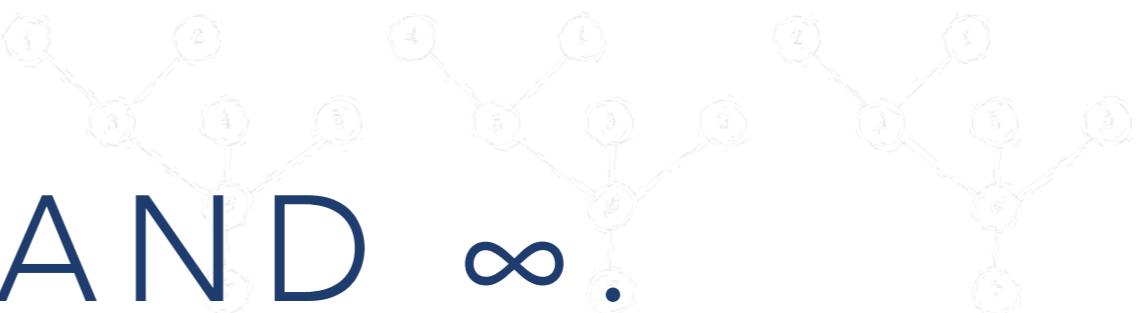
$$1 \rightarrow 1, 2 \rightarrow 3, 3 \rightarrow 2, 4 \rightarrow 4, 5 \rightarrow 6, 6 \rightarrow 8.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges  $\{(u_i, v_i)\}_{i=1}^r$  in  $\mathbb{G}$  is called a path joining the vertices  $u_1$  and  $u_r$  if  $u_i \in \{u_1, u_2\}$ ,  $v_i \in \{v_1, v_2\}$  and for every  $i = 1, 2, \dots, r-1$  the set  $\{u_i, v_i\} \cap \{u_{i+1}, v_{i+1}\} = \emptyset$ . A path is called simple if it does not visit any vertex more than once. We say that a path is closed if all its vertices are joined by a single simple path. Below we present two examples. Matrices corresponding to these, but this is not the case with either quidiagonal or cyclic matrices when  $p \geq 3$ .

Given a tree  $\mathbb{G}$  and an arbitrary vertex  $r \in \mathbb{V}$ , the pair  $T = (G, r)$  is called a rooted tree, while  $r$  is said to be the root. Unlike in a ordinary graph,  $T$  admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root  $r$  is the ancestor of the vertices in  $T \setminus \{r\}$  and these vertices are successors of  $r$ . Moreover, every  $a \in \mathbb{V} \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex along this path, except for  $r$  and  $a$ , as a predecessor of  $a$  and a successor of  $r$ . We say that the rooted tree  $T$  is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we label the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree.



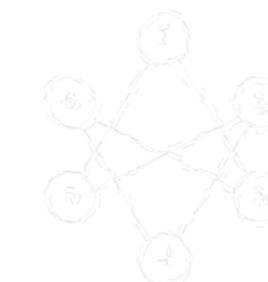
**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $\mathbb{G}$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  are then arranged so that  $T = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

tridiagonal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix} \rightarrow \text{Graph: } 1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 6$$



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, first, re-label the weights as follows:

$$w_{12} = w_{23} = w_{34} = w_{45} = w_{56} = w_{61} = 1, \quad w_{13} = w_{24} = w_{35} = w_{46} = 2, \quad w_{14} = w_{25} = w_{36} = 3$$

\_\_\_\_\_

$$\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & * & 0 \\ * & 0 & * & 0 & 0 & * \\ 0 & * & 0 & * & 0 & * \\ * & 0 & 0 & 0 & * & 0 \\ 0 & * & * & * & * & * \end{bmatrix}.$$



**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  are then arranged so, that  $\mathbb{T} = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i \sim j} a_{ki}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

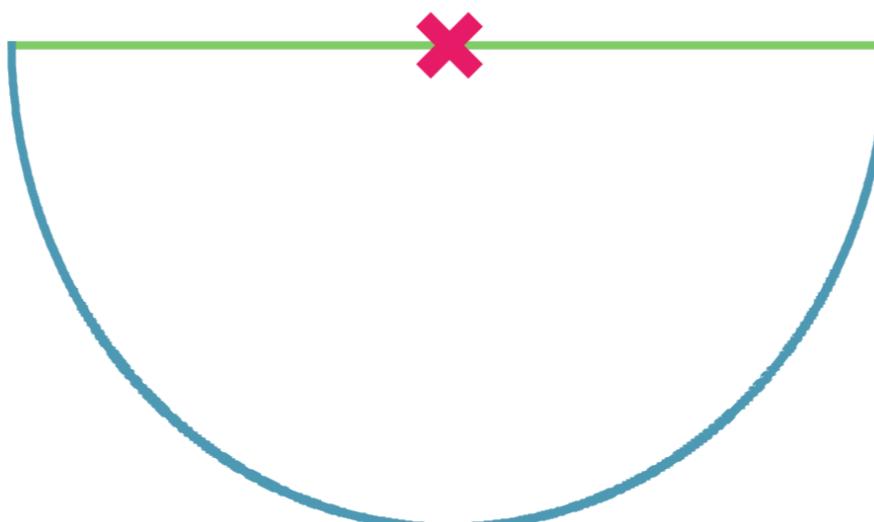
At a first glance, there is nothing to think in the case of the first matrices that we have just displayed, but its graph.

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

tridiagonal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix} \rightarrow \text{Graph: } 1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 6$$

tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, first, re-label the vertices as follows:



$$\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & * & 0 \\ * & 0 & * & 0 & 0 & * \\ 0 & * & 0 & * & 0 & * \\ * & 0 & * & 0 & 0 & * \\ 0 & * & * & * & * & * \end{bmatrix}$$

At a first glance, there is nothing to think in the case of the first matrices that we have just displayed, but its graph:



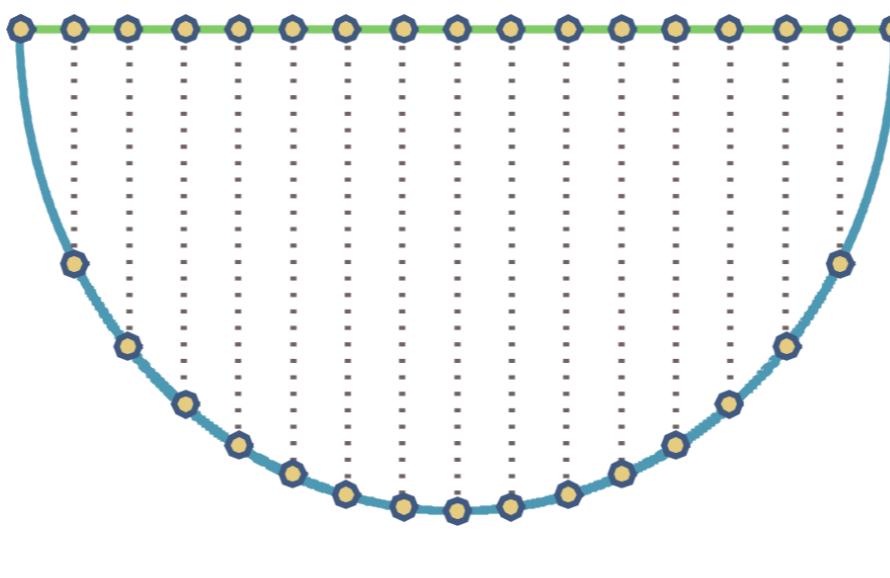
**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  are then arranged so, that  $\mathbb{T} = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ii}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just re-label the weights as follows:



At a first glance, there is nothing to link in the case of the four matrices that we have just displayed, but its graph.



**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  are then arranged so that  $T = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

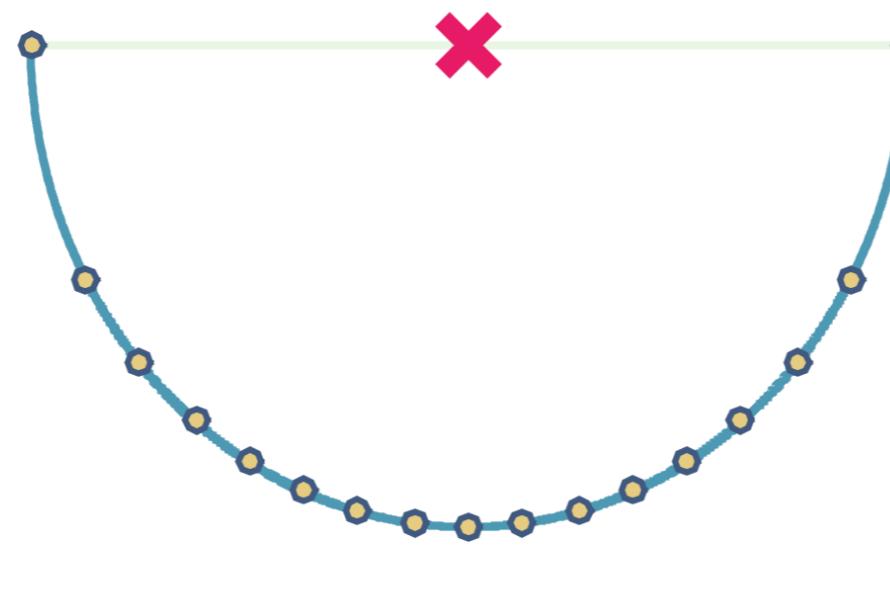
$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

tridiagonal:

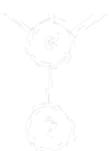
$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix} \rightarrow \text{Graph: } 1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 6$$

tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, first recall the weights as follows:



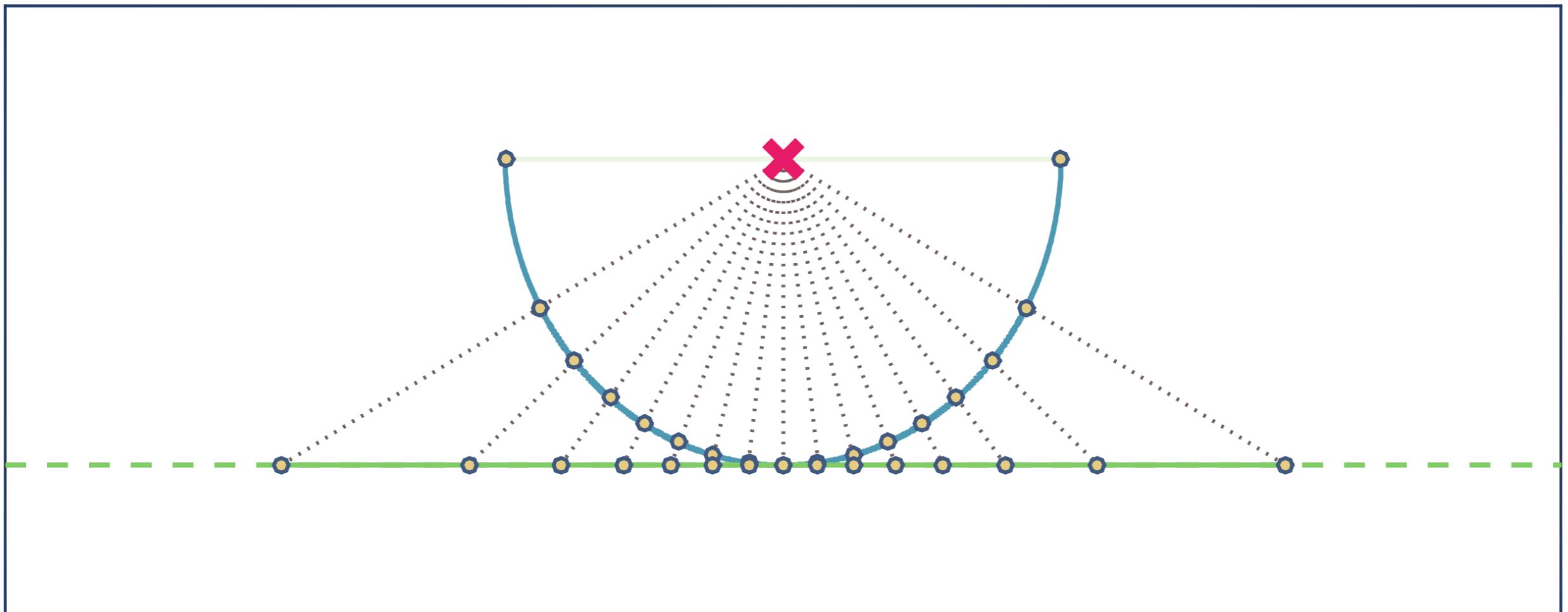
$$\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & * & 0 \\ * & 0 & * & 0 & 0 & * \\ 0 & * & 0 & * & 0 & * \\ * & 0 & * & 0 & 0 & * \\ 0 & * & * & * & * & * \end{bmatrix}$$

At a first glance, there is nothing to think in the case of the four matrices that we have just displayed, but its graph:



**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  are then arranged so, that  $\mathbb{T} = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ii}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$



Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

tridiagonal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix} \rightarrow \text{Graph: } 1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 6$$



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just re-label the weights as follows:

1 2 3 4 5 6 7 8 9 10 11 12

$$\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & * & 0 \\ * & 0 & * & 0 & 0 & * \\ 0 & * & 0 & * & 0 & * \\ * & 0 & 0 & 0 & * & 0 \\ 0 & * & * & * & * & * \end{bmatrix}.$$

At a first glance, there is nothing to think in the case of the first matrices that we have just displayed, but its graph:



**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  are then arranged so, that  $\mathbb{T} = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ii}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

# 5 Rules of Writing Tests

Filter:

Module: Acceptance | Outline



EmberFest 18.10.2019

Running:

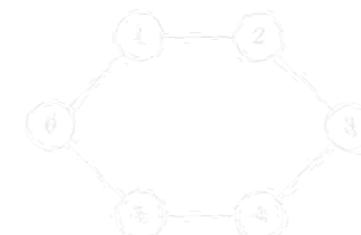
asymmetric:

$$\begin{bmatrix} 0 & 2 & 3 & 2 & 2 \\ 2 & 0 & 2 & 2 & 2 \\ 3 & 2 & 0 & 2 & 2 \\ 2 & 2 & 2 & 0 & 2 \\ 2 & 2 & 2 & 2 & 0 \end{bmatrix} \Rightarrow$$



cyclic:

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \Rightarrow$$



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} 0 & 2 & * & 0 & * & 0 \\ 2 & 0 & 0 & * & x & 0 \\ * & 0 & 0 & 0 & 0 & x \\ 0 & * & 0 & 0 & 0 & 0 \\ * & x & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

At a first glance, there is nothing to think in the case of the four matrices that we have just displayed, but its graph,

which can best be explained by an analogy with a family tree. Thus, the root  $r$  is the predecessor of all the vertices in  $T \setminus \{r\}$  and these vertices are successors of  $r$ . Moreover, every  $v \in V \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex along this path, except for  $r$  and  $v$ , as a predecessor of  $v$  and a successor of  $r$ . We say that the rooted tree  $T$  is *monotonically ordered* if each vertex is labelled before all its predecessors; in other words, we label the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three consecutive orderings of the same rooted tree:



**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  are then arranged so that  $T = (G, r)$  is monotonically ordered. Given that  $A = LL^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

# 5 Rules of Writing Tests

Filter:

Module: Acceptance | Outline

EmberFest 18.10.2019

1 assertion of 1 passed, 0 failed.

## If, if, if

cyclic:

$$\begin{bmatrix} * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \end{bmatrix} \Rightarrow$$



before all its predecessors in other words: we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three monotone orderings of the same rooted tree:



**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  are then arranged so, that  $T = (G, r)$  is monotonically ordered. Then, that  $A = L U$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ki}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

At a first glance, there is nothing to think in the case of the four matrices that we have just displayed, but its graph.

# 5 Rules of Writing Tests

Filter:

Module: Acceptance | Outline

EmberFest 18.10.2019

2 assertions of 2 passed, 0 failed.

If, if, if

Use common, everyday words

# 5 Rules of Writing Tests

Filter:

Module:

Acceptance | Outline



EmberFest 18.10.2019

3 assertions of 3 passed, 0 failed.

! If, if, if

! Use common, everyday words

! Write less with theorems and new terms

$$\begin{bmatrix} * & * & * & 0 & * & 0 \\ 0 & * & 0 & * & * & 0 \\ * & 0 & * & * & 0 & * \\ 0 & * & 0 & * & * & 0 \\ * & * & 0 & * & * & 0 \\ 0 & * & * & 0 & * & * \end{bmatrix}.$$

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph:



**Theorem 11.1** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  are then arranged so, that  $\mathbf{T} = (G, r)$  is monotonically ordered. Then, that  $A = \mathbf{L}\mathbf{U}^T$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ii}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

# 5 Rules of Writing Tests

Filter:

Module:

Acceptance | Outline



EmberFest 18.10.2019

4 assertions of 4 passed, 0 failed.

! If, if, if

! Use common, everyday words

! Write less with theorems and new terms

! All your basis are belong to us

$$\begin{bmatrix} 0 & 3 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,

**Theorem 11.1** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Choose a root  $r \in \{1, 2, \dots, d\}$  and assume that the rows and columns of  $A$  are then arranged so, that  $\tilde{T} = (G, r)$  is monotonically ordered. Then, that  $A = \tilde{L}\tilde{U}$  is a Cholesky factorization, it is true that

$$t_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ki}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

# 5 Rules of Writing Tests

Filter:

Module: [Acceptance](#) | [Outline](#) ▾

EmberFest 18.10.2019

5 assertions of 5 passed, 0 failed.

! If, if, if

! Use common, everyday words

! Write less with theorems and new terms

! All your basis are belong to us

! 1 picture = 1000 words

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

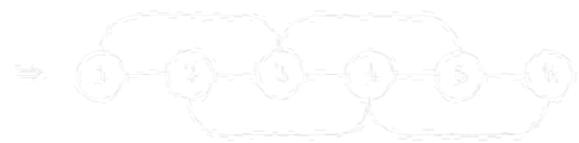
tridiagonal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



qundiagonal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ * & * & * & * & * & * \\ * & * & * & * & * & * \end{bmatrix}$$



# crunchingnumbers.live

@ijlee2

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & x & 0 \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & * \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & * & * \end{bmatrix}.$$

At a first glance, there is nothing to think in the case of the four matrices that we have just displayed, but its graph,



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

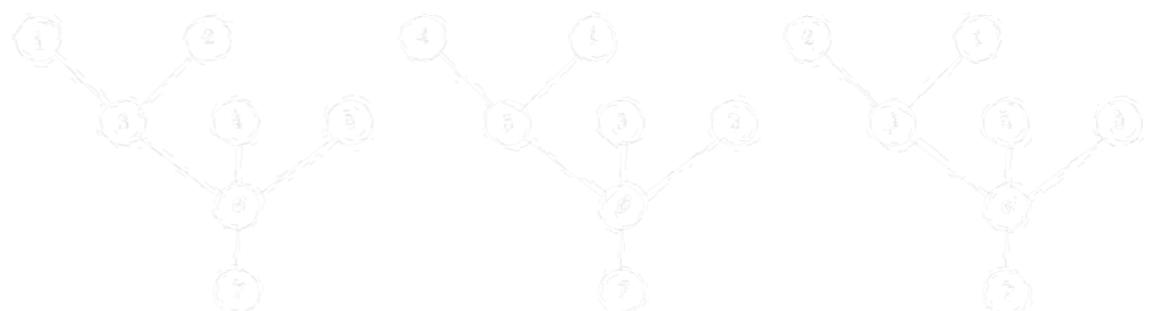
$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges  $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$  is called a path joining the vertices  $i_0$  and  $j_0$  if  $i_0 \in \{i_0, j_0\}$ ,  $j_0 \in \{i_0, j_0\}$  and for every  $k = 1, 2, \dots, n-1$  the set  $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$  contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that  $G$  is a tree if each two members of  $V$  are joined by a unique simple path. Both tridiagonal and qundiagonal matrices correspond to trees, but this is not the case with either qundiagonal or cyclic matrices when  $n \geq 3$ .

Given a tree  $T$  and an arbitrary vertex  $r \in V$ , the pair  $T - (r, \cdot)$  is called a rooted tree, where the vertex  $r$  is the root. Unlike in ordinary graph,  $T$  admits a natural partial order which should be explained by an analogy with a family tree. Thus, the vertex  $r$  is the predecessor of all the vertices in  $T \setminus \{r\}$  and these vertices are successors of  $r$ . Moreover, every  $a \in V \setminus \{r\}$  is joined to  $r$  by a simple path and we designate each vertex along this path, except for  $r$  and  $a$ , as a predecessor of  $a$  and a successor of  $r$ . We say that the rooted tree  $T$  is monotonically ordered if each vertex is labelled before all its predecessors in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three consecutive orderings of the sample rooted tree.



**Theorem 11.1.** Let  $A$  be a symmetric matrix whose graph  $G$  is a tree. Consider a root  $r \in \{1, 2, \dots, n\}$  and assume that the rows and columns of  $A$  are now arranged so, that  $T = (G, r)$  is monotonically ordered. Given that  $A = PLU$  is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{p_{jj}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

**Q.E.D.**