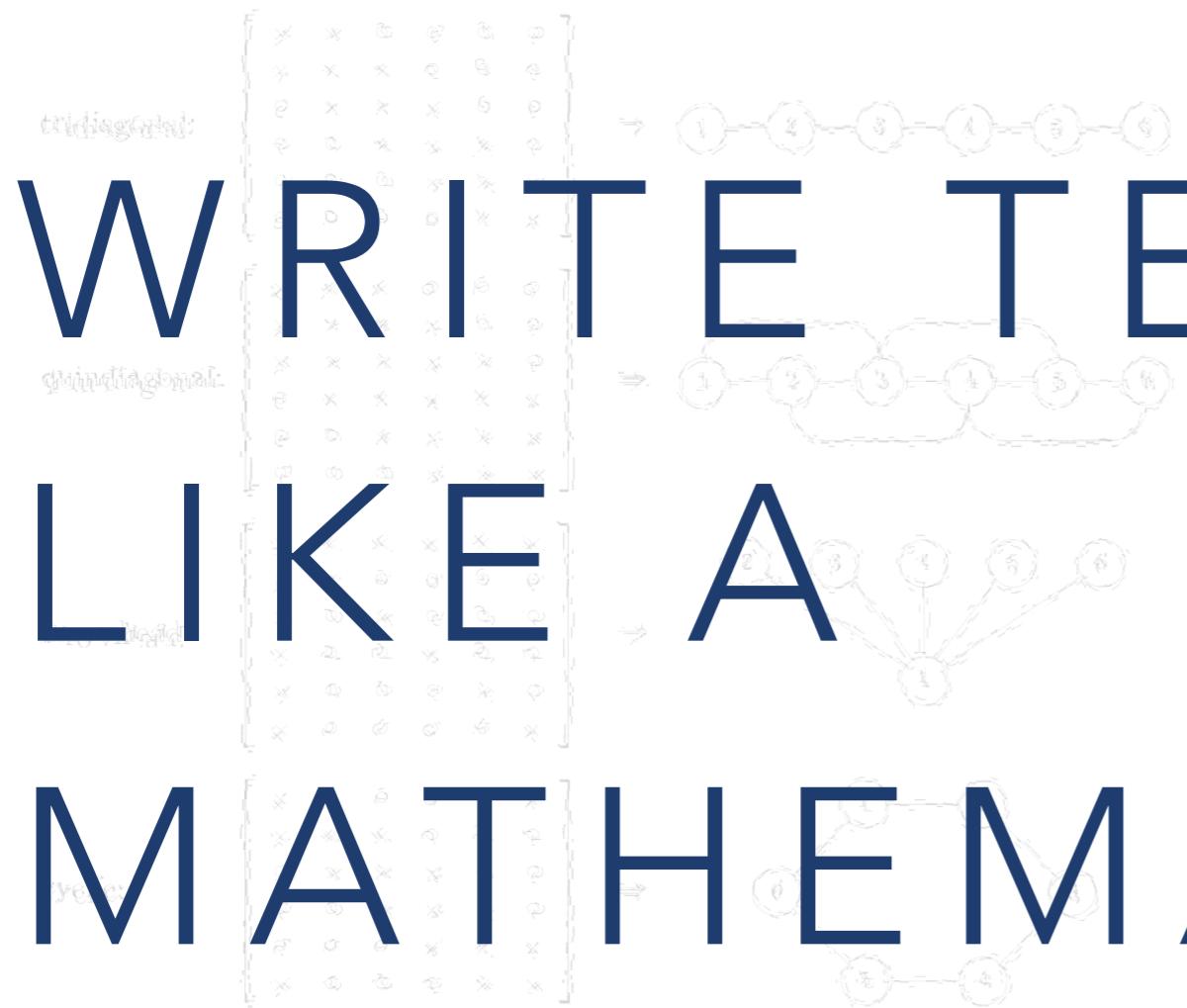


Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

ISAAC J. LEE

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, first recall the relation as follows

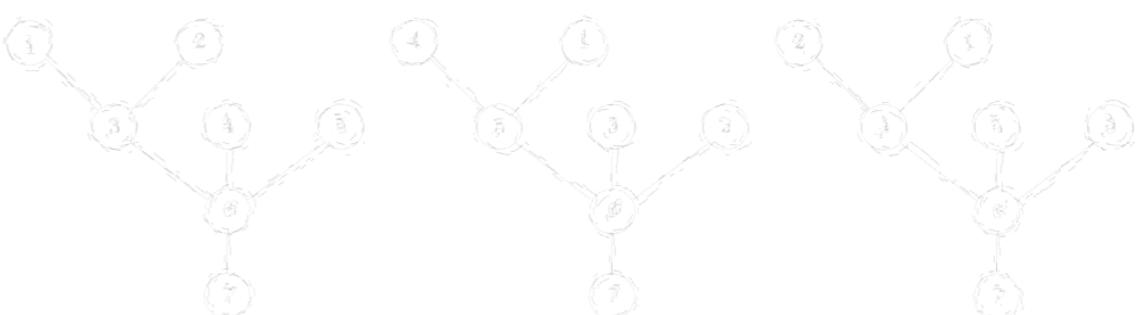
$$1 \rightarrow 2, 2 \rightarrow 3, 3 \rightarrow 2, 4 \rightarrow 5, 5 \rightarrow 6, 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

Let α be an ordered set of vertices $\{\alpha_i\}_{i=1}^n$, $\beta \in \mathbb{S}$ is called a path joining the vertices α and β if $\alpha \in \{\alpha_i\}_{i=1}^n$, $\beta \in \{\alpha_i\}_{i=1}^n$ and for every $k = 1, 2, \dots, n-1$ the set $\{\alpha_k, \alpha_{k+1}\} \cap \{\beta_{k+1}, \beta_{k+2}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that \mathbb{G} is a tree if each two members of \mathbb{V} are joined by a unique simple path. Both tridiagonal and quadiagonal matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $n \geq 3$.

Given a tree \mathbb{G} and an arbitrary vertex $r \in \mathbb{V}$, the pair $\mathbb{T} = (\mathbb{G}, r)$ is called a rooted tree, while r is said to be the root. Unlike an ordinary graph, \mathbb{T} admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $\mathbb{V} \setminus \{r\}$ and these vertices are successors of r . Moreover, every $\alpha \in \mathbb{V} \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a predecessor of α and a successor of r . We say that the rooted tree \mathbb{T} is monotonically ordered if each vertex is labelled before all its predecessors (or, more precisely, we have the relation from the top of the tree to the root. If we have already said it, relabeling a graph is tantamount to relabeling the rows and the columns of the underlying matrix).

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three monotone orderings of the sample rooted tree:



Theorem III.1. Let \mathbb{A} be a symmetric matrix whose graph \mathbb{G} is a tree. Choose a root $r \in \{1, 2, \dots, n\}$ and assume that the rows and columns of \mathbb{A} have been arranged so that $\mathbb{T} = (\mathbb{G}, r)$ is monotonically ordered. Given that $\mathbb{A} = \mathbb{L}\mathbb{U}^T$ is a Cholesky factorization, it is true that

$$a_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ii}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

5 Rules of Writing Tests

Filter:

Module: Acceptance | Outline



EmberFest 18.10.2019

Running:

asymmetric:

$$\begin{bmatrix} 0 & 2 & 3 & 2 & 2 \\ 2 & 0 & 2 & 2 & 2 \\ 3 & 2 & 0 & 2 & 2 \\ 2 & 2 & 2 & 0 & 2 \\ 2 & 2 & 2 & 2 & 0 \end{bmatrix} \Rightarrow$$



cyclic:

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \Rightarrow$$



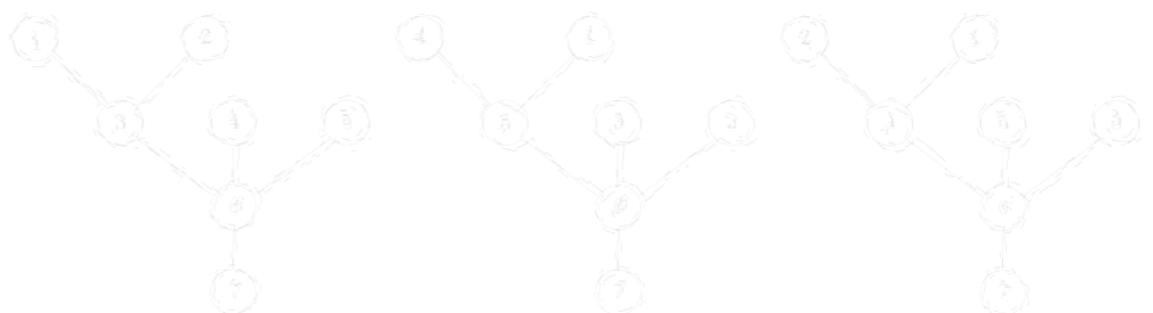
The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} 0 & 2 & * & 0 & * & 0 \\ 2 & 0 & 0 & * & x & 0 \\ * & 0 & 2 & 0 & 0 & * \\ 0 & * & 0 & 0 & * & 0 \\ * & x & 0 & 0 & 0 & * \\ 0 & 0 & * & 0 & * & 0 \end{bmatrix}.$$

At a first glance, there is nothing to think in the case of the four matrices that we have just displayed, but its graph,

which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $V \setminus \{r\}$ and these vertices are successors of r . Moreover, every $v \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and v , as a predecessor of v and a successor of r . We say that the rooted tree T is *monotonically ordered* if each vertex is labelled before all its predecessors; in other words, we label the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three consecutive orderings of the same rooted tree.



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so that $T = (G, r)$ is monotonically ordered. Then, that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ki}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

5 Rules of Writing Tests

Filter:

Module: Acceptance | Outline

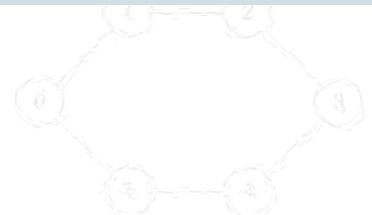
EmberFest 18.10.2019

1 assertion of 1 passed, 0 failed.

If, if, if

cyclic:

$$\begin{bmatrix} * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \end{bmatrix} \Rightarrow$$



before all its predecessors in other words: we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three monotone orderings of the same rooted tree:



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so, that $T = (G, r)$ is monotonically ordered. Then, that $A = L U$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ki}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

At a first glance, there is nothing to think in the case of the four matrices that we have just displayed, but its graph.

5 Rules of Writing Tests

Filter:

Module: Acceptance | Outline

EmberFest 18.10.2019

2 assertions of 2 passed, 0 failed.

If, if, if

Use common, everyday words

5 Rules of Writing Tests

Filter:

Module:

Acceptance | Outline



EmberFest 18.10.2019

3 assertions of 3 passed, 0 failed.

! If, if, if

! Use common, everyday words

! Write less with theorems and new terms

$$\begin{bmatrix} * & * & * & 0 & * & 0 \\ 0 & * & 0 & * & * & 0 \\ * & 0 & * & * & 0 & * \\ 0 & * & 0 & * & * & 0 \\ * & * & 0 & * & * & 0 \\ 0 & * & * & 0 & * & * \end{bmatrix}.$$

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph:



Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so, that $\mathbf{T} = (G, r)$ is monotonically ordered. Then, that $A = \mathbf{L}\mathbf{U}^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ii}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

5 Rules of Writing Tests

Filter:

Module:

Acceptance | Outline



EmberFest 18.10.2019

4 assertions of 4 passed, 0 failed.

! If, if, if

! Use common, everyday words

! Write less with theorems and new terms

! All your basis are belong to us

$$\begin{bmatrix} 0 & 3 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,

Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so, that $\tilde{T} = (G, r)$ is monotonically ordered. Then, that $A = \tilde{L}\tilde{U}$ is a Cholesky factorization, it is true that

$$t_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ki}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

5 Rules of Writing Tests

Filter:

Module: [Acceptance](#) | [Outline](#) ▾

EmberFest 18.10.2019

5 assertions of 5 passed, 0 failed.

! If, if, if

! Use common, everyday words

! Write less with theorems and new terms

! All your basis are belong to us

! 1 picture = 1000 words

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} * & * & * & 0 & * & * \\ 0 & * & 0 & * & * & * \\ * & 0 & * & 0 & * & * \\ 0 & * & 0 & * & 0 & * \\ * & * & 0 & 0 & * & * \\ 0 & * & * & * & 0 & * \end{bmatrix}$$

At a first glance, there is nothing to think in the case just displayed, but its graph,



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

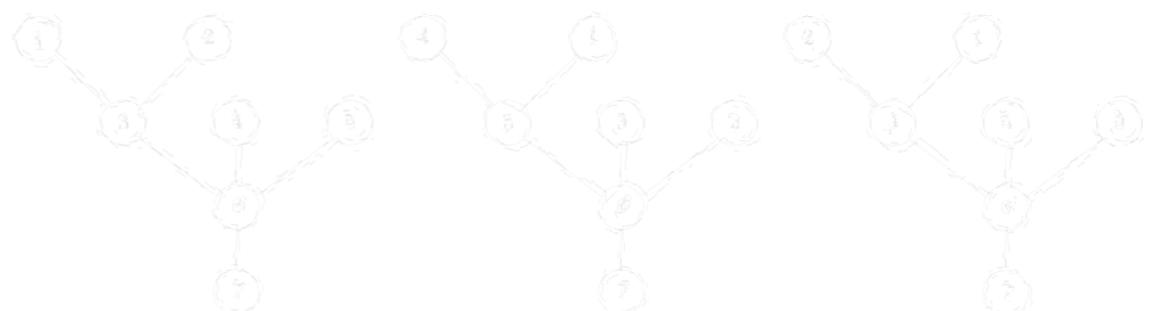
$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 3, \quad 5 \rightarrow 6, \quad 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$ is called a path joining the vertices i_0 and j_0 if $i_0 \in \{i_0, j_0\}$, $j_0 \in \{i_0, j_0\}$ and for every $k = 1, 2, \dots, n-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of V are joined by a unique simple path. Both tridiagonal and quidiagonal matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $n \geq 3$.

Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $T \setminus \{r\}$ and these vertices are successors of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a predecessor of α and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we label the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

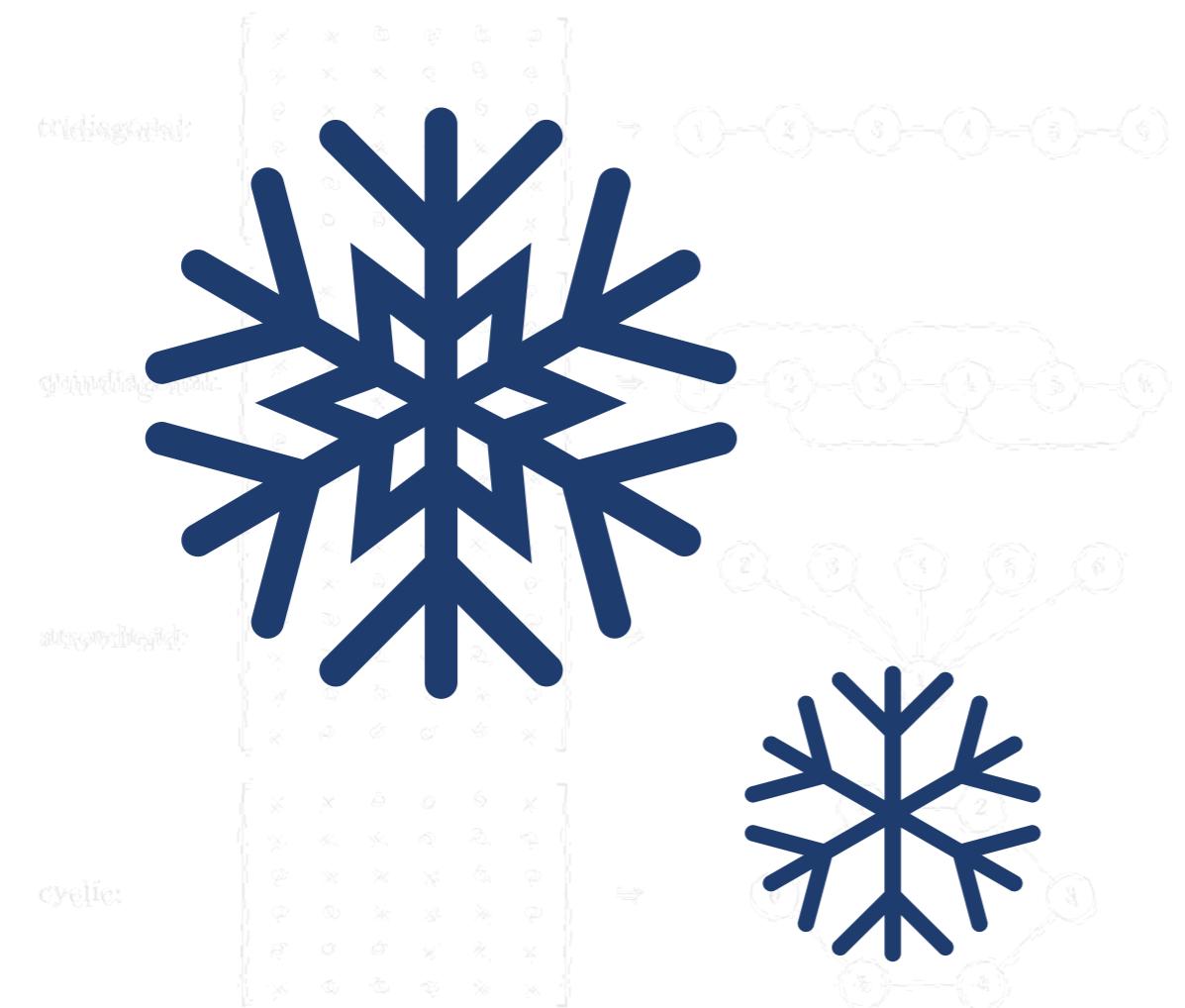
Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three consecutive orderings of the same rooted tree.



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, n\}$ and assume that the rows and columns of A are then arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\text{IF} \quad \begin{bmatrix} * & * & * & * & * \\ * & 0 & * & * & * \\ * & * & 0 & * & * \\ * & * & * & 0 & * \\ * & * & * & * & 0 \end{bmatrix}.$$

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 3, \quad 4 \rightarrow 6, \quad 5 \rightarrow 2, \quad 6 \rightarrow 4.$$

This, of course, is equivalent to reordering the equations and variables.

An ordered set of edges $\{(i_1, j_1), (i_2, j_2), \dots, (i_r, j_r)\}$ is a path in the matrix A if $i_1, i_2, \dots, i_r \in \{1, 2, \dots, n\}$, $j_1, j_2, \dots, j_r \in \{1, 2, \dots, n\}$ and $\{i_1, i_2, \dots, i_r, i_{r+1}\}$ contains exactly one member. We say that a vertex v is *reachable* from a vertex u if there is a path from u to v . We say that a vertex v is *adjacent* to a vertex u if each two vertices u and v are joined by a unique simple path. Both the paths and the adjacencies correspond to trees, but this is not the case with cycles.

Given a rooted tree T and a vertex $r \in V$, the pair (T, r) is called a *rooted tree*, while r is called the *root*. Unlike an ordinary graph, T is a natural partial ordering, which can be easily obtained by an analogy with a family tree. Thus, the vertex r is the *predecessor* of v , if v is a vertex in $T \setminus \{r\}$ and these vertices are *successors* of r . Moreover, every $v \in V \setminus \{r\}$ is connected to r by a simple path and we designate each vertex along this path, except for r , as a *predecessor* of v and a *successor* of r . We say that the rooted tree T is *monotonic* or *monotonically ordered* if each vertex is labelled before all its predecessors in other words, we go from the top of the tree to the root. (As we have already said it, relatively speaking, a graph is *transversal* to permuting the rows and the columns of the underlying matrix.)

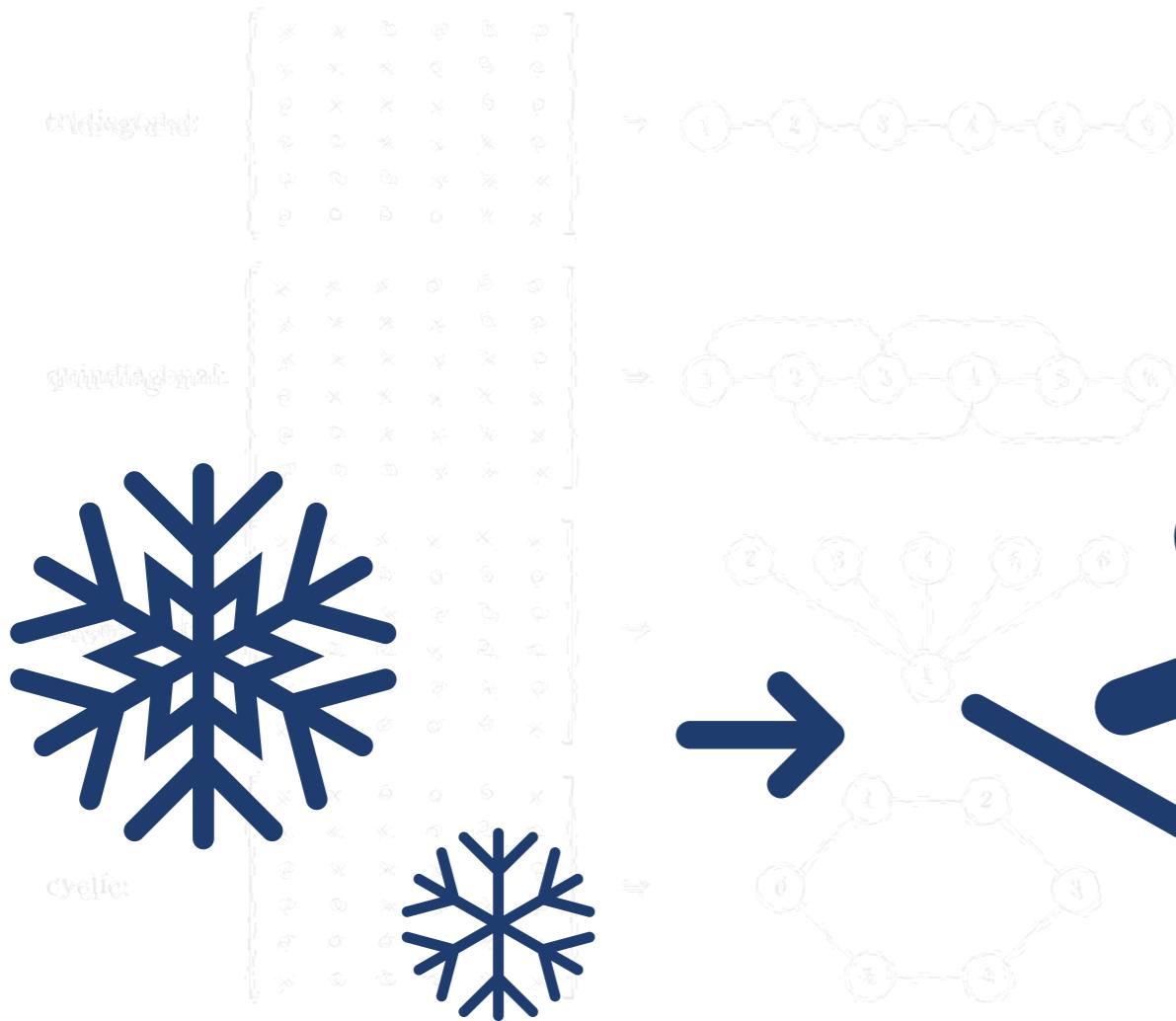
Every rooted tree will be monotonically ordered and, in general, an ordering is not unique. We now give three monotonic orderings of the sample rooted tree:



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, n\}$ and assume that the rows and columns of A are then arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} * & * & * & 0 & * & 0 \\ 0 & * & 0 & * & * & 0 \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & 0 & * & * \\ * & * & 0 & * & * & 0 \\ 0 & 0 & * & * & * & * \end{bmatrix}.$$

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,

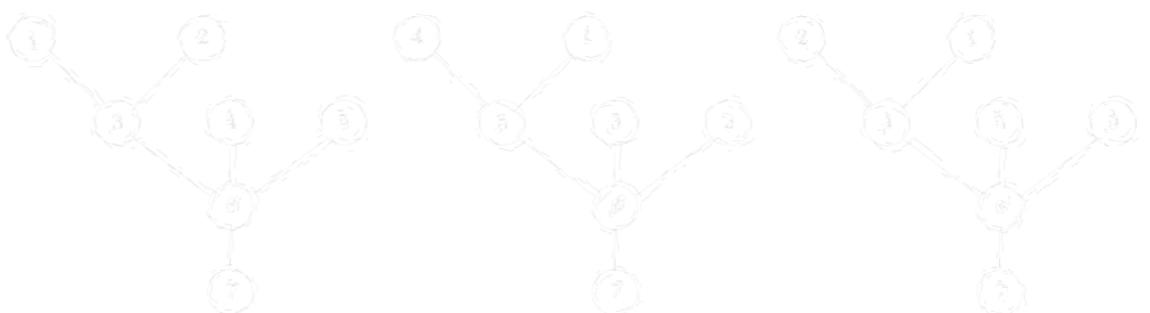
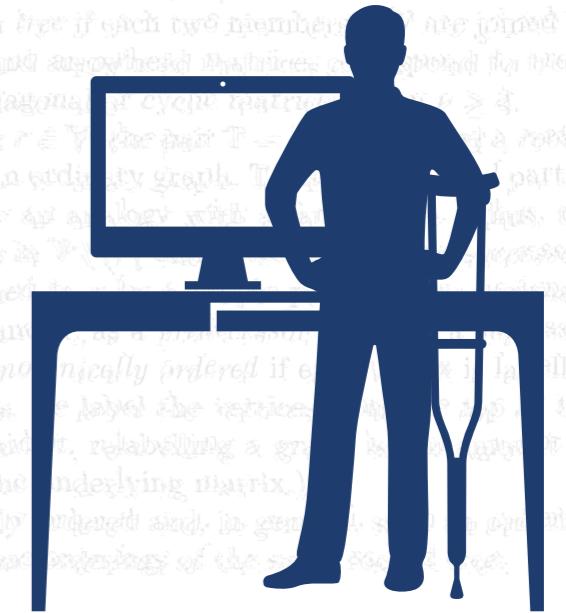


tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 8.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_0, j_0)\}_{i_0, j_0 \in \mathbb{N}_0}$ in \mathbb{G} is called a *path* joining the vertices i_0 and j_0 if $i_0 \in \{i_0, j_0\}$, $j_0 \in \{i_0, j_0\}$ and for every $k = 1, 2, \dots, n-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a *simple path* if it does not visit any vertex more than once. We say that \mathbb{G} is a *tree* if each two members i, j are joined by a unique simple path. Both tridiagonal and quadiagonal matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices. For example, a tree \mathbb{G} and an arbitrary vertex $r \in V$ the pair $T = (\mathbb{G}, r)$ is a *rooted tree* with r to be the root. Unlike in a cyclic graph, the root r in a rooted tree can best be explained by an *ancestor* of r , which is a vertex t such that r is a predecessor of t . The *predecessor* of all the vertices in \mathbb{G} is the root r . Moreover, every $t \in V \setminus \{r\}$ is joined to r and to each vertex along this path there is exactly one predecessor, t , which is a predecessor of r . We say that the rooted tree T is *monotonically ordered* if every vertex t is a predecessor of r before all its predecessors in other words, t is *lower* than r with respect to the root. (As we have already said, t , relabelling a graph by reordering the rows and the columns of the underlying matrix.) It is clear that a rooted tree will be monotonically ordered and, in general, it is not a complete tree. We now give three consecutive examples of the construction of such trees.



Theorem 11.1. Let A be a symmetric matrix whose graph \mathbb{G} is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so that $T = (\mathbb{G}, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their analysis.

1

```
assert.speaker().getsPersonal();  
  
await sing('Happy Birthday');  
  
assert.audience().isHappy();
```

just displayed, but its graph:

$$t_{k,j} = \frac{q_{kj}}{q_j}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

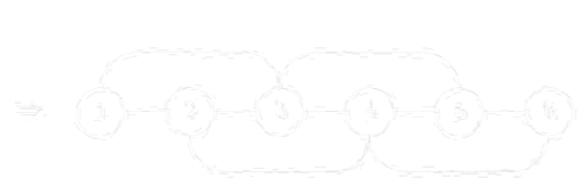
Therefore we will now give a few examples of matrices (represented by their sparsity)

and their graphs.

SUSPECT 1

quindiagonal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$



symmetric:

$$\begin{bmatrix} * & * & * & * & * & * \\ * & * & 0 & 0 & 0 & 0 \\ * & 0 & * & 0 & 0 & 0 \\ * & 0 & 0 & * & 0 & 0 \\ * & 0 & 0 & 0 & * & 0 \\ * & 0 & 0 & 0 & 0 & * \end{bmatrix}$$



cyclic:

$$\begin{bmatrix} * & 0 & 0 & 0 & * & * \\ * & * & 0 & 2 & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$

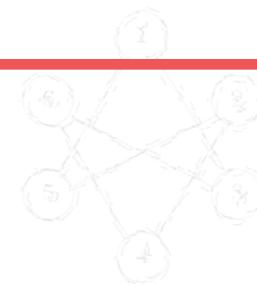


OBSERVER /

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

COMPUTED

At a first glance, the matrix A and its structure of the form of a triangular matrix just displayed, but its graph,



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 3, \quad 5 \rightarrow 6, \quad 6 \rightarrow 8.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$ is called a path joining the vertices i_0 and j_0 if $i_0 \in \{i_0, j_0\}$, $j_0 \in \{i_0, j_0\}$ and for every $k = 1, 2, \dots, n-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of V are joined by a unique simple path. Both tri-diagonal and any other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $n \geq 3$.

Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $T \setminus \{r\}$ and these vertices are successors of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a predecessor of α and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree.



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

and their graphs

SUSPECT 2

quindiagonal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$



superdiagonal:

$$\begin{bmatrix} * & * & * & * & * & * \\ * & * & 0 & 0 & 0 & 0 \\ * & 0 & * & 0 & 0 & 0 \\ 0 & 0 & * & 0 & 0 & 0 \\ 0 & 0 & 0 & * & 0 & 0 \\ 0 & 0 & 0 & 0 & * & 0 \end{bmatrix}$$



cyclic:

$$\begin{bmatrix} * & 0 & 0 & 0 & * & 0 \\ * & * & 0 & 2 & * & 0 \\ * & 0 & * & * & 0 & 0 \\ 0 & 2 & * & * & * & 0 \\ 0 & * & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$



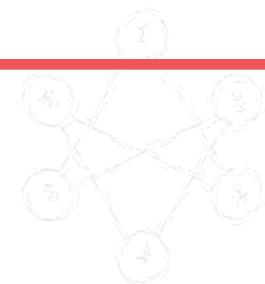
EMBER DATA / FORM BUILDER

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & * & 0 \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$

At a first glance, this is nothing but the structure of the form builder matrix

just displayed, but its graph



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 3, \quad 5 \rightarrow 6, \quad 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$ is called a path joining the vertices i_0 and j_0 if $i_0 \in \{i_0, j_0\}$, $j_0 \in \{i_0, j_0\}$ and for every $k = 1, 2, \dots, n-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of V are joined by a unique simple path. Both tri-diagonal and any other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $n \geq 3$.

Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $T \setminus \{r\}$ and these vertices are successors of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a predecessor of α and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree:



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so, that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

and their graphs.

SUSPECT 3



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

STATE

At a first glance, this is nothing but a list of the elements of the matrix, but this is not the case. The graph of the matrix is

just displayed, but its graph:



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$ $\subseteq \mathbb{E}$ is called a path joining the vertices i_0 and j_0 if $i_0 \in \{i_0, j_0\}$, $j_0 \in \{i_0, j_0\}$ and for every $k = 1, 2, \dots, n-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that \mathbb{G} is a tree if each two members of \mathbb{V} are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $n \geq 3$.

Given a tree \mathbb{T} and an arbitrary vertex $r \in \mathbb{V}$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in a ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $\mathbb{V} \setminus \{r\}$ and these vertices are successors of r . Moreover, every $\alpha \in \mathbb{V} \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a predecessor of α and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three consecutive orderings of the same rooted tree.



Theorem 11.1. Let A be a symmetric matrix whose graph \mathbb{G} is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A were been arranged so, that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

and their graphs.

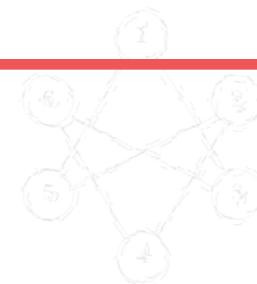
SUSPECT 4



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

LEAKAGE

At a first glance, this is nothing but a collection of the four matrices above, but just displayed, but its graph:



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

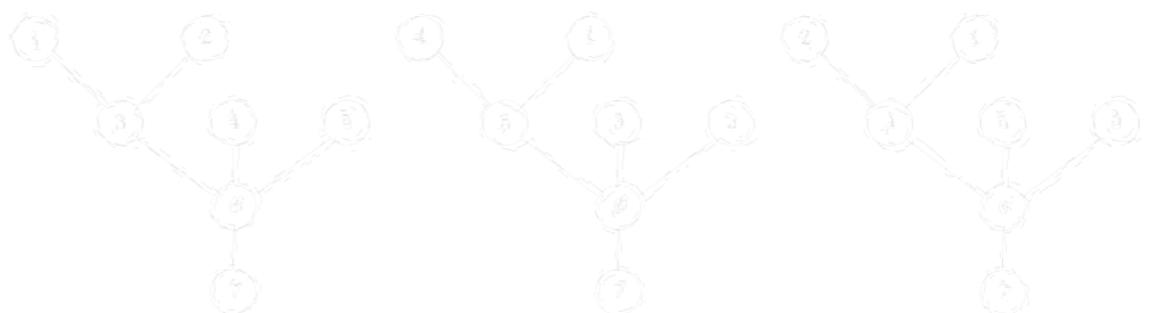
$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 1.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$ is called a path joining the vertices i_0 and j_0 if $i_0 \in \{i_0, j_0\}$, $j_0 \in \{i_0, j_0\}$ and for every $k = 1, 2, \dots, n-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of V are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $n \geq 3$.

Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in a ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $T \setminus \{r\}$ and these vertices are successors of r . Moreover, every $a \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and a , as a predecessor of a and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three consecutive orderings of the same rooted tree:



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A were been arranged so, that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

and their graphs.

SUSPECT 5



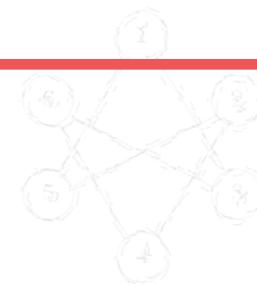
ADMIN

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

PRIVILEGE

At a first glance, this is nothing but a list of the four columns of the matrix

just displayed, but its graph,



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

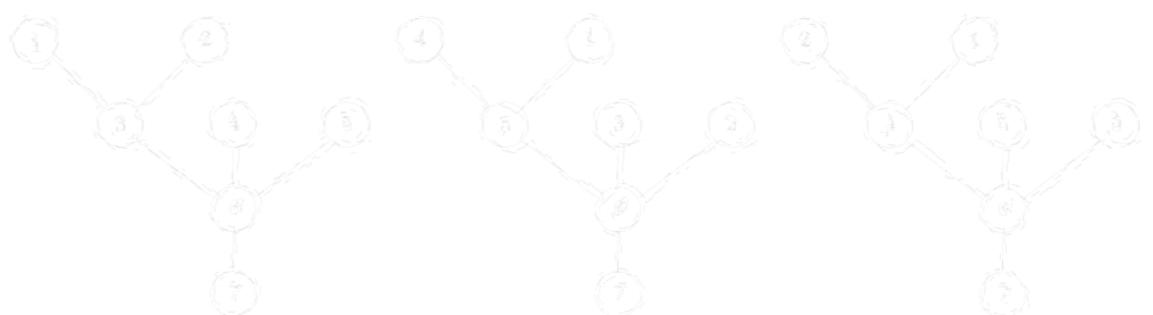
$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 8.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_0, j_0)\}_{i_0, j_0 \in \mathbb{N}_0, i_0 \neq j_0}$ is called a path joining the vertices i_0 and j_0 if $i_0 \in \{i_0, j_0\}$, $j_0 \in \{i_0, j_0\}$ and for every $k = 1, 2, \dots, n-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of V are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $n \geq 3$.

Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in a ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $T \setminus \{r\}$ and these vertices are successors of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a predecessor of α and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

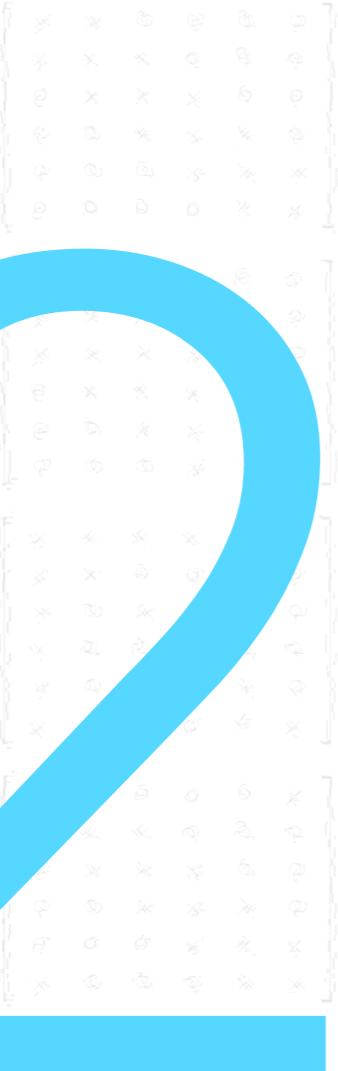
Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three consecutive orderings of the same rooted tree.

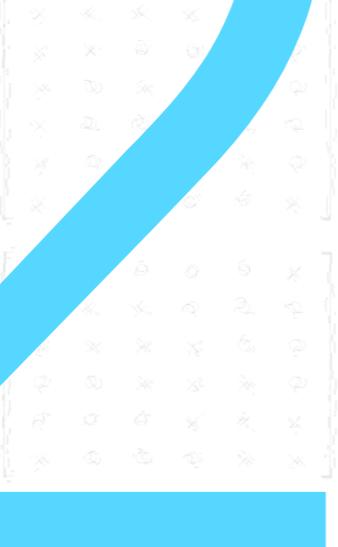


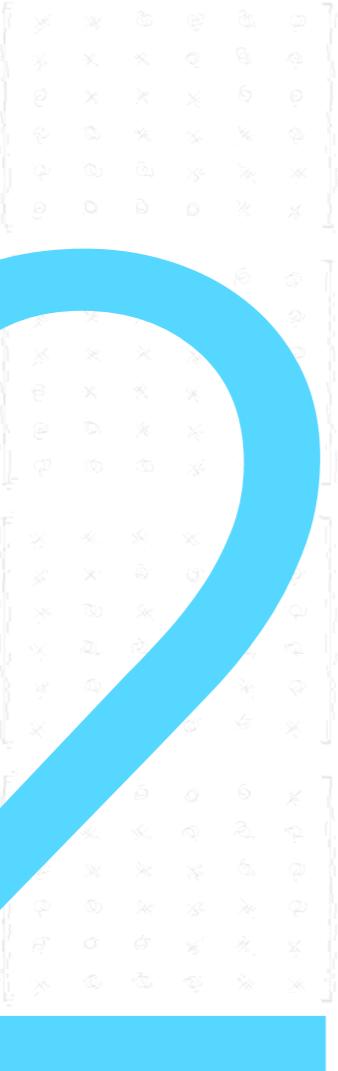
Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A were been arranged so, that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

tridiagonal:


symmetric:


cyclic:


2

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix



At a first glance, there is nothing to link it to any of the three structures just displayed, but its graph,

USE COMMON EVERYDAY WORDS



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 1.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{v_1, v_2, \dots, v_p\} \subseteq E$ is called a *path* joining the vertices v_1 and v_p ($v_i \in V$). If $v_1 = v_p$ and for every $i = 1, \dots, p-1$ the set $\{v_i, v_{i+1}\} \cap E$ contains exactly one member, it is a *simple path* if it does not visit any vertex more than once. We say that G is *tree* if each two members of V are joined by a unique simple path. (In this case, all other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $p \geq 3$.)

Given a tree G and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a *rooted tree*, while r is said to be the *root*. Unlike in ordinary graph, T adapts a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the *predecessor* of all the vertices in $V \setminus \{r\}$ and these vertices are *successors* of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a *predecessor* of α and a *successor* of r . We say that the rooted tree T is *monotonically ordered* if each vertex is labelled before all its predecessors; in other words, we *label* the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is unique. To give some concrete examples of the simple rooted trees,



Theorem 1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so, that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

and their graphs

triangular: $\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ 0 & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & * & 0 \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix} \Rightarrow \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \xrightarrow{1 \rightarrow 1, 2 \rightarrow 3, 3 \rightarrow 5, 4 \rightarrow 6, 5 \rightarrow 6, 6 \rightarrow 6}$

quidiagonal: $\begin{bmatrix} * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ 0 & * & * & * & * & * \\ 0 & 0 & * & * & * & * \\ 0 & 0 & 0 & * & * & * \end{bmatrix} \Rightarrow \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \xrightarrow{1 \rightarrow 1, 2 \rightarrow 3, 3 \rightarrow 4, 4 \rightarrow 5, 5 \rightarrow 6, 6 \rightarrow 6}$

superdiagonal: $\begin{bmatrix} * & * & * & * & * & * \\ * & 0 & 0 & 0 & 0 & 0 \\ * & 2 & 0 & 0 & 0 & 0 \\ * & 2 & 2 & 0 & 0 & 0 \\ * & 2 & 2 & 2 & 0 & 0 \\ * & 2 & 2 & 2 & 2 & 0 \end{bmatrix} \Rightarrow \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \xrightarrow{1 \rightarrow 1, 2 \rightarrow 2, 3 \rightarrow 3, 4 \rightarrow 4, 5 \rightarrow 5, 6 \rightarrow 6}$

cyclic: $\begin{bmatrix} * & 0 & 0 & 0 & * & * \\ * & * & 0 & 0 & 2 & 2 \\ * & * & * & 0 & 2 & 2 \\ * & * & * & * & * & 2 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix} \Rightarrow \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \xrightarrow{1 \rightarrow 1, 2 \rightarrow 2, 3 \rightarrow 3, 4 \rightarrow 4, 5 \rightarrow 5, 6 \rightarrow 6}$

CONVENTION;

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

AGREED ON BY MANY

At a first glance, this is not a triangular matrix, but it is a tree structure, as the one just displayed, but its graph:



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 3, 3 \rightarrow 2, 4 \rightarrow 4, 5 \rightarrow 6, 6 \rightarrow 3.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$ $\subseteq \mathbb{E}$ is called a path joining the vertices i_0 and j_0 if $i_0, j_0 \in \{1, \dots, n\}$, $i_0 \neq j_0$, $\{i_0, j_0\} \in \mathbb{E}$ and for every $k = 1, 2, \dots, n-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of V are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $n \geq 3$.

Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in ordinary graph, T adapts a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $T \setminus \{r\}$ and these vertices are successors of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a predecessor of α and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree:



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, n\}$ and assume that the rows and columns of A are then arranged so, that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

tridiagonal:

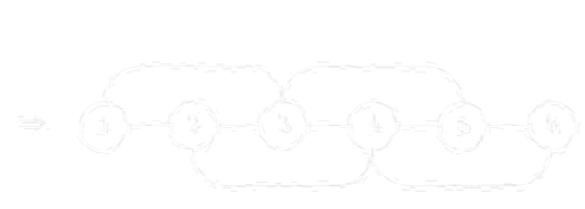
$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & 0 & * & * & 0 & 0 \\ 0 & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & * & * \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$

EVERYDAY



quindiagonal:

$$\begin{bmatrix} * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



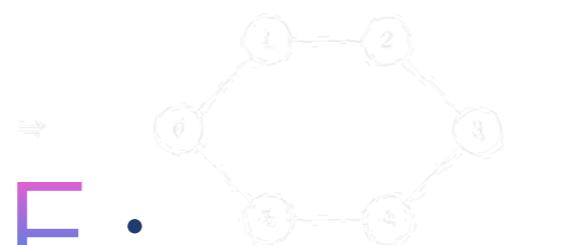
superdiagonal:

$$\begin{bmatrix} * & * & * & * & * & * \\ * & * & 0 & 0 & 0 & 0 \\ 0 & 0 & * & * & * & * \\ 0 & 0 & * & * & * & * \\ 0 & 0 & * & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



cyclic:

$$\begin{bmatrix} * & 0 & 0 & 0 & * & * \\ * & * & 0 & 2 & * & * \\ 0 & 0 & * & * & * & * \\ 0 & 2 & * & * & * & * \\ 0 & 0 & * & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



SIMPLE;

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

FAMILIAR TO MANY

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & * & * & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & * & * & * & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & * & * & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & * & * & * & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$

At a first glance this is not a matrix that is familiar to many, but it is the matrix just displayed, but its graph.



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

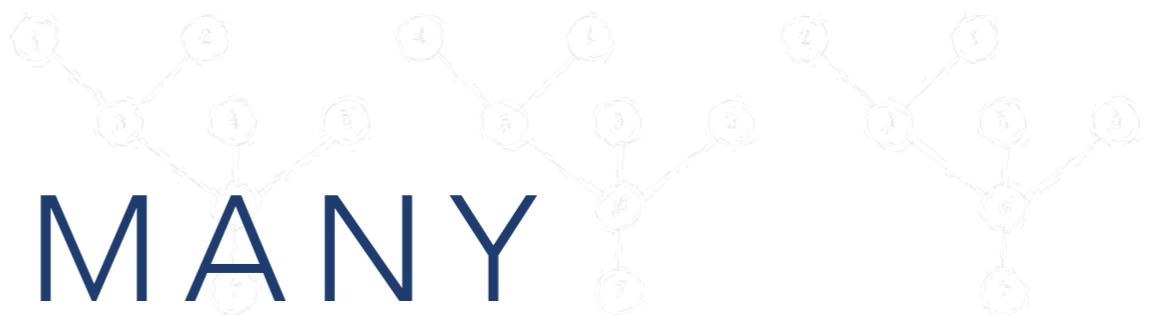
$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 3, \quad 5 \rightarrow 6, \quad 6 \rightarrow 8.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$ $\subseteq \mathbb{E}$ is called a path joining the vertices i_0 and j_0 if $i_0, j_0 \in \{1, \dots, n\}$, $i_0 \neq j_0$, $\beta \in \{i_0, j_0\}$ and for every $k = 1, 2, \dots, \nu - 1$ the set $\{i_{k+1}, j_{k+1}\} \cap \{i_k, j_k\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of V are joined by a unique simple path. Both tridiagonal and quidiagonal matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $n \geq 3$.

Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in a ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $T \setminus \{r\}$ and these vertices are successors of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a predecessor of α and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree.



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

triangular:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$

quadrangular:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$

cyclic:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$

At a first glance, the last matrix is not symmetric, but it is a triangular matrix, as we have just displayed, but its graph,



IF f IS CONTINUOUS IN $[a, b]$, AND IF $f(a)f(b) < 0$,

THEN f MUST HAVE A ZERO IN (a, b) .



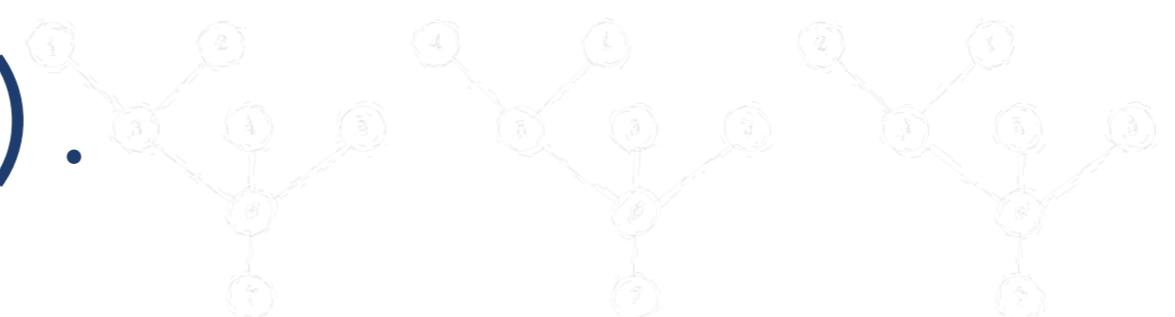
tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 3 \rightarrow 5, \quad 2 \rightarrow 4 \rightarrow 6, \quad 3 \rightarrow 1, \quad 4 \rightarrow 2.$$

Of course, it is equivalent to reordering simultaneously the equations and variables. As a result, we get the following graph G . It is called a tree spanning the vertices a and b ($a \in \{1, 2, \dots, n\}$, $b \in \{1, 2, \dots, n\}$) and for every $k = 1, 2, \dots, n-1$ the set $\{v_k, w_k\} \cap \{v_{k+1}, w_{k+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of V are joined by a unique simple path. Both triangular and quadrangular matrices correspond to trees, but this is not the case with either of tridiagonal or cyclic matrices when $n \geq 3$.

It is natural to call the tree $T = (G, r)$ a rooted tree, where r is the root vertex. Unlike in a binary tree, there is no natural partial order in which vertices can be compared. This is why we say that T is a partially ordered tree. Thus, the root r is the predecessor of all the vertices in $V \setminus \{r\}$ and these vertices are successors of r . Moreover, every $v \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and v , as a predecessor of v and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled by an integer from 1 to n in such a way that the vertices from the top of the tree to the bottom are in increasing order. (In other words, we say the vertices from the top of the tree to the bottom are in increasing order, because from the top to the bottom, the labels increase.) Labelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree:



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

```
assert.dom('[data-test-message]')  
  .hasText(  
    'Thanks for signing up!',  
    'The user sees a welcome message.'  
  );
```

Therefore we will now give a few examples of matrices (represented by their sparsity)

triangular:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



quadrangular:

supersingular:

$$\begin{bmatrix} * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & * & * & * \end{bmatrix}$$



cyclic:

$$\begin{bmatrix} * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \end{bmatrix}$$



ZERO IN (a, b) .

$$\begin{bmatrix} * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \end{bmatrix}$$



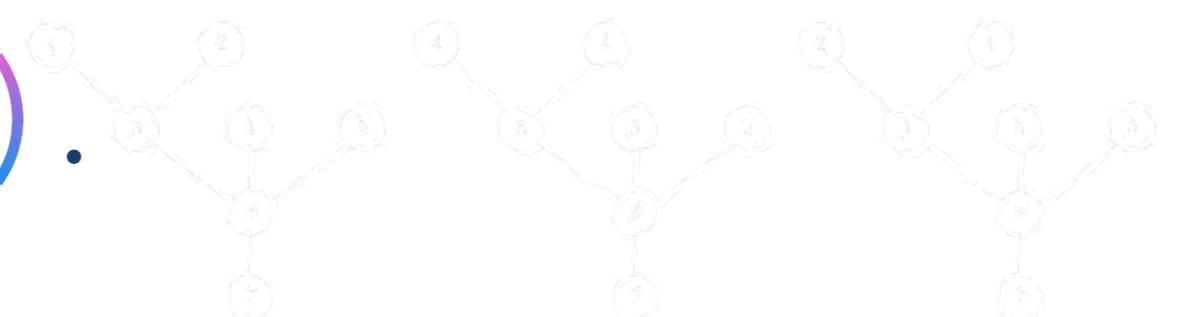
tells a different story - it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 2, 3 \rightarrow 3, 4 \rightarrow 4, 5 \rightarrow 5, 6 \rightarrow 6.$$

Of course, it is equivalent to relabeling the columns and variables. As you can see from the figure, the graph G is called a tree, since the vertices v and α ($v \in V \setminus \{\alpha\}$, $\beta \in V \setminus \{\alpha\}$) and for every $k = 1, 2, \dots, n-1$ the set $\{v, \alpha\} \cap \{v_k, \beta_k\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of V are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees, but this is not the case with either of banded or cyclic matrices when $n \geq 3$.

treating T as a binary tree, the root $r \in T$ is called a *rooted tree*. Unlike in ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $T \setminus \{r\}$ and these vertices are successors of r . Moreover, every $v \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and v , as a predecessor of v and a successor of r . We say that the rooted tree T is *monotonically ordered* if each vertex is labelled by an integer from 1 to n in such a way that the vertices from the top of the tree to the bottom are in increasing order (in other words, we say the vertices from the top of the tree to the bottom are in increasing order). Labeling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree:



Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, n\}$ and assume that the rows and columns of A are then arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = L U$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{a_{jj}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

At a first glance, this is not a very useful formula, but it is the basis for the following algorithm.

just displayed, but its graph.

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, first relabel the vertices as follows:



Dashboard

Explore

Settings



vertices more than once. We say that G is a *tree* if each two members of V are joined by a unique simple path. Both tridiagonal and super-diagonal matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $p \geq 3$.

Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a *rooted tree*, while r is said to be the *root*. Unlike in ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the *predecessor* of all the vertices in $T \setminus \{r\}$ and these vertices are *successors* of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a *predecessor* of α and a *successor* of r . We say that the rooted tree T is *monotonically ordered* if each vertex is labelled before all its predecessors. In other words, if label the vertices from the root of the

'[data-test-link="Dashboard"]'

'[data-test-link="Explore"]'

'[data-test-link="Settings"]'

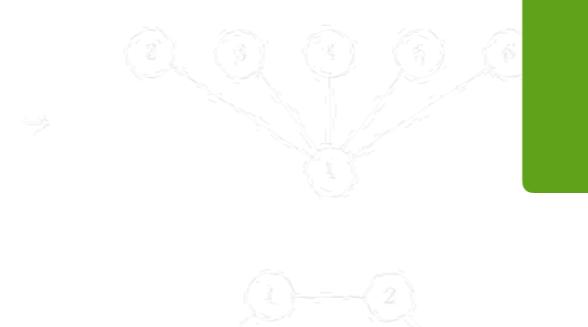
just displayed, but its graph.

$$t_{k,j} = \frac{a_{k,j}}{a_{j,j}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

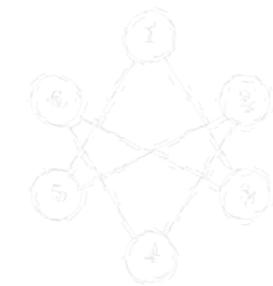
Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

Save

tridiagonal:	$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$
quindiagonal:	$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ 0 & * & * & * & * & 0 \\ 0 & 0 & * & * & * & * \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$
superdiagonal:	$\begin{bmatrix} * & * & * & * & * & * \\ * & * & 0 & 0 & 0 & 0 \\ 0 & * & * & 0 & 0 & 0 \\ 0 & 0 & * & * & 0 & 0 \\ 0 & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$



Cancel



tells a different story - it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$1 \rightarrow 5, 2 \rightarrow 3, 3 \rightarrow 4, 4 \rightarrow 6, 6 \rightarrow 1$.

or reordering (simultaneously) the equations and variables.

$\{(v_i, j)\}_{i,j} \subseteq \mathbb{S}$ is called a path joining the vertices v_i and v_j , and for every $k = 1, 2, \dots, n-1$ the set $\{v_i, j_k\} \cap$

contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that \mathbb{G} is a tree if each two members of \mathbb{V} are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees. This is not the case with either quindiagonal or cyclic matrices when $n \geq 3$.

Let a tree \mathbb{T} and an arbitrary vertex $r \in \mathbb{V}$, the pair $\mathbb{T} = (Q, r)$ is called a rooted tree, while r is said to be the root. Unlike in ordinary graph, \mathbb{T} admits a natural partial order, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $\mathbb{V} \setminus \{r\}$ and these vertices are successors of r . Moreover, every $\alpha \in \mathbb{V} \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a predecessor of α and a successor of r . We say that the rooted tree \mathbb{T} is monotonically ordered if each vertex is labelled before all its predecessors. In other words, if label the vertices from the root of the

'[data-test-button="Save"]'

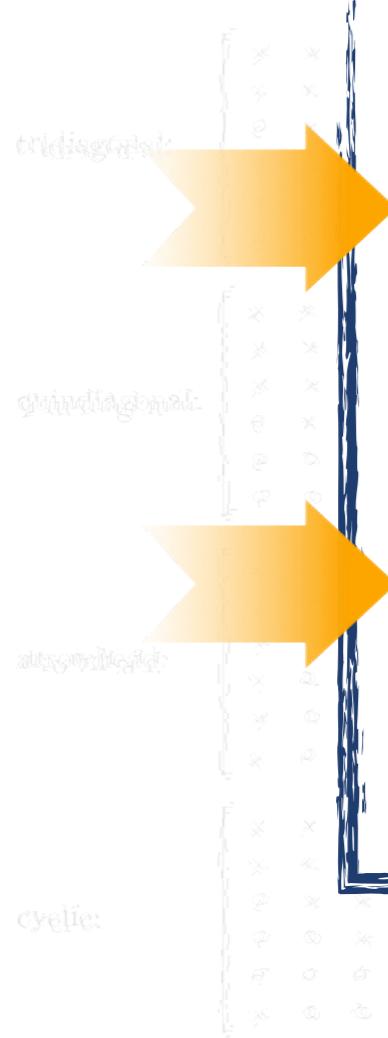
'[data-test-button="Cancel"]'

'[data-test-button="Add item"]'

just displayed, but its graph.

$$t_{k,j} = \frac{a_{k,j}}{a_{jj}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



Name*

Description

permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three consecutive orderings of the sample rooted tree.

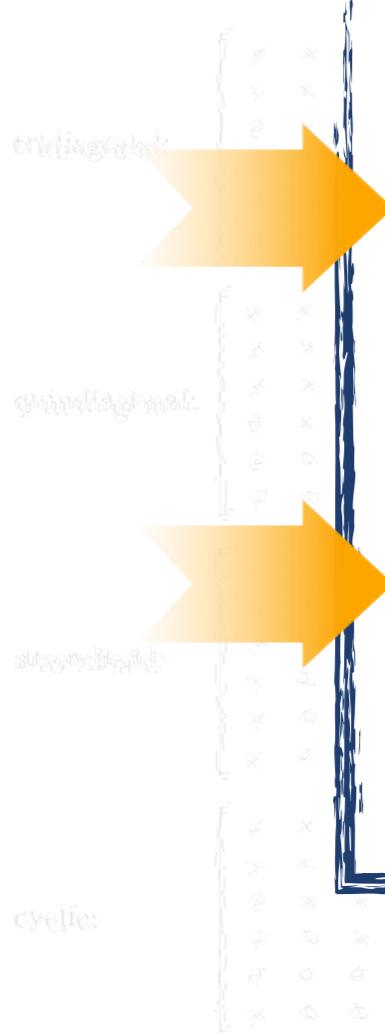
```
[data-test-field="Name"]
```

```
[data-test-field="Description"]
```

just displayed, but its graph.

$$t_{k,j} = \frac{a_{kj}}{a_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1, \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



Name

Little Bobby Tables

Description

Better not drop me!

matrix in disguised. To see
 $6 \Rightarrow 3$.
equations and vegetables.
th joining the vertices of
 $\{v_1, v_2, \dots, v_p\} \cap V$ in
ch if it does not visit any
numbers of V are joined by
edges correspond to trees.
atries when $p \geq 3$.
- (G, α) is called a rooted
digraph with natural partial
order a family tree. Thus, the
se vertices are *successors* or
e path and we designate
son of α and a *successor*
if each vertex is labelled
from the top of the
graph is *transient* to
permuting the rows and the columns of the underlying matrix.)
Every rooted tree will be monotonically ordered and, in general, such an ordering
is not unique. We now give three monotonic orderings of the sample rooted tree.

'[data-test-field="Name"]'

'[data-test-field="Description"]'

just displayed, but its graph.

$$t_{k,j} = \frac{a_{kj}}{a_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix



At a first glance, there is nothing to link it to any of the four structures just displayed, but its graph,

WRITE LESS WITH THEOREMS AND NEW TERMS

Theorem 11.1. Let A be a $n \times n$ matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, n\}$ and assume that the rows and columns of A are then arranged so, that $T = (G, r)$ is monotonically ordered. Then, that $A = LU$ is a Cholesky factorization, it is true that

$$l_{kj} = \frac{a_{kj}}{L_{jj}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

triangular:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



quadrangular:



supernodal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



cyclic:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



ZERO IN (a, b) .

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$

At a first glance this is not a matrix, but it is a little bit more than that. It is the matrix that was just displayed, but its graph:



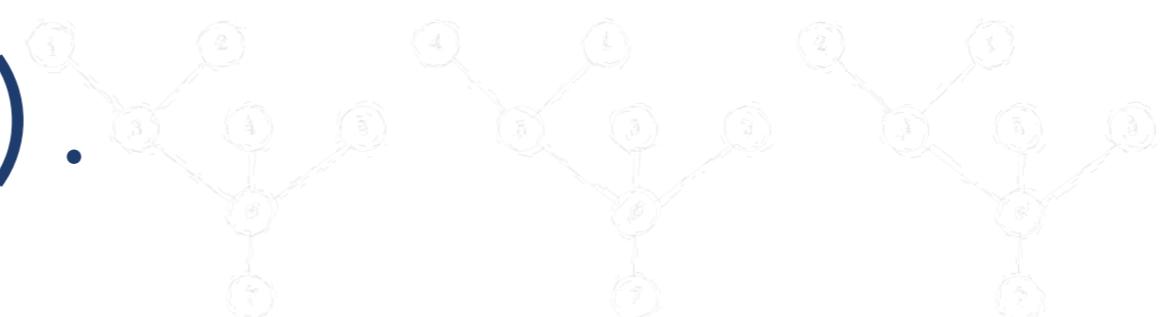
tells a different story - it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 2, 3 \rightarrow 3, 4 \rightarrow 4, 5 \rightarrow 5, 6 \rightarrow 6.$$

Of course, it is equivalent to relabeling the equations and variables. As you can see from the graph, G is a tree spanning the vertices $\{1, 2, \dots, 6\}$ and $\{i, j \in \{1, 2, \dots, 6\}, i \neq j\}$, and for every $k = 1, 2, \dots, n-1$ the set $\{v_k, w_k\} \cap \{v_{k+1}, w_{k+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of V are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees, but this is not the case with either of banded or cyclic matrices when $p \geq 3$.

As you can see from the graph, T is a rooted tree with root $r = 1$. Unlike in a binary tree, T is not a natural partial order, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $V \setminus \{r\}$ and these vertices are successors of r . Moreover, every $v \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and v , as a predecessor of v and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled by an integer from $\{1, 2, \dots, n\}$ in such a way that the vertices from the top of the tree to the bottom are in increasing order. (In other words, we say the vertices from the top of the tree to the bottom are in increasing order, because from the top to the bottom the labels increase.) Labelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree:



Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, n\}$ and assume that the rows and columns of A are then arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

PROOF.

USE THE INTERMEDIATE
VALUE THEOREM.

cyclic:

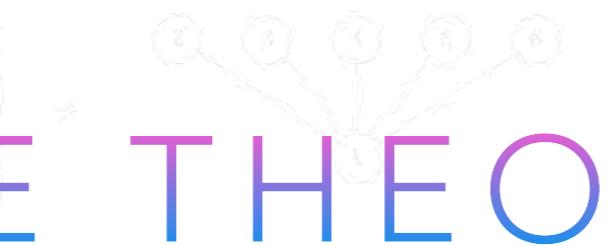
$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$



$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$



$$\begin{bmatrix} * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & 0 \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & 0 & * & 0 \\ * & 0 & * & 0 & * & 0 \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$

At a first glance this is not a cyclic matrix, but it is. Its graph is the one just displayed, but its graph:



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

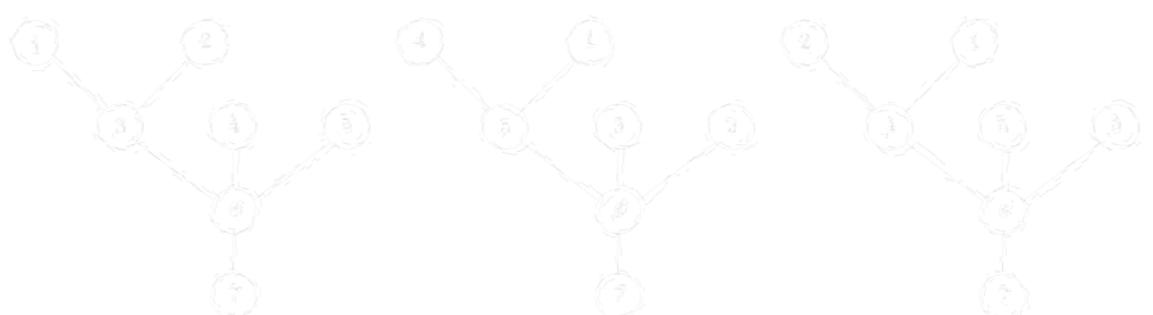
$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_0, j_0)\}_{i_0, j_0 \in \mathbb{N}_0, i_0 \neq j_0}$ in \mathbb{G} is called a path joining the vertices i_0 and j_0 if $i_0 \in \{i_0, j_0\}$, $j_0 \in \{i_0, j_0\}$ and for every $p = 1, 2, \dots, n-1$ the set $\{i_p, j_p\} \cap \{i_{p-1}, j_{p-1}\}$ contains exactly one member. It is a simple path if it does not visit any vertices more than once. We say that \mathbb{G} is a tree if in two members of \mathbb{V} are joined by a unique simple path. Known tridiagonal and cyclic matrices correspond to trees, but this is not the case with either quadiagonal or cyclic matrices when $n \geq 3$.

Given a tree \mathbb{T} and an arbitrary vertex $r \in \mathbb{V}$, the pair $\mathbb{T} = (\mathbb{G}, r)$ is called a rooted tree, while r is said to be the root. Unlike in ordinary graph, \mathbb{T} admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root is the ancestor of all the vertices in $\mathbb{T} \setminus \{r\}$ and these vertices are successors of r . Moreover, every $a \in \mathbb{V} \setminus \{r\}$ is joined to r by a simple path and we designate any vertex along this path, except for r and a , as a predecessor of a and a successor of r . We say that the rooted tree \mathbb{T} is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree:



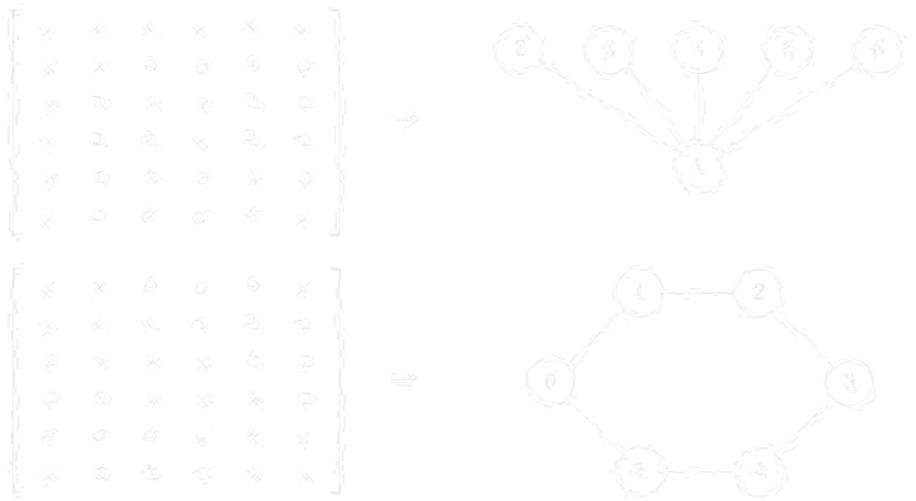
Theorem 11.1. Let A be a symmetric matrix whose graph \mathbb{G} is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so that $\mathbb{T} = (\mathbb{G}, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

PROOF.
USE IVT.

triangular:
quadratic:
cyclic:



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & x & 0 \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & 0 & * & 0 \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & * & 0 \end{bmatrix}.$$

At a first glance this is not immediately clear what the structure of the matrix is. The graph, however, is just displayed, but its graph:



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 3, \quad 5 \rightarrow 6, \quad 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$ is called a path joining the vertices i_0 and j_0 if $i_0 \in \{i_0, j_0\}$, $j_0 \in \{i_0, j_0\}$ and for every $k = 1, 2, \dots, n-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of V are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $n \geq 3$.

Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in ordinary graph, T adapts a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $T \setminus \{r\}$ and these vertices are successors of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a predecessor of α and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we lay out the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three consecutive orderings of the same rooted tree:

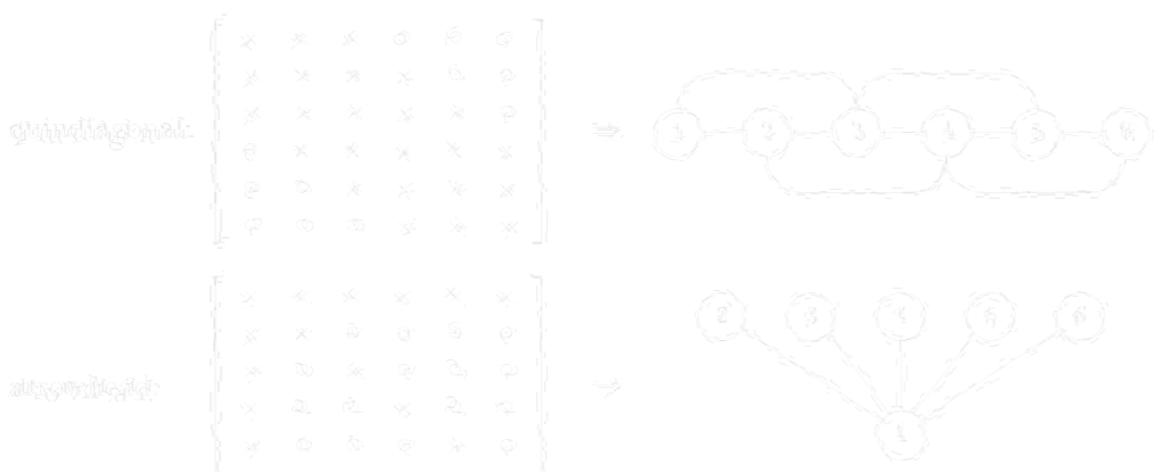


Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so, that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{j,j}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

THEOREM



PROVEN

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

TO BE TRUE

At a first glance this is a random matrix, but it is in fact a triangular matrix, just displayed, but its graph,



tells a different story – it is nothing other than the cyclic matrix in discussed. To see this, just relabel the vertices as follows:

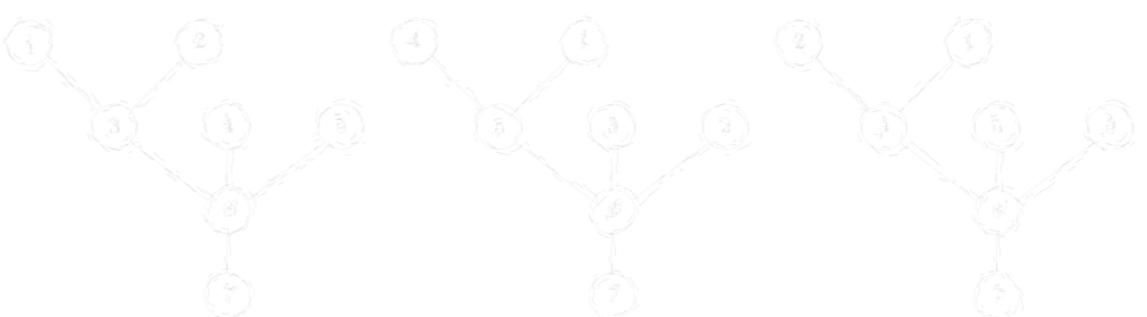
$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$ is called a path joining the vertices i_0 and j_0 if $i_0 \in \{i_0, j_0\}$, $j_0 \in \{i_0, j_0\}$ and for every $k = 1, 2, \dots, n-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of V are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $n \geq 3$.

Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in a ordinary graph, T adapts a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $T \setminus \{r\}$ and these vertices are successors of r . Moreover, every $a \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and a , as a predecessor of a and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree:



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A were been arranged so, that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

```
import { fillIn } from '@ember/test-helpers';

export async function fillForm(fields) {
  for (const { label, value } of fields) {
    // input, textarea
    await fillIn(`[data-test-field="${label}"`, value);
  }
};
```

```
import { fillForm } from '../helpers/my-test-helpers';

...
test('User can create account', async function(assert) {
  await visit('/signup');
  await fillForm([
    { label: 'Name', value: 'Little Bobby Tables' },
    { label: 'Email', value: 'little.bobby@gmail.com' }
  ]);
  ...
});
```

Therefore we will now give a few examples of matrices (represented by their sparsity)

NEW TERM

quidiagonal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ 0 & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$



superdiagonal:

$$\begin{bmatrix} * & * & * & * & * & * \\ * & 0 & 0 & 0 & 0 & 0 \\ 0 & * & 0 & 0 & 0 & 0 \\ 0 & 0 & * & 0 & 0 & 0 \\ 0 & 0 & 0 & * & 0 & 0 \\ 0 & 0 & 0 & 0 & * & 0 \end{bmatrix}$$



cyclic:

$$\begin{bmatrix} * & 0 & 0 & 0 & * & * \\ * & * & 0 & 0 & 0 & 0 \\ 0 & * & * & 0 & 0 & 0 \\ 0 & 0 & * & * & 0 & 0 \\ 0 & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



UBIQUITOUS

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

IDEA

$$\begin{bmatrix} * & * & 0 & 0 & * & * \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$

At a first glance, this is not a matrix that looks like the three structures just displayed, but its graph,



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 3, \quad 5 \rightarrow 6, \quad 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$ is called a path joining the vertices i_0 and j_0 if $i_0 \in \{i_0, j_0\}$, $j_0 \in \{i_0, j_0\}$ and for every $k = 1, 2, \dots, n-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of V are joined by a unique simple path. Both tri-diagonal and any other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $n \geq 3$.

Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in a ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $T \setminus \{r\}$ and these vertices are successors of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a predecessor of α and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree:



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, n\}$ and assume that the rows and columns of A are then arranged so, that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

triangular:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$

quadrangular:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$

cyclic:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$

At a first glance this is not a triangular or quadrangular matrix, but it is a triangular matrix just displayed, but its graph,



IF f IS CONTINUOUS IN $[a, b]$, AND IF $f(a)f(b) < 0$,

THEN f MUST HAVE A

ZERO IN (a, b) .



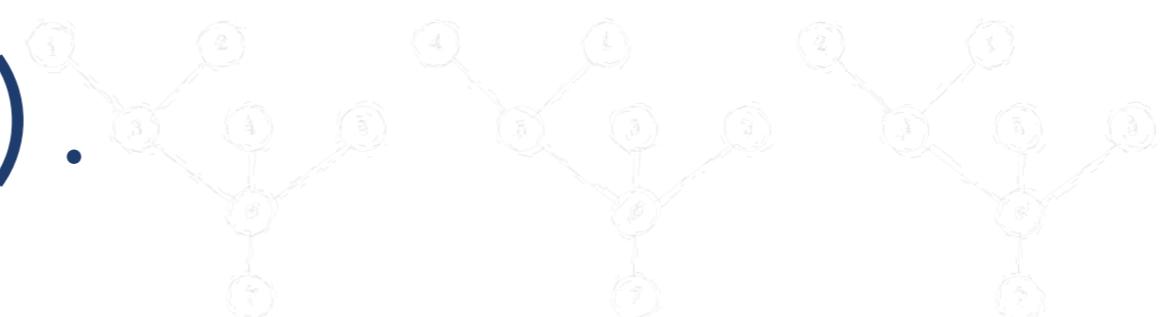
tells a different story - it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 3 \rightarrow 5, \quad 2 \rightarrow 4 \rightarrow 6, \quad 3 \rightarrow 1, \quad 4 \rightarrow 2.$$

Of course, this is equivalent to reordering (or relabeling) the equations and variables. As a result, the set of edges $\{v_i, v_j\}$ is called a *tree* spanning the vertices v_i and v_j ($i, j \in \{1, 2, \dots, n\}$, $i \neq j$) and for every $k = 1, 2, \dots, n-1$ the set $\{v_{k+1}, v_{k+2}\} \cap \{v_{k+2}, v_{k+3}\}$ contains exactly one member. It is a *simple path* if it does not visit any vertex more than once. We say that G is a *tree* if each two members of V are joined by a unique simple path. Both triangular and quadrangular matrices correspond to trees, but this is not the case with either quadrangular or cyclic matrices when $n \geq 3$.

It is natural to call a tree $T = (G, r)$ a *rooted tree*, where $r \in V$ is called a *root*. This definition is due to the analogy with a family tree. To date, a natural partial ordering which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $V \setminus \{r\}$ and these vertices are successors of r . Moreover, every $v \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and v , as a predecessor of v and a successor of r . We say that the rooted tree T is *monotonically ordered* if each vertex is labelled by an integer from 1 to n in such a way that the vertices from the top of the tree to the bottom are arranged in increasing order. (In other words, we say the vertices from the top of the tree to the bottom are arranged in increasing order. Labeling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree:



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, n\}$ and assume that the rows and columns of A are then arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{k,j}}{l_{j,j}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

triangular:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



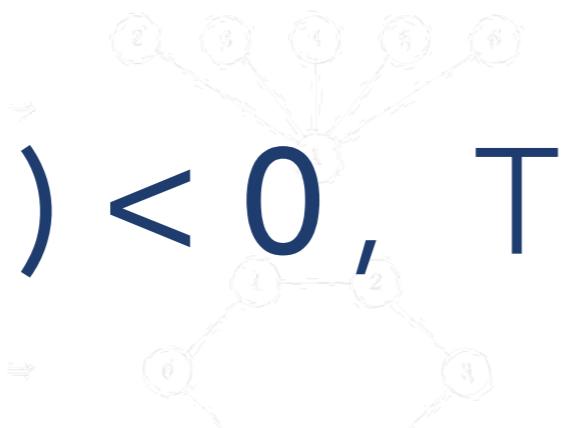
quidiagonal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



symmetric:

$$\begin{bmatrix} * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ 0 & * & * & * & * & * \\ 0 & 0 & * & * & * & * \\ 0 & 0 & 0 & * & * & * \end{bmatrix}$$



cyclic:

$$\begin{bmatrix} * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ 0 & * & * & * & * & * \\ 0 & 0 & * & * & * & * \\ 0 & 0 & 0 & * & * & * \end{bmatrix}$$



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} * & * & * & 0 & * & 0 \\ 0 & * & 0 & * & * & 0 \\ * & * & * & 0 & 0 & * \\ 0 & 0 & 0 & * & * & * \\ * & * & 0 & 0 & * & * \\ 0 & * & * & * & * & * \end{bmatrix}$$

At a first glance, this is not a symmetric matrix, but it is. It is a tridiagonal matrix, just displayed, but its graph,



tells a different story - it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_1, j_1)\}_{i_1, j_1 \in \mathbb{N}_0}$ in \mathbb{G} is called a path joining the vertices i_1 and j_1 if $i_1, j_1 \in \{1, \dots, n\}$, $i_1 \neq j_1$, $i_1, j_1 \in \mathbb{N}_0$ and for every $p = 1, 2, \dots, \nu - 1$ the set $\{i_p, j_p\} \cap \{i_{p+1}, j_{p+1}\}$ contains only one member. It is a simple path if it does not visit any vertex more than once. We say that \mathbb{G} is a tree if each two members of \mathbb{V} are joined by a unique simple path. Both quidiagonal and symmetric matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $n \geq 3$.

Given a tree \mathbb{G} and an arbitrary vertex $r \in \mathbb{V}$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in a digraph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the predecessor of the vertex v is the vertex u if v is a successor of u in the tree T . Moreover, every $u \in \mathbb{V}$ is the predecessor of v in the path and we designate u as a parent along this path, except for r , which is a predecessor of v and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we lay out the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We will give three examples consisting of the same rooted tree:

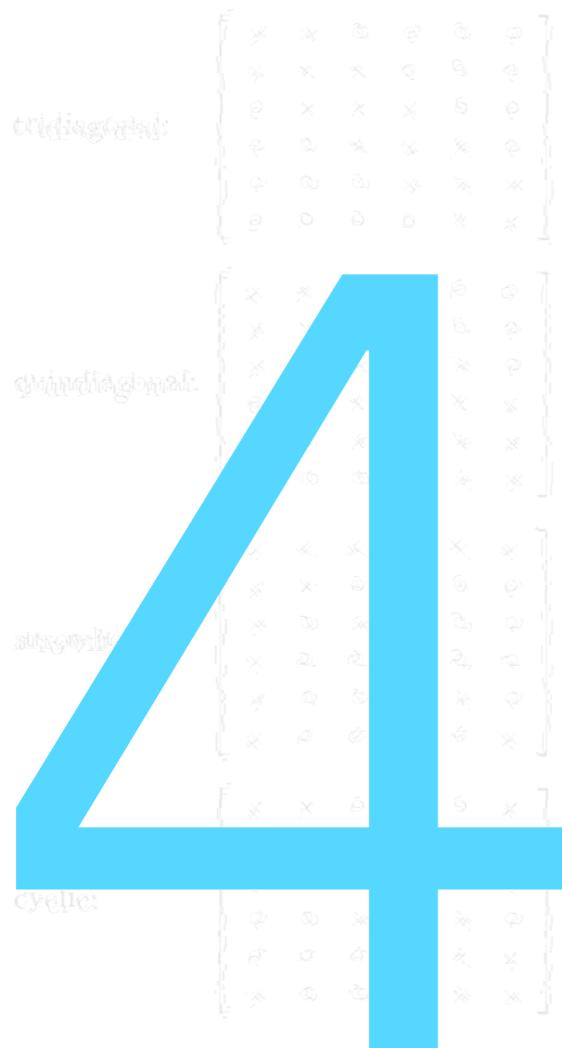


Theorem 11.1 Let A be a symmetric matrix whose graph \mathbb{G} is a tree. Choose a root $r \in \{1, 2, \dots, n\}$ and assume that the rows and columns of A are then arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = L U$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{k,j}}{a_{jj}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

```
hooks.beforeEach(function(assert) {  
  ...  
  // Example: assert.isEnabled('Submit', 'Woot!');  
  assert.isEnabled = (label, message) => {  
    assert.dom(`[data-test-button="${label}"]`)  
      .doesNotHaveAttribute('disabled', message);  
  };  
  ...  
});
```

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & * \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & * \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & * \end{bmatrix}$$

At a first glance, there is nothing to link it to any of the four structures just displayed, but its graph,

ALL YOUR BASIS ARE BELONG TO US



Let us illustrate it by a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{kj} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

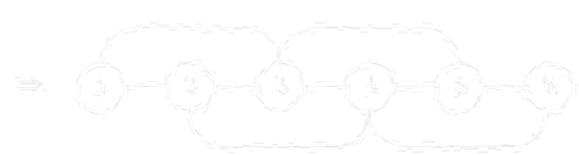
triangular:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ 0 & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



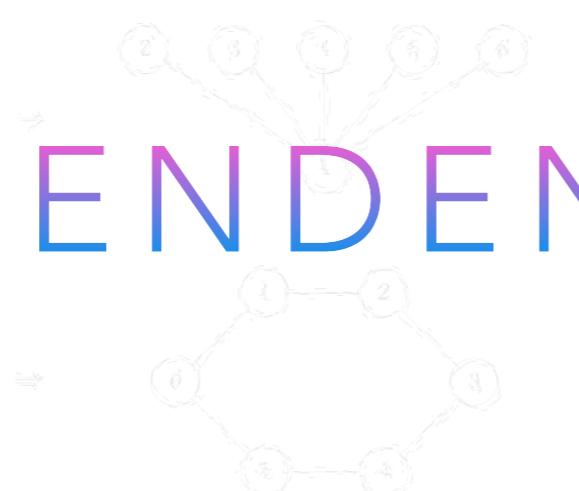
quidiagonal:

$$\begin{bmatrix} * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ 0 & * & * & * & * & * \\ 0 & * & * & * & * & * \\ 0 & 0 & * & * & * & * \end{bmatrix}$$



symmetric:

$$\begin{bmatrix} * & * & * & * & * & * \\ * & * & 0 & 0 & 0 & 0 \\ * & 0 & * & 0 & 0 & 0 \\ * & 0 & 0 & * & 0 & 0 \\ * & 0 & 0 & 0 & * & 0 \\ * & 0 & 0 & 0 & 0 & * \end{bmatrix}$$



THAT

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

SPAN THE ENTIRE SPACE

At a first glance, the matrix A in (11.1) does not look like it has a triangular structure, just displayed, but its graph,



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 3, \quad 5 \rightarrow 6, \quad 6 \rightarrow 1.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(v_i, j)\}_{i=1}^n$ in \mathbb{G} is called a path joining the vertices v_i and v_j if $i \in \{1, \dots, n\}$, $j \in \{1, \dots, n\}$ and for every $k = 1, 2, \dots, n-1$ the set $\{v_i, j\} \cap \{v_{i+k}, v_{i+k+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that \mathbb{G} is a tree if each two members of V are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $n \geq 3$.

Given a tree \mathbb{G} and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the only vertex of all the vertices and edges are successive generations of r . Moreover, every $v \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex alone, throughout, except for r and v , as a predecessor of v and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree:



Theorem 11.1. Let A be a symmetric matrix whose graph \mathbb{G} is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A were then arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph.

tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 3, 3 \rightarrow 2, 4 \rightarrow 4, 5 \rightarrow 6, 6 \rightarrow 3.$$

Of course, it is equivalent to renumbering (cyclic shift) the vertices and to consider a different set of edges $\{ (i, j) \mid i, j \in \{1, 2, \dots, 6\} \}$. It is called a *cycle* if the vertices $i, j \in \{1, 2, \dots, n\}$ and for every $\ell = 1, 2, \dots, n-1$ the set $\{i + \ell, j + \ell\}$ contains exactly one member. It is a *simple path* if it does not visit any vertex more than once. We say that G is a *tree* if each two members of V are joined by a unique path. Both tridiagonal and antidiagonal matrices correspond to the case with either quidiagonal or cyclic matrices when $\ell = 1, 2, \dots, n-1$ and no arbitrary vertex $v \in V \setminus \{r\}$ is called a *rooted tree* and r is called the *root*. Unlike in a ordinary graph, T admits a natural partial order that must be explained by an analogy with a family tree. Thus, the ancestor of all the vertices in $T \setminus \{r\}$ and these vertices are *successors*. Every $a \in V \setminus \{r\}$ is joined to r by a simple path and we designate this path, except for r and a , as a *predecessor* of a and a *successor*. The rooted tree T is *monotonically ordered* if each vertex is labelled with a natural number in other words, we label the vertices from the top of the tree to the bottom (we have already done it, relabeling a graph to a tree). (This is another reason to call T a *tree* and not a *forest* in general, such a labeling does not give three separate components of the same source tree.)



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so, that $T = (G, r)$ is monotonically ordered. Then, that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{j,j}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph.

tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 3, 3 \rightarrow 2, 4 \rightarrow 4, 5 \rightarrow 6, 6 \rightarrow 3.$$

Of course, it is equivalent to renumbering (cyclic shift) the vertices and the edges. The set of edges $\{(1, 2), (2, 3), (3, 1), (4, 5), (5, 6), (6, 4)\}$ is called a *cycle* in the graph G . It is a simple path if it does not visit any vertex more than once. We say that G is a *tree* if each two members of V are joined by a path. Both tridiagonal and cyclic matrices correspond to trees. There are two types of trees with either quidiagonal or cyclic matrices: $P(3)$ and $P(4)$. The first one has no arbitrary vertex $v \in V$ the part $T = G \setminus \{v\}$ is called a *rooted tree* and v is called the *root*. Unlike in a tridiagonal graph, T admits a natural partial order that must be explained by an analogy with a family tree. Thus, the ancestor of all the vertices in $T \setminus \{r\}$ and these vertices are successive. Every $a \in V \setminus \{r\}$ is joined to r by a simple path and we designate this path, except for r and a , as a *predecessor* of a and a *successor* of r . The rooted tree T is *monotonically ordered* if each vertex is labelled with a natural number. We label the vertices from the top of the tree to the bottom. (This is the same as the rows and the columns of the underlying matrix.)

LATITUDE
 30.267°

LONGITUDE
 -97.743°



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

The graph illustrates the relationship between the variables on the x-axis and the categories on the y-axis. The bars show a clear trend where the value increases as the category moves from left to right. A large black arrow at the bottom indicates this trend.

Red	triangular	quadratic	cubic	quartic
Red	Low	Medium	Medium	Medium
Blue	Medium	High	Very High	Very High

tells a different story - it is nothing other than the credit matrix in disguised. To see this, just replace the weights as follows:

3 → 3, 3 → 3, 3 → 3, 3 → 3, 6 → 6 // 4

GREEN 78

Red

Blue

Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, n\}$ and assume that the rows and columns of A have been permuted so that $T = (t_{ij})_{n \times n}$ is non-lexicographically oriented. Given that $A = LCL^T$ is a Cholesky factorization, it is true that

Therefore we will now give a few examples of matrices (represented by their sparsity)

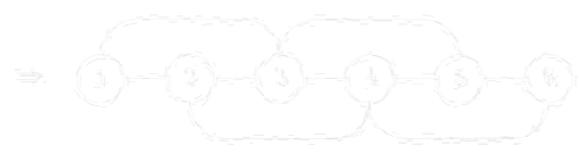
and their graphs

tridiagonal:
$$\begin{bmatrix} * & * & 0 & 0 & 0 \\ * & * & * & 0 & 0 \\ 0 & * & * & * & 0 \\ 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & * & * \end{bmatrix}$$



BASIS

quindiagonal:

$$\begin{bmatrix} * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ 0 & * & * & * & * & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$


superdiagonal:

$$\begin{bmatrix} * & * & * & * & * & * \\ * & * & 0 & 0 & 0 & 0 \\ 0 & * & * & 0 & 0 & 0 \\ 0 & 0 & * & * & 0 & 0 \\ 0 & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$


cyclic:

$$\begin{bmatrix} * & 0 & 0 & 0 & * & * \\ * & * & 0 & 2 & * & * \\ 0 & * & * & * & * & 0 \\ 0 & 2 & * & * & * & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$


BUILDING BLOCKS

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

OF TESTS

At a first glance, this is not a matrix, but a collection of little boxes containing numbers just displayed, but its graph:



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$ is called a path joining the vertices i_0 and j_0 if $i_0 \in \{1, \dots, n\}$, $j_0 \in \{1, \dots, n\}$ and for every $k = 1, 2, \dots, n-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of V are joined by a unique simple path. Both tridiagonal and superdiagonal matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $n \geq 3$.

Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in a ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $T \setminus \{r\}$ and these vertices are successors of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a predecessor of α and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is unique. The following diagram shows three consecutive orderings of the same rooted tree.



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, n\}$ and assume that the rows and columns of A were been arranged so, that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just re-label the weights as follows:



Create Edit Delete Clone Import

a single simple path. Both tridiagonal and antidiagonal matrices correspond to trees, but this is not the case with either anti-diagonal or cyclic matrices when $n > 3$.

	Name	Description
<input checked="" type="checkbox"/>	Little Bobby Tables	Better not drop me!
<input type="checkbox"/>	Big Bobby Tables	
<input type="checkbox"/>	Foo Bar	Making up names is hard.

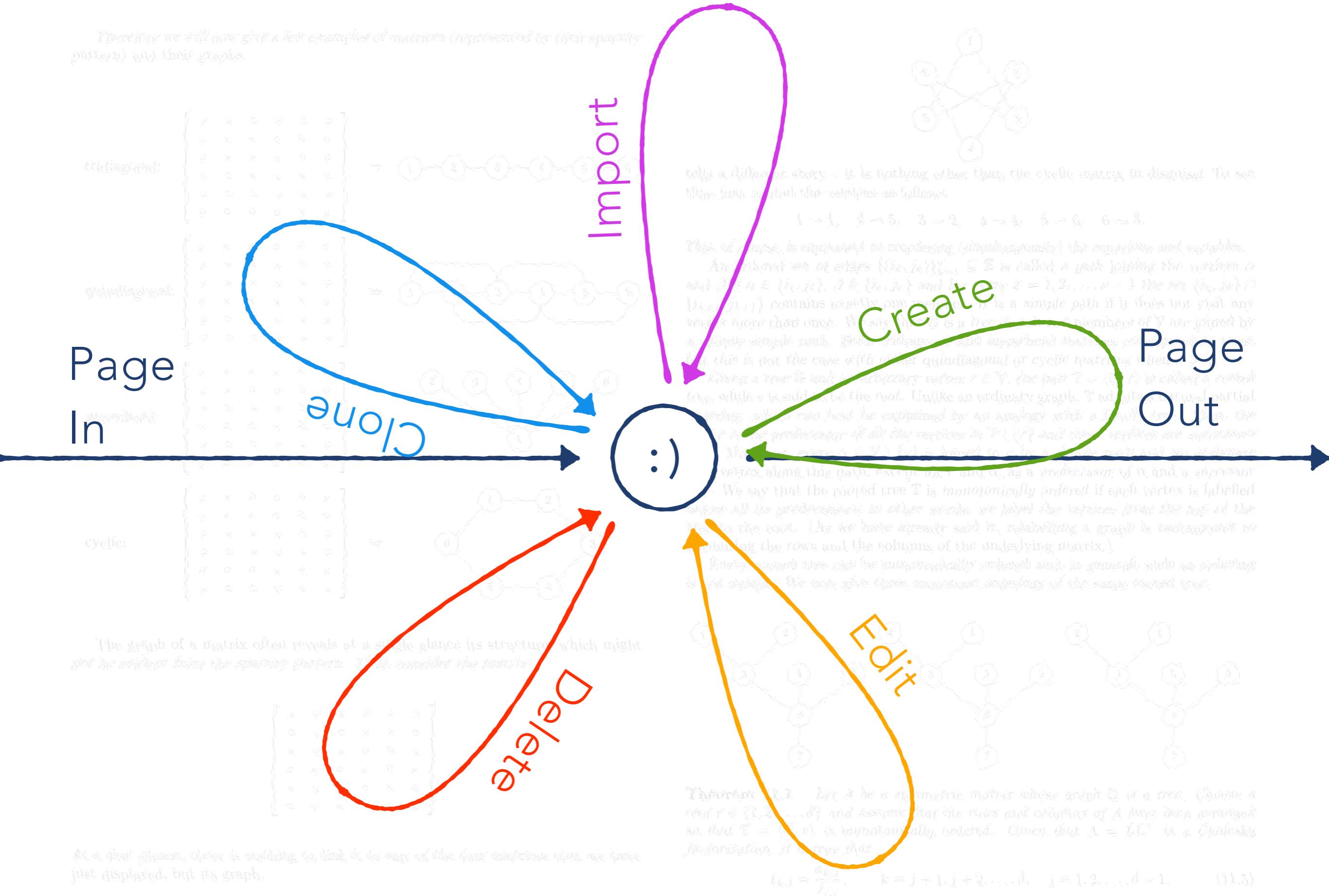


At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph:

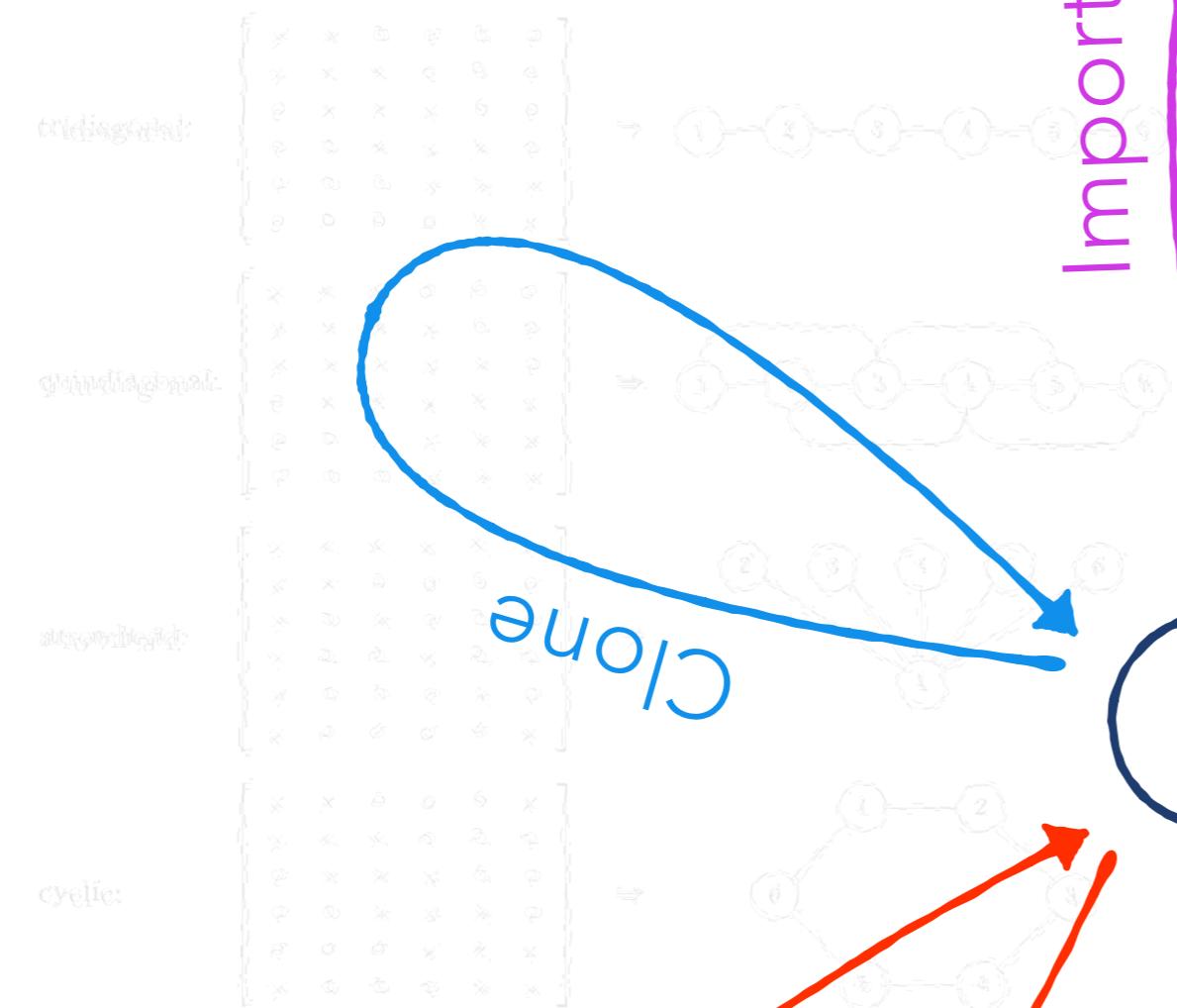


Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so that $\mathbf{T} = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ii}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$



Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix



At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,

tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just read the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 1.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_0, j_0), \dots, (i_{v-1}, j_{v-1})\}$ in \mathbb{G} is called a path joining the vertices i_0 and i_{v-1} ($i_0, i_1, \dots, i_{v-1}, i_v \in V$), $j_0 \in \{1, \dots, n\}$ and for any $k = 1, 2, \dots, v-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one vertex. It is a simple path if it does not visit any vertex more than once. We say that \mathbb{G} is a tree if no two members of V are joined by a single simple path. Both tridiagonal and quadiagonal matrices correspond to trees, but this is not the case with either quadiagonal or cyclic matrices when $n \geq 3$.

Create a tree T and a arbitrary vertex $r \in V$, the pair $T \sim (r, r)$ is called a rooted tree, while r is said to be the root. Unlike in a ordinary graph, T has a natural partial order, which can best be explained by an analogy with a family tree. Thus, the vertex r is the predecessor of all the vertices in $T \setminus \{r\}$ and these vertices are successors of r . Moreover, every $a \in V \setminus \{r\}$ is joined to r by a simple path and we designate a as a vertex along this path, except for r and a , as a predecessor of a and a successor of r respectively. We say that the rooted tree T is monotonically ordered if each vertex is labelled with all its predecessors in other words, we label the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to relabelling the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We will give three consecutive drawings of the same rooted tree.



Theorem 11.1. Let A be a symmetric matrix whose graph \mathbb{G} is a tree. Choose a root $r \in \{1, 2, \dots, n\}$ and assume that the rows and columns of A are now arranged so, that $T = (r, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{k,j}}{l_{j,j}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

Page
In

quidiagonal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 & 0 \\ 0 & * & * & 0 & 0 & 0 \\ 0 & 0 & * & * & 0 & 0 \\ 0 & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



quidiagonal:

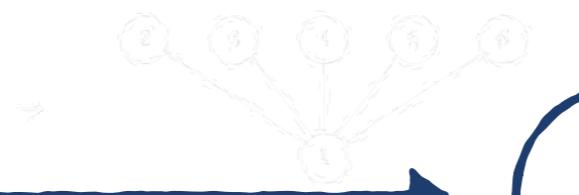
$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$



Page
Out

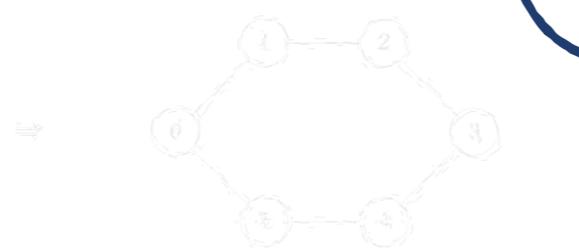
triangular:

$$\begin{bmatrix} * & * & * & * & * & * \\ * & * & 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$



cyclic:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & * \\ * & * & 0 & 0 & 0 & 0 \\ 0 & * & * & 0 & 0 & 0 \\ 0 & 0 & * & * & 0 & 0 \\ 0 & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & 0 \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & 0 \\ * & 0 & * & 0 & * & 0 \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 4.$$

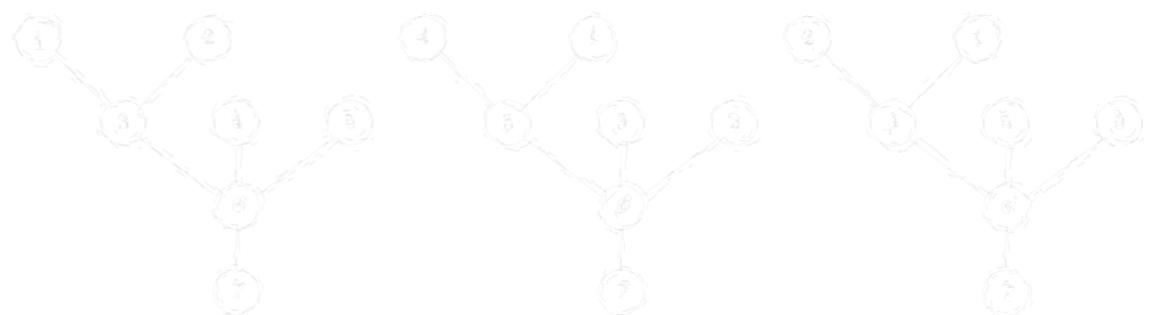
This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(v_i, j_i)\}_{i=1}^n$ in \mathbb{G} is called a path joining the vertices v_i and j_i if $i \in \{1, \dots, n\}$, $j_i \in \{1, \dots, n\}$ and for every $k = 1, 2, \dots, n-1$ the set $\{v_i, j_i\} \cap \{v_{i+1}, j_{i+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that \mathbb{G} is a tree if each two members of \mathbb{V} are joined by a unique simple path. Both tri-diagonal and any other matrices A are not trees, but this is not the case with either quidiagonal or cyclic matrices.

Given a tree \mathbb{T} and an arbitrary vertex $r \in \mathbb{V}$, the pair $\mathbb{T} = (\mathbb{G}, r)$ is called a rooted tree, while r is said to be the root. Unlike in ordinary graph, \mathbb{T} admits a natural partial order, which can best be explained by an analogy with a family tree. The vertex r is the predecessor of all the vertices in $\mathbb{V} \setminus \{r\}$ and these vertices are successors. Moreover, every $\alpha \in \mathbb{V} \setminus \{r\}$ has a unique predecessor α' and an immediate vertex along this path, except for r and α , as a predecessor of α and a successor.

We say that the rooted tree \mathbb{T} is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree.



Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their ranks.

$(|(&(a>=b)(c\sim=d))$
 $(| (A:dn:=B)(c=^*)$
 $(!(&(e<=f)$
 $(:caseExact:=h)$
 $(| (i=j)(!(k<=l))))))))$



just displayed, but its graph.

$$t_{k,j} = \frac{a_{k,j}}{b_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1, \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

(| (! (a>=b) (c~d)))
(| (A:dn := B) (c=*))
(!(&(e<=f))
(:caseExact:=h))
(| (i=j) (! (k<=l)))))))

! (| (! (a>=b) (c~d))) (| (A:dn := B) (c=*)) (!(&(e<=f)) (:caseE>))

OR FILTER GROUP

NOT FILTER GROUP

You can negate only 1 filter in a group.

(a>=b)

(c~d)

You need to use ~=.

OR FILTER GROUP

(A:dn := B)

You need to trim the attribute and filter type.

(c=*)

NOT FILTER GROUP

AND FILTER GROUP

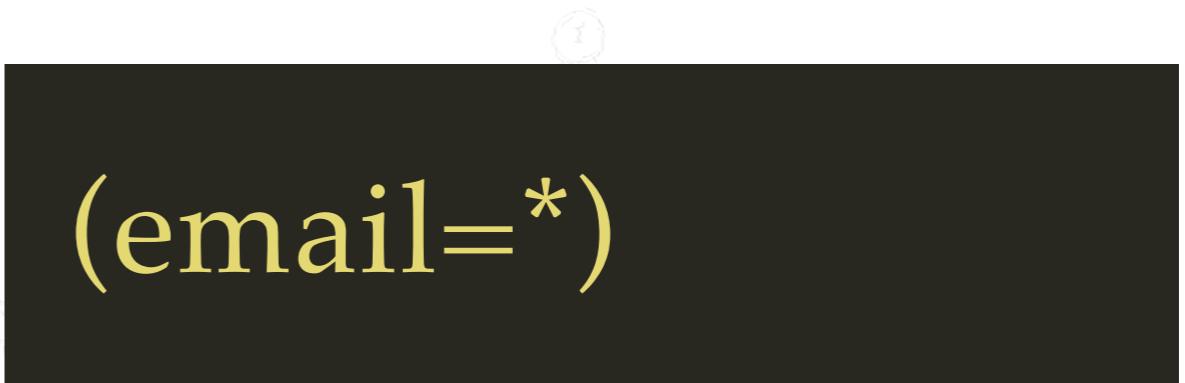
(e<=f)

(:caseExact:=h)

just displayed, but its graph.

$$t_{k,j} = \frac{a_{k,j}}{b_{j,j}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1, \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparser pattern. Thus, consider the matrix

SUBSTRING

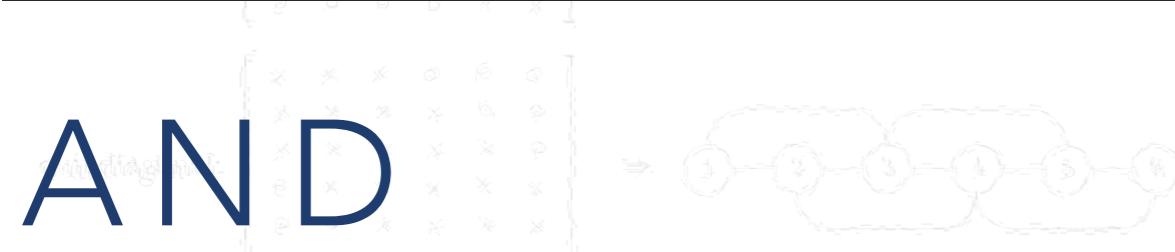
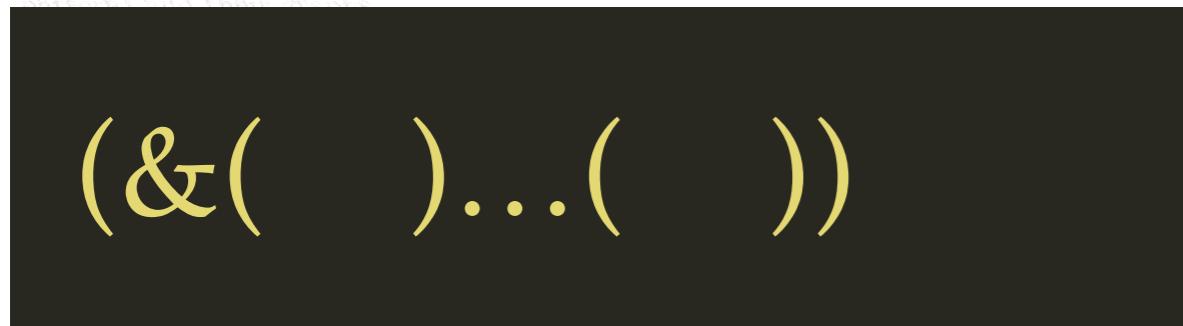
At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,



Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ii}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

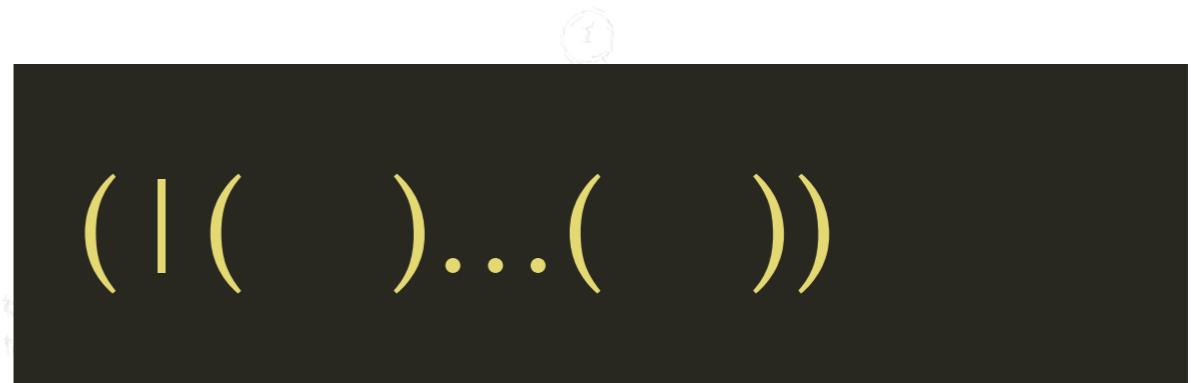
Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix



At a first glance, there is nothing to think in the case of the four matrices that we have just displayed, but its graph.



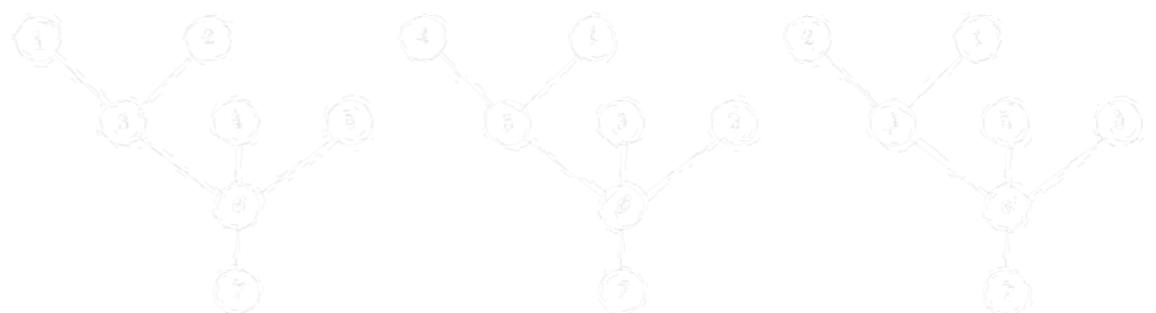
$1 \rightarrow 1, 2 \rightarrow 3, 3 \rightarrow 2, 4 \rightarrow 5, 5 \rightarrow 6, 6 \rightarrow 8$.

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(v_i, j_i)\}_{i=1}^n$ in \mathbb{G} is called a path joining the vertices v_i and v_{i+1} ($i = 1, 2, \dots, n-1$), $j_i \in \{1, \dots, n\}$ and for every $i = 1, 2, \dots, n-1$ the set $\{v_i, j_i\} \cap \{v_{i+1}, j_{i+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that \mathbb{G} is a tree if each two members of V are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $p \geq 3$.

Given a tree \mathbb{G} and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $V \setminus \{r\}$ and these vertices are successors of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a predecessor of α and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree.



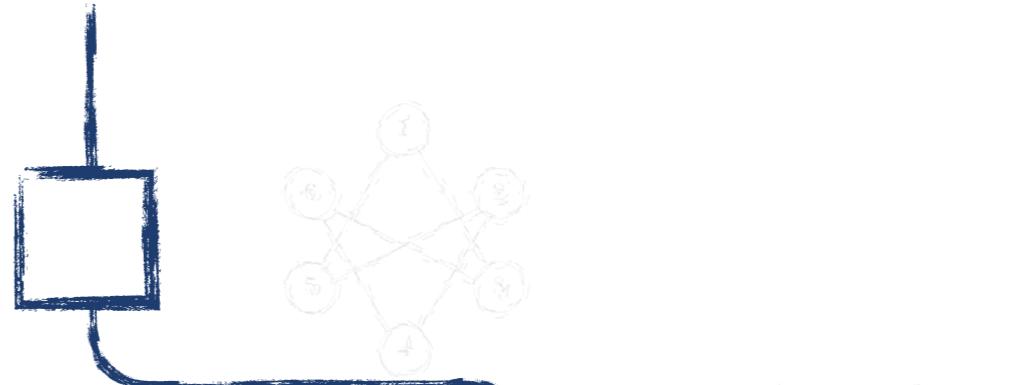
Theorem 11.1. Let A be a symmetric matrix whose graph \mathbb{G} is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ii}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

```
function addOne(x) {
  if (Number.isFinite(x)) {
    x = x + 1;
  } else {
    console.log('error');
  }
  return x;
}
```

just displayed, but its graph:



tells a different story – \mathbf{g} is not a tree. In a cyclic matrix in discussed. To see this, just re-label the vertices as follows:

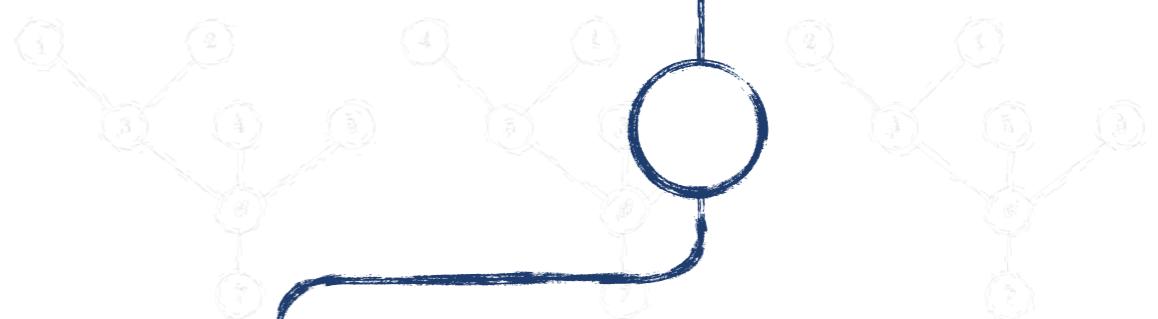
$$1 \rightarrow 1, \quad 2 \rightarrow 3, \quad 3 \rightarrow 2, \quad 4 \rightarrow 5, \quad 5 \rightarrow 6, \quad 6 \rightarrow 3.$$

This, of course, is equivalent to reordering (swapping) the equations and variables.

An ordered set of edges $\{(i_1, j_1)\}_{i_1, j_1 \in \{1, \dots, n\}}$ is a path joining the vertices i_1 and j_1 if $i_1 \in \{1, \dots, n\}$, $j_1 \in \{1, \dots, n\}$ and for every $k = 1, 2, \dots, n-1$ the set $\{i_{k+1}, j_{k+1}\} \cap \{i_k, j_k\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that \mathbf{G} is a tree if each two members of \mathbf{V} are joined by a unique simple path. Both tridiagonal and banded matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $n > 3$.

Given a tree \mathbf{G} and an arbitrary vertex $r \in \mathbf{V}$, the path $T = (Q, \pi)$ is called a rooted tree, while r is said to be the root. Unlike in an ordinary tree, the root r is not necessarily unique, which can best be explained by an analogy with a family tree. The root r is the predecessor of all the vertices in $\mathbf{V} \setminus \{r\}$ and these vertices are successors of r . Moreover, every $a \in \mathbf{V} \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and a , as a predecessor of a and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors; in other words, a vertex a is to the right of r if it is further from the root to the left. (As we have already said, we are permuting the rows and the columns of the matrix, so the order of the vertices in the matrix is not unique.)

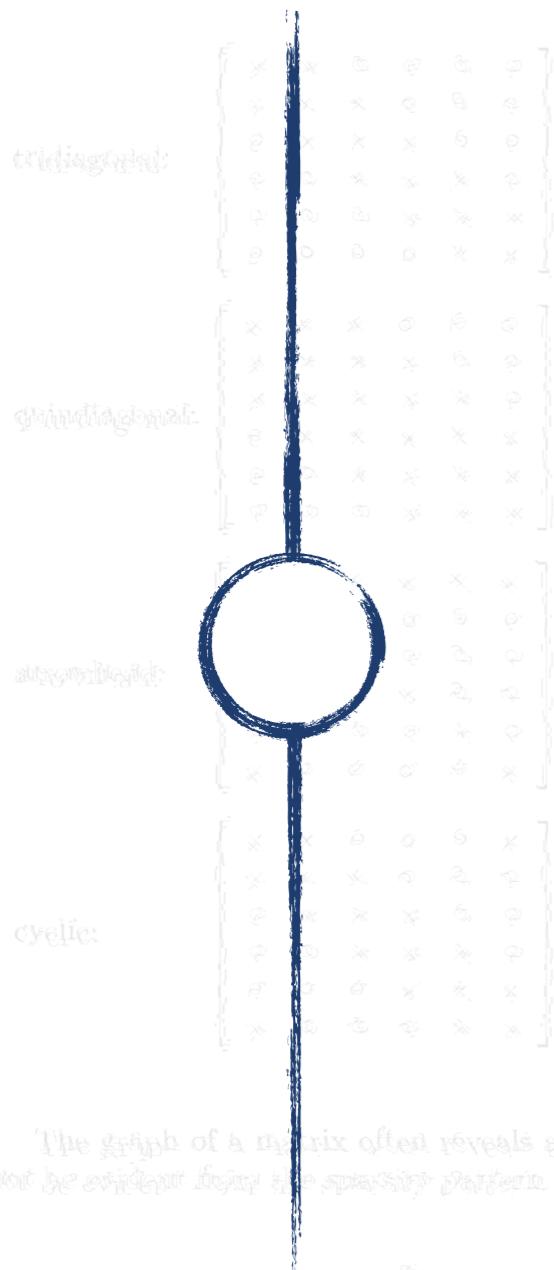
Every rooted tree will be monotonically ordered, but in general such an ordering is not unique. We now give three consecutive stages of the same rooted tree:



Theorem 2.1 Let \mathbf{A} be a symmetric matrix whose graph \mathbf{G} is a tree. Choose a root $r \in \{1, 2, \dots, n\}$. Assume that the rows and columns of \mathbf{A} have been arranged so that $T = (Q, \pi)$ is monotonically ordered. Then the Cholesky factorization of \mathbf{A} is

$$L_{k,j} = \frac{a_{k,j}}{a_{kk}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (1.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

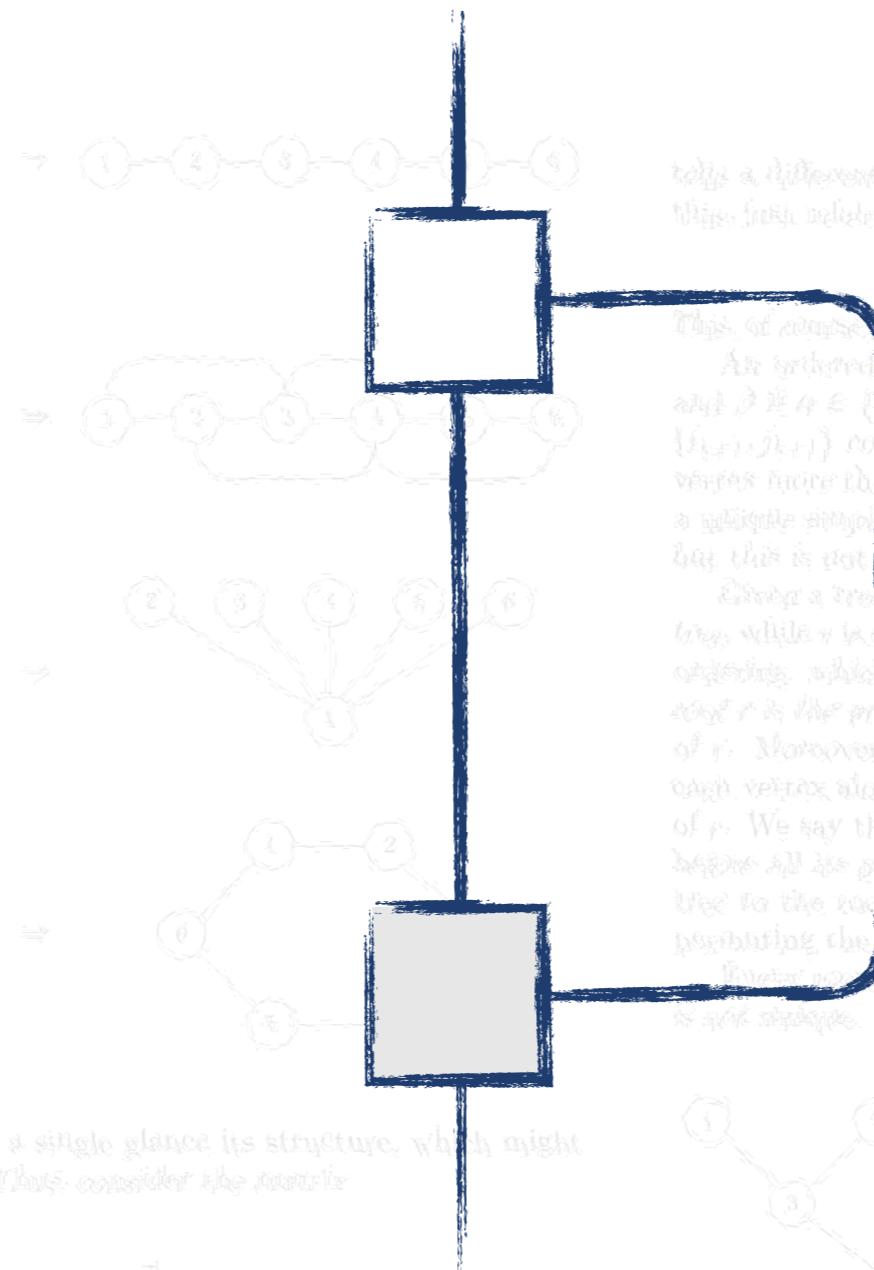


The graph of a matrix often reveals at a single glance its structure, which might not be evident from its sparsity pattern. Thus, consider the matrix

$$\text{LOG} \quad \begin{bmatrix} * & 0 & * & 0 & * & 0 \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & * \\ 0 & * & 0 & * & 0 & * \\ * & 0 & * & 0 & * & 0 \\ * & 0 & * & 0 & * & 0 \end{bmatrix}$$

| F

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, first recall that the vertices are

$$1 \rightarrow 1, \quad 2 \rightarrow 2, \quad \dots, \quad 5 \rightarrow 5, \quad 6 \rightarrow 6.$$

This, of course, is equivalent to relabelling (simultaneously) the equations and variables.

An ordered set of edges $\{(v_i, w_i)\}_{i=1}^n$ in $G = (V, E)$ is called a path joining the vertices v_1 and v_n if $i \in \{1, \dots, n\}$, $v_i \in V$, $w_i \in V$, and for every $k = 1, 2, \dots, n-1$ the set $\{v_k, w_k\} \cap \{v_{k+1}, w_{k+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of V are joined by a unique simple path. Both tridiagonal and quadiagonal matrices correspond to trees, but this is not the case with either pentadiagonal or cyclic matrices when $n \geq 3$.

Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $T \setminus \{r\}$ and these vertices are successors of r . Moreover, every $v \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r itself, as a predecessor of v and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors in other words, we layed the vertices from the top of the tree to the root. (As we have mentioned it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Even a binary tree will be monotonically ordered and, in general, such an ordering is not unique. We will give the



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so that $T = (G, r)$ is monotonically ordered. Then, that $A = LL^T$ is a Cholesky factorization, it is true that

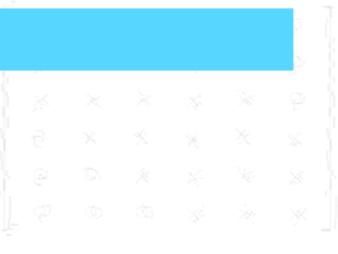
$$l_{k,j} = \frac{a_{kj}}{a_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

triangular:



quadratic:



symmetric:



cyclic:



The graph of a matrix often reveals at a single glance its structure, which may not be evident from the sparsity pattern. Thus, consider the matrix



At a first glance, there is nothing to think in the case just displayed, but its graph,

1 PICTURE = 1 000 WORDS



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_1, j_1), (i_2, j_2), \dots, (i_v, j_v)\}$ in \mathbb{G} is called a *path* joining the vertices i_1, i_2, \dots, i_v (j_1, j_2, \dots, j_v) and, for every $k = 1, 2, \dots, v-1$ the set $\{(i_k, j_k), (i_{k+1}, j_{k+1})\}$ contains exactly one edge. It is a *simple path* if it does not visit any vertex more than once. We say that \mathbb{G} is a *tree* if each two members of \mathbb{V} are joined by a single simple path. Back to graphs, the quadiagonal matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $p \geq 3$.

Given a tree \mathbb{G} and an arbitrary vertex $r \in \mathbb{V}$, the pair $T = (G, r)$ is called a *rooted tree*, while r is said to be the *root*. Unlike in ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the *predecessor* of all the vertices in $\mathbb{V} \setminus \{r\}$ and these vertices are *successors* of r . Moreover, every $\alpha \in \mathbb{V} \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a *predecessor* of α and a *successor* of r . We say that the rooted tree T is *monotonically ordered* if each vertex is labelled before all its predecessors; in other words, we *layer* the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is unique. We now give three consecutive analogies of the same rooted tree:



Theorem 1. Let A be a symmetric matrix whose graph \mathbb{G} is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so, that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

```
// TODO: Write tests later

test('it renders', async function(assert) {
  await render(hbs`<ComplexComponent />`);
  assert.ok(true);
});
```

```
import { percySnapshot } from 'ember-percy';

...
// TODO: Write tests later

test('complex workflow', async function(assert) {
  await visit('/complex-page');
  await percySnapshot(assert);
});
```

Therefore we will now give a few examples of matrices (represented by their sparsity)

triangular:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



quadrangular:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



supernodal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



cyclic:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



IF f IS CONTINUOUS IN $[a, b]$, AND IF $f(a)f(b) < 0$,

THEN f MUST HAVE A

ZERO IN (a, b) .



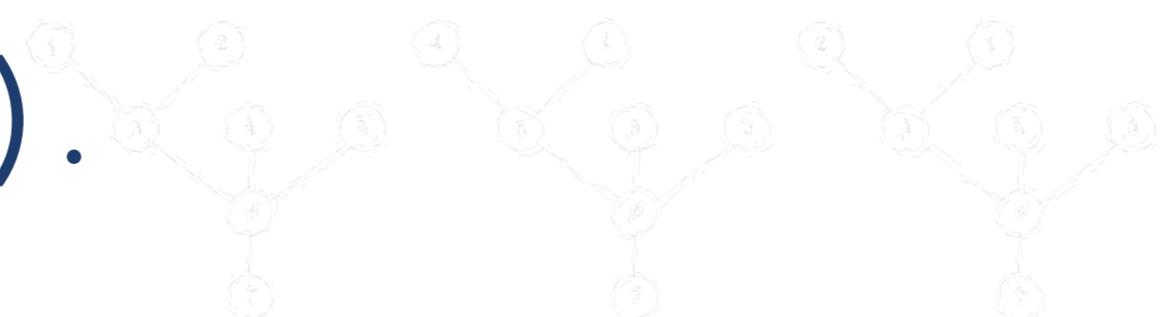
tells a different story - it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 2, 3 \rightarrow 3, 4 \rightarrow 4, 5 \rightarrow 5, 6 \rightarrow 3.$$

Of course, it is equivalent to replacing (arbitrarily) the equations and variables. As you can see from the figure, the graph G is called a tree, since the vertices a and b ($a \in V \setminus \{b\}$, $b \in V \setminus \{a\}$) and for every $k = 1, 2, \dots, n-1$ the set $\{v_1, v_2, \dots, v_k\} \cap \{v_{k+1}, v_{k+2}, \dots, v_{n-1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of V are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees, but this is not the case with either of banded or cyclic matrices when $n \geq 3$.

Of course, a rooted tree is a tree with a root vertex. Unlike in a tree, in a rooted tree there is a natural partial ordering which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $V \setminus \{r\}$ and these vertices are successors of r . Moreover, every $v \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and v , as a predecessor of v and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled by an integer from 1 to n in such a way that the vertices from the top of the tree down to the bottom are in increasing order (in other words, if we read the vertices from the top of the tree down to the bottom, we have a strictly increasing sequence). Labeling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree:

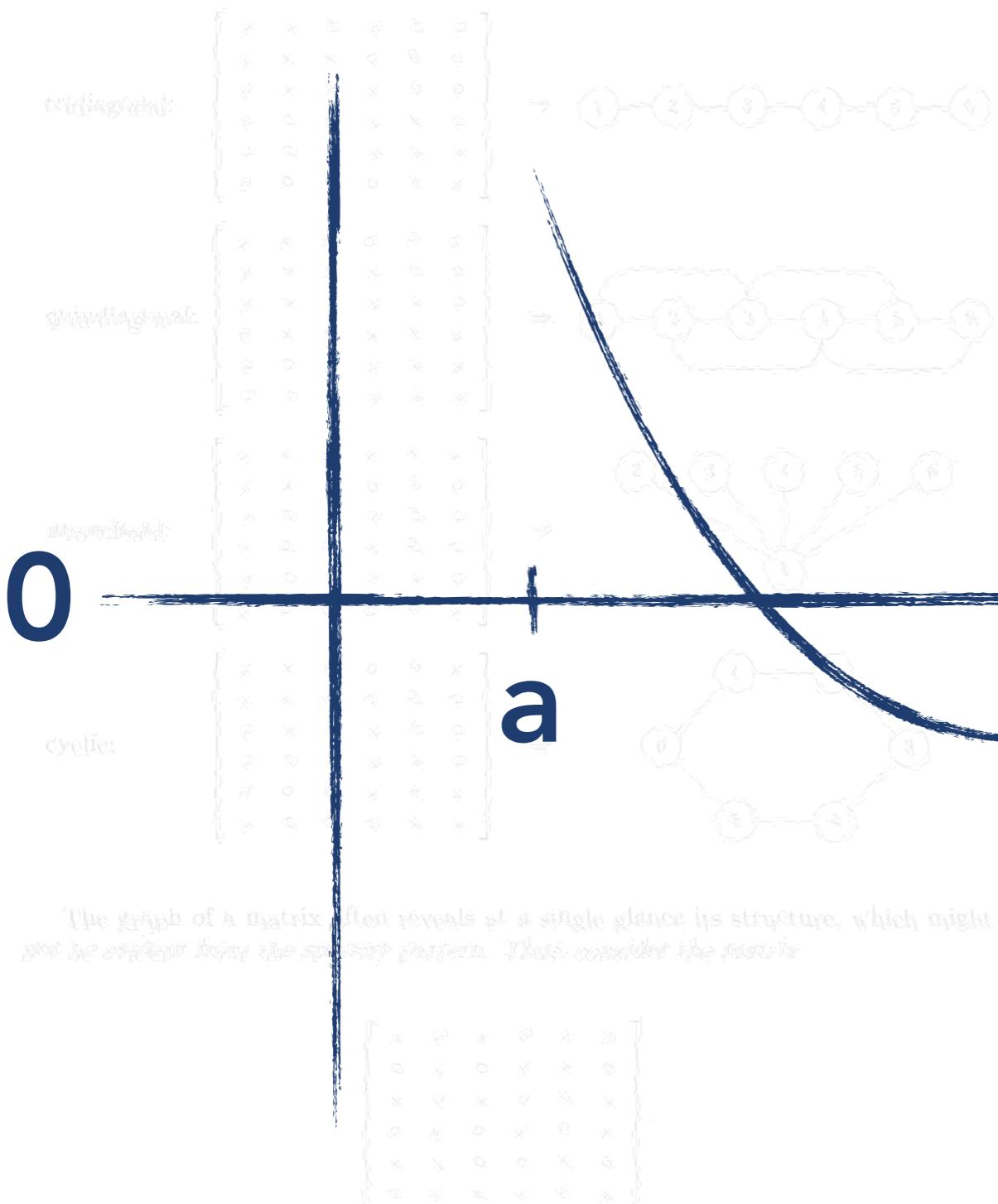


Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, n\}$ and assume that the rows and columns of A are then arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

At a first glance, this is not a very surprising fact, since this is what we have just displayed, but its graph.

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



At a first glance, there is nothing to link in the case of the four matrices that we have just displayed, but its graph.



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 3, \quad 5 \rightarrow 6, \quad 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$ is called a path joining the vertices i_0 and j_0 if $i_0 \in \{i_0, j_0\}$, $j_0 \in \{i_0, j_0\}$ and for every $k = 1, 2, \dots, n-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of V are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $p \geq 3$.

Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $T \setminus \{r\}$ and these vertices are successors of r . The set $\{v \in V \mid v \neq r\text{ and }v \text{ is a successor of }r\}$ is called an r -successor. We designate each vertex along this path, except for r and α , as a predecessor of α and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its descendants. In other words, we layed the tree down the top at the tree's root. (As we have seen, it is said it, relabelling α is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree:



Therefore we will now give a few examples of matrices (represented by their sparsity)

YOU CAN FIND EQUALLY MANY NUMBERS BETWEEN 0 AND 1 AS YOU CAN

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

BETWEEN $-\infty$ AND ∞ .

At a first glance this is not a matrix, but it is a little known triangular matrix, just displayed, but its graph,



It is a different story – it is nothing other than the cyclic matrix in disguised. To see this, let us relabel the vertices as follows:

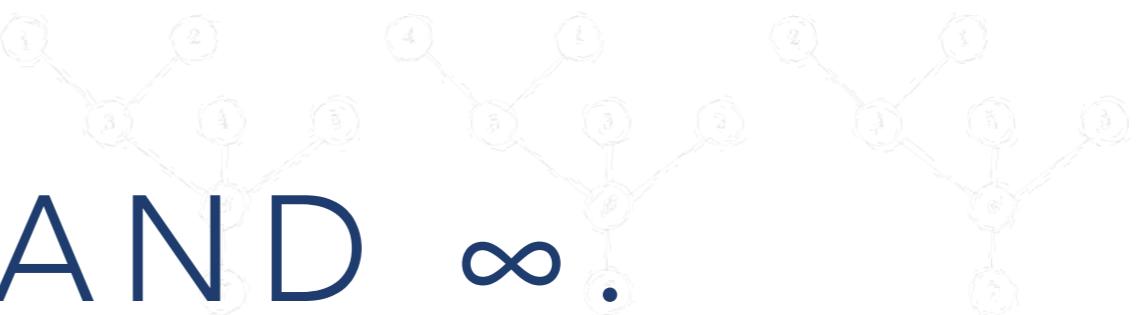
$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 8.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(u_i, v_i)\}_{i=1}^r$ in \mathbb{G} is called a path joining the vertices u_1 and u_r if $u_i \in \{u_1, u_2\}$, $v_i \in \{v_1, v_2\}$ and for every $i = 1, 2, \dots, r-1$ the set $\{u_i, v_i\} \cap \{u_{i+1}, v_{i+1}\} = \emptyset$. If a path contains only one edge, it is a simple path if it does not visit any vertex more than once. We say that a path of length r is a r -cycle if it visits the same vertex v more than once. (Note that a path of length r corresponds to a $(r+1)$ -cycle in the underlying matrix, but this is not the case with either quidiagonal or cyclic matrices when $r \geq 3$.)

Given a tree \mathbb{G} and an arbitrary vertex $r \in \mathbb{V}$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in a ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the ancestor of the vertices in $T \setminus \{r\}$ and these vertices are successors of r . Moreover, every $a \in \mathbb{V} \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and a , as a predecessor of a and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree:



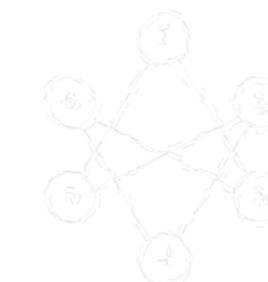
Theorem 11.1. Let A be a symmetric matrix whose graph \mathbb{G} is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A were been arranged so, that $T = (\mathbb{G}, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

tridiagonal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix} \rightarrow \text{Graph: } 1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 6$$



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, first, re-label the weights as follows:

$$w_{12} = w_{23} = w_{34} = w_{45} = w_{56} = w_{61} = 1, \quad w_{13} = w_{24} = w_{35} = w_{46} = 2, \quad w_{14} = w_{25} = w_{36} = 3$$

$$\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & * & 0 \\ * & 0 & * & 0 & 0 & * \\ 0 & * & 0 & * & 0 & * \\ * & 0 & 0 & 0 & * & 0 \\ 0 & * & * & * & * & * \end{bmatrix}.$$



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, n\}$ and assume that the rows and columns of A are then arranged so, that $\mathbb{T} = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ii}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

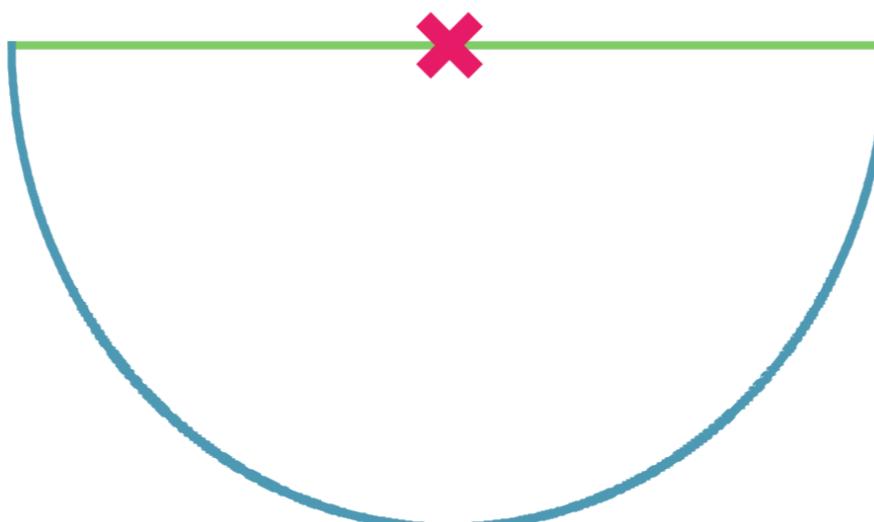
At a first glance, there is nothing to think in the case of the four matrices that we have just displayed, but its graph.

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

tridiagonal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix} \rightarrow \text{Graph: } 1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 6$$

tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, first, re-label the vertices as follows:



$$\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & * & 0 \\ * & 0 & * & 0 & 0 & * \\ 0 & * & 0 & * & 0 & * \\ * & 0 & * & 0 & 0 & * \\ 0 & * & * & * & * & * \end{bmatrix}$$

At a first glance, there is nothing to think in the case of the first matrices that we have just displayed, but its graph:



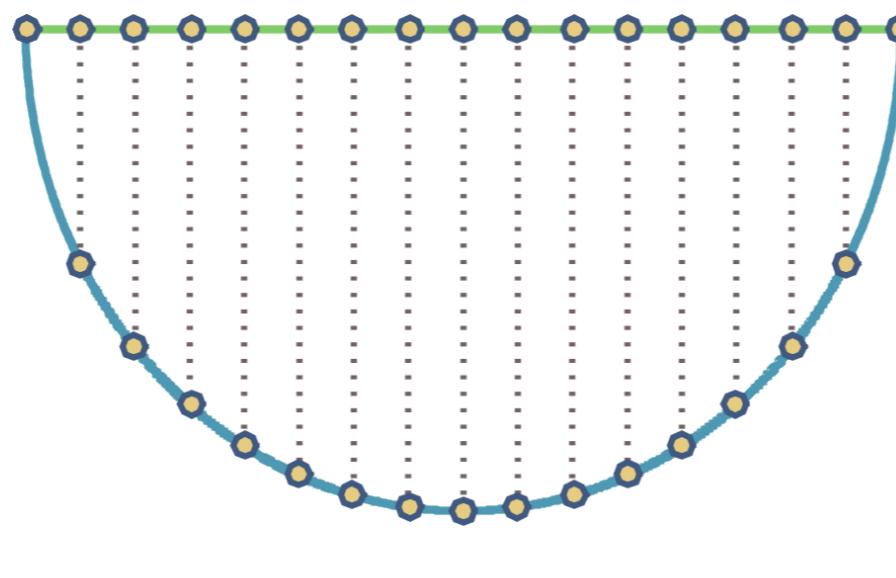
Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so, that $\mathbb{T} = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ii}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, first recall the weights as follows:



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, n\}$ and assume that the rows and columns of A are then arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

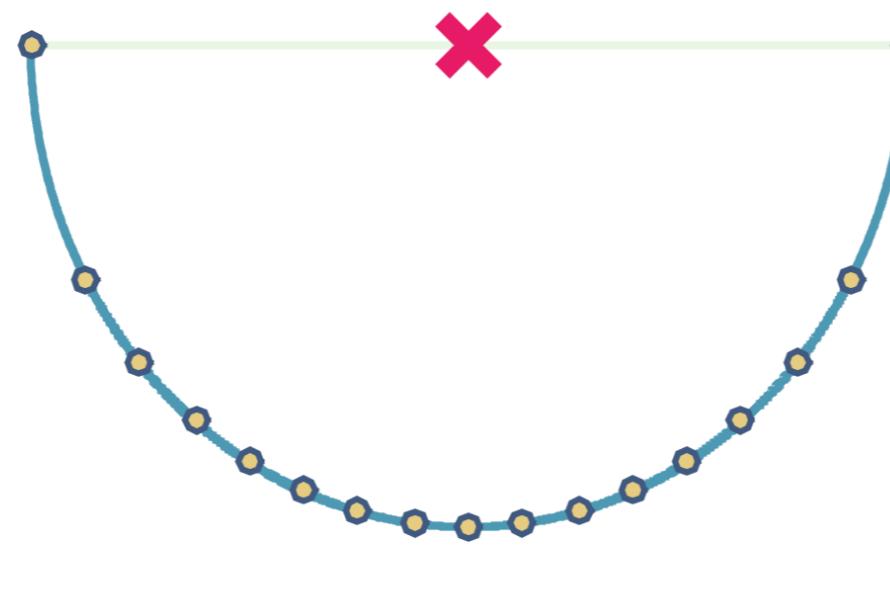
At a first glance, there is nothing to think in the case of the four matrices that we have just displayed, but its graph.

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

tridiagonal:

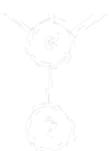
$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix} \rightarrow \text{Graph: } 1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 6$$

tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, first recall the weights as follows:



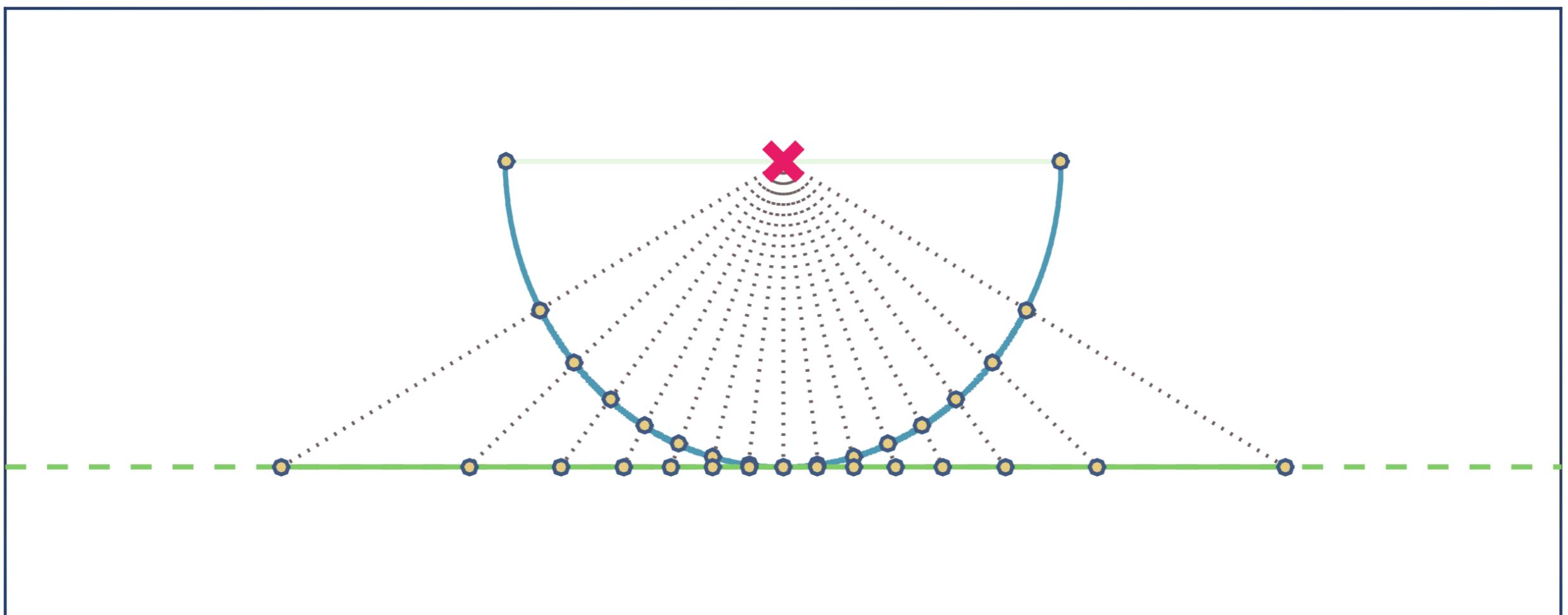
$$\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & * & 0 \\ * & 0 & * & 0 & 0 & * \\ 0 & * & 0 & * & 0 & * \\ * & 0 & * & 0 & 0 & * \\ 0 & * & * & * & * & * \end{bmatrix}$$

At a first glance, there is nothing to think in the case of the four matrices that we have just displayed, but its graph:



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so, that $\mathbb{T} = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ii}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$



5 Rules of Writing Tests

Filter:

Module: Acceptance | Outline



EmberFest 18.10.2019

Running:

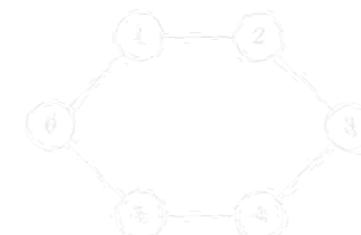
asymmetric:

$$\begin{bmatrix} 0 & 2 & 3 & 2 & 2 \\ 2 & 0 & 2 & 2 & 2 \\ 3 & 2 & 0 & 2 & 2 \\ 2 & 2 & 2 & 0 & 2 \\ 2 & 2 & 2 & 2 & 0 \end{bmatrix} \Rightarrow$$



cyclic:

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \Rightarrow$$



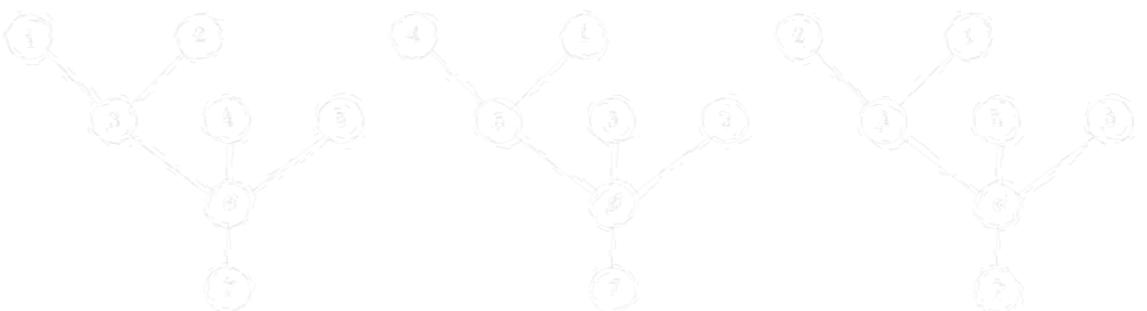
The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} 0 & 2 & * & 0 & * & 0 \\ 2 & 0 & 0 & * & x & 0 \\ * & 0 & 0 & 0 & 0 & x \\ 0 & * & 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & x & 0 & 0 & 0 \end{bmatrix}.$$

At a first glance, there is nothing to think in the case of the four matrices that we have just displayed, but its graph,

which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $T \setminus \{r\}$ and these vertices are successors of r . Moreover, every $v \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and v , as a predecessor of v and a successor of r . We say that the rooted tree T is *monotonically ordered* if each vertex is labelled before all its predecessors; in other words, we label the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three consecutive orderings of the same rooted tree.



Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

5 Rules of Writing Tests

Filter:

Module: Acceptance | Outline

EmberFest 18.10.2019

1 assertion of 1 passed, 0 failed.

If, if, if

cyclic:

$$\begin{bmatrix} * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \end{bmatrix} \Rightarrow$$



before all its predecessors in other words: we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three monotone orderings of the same rooted tree:



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so, that $T = (G, r)$ is monotonically ordered. Then, that $A = L U$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ki}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

At a first glance, there is nothing to think in the case of the four matrices that we have just displayed, but its graph.

5 Rules of Writing Tests

Filter:

Module: Acceptance | Outline

EmberFest 18.10.2019

2 assertions of 2 passed, 0 failed.

If, if, if

Use common, everyday words

5 Rules of Writing Tests

Filter:

Module:

Acceptance | Outline



EmberFest 18.10.2019

3 assertions of 3 passed, 0 failed.

! If, if, if

! Use common, everyday words

! Write less with theorems and new terms

$$\begin{bmatrix} * & * & * & 0 & * & 0 \\ 0 & * & 0 & * & * & 0 \\ * & 0 & * & * & 0 & * \\ 0 & * & 0 & * & * & 0 \\ * & * & 0 & * & * & 0 \\ 0 & * & * & 0 & * & * \end{bmatrix}.$$

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph:



Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so, that $\mathbf{T} = (G, r)$ is monotonically ordered. Then, that $A = \mathbf{L}\mathbf{U}^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ii}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

5 Rules of Writing Tests

Filter:

Module:

Acceptance | Outline



EmberFest 18.10.2019

4 assertions of 4 passed, 0 failed.

! If, if, if

! Use common, everyday words

! Write less with theorems and new terms

! All your basis are belong to us

$$\begin{bmatrix} 0 & 3 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,

Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so, that $\tilde{T} = (G, r)$ is monotonically ordered. Then, that $A = \tilde{L}\tilde{U}$ is a Cholesky factorization, it is true that

$$t_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ki}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

5 Rules of Writing Tests

Filter:

Module: [Acceptance](#) | [Outline](#) ▾

EmberFest 18.10.2019

5 assertions of 5 passed, 0 failed.

! If, if, if

! Use common, everyday words

! Write less with theorems and new terms

! All your basis are belong to us

! 1 picture = 1000 words

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

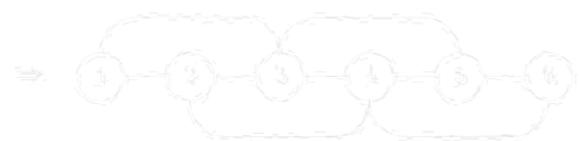
tridiagonal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



qundiagonal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ * & * & * & * & * & * \\ * & * & * & * & * & * \end{bmatrix}$$



crunchingnumbers.live

@ijlee2

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & x & 0 \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & * \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & * & * \end{bmatrix}.$$

At a first glance, there is nothing to think in the case of the four matrices that we have just displayed, but its graph,



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

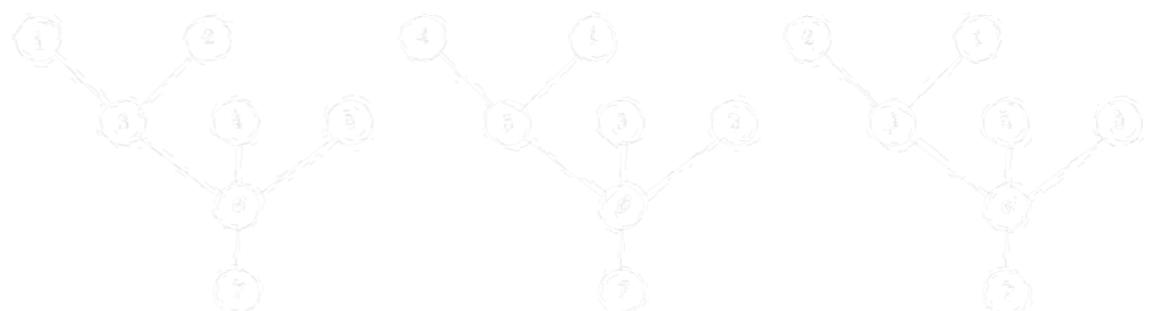
$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 3, \quad 5 \rightarrow 6, \quad 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_0, j_0)\}_{i_0, j_0 \in \mathbb{N}_0}$ is called a path joining the vertices i_0 and j_0 if $i_0, j_0 \in \{i_0, j_0\}$, $i_0 \neq j_0$, $i_0 \in \{i_0, j_0\}$, $j_0 \in \{i_0, j_0\}$ and for every $k = 1, 2, \dots, n-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of V are joined by a unique simple path. Both tridiagonal and qundiagonal matrices correspond to trees, but this is not the case with either qundiagonal or cyclic matrices when $n \geq 3$.

Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a rooted tree, where the vertex r is the root. Unlike in ordinary graph, T adapts a natural partial order which should be explained by an analogy with a family tree. Thus, the vertex r is the predecessor of all the vertices in $T \setminus \{r\}$ and these vertices are successors of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a predecessor of α and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three consecutive orderings of the sample rooted tree.



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Consider a vector $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are now arranged so, that $T = (G, r)$ is monotonically ordered. Given that $A = PLU$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{p_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Q.E.D.