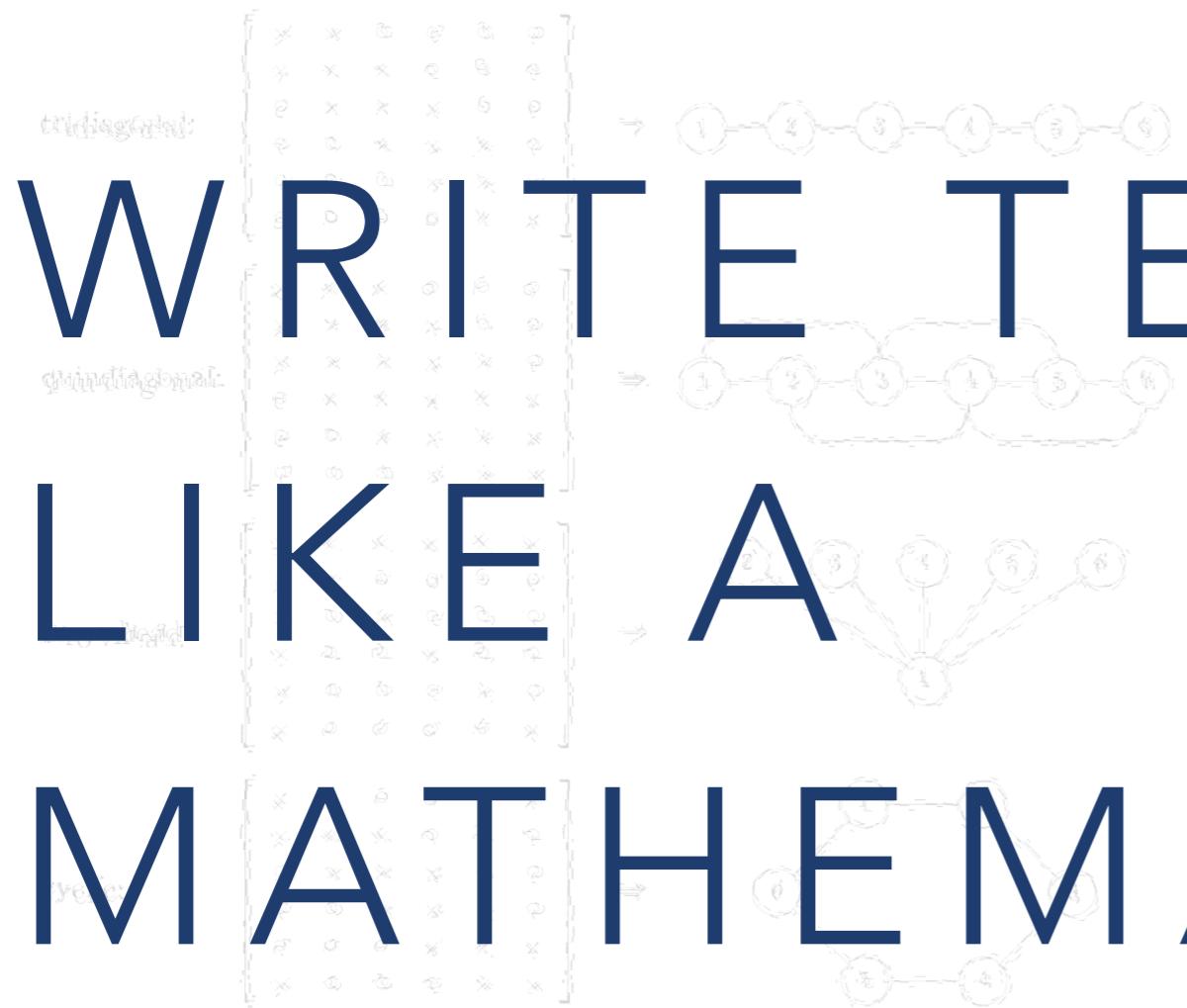


Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

ISAAC J. LEE

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, first recall the relation as follows

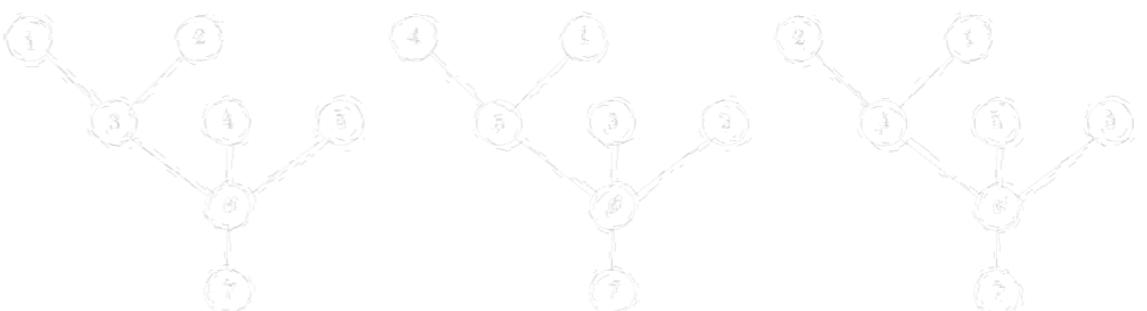
$$1 \rightarrow 2, 2 \rightarrow 3, 3 \rightarrow 2, 4 \rightarrow 5, 5 \rightarrow 6, 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

Let α be an ordered set of vertices $\{\alpha_i\}_{i=1}^n$, $\beta \in \mathbb{S}$ is called a path joining the vertices α and β if $\alpha \in \{\alpha_i, \alpha_{i+1}\}_{i=1}^n$, $\beta \in \{\alpha_i, \alpha_{i+1}\}_{i=1}^n$ and for every $k = 1, 2, \dots, n-1$ the set $\{\alpha_k, \alpha_{k+1}\} \cap \{\alpha_{k+1}, \alpha_{k+2}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that \mathbb{G} is a tree if each two members of \mathbb{V} are joined by a unique simple path. Both tridiagonal and pentadiagonal matrices correspond to trees, but this is not the case with either quadiagonal or cyclic matrices when $n \geq 3$.

Given a tree \mathbb{G} and an arbitrary vertex $r \in \mathbb{V}$, the pair $\mathbb{T} = (\mathbb{G}, r)$ is called a rooted tree, while r is said to be the root. Unlike an ordinary graph, \mathbb{T} admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $\mathbb{V} \setminus \{r\}$ and these vertices are successors of r . Moreover, every $\alpha \in \mathbb{V} \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a predecessor of α and a successor of r . We say that the rooted tree \mathbb{T} is monotonically ordered if each vertex is labelled before all its predecessors (or, more precisely, we have the relation from the top of the tree to the root. If we have already said it, relabeling a graph is tantamount to relabeling the rows and the columns of the underlying matrix).

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three monotone orderings of the sample rooted tree:



Theorem III.1. Let \mathbb{A} be a symmetric matrix whose graph \mathbb{G} is a tree. Choose a root $r \in \{1, 2, \dots, n\}$ and assume that the rows and columns of \mathbb{A} have been arranged so that $\mathbb{T} = (\mathbb{G}, r)$ is monotonically ordered. Given that $\mathbb{A} = \mathbb{L}\mathbb{U}^T$ is a Cholesky factorization, it is true that

$$a_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ii}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

5 Rules of Writing Tests

Filter:

Module: Acceptance | Outline



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Running:

asymmetric:

$$\begin{bmatrix} 0 & 2 & 3 & 2 & 2 \\ 2 & 0 & 2 & 2 & 2 \\ 3 & 2 & 0 & 2 & 2 \\ 2 & 2 & 2 & 0 & 2 \\ 2 & 2 & 2 & 2 & 0 \end{bmatrix} \Rightarrow$$



cyclic:

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \Rightarrow$$



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} 0 & 2 & * & 0 & * & 0 \\ 2 & 0 & 0 & * & x & 0 \\ * & 0 & 0 & 0 & 0 & * \\ 0 & * & 0 & 0 & 0 & * \\ * & 0 & 0 & 0 & 0 & * \\ 0 & * & * & 0 & 0 & 0 \end{bmatrix}.$$

At a first glance, there is nothing to think in the case of the four matrices that we have just displayed, but its graph,

which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $T \setminus \{r\}$ and these vertices are successors of r . Moreover, every $v \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and v , as a predecessor of v and a successor of r . We say that the rooted tree T is *monotonically ordered* if each vertex is labelled before all its predecessors; in other words, we label the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three consecutive orderings of the same rooted tree.



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

5 Rules of Writing Tests

Filter:

Module: Acceptance | Outline

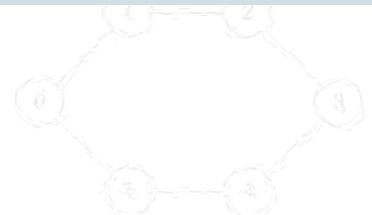
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1 assertion of 1 passed, 0 failed.

If, if, if

cyclic:

$$\begin{bmatrix} * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \end{bmatrix} \Rightarrow$$



before all its predecessors in other words: we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three monotone orderings of the same rooted tree:



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so, that $T = (G, r)$ is monotonically ordered. Then, that $A = L U$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ki}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

At a first glance, there is nothing to think in the case of the four matrices that we have just displayed, but its graph.

5 Rules of Writing Tests

Filter:

Module: Acceptance | Outline

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2 assertions of 2 passed, 0 failed.

If, if, if

Use common, everyday words

5 Rules of Writing Tests

Filter:

Module:

Acceptance | Outline



EmberFest 18.10.2019

3 assertions of 3 passed, 0 failed.

! If, if, if

! Use common, everyday words

! Write less with theorems and new terms

$$\begin{bmatrix} * & * & * & 0 & * & 0 \\ 0 & * & 0 & * & * & 0 \\ * & 0 & * & * & 0 & * \\ 0 & * & 0 & * & * & 0 \\ * & * & 0 & * & * & 0 \\ 0 & * & * & 0 & * & * \end{bmatrix}.$$

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph:



Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so, that $\mathbf{T} = (G, r)$ is monotonically ordered. Then, that $A = \mathbf{L}\mathbf{U}^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ii}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

5 Rules of Writing Tests

Filter:

Module:

Acceptance | Outline



EmberFest 18.10.2019

4 assertions of 4 passed, 0 failed.

! If, if, if

! Use common, everyday words

! Write less with theorems and new terms

! All your basis are belong to us

$$\begin{bmatrix} 0 & 3 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,

Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so, that $\tilde{T} = (G, r)$ is monotonically ordered. Then, that $A = \tilde{L}\tilde{U}$ is a Cholesky factorization, it is true that

$$t_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ki}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

5 Rules of Writing Tests

Filter:

Module: [Acceptance](#) | [Outline](#) ▾

EmberFest 18.10.2019

5 assertions of 5 passed, 0 failed.

! If, if, if

! Use common, everyday words

! Write less with theorems and new terms

! All your basis are belong to us

! 1 picture = 1000 words

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} * & * & * & 0 & * & * \\ 0 & * & 0 & * & * & * \\ * & 0 & * & 0 & * & * \\ 0 & * & 0 & * & 0 & * \\ * & * & 0 & 0 & * & * \\ 0 & * & * & * & 0 & * \end{bmatrix}$$

At a first glance, there is nothing to think in the case just displayed, but its graph,



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 3, \quad 5 \rightarrow 6, \quad 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_0, j_0)\}_{i_0, j_0 \in \mathbb{N}_0, i_0 \neq j_0}$ is called a path joining the vertices i_0 and j_0 if $i_0 \in \{i_0, j_0\}$, $j_0 \in \{i_0, j_0\}$ and for every $k = 1, 2, \dots, n-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of V are joined by a unique simple path. Both tridiagonal and quidiagonal matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $n \geq 3$.

Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in a ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $T \setminus \{r\}$ and these vertices are successors of r . Moreover, every $a \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and a , as a predecessor of a and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three consecutive orderings of the same rooted tree.



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, n\}$ and assume that the rows and columns of A are then arranged so, that $T = (G, r)$ is monotonically ordered. Given that $A = L U$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\text{IF} \quad \begin{bmatrix} * & * & 0 & 0 & 0 \\ 0 & * & * & 0 & 0 \\ * & 0 & * & * & 0 \\ 0 & * & 0 & * & * \\ 0 & 0 & * & * & * \end{bmatrix}.$$

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,



tells a different story – it is nothing other than a cyclic matrix in disguise. To see this, just read the weights as follows:

$$1 \rightarrow 1, \quad 2 \rightarrow 3, \quad 3 \rightarrow 2, \quad 4 \rightarrow 6, \quad 5 \rightarrow 4, \quad 6 \rightarrow 3.$$

This, of course, is equivalent to requiring that the matrix A has a set of steps $\{(i, j)\}_{i, j \in V}$ such that $i \neq j$, $i \in V \setminus \{r\}$, $j \in V \setminus \{r\}$ and for $i \in V \setminus \{r\}$, $j \in V \setminus \{r\}$ the set $\{i, j\} \cap \{r\}$ contains exactly one member. It is also required that A does not visit any vertex more than once. We say that G is a *cycle* if two different vertices of V are joined by a single simple path. Both tridiagonal and symmetric matrices correspond to trees, the case with either quadrangular or cyclic matrices when $p \geq 3$.

Given a tree, while it is possible to order its vertices in many ways, it can best be explained by an analogy with a family tree. Thus, the predecessor of all the vertices in $V \setminus \{r\}$ are those vertices and predecessors of r , every $a \in V \setminus \{r\}$ is joined to r by a single path and we designate this path, except for r and a , as a predecessor of a and a successor of r . Note each vertex of r . We say that r is the root of T . We say that r is the parent of all its children and the children of r are the offspring of r . The rooted tree T is monotonically ordered, if each vertex is labelled $r, 1, 2, \dots, n$. In other words, we label the vertices from the top of the tree. (As we have already said in relabelling a graph it is tantamount to rows and columns of the underlying matrix.)

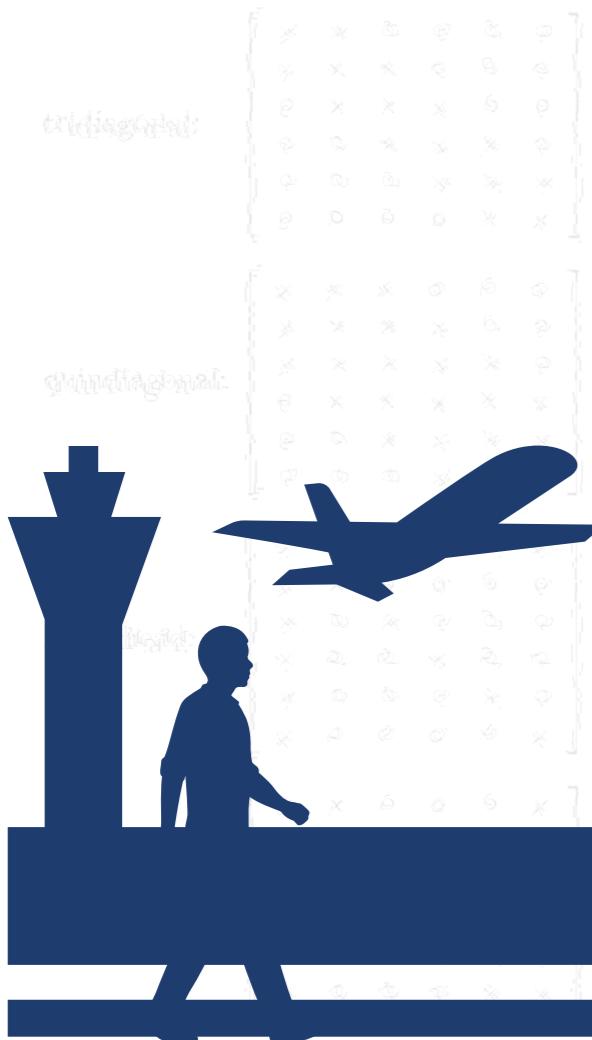
Every node in a tree is not empty and the monotonically ordered and, in general, the ordering is not unique.



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, n\}$ and assume that the rows and columns of A are then arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & 0 \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & * \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & * \end{bmatrix}.$$

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,



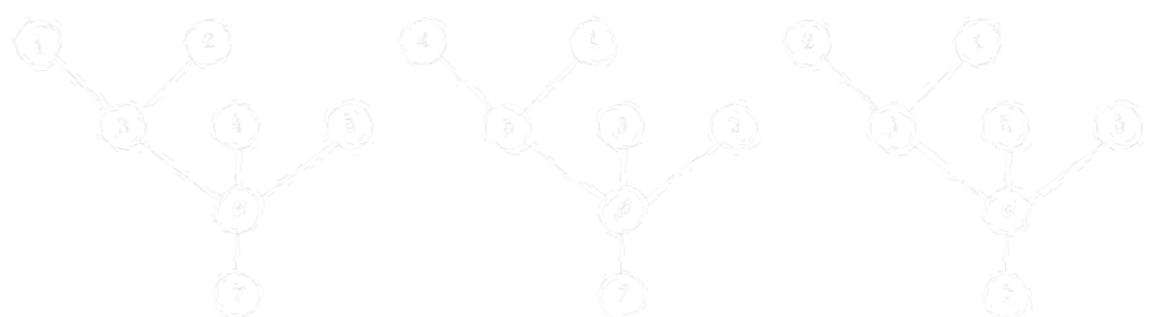
tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_0, j_0)\}_{i_0, j_0 \in \mathbb{N}_0}$ in \mathbb{G} is called a path joining the vertices i_0 and j_0 if $i_0 \in \{i_0, j_0\}$, $j_0 \in \{i_0, j_0\}$ and for every $k = 1, 2, \dots, n-1$ the set $\{i_{k-1}, j_k\} \cap \{i_k, j_{k+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that \mathbb{G} is a tree if each two members i_0, j_0 are joined by a unique simple path. Both tridiagonal and quindiagonal matrices correspond to trees; this is not the case with either quidiagonal or cyclic matrices.

Let \mathbb{G} be a tree and an arbitrary vertex $r \in V$. The pair $T = (\mathbb{G}, r)$ is called a rooted tree, where r is said to be the root. Unlike in a cyclic graph, there is no unique path from a vertex to the root. This can best be explained by an example. Let \mathbb{G} be a tree with 10 vertices, r being the root. Then r is the predecessor of all the vertices in $V \setminus \{r\}$. Moreover, every $v \in V \setminus \{r\}$ is joined to r along a path consisting of v and some vertices along this path. We say that the rooted tree T is monotonically ordered if every vertex v has a unique predecessor in \mathbb{G} ; in other words, v is layered above r and v is its root. (As we have already said, r , relabelling a graph, is not its root. We can, however, choose r to be the root of T .) Every rooted tree will be monotonically ordered and, in general, we can choose r to be any vertex. This will give three consecutive snapshots of the same rooted tree T at different stages of its growth.



Theorem 11.1. Let A be a symmetric matrix whose graph \mathbb{G} is a tree. Choose a root $r \in \{1, 2, \dots, n\}$ and assume that the rows and columns of A are then arranged so that $\mathbb{T} = (\mathbb{G}, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{j,j}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their analysis.

1

```
assert.speaker().getsPersonal();  
  
await sing('Happy Birthday');  
  
assert.audience().isHappy();
```

just displayed, but its graph:

$$t_{k,j} = \frac{q_{kj}}{q_j}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

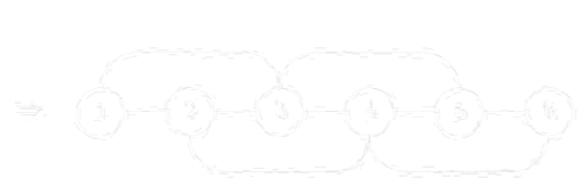
Therefore we will now give a few examples of matrices (represented by their sparsity)

and their graphs.

SUSPECT 1

quindiagonal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$



symmetric:

$$\begin{bmatrix} * & * & * & * & * & * \\ * & * & 0 & 0 & 0 & 0 \\ * & 0 & * & 0 & 0 & 0 \\ * & 0 & 0 & * & 0 & 0 \\ * & 0 & 0 & 0 & * & 0 \\ * & 0 & 0 & 0 & 0 & * \end{bmatrix}$$



cyclic:

$$\begin{bmatrix} * & 0 & 0 & 0 & * & * \\ * & * & 0 & 2 & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



OBSERVER /

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

COMPUTED

At a first glance, this is nothing but the structure of the following matrix, which was just displayed, but its graph:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 3, \quad 5 \rightarrow 6, \quad 6 \rightarrow 8.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$ is called a path joining the vertices i_0 and j_0 if $i_0 \in \{1, \dots, n\}$, $j_0 \in \{1, \dots, n\}$ and for every $k = 1, 2, \dots, v-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of V are joined by a unique simple path. Both tri-diagonal and any other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $n \geq 3$.

Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $T \setminus \{r\}$ and these vertices are successors of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a predecessor of α and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree:



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, n\}$ and assume that the rows and columns of A were then arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

and their graphs.

SUSPECT 2

quindiagonal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$



superdiagonal:

$$\begin{bmatrix} * & * & * & * & * & * \\ * & * & 0 & 0 & 0 & 0 \\ * & 0 & * & 0 & 0 & 0 \\ 0 & 0 & * & 0 & 0 & 0 \\ 0 & 0 & 0 & * & 0 & 0 \\ 0 & 0 & 0 & 0 & * & 0 \end{bmatrix}$$



cyclic:

$$\begin{bmatrix} * & 0 & 0 & 0 & * & 0 \\ * & * & 0 & 2 & * & 0 \\ * & 0 & * & * & 0 & 0 \\ 0 & 2 & * & * & * & 0 \\ 0 & * & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$



EMBER DATA / FORM BUILDER

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & * & 0 \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$

At a first glance, this is nothing but the structure of the form builder matrix

just displayed, but its graph:



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 3, \quad 5 \rightarrow 6, \quad 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$ is called a path joining the vertices i_0 and j_0 if $i_0 \in \{1, \dots, n\}$, $j_0 \in \{1, \dots, n\}$ and for every $k = 1, 2, \dots, n-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of V are joined by a unique simple path. Both tri-diagonal and any other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $n \geq 3$.

Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $T \setminus \{r\}$ and these vertices are successors of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a predecessor of α and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree:



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so, that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

and their graphs.

SUSPECT 3



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

STATE

At a first glance, this is nothing but the structure of the lower triangular matrix

just displayed, but its graph:



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$ $\subseteq \mathbb{E}$ is called a path joining the vertices i_0 and j_0 if $i_0 \in \{i_0, j_0\}$, $j_0 \in \{i_0, j_0\}$ and for every $k = 1, 2, \dots, n-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of V are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $n \geq 3$.

Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in a ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $T \setminus \{r\}$ and these vertices are successors of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a predecessor of α and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three consecutive orderings of the same rooted tree:



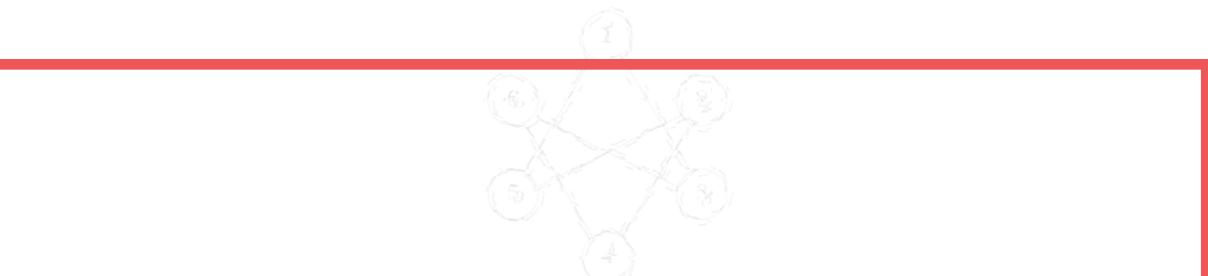
Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A were been arranged so, that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

and their graphs.

SUSPECT 4



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

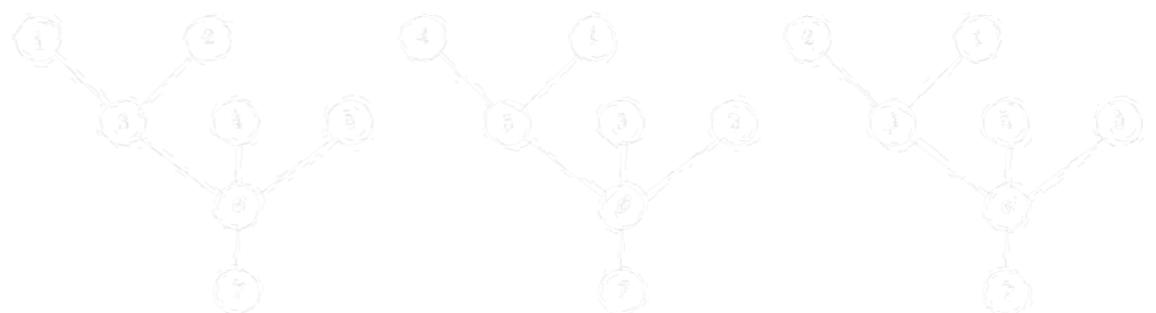
$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 3, \quad 5 \rightarrow 6, \quad 6 \rightarrow 8.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$ $\subseteq \mathbb{E}$ is called a path joining the vertices i_0 and j_0 if $i_0 \in \{i_0, j_0\}$, $j_0 \in \{i_0, j_0\}$ and for every $k = 1, 2, \dots, n-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that \mathbb{G} is a tree if each two members of \mathbb{V} are joined by a unique simple path. Both tri-diagonal and any other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $n \geq 3$.

Given a tree \mathbb{G} and an arbitrary vertex $r \in \mathbb{V}$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in ordinary graph, T adapts a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $\mathbb{V} \setminus \{r\}$ and these vertices are successors of r . Moreover, every $a \in \mathbb{V} \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and a , as a predecessor of a and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we lay out the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three consecutive drawings of the same rooted tree.



Theorem 11.1. Let A be a symmetric matrix whose graph \mathbb{G} is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

At a first glance, this is nothing but the definition of the lower triangular matrix just displayed, but its graph.

Therefore we will now give a few examples of matrices (represented by their sparsity)

and their graphs.

SUSPECT 5



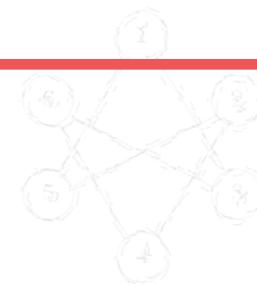
ADMIN

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

PRIVILEGE

At a first glance, this is nothing but a list of the four columns of the matrix

just displayed, but its graph,



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

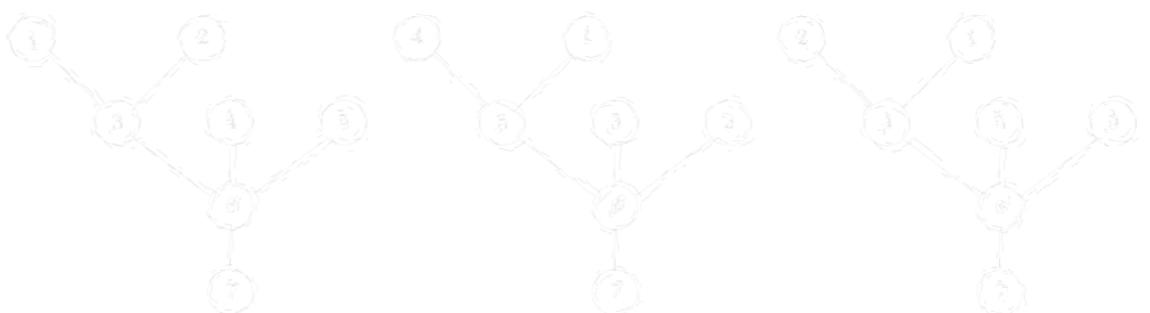
$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 8.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_0, j_0)\}_{i_0, j_0 \in \mathbb{N}_0, i_0 \neq j_0}$ is called a path joining the vertices i_0 and j_0 if $i_0 \in \{i_0, j_0\}$, $j_0 \in \{i_0, j_0\}$ and for every $k = 1, 2, \dots, n-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of V are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $n \geq 3$.

Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in a ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $T \setminus \{r\}$ and these vertices are successors of r . Moreover, every $a \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and a , as a predecessor of a and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

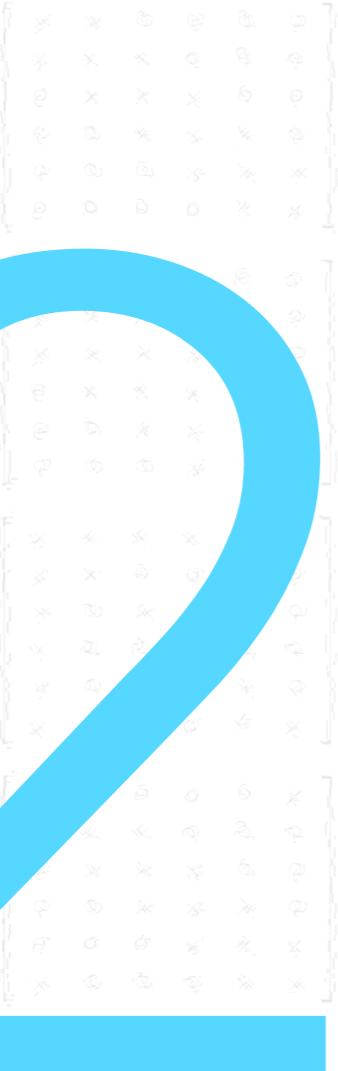
Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three consecutive orderings of the same rooted tree.

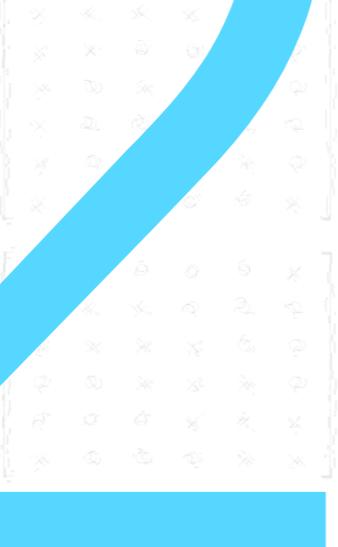


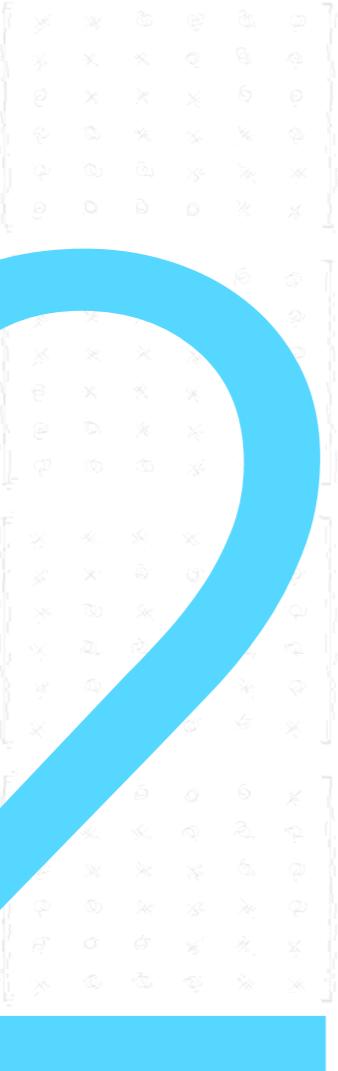
Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A were been arranged so, that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

tridiagonal:


symmetric:


cyclic:


2

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix



At a first glance, there is nothing to link it to any of the three structures just displayed, but its graph,

USE COMMON EVERYDAY WORDS



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{v_i, v_j\}$ in \mathbb{G} is called a path joining the vertices v_i and v_j ($i, j \in \mathbb{N}$). If v_i and v_j are even ($i = 1, \dots, n-1$, $j = 1, \dots, n-1$) the set $\{v_i, v_j\}$ is called a *bridge* and v_i and v_j are *members*. It is a simple path if it does not visit any vertex more than once. We say that \mathbb{G} is *tree* if each two members of \mathbb{V} are joined by a unique simple path. Both tridiagonal and cyclic matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $p \geq 3$.

Given a tree \mathbb{T} and an arbitrary vertex $r \in \mathbb{V}$, the pair $T = (G, r)$ is called a *rooted tree*, while r is said to be the *root*. Unlike in ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the *predecessor* of all the vertices in $\mathbb{V} \setminus \{r\}$ and these vertices are *successors* of r . Moreover, every $\alpha \in \mathbb{V} \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a *predecessor* of α and a *successor* of r . We say that the rooted tree T is *monotonically ordered* if each vertex is labelled before all its predecessors; in other words, we *label* the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is unique. To give the concrete example of the same rooted tree,



Theorem 1. Let A be a symmetric matrix whose graph \mathbb{G} is a tree. Choose a root $r \in \{1, 2, \dots, n\}$ and assume that the rows and columns of A are then arranged so, that $T = (\mathbb{G}, r)$ is monotonically ordered. Given that $A = L U$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{k,j}}{l_{j,j}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

triangular: $\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ 0 & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & * & 0 \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix} \Rightarrow \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \rightarrow 1-2-3-4-5-6$

quidiagonal: $\begin{bmatrix} * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ 0 & * & * & * & * & * \\ 0 & 0 & * & * & * & * \\ 0 & 0 & 0 & * & * & * \end{bmatrix} \Rightarrow \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \rightarrow 1-2-3-4-5-6$

superdiagonal: $\begin{bmatrix} * & * & * & * & * & * \\ * & 0 & 0 & 0 & 0 & 0 \\ * & 2 & 0 & 0 & 0 & 0 \\ * & 2 & 2 & 0 & 0 & 0 \\ * & 2 & 2 & 2 & 0 & 0 \\ * & 2 & 2 & 2 & 2 & 0 \end{bmatrix} \Rightarrow \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \rightarrow 1-2-3-4-5-6$

cyclic: $\begin{bmatrix} * & 0 & 0 & 0 & * & * \\ * & * & 0 & 0 & 2 & 2 \\ * & * & * & 0 & 2 & 2 \\ * & * & * & * & 0 & 2 \\ * & * & * & * & 2 & 0 \\ * & * & * & * & 2 & 0 \end{bmatrix} \Rightarrow \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \rightarrow 1-2-3-4-5-6$

CONVENTION;

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

AGREED ON BY MANY

At a first glance, this is not a triangular matrix, but it is a tree structure, as the one just displayed, but its graph:



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$ $\subseteq \mathbb{E}$ is called a path joining the vertices i_0 and j_0 if $i_0, j_0 \in \{1, \dots, n\}$, $i_0 \neq j_0$, $\beta \in \{1, \dots, n\}$ and for every $k = 1, 2, \dots, \nu - 1$ the set $\{i_{k+1}, j_{k+1}\} \cap \{i_0, \dots, i_k, j_0, \dots, j_k\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of V are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $n \geq 3$.

Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $T \setminus \{r\}$ and these vertices are successors of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a predecessor of α and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree:



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so, that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

tridiagonal:

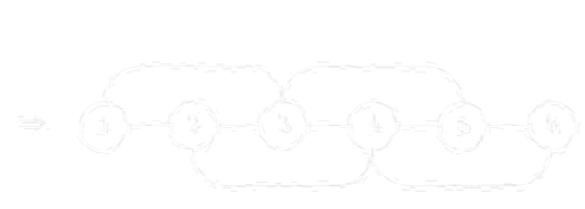
$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & 0 & * & * & 0 & 0 \\ 0 & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & * & * \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$

EVERYDAY



quindiagonal:

$$\begin{bmatrix} * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



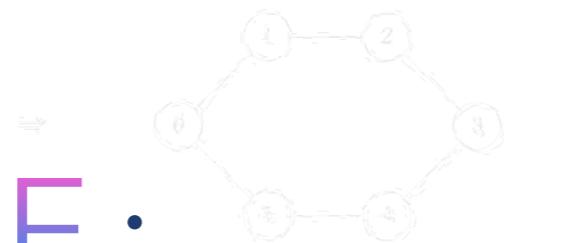
superdiagonal:

$$\begin{bmatrix} * & * & * & * & * & * \\ * & * & 0 & 0 & 0 & 0 \\ 0 & 0 & * & * & * & * \\ 0 & 0 & * & * & * & * \\ 0 & 0 & * & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



cyclic:

$$\begin{bmatrix} * & 0 & 0 & 0 & * & * \\ * & * & 0 & 2 & * & * \\ 0 & 0 & * & * & * & * \\ 0 & 2 & * & * & * & * \\ 0 & 0 & * & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



SIMPLE;

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

FAMILIAR TO MANY

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & * & * & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & * & * & * & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & * & * & * & 0 & 0 & 0 \\ 0 & 0 & 0 & * & * & * & * & 0 & 0 \\ 0 & 0 & 0 & 0 & * & * & * & * & 0 \\ 0 & 0 & 0 & 0 & * & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & * & * & * & * \end{bmatrix}$$

At a first glance this is not a matrix that is familiar to many, but it is the matrix just displayed, but its graph.



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

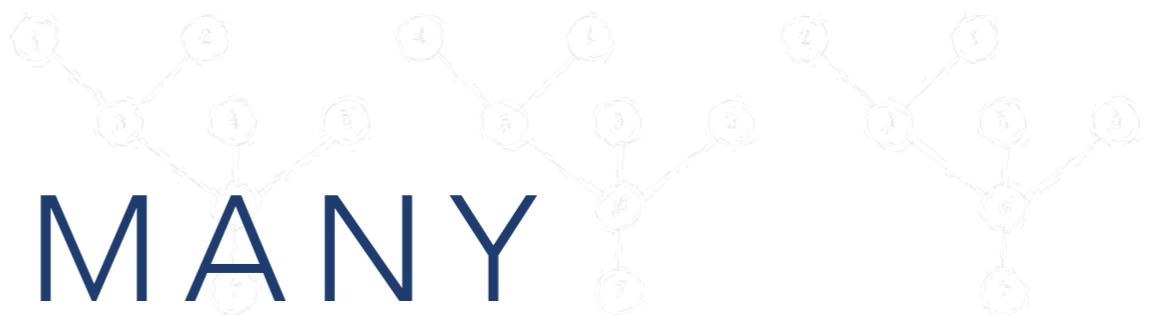
$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 3, \quad 5 \rightarrow 6, \quad 6 \rightarrow 8.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$ $\subseteq \mathbb{E}$ is called a path joining the vertices i_0 and j_0 if $i_0, j_0 \in \{1, \dots, n\}$, $i_0 \neq j_0$ and for every $k = 1, 2, \dots, n-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of V are joined by a unique simple path. Both tridiagonal and quidiagonal matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $n \geq 3$.

Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in a ordinary graph, T adapts a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $T \setminus \{r\}$ and these vertices are successors of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a predecessor of α and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree.



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

triangular:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$

quadrangular:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$

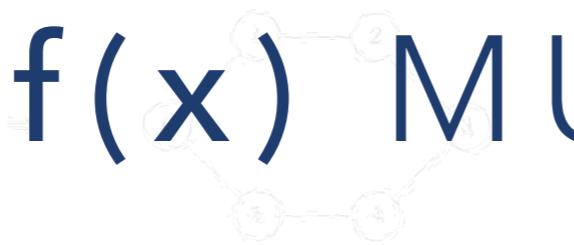
cyclic:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$

IF f IS CONTINUOUS IN $[a, b]$, AND IF $f(a)f(b) < 0$,

THEN $f(x)$ MUST HAVE A

ZERO IN (a, b) .



$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$

At a first glance, this is not a triangular matrix, but it is a triangular matrix, just displayed, but its graph,



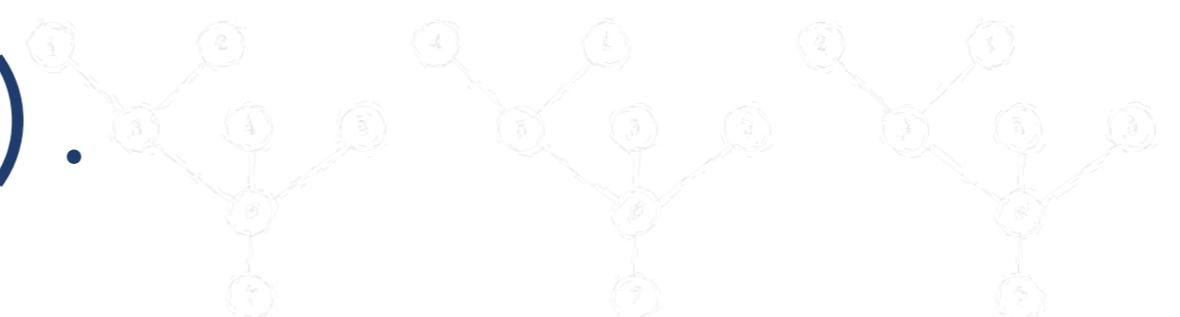
tells a different story - it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 2, 3 \rightarrow 3, 4 \rightarrow 4, 5 \rightarrow 5, 6 \rightarrow 6.$$

Of course, it is equivalent to relabeling the equations and variables. As a result, we get the following: G is called a *tree* if visiting the vertices a and b ($a \in V, b \in V, a \neq b$), $\beta \in V \setminus \{a, b\}$ and for every $k = 1, 2, \dots, n-1$ the set $\{V_k, \beta\} \cap V_{k+1, k+1}$ contains exactly one member. It is a *simple path* if it does not visit any vertex more than once. We say that G is a *tree* if each two members of V are joined by a unique simple path. Both triangular and any other matrices correspond to trees, but this is not the case with either of tridiagonal or cyclic matrices when $n \geq 3$.

As a result, we get the following: $T = (G, r)$ is called a *rooted tree* if $r \in V$ and r is the root of T . Unlike in a binary tree, T does not have a natural partial order, which can be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $V \setminus \{r\}$ and these vertices are successors of r . Moreover, every $a \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and a , as a predecessor of a and a successor of r . We say that the rooted tree T is *monotonically ordered* if each vertex is labelled with an integer r such that the label of a is less than the label of b if and only if a is an ancestor of b (i.e. the rows and the columns of the underlying matrix).

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree:



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are now arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

```
assert.dom('[data-test-message]')  
  .hasText(  
    'Thanks for signing up!',  
    'The user sees a welcome message.'  
  );
```

Therefore we will now give a few examples of matrices (represented by their sparsity)

triangular:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



quadrangular:

supersingular:

$$\begin{bmatrix} * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & * & * & * \end{bmatrix}$$



cyclic:

$$\begin{bmatrix} * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \end{bmatrix}$$



THEN f MUST HAVE A

ZERO IN (a, b) .

$$\begin{bmatrix} * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \end{bmatrix}$$

At a first glance this is not a triangular matrix, but it is a tree.

just displayed, but its graph:



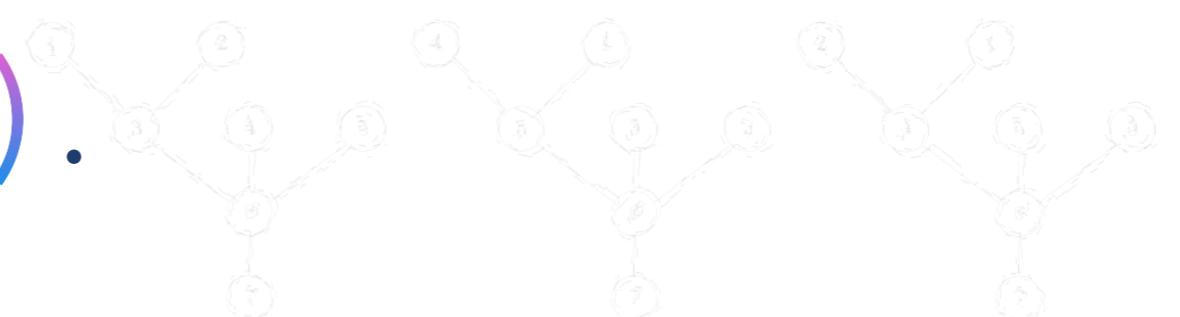
tells a different story - it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 2, 3 \rightarrow 3, 4 \rightarrow 4, 5 \rightarrow 5, 6 \rightarrow 6.$$

Of course, it is equivalent to relabeling the equations and variables. As you can see from the graph, G is a tree spanning the vertices α and β ($\alpha, \beta \in \{1, 2, \dots, 6\}$, $\beta \in \{1, 2, \dots, 6\}$) and for every $k = 1, 2, \dots, n-1$ the set $\{v_k, \beta\} \cap \{v_{k+1}, \beta\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of V are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees, but this is not the case with either of bidiagonal or cyclic matrices when $n \geq 3$.

treating T as a tree, the vertex $r \in V \setminus \{\beta\}$ is called a *rooted tree*. Unlike in ordinary graph, T does not have a natural partial ordering which can best be explained by an analogy with a family tree. Thus, the vertex r is the predecessor of all the vertices in $T \setminus \{r\}$ and these vertices are successors of r . Moreover, every $v \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a predecessor of α and a successor of r . We say that the rooted tree T is *monotonically ordered* if each vertex is labelled v_1, v_2, \dots, v_n in such a way that the vertices from the top of the tree down to the bottom are in increasing order (in other words, if we travel from the top of the tree down to the bottom, the labels increase). Labeling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree:



Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, first relabel the vertices as follows:



Dashboard

Explore

Settings



vertices more than once. We say that G is a *tree* if each two members of V are joined by a unique simple path. Both tridiagonal and super-diagonal matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $p \geq 3$.

Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a *rooted tree*, while r is said to be the *root*. Unlike in ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the *predecessor* of all the vertices in $T \setminus \{r\}$ and these vertices are *successors* of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a *predecessor* of α and a *successor* of r . We say that the rooted tree T is *monotonically ordered* if each vertex is labelled before all its predecessors. In other words, if label the vertices from the root of the

'[data-test-link="Dashboard"]'

'[data-test-link="Explore"]'

'[data-test-link="Settings"]'

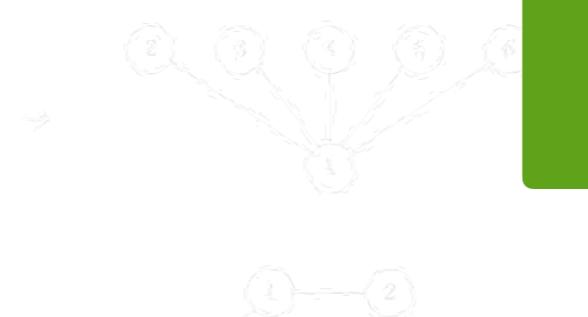
just displayed, but its graph.

$$t_{k,j} = \frac{a_{kj}}{a_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

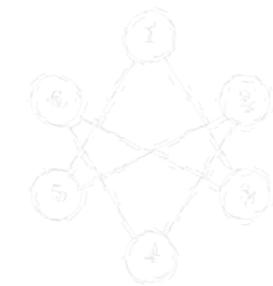
Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

Save

tridiagonal:	$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$
qundiagonal:	$\begin{bmatrix} * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \end{bmatrix}$
superdiagonal:	$\begin{bmatrix} * & * & * & * & * & * \\ * & * & 0 & 0 & 0 & 0 \\ * & 0 & * & 0 & 0 & 0 \\ 0 & * & * & 0 & 0 & 0 \\ 0 & 0 & * & * & 0 & 0 \\ 0 & 0 & 0 & * & * & * \end{bmatrix}$



Cancel



tells a different story - it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$1 \rightarrow 5, 2 \rightarrow 3, 3 \rightarrow 4, 4 \rightarrow 6, 6 \rightarrow 5.$

or reordering (simultaneously) the equations and variables.

$\{(v_i, j)\}_{i,j=1}^n \subseteq \mathbb{S}$ is called a path joining the vertices v_i and v_j , and for every $k = 1, 2, \dots, n-1$ the set $\{v_i, j_k\} \cap$

$\{v_i, j_{k+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of V are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees. This is not the case with either qundiagonal or cyclic matrices when $n \geq 3$.

Fix a tree T and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in ordinary graph, T admits a natural partial order, which can best be explained by an analogy with a family tree. Thus, the vertex r is the predecessor of all the vertices in $T \setminus \{r\}$ and these vertices are successors of r . Moreover, every $a \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and a , as a predecessor of a and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors. In other words, if travel the vertices from the root of the

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'[data-test-button="Cancel"]'

'[data-test-button="Add item"]'

just displayed, but its graph.

$$t_{k,j} = \frac{a_{k,j}}{a_{j,j}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (specified by their sparsity pattern) and their graphs.

三

Name*

× × × 0 0 0 × × × 0 0 0 × × × 0 0 0
× × × 0 0 0 × × × 0 0 0 × × × 0 0 0

W W D D D D X X X D D D

Description

atrix in disguised. To see

equations and variables. By joining the vertices of $\mathcal{V} = \mathcal{V}_1 \cup \mathcal{V}_2$ the set $\{p_1, p_2\}$ is reached if it does not visit any members of \mathcal{V} are joined by lines corresponding to vertices which will be \mathcal{V}_1 .

- (G, α) is called a *rooted* α *admits a natural partial*
a family tree. Thus, the
se vertices are predecessors
a path and we designate
son of α and a α successor
if each vertex is labelless
tree from the top of the
graph is tantamount to

```
'[data-test-field="Name"]'
```

'[data-test-field="Description"]'

just displayed, but its graph,

$$t_{k,j} = \frac{a_{k,j}}{f_{k,j}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparse pattern) and their graphs.

三

tridimensional
cyclic

Name

Little Bobby Tables

Description

Better not drop me!

permuting the rows and the columns of the underlying matrix.)

```
'[data-test-field="Name"]'
```

```
'[data-test-field="Description"]'
```

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix



At a first glance, there is nothing to link it to any of the four structures just displayed, but its graph,

WRITE LESS WITH THEOREMS AND NEW TERMS

Theorem 11.1. Let A be a $n \times n$ matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, n\}$ and assume that the rows and columns of A are then arranged so, that $T = (G, r)$ is monotonically ordered. Then, that $A = L U$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{k,j}}{l_{j,j}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

triangular:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



quadrangular:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



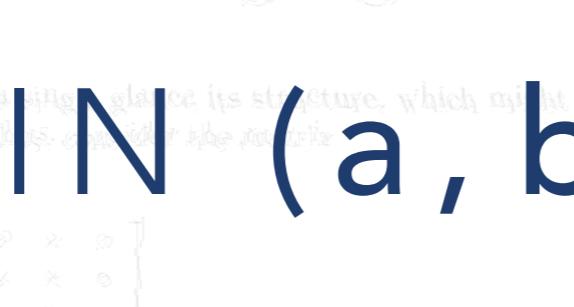
supernodal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



cyclic:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



IF f IS CONTINUOUS IN $[a, b]$, AND IF $f(a)f(b) < 0$,

THEN f MUST HAVE A

ZERO IN (a, b) .



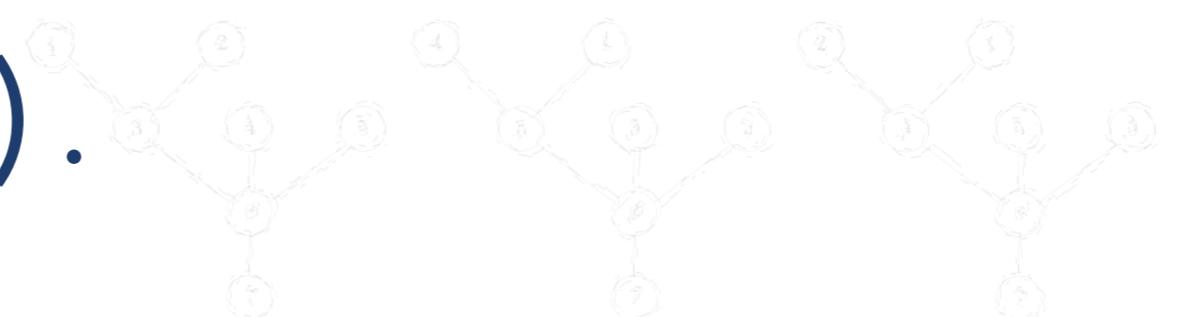
tells a different story - it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 2, 3 \rightarrow 3, 4 \rightarrow 4, 5 \rightarrow 5, 6 \rightarrow 3.$$

Of course, it is equivalent to replacing (arbitrarily) the equations and variables. As you can see from the figure, the graph G is called a tree, since the vertices a and b ($a \in V \setminus \{b\}$, $b \in V \setminus \{a\}$) and for every $k = 1, 2, \dots, n-1$ the set $\{v_1, v_2, \dots, v_k\} \cap \{v_{k+1}, v_{k+2}, \dots, v_n\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of V are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees, but this is not the case with either of banded or cyclic matrices when $n \geq 3$.

Of course, a rooted tree is a tree with a root vertex r . Unlike in a general graph, there is a natural partial ordering which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $V \setminus \{r\}$ and these vertices are successors of r . Moreover, every $v \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and v , as a predecessor of v and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled by an integer $r \in \{1, 2, \dots, n\}$ in such a way that the vertices from the top of the tree to the bottom are in increasing order (in other words, if we read the vertices from the top of the tree to the bottom, we have a strictly increasing sequence). Labeling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree:



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, n\}$ and assume that the rows and columns of A are then arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

At a first glance, this is not a very useful formula, but it is the basis for the algorithm just displayed, but its graph.

Therefore we will now give a few examples of matrices (represented by their sparsity)

PROOF. USE THE INTERMEDIATE VALUE THEOREM.

cyclic:

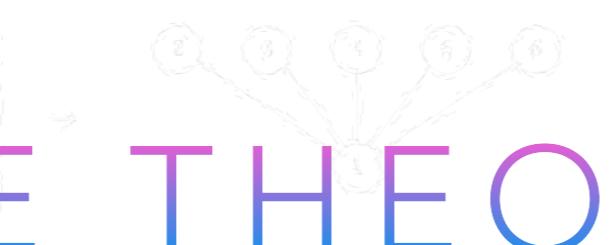
$$\begin{bmatrix} * & * & 0 & 0 & 0 \\ 0 & * & * & 0 & 0 \\ * & 0 & * & 0 & 0 \\ 0 & 0 & 0 & * & 0 \\ 0 & 0 & 0 & 0 & * \end{bmatrix}$$



$$\begin{bmatrix} * & * & 0 & 0 & 0 \\ 0 & * & * & 0 & 0 \\ * & 0 & * & 0 & 0 \\ 0 & 0 & 0 & * & 0 \\ 0 & 0 & 0 & 0 & * \end{bmatrix}$$



$$\begin{bmatrix} * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ * & 0 & * & * & 0 & 0 \\ 0 & 0 & 0 & * & 0 & 0 \\ 0 & 0 & 0 & 0 & * & 0 \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & 0 \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & * \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & * \end{bmatrix}$$

At a first glance this is not a cyclic matrix, but it is. It is a 6x6 matrix with the same structure as the one just displayed, but its graph



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

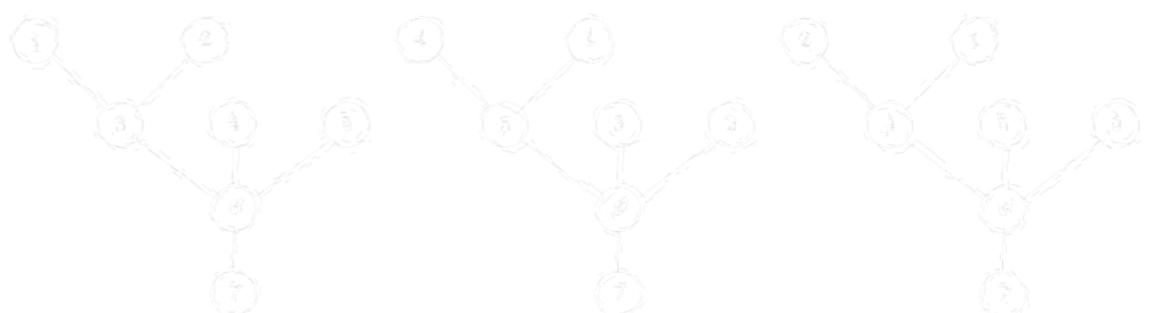
$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_0, j_0)\}_{i_0, j_0 \in \mathbb{N}_0, i_0 \neq j_0}$ in \mathbb{G} is called a path joining the vertices i_0 and j_0 if $i_0 \in \{i_0, j_0\}$, $j_0 \in \{i_0, j_0\}$ and for every $p = 1, 2, \dots, v-1$ the set $\{i_p, j_p\} \cap \{i_{p+1}, j_{p+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertices more than once. We say that \mathbb{G} is a tree if in two members of \mathbb{V} are joined by a unique simple path. Known tridiagonal and cyclic matrices correspond to trees, but this is not the case with either quadiagonal or cyclic matrices when $v \geq 3$.

Given a tree \mathbb{T} and an arbitrary vertex $r \in \mathbb{V}$, the pair $\mathbb{T} = (\mathbb{G}, r)$ is called a rooted tree, while r is said to be the root. Unlike in ordinary graph, \mathbb{T} adapts a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root is the ancestor of all the vertices in $\mathbb{V} \setminus \{r\}$ and these vertices are successors of r . Moreover, every $\alpha \in \mathbb{V} \setminus \{r\}$ is joined to r by a simple path and we designate any vertex along this path, except for r and α , as a predecessor of α and a successor of r . We say that the rooted tree \mathbb{T} is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree:



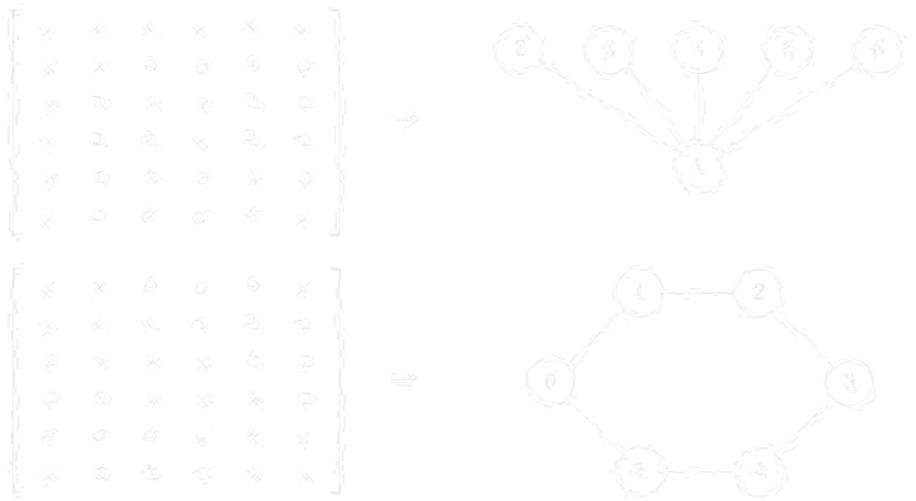
Theorem 11.1. Let \mathbb{A} be a symmetric matrix whose graph \mathbb{G} is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of \mathbb{A} were then arranged so that $\mathbb{T} = (\mathbb{G}, r)$ is monotonically ordered. Given that $\mathbb{A} = \mathbb{L}\mathbb{U}^T$ is a Cholesky factorization, it is true that

$$a_{k,j} = \frac{a_{k,j}}{a_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

PROOF.
USE IVT.

triangular:
quadratic:
cyclic:



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & x & 0 \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & 0 & * & 0 \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & * & 0 \end{bmatrix}.$$

At a first glance this is not immediately clear what the structure of the matrix is. The graph, however, is just displayed, but its graph:



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 3, \quad 5 \rightarrow 6, \quad 6 \rightarrow 3.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$ is called a path joining the vertices i_0 and j_0 if $i_0 \in \{i_0, j_0\}$, $j_0 \in \{i_0, j_0\}$ and for every $k = 1, 2, \dots, n-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of V are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $n \geq 3$.

Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in ordinary graph, T adapts a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $T \setminus \{r\}$ and these vertices are successors of r . Moreover, every $a \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and a , as a predecessor of a and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three consecutive orderings of the same rooted tree:

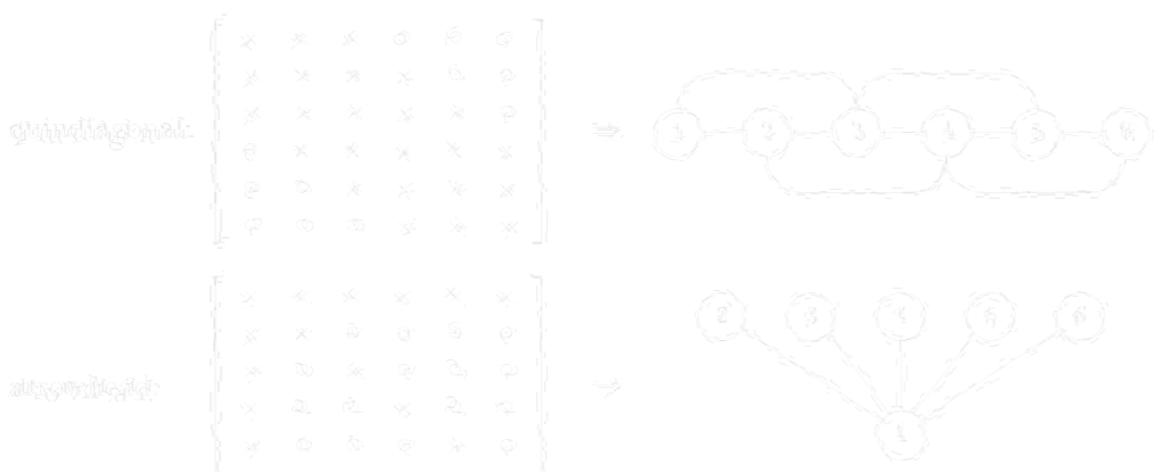


Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so, that $T = (G, r)$ is monotonically ordered. Given that $A = L U$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{j,j}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

THEOREM



PROVEN

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

TO BE TRUE

At a first glance this is a random matrix, but it is in fact a triangular matrix, just displayed, but its graph,



tells a different story – it is nothing other than the cyclic matrix in discussed. To see this, just relabel the vertices as follows:

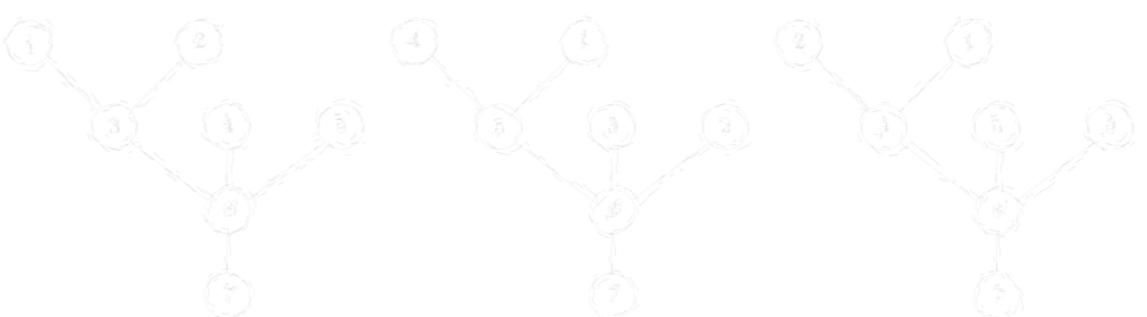
$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$ is called a path joining the vertices i_0 and j_0 if $i_0 \in \{i_0, j_0\}$, $j_0 \in \{i_0, j_0\}$ and for every $k = 1, 2, \dots, n-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of V are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $n \geq 3$.

Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in a ordinary graph, T adapts a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $T \setminus \{r\}$ and these vertices are successors of r . Moreover, every $a \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and a , as a predecessor of a and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree:



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A were been arranged so, that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

```
import { fillIn } from '@ember/test-helpers';

export async function fillForm(fields) {
  for (const { label, value } of fields) {
    // input, textarea
    await fillIn(`[data-test-field="${label}"`, value);
  }
};
```

```
import { fillForm } from '../helpers/my-test-helper';

...
test('User can create account', async function(assert) {
  await visit('/signup');
  await fillForm([
    { label: 'Name', value: 'Little Bobby Tables' },
    { label: 'Email', value: 'little.bobby@gmail.com' }
  ]);
  ...
});
```

Therefore we will now give a few examples of matrices (represented by their sparsity)

NEW TERM

quidiagonal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ 0 & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$



superdiagonal:

$$\begin{bmatrix} * & * & * & * & * & * \\ * & 0 & 0 & 0 & 0 & 0 \\ 0 & * & 0 & 0 & 0 & 0 \\ 0 & 0 & * & 0 & 0 & 0 \\ 0 & 0 & 0 & * & 0 & 0 \\ 0 & 0 & 0 & 0 & * & 0 \end{bmatrix}$$



cyclic:

$$\begin{bmatrix} * & 0 & 0 & 0 & * & * \\ * & * & 0 & 0 & 0 & 0 \\ 0 & * & * & 0 & 0 & 0 \\ 0 & 0 & * & * & 0 & 0 \\ 0 & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



UBIQUITOUS

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

IDEA

$$\begin{bmatrix} * & * & 0 & 0 & * & * \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$

At a first glance, this is not a matrix that looks like the three structures just displayed, but its graph,



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 3, \quad 5 \rightarrow 6, \quad 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$ is called a path joining the vertices i_0 and j_0 if $i_0 \in \{i_0, j_0\}$, $j_0 \in \{i_0, j_0\}$ and for every $k = 1, 2, \dots, n-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of V are joined by a unique simple path. Both tri-diagonal and any other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $n \geq 3$.

Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in a ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $T \setminus \{r\}$ and these vertices are successors of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a predecessor of α and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree:



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, n\}$ and assume that the rows and columns of A are then arranged so, that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

triangular:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$

quadrangular:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$

cyclic:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$

At a first glance this is not a triangular or quadrangular matrix, but it is a triangular matrix just displayed, but its graph,



IF f IS CONTINUOUS IN $[a, b]$, AND IF $f(a)f(b) < 0$,

THEN f MUST HAVE A

ZERO IN (a, b) .



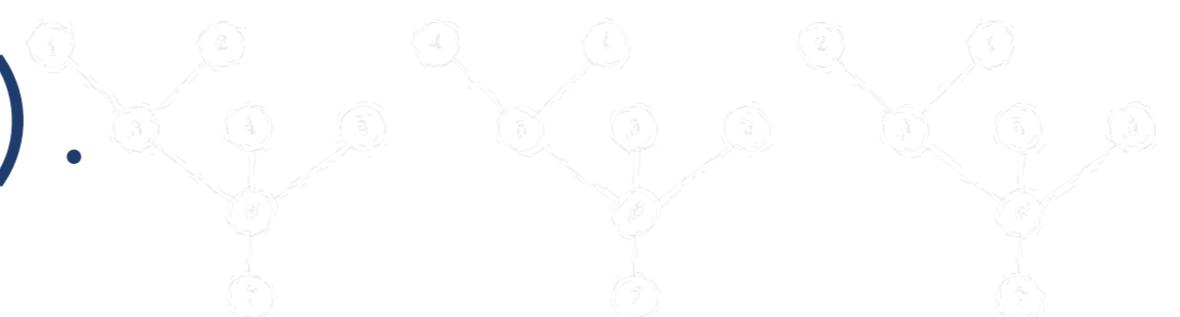
tells a different story - it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 3 \rightarrow 5, \quad 2 \rightarrow 4 \rightarrow 6, \quad 3 \rightarrow 1, \quad 4 \rightarrow 2.$$

Of course, it is equivalent to reordering simultaneously the equations and variables. As a result, we get the following graph G . It is called a tree, since the vertices a and b ($a \in V \setminus \{r\}$, $b \in V \setminus \{r\}$) and for every $k = 1, 2, \dots, n-1$ the set $\{v_1, v_2\} \cap \{v_{k+1}, v_{k+2}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of V are joined by a unique simple path. Both triangular and quadrangular matrices correspond to trees, but this is not the case with either quadrangular or cyclic matrices when $n \geq 3$.

It is natural to call a tree T a rooted tree, if there is a vertex $r \in V$ (the root) such that it is not joined to any other vertex. Unlike in a binary tree, there is a natural partial order, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $V \setminus \{r\}$ and these vertices are successors of r . Moreover, every $a \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and a , as a predecessor of a and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled by an integer from 1 to n in such a way that the vertices from the top of the tree to the bottom are arranged in increasing order. (In other words, we say the vertices from the top of the tree to the bottom are arranged in increasing order, because the top vertex is the root, and the bottom vertex is the leaf.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree:



Therefore we will now give a few examples of matrices (represented by their sparsity)

triangular:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



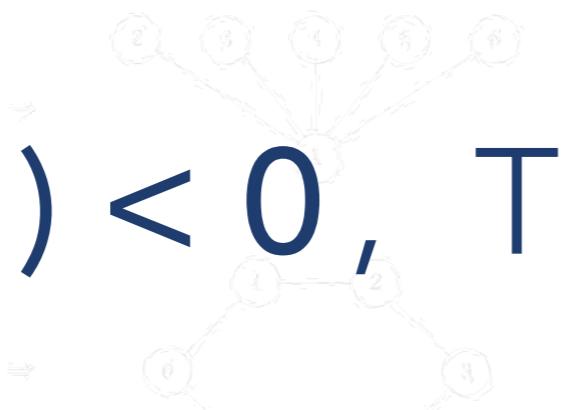
quadratic:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



symmetric:

$$\begin{bmatrix} * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ 0 & * & * & * & * & * \\ 0 & 0 & * & * & * & * \\ 0 & 0 & 0 & * & * & * \end{bmatrix}$$



cyclic:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



IF $f \in C([a, b])$, AND $f(a)f(b) < 0$, THEN f MUST HAVE A ZERO IN (a, b) .

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} * & * & * & 0 & * & 0 \\ 0 & * & 0 & * & * & 0 \\ * & * & * & 0 & 0 & * \\ 0 & 0 & * & * & * & * \\ * & * & 0 & * & * & * \\ 0 & * & * & * & * & * \end{bmatrix}$$

At a first glance, this is not a triangular matrix, but it is a tridiagonal matrix, just displayed, but its graph:



tells a different story - it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_0, j_0)\}_{i_0, j_0 \in \mathbb{N}_0, i_0 \neq j_0}$ in \mathbb{G} is called a path joining the vertices i_0 and j_0 if $i_0, j_0 \in \{1, 2, \dots, n\}$, $i_0 \neq j_0$, and for every $p = 1, 2, \dots, v-1$ the set $\{i_p, j_p\} \cap \{i_{p+1}, j_{p+1}\}$ contains only one member. It is a simple path if it does not visit any vertex more than once. We say that \mathbb{G} is a tree if each two members of \mathbb{V} are joined by a unique simple path. Both quadiagonal and symmetric matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $n \geq 3$.

Given a tree \mathbb{G} and an arbitrary vertex $r \in \mathbb{V}$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in a digraph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the predecessor of the vertex v is the vertex u if v is a successor of u in the tree T . Moreover, every $u \in \mathbb{V}$ is the predecessor of v in the path and we designate u as a parent along this path, except for r , which is a predecessor of v and a successor of u . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we lay out the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We will give three examples consisting of the same rooted tree:

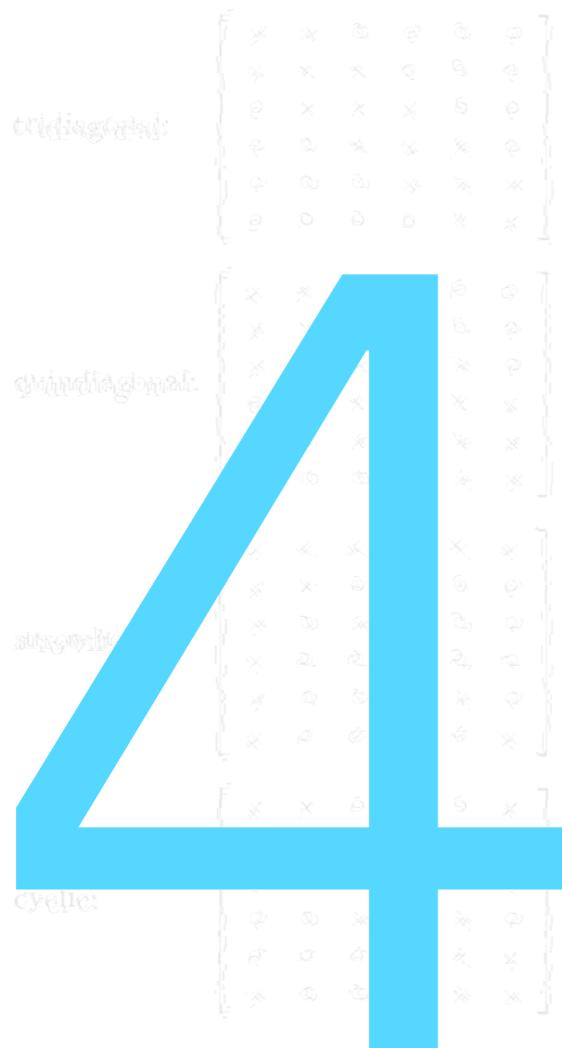


Theorem 11.1 Let A be a symmetric matrix whose graph \mathbb{G} is a tree. Choose a root $r \in \{1, 2, \dots, n\}$ and assume that the rows and columns of A are then arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

```
hooks.beforeEach(function(assert) {  
  ...  
  // Example: assert.isEnabled('Submit', 'Woot!');  
  assert.isEnabled = (label, message) => {  
    assert.dom(`[data-test-button="${label}"`)  
      .hasNoAttribute('disabled', message);  
  };  
  ...  
});
```

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



ALL YOUR BASIS ARE BELONG TO US

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & * \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & * \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & * \end{bmatrix}$$

At a first glance, there is nothing to link it to any of the four structures just displayed, but its graph,

tells a different story. It is nothing other than the cyclic matrix in disguised. To see this, just read the vertices in binary:

$$1 \rightarrow 1, 2 \rightarrow 3, 3 \rightarrow 2, 4 \rightarrow 5, 5 \rightarrow 6, 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_0, j_0)\}_{i_0, j_0 \in \mathbb{S}}$ is called a path joining the vertices i_0 and j_0 if $i_0 \in \{1, 2, \dots, n\}$, $j_0 \in \{1, 2, \dots, n\}$ and for every $k = 1, 2, \dots, n-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of \mathbb{V} are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees, but this is not the case with either quadiagonal or cyclic matrices when $n \geq 3$.

Given a tree T and an arbitrary vertex $r \in \mathbb{V}$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $T \setminus \{r\}$ and these vertices are successors of r . Moreover, every $a \in \mathbb{V} \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and a , as a predecessor of a and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Any rooted tree and the monotonically ordered and, in general, such an ordering of the vertices of the same rooted tree.

Therefore let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, n\}$ and assume that the rows and columns of A were been arranged so, that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{k,j}}{l_{j,j}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

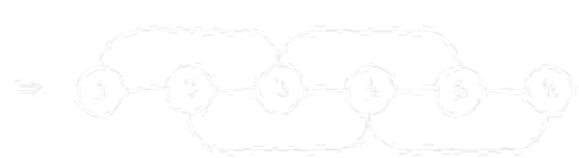
triangular:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$



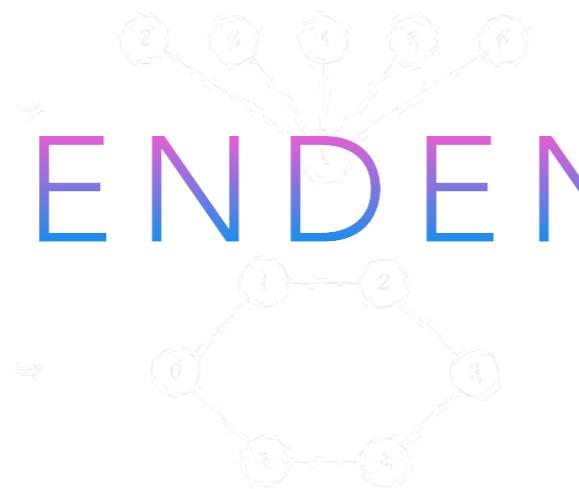
quidiagonal:

$$\begin{bmatrix} * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$



diagonal:

$$\begin{bmatrix} * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

SPAN THE ENTIRE SPACE

At a first glance, this is not a matrix that looks like the basis matrices we have just displayed, but its graph,



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(v_i, j)\}_{i=1}^n$ in \mathbb{G} is called a path joining the vertices v_i and v_j if $i \in \{1, \dots, n\}$, $j \in \{1, \dots, n\}$ and for every $k = 1, 2, \dots, n-1$ the set $\{v_i, j\} \cap \{v_{i+k}, v_{i+k+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that \mathbb{G} is a tree if each two members of V are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $n \geq 3$.

Given a tree \mathbb{G} and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the only vertex that has no predecessors and all other vertices are successors of r . Moreover, every $v \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex alone, throughout, except for r and v , as a predecessor of v and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree:



Theorem 11.1. Let A be a symmetric matrix whose graph \mathbb{G} is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph.

tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 3, 3 \rightarrow 2, 4 \rightarrow 4, 5 \rightarrow 6, 6 \rightarrow 3.$$

Of course, it is equivalent to renumbering (cyclic shift) the vertices and to consider a different set of edges $\{ (i, j) \mid i, j \in \{1, 2, \dots, 6\} \}$. It is called a *cycle* if the vertices $i, j \in \{1, 2, \dots, n\}$ and for every $\ell = 1, 2, \dots, n-1$ the set $\{i + \ell, j + \ell\}$ contains exactly one member. It is a *simple path* if it does not visit any vertex more than once. We say that G is a *tree* if each two members of V are joined by a unique path. Both tridiagonal and antidiagonal matrices correspond to the case with either quidiagonal or cyclic matrices when $\ell = 1, 2, \dots, n-1$ and no arbitrary vertex $v \in V \setminus \{r\}$ is called a *root* and r is called the *root*. Unlike in a ordinary graph, T admits a natural partial order that must be explained by an analogy with a family tree. Thus, the ancestor of all the vertices in $T \setminus \{r\}$ and these vertices are *successors*. Every $a \in V \setminus \{r\}$ is joined to r by a simple path and we designate this path, except for r and a , as a *predecessor* of a and a *successor*. The rooted tree T is *monotonically ordered* if each vertex is labelled with a natural number in other words, we label the vertices from the top of the tree to the bottom (we have already done it, relabeling a graph to obtain a tree). This is another way to give three examples of the same concept.



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so, that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{j,j}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,

tridiagonal:

$$\begin{bmatrix} * & * & 0 & & \\ * & * & * & 0 & \\ 0 & & & & \end{bmatrix}$$

quasi-tridiagonal:

$$\begin{bmatrix} * & * & 0 & & & \\ * & * & * & 0 & & \\ 0 & & & & & \\ & & & * & * & 0 \\ & & & 0 & * & * \\ & & & & 0 & * \end{bmatrix}$$

general:

$$\begin{bmatrix} * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * \end{bmatrix}$$

Toeplitz:

$$\begin{bmatrix} * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * \end{bmatrix}$$

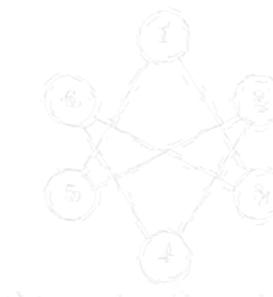
block-diagonal:

$$\begin{bmatrix} * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * \end{bmatrix}$$

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,

graph:

$$\begin{array}{c} \text{graph} \\ \text{---} \end{array}$$



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 3, 3 \rightarrow 2, 4 \rightarrow 4, 5 \rightarrow 6, 6 \rightarrow 8.$$

Of course, it is equivalent to renumbering (cyclic shift) the vertices and edges. Consider a set of sites $\{1, 2, \dots, n\}$. It is called a *root* if the vertex $r \in \{1, 2, \dots, n\}$ and for every $i = 1, 2, \dots, n-1$ the set $\{r+i\}$ contains exactly one member. It is a *simple path* if it does not visit any vertex more than once. We say that G is a *tree* if each two members of V are joined by a simple path. Both tridiagonal and quasi-tridiagonal correspond to trees. A tree with either quidiagonal or cyclic pattern is a *cycle*. A tree with an arbitrary vertex r is called a *rooted tree*. Unlike in a ordinary graph, T admits a natural partial order that must be explained by an analogy with a family tree. Thus, the ancestor of all the vertices in $T \setminus \{r\}$ and these vertices are *successors* of r . Every $a \in V \setminus \{r\}$ is joined to r by a simple path and we designate this path, except for r and a , as a *predecessor* of a and a *successor* of r . The rooted tree T is *monotonically ordered* if each vertex is labelled with its *depth* in other words, we label the vertices from the top of the tree to the bottom. (In other words, in the underlying matrix, the rows and columns are monotonically ordered and, in general, such a ordering does not give three consecutive entries of the same nonzero entry.)

Now, we give three examples of the simple rooted tree.



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, n\}$ and assume that the rows and columns of A are arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{j,j}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

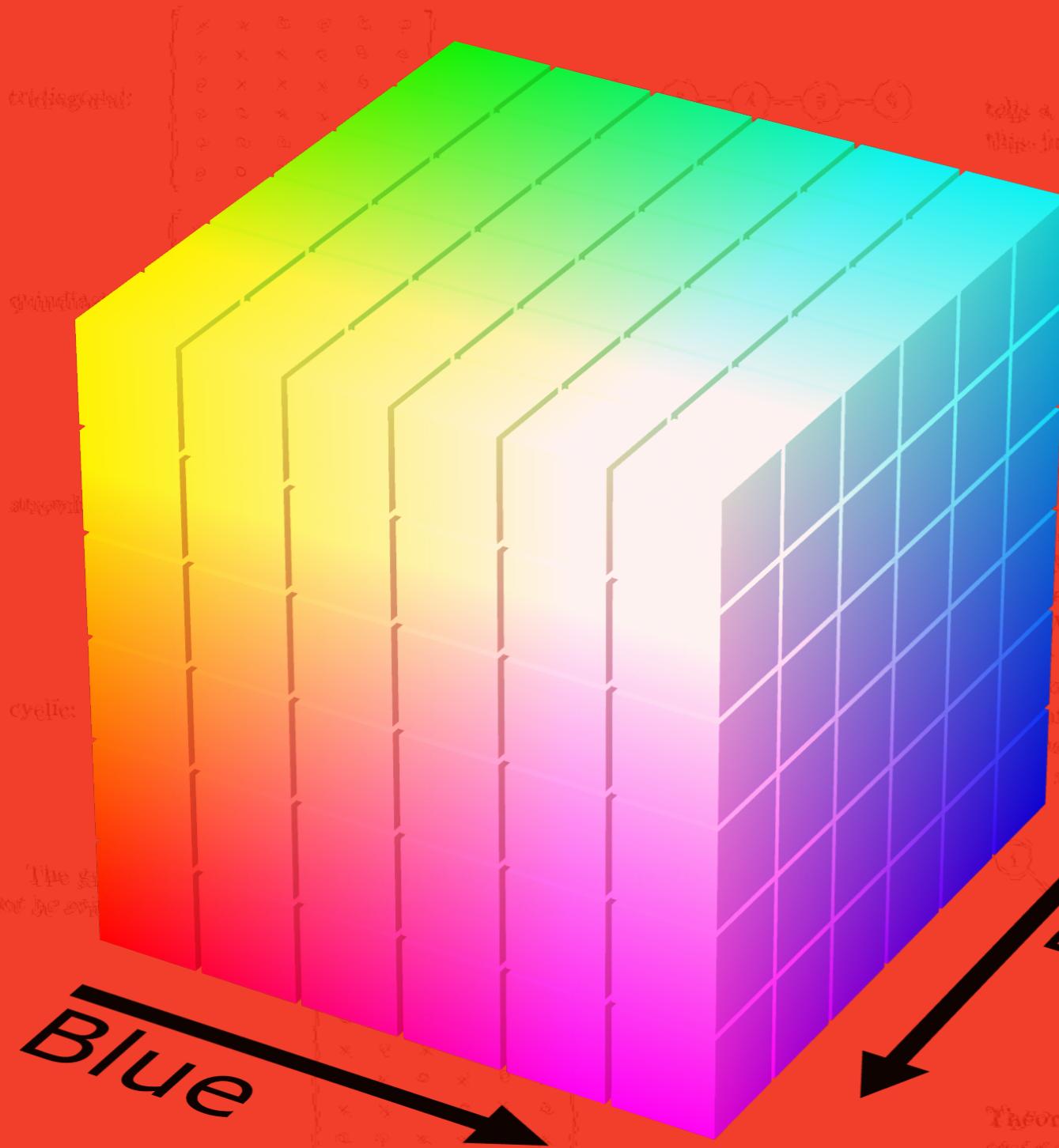


Theorem 11.2. Let A be a symmetric matrix whose graph G is a cycle. Choose a root $r \in \{1, 2, \dots, n\}$ and assume that the rows and columns of A are arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sqrt{a_{rr}}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.6)$$



Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



At a first glance, there is nothing to link in to any of the four matrices that we have just displayed, but its graph.



RED

224

tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, first relabel the vertices as follows:

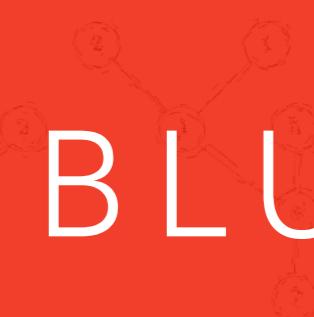
$$1 \rightarrow 1, 2 \rightarrow 3, 3 \rightarrow 2, 4 \rightarrow 4, 5 \rightarrow 6, 6 \rightarrow 5$$

Of course, it is equivalent to reordering (simultaneously) the columns and rows. A *partial set of steps* $\{(v_i, j_i)\}_{i=1}^n \subseteq \mathbb{E}$ is called a *path* joining the vertices v_i and v_j if $(v_i, j_i), j_i \in \{1, \dots, n\}$ and for every $k = 1, 2, \dots, n-1$ the set $\{v_{k+1}, j_{k+1}\} \cap \{v_i, j_i\}$ contains exactly one member. It is a *simple path* if it does not visit any vertex more than once. We say that G is a *tree* if each two members of V are joined by a simple path. Both tri-diagonal and quasi-diagonal matrices correspond to trees. (This is not the case with either quidiagonal or cyclic matrices when $n \geq 3$.)

Let's start with an arbitrary vertex $r \in V$. The set $T = \{v_i \in V \mid v_i \text{ is joined to } r \text{ by a simple path}\}$ is called a *rooted tree* with r added to be the root. Unlike a ordinary tree, T does not have a unique root, which can best be explained by an example of a family tree. This is the reason why all the vertices in T and their children are *descendants* of r . Moreover, if $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate α as the *parent* of r , then r is the *child* of α , except for r and α , as a *predecessor* of α and a *successor* of r . We say that a rooted tree T is *monotonically ordered* if each vertex is labelled with a unique integer i in other words we label the vertices from the top of T to the bottom. (We have already said it, relabelling a graph is the same as relabelling the rows and the columns of the underlying matrix.)

Every vertex in T will be monotonically ordered and, in general, such an ordering is unique. We can give three consecutive orderings of the same rooted tree:

Red



BLUE

57

Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, n\}$ and assume that the rows and columns of A are re-arranged so, that $T = (G, r)$ is monotonically ordered. Then, that $A = L U$ is a Cholesky factorization. It is true that

$$l_{k,j} = \frac{a_{k,j}}{a_{j,j}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

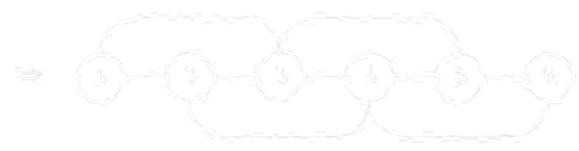
and their graphs

tridiagonal:
$$\begin{bmatrix} * & * & 0 & 0 & 0 \\ * & * & * & 0 & 0 \\ 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & * & 0 \\ 0 & 0 & 0 & 0 & * \end{bmatrix}$$



BASIS

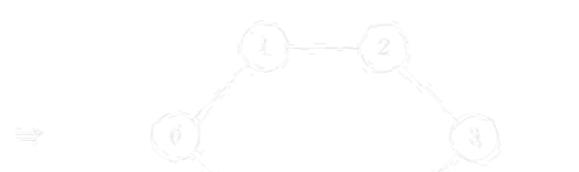
quindiagonal:

$$\begin{bmatrix} * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & * & 0 \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$


superdiagonal:

$$\begin{bmatrix} * & * & * & * & * & * \\ * & * & 0 & 0 & 0 & 0 \\ 0 & 0 & * & 0 & 0 & 0 \\ 0 & 0 & 0 & * & 0 & 0 \\ 0 & 0 & 0 & 0 & * & 0 \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$


cyclic:

$$\begin{bmatrix} * & 0 & 0 & 0 & * & * \\ * & * & 0 & 0 & 0 & 0 \\ 0 & 0 & * & 0 & 0 & 0 \\ 0 & 0 & 0 & * & 0 & 0 \\ 0 & 0 & 0 & 0 & * & 0 \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$


BUILDING BLOCKS

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

OF TESTS

At a first glance, this is not a matrix, but a graph of 10 nodes. It is, however, a matrix, just displayed, but its graph:



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$ is called a path joining the vertices i_0 and j_0 if $i_0 \in \{1, \dots, n\}$, $j_0 \in \{1, \dots, n\}$ and for every $k = 1, 2, \dots, n-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of V are joined by a unique simple path. Both tridiagonal and superdiagonal matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $n \geq 3$.

Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in a ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $T \setminus \{r\}$ and these vertices are successors of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a predecessor of α and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is unique. The following diagram shows three consecutive orderings of the same rooted tree.



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A were been arranged so, that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just re-label the weights as follows:

Create Edit Delete Clone Import

[[*, 0, 0, 0, 0, 0], [0, *, 0, 0, 0, 0], [0, 0, *, 0, 0, 0], [0, 0, 0, *, 0, 0], [0, 0, 0, 0, *, 0], [0, 0, 0, 0, 0, *]]

a single simple path. Both tridiagonal and any banded matrices correspond to trees, but this is not the case with either anti-diagonal or cyclic matrices when $n > 3$.

	Name	Description
<input checked="" type="checkbox"/>	Little Bobby Tables	Better not drop me!
<input type="checkbox"/>	Big Bobby Tables	
<input type="checkbox"/>	Foo Bar	Making up names is hard.

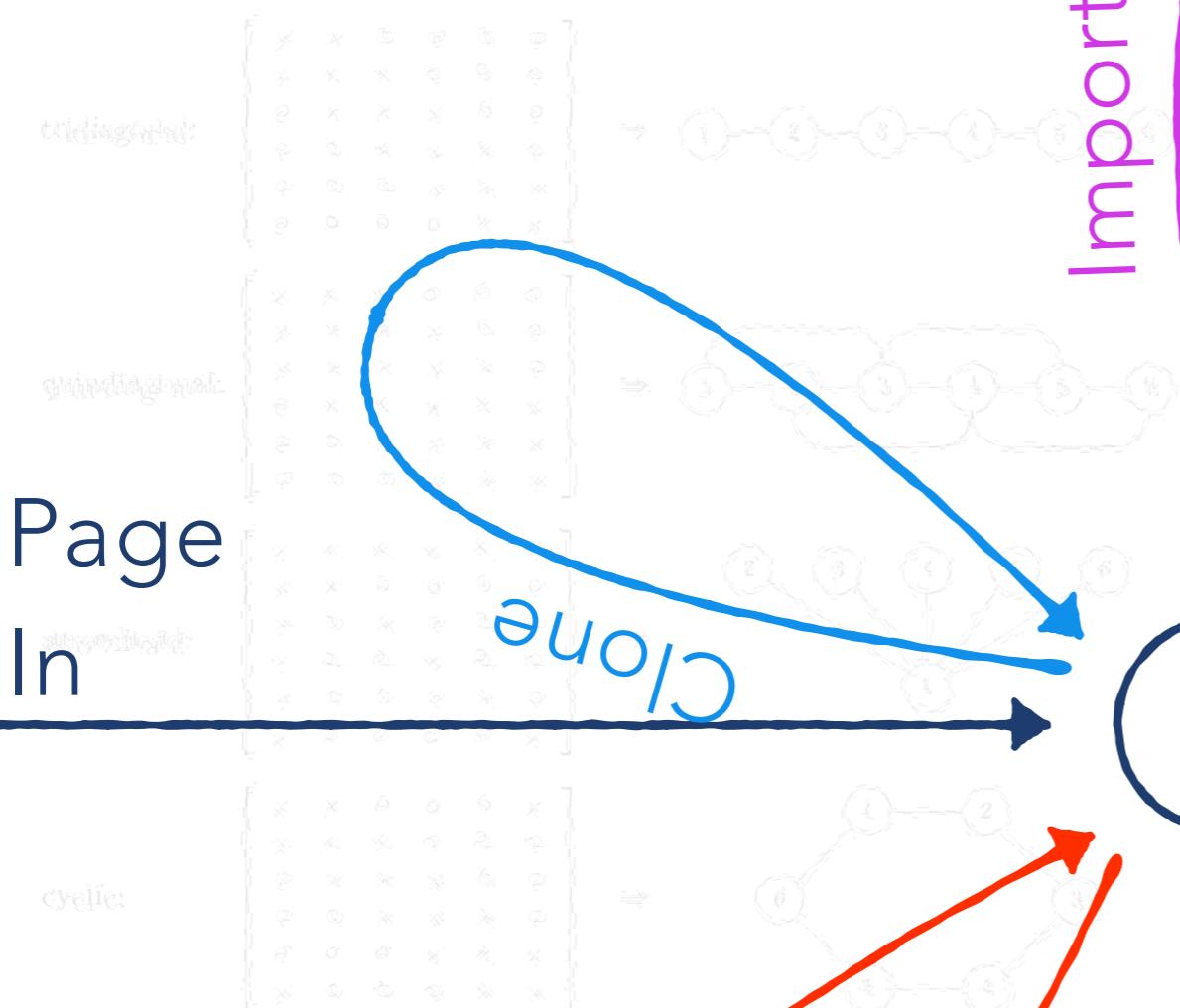


Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so that $\mathbf{T} = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph:

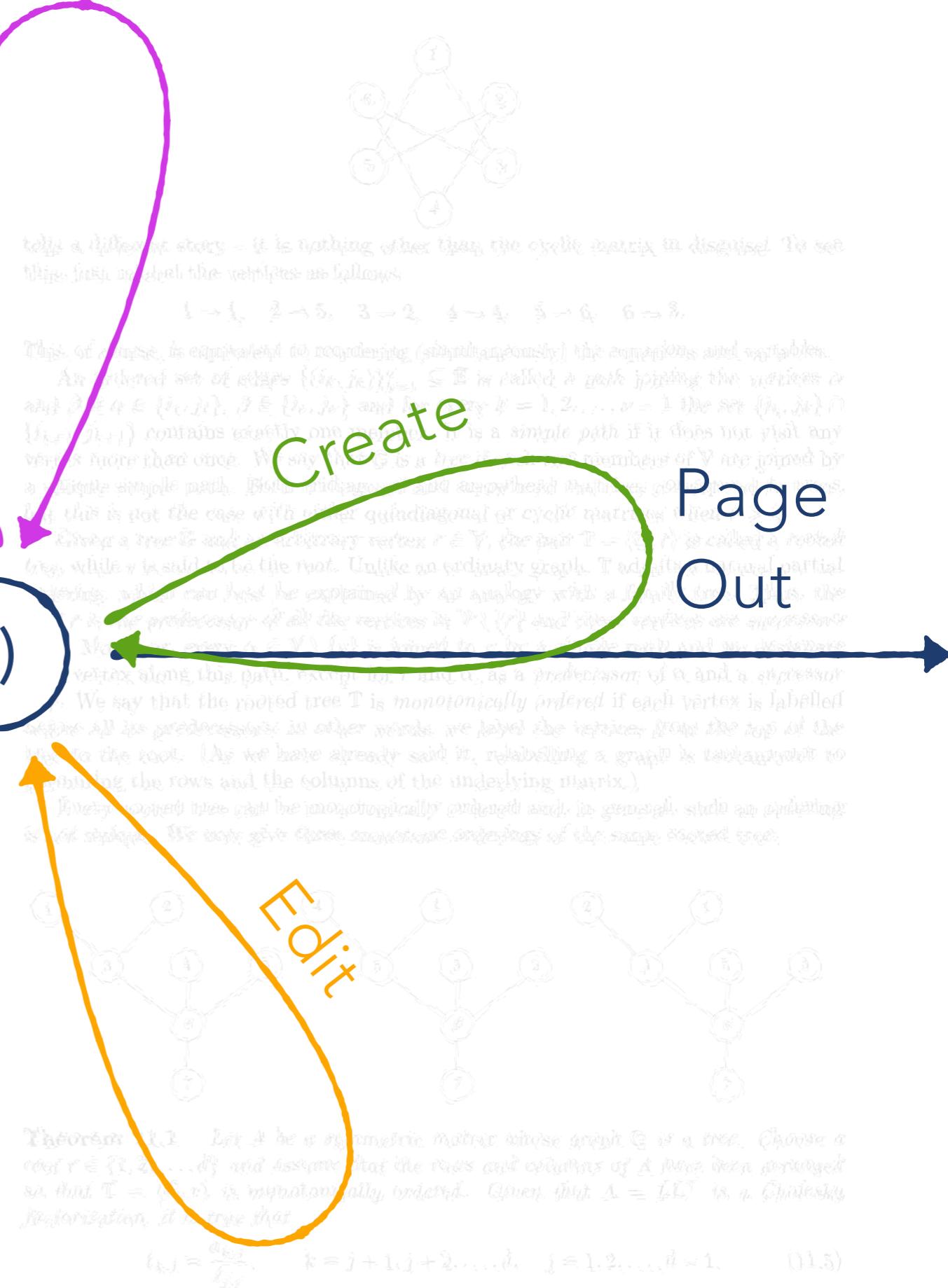
Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



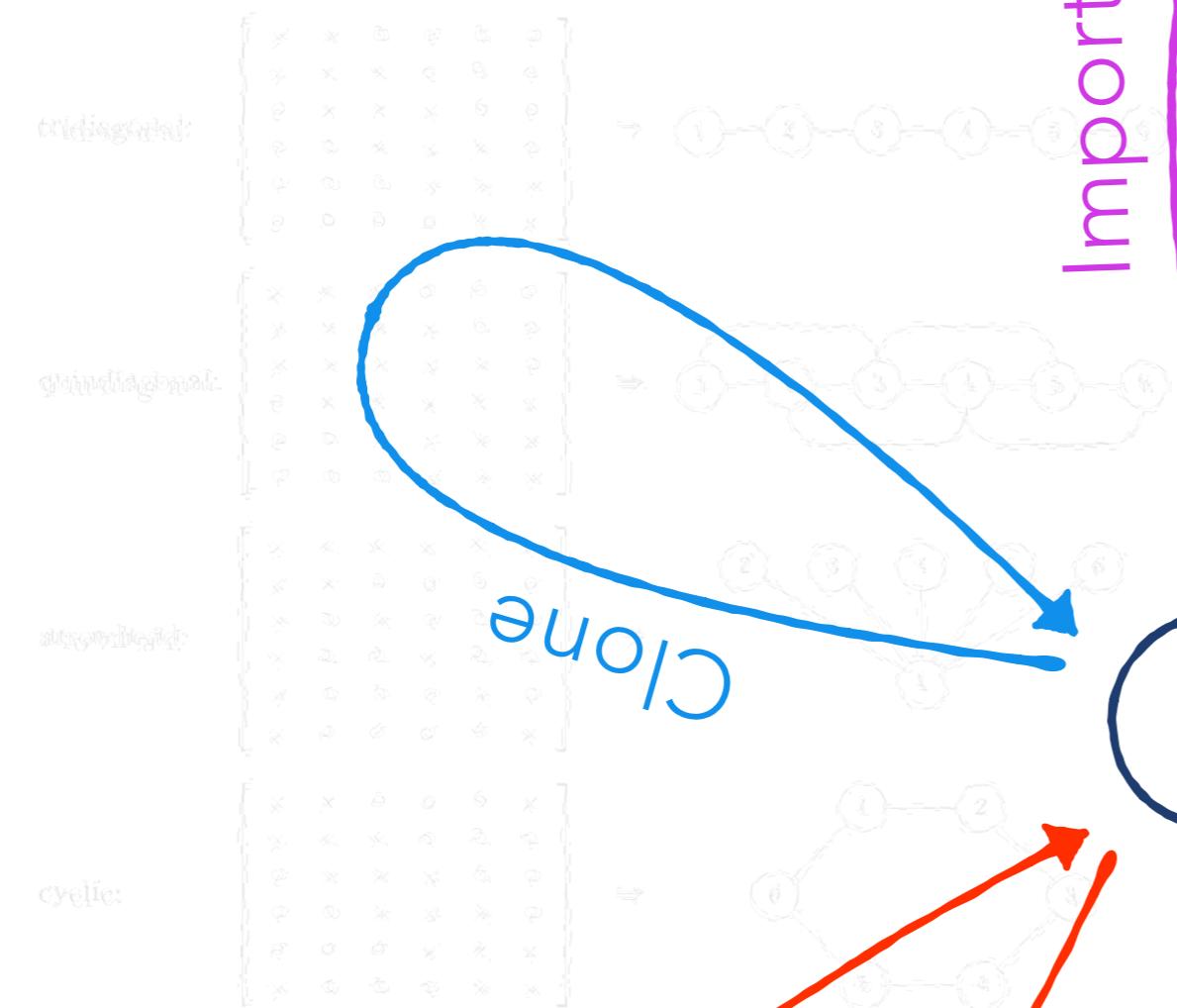
The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix



At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,



Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



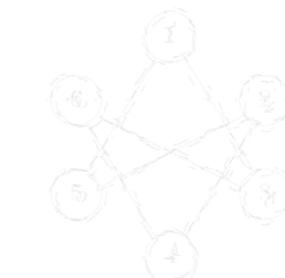
The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$

Delete

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,

Import



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just read the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 3, 3 \rightarrow 2, 4 \rightarrow 4, 5 \rightarrow 6, 6 \rightarrow 8.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_0, j_0), \dots, (i_{v-1}, j_{v-1})\}$ is called a path joining the vertices i_0 and i_{v-1} ($i_0, i_1, \dots, i_{v-1}, i_v \in V$, $j_0, j_1, \dots, j_{v-1} \in V$) if for every $k = 1, 2, \dots, v-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one vertex. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if no two members of V are joined by a single simple path. Both quadiagonal and pentadiagonal matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $p \geq 3$.

Create a tree T and a arbitrary vertex $r \in V$, the pair $T - \{r\}$ is called a rooted tree, while r is said to be the root. Unlike in a ordinary graph, T has a natural partial order, which can best be explained by an analogy with a family tree. Thus, the vertex v is a predecessor of all the vertices in $T \setminus \{v\}$ and these vertices are successors of v . Moreover, every $a \in V \setminus \{r\}$ is joined to r by a simple path and we designate a as a predecessor of r and r as a predecessor of a and a successor of a along this path, except in r and a , as a predecessor of a and a successor of a . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to relabelling the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We will give three consecutive drawings of the same rooted tree.



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, n\}$ and assume that the rows and columns of A are now arranged so, that $T = (r, n)$ is monotonically ordered. Then, that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{k,j}}{l_{j,j}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

Page
In

quidiagonal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 & 0 \\ 0 & * & * & 0 & 0 & 0 \\ 0 & 0 & * & * & 0 & 0 \\ 0 & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



quidiagonal:

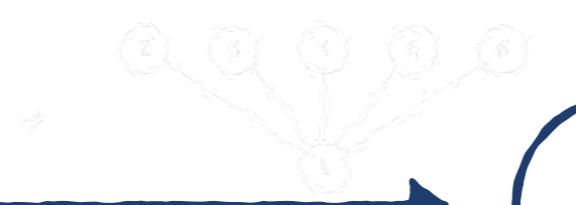
$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ 0 & 0 & * & * & * & * \\ 0 & 0 & 0 & * & * & * \end{bmatrix}$$



Page
Out

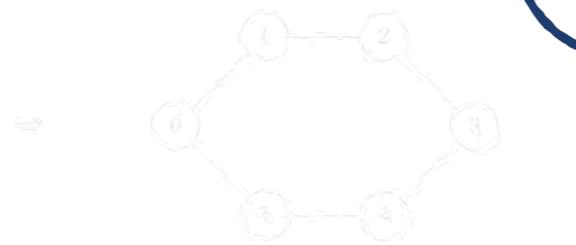
triangular:

$$\begin{bmatrix} * & * & * & * & * & * \\ * & * & 0 & 0 & 0 & 0 \\ * & 0 & * & 0 & 0 & 0 \\ 0 & * & * & 0 & 0 & 0 \\ 0 & 0 & * & * & 0 & 0 \\ 0 & 0 & 0 & * & * & 0 \end{bmatrix}$$



cyclic:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & * \\ * & * & 0 & 0 & 0 & 0 \\ 0 & * & * & 0 & 0 & 0 \\ 0 & 0 & * & * & 0 & 0 \\ 0 & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & 0 \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & * \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & * \end{bmatrix}$$

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 4.$$

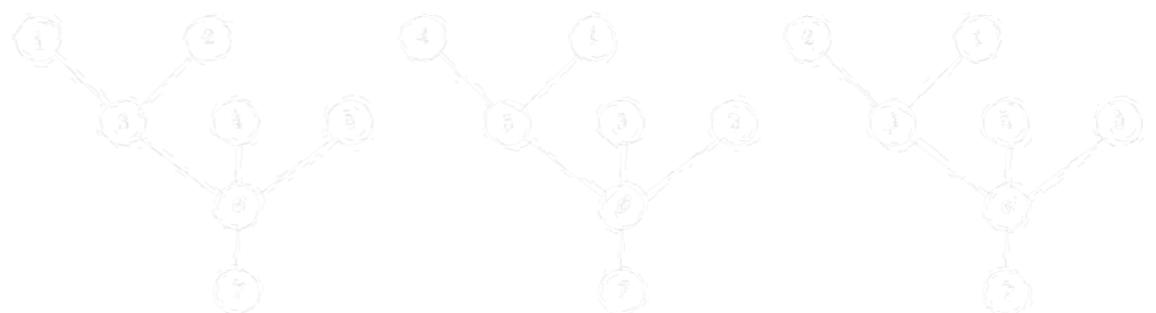
This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$ is called a path joining the vertices i_0 and j_0 if $i_0 \in \{i_0, j_0\}$, $j_0 \in \{i_0, j_0\}$ and for every $k = 1, 2, \dots, n-1$ the set $\{i_{k+1}, j_{k+1}\} \cap \{i_0, j_0\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of V are joined by a unique simple path. Both tri-diagonal and any other matrices A are not trees, but this is not the case with either quidiagonal or cyclic matrices.

Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in ordinary graph, T admits a natural partial order, which can best be explained by an analogy with a family tree. The vertex r is the predecessor of all the vertices in $T \setminus \{r\}$ and these vertices are successors. Moreover, every $\alpha \in V \setminus \{r\}$ has a unique predecessor α' and an immediate vertex along this path, except for r and α , as a predecessor of α and a successor.

We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree.

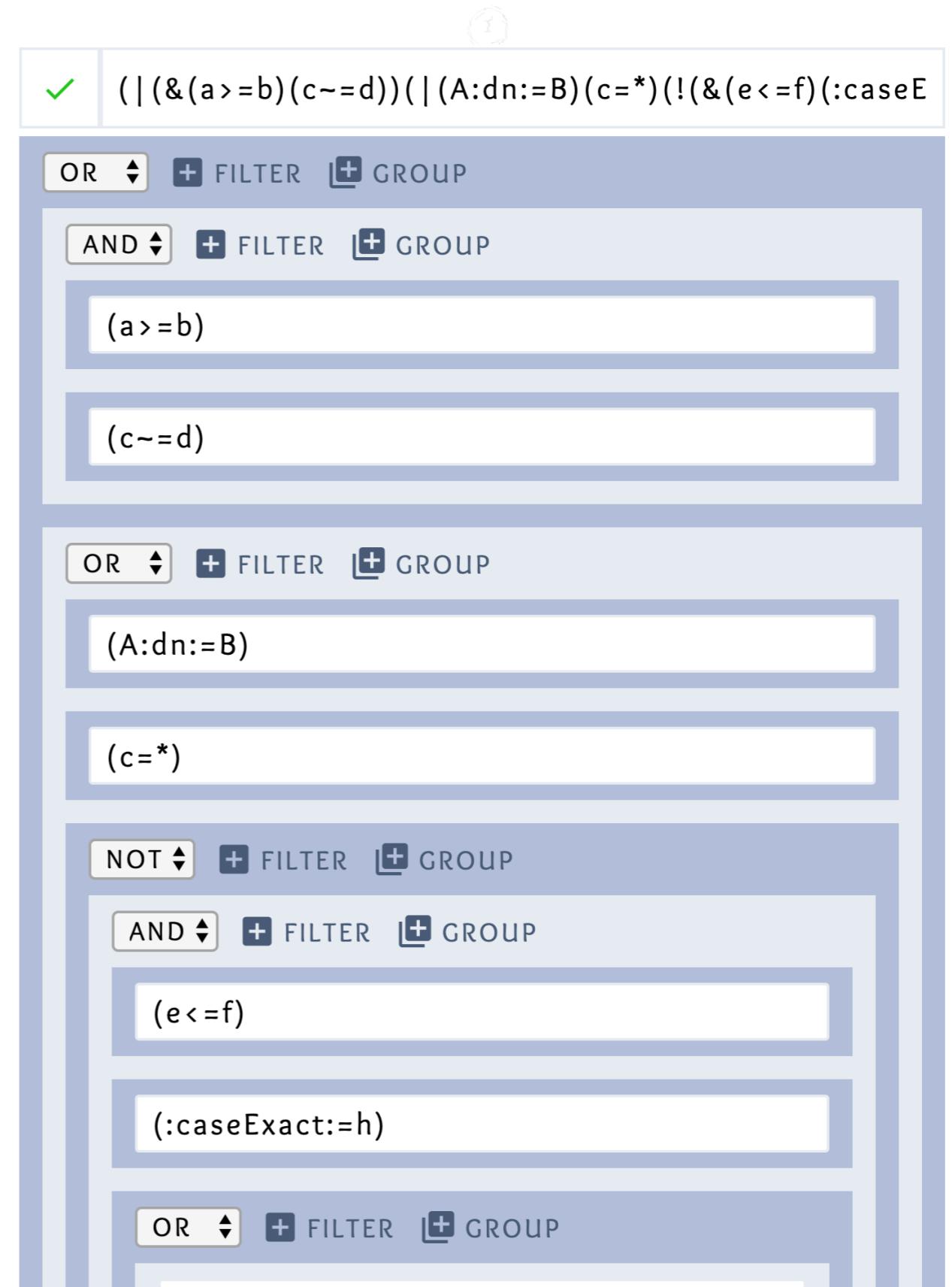


Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, n\}$ and assume that the rows and columns of A are then arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ii}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their ranks.

$(|(&(a>=b)(c\sim=d))$
 $(| (A:dn:=B)(c=^*)$
 $(!(&(e<=f)$
 $(:caseExact:=h)$
 $(| (i=j)(!(k<=l))))))))$



just displayed, but its graph.

$$t_{k,j} = \frac{a_{k,j}}{b_{j,j}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1, \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

(| (! (a>=b) (c~d)))
(| (A:dn := B) (c=*))
(!(&(e<=f))
(:caseExact:=h))
(| (i=j) (! (k<=l)))))))

! (| (! (a>=b) (c~d))) (| (A:dn := B) (c=*)) (!(&(e<=f)) (:caseE>))

OR FILTER GROUP

NOT FILTER GROUP

You can negate only 1 filter in a group.

(a>=b)

(c~d)

You need to use ~=.

OR FILTER GROUP

(A:dn := B)

You need to trim the attribute and filter type.

(c=*)

NOT FILTER GROUP

AND FILTER GROUP

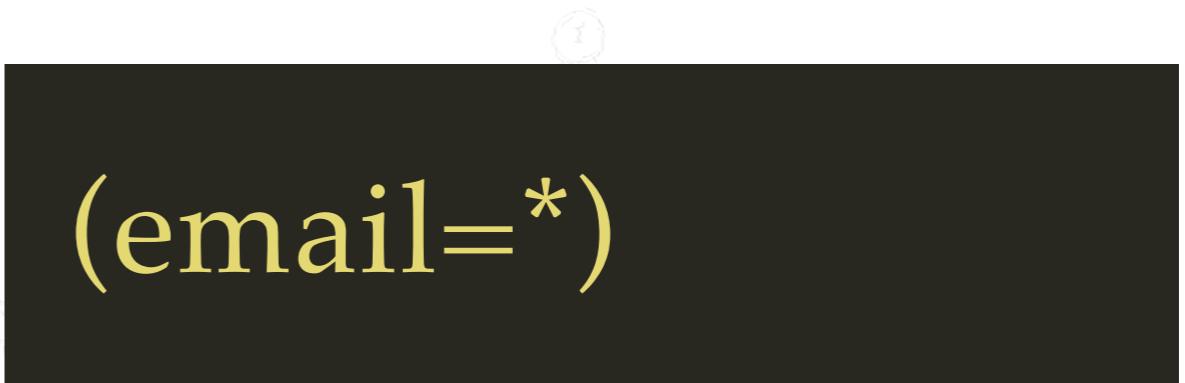
(e<=f)

(:caseExact:=h)

just displayed, but its graph.

$$t_{k,j} = \frac{a_{k,j}}{b_{j,j}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1, \quad (11.5)$$

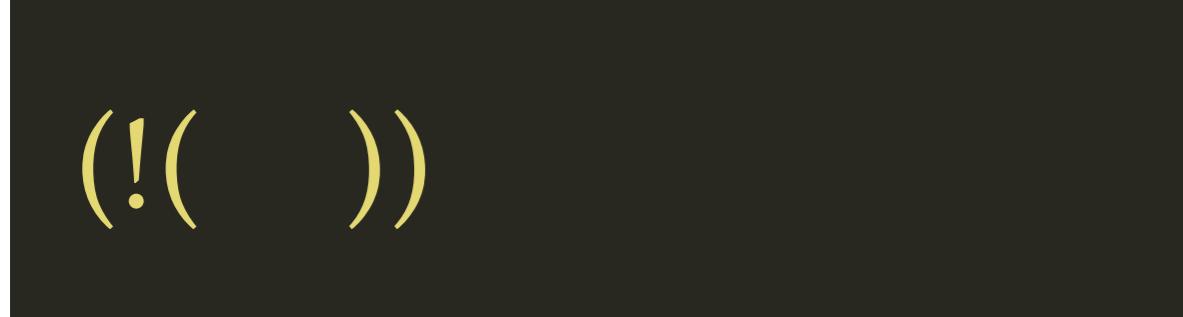
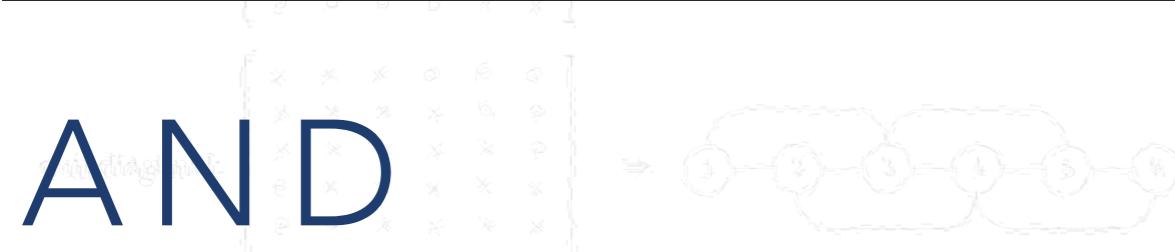
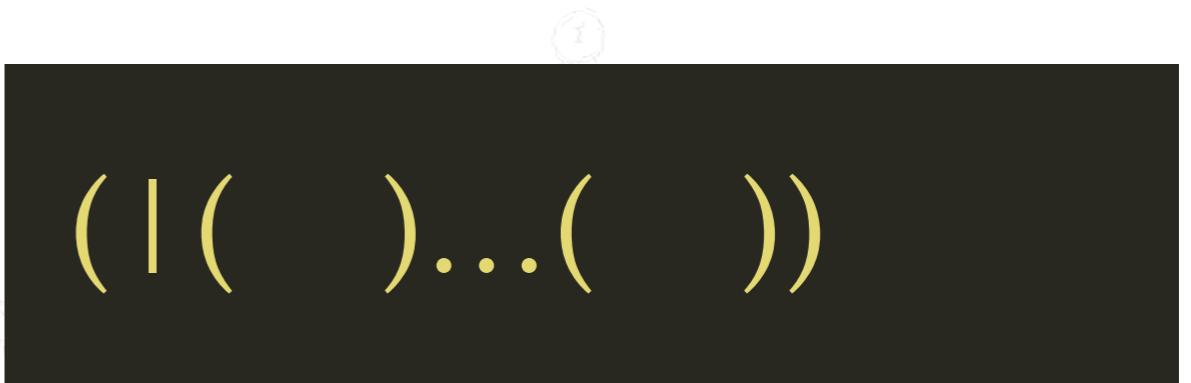
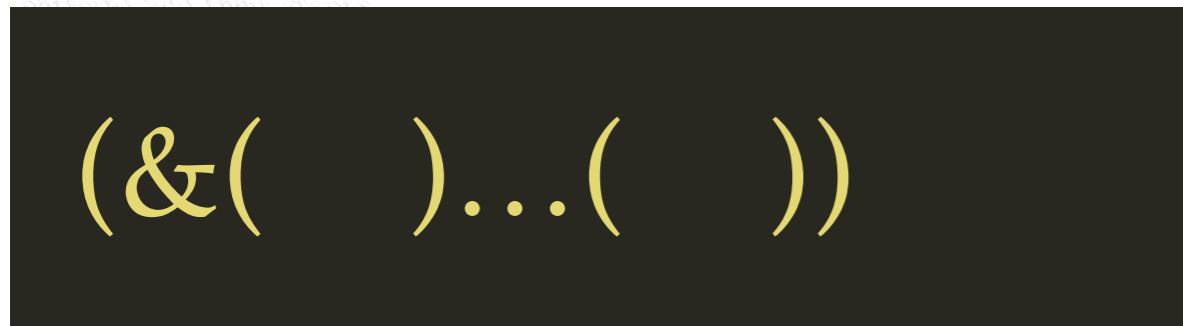
Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ki}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix



At a first glance, there is nothing to think in the case of the four matrices that we have just displayed, but its graph,

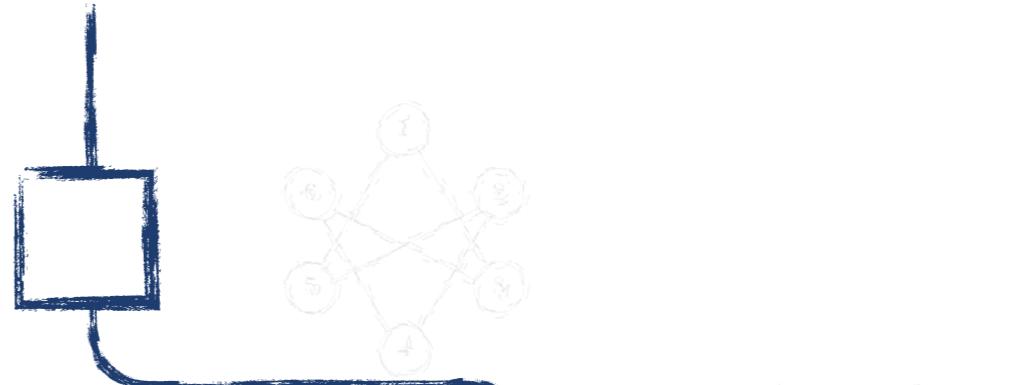


$$l_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ii}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

```
function addOne(x) {
  if (Number.isFinite(x)) {
    x = x + 1;
  } else {
    console.log('error');
  }
  return x;
}
```

just displayed, but its graph:



tells a different story – \mathbf{g} is not a tree. In a cyclic matrix in discussed. To see this, just re-label the vertices as follows:

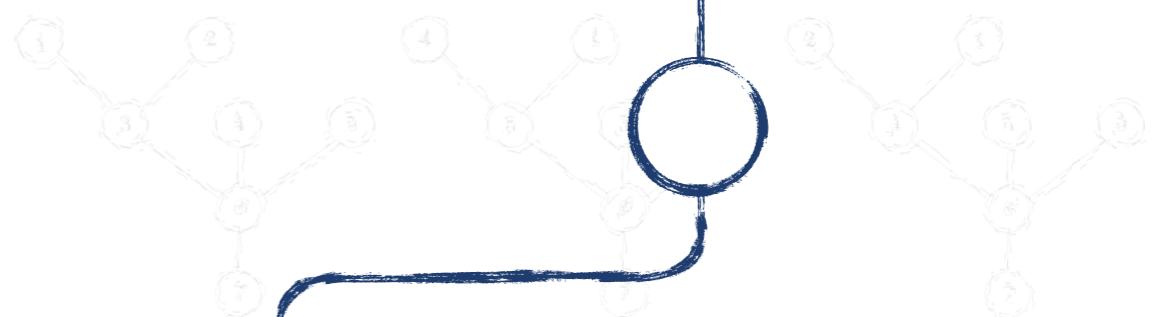
$$1 \rightarrow 1, \quad 2 \rightarrow 3, \quad 3 \rightarrow 2, \quad 4 \rightarrow 5, \quad 5 \rightarrow 6, \quad 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (swapping) the equations and variables.

An ordered set of edges $\{(i_1, j_1)\}_{i_1, j_1 \in \{1, \dots, n\}}$ is a path joining the vertices i_1 and j_1 if $i_1 \in \{1, \dots, n\}$, $j_1 \in \{1, \dots, n\}$ and for every $k = 1, 2, \dots, n-1$ the set $\{i_{k+1}, j_{k+1}\} \cap \{i_k, j_k\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that \mathbf{G} is a tree if each two members of \mathbf{V} are joined by a unique simple path. Both tridiagonal and banded matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $n > 3$.

Given a tree \mathbf{G} and an arbitrary vertex $r \in \mathbf{V}$, the path $T = (Q, \pi)$ is called a rooted tree, while r is said to be the root. Unlike in an ordinary tree, the root r is not necessarily unique, which can best be explained by an analogy with a family tree. The root r is the predecessor of all the vertices in $\mathbf{V} \setminus \{r\}$ and these vertices are successors of r . Moreover, every $a \in \mathbf{V} \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and a , as a predecessor of a and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors; in other words, a vertex a is to the right of r if it is further from the root to the root. (As we have already said, we are permuting the rows and the columns of the matrix, so the order of the vertices in the matrix is not unique.)

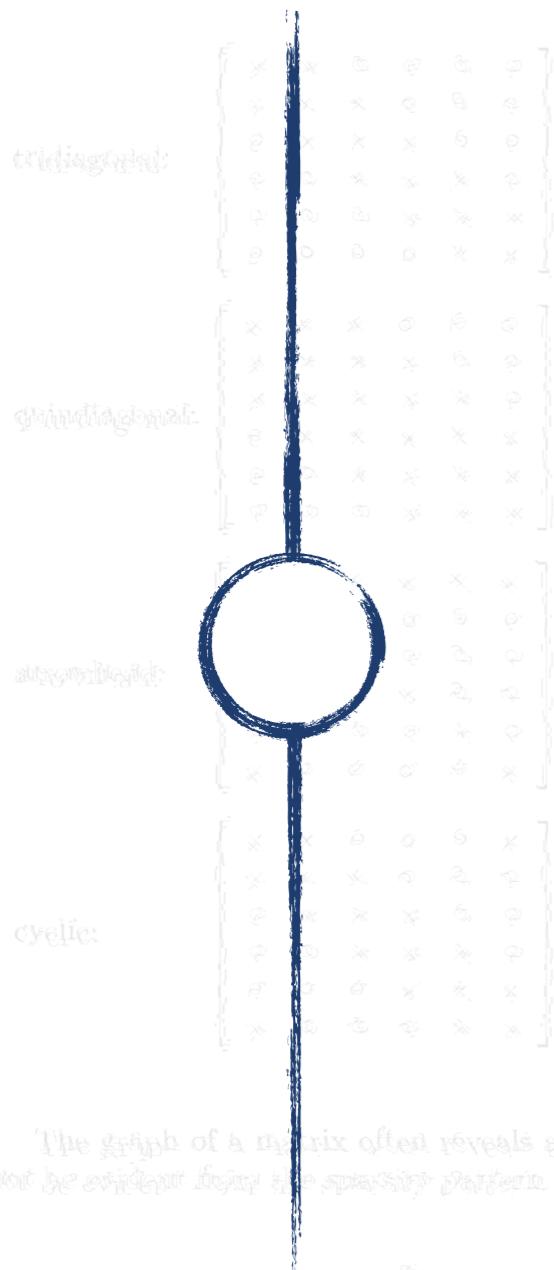
Every rooted tree will be monotonically ordered, but in general such an ordering is not unique. We now give three consecutive stages of the same rooted tree:



Theorem 2.1 Let \mathbf{A} be a symmetric matrix whose graph \mathbf{G} is a tree. Choose a root $r \in \{1, 2, \dots, n\}$. Assume that the rows and columns of \mathbf{A} have been arranged so that $T = (Q, \pi)$ is monotonically ordered. Then the Cholesky factorization of \mathbf{A} is

$$L_{k,j} = \frac{a_{k,j}}{a_{kk}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (1.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

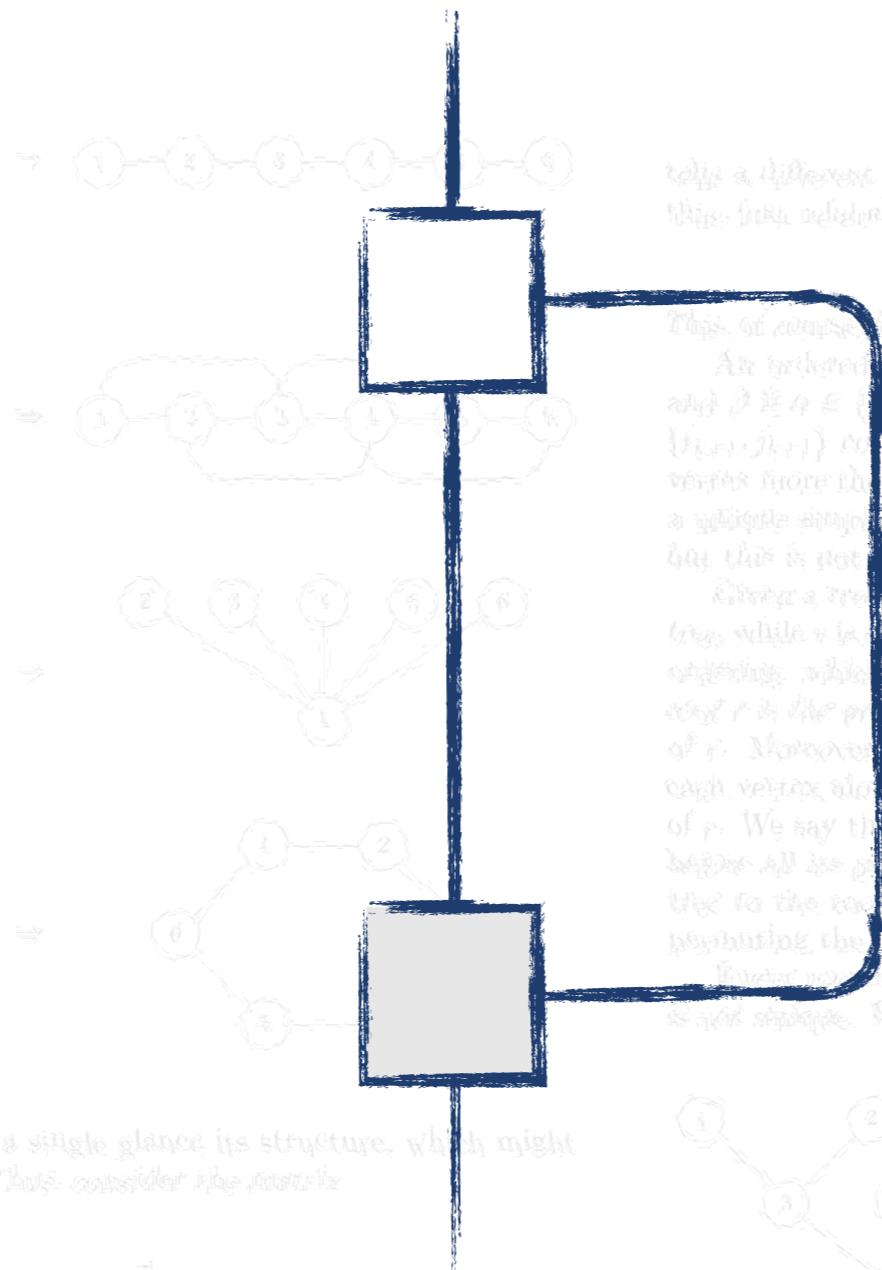


The graph of a matrix often reveals at a single glance its structure, which might not be evident from its sparsity pattern. Thus, consider the matrix

$$\text{LOG} \quad \begin{bmatrix} * & 0 & * & 0 & * & 0 \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & 0 & * & 0 \\ 0 & 0 & * & 0 & * & 0 \\ * & * & 0 & * & 0 & 0 \\ 0 & 0 & 0 & * & 0 & 0 \end{bmatrix}$$

IF

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, first recall that the vertices are

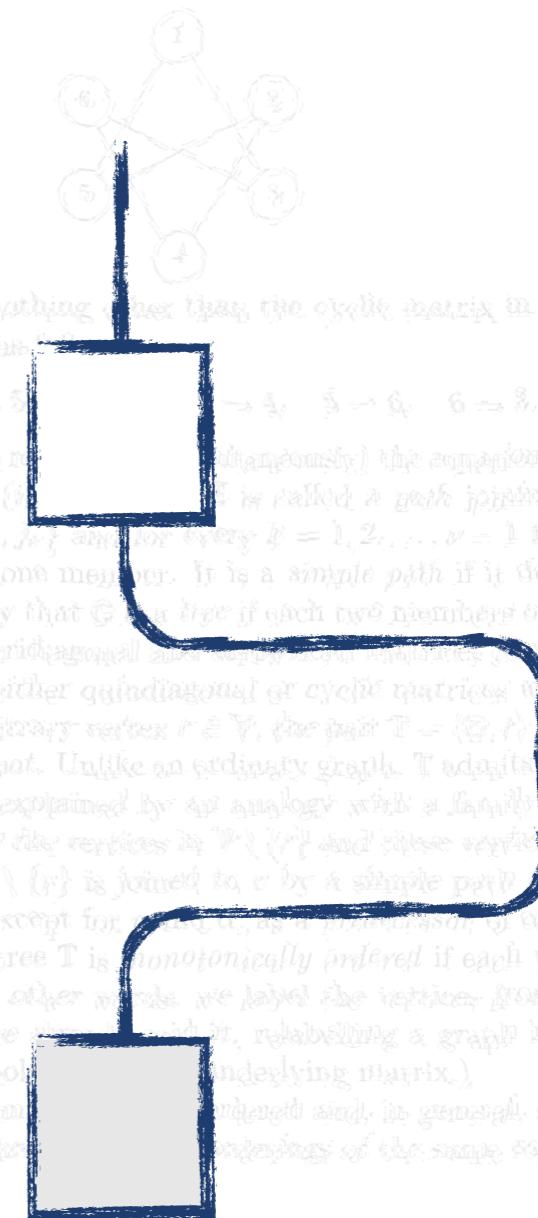
$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad \dots \rightarrow 5, \quad 6 \rightarrow 3.$$

This, of course, is equivalent to renumbering (simultaneously) the equations and variables.

An ordered set of edges $\{(v_i, w_i)\}_{i=1}^n$ in $G = (V, E)$ is called a path joining the vertices v_1 and v_n if $i \in \{1, \dots, n\}$, $v_i \in V$, $w_i \in V$, and for every $k = 1, 2, \dots, n-1$ the set $\{v_{k+1}, w_k\} \cap \{v_k, w_{k+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of V are joined by a unique simple path. Both tridiagonal and pentadiagonal matrices correspond to trees, but this is not the case with either quadiagonal or cyclic matrices when $n \geq 3$.

Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $T \setminus \{r\}$ and these vertices are successors of r . Moreover, every $v \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r itself, as a predecessor of v and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors in other words, we layed the vertices from the top of the tree to the root. (As we have mentioned it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Even a binary tree will be monotonically ordered and, in general, such an ordering is not unique. We will give the



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so that $T = (G, r)$ is monotonically ordered. Then, that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{a_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

triangular:



quadratic:



symmetric:



cyclic:



The graph of a matrix often reveals at a single glance its structure, which may not be evident from the sparsity pattern. Thus, consider the matrix



At a first glance, there is nothing to think in the case just displayed, but its graph,

1

PICTURE

1000

WORDS



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

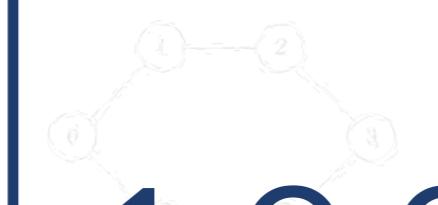
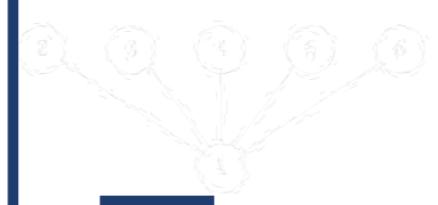
$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_1, j_1), (i_2, j_2), \dots, (i_v, j_v)\}$ in \mathbb{G} is called a *path* joining the vertices i_1, i_2, \dots, i_v (j_1, j_2, \dots, j_v) and, for every $k = 1, 2, \dots, v-1$ the set $\{(i_k, j_k), (i_{k+1}, j_{k+1})\}$ contains exactly one edge. It is a *simple path* if it does not visit any vertex more than once. We say that \mathbb{G} is a *tree* if each two members of \mathbb{V} are joined by a single simple path. Back to graphs, the quadiagonal matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $p \geq 3$.

Given a tree \mathbb{G} and an arbitrary vertex $r \in \mathbb{V}$, the pair $T = (G, r)$ is called a *rooted tree*, while r is said to be the *root*. Unlike in ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the *predecessor* of all the vertices in $\mathbb{V} \setminus \{r\}$ and these vertices are *successors* of r . Moreover, every $\alpha \in \mathbb{V} \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a *predecessor* of α and a *successor* of r . We say that the rooted tree T is *monotonically ordered* if each vertex is labelled before all its predecessors; in other words, we *layer* the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is unique. We now give three consecutive endings of the same rooted tree:



1 000 000



Theorem 1.1. Let A be a symmetric matrix whose graph \mathbb{G} is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (1.5)$$

```
// TODO: Write tests later
test('it renders', async function(assert) {
  await render(hbs`<ComplexComponent />`);
  assert.ok(true);
});
```

```
import { percySnapshot } from 'ember-percy';

...
// TODO: Write tests later

test('a complex workflow', async function(assert) {
  await visit('/complex-page');
  await percySnapshot(assert);
});
```

Therefore we will now give a few examples of matrices (represented by their sparsity)

triangular:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



quadrangular:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



supernodal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



cyclic:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



IF f IS CONTINUOUS IN $[a, b]$, AND IF $f(a)f(b) < 0$,

THEN f MUST HAVE A

ZERO IN (a, b) .



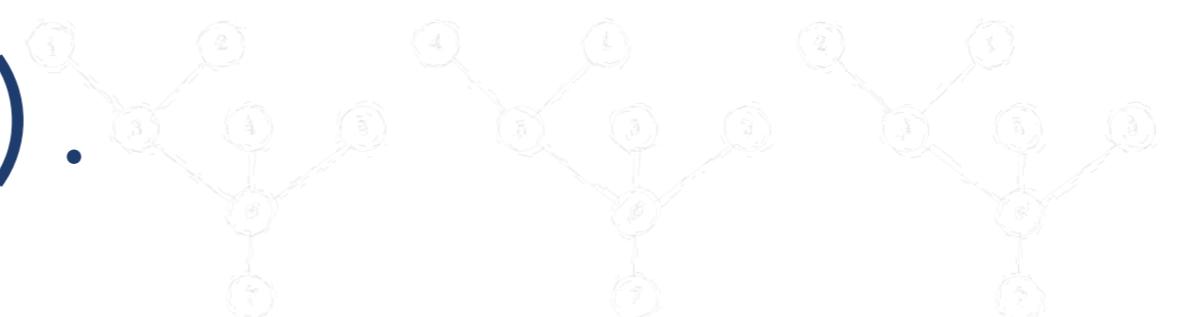
tells a different story - it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 2, 3 \rightarrow 3, 4 \rightarrow 4, 5 \rightarrow 5, 6 \rightarrow 6.$$

Of course, it is equivalent to renumbering the equations and variables. As you can see from the figure, G is called a tree, since the vertices v and α ($v \in V \setminus \{\alpha\}$, $\beta \in V \setminus \{\alpha\}$) and for every $k = 1, 2, \dots, n-1$ the set $\{v_1, v_2, \dots, v_k\} \cap \{v_{k+1}, v_{k+2}, \dots, v_{k+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of V are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees, but this is not the case with either of banded or cyclic matrices when $p \geq 3$.

Of course, it is equivalent to renumbering the equations and variables. As you can see from the figure, G is called a tree, since the vertices v and α ($v \in V \setminus \{\alpha\}$, $\beta \in V \setminus \{\alpha\}$) and for every $k = 1, 2, \dots, n-1$ the set $\{v_1, v_2, \dots, v_k\} \cap \{v_{k+1}, v_{k+2}, \dots, v_{k+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of V are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees, but this is not the case with either of banded or cyclic matrices when $p \geq 3$.

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three consecutive orderings of the same rooted tree:



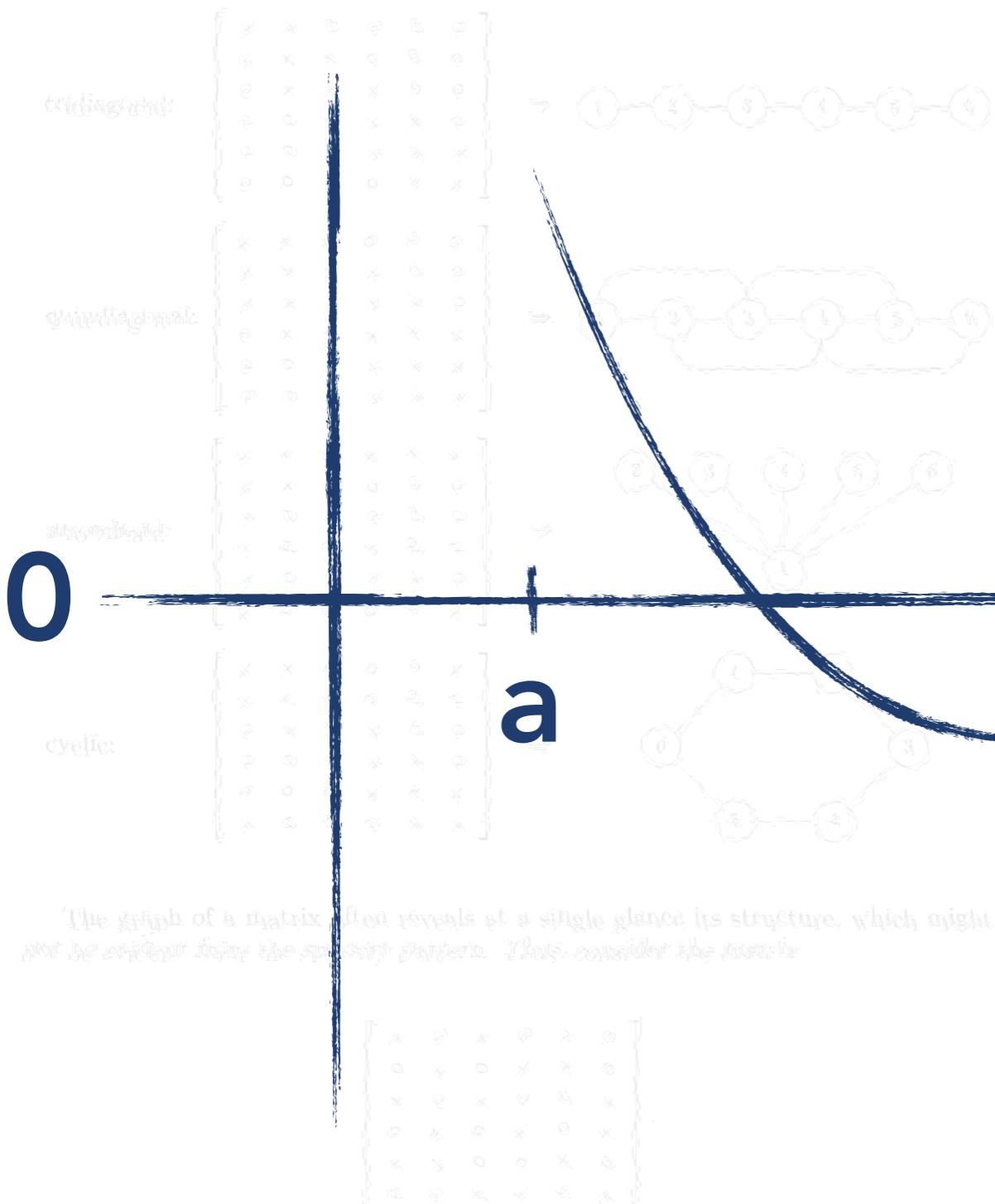
Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

At a first glance, this is not a very useful formula, but it is the basis for the following algorithm.

just displayed, but its graph:

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & * \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & 0 & * & 0 \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & * \end{bmatrix}.$$

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 3, \quad 5 \rightarrow 6, \quad 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$ is called a path joining the vertices i_0 and j_0 if $i_0 \in \{i_0, j_0\}$, $j_0 \in \{i_0, j_0\}$ and for every $k = 1, 2, \dots, n-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of V are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $n \geq 3$.

Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $T \setminus \{r\}$ and these vertices are successors of r . More generally, if α is a vertex in T , then α is a predecessor of all the vertices along the path, except for r and α , as a predecessor of α and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its descendants. In other words, we layed the tree down the top at the tree's root. (As we have seen, it is said it, relabelling α is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree:



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, n\}$ and assume that the rows and columns of A are then arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

YOU CAN FIND EQUALLY MANY NUMBERS BETWEEN 0 AND 1 AS YOU CAN

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

BETWEEN $-\infty$ AND ∞ .

At a first glance this is not a matrix, but it is a little known triangular matrix, just displayed, but its graph,



It is a different story – it is nothing other than the cyclic matrix in disguised. To see this, let us relabel the vertices as follows:

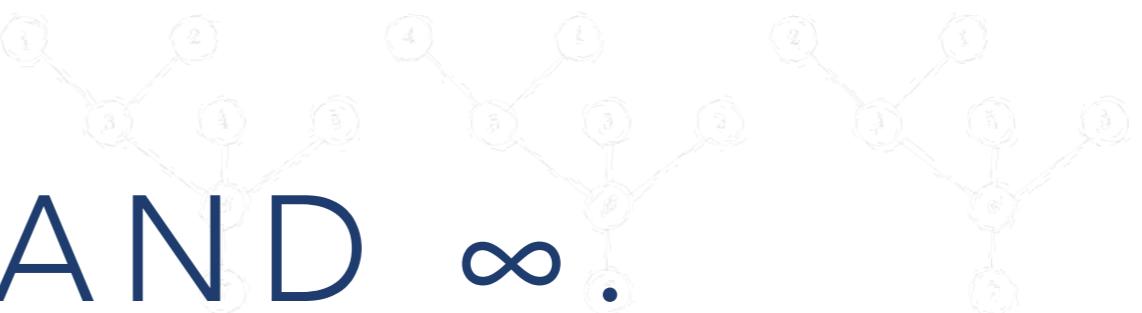
$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 8.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(u_i, v_i)\}_{i=1}^r$ in \mathbb{G} is called a path joining the vertices u_1 and u_r if $u_i \in \{u_1, u_2\}$, $v_i \in \{v_1, v_2\}$ and for every $i = 1, 2, \dots, r-1$ the set $\{u_i, v_i\} \cap \{u_{i+1}, v_{i+1}\} = \emptyset$. If a path contains exactly one segment, then it is a simple path if it does not visit any vertex more than once. We say that a path of length r is a r -cycle if it visits every vertex v_i exactly once and $v_r = v_1$. This is the case with the matrix A above, but this is not the case with either quidiagonal or cyclic matrices when $p \geq 3$.

Given a tree \mathbb{G} and an arbitrary vertex $r \in \mathbb{V}$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in a ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the ancestor of the vertices in $T \setminus \{r\}$ and these vertices are successors of r . Moreover, every $a \in \mathbb{V} \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and a , as a predecessor of a and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we label the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree:



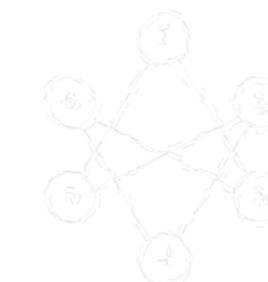
Theorem 11.1. Let A be a symmetric matrix whose graph \mathbb{G} is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

tridiagonal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix} \rightarrow \text{Graph: } 1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 6$$



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, first, re-label the weights as follows:

$$w_{12} = w_{23} = w_{34} = w_{45} = w_{56} = w_{61} = 1, \quad w_{13} = w_{24} = w_{35} = w_{46} = 2, \quad w_{14} = w_{25} = w_{36} = 3$$

$$\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & * & 0 \\ * & 0 & * & 0 & 0 & * \\ 0 & * & 0 & * & 0 & * \\ * & 0 & 0 & 0 & * & 0 \\ 0 & * & * & * & * & * \end{bmatrix}.$$



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so, that $\mathbb{T} = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i \sim j} a_{ki}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

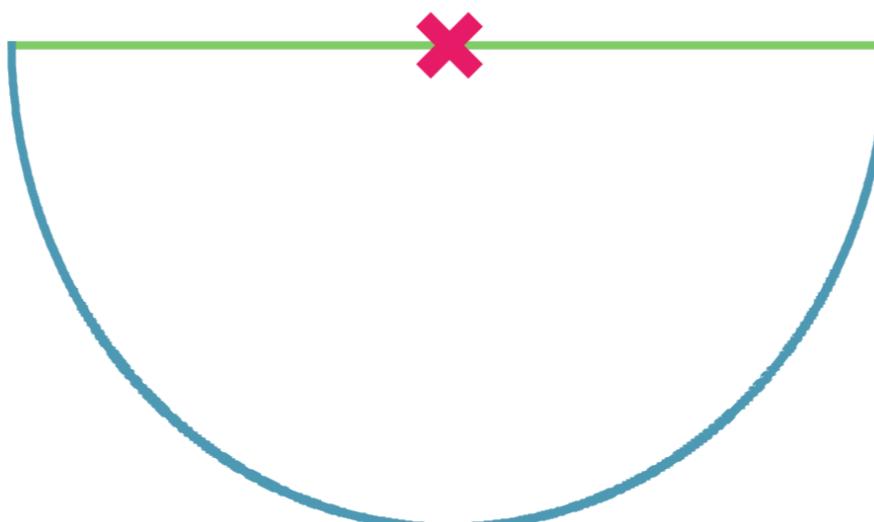
At a first glance, there is nothing to think in the case of the first matrices that we have just displayed, but its graph.

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

tridiagonal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix} \rightarrow \text{Graph: } 1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 6$$

tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, first, re-label the vertices as follows:



$$\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & * & 0 \\ * & 0 & * & 0 & 0 & * \\ 0 & * & 0 & * & 0 & * \\ * & 0 & * & 0 & 0 & * \\ 0 & * & * & * & * & * \end{bmatrix}$$

At a first glance, there is nothing to think in the case of the four matrices that we have just displayed, but its graph:



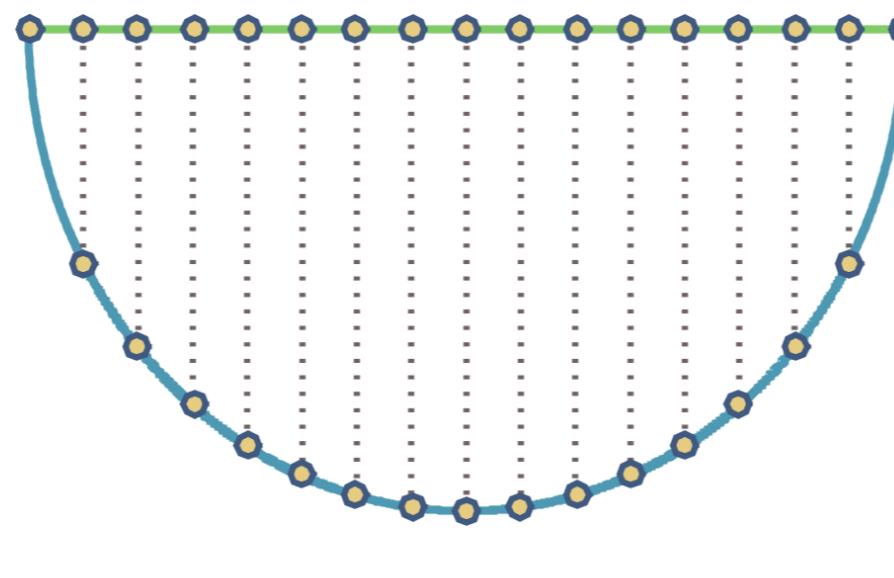
Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so, that $\mathbb{T} = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ii}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, first recall the weights as follows:



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, n\}$ and assume that the rows and columns of A are then arranged so, that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

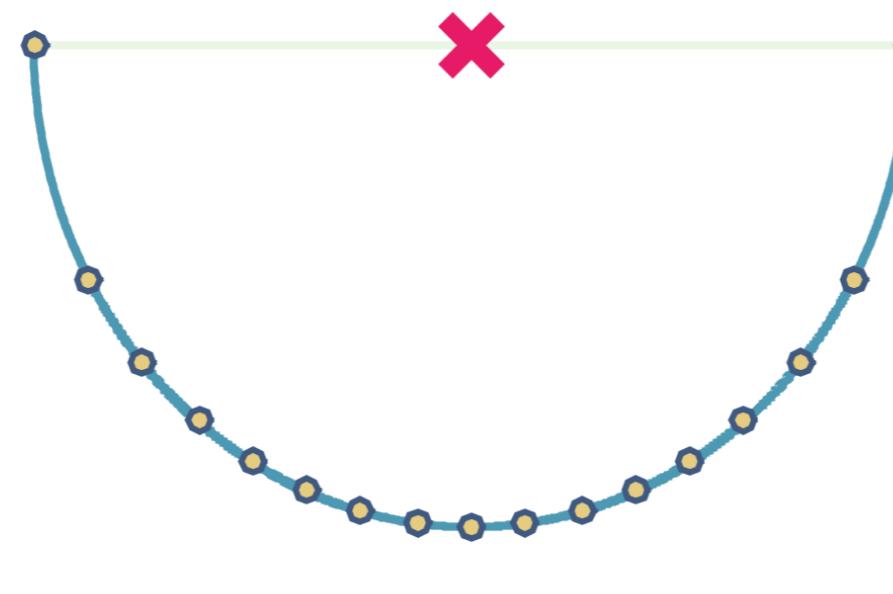
At a first glance, there is nothing to think in the case of the four matrices that we have just displayed, but its graph.

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

tridiagonal:

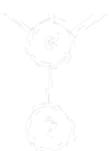
$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix} \rightarrow \text{Graph: } 1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 6$$

tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, first recall the weights as follows:



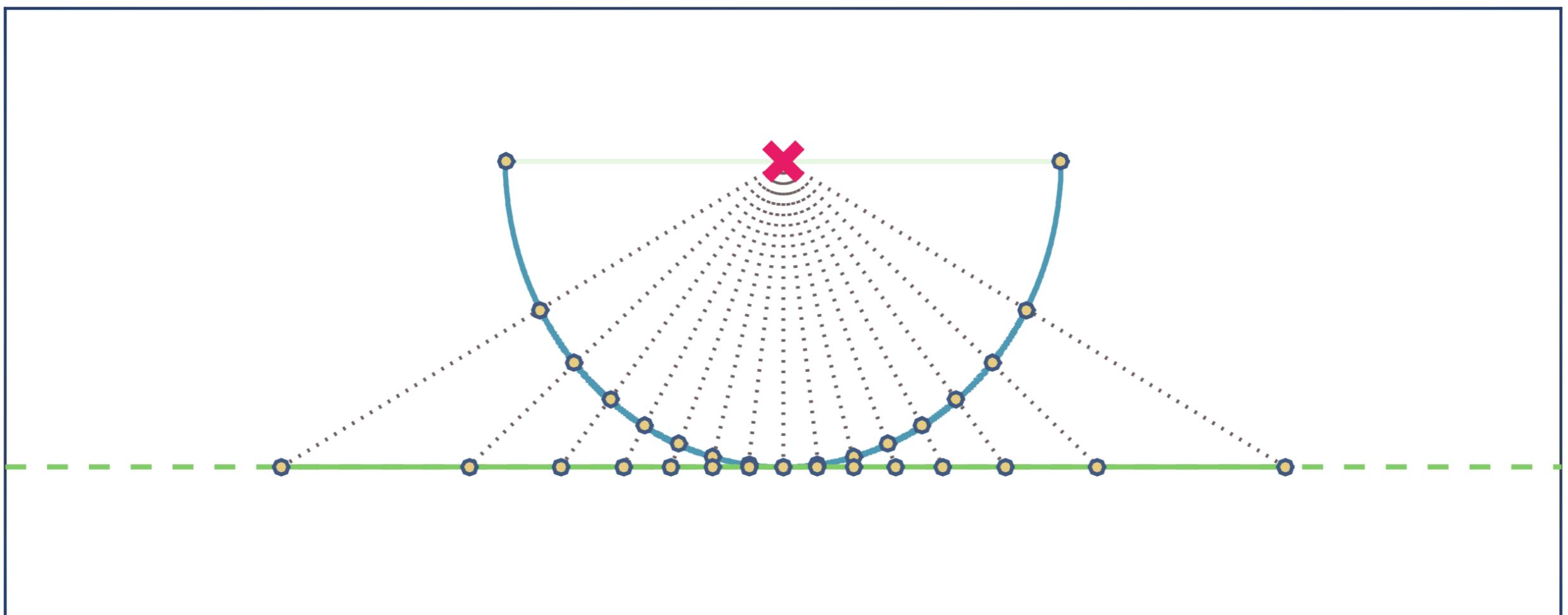
$$\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & * & 0 \\ * & 0 & * & 0 & 0 & * \\ 0 & * & 0 & * & 0 & * \\ * & * & 0 & 0 & * & 0 \\ 0 & * & * & * & * & * \end{bmatrix}.$$

At a first glance, there is nothing to think in the case of the four matrices that we have just displayed, but its graph:



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so, that $\mathbb{T} = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ii}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$



Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

tridiagonal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix} \rightarrow \text{Graph: } 1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 6$$



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just re-label the weights as follows:

1 2 3 4 5 6 7 8 9 10 11 12

$$\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & * & 0 \\ * & 0 & * & 0 & 0 & * \\ 0 & * & 0 & * & 0 & * \\ * & 0 & * & 0 & 0 & * \\ 0 & * & * & * & * & * \end{bmatrix}.$$

At a first glance, there is nothing to think in the case of the first matrices that we have just displayed, but its graph:



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so, that $\mathbb{T} = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ii}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

5 Rules of Writing Tests

Filter:

Module: Acceptance | Outline



EmberFest 18.10.2019

Running:

asymmetric:

$$\begin{bmatrix} 0 & 2 & 3 & 2 & 2 \\ 2 & 0 & 2 & 2 & 2 \\ 3 & 2 & 0 & 2 & 2 \\ 2 & 2 & 2 & 0 & 2 \\ 2 & 2 & 2 & 2 & 0 \end{bmatrix} \Rightarrow$$



cyclic:

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \Rightarrow$$



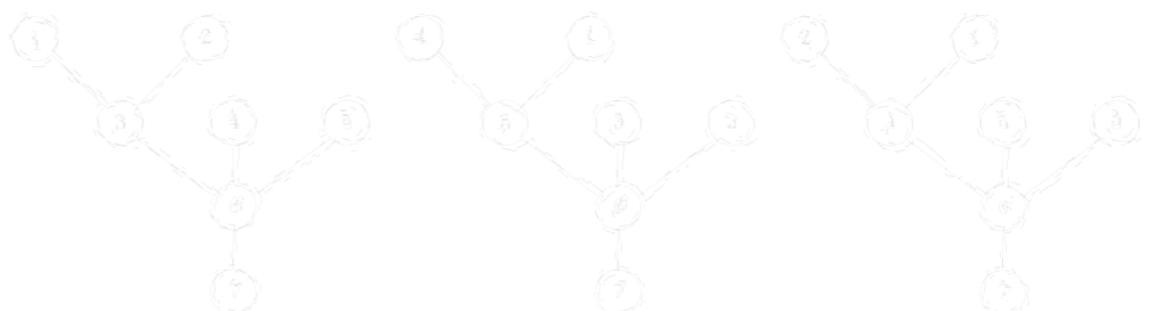
The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} 0 & 2 & * & 0 & * & 0 \\ 2 & 0 & 0 & * & x & 0 \\ * & 0 & 0 & 0 & 0 & x \\ 0 & * & 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & x & 0 & 0 & 0 \end{bmatrix}.$$

At a first glance, there is nothing to think in the case of the four matrices that we have just displayed, but its graph,

which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $T \setminus \{r\}$ and these vertices are successors of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a predecessor of α and a successor of r . We say that the rooted tree T is *monotonically ordered* if each vertex is labelled before all its predecessors; in other words, we label the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three consecutive orderings of the same rooted tree.



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ii}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

5 Rules of Writing Tests

Filter:

Module: Acceptance | Outline

EmberFest 18.10.2019

1 assertion of 1 passed, 0 failed.

If, if, if

cyclic:

$$\begin{bmatrix} * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \end{bmatrix} \Rightarrow$$



before all its predecessors in other words: we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three monotone orderings of the same rooted tree:



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so, that $T = (G, r)$ is monotonically ordered. Then, that $A = L U$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ki}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

At a first glance, there is nothing to think in the case of the four matrices that we have just displayed, but its graph.

5 Rules of Writing Tests

Filter:

Module: Acceptance | Outline

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2 assertions of 2 passed, 0 failed.

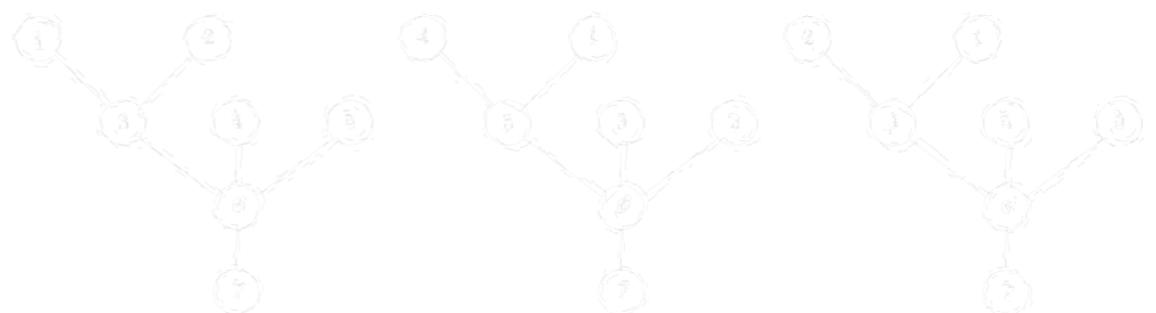
! If, if, if

! Use common, everyday words

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} * & * & * & 0 & * & 0 \\ 0 & * & 0 & * & * & 0 \\ * & 0 & * & * & 0 & * \\ 0 & * & 0 & * & * & 0 \\ * & * & 0 & 0 & * & * \\ 0 & 0 & * & * & * & * \end{bmatrix}.$$

At a first glance, there is nothing to think in to any of the four matrices that we have just displayed, but its graph.



Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so, that $\mathbf{T} = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ii}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

5 Rules of Writing Tests

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Acceptance | Outline



EmberFest 18.10.2019

3 assertions of 3 passed, 0 failed.

! If, if, if

! Use common, everyday words

! Write less with theorems and new terms

$$\begin{bmatrix} * & * & * & 0 & * & 0 \\ 0 & * & 0 & * & * & 0 \\ * & 0 & * & * & 0 & * \\ 0 & * & 0 & * & * & 0 \\ * & * & 0 & * & * & 0 \\ 0 & * & * & 0 & * & * \end{bmatrix}.$$

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph:



Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so, that $\mathbf{T} = (G, r)$ is monotonically ordered. Then, that $A = \mathbf{L}\mathbf{U}^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ii}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

5 Rules of Writing Tests

Filter:

Module:

Acceptance | Outline



EmberFest 18.10.2019

4 assertions of 4 passed, 0 failed.

! If, if, if

! Use common, everyday words

! Write less with theorems and new terms

! All your basis are belong to us

$$\begin{bmatrix} 0 & 3 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph.

Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so, that $\tilde{T} = (G, r)$ is monotonically ordered. Then, that $A = \tilde{L}\tilde{U}$ is a Cholesky factorization, it is true that

$$t_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ki}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

5 Rules of Writing Tests

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EmberFest 18.10.2019

5 assertions of 5 passed, 0 failed.

! If, if, if

! Use common, everyday words

! Write less with theorems and new terms

! All your basis are belong to us

! 1 picture = 1000 words

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

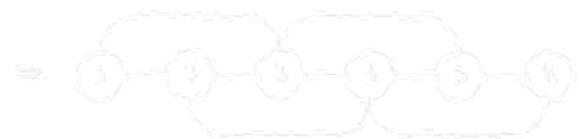
tridiagonal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



qundiagonal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ * & * & * & * & * & * \\ * & * & * & * & * & * \end{bmatrix}$$



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The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & 0 \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & * \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & * \end{bmatrix}.$$

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

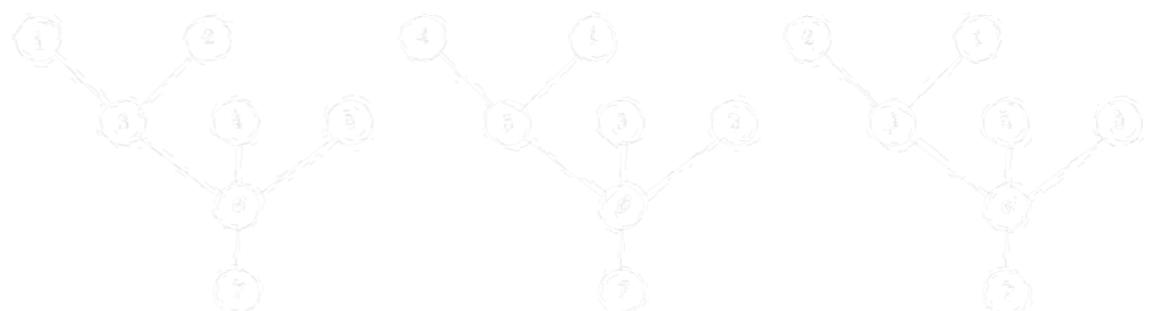
$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$ is called a path joining the vertices i_0 and j_0 if $i_0 \in \{i_0, j_0\}$, $j_0 \in \{i_0, j_0\}$ and for every $k = 1, 2, \dots, n-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of V are joined by a unique simple path. Both tridiagonal and qundiagonal matrices correspond to trees, but this is not the case with either qundiagonal or cyclic matrices when $n \geq 3$.

Given a tree T and an arbitrary vertex $r \in V$, the pair $T - (r, \cdot)$ is called a rooted tree, where the vertex r is the root. Unlike in ordinary graph, T admits a natural partial order which should be explained by an analogy with a family tree. Thus, the vertex r is the predecessor of all the vertices in $T \setminus \{r\}$ and these vertices are successors of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a predecessor of α and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three consecutive orderings of the same rooted tree.



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Consider a root $r \in \{1, 2, \dots, n\}$ and assume that the rows and columns of A are now arranged so, that $T = (G, r)$ is monotonically ordered. Given that $A = PLU$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{p_{jj}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

Q.E.D.