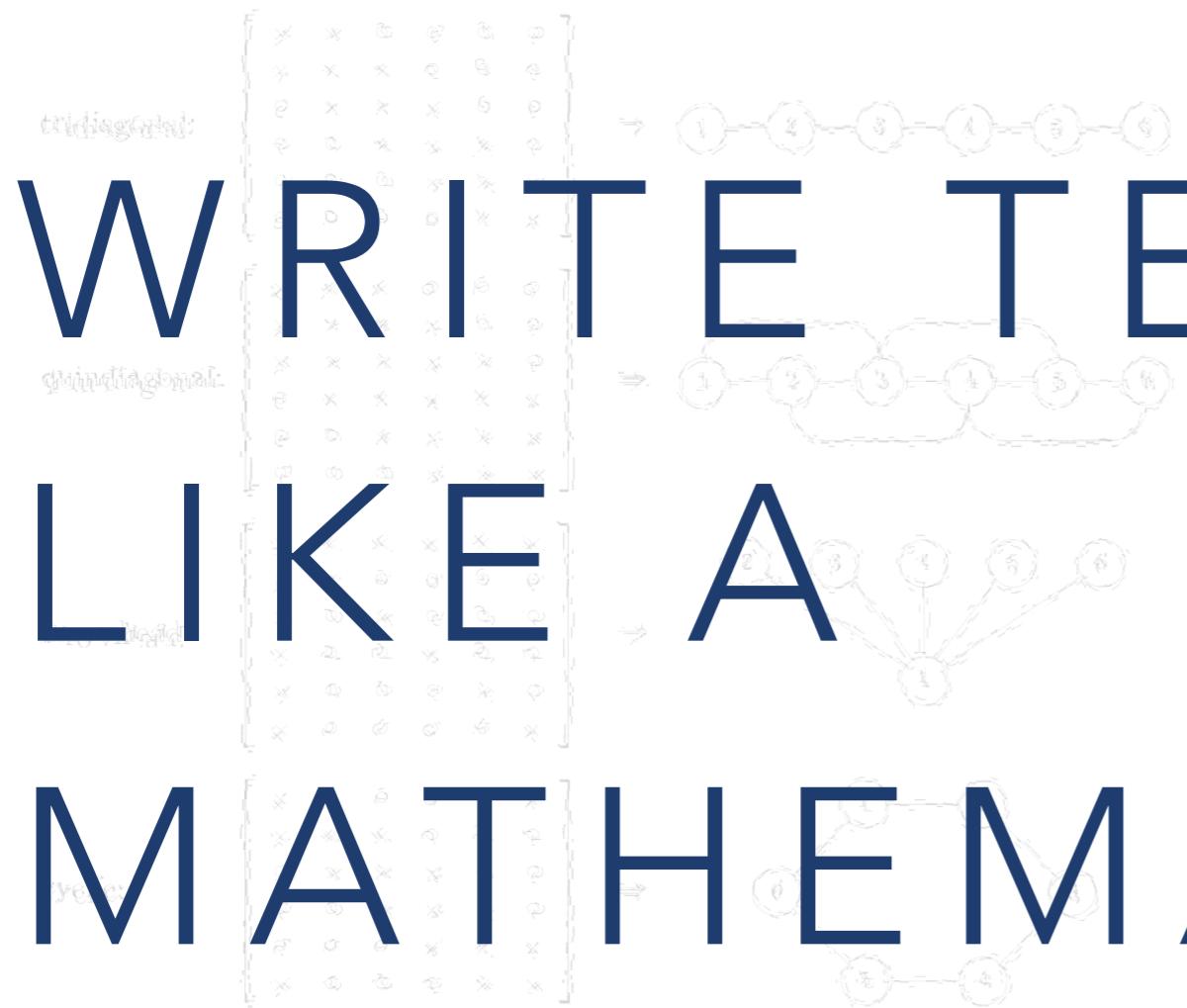


Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

ISAAC J. LEE

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, first recall the relation as follows

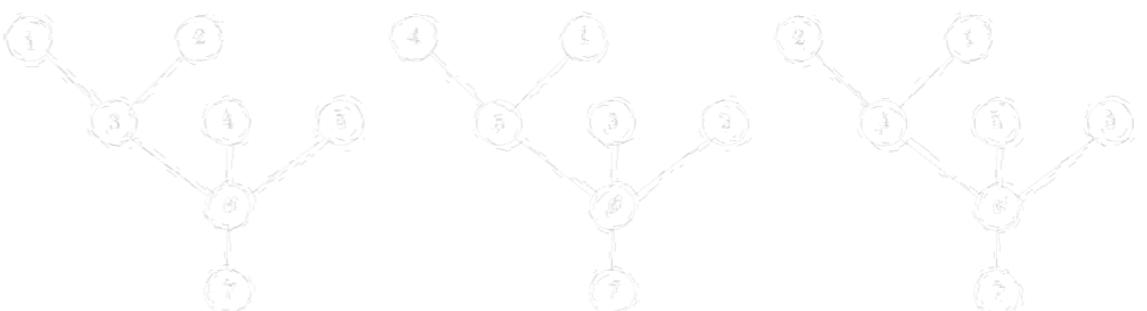
$$1 \rightarrow 2, 2 \rightarrow 3, 3 \rightarrow 4, 4 \rightarrow 5, 5 \rightarrow 6, 6 \rightarrow 1.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

Let α be an ordered set of vertices $\{\alpha_i\}_{i=1}^n$, $\beta \in \mathbb{S}$ is called a path joining the vertices α and β if $\alpha \in \{\alpha_i, \alpha_{i+1}\}_{i=1}^n$, $\beta \in \{\alpha_i, \alpha_{i+1}\}_{i=1}^n$ and for every $k = 1, 2, \dots, n-1$ the set $\{\alpha_i, \alpha_{i+1}\} \cap \{\alpha_{k+1}, \alpha_{k+2}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that \mathbb{G} is a tree if each two members of \mathbb{V} are joined by a unique simple path. Both tridiagonal and pentadiagonal matrices correspond to trees, but this is not the case with either quadiagonal or cyclic matrices when $n \geq 3$.

Given a tree \mathbb{G} and an arbitrary vertex $r \in \mathbb{V}$, the pair $\mathbb{T} = (\mathbb{G}, r)$ is called a rooted tree, while r is said to be the root. Unlike an ordinary graph, \mathbb{T} admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $\mathbb{V} \setminus \{r\}$ and these vertices are successors of r . Moreover, every $\alpha \in \mathbb{V} \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a predecessor of α and a successor of r . We say that the rooted tree \mathbb{T} is monotonically ordered if each vertex is labelled before all its predecessors (or, more precisely, before the root of the subtree to which it belongs). (If we take already said it, relabeling a graph is tantamount to relabeling the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three monotone orderings of the sample rooted tree:



Theorem III.1. Let \mathbb{A} be a symmetric matrix whose graph \mathbb{G} is a tree. Choose a root $r \in \{1, 2, \dots, n\}$ and assume that the rows and columns of \mathbb{A} have been arranged so, that $\mathbb{T} = (\mathbb{G}, r)$ is monotonically ordered. Given that $\mathbb{A} = \mathbb{L}\mathbb{U}^T$ is a Cholesky factorization, it is true that

$$a_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ii}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

5 Rules of Writing Tests

Filter:

Module: Acceptance | Outline



EmberFest 18.10.2019

Running:

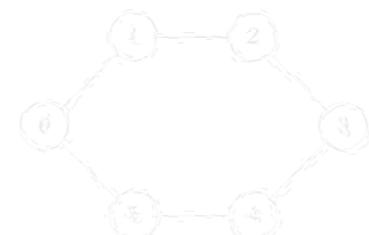
asymmetric:

$$\begin{bmatrix} 0 & 2 & 3 & 2 & 2 \\ 2 & 0 & 2 & 2 & 2 \\ 3 & 2 & 0 & 2 & 2 \\ 2 & 2 & 2 & 0 & 2 \\ 2 & 2 & 2 & 2 & 0 \end{bmatrix} \Rightarrow$$



cyclic:

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \Rightarrow$$



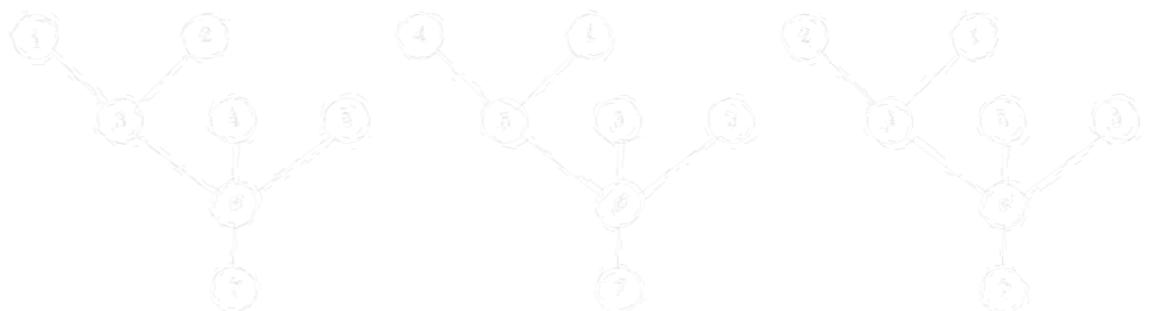
The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} 0 & 2 & * & 0 & * & 0 \\ 2 & 0 & 0 & * & x & 0 \\ * & 0 & 2 & 0 & 0 & x \\ 0 & * & 0 & 0 & x & 0 \\ * & x & 0 & 0 & 0 & 0 \\ 0 & 0 & x & 0 & 0 & 0 \end{bmatrix}.$$

At a first glance, there is nothing to think in the case of the four matrices that we have just displayed, but its graph,

which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $T \setminus \{r\}$ and these vertices are successors of r . Moreover, every $v \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and v , as a predecessor of v and a successor of r . We say that the rooted tree T is *monotonically ordered* if each vertex is labelled before all its predecessors; in other words, we label the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three consecutive orderings of the same rooted tree:



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so that $T = (G, r)$ is monotonically ordered. Then, that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ki}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

5 Rules of Writing Tests

Filter:

Module: Acceptance | Outline

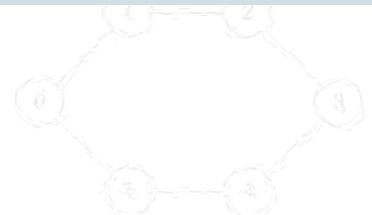
EmberFest 18.10.2019

1 assertion of 1 passed, 0 failed.

If, if, if

cyclic:

$$\begin{bmatrix} * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \end{bmatrix} \Rightarrow$$



before all its predecessors in other words: we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three monotone orderings of the same rooted tree:



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so, that $T = (G, r)$ is monotonically ordered. Then, that $A = L U$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ki}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

At a first glance, there is nothing to think in the case of the four matrices that we have just displayed, but its graph.

5 Rules of Writing Tests

Filter:

Module: Acceptance | Outline

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2 assertions of 2 passed, 0 failed.

If, if, if

Use common, everyday words

5 Rules of Writing Tests

Filter:

Module:

Acceptance | Outline



EmberFest 18.10.2019

3 assertions of 3 passed, 0 failed.

! If, if, if

! Use common, everyday words

! Write less with theorems and new terms

$$\begin{bmatrix} * & * & * & 0 & * & 0 \\ 0 & * & 0 & * & * & 0 \\ * & 0 & * & * & 0 & * \\ 0 & * & 0 & * & * & 0 \\ * & * & 0 & * & * & 0 \\ 0 & * & * & 0 & * & * \end{bmatrix}.$$

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph:



Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so, that $\mathbf{T} = (G, r)$ is monotonically ordered. Then, that $A = \mathbf{L}\mathbf{U}^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ii}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

5 Rules of Writing Tests

Filter:

Module:

Acceptance | Outline



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4 assertions of 4 passed, 0 failed.

! If, if, if

! Use common, everyday words

! Write less with theorems and new terms

! All your basis are belong to us

$$\begin{bmatrix} 0 & 3 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph.

Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so, that $\tilde{T} = (G, r)$ is monotonically ordered. Then, that $A = \tilde{L}\tilde{U}$ is a Cholesky factorization, it is true that

$$t_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ki}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

5 Rules of Writing Tests

Filter:

Module: [Acceptance](#) | [Outline](#) ▾

EmberFest 18.10.2019

5 assertions of 5 passed, 0 failed.

! If, if, if

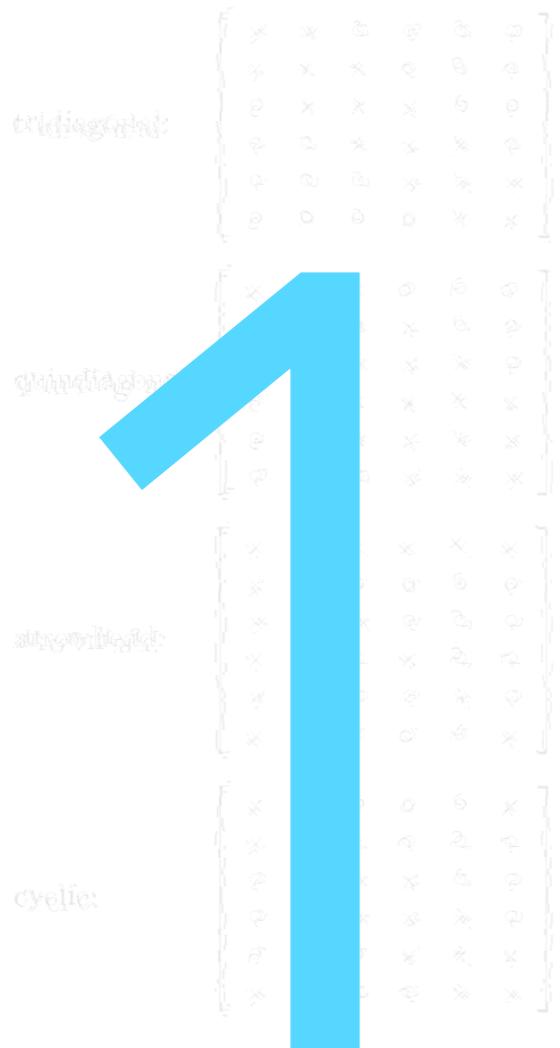
! Use common, everyday words

! Write less with theorems and new terms

! All your basis are belong to us

! 1 picture = 1000 words

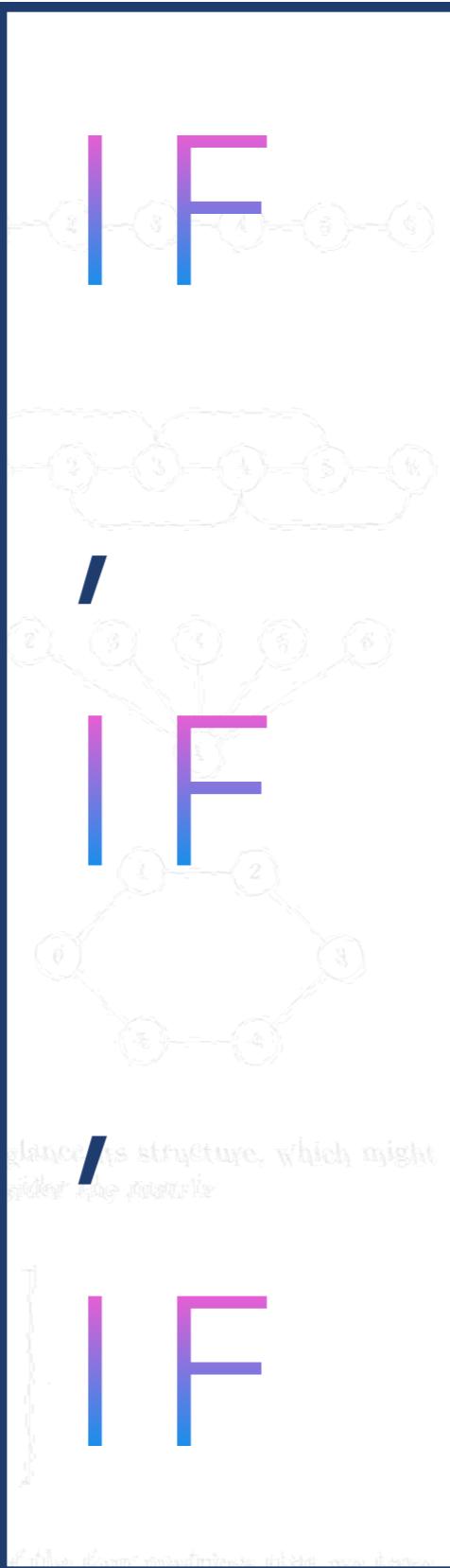
Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} * & * & * & 0 & * & * \\ 0 & * & 0 & * & * & * \\ * & 0 & * & 0 & * & * \\ 0 & * & 0 & * & 0 & * \\ * & * & 0 & 0 & * & * \\ 0 & * & * & * & 0 & * \end{bmatrix}$$

At a first glance, there is nothing to think in the matrix just displayed, but its graph,



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

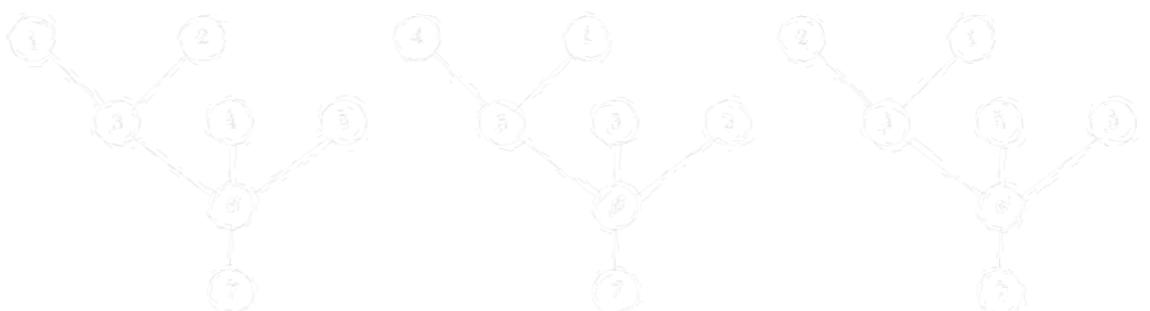
$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 3, \quad 5 \rightarrow 6, \quad 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$ is called a path joining the vertices i_0 and j_0 if $i_0 \in \{i_0, j_0\}$, $j_0 \in \{i_0, j_0\}$ and for every $k = 1, 2, \dots, n-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of V are joined by a unique simple path. Both tridiagonal and quidiagonal matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $n \geq 3$.

Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $T \setminus \{r\}$ and these vertices are successors of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a predecessor of α and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we label the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three consecutive orderings of the same rooted tree.



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, n\}$ and assume that the rows and columns of A are then arranged so, that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\text{IF} \quad \begin{bmatrix} * & * & 0 & 0 & 0 \\ 0 & * & * & 0 & 0 \\ * & 0 & * & 0 & 0 \\ 0 & 0 & 0 & * & * \\ 0 & 0 & 0 & * & * \end{bmatrix}.$$

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,



tells a different story – it is nothing other than a cyclic matrix in disguise. To see this, just read the weights as follows:

$$1 \rightarrow 1, 2 \rightarrow 3, 3 \rightarrow 2, 4 \rightarrow 1, 5 \rightarrow 6, 6 \rightarrow 8.$$

This, of course, is equivalent to reading the weights of the edges of the graph. An *undirected graph* $G = (V, E)$ is a set of vertices $V = \{v_1, v_2, \dots, v_n\}$ and edges $E \subseteq \{v_i, v_j\} \mid 1 \leq i, j \leq n\}$. G is called *connected* if the set $\{v_i, v_j\} \cap E$ contains exactly one member if and only if $v_i \neq v_j$. G is called *acyclic* if it does not visit any vertex more than once. We say that G is a *tree* if it has two members of V are joined by a unique simple path. Both tridiagonal and symmetric matrices correspond to trees, the case with either quadrangular or cyclic matrices when $n \geq 3$.

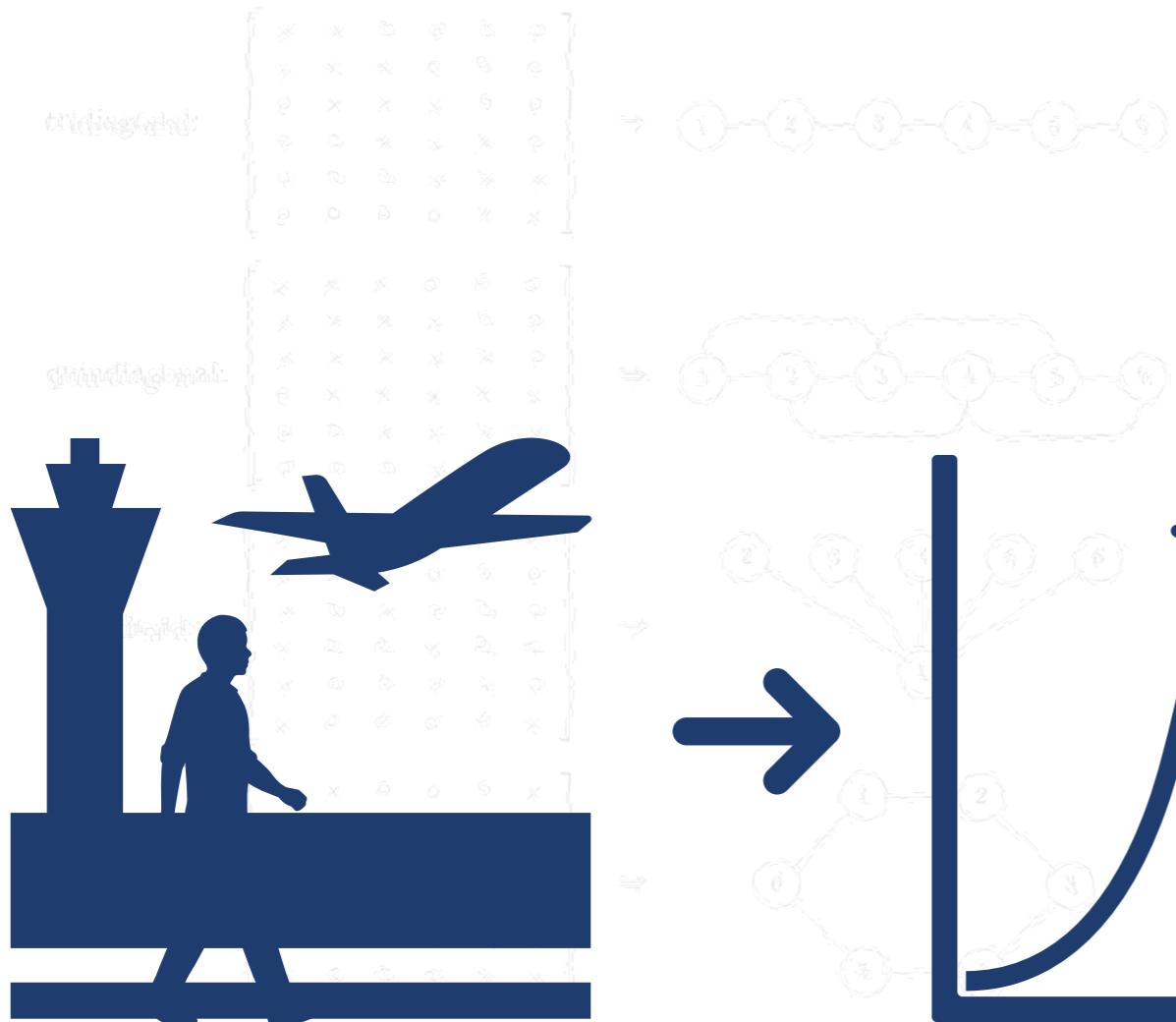
Given a tree, while it is not necessarily a *rooted tree*, it can best be explained by an analogy with a family tree. Thus, the *ancestor* of all the vertices in $V \setminus \{r\}$ is the vertex r , every $a \in V \setminus \{r\}$ is joined to r by a simple path and we designate this path, except for r and a , as a *predecessor* of a and a *successor* of r . Note that the rooted tree is *monotonically ordered*, that is, each vertex is labelled *predecessor* in other words, we label the tree from the top to the bottom. (As we have already said in relabelling a graph is tantamount to rows and columns of the underlying matrix.)

Every node in a tree is *not a leaf* and, in general, a *non-rooting* node is not a *parent*.



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, n\}$ and assume that the rows and columns of A are then arranged so that $\mathbf{T} = (G, r)$ is monotonically ordered. Given that $A = \mathbf{L}\mathbf{U}$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$



Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their analysis.

1

```
assert.speaker().getsPersonal();  
  
await sing('Happy Birthday');  
  
assert.audience().isHappy();
```

just displayed, but its graph:

$$t_{k,j} = \frac{q_{kj}}{q_j}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

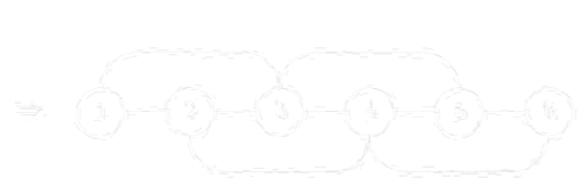
Therefore we will now give a few examples of matrices (represented by their sparsity)

and their graphs.

SUSPECT 1

quindiagonal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$



symmetric:

$$\begin{bmatrix} * & * & * & * & * & * \\ * & * & 0 & 0 & 0 & 0 \\ * & 0 & * & 0 & 0 & 0 \\ * & 0 & 0 & * & 0 & 0 \\ * & 0 & 0 & 0 & * & 0 \\ * & 0 & 0 & 0 & 0 & * \end{bmatrix}$$



cyclic:

$$\begin{bmatrix} * & 0 & 0 & 0 & * & * \\ * & * & 0 & 2 & * & * \\ * & 0 & * & * & * & * \\ * & * & * & * & * & * \\ * & 0 & * & * & * & * \\ 0 & * & * & * & * & * \end{bmatrix}$$

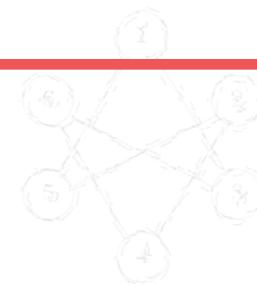


OBSERVER /

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

COMPUTED

At a first glance, the matrix A and its structure of the form of a symmetric matrix just displayed, but its graph,



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

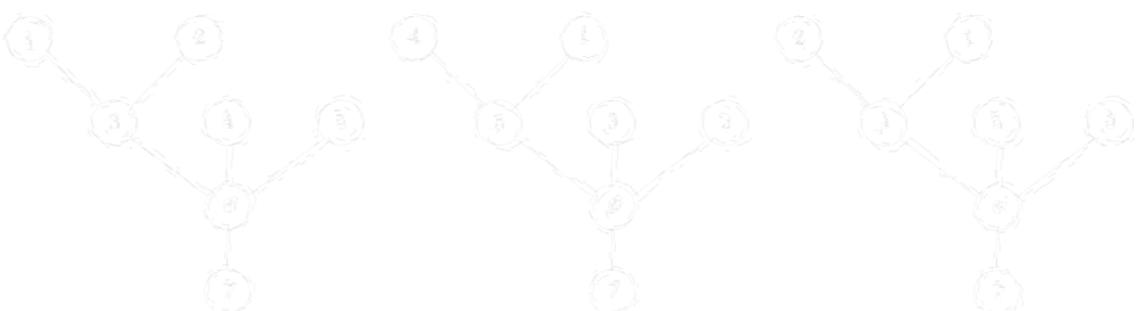
$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 3, \quad 5 \rightarrow 6, \quad 6 \rightarrow 8.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$ is called a path joining the vertices i_0 and j_0 if $i_0 \in \{i_0, j_0\}$, $j_0 \in \{i_0, j_0\}$ and for every $k = 1, 2, \dots, n-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of V are joined by a unique simple path. Both tri-diagonal and any other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $n \geq 3$.

Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $T \setminus \{r\}$ and these vertices are successors of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a predecessor of α and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree.



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so, that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

and their graphs

SUSPECT 2

quindiagonal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$



superdiagonal:

$$\begin{bmatrix} * & * & * & * & * & * \\ * & * & 0 & 0 & 0 & 0 \\ * & 0 & * & 0 & 0 & 0 \\ 0 & 0 & * & 0 & 0 & 0 \\ 0 & 0 & 0 & * & 0 & 0 \\ 0 & 0 & 0 & 0 & * & 0 \end{bmatrix}$$



cyclic:

$$\begin{bmatrix} * & 0 & 0 & 0 & * & 0 \\ * & * & 0 & 2 & * & 0 \\ * & 0 & * & * & 0 & 0 \\ 0 & 2 & * & * & * & 0 \\ 0 & * & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$



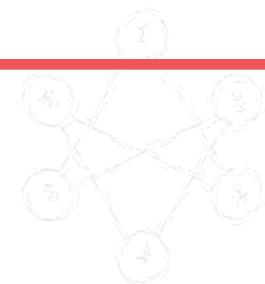
EMBER DATA / FORM BUILDER

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & * & 0 \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$

At a first glance, this is nothing but the structure of the form builder matrix

just displayed, but its graph



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

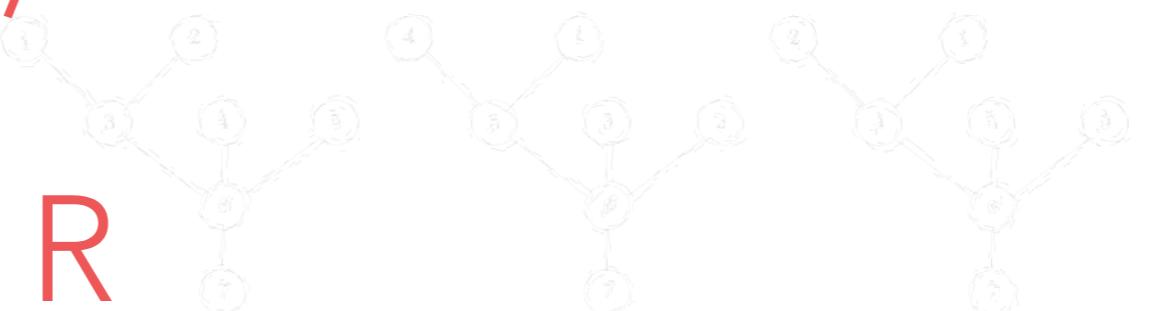
$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 8.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$ is called a path joining the vertices i_0 and j_0 if $i_0 \in \{1, \dots, n\}$, $j_0 \in \{1, \dots, n\}$ and for every $k = 1, 2, \dots, n-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of V are joined by a unique simple path. Both tri-diagonal and any other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $n \geq 3$.

Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $T \setminus \{r\}$ and these vertices are successors of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a predecessor of α and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree.



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, n\}$ and assume that the rows and columns of A are then arranged so, that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

and their graphs.

SUSPECT 3



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

STATE

At a first glance, this is nothing but the structure of the lower triangular matrix

just displayed, but its graph:



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$ $\subseteq \mathbb{E}$ is called a path joining the vertices i_0 and j_0 if $i_0 \in \{i_0, j_0\}$, $j_0 \in \{i_0, j_0\}$ and for every $k = 1, 2, \dots, n-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of V are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $n \geq 3$.

Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in a ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $T \setminus \{r\}$ and these vertices are successors of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a predecessor of α and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three consecutive orderings of the same rooted tree:



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A were been arranged so, that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

and their graphs.

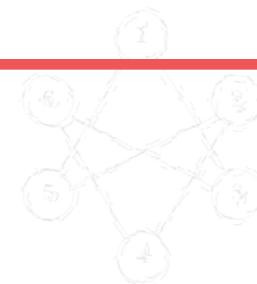
SUSPECT 4



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

LEAKAGE

At a first glance, this is nothing but a collection of the four matrices above, just displayed, but its graph,



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

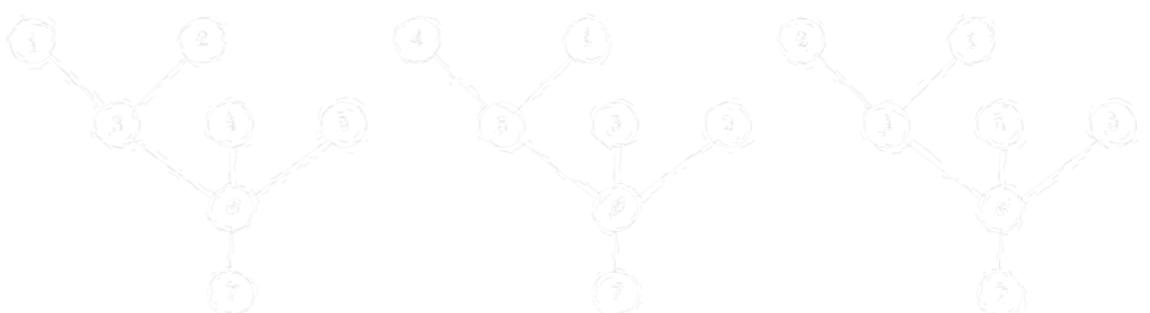
$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 1.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$ $\subseteq \mathbb{E}$ is called a path joining the vertices i_0 and j_0 if $i_0 \in \{i_0, j_0\}$, $j_0 \in \{i_0, j_0\}$ and for every $k = 1, 2, \dots, n-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that \mathbb{G} is a tree if each two members of \mathbb{V} are joined by a unique simple path. Both tri-diagonal and any other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $n \geq 3$.

Given a tree \mathbb{G} and an arbitrary vertex $r \in \mathbb{V}$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in ordinary graph, T adapts a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $\mathbb{V} \setminus \{r\}$ and these vertices are successors of r . Moreover, every $\alpha \in \mathbb{V} \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a predecessor of α and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three consecutive orderings of the same rooted tree.



Theorem 11.1. Let A be a symmetric matrix whose graph \mathbb{G} is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A were been arranged so, that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

and their graphs.

SUSPECT 5

quindiagonal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ 0 & * & * & 0 & 0 & 0 \\ 0 & 0 & * & * & 0 & 0 \\ 0 & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & * & * \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$



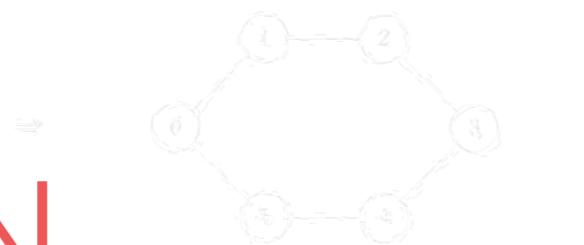
superdiagonal:

$$\begin{bmatrix} * & * & * & * & * & * \\ 0 & * & 0 & 0 & 0 & 0 \\ 0 & 0 & * & 0 & 0 & 0 \\ 0 & 0 & 0 & * & 0 & 0 \\ 0 & 0 & 0 & 0 & * & 0 \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$



cyclic:

$$\begin{bmatrix} * & 0 & 0 & 0 & * & * \\ 0 & * & 0 & 0 & 2 & 2 \\ 0 & 0 & * & 0 & 2 & 2 \\ 0 & 0 & 0 & * & 2 & 2 \\ 0 & 0 & 0 & 0 & * & 2 \\ 0 & 0 & 0 & 0 & 2 & * \end{bmatrix}$$

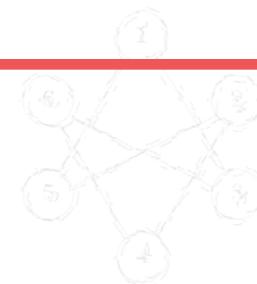


ADMIN

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

PRIVILEGE

At a first glance, this is nothing but the structure of the famous 0-1 matrix just displayed, but its graph:



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 3, \quad 5 \rightarrow 6, \quad 6 \rightarrow 8.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$ is called a path joining the vertices i_0 and j_0 if $i_0 \in \{i_0, j_0\}$, $j_0 \in \{i_0, j_0\}$ and for every $k = 1, 2, \dots, n-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of V are joined by a unique simple path. Both tri-diagonal and any other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $n \geq 3$.

Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in a ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $T \setminus \{r\}$ and these vertices are successors of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a predecessor of α and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

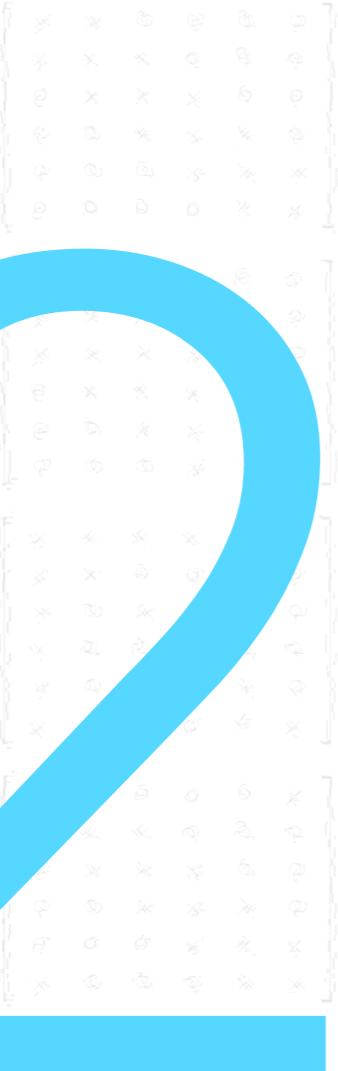
Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three consecutive orderings of the same rooted tree:

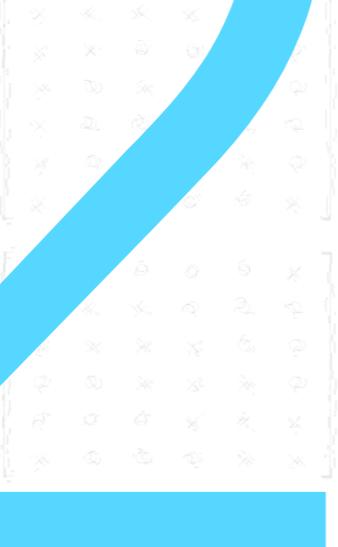


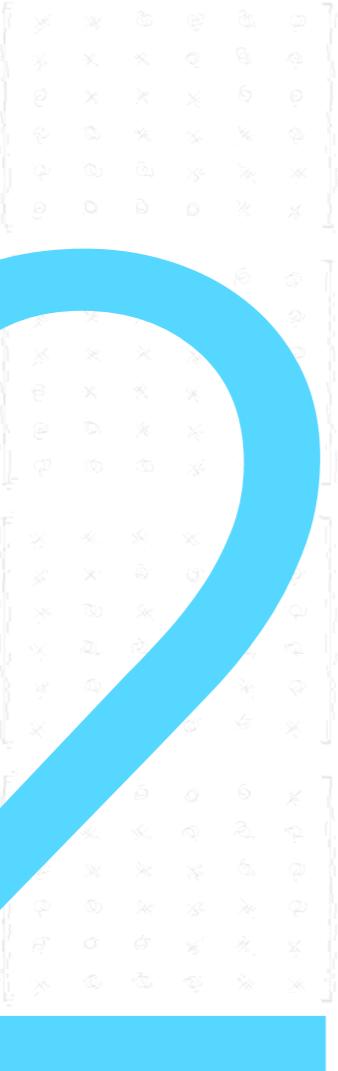
Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so, that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

tridiagonal:


symmetric:


cyclic:


2

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix



At a first glance, there is nothing to link it to any of the three structures just displayed, but its graph,

USE COMMON EVERYDAY WORDS



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 1.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{v_1, v_2, \dots, v_p\} \subseteq E$ is called a path joining the vertices v_1 and v_p ($v_1, v_p \in V$). If $v_1 = v_p$ and for every $i = 1, \dots, p-1$ the set $\{v_i, v_{i+1}\} \cap E$ contains exactly one member, it is a simple path if it does not visit any vertex more than once. We say that G is tree if each two members of V are joined by a unique simple path. (In this case, all other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $p \geq 3$.)

Given a tree G and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $V \setminus \{r\}$ and these vertices are successors of r . Moreover, every $v \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and v , as a predecessor of v and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we label the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is unique. To give the precise definition of the unique rooted tree,



Theorem 1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so, that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

and their graphs

triangular: $\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ 0 & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & * & 0 \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix} \Rightarrow \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \xrightarrow{1 \rightarrow 1, 2 \rightarrow 3, 3 \rightarrow 5, 4 \rightarrow 6, 5 \rightarrow 6, 6 \rightarrow 6}$

quidiagonal: $\begin{bmatrix} * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ 0 & * & * & * & * & * \\ 0 & 0 & * & * & * & * \\ 0 & 0 & 0 & * & * & * \end{bmatrix} \Rightarrow \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \xrightarrow{1 \rightarrow 1, 2 \rightarrow 3, 3 \rightarrow 4, 4 \rightarrow 5, 5 \rightarrow 6, 6 \rightarrow 6}$

superdiagonal: $\begin{bmatrix} * & * & * & * & * & * \\ * & 0 & 0 & 0 & 0 & 0 \\ * & 2 & 0 & 0 & 0 & 0 \\ * & 2 & 2 & 0 & 0 & 0 \\ * & 2 & 2 & 2 & 0 & 0 \\ * & 2 & 2 & 2 & 2 & 0 \end{bmatrix} \Rightarrow \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \xrightarrow{1 \rightarrow 1, 2 \rightarrow 2, 3 \rightarrow 3, 4 \rightarrow 4, 5 \rightarrow 5, 6 \rightarrow 6}$

cyclic: $\begin{bmatrix} * & 0 & 0 & 0 & * & * \\ * & * & 0 & 0 & 2 & 2 \\ * & * & * & 0 & 2 & 2 \\ * & * & * & * & * & 2 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix} \Rightarrow \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \xrightarrow{1 \rightarrow 1, 2 \rightarrow 2, 3 \rightarrow 3, 4 \rightarrow 4, 5 \rightarrow 5, 6 \rightarrow 6}$

CONVENTION;

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

AGREED ON BY MANY

At a first glance, this is not a triangular matrix, but it is a tree structure, as the one just displayed, but its graph:



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 3, 3 \rightarrow 2, 4 \rightarrow 4, 5 \rightarrow 6, 6 \rightarrow 8.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$ $\subseteq \mathbb{E}$ is called a path joining the vertices i_0 and j_0 if $i_0, j_0 \in \{1, \dots, n\}$, $i_0 \neq j_0$, $\{i_0, j_0\} \in \mathbb{E}$ and for every $k = 1, 2, \dots, n-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of V are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $n \geq 3$.

Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in ordinary graph, T adapts a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $T \setminus \{r\}$ and these vertices are successors of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a predecessor of α and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree:



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, n\}$ and assume that the rows and columns of A are then arranged so, that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

tridiagonal:

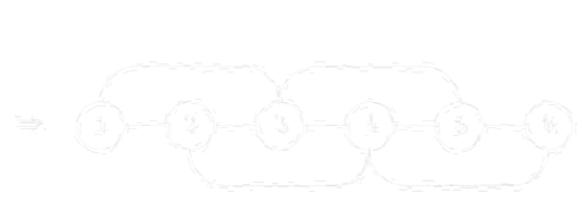
$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & 0 & * & * & 0 & 0 \\ 0 & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & * & * \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$

EVERYDAY



quindiagonal:

$$\begin{bmatrix} * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



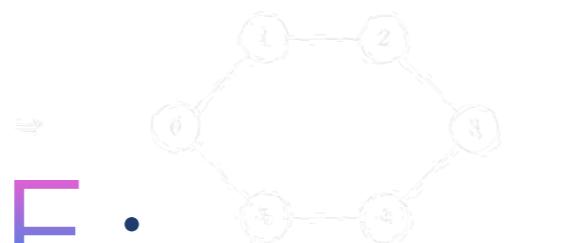
superdiagonal:

$$\begin{bmatrix} * & * & * & * & * & * \\ * & * & 0 & 0 & 0 & 0 \\ 0 & 0 & * & * & * & * \\ 0 & 0 & * & * & * & * \\ 0 & 0 & * & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



cyclic:

$$\begin{bmatrix} * & 0 & 0 & 0 & * & * \\ * & * & 0 & 2 & * & * \\ 0 & 0 & * & * & * & * \\ 0 & 2 & * & * & * & * \\ 0 & 0 & * & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



SIMPLE;

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

FAMILIAR TO MANY

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & * & * & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & * & * & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & * & * & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & * & * & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$

At a first glance this is not a matrix that is familiar to many, but it is the matrix just displayed, but its graph.



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

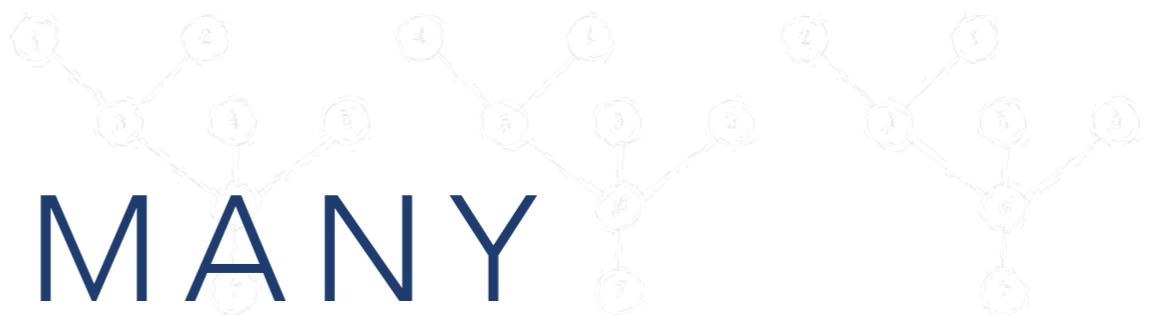
$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 3, \quad 5 \rightarrow 6, \quad 6 \rightarrow 8.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$ $\subseteq \mathbb{E}$ is called a path joining the vertices i_0 and j_0 if $i_0 \in \{i_0, j_0\}$, $j_0 \in \{i_0, j_0\}$ and for every $k = 1, 2, \dots, n-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of V are joined by a unique simple path. Both tridiagonal and quidiagonal matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $n \geq 3$.

Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in a ordinary graph, T adapts a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $T \setminus \{r\}$ and these vertices are successors of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a predecessor of α and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree.



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, n\}$ and assume that the rows and columns of A are then arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

triangular:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$

quadrangular:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$

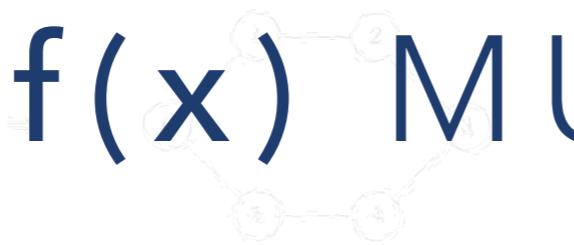
cyclic:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$

IF f IS CONTINUOUS IN $[a, b]$, AND IF $f(a)f(b) < 0$,

THEN $f(x)$ MUST HAVE A

ZERO IN (a, b) .



$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$

At a first glance, this is not a triangular matrix, but it is a triangular matrix, just displayed, but its graph,



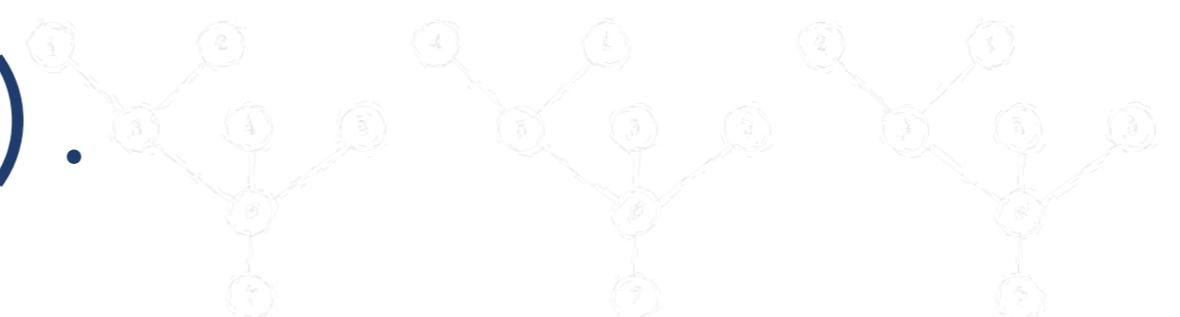
tells a different story - it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 2, 3 \rightarrow 3, 4 \rightarrow 4, 5 \rightarrow 5, 6 \rightarrow 6.$$

Of course, it is equivalent to relabeling the equations and variables. As a result, we get the following: G is called a *tree* if visiting the vertices a and b ($a \in V, b \in V, a \neq b$), $\beta \in V \setminus \{a, b\}$, and for every $k = 1, 2, \dots, n-1$ the set $\{V_k, \beta\} \cap V_{k+1, k+1}$ contains exactly one member. It is a *simple path* if it does not visit any vertex more than once. We say that G is a *tree* if each two members of V are joined by a unique simple path. Both triangular and any other matrices correspond to trees, but this is not the case with either of tridiagonal or cyclic matrices when $n \geq 3$.

As a result, we get the following: $T = (G, r)$ is called a *rooted tree* if $r \in V$ and r is the root of T . Unlike in a binary tree, there is a natural partial order, which can be explained by analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $V \setminus \{r\}$ and these vertices are successors of r . Moreover, every $a \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and a , as a predecessor of a and a successor of r . We say that the rooted tree T is *monotonically ordered* if each vertex is labelled with an integer r such that the label of a is less than the label of b if and only if a is an ancestor of b (i.e., the row a is to the left of the row b in the underlying matrix).

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree:



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

```
assert.dom('[data-test-message]')  
.hasText(  
  'Thanks for signing up!',  
  'The user sees a welcome message.'  
);
```

Therefore we will now give a few examples of matrices (represented by their sparsity)

triangular:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



quadrangular:

supersingular:

$$\begin{bmatrix} * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & * & * & * \end{bmatrix}$$



cyclic:

$$\begin{bmatrix} * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \end{bmatrix}$$



ZERO IN (a, b) .

$$\begin{bmatrix} * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \end{bmatrix}$$

At a first glance this is not a triangular matrix, but it is a triangular matrix, just displayed, but its graph:



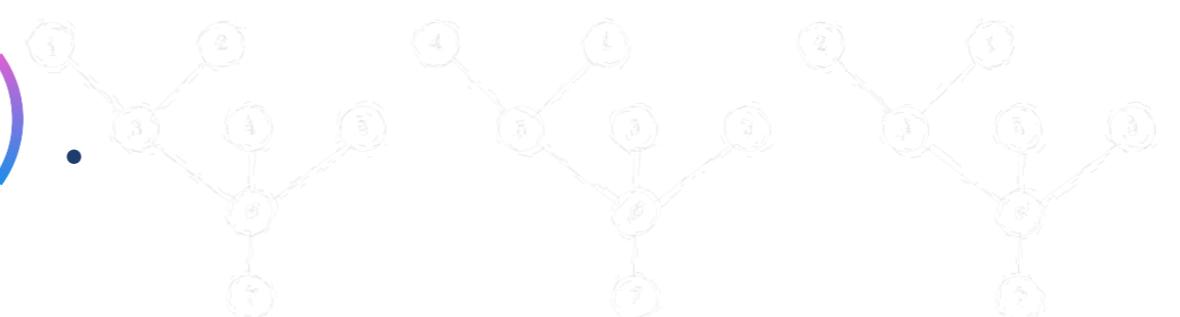
tells a different story - it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 2, 3 \rightarrow 3, 4 \rightarrow 4, 5 \rightarrow 5, 6 \rightarrow 6.$$

Of course, it is equivalent to relabeling the equations and variables. As you can see from the figure, the graph G is called a tree, since the vertices v and α ($v \in V \setminus \{\alpha\}$, $\beta \in V \setminus \{\alpha\}$) and for every $k = 1, 2, \dots, n-1$ the set $\{v, \alpha, \beta\} \cap \{v_k, \alpha_k, \beta_k\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of V are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees, but this is not the case with either of banded or cyclic matrices when $p \geq 3$.

treating T as a binary tree, the root r of T is called a *rooted tree*. Unlike in ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $T \setminus \{r\}$ and these vertices are successors of r . Moreover, every $v \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and v , as a predecessor of v and a successor of r . We say that the rooted tree T is *monotonically ordered* if each vertex is labelled by an integer from 1 to n (in other words, we say the vertices from the top of the tree to the bottom). (As we have already said, labelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

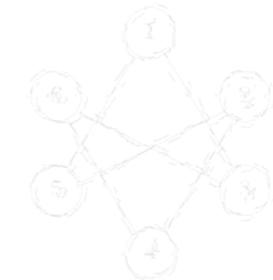
Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree:



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = L U$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{a_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, first relabel the vertices as follows:



Dashboard

Explore

Settings



vertices more than once. We say that G is a *tree* if each two members of V are joined by a unique simple path. Both tridiagonal and super-diagonal matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $p \geq 3$.

Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a *rooted tree*, while r is said to be the *root*. Unlike in ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the *predecessor* of all the vertices in $T \setminus \{r\}$ and these vertices are *successors* of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a *predecessor* of α and a *successor* of r . We say that the rooted tree T is *monotonically ordered* if each vertex is labelled before all its predecessors. In other words, if label the vertices from the root of the

'[data-test-link="Dashboard"]'

'[data-test-link="Explore"]'

'[data-test-link="Settings"]'

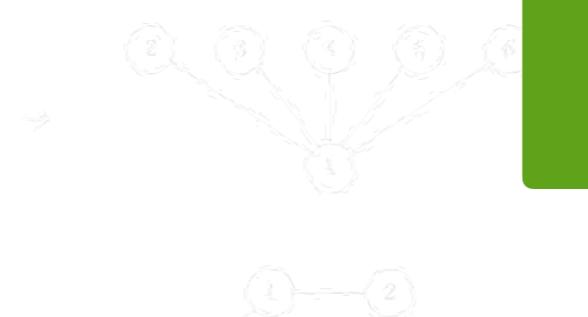
just displayed, but its graph.

$$t_{k,j} = \frac{a_{kj}}{a_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

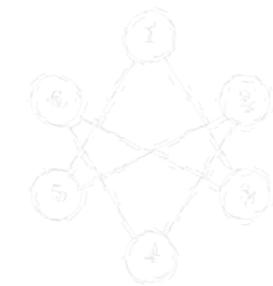
Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

Save

tridiagonal:	$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$
quindiagonal:	$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$
superdiagonal:	$\begin{bmatrix} * & * & * & * & * & * \\ * & * & 0 & 0 & 0 & 0 \\ * & 0 & * & 0 & 0 & 0 \\ 0 & * & * & 0 & 0 & 0 \\ 0 & 0 & * & * & 0 & 0 \\ 0 & 0 & 0 & * & * & * \end{bmatrix}$



Cancel



tells a different story - it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$1 \rightarrow 5, 2 \rightarrow 3, 3 \rightarrow 4, 4 \rightarrow 6, 6 \rightarrow 5.$

or reordering (simultaneously) the equations and variables.

$\{(v_i, j)\}_{i,j=1}^n \subseteq \mathbb{S}$ is called a path joining the vertices v_i and v_j , and for every $k = 1, 2, \dots, n-1$ the set $\{v_i, j\} \cap$

$\{v_i, j+k\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that \mathbb{S} is a tree if each two members of \mathbb{V} are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees. This is not the case with either quindiagonal or cyclic matrices when $n \geq 3$.

Fix a tree \mathbb{T} and an arbitrary vertex $r \in \mathbb{V}$, the pair $\mathbb{T} = (Q, r)$ is called a rooted tree, while r is said to be the root. Unlike in ordinary graph, \mathbb{T} admits a natural partial order, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $\mathbb{V} \setminus \{r\}$ and these vertices are successors of r . Moreover, every $\alpha \in \mathbb{V} \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a predecessor of α and a successor of r . We say that the rooted tree \mathbb{T} is monotonically ordered if each vertex is labelled before all its predecessors. In other words, if α is the vertex from the set of the

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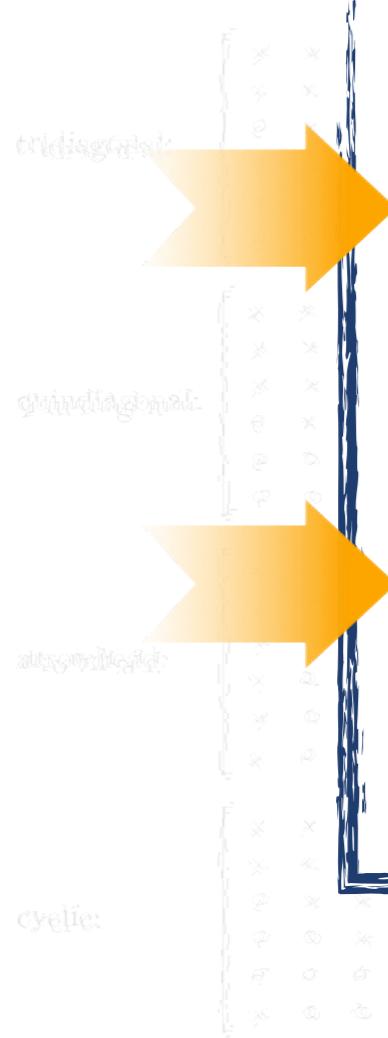
'[data-test-button="Cancel"]'

'[data-test-button="Add item"]'

just displayed, but its graph.

$$t_{k,j} = \frac{a_{k,j}}{a_{j,j}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



Name*

Description

permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three consecutive orderings of the sample rooted tree.

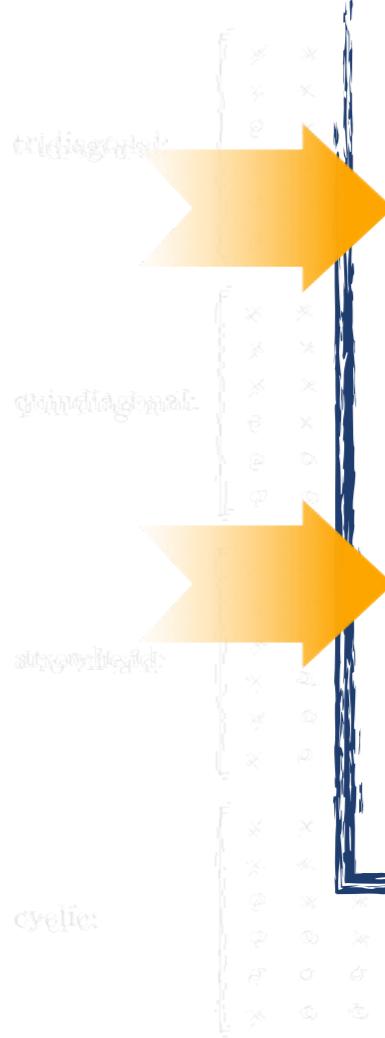
```
[data-test-field="Name"]
```

```
[data-test-field="Description"]
```

just displayed, but its graph.

$$t_{k,j} = \frac{a_{kj}}{a_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1, \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



Name

Little Bobby Tables

Description

Better not drop me!

matrix in disguised. To see
 $6 \Rightarrow 3$.
equations and vegetables.
th joining the vertices of
 $\{v_1, v_2, \dots, v_p\} \cap V$ in
ch if it does not visit any
numbers of V are joined by
edges correspond to trees.
atries when $p \geq 3$.
- (G, α) is called a rooted
digraph with natural partial
order a family tree. Thus, the
se vertices are *successors* or
e path and we designate
son of α and a *successor*
if each vertex is labelled
from the top of the
graph is *transient* to
permuting the rows and the columns of the underlying matrix.)
Every rooted tree will be monotonically ordered and, in general, such an ordering
is not unique. We now give three monotonic orderings of the sample rooted tree.

'[data-test-field="Name"]'

'[data-test-field="Description"]'

just displayed, but its graph.

$$t_{k,j} = \frac{a_{kj}}{a_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix



At a first glance, there is nothing to link it to any of the four structures just displayed, but its graph,

WRITE LESS WITH THEOREMS AND NEW TERMS

Theorem 11.1. Let A be a $n \times n$ matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, n\}$ and assume that the rows and columns of A are then arranged so, that $T = (G, r)$ is monotonically ordered. Given that $A = LU$ is a Cholesky factorization, it is true that

$$l_{kj} = \frac{a_{kj}}{L_{jj}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

triangular:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



quadrangular:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



supernodal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



cyclic:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



IF f IS CONTINUOUS IN $[a, b]$, AND IF $f(a)f(b) < 0$,

THEN f MUST HAVE A

ZERO IN (a, b) .



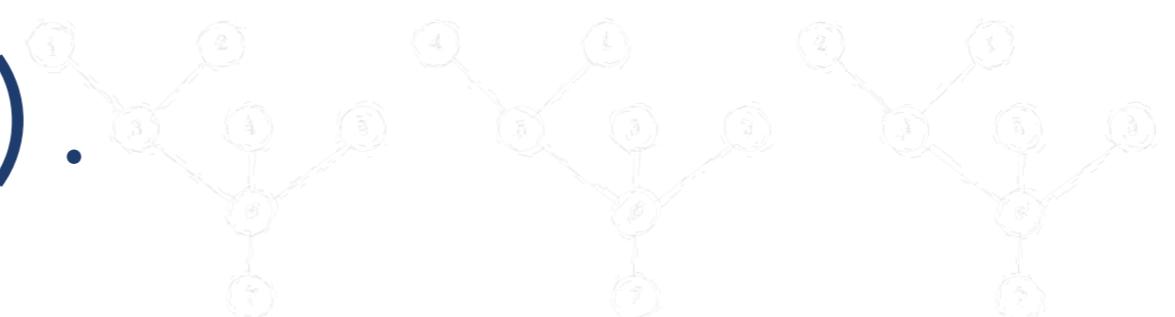
tells a different story - it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 2, 3 \rightarrow 3, 4 \rightarrow 4, 5 \rightarrow 5, 6 \rightarrow 6.$$

Of course, it is equivalent to replacing (arbitrarily) the equations and variables. As you can see from the figure, the graph G is called a tree, since the vertices a and b ($a \in V \setminus \{b\}$, $b \in V \setminus \{a\}$) and for every $k = 1, 2, \dots, n-1$ the set $\{v_1, v_2, \dots, v_k\} \cap \{v_{k+1}, v_{k+2}, \dots, v_n\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of V are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees, but this is not the case with either of banded or cyclic matrices when $n \geq 3$.

Of course, a rooted tree is a tree with a root vertex r . Unlike in a binary tree, there is a natural partial order in which each vertex has an unique parent. Thus, the vertex r is the predecessor of all the vertices in $V \setminus \{r\}$ and these vertices are successors of r . Moreover, every $v \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and v , as a predecessor of v and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled by an integer r in such a way that the vertices from the top of the tree to the bottom are in increasing order (in other words, we say the vertices from the top of the tree to the bottom are in increasing order). Labelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree:



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

At a first glance, this is not a very useful formula, but it is the basis for the algorithm just displayed, but its graph.

Therefore we will now give a few examples of matrices (represented by their sparsity)

PROOF. USE THE INTERMEDIATE VALUE THEOREM.

cyclic:

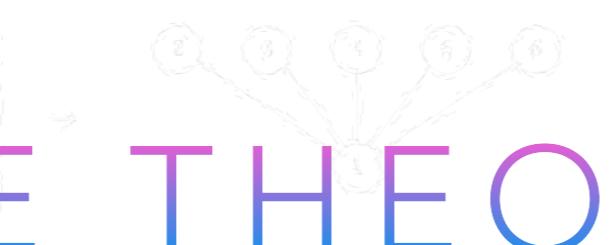
$$\begin{bmatrix} * & * & 0 & 0 & 0 \\ 0 & * & * & 0 & 0 \\ * & 0 & * & 0 & 0 \\ 0 & 0 & 0 & * & 0 \\ 0 & 0 & 0 & 0 & * \end{bmatrix}$$



$$\begin{bmatrix} * & * & 0 & 0 & 0 \\ 0 & * & * & 0 & 0 \\ * & 0 & * & 0 & 0 \\ 0 & 0 & 0 & * & 0 \\ 0 & 0 & 0 & 0 & * \end{bmatrix}$$



$$\begin{bmatrix} * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ * & 0 & * & * & 0 & 0 \\ 0 & 0 & 0 & * & 0 & 0 \\ 0 & 0 & 0 & 0 & * & 0 \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & 0 \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & * \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & * \end{bmatrix}$$

At a first glance, this is not a cyclic matrix, but it is. It is a 6x6 matrix, which is not just displayed, but its graph.



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

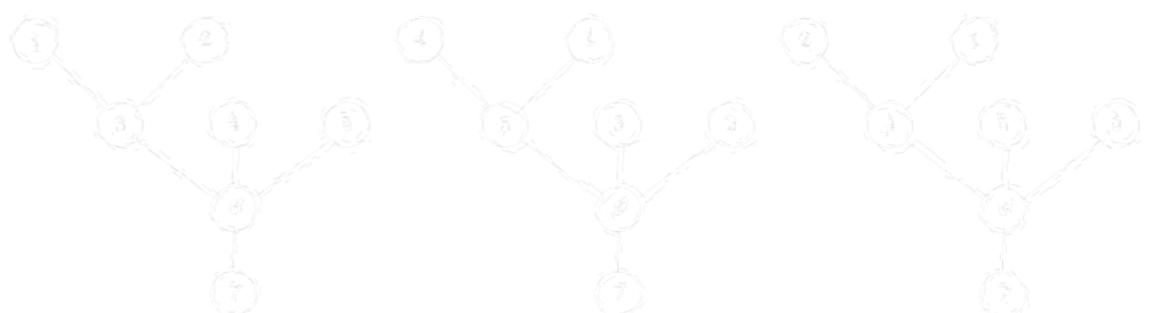
$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_0, j_0)\}_{i_0, j_0 \in \mathbb{N}_0, i_0 \neq j_0}$ in \mathbb{G} is called a path joining the vertices i_0 and j_0 if $i_0 \in \{i_0, j_0\}$, $j_0 \in \{i_0, j_0\}$ and for every $p = 1, 2, \dots, v-1$ the set $\{i_p, j_p\} \cap \{i_{p+1}, j_{p+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertices more than once. We say that \mathbb{G} is a tree if in two members of \mathbb{V} are joined by a unique simple path. Known tridiagonal and cyclic matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $p \geq 3$.

Given a tree \mathbb{T} and an arbitrary vertex $r \in \mathbb{V}$, the pair $\mathbb{T} = (\mathbb{G}, r)$ is called a rooted tree, while r is said to be the root. Unlike in ordinary graph, \mathbb{T} adapts a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root is the ancestor of all the vertices in $\mathbb{V} \setminus \{r\}$ and these vertices are successors of r . Moreover, every $a \in \mathbb{V} \setminus \{r\}$ is joined to r by a simple path and we designate any vertex along this path, except for r and a , as a predecessor of a and a successor of r . We say that the rooted tree \mathbb{T} is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

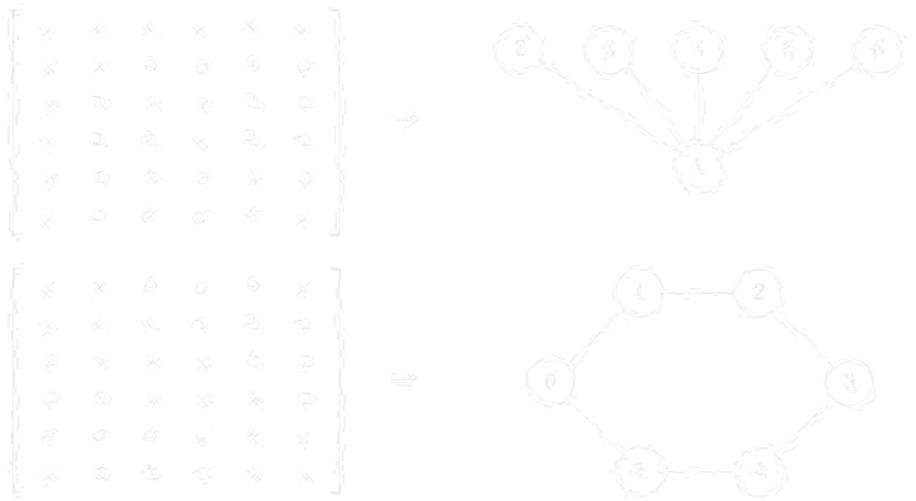
Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree.



Therefore we will now give a few examples of matrices (represented by their sparsity)

PROOF.
USE IVT.

triangular:
quadratic:
cyclic:



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & x & 0 \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & 0 & * & 0 \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & * & 0 \end{bmatrix}.$$

At a first glance this is not immediately clear what the structure of the matrix is. The graph, however, is just displayed, but its graph:



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 3, \quad 5 \rightarrow 6, \quad 6 \rightarrow 8.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$ is called a path joining the vertices i_0 and j_0 if $i_0 \in \{i_0, j_0\}$, $j_0 \in \{i_0, j_0\}$ and for every $k = 1, 2, \dots, n-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of V are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $n \geq 3$.

Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in a ordinary graph, T adapts a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $T \setminus \{r\}$ and these vertices are successors of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a predecessor of α and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three consecutive orderings of the same rooted tree:

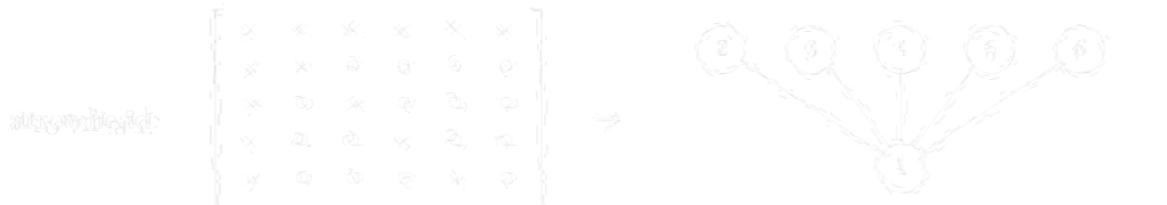
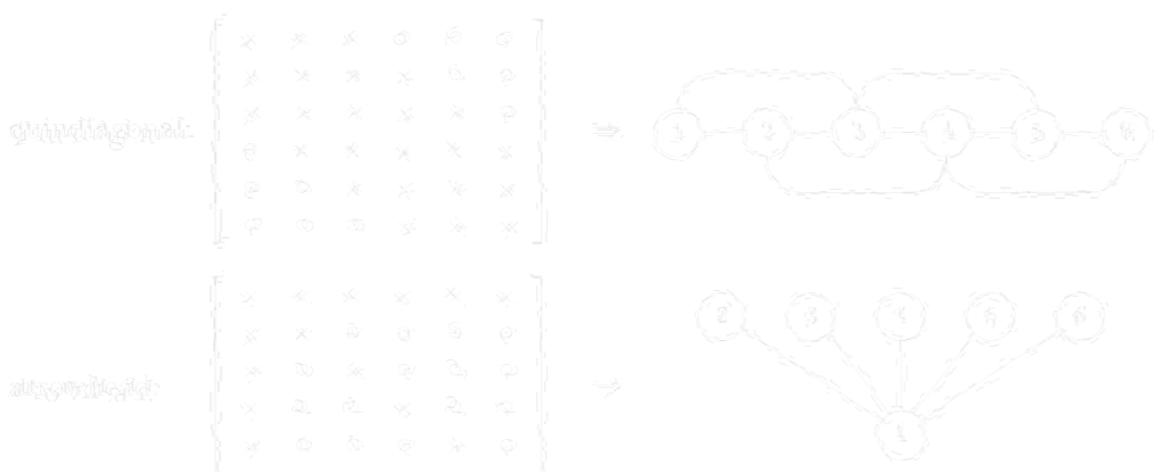


Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A were been arranged so, that $T = (G, r)$ is monotonically ordered. Given that $A = L U$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

THEOREM



PROVEN

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

TO BE TRUE

At a first glance this is not a matrix, but it is a little known triangular matrix, just displayed, but its graph,



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 3, \quad 5 \rightarrow 6, \quad 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$ $\subseteq \mathbb{E}$ is called a path joining the vertices i_0 and j_0 if $i_0 \in \{i_0, j_0\}$, $j_0 \in \{i_0, j_0\}$ and for every $k = 1, 2, \dots, n-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that \mathbb{G} is a tree if each two members of \mathbb{V} are joined by a unique simple path. Both tri-diagonal and any other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $n \geq 3$.

Given a tree \mathbb{T} and an arbitrary vertex $r \in \mathbb{V}$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in a ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $\mathbb{V} \setminus \{r\}$ and these vertices are successors of r . Moreover, every $a \in \mathbb{V} \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and a , as a predecessor of a and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three consecutive orderings of the same rooted tree:



```
import { fillIn } from '@ember/test-helpers';

export async function fillForm(fields) {
  for (const { label, value } of fields) {
    // input, textarea
    await fillIn(`[data-test-field="${label}"`, value);
  }
};
```

```
import { fillForm } from '../helpers/my-test-helper';

...
test('User can create account', async function(assert) {
  await visit('/signup');
  await fillForm([
    { label: 'Name', value: 'Little Bobby Tables' },
    { label: 'Email', value: 'little.bobby@gmail.com' }
  ]);
  ...
});
```

Therefore we will now give a few examples of matrices (represented by their sparsity)

tri-diagonal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & 0 & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix} \Rightarrow \text{graph: } \begin{array}{c} 1 \\ \text{---} \quad \text{---} \quad \text{---} \\ 2 \quad 3 \quad 4 \quad 5 \quad 6 \end{array}$$

quindiagonal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & 0 & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix} \Rightarrow \text{graph: } \begin{array}{c} 1 \\ \text{---} \quad \text{---} \quad \text{---} \\ 2 \quad 3 \quad 4 \quad 5 \quad 6 \end{array}$$

super-diagonal:

$$\begin{bmatrix} * & * & * & * & * & * \\ * & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5 & 0 \end{bmatrix} \Rightarrow \text{graph: } \begin{array}{c} 1 \\ \text{---} \quad \text{---} \quad \text{---} \\ 2 \quad 3 \quad 4 \quad 5 \quad 6 \end{array}$$

cyclic:

$$\begin{bmatrix} * & 0 & 0 & 0 & * & * \\ * & * & 0 & 0 & 0 & 0 \\ 0 & 0 & * & 0 & 0 & 0 \\ 0 & 0 & 0 & * & 0 & 0 \\ 0 & 0 & 0 & 0 & * & 0 \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix} \Rightarrow \text{graph: } \begin{array}{c} 1 \\ \text{---} \quad \text{---} \quad \text{---} \\ 2 \quad 3 \quad 4 \quad 5 \quad 6 \end{array}$$

UBIQUITOUS

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

IDEA

At a first glance this is not a matrix, but a graph. But it is a matrix, as we have just displayed, but its graph:



tells a different story - it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 3, \quad 5 \rightarrow 6, \quad 6 \rightarrow 8.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$ is called a path joining the vertices i_0 and j_0 if $i_0 \in \{i_0, j_0\}$, $j_0 \in \{i_0, j_0\}$ and for every $k = 1, 2, \dots, n-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of V are joined by a unique simple path. Both tri-diagonal and super-diagonal matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $n \geq 3$.

Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in a ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $T \setminus \{r\}$ and these vertices are successors of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a predecessor of α and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree:



Therefore we will now give a few examples of matrices (represented by their sparsity)

triangular:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$

quadrangular:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$

cyclic:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$

At a first glance this is not a triangular or quadrangular matrix, but it is a triangular matrix just displayed, but its graph,



IF f IS CONTINUOUS IN $[a, b]$, AND IF $f(a)f(b) < 0$,

THEN f MUST HAVE A

ZERO IN (a, b) .



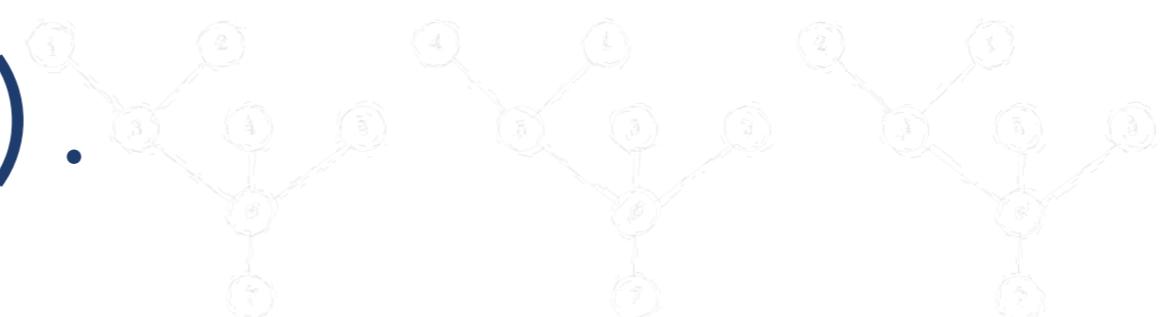
tells a different story - it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 3 \rightarrow 5, \quad 2 \rightarrow 4 \rightarrow 6, \quad 3 \rightarrow 1, \quad 4 \rightarrow 2.$$

Of course, this is equivalent to reordering (or relabeling) the equations and variables. As a result, the set of edges $\{v_i, v_j\}$ is called a *tree* spanning the vertices v_i and v_j ($i, j \in \{1, 2, \dots, n\}$, $i \neq j$) and for every $k = 1, 2, \dots, n-1$ the set $\{v_{k+1}, v_{k+2}\} \cap \{v_{k+2}, v_{k+3}\}$ contains exactly one member. It is a *simple path* if it does not visit any vertex more than once. We say that G is a *tree* if each two members of V are joined by a unique simple path. Both triangular and quadrangular matrices correspond to trees, but this is not the case with either quadrangular or cyclic matrices when $n \geq 3$.

Finally, in order to make the theory more general, we introduce a natural partial ordering which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $V \setminus \{r\}$ and these vertices are successors of r . Moreover, every $v \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and v , as a predecessor of v and a successor of r . We say that the rooted tree T is *monotonically ordered* if each vertex is labelled by an integer $r \in \{1, 2, \dots, n\}$ in such a way that the vertices from the top of the tree down to the bottom are in increasing order (in other words, we say the vertices from the top of the tree down to the bottom are in increasing order). Labelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree:



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, n\}$ and assume that the rows and columns of A are then arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

triangular:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



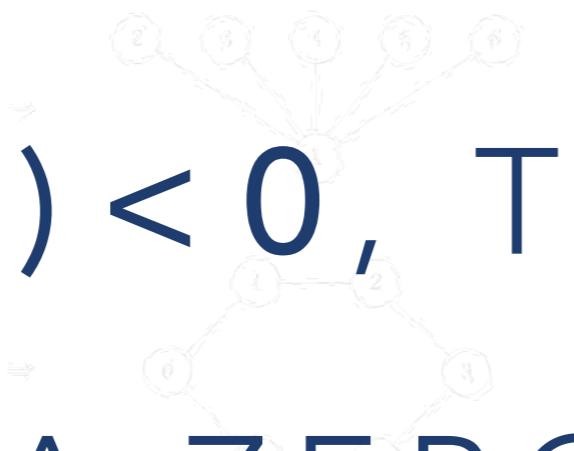
quadratic:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



symmetric:

$$\begin{bmatrix} * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ 0 & * & * & * & * & * \\ 0 & 0 & * & * & * & * \\ 0 & 0 & 0 & * & * & * \end{bmatrix}$$



cyclic:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} * & * & * & 0 & * & 0 \\ 0 & * & 0 & * & * & 0 \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & * & 0 \\ * & * & 0 & 0 & * & * \\ 0 & * & * & 0 & * & * \end{bmatrix}$$

At a first glance, this is not a triangular, quadratic, or cyclic matrix. But it is a matrix that has just been displayed, but its graph,



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 4, 6 \rightarrow 6.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_1, j_1)\}_{i_1, j_1 \in \mathbb{N}_0}$ $\subseteq \mathbb{E}$ is called a path joining the vertices i_1 and j_1 if $i_1, j_1 \in \{1, \dots, n\}$, $i_1 \neq j_1$, $i_1, j_1 \in \mathbb{N}_0$ and for every $p = 1, 2, \dots, \nu - 1$ the set $\{i_p, j_p\} \cap \{i_{p+1}, j_{p+1}\}$ contains only one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of V are joined by a unique simple path. Both quadiagonal and symmetric matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $n \geq 3$.

Given a tree G and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in a digraph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the predecessor of the vertex v is the vertex u if v is a successor of u in the tree T . Moreover, every $u \in G$ is the predecessor of v in the path and we designate u as a parent along this path, except for v , which is a predecessor of v and a successor of u . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we lay out the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We will give three examples consisting of the same rooted tree:

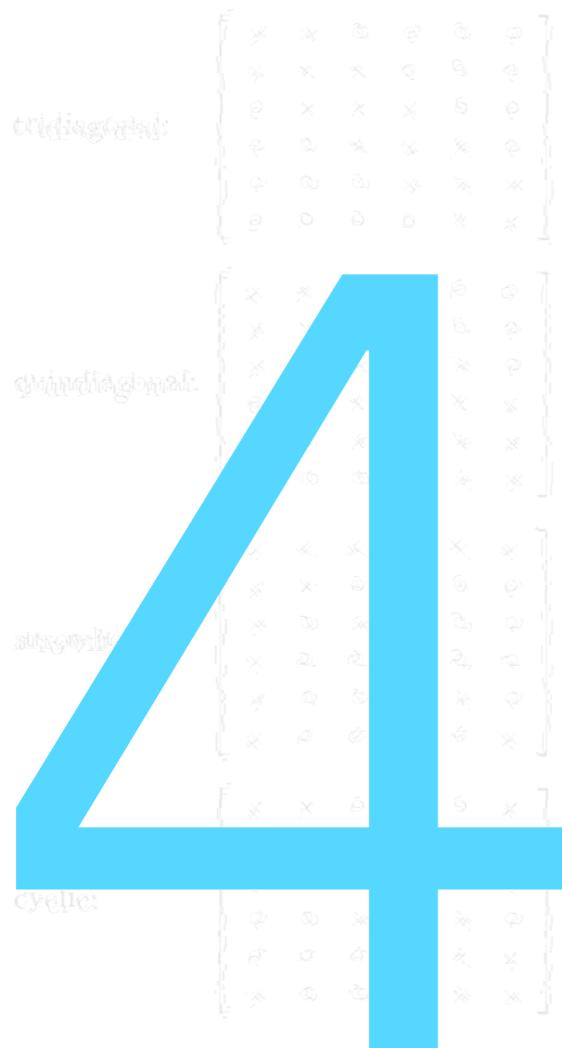


Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

```
hooks.beforeEach(function(assert) {  
  ...  
  // Example: assert.isEnabled('Submit', 'Woot!');  
  assert.isEnabled = (label, message) => {  
    assert.dom(`[data-test-button="${label}"]`)  
      .doesNotHaveAttribute('disabled', message);  
  };  
  ...  
});
```

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & * \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & * \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & * \end{bmatrix}$$

At a first glance, there is nothing to link it to any of the four structures just displayed, but its graph,

ALL YOUR BASIS ARE BELONG TO US



Let us now let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

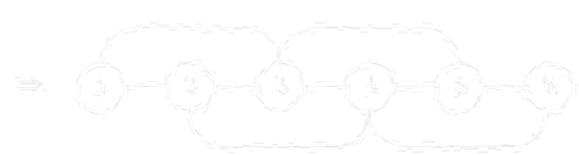
triangular:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$



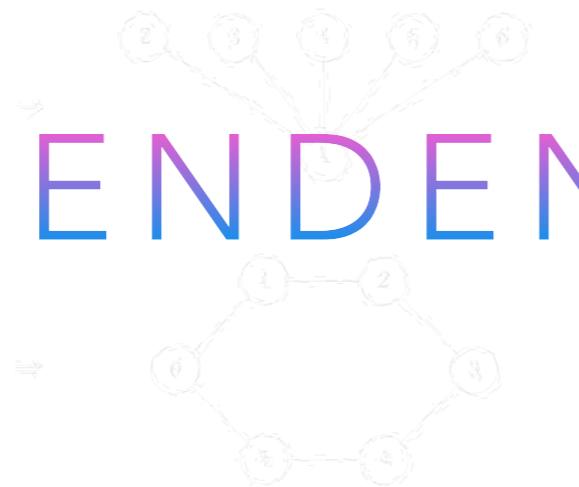
quidiagonal:

$$\begin{bmatrix} * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$



acyclic:

$$\begin{bmatrix} * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$



THAT

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

SPAN THE ENTIRE SPACE

At a first glance, this is not a matrix that looks like the basis matrices that we have just displayed, but its graph:



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(v_i, j_i)\}_{i=1}^n$ in \mathbb{G} is called a path joining the vertices v_i and v_j if $i \in \{1, \dots, n\}$, $j \in \{1, \dots, n\}$ and for every $k = 1, 2, \dots, n-1$ the set $\{v_i, j_k\} \cap \{v_{i+k}, j_{k+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that \mathbb{G} is a tree if each two members of V are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $n \geq 3$.

Given a tree \mathbb{G} and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the root of all the edges and every vertex is an ancestor of r . Moreover, every $v \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex alone, throughout, except for r and a, as a predecessor of v and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

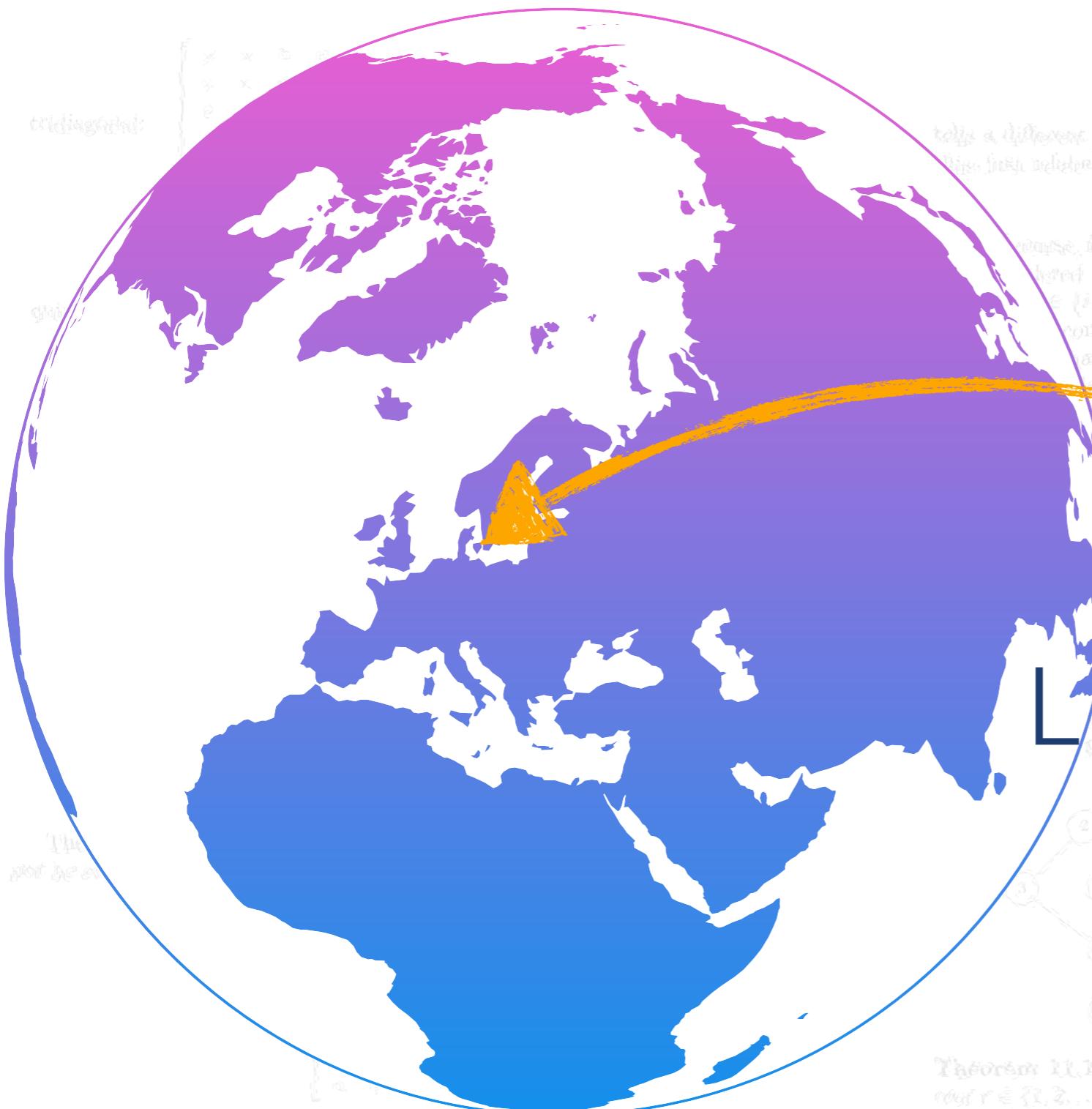
Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree:



Theorem 11.1. Let A be a symmetric matrix whose graph \mathbb{G} is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A were been arranged so, that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph.

tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 3, 3 \rightarrow 2, 4 \rightarrow 4, 5 \rightarrow 6, 6 \rightarrow 3.$$

Of course, it is equivalent to renumbering (cyclic shift) the vertices and to consider a different set of edges $\{ (i, j) \mid i, j \in \{1, 2, \dots, 6\} \}$. It is called a *cycle* if the vertices $i, j \in \{1, 2, \dots, n\}$ and for every $\ell = 1, 2, \dots, n-1$ the set $\{i + \ell, j + \ell\}$ contains exactly one member. It is a *simple path* if it does not visit any vertex more than once. We say that G is a *tree* if each two members of V are joined by a unique path. Both tridiagonal and antidiagonal matrices correspond to the case with either quidiagonal or cyclic matrices when $\ell = 1, 2, \dots, n-1$ and no arbitrary vertex $v \in V \setminus \{r\}$ is called a *rooted tree* and r is called the *root*. Unlike in a ordinary graph, T admits a natural partial order that must be explained by an analogy with a family tree. Thus, the ancestor of all the vertices in $T \setminus \{r\}$ and these vertices are *successors*. Every $a \in V \setminus \{r\}$ is joined to r by a simple path and we designate this path, except for r and a , as a *predecessor* of a and a *successor*. The rooted tree T is *monotonically ordered* if each vertex is labelled with a natural number in other words, we label the vertices from the top of the tree to the bottom (we have already done it, relabeling a graph to a tree). (This is another reason to call T a *tree* and not a *forest* in general, such a labeling does not give three separate components of the same source tree.)



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so, that $T = (G, r)$ is monotonically ordered. Then, that $A = LL^T$ is a Cholesky factorization, it is true that

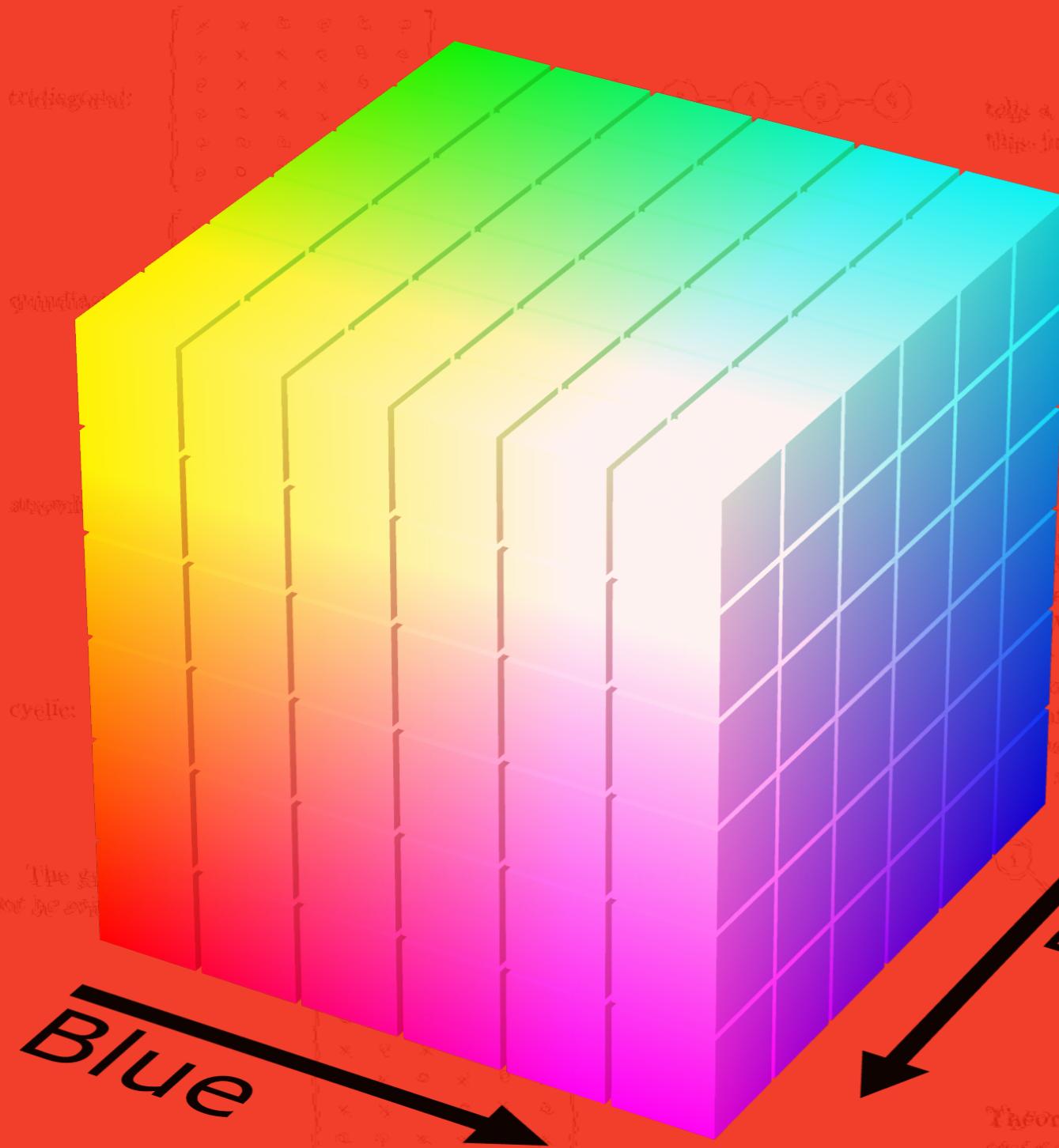
$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

A world map showing the Americas. A thick orange arrow originates in the central United States and points southeast towards Mexico. The map is color-coded: the United States and Canada are in purple, Mexico and Central America are in blue, and South America is in dark blue. The oceans are white. The arrow is the most prominent feature, highlighting the movement or flow from the United States into Mexico.

A graphic featuring the words "LATITUDE" and "LONGITUDE" in large, bold, dark blue capital letters. Between them is a large, stylized number "30.267°". A vertical purple line with a blue arrow at the bottom extends from the "L" in LATITUDE to the "L" in LONGITUDE. A horizontal orange line with a blue arrow at the end extends from the "3" in 30.267 to the "7" in 30.267. The background is a light gray with a faint, repeating grid pattern.

LONGITUDE

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



At a first glance, there is nothing to link in to any of the four matrices that we have just displayed, but its graph.



RED

224

tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, first relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 4, 5 \rightarrow 6, 6 \rightarrow 3$$

Of course, it is equivalent to reordering (simultaneously) the columns and rows. A *partial set of steps* $\{(v_i, j_i)\}_{i=1}^n \subseteq \mathbb{E}$ is called a *path* joining the vertices v_i and v_j if $(v_i, j_i), j_i \in \{1, \dots, n\}$ and for every $k = 1, 2, \dots, n-1$ the set $\{v_{k+1}, j_{k+1}\} \cap \mathbb{E}$ contains exactly one member. It is a *simple path* if it does not visit any vertex more than once. We say that G is a *tree* if each two members of V are joined by a simple path. Both tri-diagonal and quasi-diagonal matrices correspond to trees. (This is not the case with either quidiagonal or cyclic matrices when $n \geq 3$.)

Let's start with an arbitrary vertex $r \in V$. If the set $T = \{v_i \in V \mid v_i \text{ is joined to } r\}$ is called a *rooted tree* with r added to be the root. Unlike a ordinary tree, T does not have a unique root, which can best be explained by an example. A family tree. This is the set of all the vertices in $V \setminus \{r\}$ and these vertices are *successors* of r . If a vertex $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate α as an *ancestor* of r , except for r and α , as a *predecessor* of α and a *successor* of r . We say that a rooted tree T is *monotonically ordered* if each vertex is labelled with a unique integer i in other words we label the vertices from the top of T to the bottom. (We have already said it, relabelling a graph is the same as relabelling the rows and the columns of the underlying matrix.)

Every vertex then will be monotonically ordered and, in general, such an ordering is called a *topological ordering*. We can give three consecutive orderings of the same rooted tree:

Red

Green

Blue

Blue

Red

Green

Therefore we will now give a few examples of matrices (represented by their sparsity)

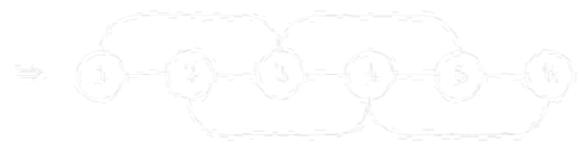
and their graphs

tridiagonal:
$$\begin{bmatrix} * & * & 0 & 0 & 0 \\ * & * & * & 0 & 0 \\ 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & * & 0 \\ 0 & 0 & 0 & 0 & * \end{bmatrix}$$



BASIS

quindiagonal:

$$\begin{bmatrix} * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & * & 0 \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$


superdiagonal:

$$\begin{bmatrix} * & * & * & * & * & * \\ * & * & 0 & 0 & 0 & 0 \\ 0 & 0 & * & 0 & 0 & 0 \\ 0 & 0 & 0 & * & 0 & 0 \\ 0 & 0 & 0 & 0 & * & 0 \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$


cyclic:

$$\begin{bmatrix} * & 0 & 0 & 0 & * & * \\ * & * & 0 & 0 & 0 & 0 \\ 0 & 0 & * & 0 & 0 & 0 \\ 0 & 0 & 0 & * & 0 & 0 \\ 0 & 0 & 0 & 0 & * & 0 \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$


BUILDING BLOCKS

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

OF TESTS

At a first glance, this is not a matrix, but a graph of 10 nodes. It is, however, a matrix, just displayed, but its graph:



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$ is called a path joining the vertices i_0 and j_0 if $i_0 \in \{1, \dots, n\}$, $j_0 \in \{1, \dots, n\}$ and for every $k = 1, 2, \dots, n-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of V are joined by a unique simple path. Both tridiagonal and superdiagonal matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $n \geq 3$.

Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in a ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $T \setminus \{r\}$ and these vertices are successors of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a predecessor of α and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is unique. The following diagram shows three consecutive orderings of the same rooted tree.



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A were been arranged so, that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



tells a different story - it is nothing other than the cyclic matrix in disguised. To see this, just replace the weights as follows:

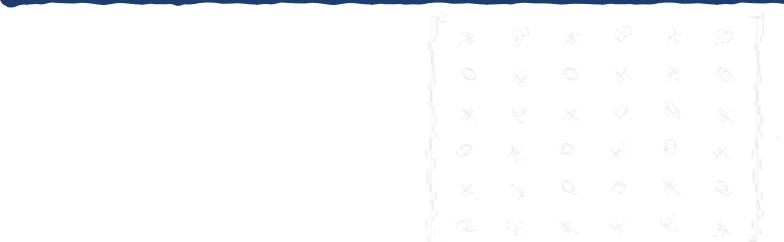


Create Edit Delete Clone Import

[[*, 0, 0, 0, 0, 0], [0, *, 0, 0, 0, 0], [0, 0, *, 0, 0, 0], [0, 0, 0, *, 0, 0], [0, 0, 0, 0, *, 0], [0, 0, 0, 0, 0, *]]

A single sparse matrix. Both tridiagonal and any banded matrices correspond to trees, but this is not the case with either anti-diagonal or cyclic matrices when $n > 3$.

	Name	Description
<input checked="" type="checkbox"/>	Little Bobby Tables	Better not drop me!
<input type="checkbox"/>	Big Bobby Tables	
<input type="checkbox"/>	Foo Bar	Making up names is hard.



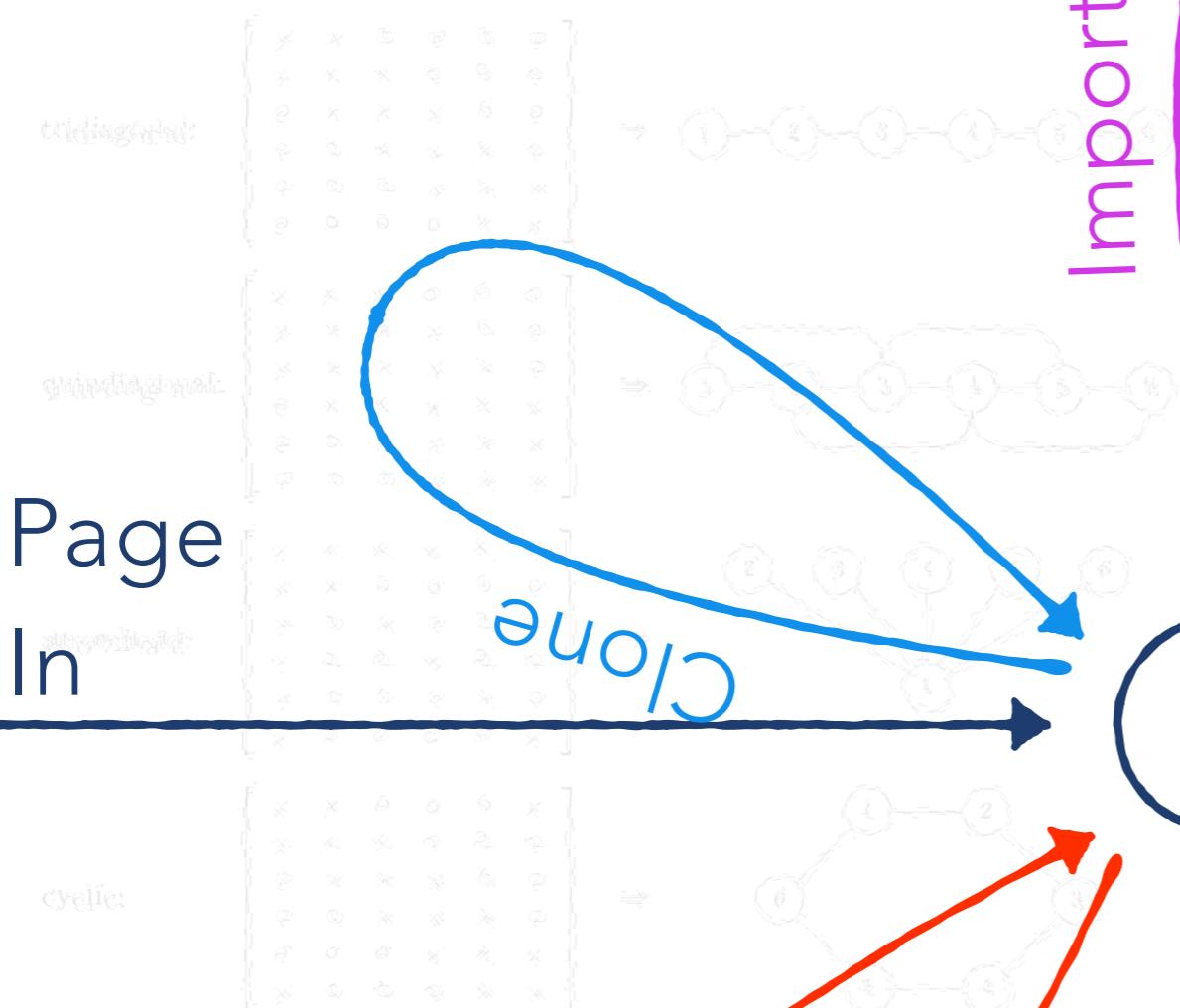
At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph:



Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so that $\mathbf{T} = (G, r)$ is monotonically ordered. Given that $A = \mathbf{L}\mathbf{U}$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,

tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just read the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 1.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(v_i, j)\}_{i,j}$, $\mathbb{S} \subseteq \mathbb{V} \times \mathbb{V}$ is called a path joining the vertices v_i and v_j if $i, j \in \{1, \dots, n\}$, $\mathbb{S} \subseteq \mathbb{V} \times \mathbb{V}$ and for any $k = 1, 2, \dots, n-1$ the set $\{v_i, j\} \cap \{v_{i+k}, j+k\}$ contains exactly one vertex. It is a simple path if it does not visit any vertex more than once. We say that \mathbb{G} is a tree if none of members of \mathbb{V} are joined by a single simple path. But tridiagonal and quadiagonal matrices contain cycles. Note this is not the case with either quadiagonal or cyclic matrices when

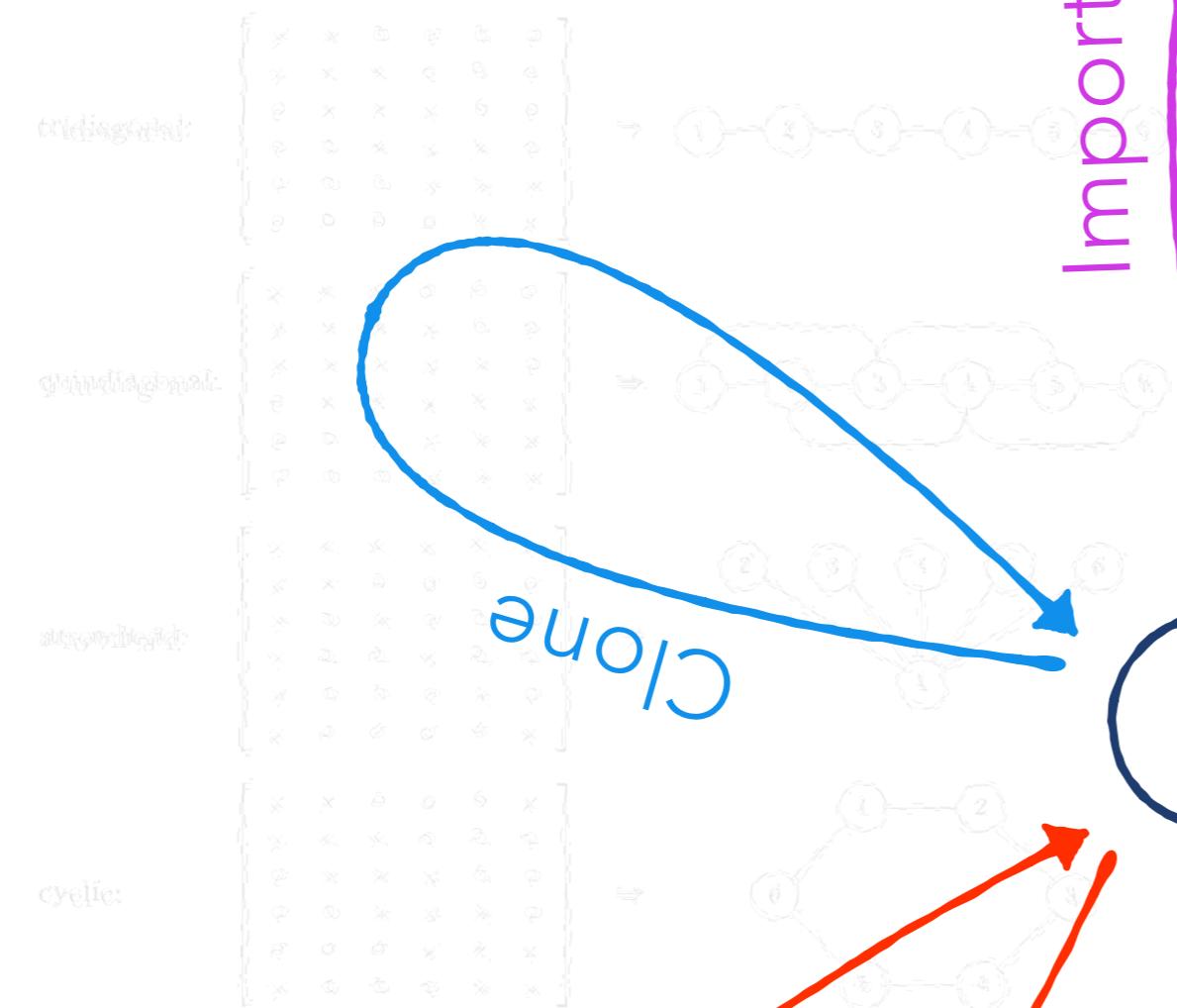
Create a tree \mathbb{T} and a necessary vertex $r \in \mathbb{V}$, the pair $\mathbb{T} - \{r\}$ is called a rooted tree, while r is said to be the root. Unlike in a ordinary graph, \mathbb{T} has its natural partial order, which can best be explained by an analogy with a family tree. The root r is the predecessor of all the vertices in $\mathbb{T} - \{r\}$ and these vertices are the successors of r . Moreover, r is the only vertex in \mathbb{T} that has no predecessor. We say that the root r is a leaf if it has no successor. We say that the rooted tree \mathbb{T} is monotonically ordered if each vertex is labelled with all its predecessors in other words we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to relabelling the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We will give three consecutive orderings of the same rooted tree.

Theorem 11.1. Let \mathbb{A} be a symmetric matrix whose graph \mathbb{G} is a tree. Choose a root $r \in \{1, 2, \dots, n\}$ and assume that the rows and columns of \mathbb{A} were been arranged so, that $\mathbb{T} = \{v_i, j\}$ is monotonically ordered. Then, that $\mathbb{A} = \mathbb{L}\mathbb{U}^T$ is a Cholesky factorization it is true that

$$l_{k,j} = \frac{a_{k,j}}{l_{j,j}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



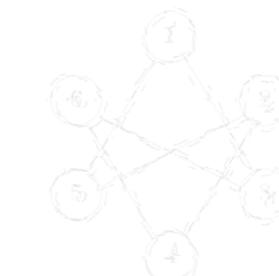
The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$

Delete

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,

Import



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just read the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 3, 3 \rightarrow 2, 4 \rightarrow 4, 5 \rightarrow 6, 6 \rightarrow 8.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_0, j_0), \dots, (i_{v-1}, j_{v-1})\}$ is called a path joining the vertices i_0 and i_{v-1} ($i_0, i_1, \dots, i_{v-1}, i_v \in V$, $j_0, j_1, \dots, j_{v-1} \in V$) and for any $k = 1, 2, \dots, v-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one vertex. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if no two members of V are joined by a single simple path. Both quadiagonal and pentadiagonal matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $p \geq 3$.

Create a tree T and a arbitrary vertex $r \in V$, the pair $T \sim (r, r)$ is called a rooted tree, while r is said to be the root. Unlike in a ordinary graph, T has a natural partial order which can best be explained by an analogy with a family tree. Thus, the vertex v is a predecessor of all the vertices in $T \setminus \{v\}$ and these vertices are successors of v . Moreover, every $a \in V \setminus \{r\}$ is joined to r by a simple path and we designate vertex along this path, except r and a , as a predecessor of a and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to relabelling the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We will give three consecutive drawings of the same rooted tree.



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a row $r \in \{1, 2, \dots, n\}$ and assume that the rows and columns of A are then arranged so, that $T = (r, r)$ is monotonically ordered. Then, that $A = LL^T$ is a Cholesky factorization it is true that

$$l_{k,j} = \frac{a_{k,j}}{l_{j,j}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

Page
In

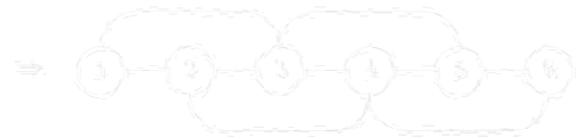
quidiagonal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 & 0 \\ 0 & * & * & 0 & 0 & 0 \\ 0 & 0 & * & * & 0 & 0 \\ 0 & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



quidiagonal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ 0 & 0 & * & * & * & * \\ 0 & 0 & 0 & * & * & * \end{bmatrix}$$

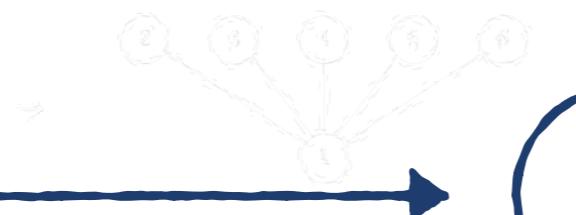


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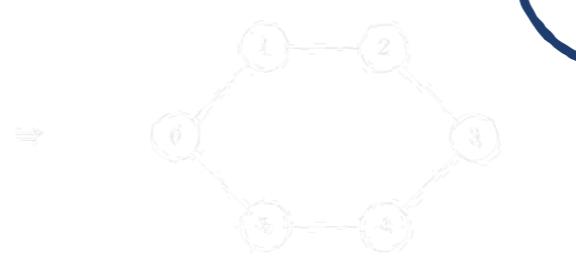
triangular:

$$\begin{bmatrix} * & * & * & * & * & * \\ * & * & 0 & 0 & 0 & 0 \\ * & 0 & * & 0 & 0 & 0 \\ 0 & * & * & 0 & 0 & 0 \\ 0 & 0 & * & * & 0 & 0 \\ 0 & 0 & 0 & * & * & 0 \end{bmatrix}$$



cyclic:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & * \\ * & * & 0 & 0 & 0 & 0 \\ 0 & * & * & 0 & 0 & 0 \\ 0 & 0 & * & * & 0 & 0 \\ 0 & 0 & 0 & * & * & 0 \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & 0 \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & * \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & * \end{bmatrix}$$

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, \quad 2 \rightarrow 5, \quad 3 \rightarrow 2, \quad 4 \rightarrow 3, \quad 5 \rightarrow 6, \quad 6 \rightarrow 4.$$

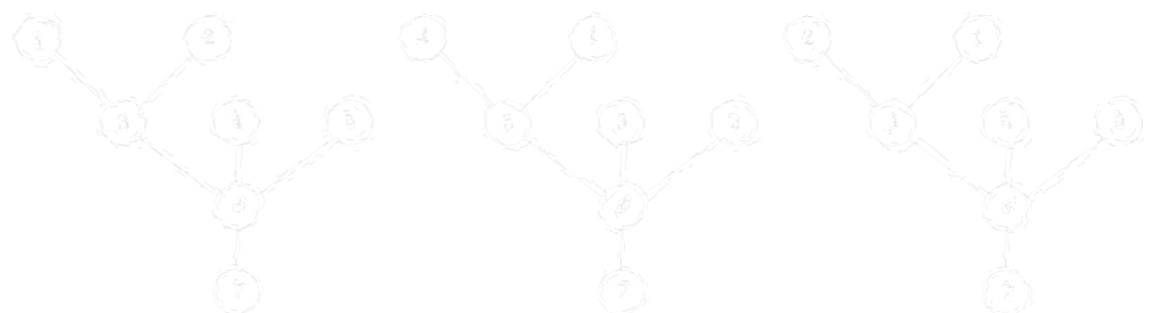
This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_0, j_0)\}_{i_0, j_0 \in \{1, \dots, n\}}$ is called a path joining the vertices i_0 and j_0 if $i_0 \in \{i_0, j_0\}$, $j_0 \in \{i_0, j_0\}$ and for every $k = 1, 2, \dots, n-1$ the set $\{i_{k+1}, j_{k+1}\} \cap \{i_k, j_k\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of V are joined by a unique simple path. Both tri-diagonal and any other matrices A are not trees, but this is not the case with either quidiagonal or cyclic matrices.

Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in ordinary graph, T admits a natural partial order, which can best be explained by an analogy with a family tree. The vertex r is the predecessor of all the vertices in $T \setminus \{r\}$ and these vertices are successors. Moreover, every $\alpha \in V \setminus \{r\}$ has a unique predecessor α' and an immediate vertex along this path, except for r and α , as a predecessor of α and a successor.

We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree.

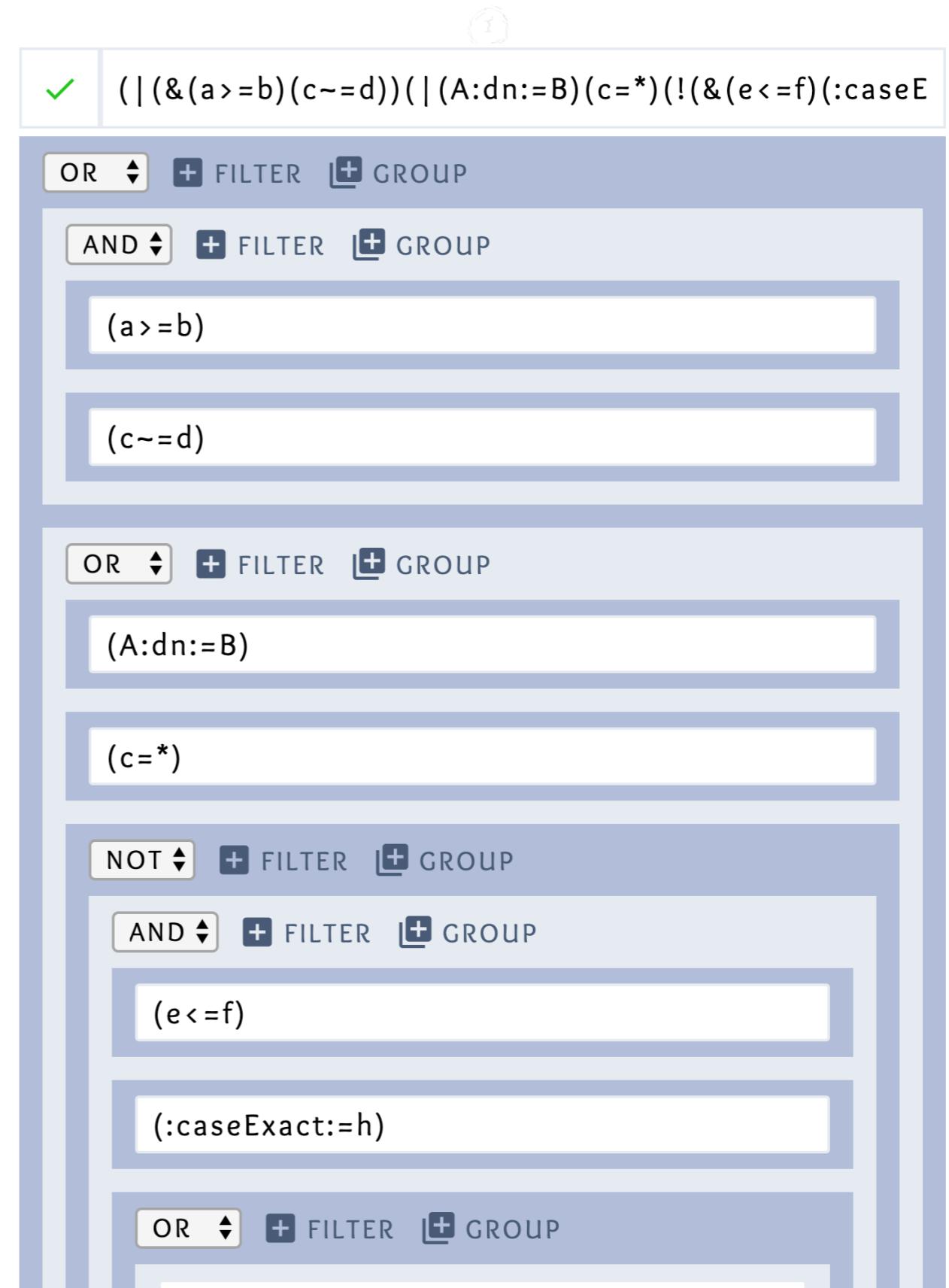


Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, n\}$ and assume that the rows and columns of A are then arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ki}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their analysis.

$(|(&(a>=b)(c\sim=d))$
 $(| (A:dn:=B)(c=^*)$
 $(!(&(e<=f)$
 $(:caseExact:=h)$
 $(| (i=j)(!(k<=l))))))))$



just displayed, but its graph.

$$t_{k,j} = \frac{a_{k,j}}{b_{k,j}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1, \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

(| (! (a>=b) (c~d)))
(| (A:dn := B) (c=*))
(!(&(e<=f))
(:caseExact:=h))
(| (i=j) (! (k<=l)))))))

! (| (! (a>=b) (c~d))) (| (A:dn := B) (c=*)) (!(&(e<=f)) (:caseE>))

OR FILTER GROUP

NOT FILTER GROUP

You can negate only 1 filter in a group.

(a>=b)

(c~d)

You need to use ~=.

OR FILTER GROUP

(A:dn := B)

You need to trim the attribute and filter type.

(c=*)

NOT FILTER GROUP

AND FILTER GROUP

(e<=f)

(:caseExact:=h)

just displayed, but its graph.

$$t_{k,j} = \frac{a_{k,j}}{b_{j,j}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1, \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity patterns) and their ranks.

(id>=123)

(email=*)

S I M P L E

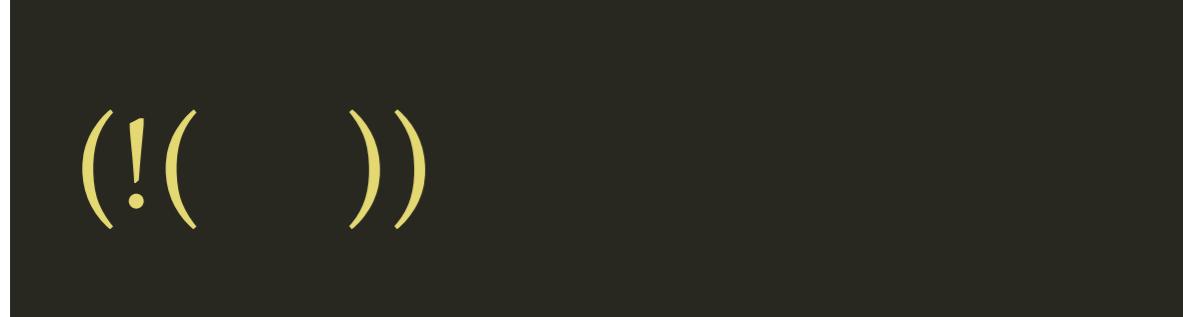
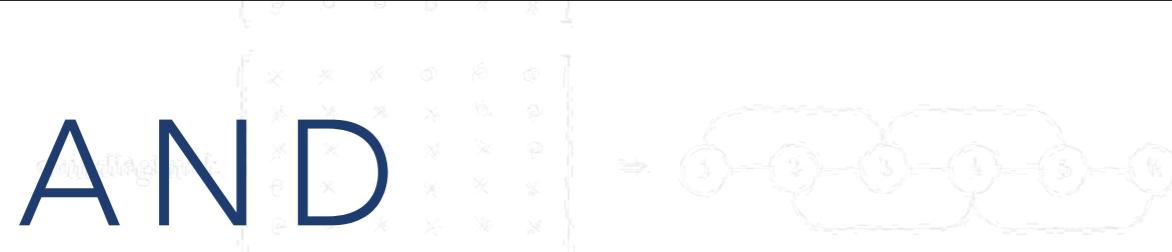
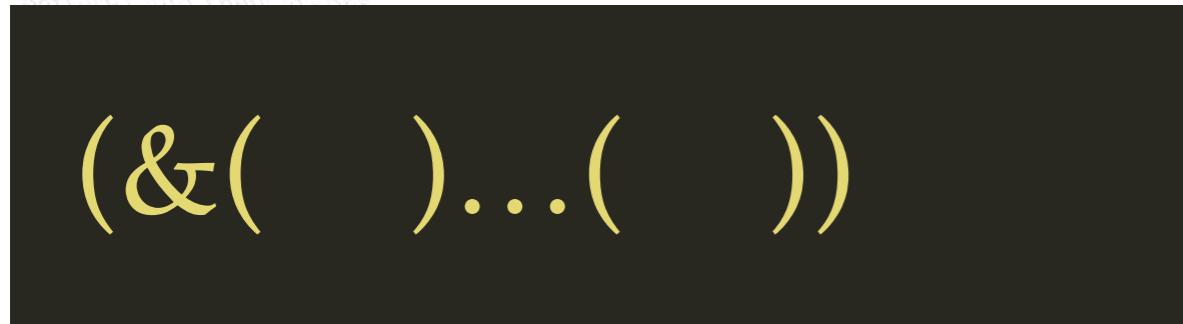
(cn=Jane*)

(:dn:1.2.3.4:=Jay)

SUBSTRING

EXTENSIBLE

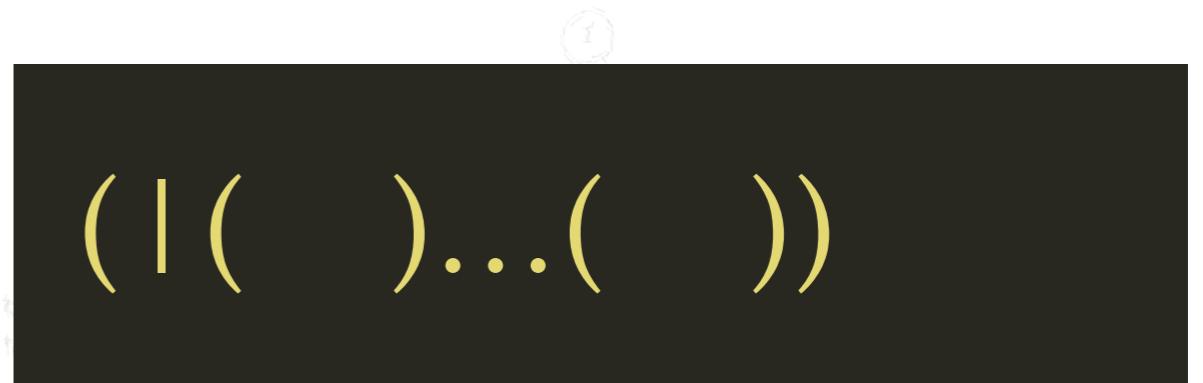
Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix



At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,



This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(v_i, j_i)\}_{i=1}^n$ in \mathbb{G} is called a path joining the vertices v_i and v_{i+1} ($i = 1, 2, \dots, n-1$), $j_i \in \{v_i, j_i\}$ and for every $k = 1, 2, \dots, n-1$ the set $\{v_i, j_i\} \cap \{v_{i+1}, j_{i+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that \mathbb{G} is a tree if each two members of \mathbb{V} are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $p \geq 3$.

Given a tree \mathbb{G} and an arbitrary vertex $r \in \mathbb{V}$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $\mathbb{V} \setminus \{r\}$ and these vertices are successors of r . Moreover, every $v \in \mathbb{V} \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and v , as a predecessor of v and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we label the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three consecutive orderings of the same rooted tree.



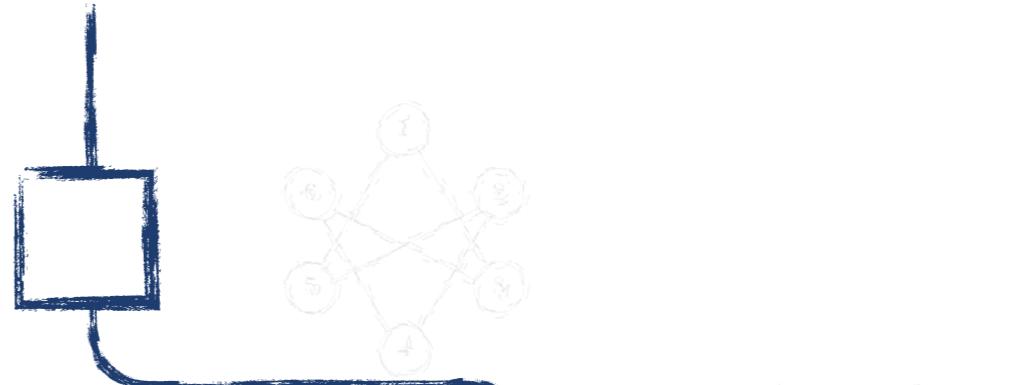
Theorem 11.1. Let A be a symmetric matrix whose graph \mathbb{G} is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so, that $T = (G, r)$ is monotonically ordered. Given that $A = L U$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{a_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

```
function addOne(x) {
  if (Number.isFinite(x)) {
    x = x + 1;
  } else {
    console.log('error');
  }
  return x;
}
```

just displayed, but its graph:



tells a different story – \mathbf{g} is not a tree. In a cyclic matrix in discussed. To see this, just re-label the vertices as follows:

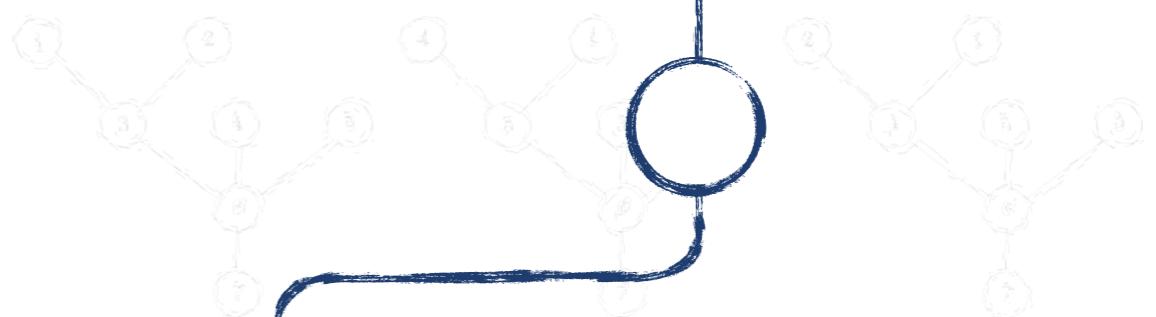
$$1 \rightarrow 1, \quad 2 \rightarrow 3, \quad 3 \rightarrow 2, \quad 4 \rightarrow 5, \quad 5 \rightarrow 6, \quad 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (swapping) the equations and variables.

An ordered set of edges $\{(i_1, j_1)\}_{i_1, j_1 \in \{1, \dots, n\}}$ is a path joining the vertices i_1 and j_1 if $i_1 \in \{1, \dots, n\}$, $j_1 \in \{1, \dots, n\}$ and for every $k = 1, 2, \dots, n-1$ the set $\{i_{k+1}, j_{k+1}\} \cap \{i_k, j_k\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that \mathbf{G} is a tree if each two members of \mathbf{V} are joined by a unique simple path. Both tridiagonal and banded matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $n > 3$.

Given a tree \mathbf{G} and an arbitrary vertex $r \in \mathbf{V}$, the path $T = (Q, \pi)$ is called a rooted tree, while r is said to be the root. Unlike in an ordinary tree, the root r is not necessarily unique, which can best be explained by an analogy with a family tree. The root r is the predecessor of all the vertices in $\mathbf{V} \setminus \{r\}$ and these vertices are successors of r . Moreover, every $a \in \mathbf{V} \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and a , as a predecessor of a and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors; in other words, a vertex a is to the right of r if it is further from the root to the left. (As we have already said, we are permuting the rows and the columns of the matrix, so the order of the columns in the matrix is not unique.)

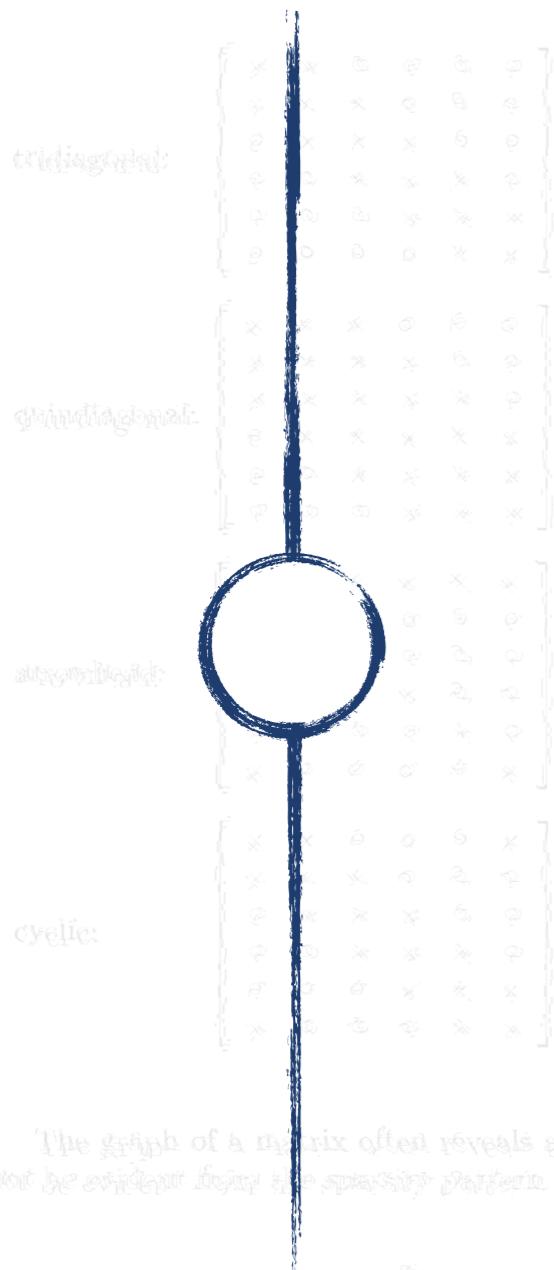
Every rooted tree will be monotonically ordered, but in general such an ordering is not unique. We now give three consecutive stages of the same rooted tree:



Theorem 2.1 Let \mathbf{A} be a symmetric matrix whose graph \mathbf{G} is a tree. Choose a root $r \in \{1, 2, \dots, n\}$. Assume that the rows and columns of \mathbf{A} have been arranged so that $T = (Q, \pi)$ is monotonically ordered. Then the Cholesky factorization of \mathbf{A} is

$$L_{k,j} = \frac{a_{k,j}}{a_{kk}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (1.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

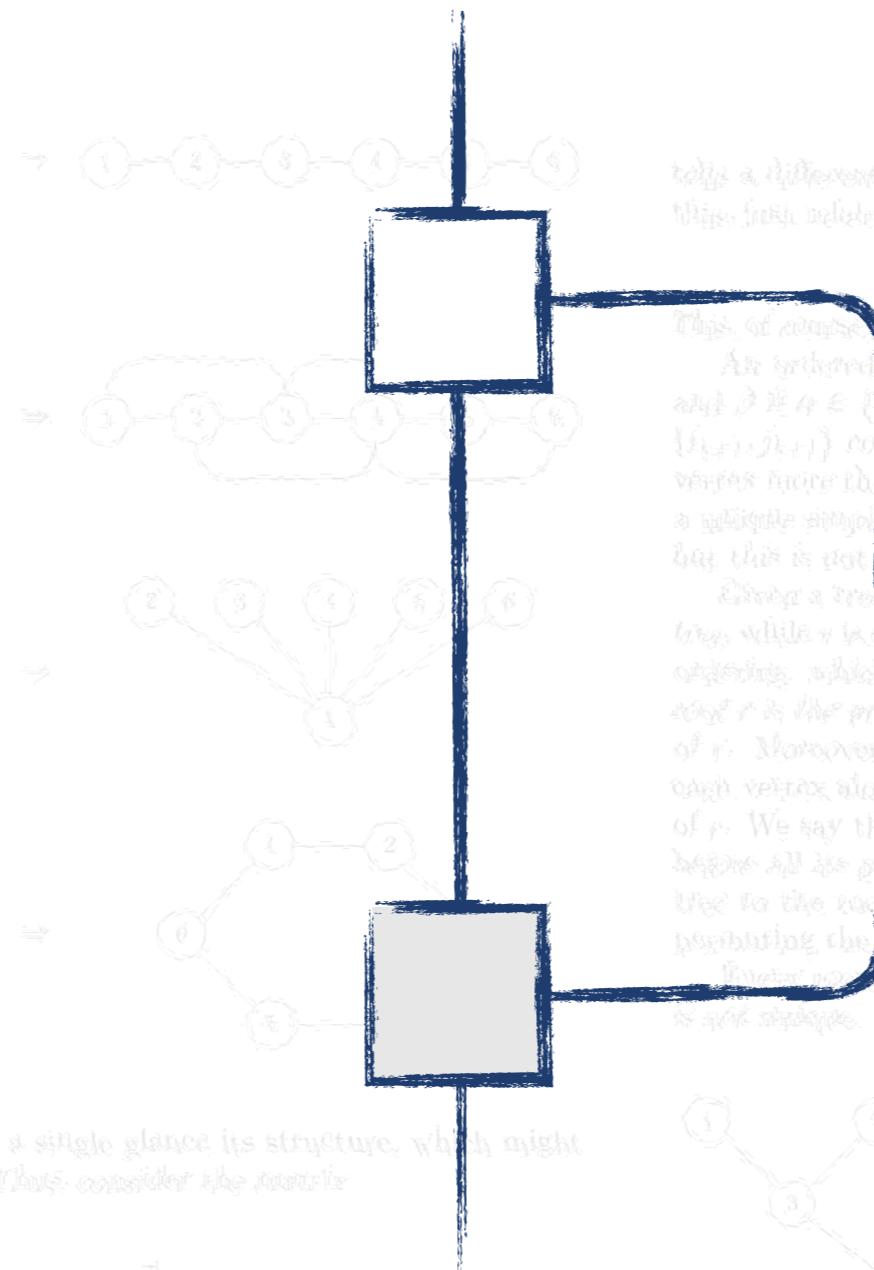


The graph of a matrix often reveals at a single glance its structure, which might not be evident from its sparsity pattern. Thus, consider the matrix

$$\text{LOG} \quad \begin{bmatrix} * & 0 & * & 0 & * & 0 \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & * \\ 0 & * & 0 & * & 0 & * \\ * & 0 & * & 0 & * & 0 \\ * & 0 & * & 0 & * & 0 \end{bmatrix}$$

| F

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, first recall that the vertices are

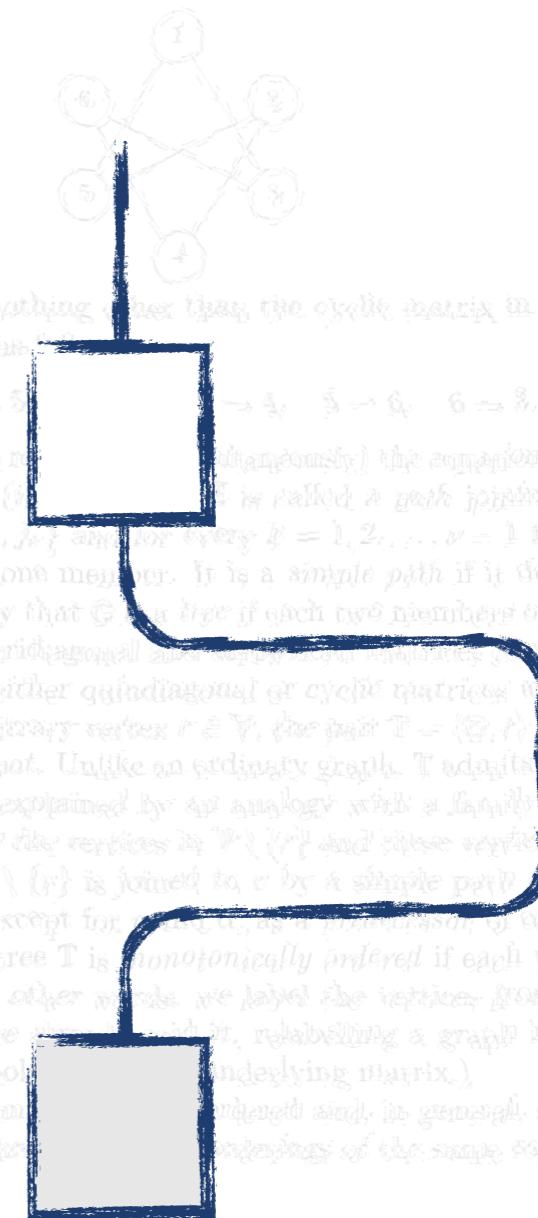
$$1 \rightarrow 1, \quad 2 \rightarrow 2, \quad \dots, \quad 5 \rightarrow 6, \quad 6 \rightarrow 1.$$

This, of course, is equivalent to renumbering (simultaneously) the equations and variables.

An ordered set of edges $\{(v_i, w_i)\}_{i=1}^n$ in $G = (V, E)$ is called a path joining the vertices v_1 and v_n if $i \in \{1, \dots, n\}$, $v_i \in V$, $w_i \in V$, and for every $k = 1, 2, \dots, n-1$ the set $\{v_k, w_k\} \cap \{v_{k+1}, w_{k+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of V are joined by a unique simple path. Both tridiagonal and pentadiagonal matrices correspond to trees, but this is not the case with either quadiagonal or cyclic matrices when $n \geq 3$.

Given a tree T and an arbitrary vertex $r \in V$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in a ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $T \setminus \{r\}$ and these vertices are successors of r . Moreover, every $v \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r itself, as a predecessor of v and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors in other words, we layed the vertices from the top of the tree to the root. (As we have mentioned it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Even a binary tree will be monotonically ordered and, in general, such an ordering is not unique. We will give the



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so that $T = (G, r)$ is monotonically ordered. Then, that $A = LL^T$ is a Cholesky factorization, it is true that

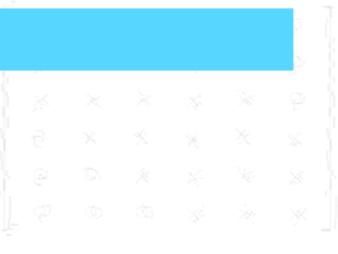
$$l_{k,j} = \frac{a_{kj}}{a_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

triangular:



quadratic:



symmetric:



cyclic:



The graph of a matrix often reveals at a single glance its structure, which may not be evident from the sparsity pattern. Thus, consider the matrix



At a first glance, there is nothing to think in the case just displayed, but its graph,

1

PICTURE

1000

WORDS



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

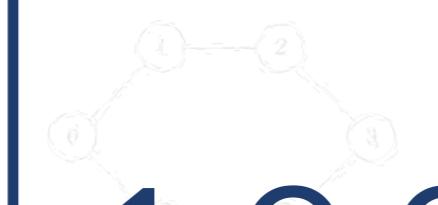
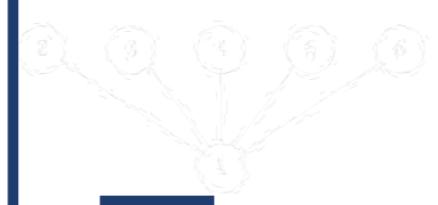
$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_1, j_1), (i_2, j_2), \dots, (i_v, j_v)\}$ in \mathbb{G} is called a *path* joining the vertices i_1, i_2, \dots, i_v (j_1, j_2, \dots, j_v) and, for every $k = 1, 2, \dots, v-1$ the set $\{(i_k, j_k), (i_{k+1}, j_{k+1})\}$ contains exactly one edge. It is a *simple path* if it does not visit any vertex more than once. We say that \mathbb{G} is a *tree* if each two members of \mathbb{V} are joined by a single simple path. Back to graphs, the quadiagonal matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $p \geq 3$.

Given a tree \mathbb{G} and an arbitrary vertex $r \in \mathbb{V}$, the pair $T = (G, r)$ is called a *rooted tree*, while r is said to be the *root*. Unlike in ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the *predecessor* of all the vertices in $\mathbb{V} \setminus \{r\}$ and these vertices are *successors* of r . Moreover, every $\alpha \in \mathbb{V} \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a *predecessor* of α and a *successor* of r . We say that the rooted tree T is *monotonically ordered* if each vertex is labelled before all its predecessors; in other words, we *layer* the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is unique. We now give three consecutive endings of the same rooted tree:



1 0 0 0

W O R D S

Theorem 1.1. Let A be a symmetric matrix whose graph \mathbb{G} is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (1.5)$$

```
// TODO: Write tests later

test('it renders', async function(assert) {
  await render(hbs`<ComplexComponent />`);
  assert.ok(true);
});
```

```
import { percySnapshot } from 'ember-percy';

...
// TODO: Write tests later

test('complex workflow', async function(assert) {
  await visit('/complex-page');
  await percySnapshot(assert);
});
```

Therefore we will now give a few examples of matrices (represented by their sparsity)

triangular:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



quadrangular:



supernodal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



cyclic:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



ZERO IN (a, b) .

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$

At a first glance this is not a matrix, but it is a little bit more than that. It is the matrix that was just displayed, but its graph:



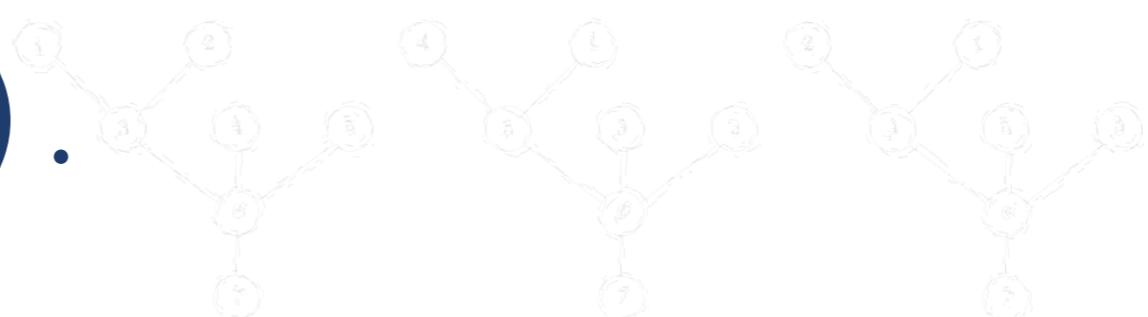
tells a different story - it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 2, 3 \rightarrow 3, 4 \rightarrow 4, 5 \rightarrow 5, 6 \rightarrow 6.$$

Of course, it is equivalent to relabeling the equations and variables. As you can see from the graph, G is a tree spanning the vertices a and b ($a, b \in \{1, 2, \dots, 6\}$, $a \neq b$) and for every $i = 1, 2, \dots, n-1$ the set $\{v_i, w_i\} \cap \{v_{i+1}, w_{i+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of V are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees, but this is not the case with either of banded or cyclic matrices when $p \geq 3$.

Of course, a rooted tree is a tree with a root vertex r . Unlike in a general tree, there is a natural partial order which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $V \setminus \{r\}$ and these vertices are successors of r . Moreover, every $v \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and v , as a predecessor of v and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled by an integer $r, r+1, \dots, n$ (in other words, we say the vertices from the top of the tree to the bottom, as we have already said). Labelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

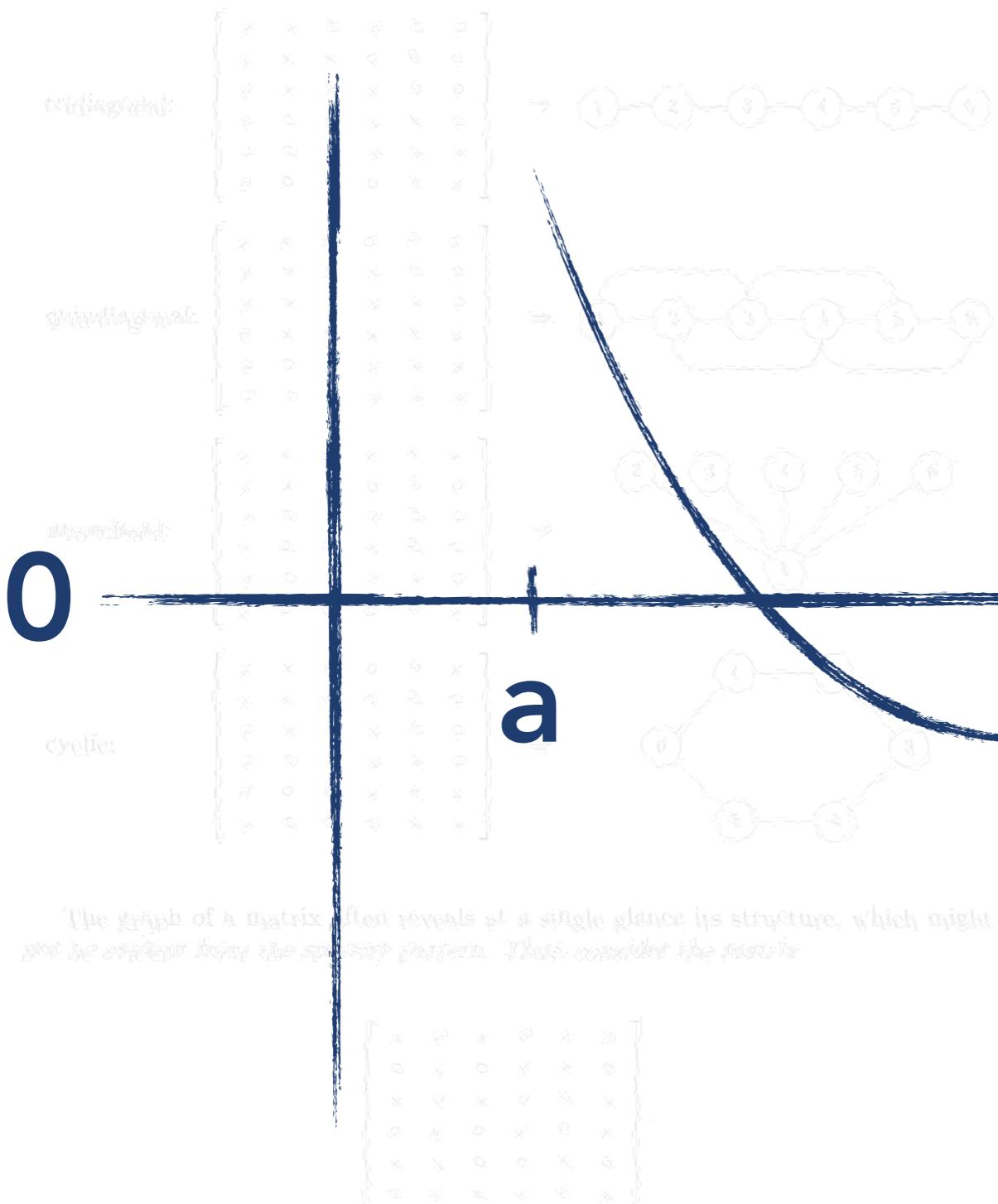
Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree:



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & * \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & 0 & * & 0 \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & * \end{bmatrix}.$$

At a first glance, there is nothing to think in the case of the four matrices that we have just displayed, but its graph,



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_k, j_k)\}_{k=1}^n$ in \mathbb{G} is called a path joining the vertices i_1 and j_n if $i_k \in \{1, \dots, n\}$, $j_k \in \{1, \dots, n\}$ and for every $k = 1, 2, \dots, n-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that \mathbb{G} is a tree if each two members of \mathbb{V} are joined by a unique simple path. Both tridiagonal and any other matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $n \geq 3$.

Given a tree \mathbb{T} and an arbitrary vertex $r \in \mathbb{V}$, the pair $\mathbb{T} = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in ordinary graph, \mathbb{T} admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $\mathbb{V} \setminus \{r\}$ and these vertices are successors of r . Moreover, any vertex $\alpha \in \mathbb{V} \setminus \{r\}$ is a child of r if r is an ancestor of α . We designate each vertex along this path, except for r and α , as a predecessor of α and a successor of r . We say that the rooted tree \mathbb{T} is monotonically ordered if each vertex is labelled before all its descendants. In other words, we layed the tree down the top at the tree's root. (As we have seen, it is said it, relabelling \mathbb{V} is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree:



Theorem 11.1. Let A be a symmetric matrix whose graph \mathbb{G} is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so that $\mathbb{T} = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity)

YOU CAN FIND EQUALLY MANY NUMBERS BETWEEN 0 AND 1 AS YOU CAN

The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

BETWEEN $-\infty$ AND ∞ .

At a first glance this is not a matrix, but it is a little known triangular matrix, just displayed, but its graph,



It is a different story – it is nothing other than the cyclic matrix in disguised. To see this, let us relabel the vertices as follows:

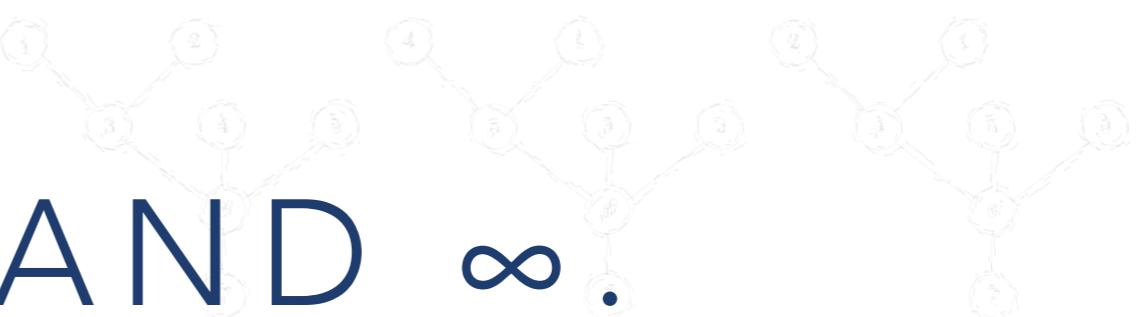
$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 1.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(u_i, v_i)\}_{i=1}^r$ in \mathbb{G} is called a path joining the vertices u_1 and u_r if $u_i \in \{u_1, u_2\}$, $v_i \in \{v_1, v_2\}$ and for every $i = 1, 2, \dots, r-1$ the set $\{u_i, v_i\} \cap \{u_{i+1}, v_{i+1}\} = \emptyset$. A path is simple if it does not visit any vertex more than once. We say that a path is closed if it starts and ends at the same vertex. A cycle is a simple path. Note that a path and a cycle may be joined by a single simple path. Both the original and simplified matrices correspond to trees, but this is not the case with either quidiagonal or cyclic matrices when $p \geq 3$.

Given a tree \mathbb{G} and an arbitrary vertex $r \in \mathbb{V}$, the pair $T = (G, r)$ is called a rooted tree, while r is said to be the root. Unlike in a ordinary graph, T admits a natural partial ordering, which can best be explained by an analogy with a family tree. Thus, the root is the ancestor of the vertices in $T \setminus \{r\}$ and these vertices are successors of r . Moreover, every $a \in \mathbb{V} \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and a , as a predecessor of a and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors; in other words, we label the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three concrete examples of the same rooted tree:



Theorem 11.1. Let A be a symmetric matrix whose graph \mathbb{G} is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{l_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

tridiagonal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix} \rightarrow \text{Graph: } 1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 6$$



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, first, re-label the weights as follows:

$$w_{12} = w_{23} = w_{34} = w_{45} = w_{56} = w_{61} = 1, \quad w_{13} = w_{24} = w_{35} = w_{46} = 2, \quad w_{14} = w_{25} = w_{36} = 3$$

$$\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & * & 0 \\ * & 0 & * & 0 & 0 & * \\ 0 & * & 0 & * & 0 & * \\ * & 0 & 0 & 0 & * & 0 \\ 0 & * & * & * & * & * \end{bmatrix}.$$



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so, that $\mathbb{T} = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i \sim j} a_{ki}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

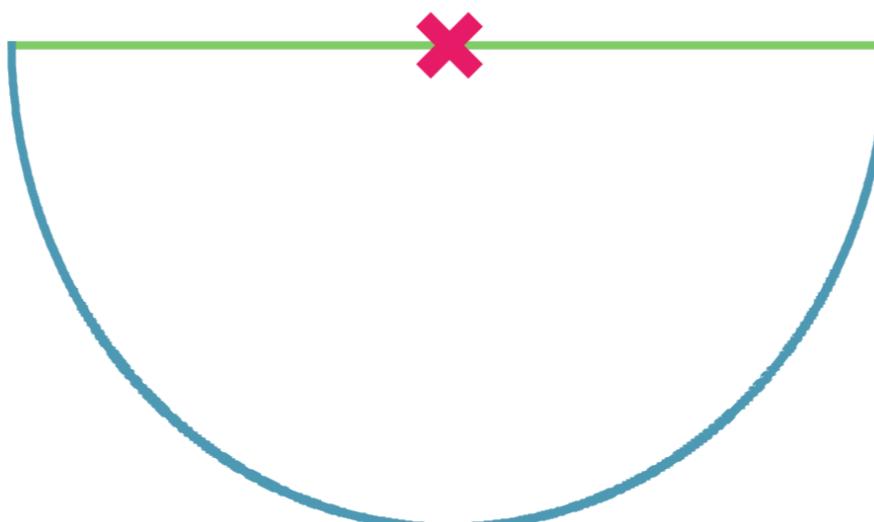
At a first glance, there is nothing to think in the case of the first matrices that we have just displayed, but its graph.

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

tridiagonal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix} \rightarrow \text{Graph: } 1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 6$$

tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, first, re-label the vertices as follows:



$$\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & * & 0 \\ * & 0 & * & 0 & 0 & * \\ 0 & * & 0 & * & 0 & * \\ * & 0 & * & 0 & 0 & * \\ 0 & * & * & * & * & * \end{bmatrix}$$

At a first glance, there is nothing to think in the case of the first matrices that we have just displayed, but its graph:



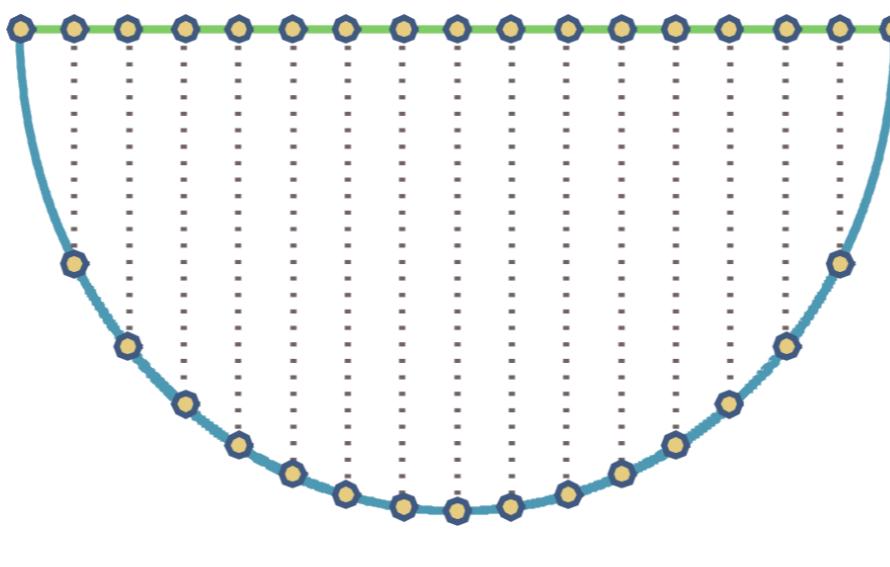
Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so, that $\mathbb{T} = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ii}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

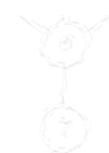
Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just re-label the weights as follows:



At a first glance, there is nothing to link in the case of the four matrices that we have just displayed, but its graph.



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so that $T = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

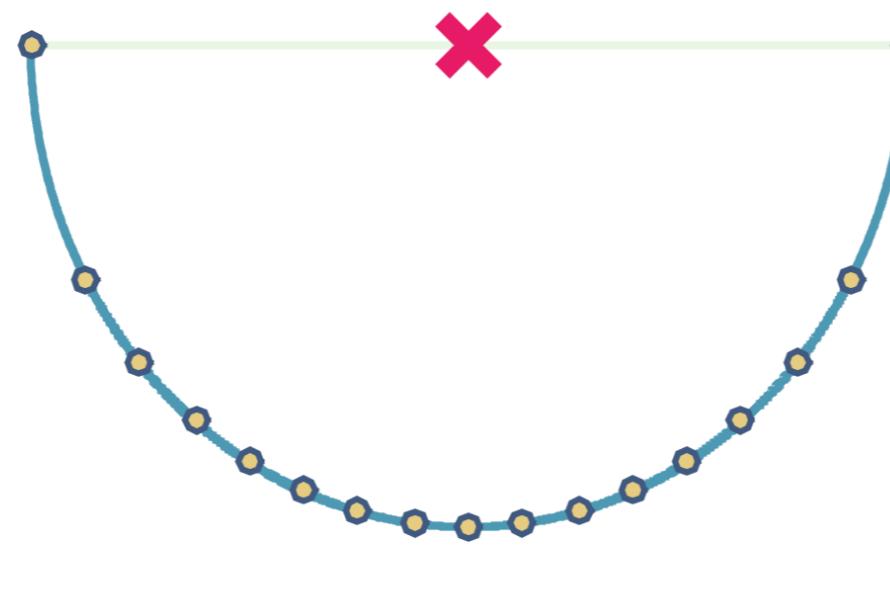
$$l_{k,j} = \frac{a_{kj}}{L_{jj}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

tridiagonal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix} \rightarrow \text{Graph: } 1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 6$$

tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, first recall the weights as follows:



$$\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & * & 0 \\ * & 0 & * & 0 & 0 & * \\ 0 & * & 0 & * & 0 & * \\ * & 0 & * & 0 & 0 & * \\ 0 & * & * & * & * & * \end{bmatrix}$$

At a first glance, there is nothing to think in the case of the four matrices that we have just displayed, but its graph:



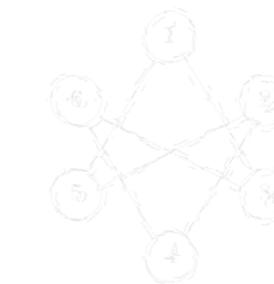
Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so, that $\mathbb{T} = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ii}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

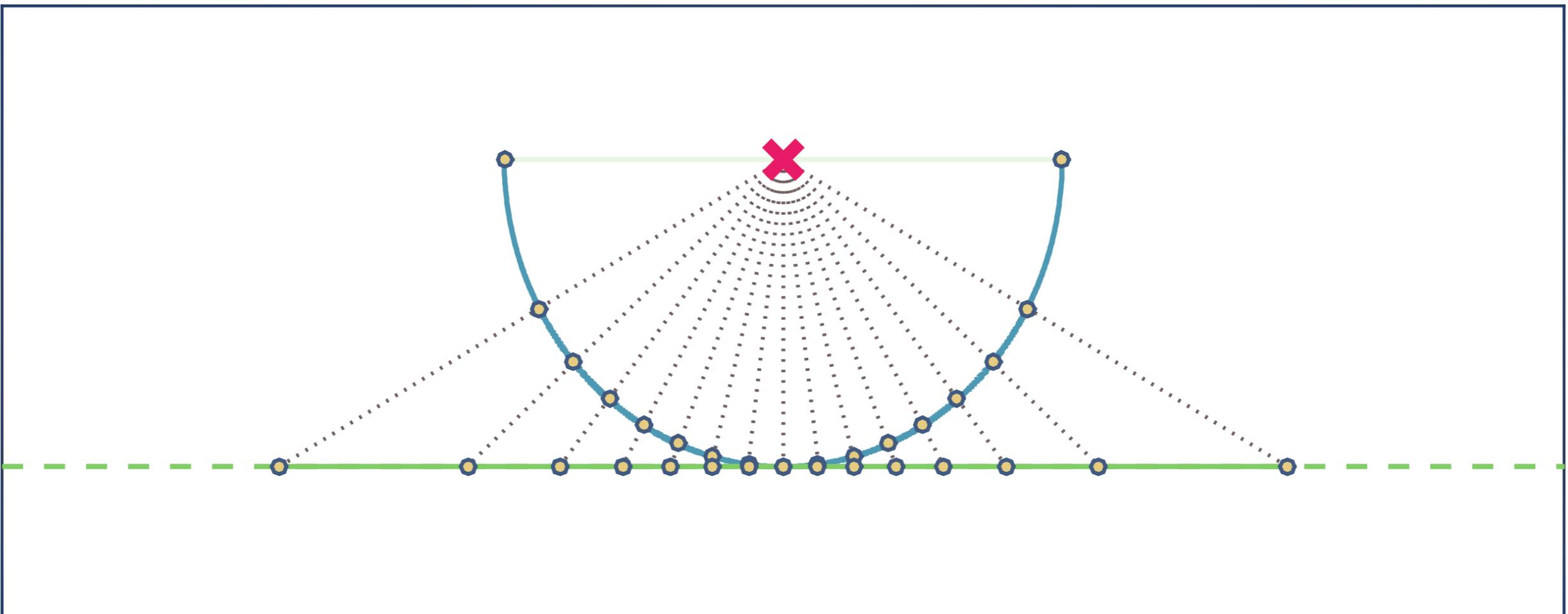
Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

tridiagonal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix} \rightarrow \text{Graph: } 1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 6$$



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just re-label the vertices as follows:



$$\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & 0 \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & 0 \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & * \end{bmatrix}.$$

At a first glance, there is nothing to think in the case of the four matrices that we have just displayed, but its graph:



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so, that $\mathbb{T} = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ii}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

tridiagonal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix} \rightarrow \text{Graph: } 1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 6$$



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just re-label the weights as follows:

1 2 3 4 5 6 7 8 9 10 11 12

$$\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & * & 0 \\ * & 0 & * & 0 & 0 & * \\ 0 & * & 0 & * & 0 & * \\ * & 0 & * & 0 & 0 & * \\ 0 & * & * & * & * & * \end{bmatrix}.$$

At a first glance, there is nothing to think in the case of the first matrices that we have just displayed, but its graph:



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so, that $\mathbb{T} = (G, r)$ is monotonically ordered. Given that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ii}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

5 Rules of Writing Tests

Filter:

Module: Acceptance | Outline



EmberFest 18.10.2019

Running:

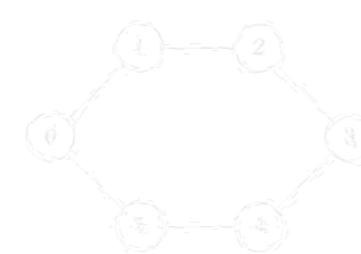
asymmetric:

$$\begin{bmatrix} 0 & 2 & 3 & 2 & 2 \\ 2 & 0 & 2 & 2 & 2 \\ 3 & 2 & 0 & 2 & 2 \\ 2 & 2 & 2 & 0 & 2 \\ 2 & 2 & 2 & 2 & 0 \end{bmatrix} \Rightarrow$$



cyclic:

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \Rightarrow$$



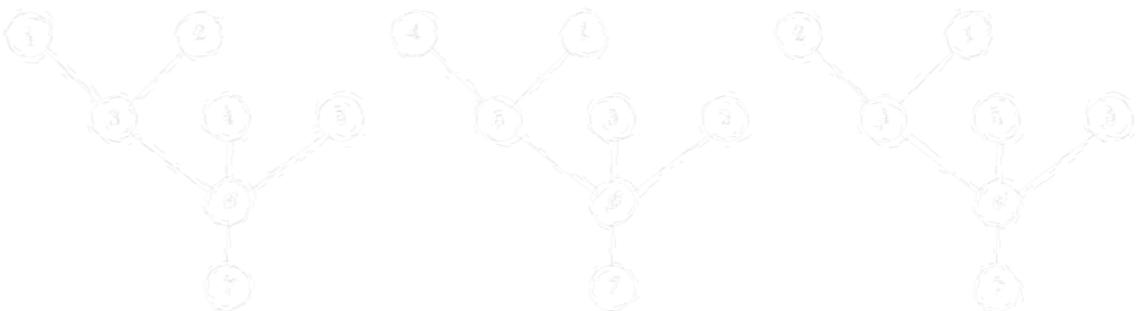
The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} 0 & 2 & * & 0 & * & 0 \\ 2 & 0 & 0 & * & x & 0 \\ * & 0 & 0 & 0 & 0 & * \\ 0 & * & 0 & 0 & 0 & 0 \\ * & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

At a first glance, there is nothing to think in the case of the four matrices that we have just displayed, but its graph,

which can best be explained by an analogy with a family tree. Thus, the root r is the predecessor of all the vertices in $T \setminus \{r\}$ and these vertices are successors of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a predecessor of α and a successor of r . We say that the rooted tree T is *monotonically ordered* if each vertex is labelled before all its predecessors; in other words, we label the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three consecutive orderings of the same rooted tree.



Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so that $T = (G, r)$ is monotonically ordered. Then, that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ki}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

5 Rules of Writing Tests

Filter:

Module: Acceptance | Outline



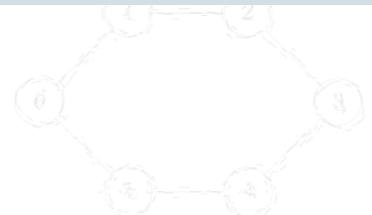
EmberFest 18.10.2019

1 assertion of 1 passed, 0 failed.

If, if, if

cyclic:

$$\begin{bmatrix} * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \end{bmatrix} \Rightarrow$$



before all its predecessors in other words: we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three monotone orderings of the same rooted tree:



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so, that $T = (G, r)$ is monotonically ordered. Then, that $A = L U$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ki}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

At a first glance, there is nothing to think in the case of the four matrices that we have just displayed, but its graph.

5 Rules of Writing Tests

Filter:

Module: Acceptance | Outline

EmberFest 18.10.2019

2 assertions of 2 passed, 0 failed.

If, if, if

Use common, everyday words

5 Rules of Writing Tests

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EmberFest 18.10.2019

3 assertions of 3 passed, 0 failed.

! If, if, if

! Use common, everyday words

! Write less with theorems and new terms

$$\begin{bmatrix} * & * & * & 0 & * & 0 \\ 0 & * & 0 & * & * & 0 \\ * & 0 & * & * & 0 & * \\ 0 & * & 0 & * & * & 0 \\ * & * & 0 & * & * & 0 \\ 0 & * & * & 0 & * & * \end{bmatrix}.$$

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph:



Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so, that $\mathbb{T} = (G, r)$ is monotonically ordered. Then, that $A = LL^T$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ki}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

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4 assertions of 4 passed, 0 failed.

! If, if, if

! Use common, everyday words

! Write less with theorems and new terms

! All your basis are belong to us

$$\begin{bmatrix} 0 & 3 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph:

Theorem 11.1 Let A be a symmetric matrix whose graph G is a tree. Choose a root $r \in \{1, 2, \dots, d\}$ and assume that the rows and columns of A are then arranged so, that $\tilde{T} = (G, r)$ is monotonically ordered. Then, that $A = \tilde{L}\tilde{U}$ is a Cholesky factorization, it is true that

$$t_{k,j} = \frac{a_{kj}}{\sum_{i=1}^j a_{ki}}, \quad k = j+1, j+2, \dots, d, \quad j = 1, 2, \dots, d-1. \quad (11.5)$$

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EmberFest 18.10.2019

5 assertions of 5 passed, 0 failed.

! If, if, if

! Use common, everyday words

! Write less with theorems and new terms

! All your basis are belong to us

! 1 picture = 1000 words

Therefore we will now give a few examples of matrices (represented by their sparsity pattern) and their graphs.

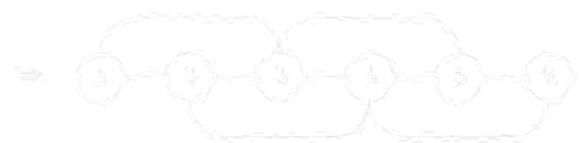
tridiagonal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ 0 & * & * & * & 0 & 0 \\ 0 & 0 & * & * & * & 0 \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & * & * \end{bmatrix}$$



qundiagonal:

$$\begin{bmatrix} * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ * & * & * & * & * & * \\ * & * & * & * & * & * \end{bmatrix}$$



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The graph of a matrix often reveals at a single glance its structure, which might not be evident from the sparsity pattern. Thus, consider the matrix

$$\begin{bmatrix} * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & 0 \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & * \\ * & 0 & * & 0 & * & 0 \\ 0 & * & 0 & * & 0 & * \end{bmatrix}.$$

At a first glance, there is nothing to link it to any of the four matrices that we have just displayed, but its graph,



tells a different story – it is nothing other than the cyclic matrix in disguised. To see this, just relabel the vertices as follows:

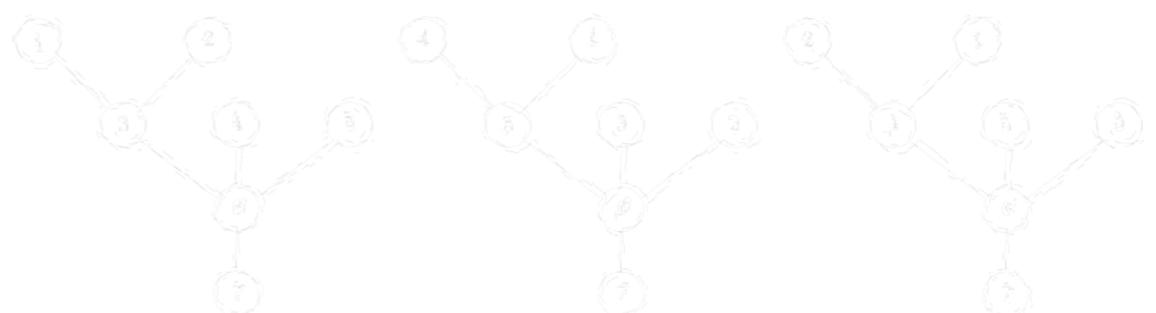
$$1 \rightarrow 1, 2 \rightarrow 5, 3 \rightarrow 2, 4 \rightarrow 3, 5 \rightarrow 6, 6 \rightarrow 4.$$

This, of course, is equivalent to reordering (simultaneously) the equations and variables.

An ordered set of edges $\{(i_0, j_0)\}_{i_0, j_0 \in \mathbb{N}_0}$ is called a path joining the vertices i_0 and j_0 if $i_0 \in \{i_0, j_0\}$, $j_0 \in \{i_0, j_0\}$ and for every $k = 1, 2, \dots, n-1$ the set $\{i_k, j_k\} \cap \{i_{k+1}, j_{k+1}\}$ contains exactly one member. It is a simple path if it does not visit any vertex more than once. We say that G is a tree if each two members of V are joined by a unique simple path. Both tridiagonal and qundiagonal matrices correspond to trees, but this is not the case with either qundiagonal or cyclic matrices when $n \geq 3$.

Given a tree T and an arbitrary vertex $r \in V$, the pair $T - (r, \cdot)$ is called a rooted tree, where the vertex r is the root. Unlike in ordinary graph, T admits a natural partial order which should be explained by an analogy with a family tree. Thus, the vertex r is the predecessor of all the vertices in $T \setminus \{r\}$ and these vertices are successors of r . Moreover, every $\alpha \in V \setminus \{r\}$ is joined to r by a simple path and we designate each vertex along this path, except for r and α , as a predecessor of α and a successor of r . We say that the rooted tree T is monotonically ordered if each vertex is labelled before all its predecessors in other words, we layed the vertices from the top of the tree to the root. (As we have already said it, relabelling a graph is tantamount to permuting the rows and the columns of the underlying matrix.)

Every rooted tree will be monotonically ordered and, in general, such an ordering is not unique. We now give three consecutive orderings of the same rooted tree.



Theorem 11.1. Let A be a symmetric matrix whose graph G is a tree. Consider a root $r \in \{1, 2, \dots, n\}$ and assume that the rows and columns of A are now arranged so, that $T = (G, r)$ is monotonically ordered. Given that $A = PLU$ is a Cholesky factorization, it is true that

$$l_{k,j} = \frac{a_{kj}}{p_{jj}}, \quad k = j+1, j+2, \dots, n, \quad j = 1, 2, \dots, n-1. \quad (11.5)$$

Q.E.D.