Exploration of Federal Funds Rate – Unemployment Rate Causal Relationship

I. Introduction.

The federal funds and the unemployment rate give snap shots of overall economic status. The federal funds rate is used to influence all other interest rates. Interest rates represent the cost of borrowed funds needed to allow continued business or expansion. The unemployment rate is an indicator of how successful people are at finding work. It provides a measure of economic hardship faced by people in the economy and the efficiency of the economy. A primary tool of the U.S. Federal Reserve Bank is manipulating the federal funds rate. Uncovering a possible connection between the federal funds rate and the level of unemployment would be useful in shaping economic policy decisions.

II. Literature Review.

Several authors have investigated a possible relationship between the federal funds rate and the unemployment rate. Bennani investigated how an expansionary monetary policy shock affects the unemployment rate of different racial groups in the US using data from 1969Q2 to 2015Q4. They found that the Black unemployment rate is most responsive to an expansionary monetary policy shock. The response associated with the Hispanic unemployment rate was uncertain. Dash used vector auto regressions (VAR) to analyze data between 1960 to 2019. Results showed that the impact of unconventional monetary policy transmits to the real economy through conventional interest rates, exchange rates and asset price channels. The responses of unemployment rate are smaller during QE1 and QE2 due to the rise in inflation uncertainty and economic policy uncertainty. Using the VAR model, Hsing finds that the federal funds rate responds positively to a shock to the output gap, the inflation gap, the long-term interest rate, or the lagged federal funds rate, and it reacts negatively to a shock to the unemployment rate gap or the exchange rate. Modeste used the cointegration technique to investigate the causal linkage between the federal funds rate and the unemployment rate between 1955 to 1999. Results suggested cointegration and a bi-directional causality between the federal funds rate and the unemployment rate.
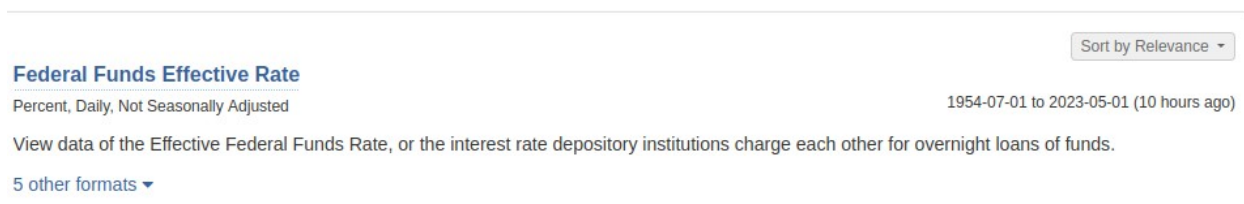
III. Economic/Financial Theory.

Before beginning the analysis, intuition suggests that the announcement of the federal funds rate should have an impact of the rate of unemployment. Workers, investors, and entrepreneurs would all be impacted. Workers might use knowledge of the funds rate to determine their level of consumption, level of investment, decision to keep current (possible non-ideal) employment if currently employed, or how aggressively to seek new employment if currently unemployed. Investor risk aversion may be impacted by the funds rate. Investors might use the federal funds rate knowledge to i) select or rank investment opportunities, ii) determine the duration of investment opportunities, and iii) determine the timing of investment opportunities. Since investor behaviors shape the opportunities of entrepreneurs, the federal funds rate announcement might determine if new companies are launched, impacting the unemployment rate. In turn, these worker, investor, and entrepreneur behaviors would be followed the Federal Reserve policy makers who set the federal funds rate. As such, intuition suggests a back and forth, circular causal relationship between the federal funds rate and the unemployment rate.

IV. Data.

Both the federal funds rate time series and the unemployment rate time series were obtained from the Federal Reserve Bank of Saint Louis Economic Research website, https://fred.stlouisfed.org/. For both monthly time series as percentage rates, the start date was July 1954 and the end date was March 2023. The federal funds rate was not seasonally adjusted. The unemployment rate was not seasonally adjusted.

To locate the federal funds rate time series on the Saint Louis Federal Reserve website, the term "Federal Funds Effective Rate (DFF)" was used with query results shown below in figure IV-1.
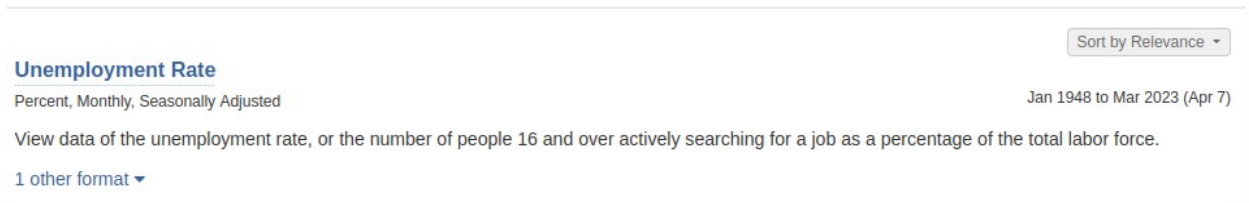

Figure IV-1. Desired federal funds rate time series query results.

A snap shot of the federal funds rate time series with needed date adjustments is shown below in figure IV-2.

Figure IV-2. Snap shot of the federal funds rate time series.

To locate the unemployment rate time series on the Saint Louis Federal Reserve website, the term "Unemployment Rate (UNRATE)" was used with query results shown below in figure IV-3.

Figure IV-3. Desired federal funds rate time series query results.

A snap shot of the unemployment rate time series with needed date adjustments is shown below in figure IV-4.

Figure IV-4. Snap shot of the unemployment rate time series.

Both time series files are submitted with this report as CSV files, "FEDFUNDS.csv" for the federal funds rate and "UNRATE.csv" for the unemployment rate.

Preliminary characterization of both times series CSV files was done using the R programming language and a free account on the website formerly known as "RStudio Cloud" but now renamed as "Posit Cloud", https://rstudio.cloud. The name change reflects inclusion of both the R and Python languages for data analysis in the cloud based platform. Figure IV-5 shows the code used to verify the two CSV times series files.

```
1   options(warn=-1)
2   install.packages("ggplot2", "lubridate", "scales", "gridExtra")
3   install.packages("ggthemes", dependencies = TRUE)
4   install.packages("uroot", "urca")
5   install.packages("tseries")
6   install.packages("lmtest")
7   install.packages("vars", "astsa", "rugarch")
8   install.packages("tidyr")
9   install.packages("tidyverse")
10  install.packages("forecast")
11  install.packages("TSstudio")
12  install.packages('bayesforecast')
13  install.packages('tseries')
14  library(tseries)
15  library(bayesforecast)
16  library(tidyverse)
17  library(forecast)
18  library(TSstudio)
19  library(tidyr)
20  library(tsDyn)
21  library(uroot)
22  library(ggplot2)
23  library(urca)
24  library(lmtest)
25  library(vars)
26  library(astsa)
27  library(rugarch)
28 ▾ ###############################################################################################
29  # Verify federal funds rate time series
30  FedFunds_data <- read.csv("FEDFUNDS.csv")
31  print(head(FedFunds_data))
32  print(tail(FedFunds_data))
33  # Verify unemployment rate time series
34  UnEmpRate_data <- read.csv("UNRATE.csv")
35  print(head(UnEmpRate_data))
36  print(tail(UnEmpRate_data))
37  # Plot two time series
38  ggplot(FedFunds_data, aes(x=  as.Date(FedFunds_data$DATE)  ) )+
39    geom_line(aes(y=FedFunds_data$FEDFUNDS, color="Federal Funds rate"), group=1) +
40    geom_line(aes(y=UnEmpRate_data$UNRATE, color="Unemployment Rate"), group=1) +
41    labs(color="Legend text") +
42    labs(x = "Year", y = "Percent", title = "Federal Funds Rate and Unemployment Rate") +
43    scale_color_manual(values=c("blue", "red")) +
44    scale_x_date(date_breaks = "10 year", date_labels = "%Y")
```

<u>Figure IV-5.</u> CSV verification code

Figure IV-6 shows the verified contents of the two CSV times series files.

```
> FedFunds_data <- read.csv("FEDFUNDS.csv")    > UnEmpRate_data <- read.csv("UNRATE.csv")
> print(head(FedFunds_data))                   > print(head(UnEmpRate_data))
        DATE FEDFUNDS                                   DATE UNRATE
1 1954-07-01     0.80                           1 1954-07-01    5.8
2 1954-08-01     1.22                           2 1954-08-01    6.0
3 1954-09-01     1.07                           3 1954-09-01    6.1
4 1954-10-01     0.85                           4 1954-10-01    5.7
5 1954-11-01     0.83                           5 1954-11-01    5.3
6 1954-12-01     1.28                           6 1954-12-01    5.0
> print(tail(FedFunds_data))                   > print(tail(UnEmpRate_data))
        DATE FEDFUNDS                                   DATE UNRATE
820 2022-10-01     3.08                         820 2022-10-01    3.7
821 2022-11-01     3.78                         821 2022-11-01    3.6
822 2022-12-01     4.10                         822 2022-12-01    3.5
823 2023-01-01     4.33                         823 2023-01-01    3.4
824 2023-02-01     4.57                         824 2023-02-01    3.6
825 2023-03-01     4.65                         825 2023-03-01    3.5
```

Figure IV-6. Verified contents of two CSV times series files

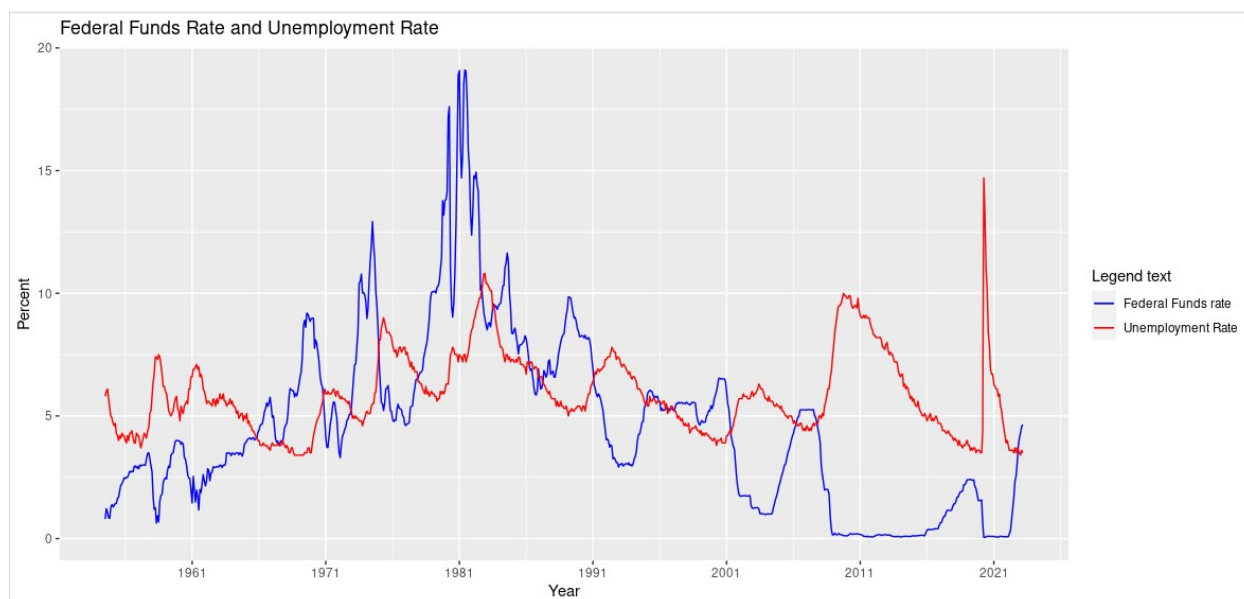Figure IV-7 shows the verified plots of the two CSV times series files.

Figure IV-7. Verified plots of two CSV times series files

Figure IV-7 reveals several basic characteristics of the two time series. The federal funds rate seems to show a upward trend from 1954 to about 1981 then a downward trend from 1981 to 2023. The unemployment rate time series may show a slight trend. Both time series seem to show non-constant variance. Neither time series shows easily noted seasonality. Both time series show abrupt changes and possible outliers around 2007 and 2020. These most likely reflect the 2007 to 2009 Great Recession and the COVID-19 pandemic of 2019 to 2020.

Figures IV-8 and IV-9 show the decomposed federal funds rate time series and unemployment rate time series, respectively.

```
46   # Decompose federal funds and unemployment time series
47   FedFunds_ts <- ts(FedFunds_data$FEDFUNDS, start = c(1954, 7), end = c(2023, 3), frequency = 12)
48   FedFunds_ts_dec <- decompose(FedFunds_ts)
49   plot(FedFunds_ts_dec, xlab = "")
50   title(main = "Federal Funds Rate", line = 1.25)
51   title(xlab = "Year", line = 2.5)
```
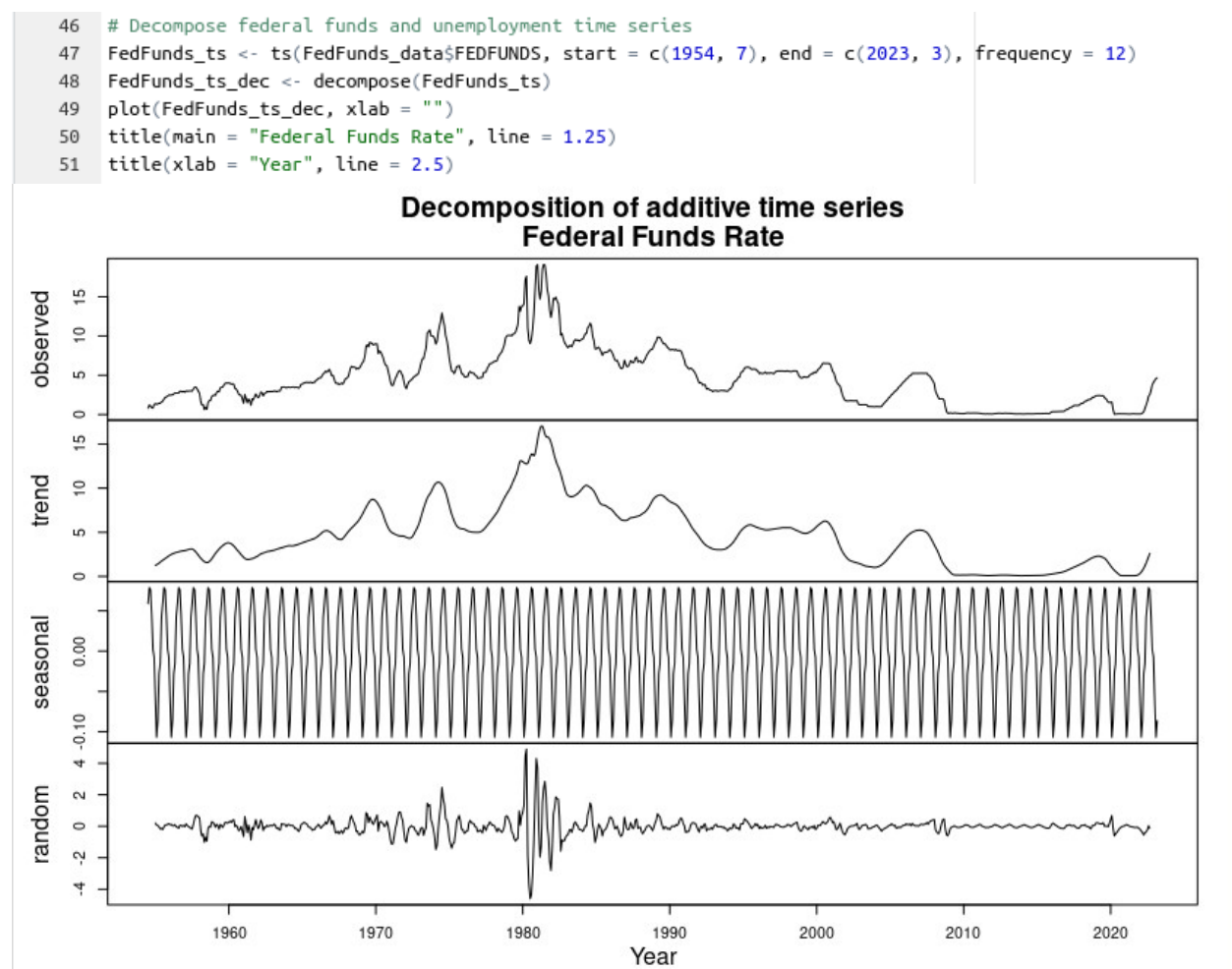

Figure IV-8. Decomposed federal funds rate time series and associated code.

```
53  UnEmpRate_ts <- ts(UnEmpRate_data$UNRATE, start = c(1954, 7), end = c(2023, 3), frequency = 12)
54  UnEmpRate_ts_dec <- decompose(UnEmpRate_ts)
55  plot(UnEmpRate_ts_dec, xlab = "")
56  title(main = "Unemployment Rate", line = 1.25)
57  title(xlab = "Year", line = 2.5)
```
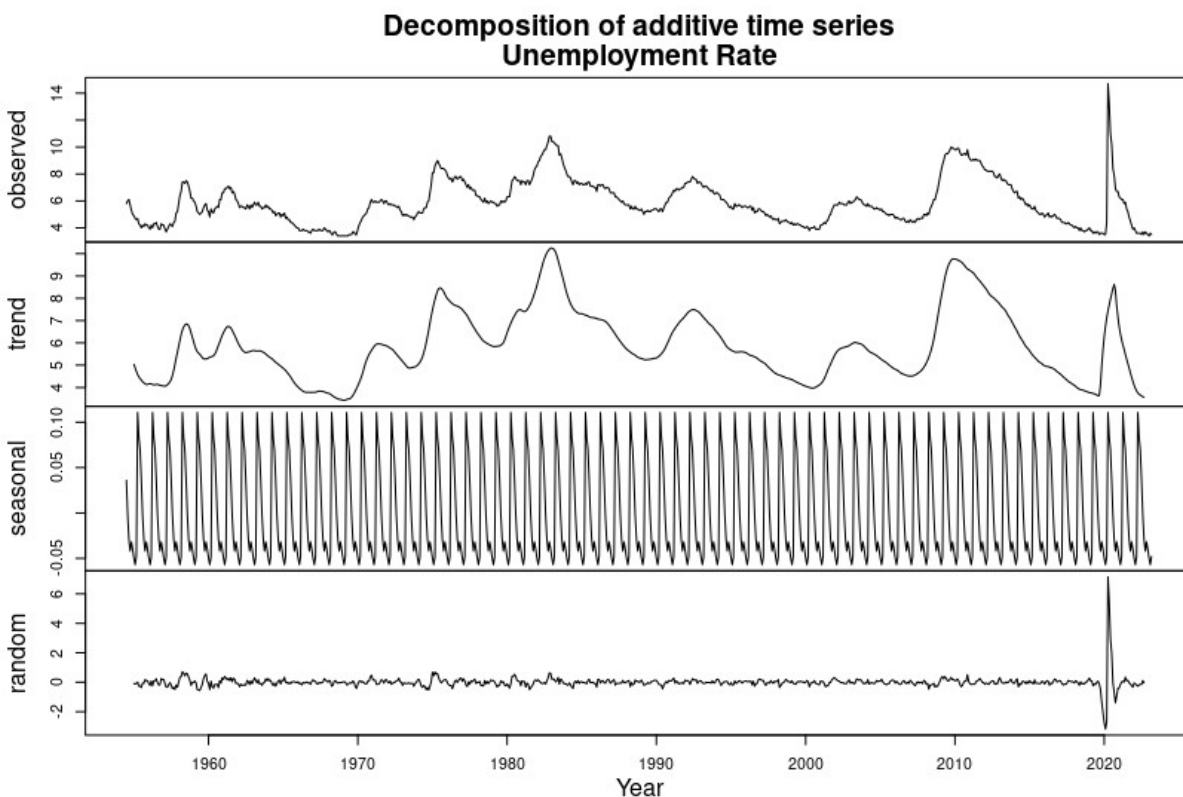

Decomposition of additive time series
Unemployment Rate

Figure IV-9. Decomposed unemployment rate time series and associated code.

Further insight into the two times series can be found using the summary() command.

```
59  # Time series summaries
60  summary(FedFunds_ts)
61  summary(UnEmpRate_ts)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  0.050   1.760   4.130   4.595   6.270  19.100
> summary(UnEmpRate_ts)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  3.400   4.700   5.600   5.874   6.900  14.700
```

Figure IV-10. Time series summaries and associated code.

Figures IV-8 and IV-9 further describe the two time series. The federal funds rate again seems to show a upward trend from 1954 to about 1981 then a downward trend from 1981 to 2023. This trend could stem from the 1970s Oil Crisis (random noise events in the early and mid-1970s) and federal funds rate changes in response to escalating inflation. The federal funds rate reached a peak (at about 19%) under Federal Reserve chair Paul Volcker who raised rates and kept them there to finally end 1970s era inflation. The unemployment rate time series seems to show slight up and down trends. Both time series shows seasonality. As described before, peaks in the random or noise component of the unemployment rate time series in 2020 reflects the the COVID-19 pandemic of 2019 to 2020 when the unemployment rate rose to about 14%. These plot interpretations are supported in the figure IV-10 summaries.

Both time series were checked for autocorrelation. Auto correlation is the correlation between a time series and the lagged version of itself. This could be the correlation between the current interval value of a time series and an earlier (or lagged) interval value of the same time series. Autocorrelation testing may reveal cyclic patterns in a time series useful for forecasting the time series. Auto-Correlation Function (ACF) plots and Partial Auto-Correlation (PACF) plots for the federal funds rate time series and unemployment rate time series are shown in figure IV-11 through figure IV-14. The "coredata()" command allowed for plotting of data with user selected x-axis values for better ACF and PACF lag visualizations.

```
63   # Autocorrelations
64   acfCoreFF_ts<-acf(coredata(FedFunds_ts) )
65   plot(acfCoreFF_ts, main="ACF for Federal Funds Rate")
66   axis(1, at = seq(0, 30, by=1), labels = 0:30 )
```
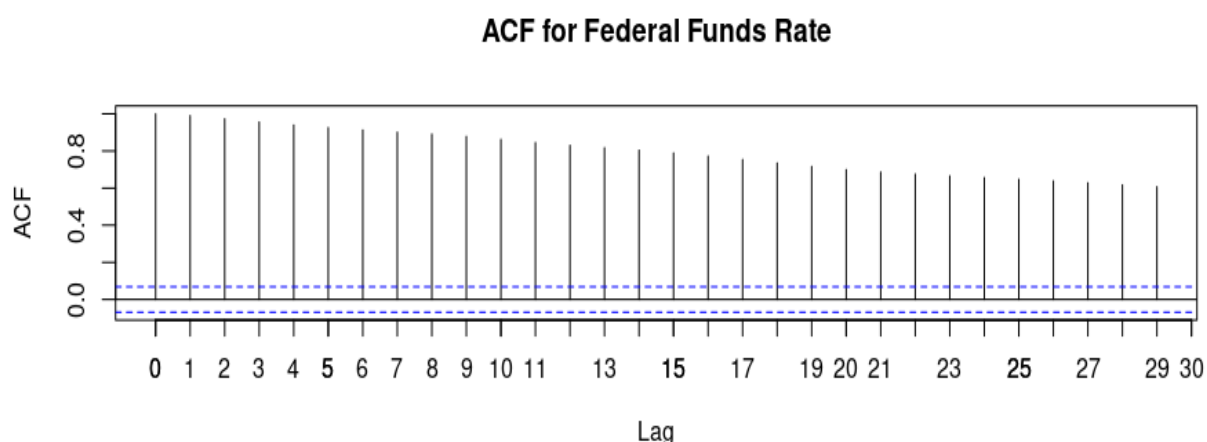

Figure IV-11. ACF for federal funds rate and associated code.

```
68  # Autocorrelations
69  acfCoreUNEMP_ts<-acf(coredata(UnEmpRate_ts) )
70  plot(acfCoreUNEMP_ts, main="ACF for Unemployment Rate")
71  axis(1, at = seq(0, 30, by=1), labels = 0:30 )
```
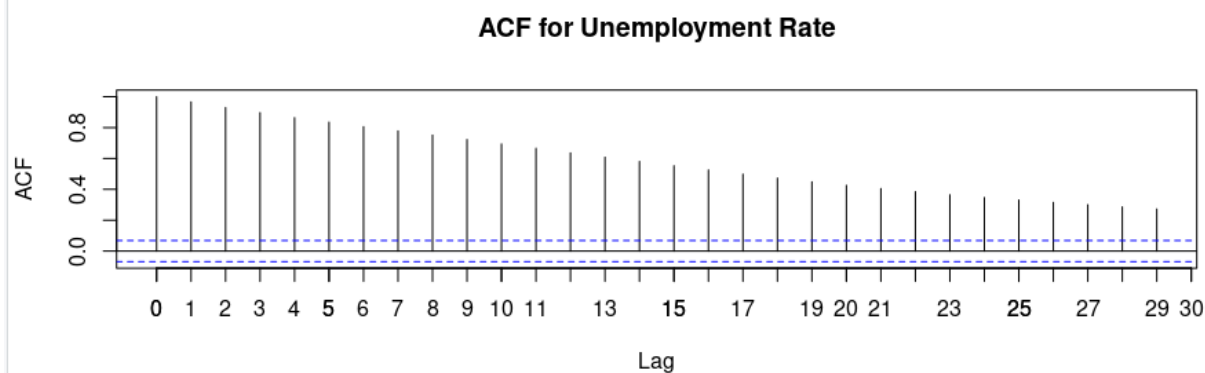
ACF for Unemployment Rate

Figure IV-12. ACF for unemployment rate and associated code.

```
73  # Autocorrelations
74  acfCoreFF_ts<-pacf(coredata(FedFunds_ts) )
75  plot(acfCoreFF_ts, main="PACF for Federal Funds Rate")
76  axis(1, at = seq(0, 30, by=1), labels = 0:30 )
```
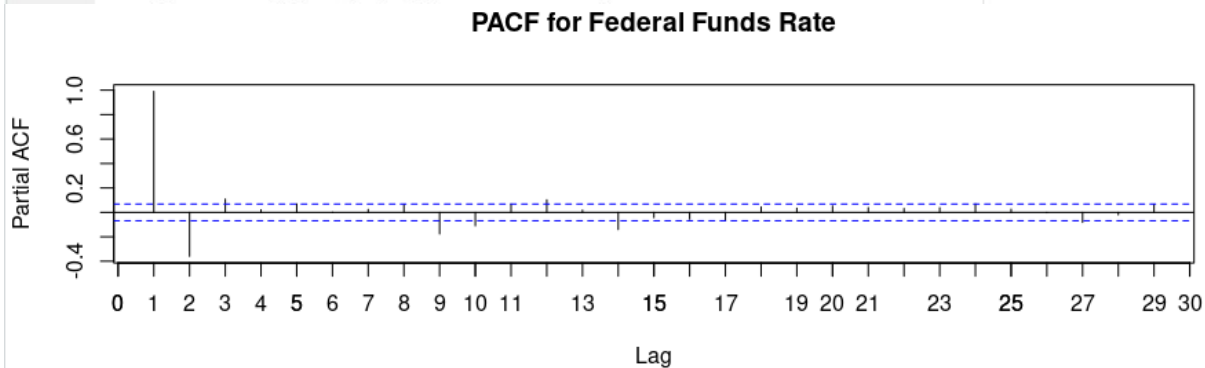
PACF for Federal Funds Rate

Figure IV-13. PACF for federal funds rate and associated code.

```
78  # Autocorrelations
79  acfCoreUNEMP_ts<-pacf(coredata(UnEmpRate_ts) )
80  plot(acfCoreUNEMP_ts, main="PACF for Unemployment Rate")
81  axis(1, at = seq(0, 30, by=1), labels = 0:30 )
```

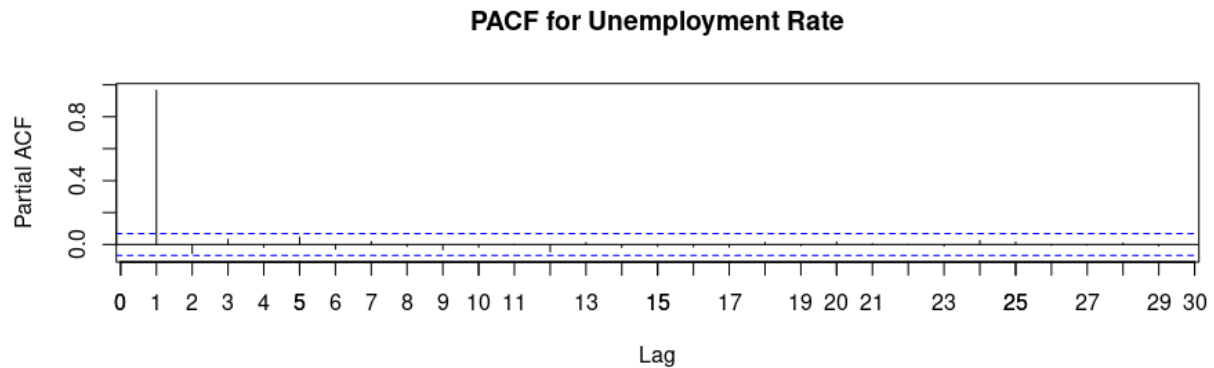**PACF for Unemployment Rate**

Figure IV-14. PACF for unemployment rate and associated code.

The ACF and PACF graphs can be used to determine the order of Auto-Regressive (AR), Moving average (MA), and ARMA models for possible forecasting. For the federal funds rate time series, figure IV-11 shows an ACF geometric decay and figure IV-13 shows PACF oscillation with strong correlations at lags 1 and 2. This suggests the federal funds time series can be modeled with an ARMA(1,1) model. For the unemployment rate time series, figure IV-12 shows an ACF geometric decay and figure IV-14 PACF shows a strong correlations at lag 1. This suggests the unemployment rate time series can be modeled with an AR(1) model.

For stationarity both time series should have constant means, constant variances (constant standard deviations), and no seasonality. Figures IV-7, -8, and -9 give visual clues that both the federal funds rate time series and unemployment time series are probably not stationary. These figures also suggest a drift and trend may be present in both time series. The next section will conduct more formal testing for stationarity using the Augmented Dickey Fuller (ADF), Phillips Perron (PP), and Kwiatkowski–Phillips–Schmidt–Shin (KPSS) tests.

Stationarity Checks for Federal Funds Rate Time Series:

For the ADF and PP tests, the null and alternative hypothesis must be defined and alpha will be set to 5% or 0.05:

Null Hypothesis: H0: The series has a unit root (value of a =1), the series is non-stationary.

Alternative Hypothesis: H1: The series has no unit root, the series is stationary.

For p-values above alpha=0.05, retain null hypothesis that series is non-stationary.

For p-values below alpha=0.05, reject null hypothesis, accept alternate hypothesis that time series is stationary.

```
83   # Augmented Dickey Fuller
84   # Test for fed funds rate time series stationarity
85   # alpha = 5% or 0.05 significance level
86   stationary.test(FedFunds_ts, method = "adf", nlag =2)

> # Augmented Dickey Fuller
> # Test for fed funds rate time series stationarity
> # alpha = 5% or 0.05 significance level
> stationary.test(FedFunds_ts, method = "adf", nlag =2)
Augmented Dickey-Fuller Test
alternative: stationary                          .

Type 1: no drift no trend
     lag    ADF p.value
[1,]   0 -1.08  0.2946
[2,]   1 -1.73  0.0834
Type 2: with drift no trend
     lag    ADF p.value
[1,]   0 -2.09  0.2914
[2,]   1 -3.01  0.0368
Type 3: with drift and trend
     lag   ADF p.value
[1,]   0 -2.41  0.4043
[2,]   1 -3.31  0.0689
----
Note: in fact, p.value = 0.01 means p.value <= 0.01
```

Interpretation: The p-values > 0.05. Therefore, federal funds rate time series is non-stationary

Figure IV-15. ADF test for federal funds rate time series and associated code.

```
88   # Phillips Perron Test
89   # Test for fed funds rate time series stationarity
90   # alpha = 5% or 0.05 significance level
91   stationary.test(FedFunds_ts, method = "pp", nlag =2)

> # Phillips Perron Test
> # Test for fed funds rate time series stationarity
> # alpha = 5% or 0.05 significance level
> stationary.test(FedFunds_ts, method = "pp", nlag =2)
Phillips-Perron Unit Root Test
alternative: stationary

Type 1: no drift no trend
 lag Z_rho p.value
  6 -3.98   0.242
-----
 Type 2: with drift no trend
 lag Z_rho p.value
  6 -11.9  0.0891
-----
 Type 3: with drift and trend
 lag Z_rho p.value
  6 -14.3   0.274
--------------
Note: p-value = 0.01 means p.value <= 0.01
```

Interpretation: The p-values > 0.05. Therefore, federal funds rate time series is non-stationary

Figure IV-16. PP test for federal funds rate time series and associated code.

For the KPSS test, the null and alternative hypothesis must be defined and alpha will be set to 5% or 0.05:

Null Hypothesis: H0: The series is stationary.

Alternative Hypothesis: H1: The series is non-stationary.

For p-values above alpha=0.05, retain null hypothesis that series is stationary.

For p-values below alpha=0.05, reject null hypothesis, accept alternate hypothesis that time series is non-stationary.

```
93  # Kwiatkowski-Phillips-Schmidt-Shin Test
94  # Test for fed funds rate time series stationarity
95  # alpha = 5% or 0.05 significance level
96  stationary.test(FedFunds_ts, method = "kpss", nlag =2)
> # Kwiatkowski-Phillips-Schmidt-Shin Test
> # Test for fed funds rate time series stationarity
> # alpha = 5% or 0.05 significance level
> stationary.test(FedFunds_ts, method = "kpss", nlag =2)
KPSS Unit Root Test
alternative: nonstationary

Type 1: no drift no trend
 lag  stat p.value
  6 0.469     0.1
-----
 Type 2: with drift no trend
 lag stat p.value
  6 0.17     0.1
-----
 Type 1: with drift and trend
 lag    stat p.value
  6 0.0518     0.1
-----------
Note: p.value = 0.01 means p.value <= 0.01
    : p.value = 0.10 means p.value >= 0.10
```

Interpretation: All p-values > 0.05. Therefore, federal funds rate time series is stationary

Figure IV-17. KPSS test for federal funds rate time series and associated code.

Stationarity Checks for Unemployment Rate Time Series:

For the ADF and PP tests, the null and alternative hypothesis must be defined and alpha will be set to 5% or 0.05:

Null Hypothesis: H0: The series has a unit root (value of a =1), the series is non-stationary.

Alternative Hypothesis: H1: The series has no unit root, the series is stationary.

For p-values above alpha=0.05, retain null hypothesis that series is non-stationary.

For p-values below alpha=0.05, reject null hypothesis, accept alternate hypothesis that time series is stationary.

```
 98  # Augmented Dickey Fuller
 99  # Test for unemployment rate time series stationarity
100  # alpha = 5% or 0.05 significance level
101  stationary.test(UnEmpRate_ts, method = "adf", nlag =2)
```

```
> # Augmented Dickey Fuller
> # Test for unemployment rate time series stationarity
> # alpha = 5% or 0.05 significance level
> stationary.test(UnEmpRate_ts, method = "adf", nlag =2)
Augmented Dickey-Fuller Test
alternative: stationary

Type 1: no drift no trend
     lag    ADF p.value
[1,]   0  -1.14   0.272
[2,]   1  -1.19   0.253
Type 2: with drift no trend
     lag    ADF p.value
[1,]   0  -3.55    0.01
[2,]   1  -3.74    0.01
Type 3: with drift and trend
     lag    ADF p.value
[1,]   0  -3.53  0.0393
[2,]   1  -3.73  0.0222
----
Note: in fact, p.value = 0.01 means p.value <= 0.01
```

Interpretation: The p-values < 0.05. Therefore, unemployment rate time series is stationary

Figure IV-18. ADF test for unemployment rate time series and associated code.

```
103  # Phillips Perron Test
104  # Test for unemployment rate time series stationarity
105  # alpha = 5% or 0.05 significance level
106  stationary.test(UnEmpRate_ts, method = "pp", nlag =2)
```

```
> # Phillips Perron Test
> # Test for unemployment rate time series stationarity
> # alpha = 5% or 0.05 significance level
> stationary.test(UnEmpRate_ts, method = "pp", nlag =2)
Phillips-Perron Unit Root Test
alternative: stationary

Type 1: no drift no trend
 lag Z_rho p.value
   6 -2.16   0.393
-----
 Type 2: with drift no trend
 lag Z_rho p.value
   6 -26.6    0.01
-----
 Type 3: with drift and trend
 lag Z_rho p.value
   6 -26.5  0.0193
------------
Note: p-value = 0.01 means p.value <= 0.01
```

Interpretation: The p-values < 0.05. Therefore, unemployment rate time series is stationary

Figure IV-19. PP test for unemployment rate time series and associated code.

For the KPSS test, the null and alternative hypothesis must be defined and alpha will be set to 5% or 0.05:

Null Hypothesis: H0: The series is stationary.

Alternative Hypothesis: H1: The series is non-stationary.

For p-values above alpha=0.05, retain null hypothesis that series is stationary.

For p-values below alpha=0.05, reject null hypothesis, accept alternate hypothesis that time series is non-stationary.

```
108  # Kwiatkowski-Phillips-Schmidt-Shin Test
109  # Test for unemployment rate time series stationarity
110  # alpha = 5% or 0.05 significance level
111  stationary.test(UnEmpRate_ts, method = "kpss", nlag =2)

> # Kwiatkowski-Phillips-Schmidt-Shin Test
> # Test for unemployment rate time series stationarity
> # alpha = 5% or 0.05 significance level
> stationary.test(UnEmpRate_ts, method = "kpss", nlag =2)
KPSS Unit Root Test
alternative: nonstationary

Type 1: no drift no trend
 lag stat p.value
  6 0.46     0.1
-----
 Type 2: with drift no trend
 lag  stat p.value
  6 0.083     0.1
-----
 Type 1: with drift and trend
 lag   stat p.value
  6 0.0827     0.1
-----------
Note: p.value = 0.01 means p.value <= 0.01
   : p.value = 0.10 means p.value >= 0.10
```

Interpretation: The p-value > 0.05. Therefore, unemployment rate time series is stationary

Figure IV-20. KPSS test for unemployment rate time series and associated code.

TEST	TIME SERIES	
	FEDERAL FUNDS RATE	UNEMPLOYMENT RATE
AUGMENTED DICKEY FULLER	non-stationary (Figure IV-15)	stationary (Figure IV-18)
PHILLIPS PERRON	non-stationary (Figure IV-16)	stationary (Figure IV-19)
KPSS	stationary (Figure IV-17)	stationary (Figure IV-20)

Table IV-A. Stationarity Test Summary

Considered together (as shown in Table IV-A), the tests for stationarity (figures IV-15 through figures IV-20) suggest that the federal funds rate time series is non-stationary and the unemployment time series is stationary. However, figures IV-7 (repeated below) along with decomposition figures IV-8

and IV-9 suggest both time series are non-stationary. For the next steps in the analysis, to err on the side of caution, both series will be treated as non-stationary.
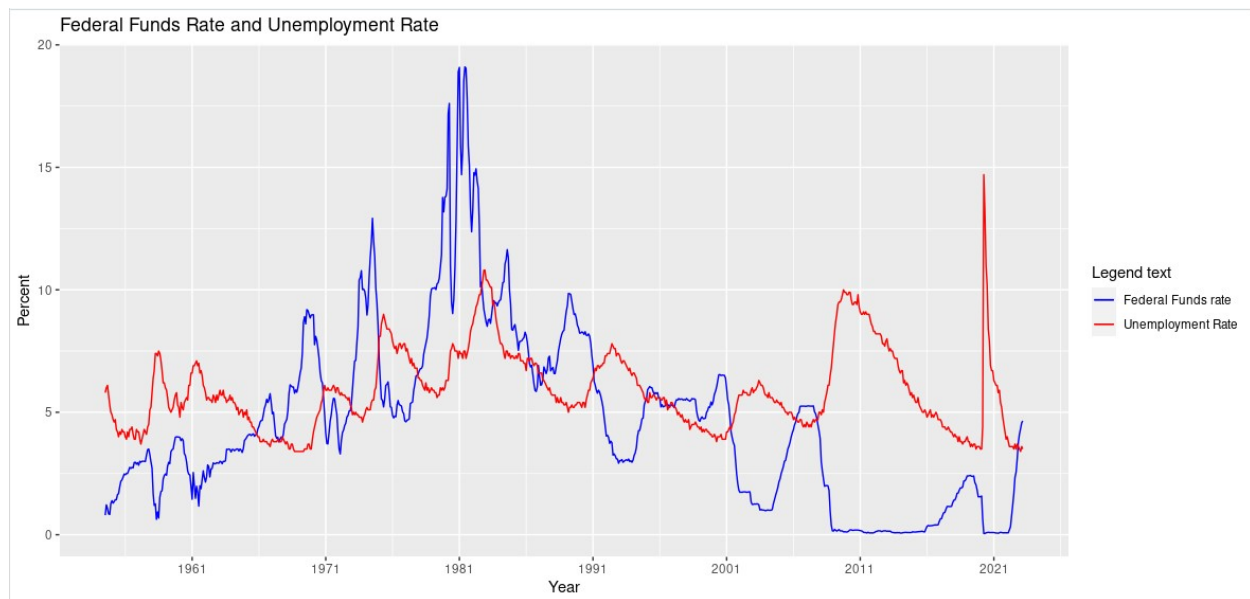

Figure IV-7 (repeated). Verified plots of two CSV times series files

V. The Model to be Estimated.

Based upon the exploratory data analysis completed in Section V, both time series seem to be non-stationary with trends and seasonality. Two differencing strategies were compared to find the best method to make both time series stationary. Single stage differencing using only lag = 2 was compared to two stage differencing using first lag = 2 then lag =12. Lag = 2 differencing was done to address noted trends while lag = 12 differencing was done to address noted seasonality. Both differencing strategies were compared using ACF.

```
113  # Difference fed funds rate time series
114  # Compare two differencing strategies:
115  # Single stage: Difference only with lag = 2 to remove trend
116  # Two stage: Difference first with lag = 2 to remove trend,
117  #    then difference with lag = 12 for monthly seasonality
118  FF_ts_d2<-diff(FedFunds_ts, difference=2 )
119  FF_ts_d12<-diff(FedFunds_ts, difference=12 )
120  FF_ts_d2d12<-diff(FF_ts_d2, difference=12 )
121
122  d2CoreFF_ts<-acf(coredata(FF_ts_d2), main="Lag = 2 Differenced Federal Funds Rate")
123  d12CoreFF_ts<-acf(coredata(FF_ts_d12), main="Lag = 12 Differenced Federal Funds Rate")
124  d2d12CoreFF_ts<-acf(coredata(FF_ts_d2d12), main="Lag = 2 then 12 Differenced Federal Funds Rate")
```
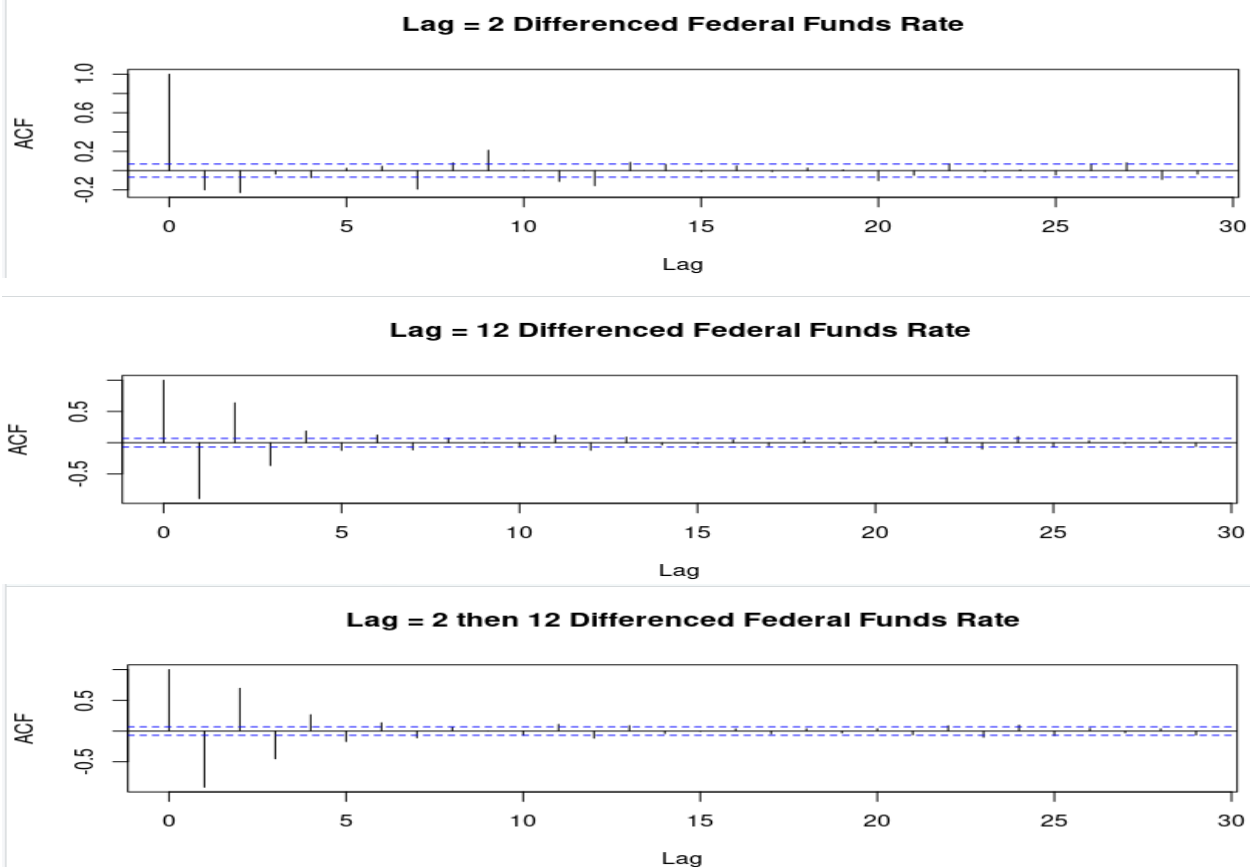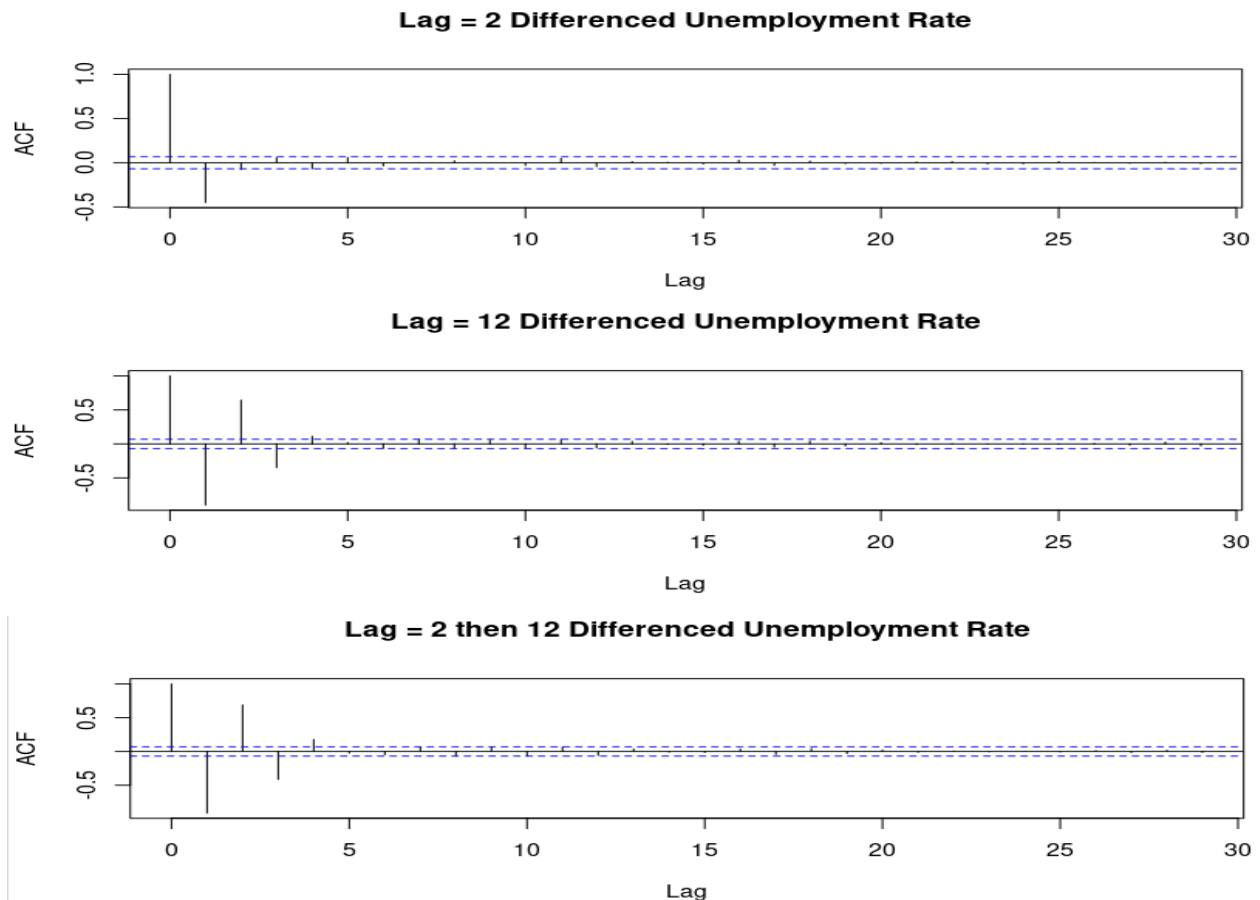

Figure V-1. Differencing strategy comparison for federal funds rate time series with code.

```
126  # Difference unemployment rate time series
127  # Compare two differencing strategies:
128  # Single stage: Difference only with lag = 2 to remove trend
129  # Two stage: Difference first with lag = 2 to remove trend,
130  #   then difference with lag = 12 for monthly seasonality
131  UMEMP_ts_d2<-diff(UnEmpRate_ts, difference=2 )
132  UMEMP_ts_d12<-diff(UnEmpRate_ts, difference=12 )
133  UMEMP_ts_d2d12<-diff(UMEMP_ts_d2, difference=12 )
134
135  d2CoreUMEMP_ts<-acf(coredata(UMEMP_ts_d2), main="Lag = 2 Differenced Unemployment Rate")
136  d12CoreUMEMP_ts<-acf(coredata(UMEMP_ts_d12), main="Lag = 12 Differenced Unemployment Rate")
137  d2d12CoreUMEMP_ts<-acf(coredata(UMEMP_ts_d2d12), main="Lag = 2 then 12 Differenced Unemployment Rate")
```


Figure V-2. Differencing strategy comparison for unemployment rate time series with code.

  Figures V-1 and V-2 show the best (though still not ideal) ACF differencing results with the single stage lag = 2 strategy. With both time series made stationary by lag =2 differencing, a cointegration analysis was performed to determine a possible relationship between the two time series. A conintegration analysis will determine if a linear combination of the two non-stationary time series under consideration could be stationary. First step in the cointegration analysis is the Johansen

Test to determine if a conintegration relationship exists between the two time series being evaluated.
In the Johansen Test, ca.jo(), K = 2 refers to the lag of 2.

```
139  # Cointegration Analysis
140  # System is composed of two time series:
141  #   1. federal funds rate
142  #   2. unemployment rate
143  # Bind two time series into single data frame
144  dset <-  cbind(FedFunds_ts, UnEmpRate_ts)
145  head(dset)
146  # Perform Johansen Testing to determine number of cointegrating relationships
147  #   present in the system
148  cointeg_test_rela <- ca.jo(dset, type = "trace", ecdet ="const", K=2)
149  summary(cointeg_test_rela)
```

```
######################
# Johansen-Procedure #
######################

Test type: trace statistic , without linear trend and constant in cointegration

Eigenvalues (lambda):
[1] 2.595023e-02 1.064292e-02 3.469447e-18

Values of teststatistic and critical values of test:

          test 10pct  5pct  1pct
r <= 1 |  8.81  7.52  9.24 12.97
r = 0  | 30.45 17.85 19.96 24.60

Eigenvectors, normalised to first column:
(These are the cointegration relations)

                FedFunds_ts.l2 UnEmpRate_ts.l2    constant
FedFunds_ts.l2        1.000000        1.000000   1.0000000
UnEmpRate_ts.l2      -3.920858        1.124706  -0.6883891
constant             18.178319      -11.379315  65.4004218

Weights W:
(This is the loading matrix)

                FedFunds_ts.l2 UnEmpRate_ts.l2        constant
FedFunds_ts.d     -0.001338116    -0.011069045 -3.711671e-19
UnEmpRate_ts.d     0.009507076    -0.000334385 -3.477786e-19
```

Figure V-3. Johansen test for number of conintegration relationships with code.

      For the Johansen Test, the null and alternative hypothesis must be defined and alpha will be set to 5% or 0.05:

Null Hypothesis:

H0: There are "r" cointegrating relationships between the tested time series

Alternative Hypothesis:

H1: There are more than "r" (or "r" +1) cointegrating relationships between the tested time series.

Relevant areas for interpreting the Johansen Test are highlighted by red boxes in figure V-3.

For the r = 0 line, the null hypothesis that there are r = 0 cointegrating relationships between the tested time series can be rejected because the test statistic value (30.45) is greater than the 5% significance value (19.96). The alternative hypothesis of r + 1 possible cointegrating relationships is accepted.

For the r <= 1 line, the null hypothesis that there are r <= 1 cointegrating relationships between the tested time series can be accepted because the test statistic value (8.81) is less than the 5% significance value (9.24). The alternative hypothesis of r + 1 cointegrating relationships is not accepted.

As a result, the overall interpretation of the Johansen Test is that there is a (r = 1) cointegrating relationships between the federal funds rate and unemployment rate time series.

VI. Empirical Results.

With the federal funds rate and unemployment rate time series explored and the Johansen Test suggesting a relationship, the nature of the relationship was explored.

```
139   # Cointegration Analysis
140   # System is composed of two time series:
141   #   1. federal funds rate
142   #   2. unemployment rate
143   # Bind two time series into single data frame
144   dset <-  cbind(FedFunds_ts, UnEmpRate_ts)
145   head(dset)
146   # Perform Johansen Testing to determine number of cointegrating relationships
147   #    present in the system
148   cointeg_test_rela <- ca.jo(dset, type = "trace", ecdet ="const", K=2)
149   summary(cointeg_test_rela)
150
151   # Vector Error Correction Model (VECM)
152   VECM_model <- VECM(dset, 2, r=1, estim=("2OLS"))
153   summary(VECM_model)
```
```
Full sample size: 825   End sample size: 822
Number of variables: 2  Number of estimated slope parameters 12
AIC -2750.755   BIC -2689.502   SSR 305.5393
Cointegrating vector (estimated by 2OLS):
    FedFunds_ts UnEmpRate_ts
r1          1   -0.7355545
```
```
                      ECT             Intercept       FedFunds_ts -1      UnEmpRate_ts -1
Equation FedFunds_ts  -0.0083(0.0042)* 0.0052(0.0156)  0.4400(0.0347)***  -0.0849(0.0370)*
Equation UnEmpRate_ts 0.0128(0.0040)** -0.0064(0.0147) -0.0979(0.0327)**  0.0287(0.0349)
                      FedFunds_ts -2    UnEmpRate_ts -2
Equation FedFunds_ts  -0.1502(0.0349)*** -0.0241(0.0371)
Equation UnEmpRate_ts -0.0084(0.0328)    -0.0673(0.0349).
```

Figure VI-1. Johansen test and Vector Error Correction Model with code.

The variables Fedfunds_ts (dependent) and UnEmpRate_ts (independent) are cointegrated. Relevant areas are highlighted by the red boxes. The topmost box shows the estimated model as

$$FedFunds\_ts = -0.7355545 * UnEmpRate\_ts.$$

The middle and lower red boxes show important lag variables. The 1 period lag independent variable FedFunds_ts is significant (***) and has a coefficient of 0.4400 with standard error of 0.0347. The 2 period lag independent variable FedFunds_ts is significant (***) and has a coefficient of - 0.1502 with standard error of 0.0349. The 1 period lag independent variable UnEmpRate_ts is significant (*) and has a coefficient of - 0.0849 with standard error of 0.0370. The full model with .L# representing number of lags (#) is

$$FedFunds\_ts = 0.0052 - 0.7355545 * UnEmpRate\_ts + 0.4400 * FedFunds\_ts.L2$$
$$- 0.1502 * FedFunds\_ts.L2 - 0.0849 * UnEmpRate.L1.$$

Each time series ability to impose a shock on the other time series was then explored.

```
155  # Impulse shock evaluations
156  # bind two time series into single data frame
157  dset <-  cbind(FedFunds_ts, UnEmpRate_ts)
158  # form vector autoregressive (VAR) model with p=2 lag order
159  FF_Unemp_VAR <- VAR(dset, p=2, type="both")
160  # calculate impulse responses (IRF)
161  IRF_FedFunds_on_Umenp <- irf(FF_Unemp_VAR, impulse = "FedFunds_ts",
162                      response="UnEmpRate_ts", n.head =200, ortho=FALSE)
163  plot(IRF_FedFunds_on_Umenp, ylab ="Umemployment Rate",
164      main="Response of Unemployment Rate to Shock from Federal Funds Rate")
```
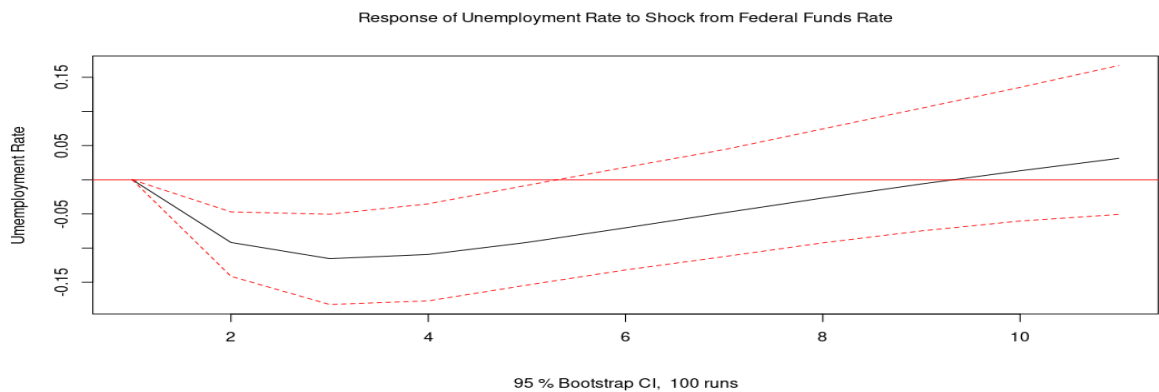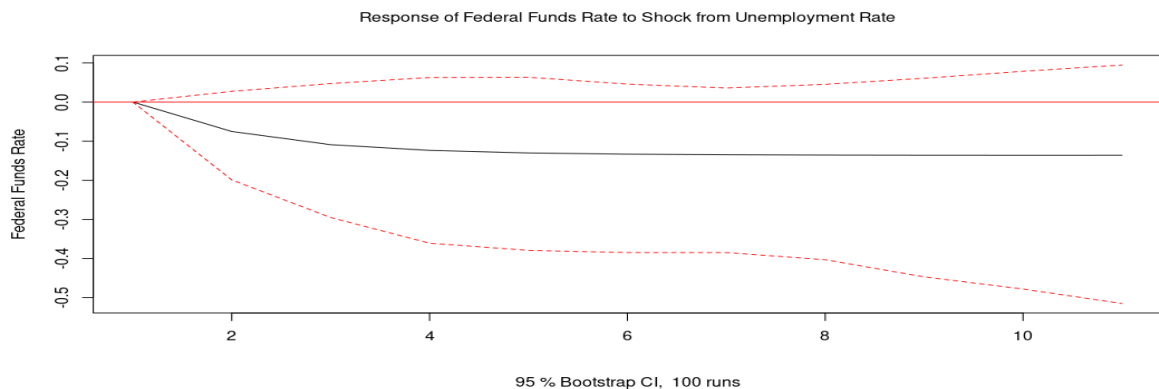

Figure VI-2. Impulse response of unemployment rate to federal funds rate shock with code.


```
165  IRF_Umenp_on_FedFunds <- irf(FF_Unemp_VAR, impulse = "UnEmpRate_ts",
166                      response="FedFunds_ts", n.head =200, ortho=FALSE)
167  plot(IRF_Umenp_on_FedFunds, ylab ="Federal Funds Rate",
168      main="Response of Federal Funds Rate to Shock from Unemployment Rate")
```

Figure VI-3. Impulse response of federal funds rate to unemployment rate shock with code.

Figure VI-2 shows the unemployment rate is sensitive to federal funds rate shocks with an approximately 10% dip requiring 10 months to recover. Figure VI-3 shows the federal funds rate is sensitive to unemployment shocks with a fall of about 10% but minimal recovery even after 10 months.

VII. Conclusion.

  Monthly Federal Funds Rate and Unemployment Rate time series from July 1954 to March 2023 were evaluated. Analysis found both time series non-stationary with trend and seasonality. The unemployment rate seemed to have trend stationary, deterministic trends with mean reverting to outside shocks. Johansen Testing revealed a cointegration relationship. Vector Error Correction Model analysis produced a possible relationship of

$$\text{FedFunds\_ts} = 0.0052 - 0.7355545 * \text{UnEmpRate\_ts} + 0.4400 * \text{FedFunds\_ts.L2}$$
$$- 0.1502 * \text{FedFunds\_ts.L2} - 0.0849 * \text{UnEmpRate.L1}.$$

In the relationship ".L#" represents a lagged version of the attached variable and "#" represents the degree of lag. Impulse analysis demonstrated that the both time series are sensitive to shocks from each other but only the unemployment rate showed signs of recovery after about 10 months.

Works Consulted

Bennani, H. (2023). Effect of monetary policy shocks on the racial unemployment rates in the   US. *Economic Systems*, *47*(1), 101058. https://doi.org/10.1016/j.ecosys.2022.101058

Dash, P. (2023). The effects of conventional and unconventional monetary policy on the unemployment rate in the US. *Journal of Economic Studies*. https://doi.org/10.1108/jes-10-2022-0555

How does the Federal Reserve Affect Inflation and employment?. Board of Governors of the Federal Reserve System. (n.d.). https://www.federalreserve.gov/faqs/money_12856.htm

Hsing, Y. (2005). Did U.S. monetary policy respond to exchange rates, long-term interest rates, and the unemployment rate gap? *The International Trade Journal*, *19*(1), 67–82. https://doi.org/10.1080/08853900590905766

Modeste, N.. (2011). Do changes in the federal funds rate cause changes in the unemployment   rate?. https://www.researchgate.net/publication/241886379_DO_CHANGES_IN_ THE_FEDERAL_FUNDS_RATE_CAUSE_CHANGES_IN_THE_UNEMPLOYMENT_RATE