Affective Agents in the Prisoner's Dilemma

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***Abstract*—The Prisoner's Dilemma (PD) is an abstract social game where two players each decide to 'cooperate' with or 'defect' from the other to earn a certain number of points. The Iterated Prisoner's Dilemma (IPD) is the PD repeatedly played between two players who retain memory of past events. Round-robin tournaments between various rational agents have been carried out to determine which strategies perform the best at the IPD, but none of these works have incorporated affective agents. This paper proposes a novel affective agent capable of playing the IPD in a manner that models emotion-driven human behavior and presents the results of such unprecedented tournaments to determine the most effective strategies against emotional human players. Overall, in an environment of diverse emotional agents, the most effective strategy is to be minimally aroused, minimally temperamental, and, most importantly, positively disposed.**

***Index Terms*—affective computing, Prisoner's Dilemma, Iterated Prisoner's Dilemma, cooperation, strategy**

I. Introduction

A. The Prisoner's Dilemma

The Prisoner's Dilemma (PD) [1] is a game that tests capacity for cooperation in the face of individual gain. Often presented in the form of a story involving two apprehended criminals choosing whether to testify against their partner to determine their prison sentences [2, p. 118], the PD may be more generally formulated as an abstract social dilemma where two players each must separately decide to 'cooperate' with or 'defect' from the other. The rewards and penalties associated with each of the four possible outcomes may be mapped onto arbitrary point values; while there is no consensus on exact assignments, the general structure is as follows:

- If one player cooperates and the other player defects, the defector receives a maximal reward (A points) and the cooperator receives a maximal penalty (B points).
- If both players cooperate, both players receive a modest reward (C points).
- If both players defect, both players receive a modest penalty (D points).

These point values must be assigned such that $A > C > D > B$ and $2C > A+B$. This formulation creates the "clash of 'individual rationality' ... and 'collective rationality' " [3, p. 245] central to the PD. With this structure, each player individually is invariably better off defecting (i.e., they obtain more points by defecting no matter what the other player decides), but both players defecting leads to the worst collective outcome (i.e., the smallest sum of points earned by both players) and the best collective outcome (i.e., the largest sum of points earned by both players) occurs when both cooperate.

A more interesting version of the game is the Iterated Prisoner's Dilemma (IPD) [4]. The IPD is an extension of the PD where the two players repeatedly play the PD with each other for multiple rounds while retaining memory of what has happened in previous rounds. This version of the game introduces further motivation to cooperate because the players know that they will interact again in the future [5]. Because the IPD models the emergence of cooperation in repeated interactions between (perhaps adversarial) entities, it may be likened to many real-world situations (e.g., what has been suggested to be the ultimate challenge to the field of affective computing: the negotiation of a peace treaty [6]). That situations as complex as geopolitical conflict can be represented by such simple models highlights the importance of obtaining a thorough understanding of the PD and IPD.

B. Axelrod's Tournaments

Robert Axelrod's 1980 tournaments [7], [8] are perhaps the most famous and influential investigations of the PD and IPD. In these works, dozens of artificial agents were compiled from various sources and matched against all others in a round-robin tournament format. The scores earned by each agent in every game were aggregated and used to rank the strategies used with the goal of determining which performed the best overall. In Axelrod's words, this made the work "a 'primer' on how to play the Prisoner's Dilemma game effectively" [7, p. 3]. The `TITFORTAT` rule, which begins by cooperating before simply copying the other player's previous decision for the rest of the game, won both tournaments [7, p. 7], [8, p. 382]. From this result and other observations, Axelrod concluded that it pays to be "nice" (i.e., cooperate at least as long as the other player does), "forgiving" (i.e., re-establish mutual cooperation after violations), and "provokable" (i.e., punish unwarranted defections) [8, p. 403].

Despite Axelrod's characterization of agent behavior using emotionally-charged terms, all participating agents were strictly rational. These agents simply followed a static set of concrete rules in a cold and calculated decision-making process. This is unsurprising; such rule-based systems reflect the conventional view of intelligence within computer science of "logic, rationality, and predictability" [9, p. 1].

All code is available at https://github.com/ijoffe/AffectivePrisonersDilemma. Generative AI was not used in any capacity for this project.

However, humans are inherently emotional beings [9] who make decisions largely based on their affective states [10], [11]. Furthermore, the PD and IPD are inherently emotional games; human players do not act purely rationally, but also based on their own emotions [12] (especially guilt [13] and anger [14]) and, in face-to-face versions of the game, their appraisals of other player's emotions [15].

Axelrod acknowledged that there is no single best strategy for the game; the most effective strategy depends "upon the nature of the other strategies with which it must interact" [7, p. 7]. Thus, these tournaments only revealed the best strategies for playing the IPD against precisely the agents in the tested sets – all of which were exclusively rational. To understand the best strategies for playing the IPD against humans, who are emotional beings, affective agents capable of playing the IPD while behaving according to models of human emotion must be examined. Such agents have been developed before [16], [17], but no Axelrod-style tournament has been carried out to analyze their performance.

C. Paper Organization

This work analyzes the performance of affective agents in the PD and IPD with the aim of understanding the most optimal strategies (or, in this case, personality configurations) against emotional human players. Novel tunable affective agents capable of playing the IPD are designed and implemented and novel tournaments studying various groups of different instantiations of these affective agents are conducted.

The remainder of this paper is organized as follows:

- Section II describes the model of human emotion utilized and details the workings of the affective agents employed.
- Section III outlines the general experimental setup and analyzes the results of the tournaments conducted.
- Section IV provides direction for future research and summarizes the main findings.

II. Methodology

A. Theories of Emotion

Countless different theories and models of human emotion have been posited, but there is little to no consensus as to which is the best one [18]. In general, each model has its strengths but also suffers from weaknesses [19], especially related to those aspects of human behavior that it does not adequately explain or account for. Models of human emotion vary greatly by complexity [20], especially in the number of features they are capable of modeling.

The present application of emotion models to constructing an affective agent capable of playing the PD and IPD does not necessitate the use of a particularly complicated nor comprehensive emotion model. This is because the domain of possible input stimuli the affective agent can experience and the range of possible output behaviors it must be able to generate are both very narrow – input to the agent consists exclusively of a sequence of binary data (i.e., the cooperations and defections of past rounds) and the output from the agent is merely a single binary value (i.e., whether to cooperate or defect in the current round). Thus, playing the PD can be fundamentally represented as making a simple binary decision: cooperating or defecting. Cooperation represents the more 'positive' choice, while defection represents the more 'negative' choice.

Ideally, some single dimension or feature of human emotion can be directly mapped onto this decision (e.g., the agent cooperates if the emotion felt has a value in this dimension that surpasses some threshold and defects otherwise). Then, simply a model of how this facet of emotion evolves throughout the IPD would be all that is required to replicate human behavior. Fortunately, this is precisely the case: "humans base their decision to cooperate primarily on how good the other interactant is" [21, p. 1]. This behavior can be understood through the lens of direct reciprocity, the mechanism underlying the operation of the two-time tournament-winning `TITFORTAT` rule [22] and underpinning the emergence of cooperation throughout the natural world [23]–[25]. Fundamentally, one player cooperating makes the other player more likely to cooperate and one player defecting makes the other player more likely to defect. This relationship may be generalized to form the assumption of how an affective agent should play the game. One agent cooperating should make the other agent experience more positive emotions and one agent defecting should make the other agent experience more negative emotions. In turn, positive emotions should cause that agent to react by cooperating, while negative emotions should cause that agent to react by defecting. Thus, the model of emotion utilized must simply define how this emotional activation evolves in a manner that follows human patterns.

More complex models of human emotion, such as Affect Control Theory (ACT) [26] or Bayesian Affect Control Theory (BayesACT) [27], are capable of modeling a wide array of environments, objects, and identities. However, these capabilities are not required to solve the problem being investigated here. These additional features, if integrated, may only serve to introduce unnecessary information to the model that may act as noise, hijacking the model's workings and directing behavior away from what should be observed.

B. Picard's Model of Emotion

Rosalind Picard's original model of human emotion [28] was employed herein to enable artificial agents to play the IPD in a way that simulates human emotional behavior. The governing equation of this model [28, p. 151] is given by:

$$y = \frac{g}{1 + e^{\frac{x - x_0}{s}}} + y_0 \tag{1}$$

where y is the emotional output, x is the input stimulus, g is the gain, x_0 is the shift, s is the steepness, and y_0 is the offset.

Equation (1) captures several important aspects of human emotion [28, pp. 159–160], many of which are critical for the present application:

- Emotions are short-lived transient experiences that eventually cease [28, p. 159]. In the context of the IPD, emotions evoked in the agent several rounds ago should have less influence on the agent's present decision.

- Repeated input stimuli build increasingly intense emotional responses [28, p. 159]. In the context of the IPD, one cooperation or defection should not change an agent's emotional state overly significantly but a continual pattern of cooperations or defections should.
- Personality has a marked effect on emotional responses evoked [28, p. 159]. In the context of the IPD, different agents should respond differently as the game evolves.
- Emotional response has a nonlinear relationship with input stimuli, meaning that activation onset may be sudden and that emotional responses must eventually saturate [28, p. 160]. In the context of the IPD, agents should have a 'breaking point' where a stimulus suddenly produces an emotional response and they should have limits on the intensity of emotions that they can experience.
- Feedback has a critical influence on how emotions are experienced [28, p. 160]. In the context of the IPD, agents should not experience emotions in a vacuum, but rather based on the complex interplay of their current mood as well as their conscious and unconscious expectations.

Picard pointed out that fundamental issues with this model arise in determining the tangible correlates of its input and output values and the assignment of its seemingly arbitrary parameter values [28, p. 154]. Nevertheless, by applying this generalized model to a specific downstream task, such as playing the IPD, these gaps can be directly filled to create an emotional model capable of completing the desired task.

C. Affective Agent

The novel affective agent applied in this work utilizes a slightly altered version of Picard's model [28] given by:

$$y = \frac{2a}{1 + e^{-b(x+c)}} - a - d \geq -e \tag{2}$$

where y is the emotional output, x is the input stimulus, a is the gain that represents arousal, b is the steepness that represents temperament, c is the shift that represents mood, d is the offset that represents cognitive expectation, and e is the threshold that represents disposition. Equations (1) and (2) model identical mathematical relationships, but the latter has a slightly altered form to allow the parameters to fit their underlying semantics more naturally. Each element of this model plays an important role, as described below.

Output, y. This reflects the subjective emotion felt by the agent; the sign of y reflects whether the emotion is positive or negative and the magnitude of y reflects how strongly the emotion is felt. This value is subsequently thresholded to produce the binary decision required to play a single PD game. The emotion experienced by the agent is constrained to usually fall within the range $[-a, a]$ (bounded by the agent's arousal with some variation due to cognitive expectation).

Input, x. This reflects the current state of the outside world that evoked the activated emotion. For this application, this is the decision made by the other player in the previous round of the IPD. If the other player just cooperated, then the input is $x = 1$ (a positive stimulus because cooperation is interpreted as a positive action), and if they just defected, the input is $x = -1$ (a negative stimulus because defection is interpreted as a negative action). Note that the effect of decisions made in previous rounds is represented by the c parameter; only the single most recent decision made by the other player is mapped as input.

Arousal, a. Changing a has the effect of vertically stretching the sigmoid-based function about the x-axis. Arousal reflects the strength or intensity of the emotions able to be felt by the agent; a large a means that the agent is capable of experiencing more powerful emotions, while a small a means that the agent is not capable of experiencing very powerful emotions (i.e., the agent is relatively stoic). This parameter is time-independent (i.e., constant throughout the IPD) and is explicitly tuned externally when the agent is instantiated. This makes sense because different people naturally experience different intensities of emotion and these maximum intensities are typically unchanging. Generally, this parameter should fall somewhere inside the range $[0.5, 5]$ (arbitrarily set to prevent excessive or deficient arousal), with a default value of 1.

Temperament, b. Changing b has the effect of horizontally stretching the sigmoid-based function about the y-axis. Temperament reflects the volatility of the emotions felt by the agent; a large b means that the agent swings wildly between emotions (i.e., the agent has a 'short fuse'), while a small b means that the agent more gradually adjusts emotions (i.e., the agent is 'unshakeable'). This parameter is time-independent and is explicitly tuned externally when the agent is instantiated. This makes sense because different people have different temperaments and these temperaments usually do not change. Generally, this parameter should also fall somewhere inside the range $[0.5, 5]$ (arbitrarily set to maintain the integrity of the sigmoidal relationship), also with a default value of 1.

Mood, c. Changing c has the effect of horizontally shifting the sigmoid-based function along the x-axis. Mood reflects the general affective state felt by the agent; a positive c means that the agent is more inclined to feel positive emotions (i.e., a less positive input stimulus can evoke a positive emotion), while a negative c means that the agent is less inclined to feel positive emotions (i.e., a more positive input stimulus is required to evoke a positive emotion). This parameter is time-dependent (i.e., evolves throughout the IPD) and is updated automatically as the game progresses. More specifically, c begins at a value of 0 (reflecting a neutral initial mood) and is subsequently updated after each round of the game to store the previously-activated emotion value. In this sense, mood is represented as the accumulation of the previous emotions felt over time, consistent with its generally agreed-upon definition in the literature (e.g., [26, p. 183]). This makes sense because mood exists in all people but changes over time depending almost solely on what emotions have recently been felt. This parameter usually falls somewhere inside the range $[-a, a]$ because it is simply assigned the value of the previous emotion activation y. Note that different people's predispositions to experiencing different moods is represented by the e parameter.

Cognitive expectation, d. Changing d has the effect of vertically shifting the sigmoid-based function along the y-axis. Cognitive expectation reflects the impact of incongruency on the emotion felt by the agent; a positive d means that the agent expects a positive outcome (causing the emotion felt to tend to be more negative), while a negative d means that the agent expects a negative outcome (causing the emotion felt to tend to be more positive). This parameter is time-dependent and is updated automatically as the game progresses. More specifically, d begins at a value of 0 (reflecting no initial expectation) and is subsequently updated after each round of the game to reflect the previous choices made by the other player in the game. In this sense, cognitive expectation is represented as the accumulation of the previous decisions made over time and is directly informed by the objective progression of the game. This makes sense because cognitive expectation exists in all people but changes over time depending solely on past experiences. This parameter always falls somewhere inside the range $[-2x, 2x]$ where x is the tuning factor that determines how impactful cognitive expectation is on the emotion felt (taken here as $x = 0.1$).

Disposition, e. Changing e does not affect the response curve of the sigmoid-based model, but it determines the mapping of the continuous emotion value felt onto the discrete decision made by the agent. The disposition reflects the general tendency of the agent; a positive e means that the agent is more positive (i.e., requires a less positive emotion to decide to cooperate), while a negative e means that the agent is more negative (i.e., requires a more positive emotion to decide to cooperate). This parameter is time-independent and is explicitly tuned externally when the agent is instantiated. This makes sense because different people have different dispositions, with some generally happy people being predisposed to cooperation and other more negative people being predisposed to defection (an effect that has been experimentally observed [29]). Generally, this parameter should fall somewhere inside the range $[-a, a]$ (because any value significantly outside this range invariably results in an always-defect or always-cooperate strategy), with a default value of 0. Note that this parameter entirely determines the agent's first move of the game (to begin, $x = c = d = 0$, so $y = 0$ and the decision made is simply the sign of e).

This affective agent architecture is capable of reproducing many aspects of human behavior in the IPD [30], [31]. The disposition parameter can be tuned to produce agents adhering to an always-cooperate or always-defect strategy, as a large fraction of humans do [30, p. 8]. In recommended parameter ranges, the agent exhibits personality-dependent "moody conditional cooperation" driven by direct reciprocity; the agent's propensity to cooperate increases both when it just cooperated (because the previously-activated positive emotional response that caused the cooperation now positively biases future emotional responses via mood) and when the other player just cooperated (because the input is now positive), like many humans do [30, p. 8]. At default parameter values, the agent approximately follows the `TITFORTAT` rule.

III. Results and Discussion

A. Experimental Setup

Now that a tunable affective agent capable of playing the IPD in a manner that reflects emotion-driven human behavior has been designed, informative tournaments may be conducted. Such tournaments serve to reveal the performance of different personality configurations (analogue of strategies) of this agent architecture to determine which performs the best.

All tournaments conducted used the same fundamental setup based closely off of Axelrod's original 1980 tournaments [7], [8]. All agents included in each tournament played exactly one game of the IPD against each other agent as well as a copy of itself, as in Axelrod's tournaments (except for the lack of a `RANDOM` agent, which will be discussed later) [7, p. 8], [8, p. 382]. Each IPD consisted of exactly 200 rounds of the regular PD (meaning that scores must fall within the range $[0, 1000]$), as in Axelrod's original tournament [7, p. 8]. Axelrod's second tournament introduced an element of randomness into the length of the IPD to avoid "endgame effects" [8, p. 383], but the affective agent employed herein has no knowledge of the length of the game and, thus, such effects were already impossible. Each PD game used a standard payoff matrix (Table I) with $A = 5$, $B = 0$, $C = 3$, and $D = 1$, as in Axelrod's tournaments [7, p. 8], [8, p. 381].

To prevent the games in the tournament from devolving into infinite mutual cooperation or infinite mutual defection due to reciprocity, a mechanism of non-determinism was introduced. This is especially important because of the nature of the affective agent used; at default parameters (i.e., for $e = 0$), the agent is inclined to always cooperate, which does not permit tournaments that provide meaningful insights. Thus, an element of noise was inserted: in each PD, each decision made by each agent has a small chance of being inverted (crucially, only the decision was modified and not the underlying emotion value). While this inclusion may seem to only introduce unnecessary noise into the data such that they no longer completely captures the agent's performance (and it did introduce variability into the results of individual tournaments, which will be discussed later), it allowed the agents' behavior to be truly tested in practice. This inclusion also accurately modeled the real-world; miscommunications between humans occur all the time, so it is reasonable to imagine that, in the context of the real-world problem the game represents, some cooperations or defections may be misinterpreted. The level of noise added into the tournament,

TABLE I
PD payoff matrix used in all tournaments. The first element of the tuple represents the number of points earned by Player #1 and the second element represents the number of points earned by Player #2.

		Player #1	
		Cooperate	Defect
Player #2	Cooperate	$(3, 3)$	$(5, 0)$
	Defect	$(0, 5)$	$(1, 1)$

n, can be defined as the fraction of decisions that are randomly overturned. This noise parameter may be set arbitrarily (within reason) because it merely altered the magnitude of the relationships being observed in the results and not the actual relationships themselves. A value of $n = 0.025$ was used for all tournaments. This means that each individual decision made throughout the IPD has a 2.5% chance of being flipped, causing around 10 total instances of the noise changing a decision to occur in each 200-round and 400-decision game of the IPD. This was not enough to poison the results obtained, but was sufficient to trigger emotional responses from the agents often enough to allow the desired behavioral patterns to emerge in experiments.

Unlike in Axelrod's tournaments, no `RANDOM` agent was incorporated [7, p. 8], [8, p. 380]. The effect of randomness on agent performance was already accounted for by the insertion of random noise; adding noise to each individual decision was preferable because it added the variability required to evoke emotional responses in every game in the entire tournament (and not just in one game per agent). Because these emotional agents were already less static and predictable than the rule-based agents of Axelrod's tournaments, agent performance against erratic players no longer needed to be explicitly probed through the inclusion of a `RANDOM` agent.

The addition of noise had the unfortunate consequence of making tournament results non-deterministic. To handle this issue, all tournaments were conducted multiple times and their results were aggregated. Specifically, each tournament was run exactly 100 times and the scores in each execution were averaged. To understand the variability in the results of the 100-sample aggregated tournaments, this procedure was repeated exactly 10 times. The scores in the 100-sample aggregated tournaments were also aggregated to produce a mean and absolute range of performance for each agent across these 10 samples. Thus, the scores reported represent the agents' average performance across 1000 executions of the tournament, limiting the effect of noisy outliers on analysis.

B. Experiment 1

The effect of varying each tunable emotional parameter (i.e., arousal, a; temperament, b; and disposition, e) individually while all others were assigned their default values (i.e., $a = 1$, $b = 1$, and $e = 0$) was tested in three separate novel tournaments.

Arousal. First, the effect of varying the agent's arousal parameter was tested (Table II; Fig. 1). For this tournament, a small population of 25 default agents differentiated only by their arousal value were included. These agents were assigned arousal values covering the entire recommended range of $[0.5, 5]$ with additional granularity examined in the range $[1, 2]$.

This tournament revealed a significant relationship between agent arousal and performance. This relationship was highly non-linear, characterized by two main levels of performance joined by a sharp discontinuity. Agents with low arousal values (i.e., $a \leq 1.5$) performed significantly worse than agents with high arousal values (i.e., $a \geq 2.5$). Between these two approximately constant regimes (i.e., $1.5 \leq a \leq 2.5$) existed a relatively linear range where performance scaled directly with arousal. Additionally, a relatively large variability in results existed between tournament runs; this suggests that the noise introduced in this experiment played a significant role in performance, but the effect of arousal still dominated. Overall, in this environment, *it payed to be emotionally aroused.*

TABLE II
TOURNAMENT RESULTS (MEAN SCORE OF EACH AGENT ACROSS ALL 10 TRIALS) FOR EXPERIMENT 1.1 (AROUSAL).

Agent Arousal, a	Average Agent Score	Agent Arousal, a	Average Agent Score	Agent Arousal, a	Average Agent Score
0.5	540.19	1.7	565.47	3.5	581.41
0.75	540.18	1.8	565.33	3.75	580.59
1	539.04	1.9	570.59	4	581.35
1.1	539.70	2	572.11	4.25	581.82
1.2	540.07	2.25	578.29	4.5	582.42
1.3	540.01	2.5	580.32	4.75	582.41
1.4	539.26	2.75	581.41	5	581.86
1.5	540.70	3	580.69		
1.6	562.12	3.25	581.43		

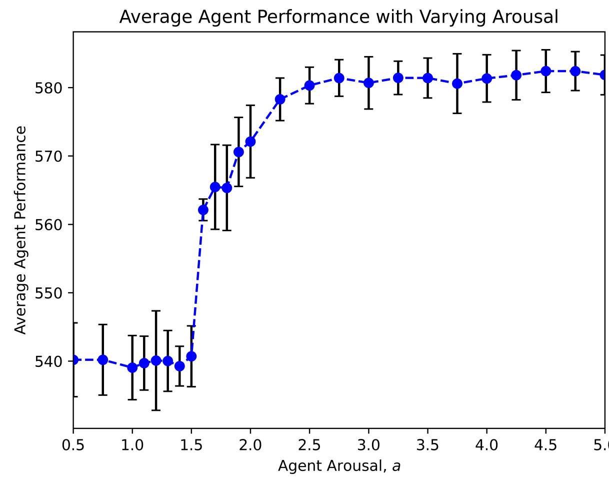

Fig. 1. Visualization of tournament results for Experiment 1.1 (arousal). Data points indicate the mean score of each agent across all 10 trials while the error bars indicate the maximum and minimum scores among these trials.

Temperament. Second, the effect of varying the agent's temperament parameter was tested (Table III; Fig. 2). For this tournament, a small population of 25 default agents differentiated only by their temperament value were included. These agents were assigned temperament values covering the entire recommended range of $[0.5, 5]$ with additional granularity examined in the range $[1, 2]$.

This tournament revealed no significant relationship between agent temperament and performance. The results were characterized by approximately constant performance. Additionally, a relatively large variability in results existed between tournament runs; this suggests that the noise introduced in this experiment played a significant role in performance and that the effect of temperament was limited. Overall, in this environment, *it neither payed nor costed to be temperamental.*

TABLE III
TOURNAMENT RESULTS FOR EXPERIMENT 1.2 (TEMPERAMENT).

Agent Temperament, b			Average Agent Score		
0.5	467.22	1.7	467.03	3.5	466.62
0.75	467.20	1.8	466.81	3.75	467.17
1	466.54	1.9	466.50	4	466.64
1.1	467.50	2	466.93	4.25	466.68
1.2	466.63	2.25	467.10	4.5	466.77
1.3	467.04	2.5	465.86	4.75	467.09
1.4	466.93	2.75	466.84	5	467.16
1.5	467.15	3	467.32		
1.6	466.20	3.25	466.87		

TABLE IV
TOURNAMENT RESULTS FOR EXPERIMENT 1.3 (DISPOSITION).

Agent Disposition, e			Average Agent Score		
-1	391.14	-0.2	419.29	0.35	483.50
-0.9	391.12	-0.15	420.04	0.4	483.61
-0.8	391.16	-0.1	419.42	0.45	459.34
-0.7	391.08	-0.05	419.40	0.5	461.03
-0.6	391.10	0	427.08	0.6	460.95
-0.5	390.90	0.05	427.46	0.7	460.84
-0.45	394.15	0.1	427.17	0.8	460.88
-0.4	337.45	0.15	427.00	0.9	460.86
-0.35	337.76	0.2	427.59	1	460.94
-0.3	342.02	0.25	435.52		
-0.25	407.56	0.3	478.37		

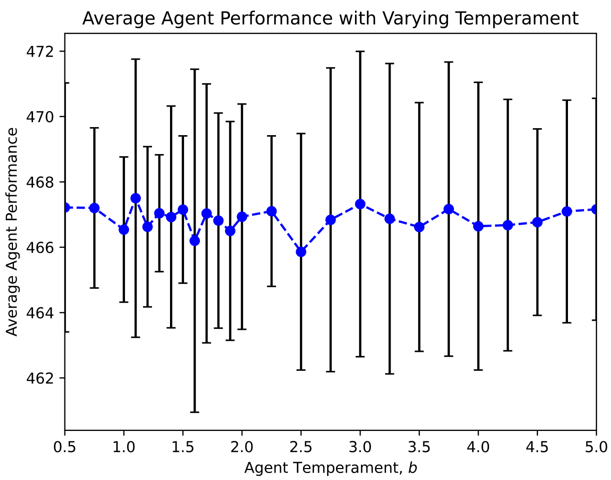

Fig. 2. Visualization of tournament results for Experiment 1.2 (temperament).

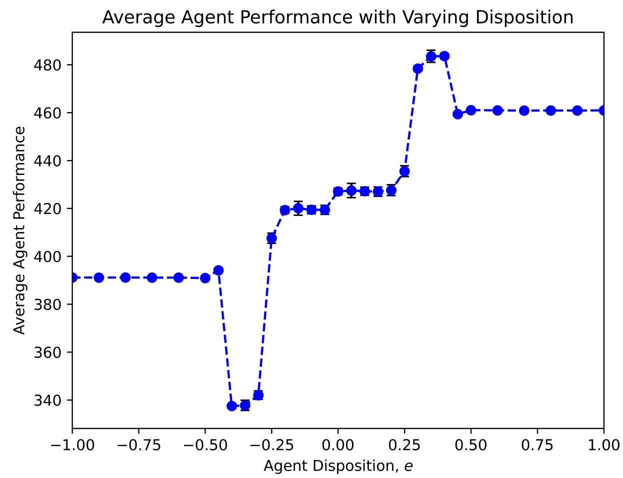

Fig. 3. Visualization of tournament results for Experiment 1.3 (disposition).

Disposition. Third, the effect of varying the agent's disposition parameter was tested (Table IV; Fig. 3). For this tournament, a small population of 31 default agents differentiated only by their disposition value were included. These agents were assigned disposition values covering the entire recommended range of $[-1, 1]$ (because $a = 1$) with additional granularity examined in the range $[-0.5, 0.5]$.

This tournament revealed an extremely significant relationship between agent disposition and performance. This relationship was highly non-linear, characterized by a multi-modal performance distribution. Multiple conclusions may be drawn from these results. First, positive dispositions produced superior performance. All agents with a positive disposition outperformed all agents with a negative disposition in a striking similarity to the "niceness" property observed in Axelrod's initial tournament [7, p. 9]. Second, extreme disposition values produced relatively constant performance. Increasing the magnitude of the agent's disposition beyond 0.5 (i.e., $|e| \geq 0.5$) did not impact performance in both the positive- and negative-disposition cases. Functionally, these agents played with an always-cooperate strategy (i.e., an altruist) or an always-defect strategy (i.e., a scrooge), so their performance is approximately constant. Altruists outperformed scrooges because of reciprocity; many of the other agents included were willing to sustain infinite mutual cooperation with the altruists, while many of the other agents resorted to infinite mutual defection with the scrooges. Third, the most dramatic performance variation was observed when the magnitude of agent's disposition was large, but not large enough to produce an always-cooperate or always-defect strategy. The best-performing agents were moderately positively disposed (i.e., $0.3 \leq e \leq 0.4$) and the worst-performing agents were moderately negatively disposed (i.e., $-0.4 \leq e \leq -0.3$). Fourth, performance was relatively constant among agents with weak dispositions. Agents with dispositions close to zero (i.e., $-0.2 \leq e \leq 0.2$) showed little performance variation, although positively disposed agents exhibited a slight advantage. Fifth, almost no variability in results existed between tournament runs; this suggests that the noise introduced in this experiment played an insignificant role in performance and that the effect of disposition dominated. Taken together, the data suggest that performance decreased as agent disposition trended from moderately positive to extremely positive, neutral, extremely negative, and moderately negative. Overall, in this environment, *it payed to be positively disposed (but not overly so).*

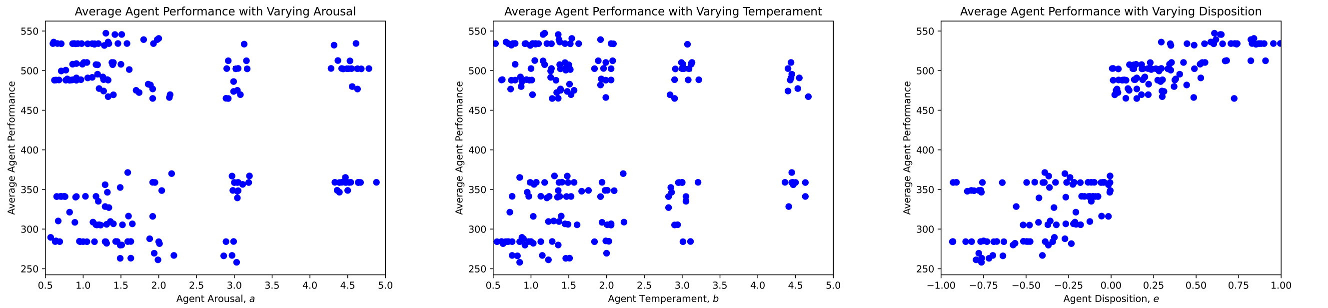

Fig. 4. Visualization of tournament results for Experiment 2 (all agent parameters). (Note that agent disposition is normalized by arousal in the third inset.)

C. Experiment 2

Axelrod pointed out that there is no single universally-optimal strategy; the performance of a strategy depends on the other strategies it plays against [7, p. 7], [8, p. 380]. Thus, the tournaments of Experiment 1 provide useful insights into the effect of each agent parameter on performance in isolation, but do not tell the whole story. The values of the other parameters were held constant at their default values, but these configurations do not necessarily represent a realistic distribution of personalities. While patterns emerged for each personality parameter, these patterns may not persist in a larger, more diverse group of agents. Therefore, a comprehensive novel tournament where all agent parameters are varied simultaneously must be conducted to capture the complex interdependencies between parameters.

The effect of varying all agent parameters simultaneously was tested (Table V; Fig. 4). For this tournament, a large population of 200 agents (constructed by randomly selecting each parameter value from a representative domain $a, b \in \{0.75, 1, 1.25, 1.5, 2, 3, 4.5\}$ and $e \in \{-0.8, -0.5, -0.25, -0.1, 0, 0.1, 0.25, 0.5, 0.8\}$ and adding a small amount of noise $\sim \mathcal{N}(\mu = 0, \sigma = 0.1)$) were included.

This tournament revealed multiple important relationships between agent parameters and performance. First, an extremely significant relationship existed between agent disposition and performance. As in Experiment 1.3, all positively disposed agents outperformed all negatively disposed agents. However, the effect here was amplified, with the best-performing negatively disposed agent and worst-performing positively disposed agent being separated by nearly a 100-point chasm. This effect occurred because the positively disposed agents mutually cooperated with the other positively disposed agents (so they consistently earned C points) while the negatively disposed agents mutually defected with the other negatively disposed agents (so they consistently earned only D points) beginning in the very first round (which is determined entirely by disposition). While Axelrod reported a similar "niceness" property [7, p. 9], the magnitude of the effect here was much larger. This is because of the purely emotional nature of the affective agents used; unlike in other environments, cooperators were less susceptible to free riders because their opponents do not act strategically and instead simply reciprocated their positive actions. In contrast to Experiment 1.3, no limiting effect was observed; always-cooperate agents were generally the best-performing agents. Performance positively correlated with disposition to some degree in both clusters of agents; very positively disposed agents performed the best among the positively disposed agents and marginally negatively disposed agents performed the best among the negatively disposed agents. Overall, agent disposition had a dominant effect on performance because a stronger tendency to cooperate improved performance. Second, a relatively insignificant relationship existed between agent arousal and performance. Among the positively disposed agents, arousal slightly negatively correlated with performance; minimally aroused agents performed the best and maximally aroused agents performed the worst. This is because minimally aroused agents were more likely to maintain cooperation when faced with noisy defections (because they could not feel emotions strong enough to sway their actions) while maximally aroused agents were more likely to defect (because they felt a strong negative emotion in response). Among the negatively disposed agents, the opposite was true and arousal slightly positively correlated with performance; maximally aroused agents performed the best and minimally aroused agents performed the worst. The explanation is similar; maximally aroused agents overcame their tendency to defect and mutually cooperated in some cases (because they felt strong positive emotions) while minimally aroused agents maintained defection (because their emotions were not strong enough to sway them). Third, a relatively insignificant relationship existed between agent temperament and performance. This relationship and the accompanying explanation are similar to the arousal case; temperament slightly negatively correlated with performance among the positively disposed agents and slightly positively correlated with performance among the negatively disposed agents. Temperament played a very similar role to arousal because, when large, it enabled the agent to overturn its disposition. Fourth, almost no variability in results existed between tournament runs; this suggests that the introduced noise played an insignificant role in agent performance and that agent parameters dominated. Overall, in this environment of diverse emotional agents, *it payed to be minimally aroused, minimally tempered, and, most importantly, positively disposed.*

IV. Conclusion

A. Future Work

This work represents a significant step in understanding the role of human emotion in the PD and IPD by uncovering the personality configurations that produce the best performance, but is only the beginning. The literature on the PD and IPD is vast, so this work may be expanded upon in numerous ways by exploring different research paradigms. Several specific factors limit the present work, but may be overcome in future work.

Fundamentally, "there is no best rule independent of the environment" [8, p. 402]; thus, blanket statements about the best strategies in all environments cannot be made. To gain a more thorough understanding of the effects of tuning the affective agent, more tournaments should be conducted incorporating more sets of agents (affective and otherwise) with more in-depth analysis. Furthermore, ecological experiments investigating evolving populations of agents (like in Axelrod's second tournament [8, pp. 399–401]) may be carried out to understand how personality configurations persist.

The model of human emotion utilized herein is (perhaps overly) simplified. While complicated models may be overkill, additional features may serve to better capture individual variation and the more subtle nuances of the game. Furthermore, like all tasks, humans do not play the game entirely emotionally nor entirely rationally. Real humans may reluctantly act against their emotions in pursuit of strategic interests, but such behavior is not supported by this solely emotional affective agent model. A model integrating both emotional impulse and deeper cognitive strategy may more accurately capture human behavior and would make for interesting tournaments.

B. Summary

This paper presented a novel affective agent leveraging a model of human emotion to play the IPD in a manner that accurately reflects emotion-driven behavior. Additionally, this paper reported the results of multiple novel tournaments analyzing the performance of various populations of these affective agents. The results of these tournaments suggest that, all else being equal, it pays to be emotionally aroused, neither pays nor costs to be temperamental, and pays to be positively disposed (but not overly so). In more diverse environments, it pays to be minimally aroused, minimally tempered, and, most importantly, positively disposed. This work expands upon Axelrod's "primer" [7, p. 3] by uncovering the optimal way to play the PD and IPD against affective agents (such as humans).

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TABLE V
TOURNAMENT RESULTS FOR EXPERIMENT 2 (ALL AGENT PARAMETERS).

Agent Parameters, (a, b, e)			Average Agent Score		
$(a = 1.30, b = 1.18, e = 0.79)$	547.21	$(a = 1.32, b = 0.62, e = 0.18)$	488.65	$(a = 3.04, b = 1.67, e = -2.57)$	347.77
$(a = 1.51, b = 1.16, e = 0.97)$	545.66	$(a = 1.27, b = 0.94, e = 0.18)$	488.65	$(a = 4.39, b = 0.95, e = -3.35)$	346.58
$(a = 1.42, b = 1.36, e = 0.92)$	545.53	$(a = 0.97, b = 2.90, e = 0.18)$	488.55	$(a = 1.32, b = 2.86, e = -0.01)$	346.49
$(a = 2.00, b = 1.54, e = 1.68)$	540.64	$(a = 0.86, b = 3.22, e = 0.26)$	488.43	$(a = 0.75, b = 1.13, e = -0.13)$	341.43
$(a = 1.98, b = 1.37, e = 1.22)$	539.14	$(a = 0.78, b = 3.08, e = 0.06)$	488.24	$(a = 0.71, b = 0.97, e = -0.09)$	341.36
$(a = 1.80, b = 1.92, e = 1.48)$	539.10	$(a = 0.68, b = 2.06, e = 0.03)$	488.17	$(a = 1.03, b = 0.75, e = -0.11)$	341.32
$(a = 1.33, b = 1.38, e = 0.78)$	536.08	$(a = 0.85, b = 0.61, e = 0.06)$	488.16	$(a = 1.17, b = 1.24, e = -0.09)$	341.30
$(a = 0.61, b = 0.67, e = 0.18)$	535.98	$(a = 0.84, b = 1.01, e = 0.07)$	488.14	$(a = 0.70, b = 1.38, e = -0.11)$	341.29
$(a = 4.61, b = 0.79, e = 4.59)$	534.26	$(a = 0.64, b = 4.43, e = 0.24)$	488.08	$(a = 0.75, b = 4.63, e = -0.10)$	341.22
$(a = 1.58, b = 0.76, e = 1.31)$	534.23	$(a = 1.33, b = 0.79, e = 0.09)$	488.06	$(a = 0.65, b = 1.53, e = -0.06)$	341.15
$(a = 1.46, b = 1.02, e = 1.03)$	534.22	$(a = 0.77, b = 0.98, e = 0.05)$	487.96	$(a = 0.91, b = 1.47, e = -0.07)$	341.12
$(a = 0.88, b = 1.06, e = 0.64)$	534.19	$(a = 0.62, b = 1.99, e = 0.17)$	487.94	$(a = 0.71, b = 3.05, e = -0.09)$	341.08
$(a = 0.66, b = 0.93, e = 0.63)$	534.19	$(a = 0.82, b = 1.33, e = 0.01)$	487.89	$(a = 0.65, b = 1.92, e = -0.07)$	341.00
$(a = 0.98, b = 1.03, e = 0.85)$	534.15	$(a = 0.91, b = 0.74, e = 0.08)$	487.84	$(a = 0.76, b = 2.82, e = -0.08)$	340.97
$(a = 1.93, b = 0.53, e = 1.36)$	534.15	$(a = 2.99, b = 0.88, e = 0.87)$	486.24	$(a = 0.90, b = 1.35, e = -0.24)$	340.30
$(a = 0.93, b = 1.45, e = 0.82)$	534.15	$(a = 1.86, b = 1.51, e = 0.41)$	482.88	$(a = 3.04, b = 1.21, e = -1.29)$	339.38
$(a = 1.20, b = 0.70, e = 0.89)$	534.11	$(a = 1.89, b = 1.92, e = 0.84)$	481.84	$(a = 1.20, b = 3.05, e = -0.14)$	335.08
$(a = 0.60, b = 1.21, e = 0.54)$	534.08	$(a = 4.56, b = 0.87, e = 1.69)$	479.87	$(a = 1.29, b = 4.41, e = -0.72)$	328.41
$(a = 1.34, b = 2.06, e = 1.17)$	534.07	$(a = 1.21, b = 4.53, e = 0.12)$	477.36	$(a = 1.34, b = 2.82, e = -0.45)$	327.14
$(a = 1.12, b = 1.10, e = 0.50)$	534.04	$(a = 1.92, b = 1.39, e = 0.29)$	476.88	$(a = 0.82, b = 0.72, e = -0.17)$	321.33
$(a = 0.86, b = 1.63, e = 0.63)$	533.98	$(a = 4.63, b = 0.73, e = 0.16)$	476.74	$(a = 1.60, b = 1.40, e = -0.09)$	316.38
$(a = 0.71, b = 1.27, e = 0.39)$	533.93	$(a = 3.04, b = 1.39, e = 0.09)$	475.11	$(a = 1.92, b = 1.03, e = -0.03)$	316.10
$(a = 1.15, b = 0.92, e = 0.40)$	533.90	$(a = 1.70, b = 1.57, e = 0.18)$	475.07	$(a = 0.67, b = 1.30, e = -0.24)$	310.23
$(a = 0.90, b = 2.09, e = 0.59)$	533.83	$(a = 1.27, b = 4.40, e = 0.38)$	474.23	$(a = 1.36, b = 1.23, e = -0.17)$	309.52
$(a = 1.06, b = 1.52, e = 0.64)$	533.81	$(a = 2.99, b = 1.48, e = 0.93)$	473.72	$(a = 1.36, b = 1.37, e = -0.17)$	309.52
$(a = 3.13, b = 1.06, e = 2.31)$	533.45	$(a = 1.74, b = 1.34, e = 0.07)$	472.38	$(a = 1.13, b = 2.23, e = -0.24)$	308.81
$(a = 1.15, b = 3.07, e = 0.98)$	533.28	$(a = 3.08, b = 1.02, e = 0.55)$	469.68	$(a = 0.89, b = 1.94, e = -0.38)$	308.54
$(a = 4.32, b = 0.74, e = 2.11)$	532.10	$(a = 2.15, b = 1.53, e = 0.47)$	469.68	$(a = 1.40, b = 2.06, e = -0.31)$	306.74
$(a = 1.28, b = 0.99, e = 0.45)$	531.79	$(a = 1.40, b = 2.84, e = 0.03)$	469.46	$(a = 1.65, b = 1.45, e = -0.44)$	306.64
$(a = 4.37, b = 1.05, e = 2.97)$	512.70	$(a = 1.33, b = 4.67, e = 0.40)$	467.13	$(a = 1.31, b = 1.49, e = -0.27)$	306.00
$(a = 2.92, b = 2.08, e = 2.40)$	512.47	$(a = 2.14, b = 1.99, e = 1.04)$	466.11	$(a = 1.21, b = 2.01, e = -0.45)$	305.51
$(a = 3.16, b = 1.89, e = 2.87)$	512.43	$(a = 2.89, b = 1.36, e = 0.25)$	465.05	$(a = 1.17, b = 2.94, e = -0.21)$	305.49
$(a = 4.53, b = 1.16, e = 3.04)$	512.31	$(a = 1.92, b = 2.90, e = 1.39)$	464.85	$(a = 1.52, b = 1.61, e = -0.49)$	305.39
$(a = 1.05, b = 4.44, e = 0.58)$	510.30	$(a = 2.92, b = 1.28, e = 0.44)$	464.83	$(a = 1.18, b = 2.90, e = -0.61)$	305.24
$(a = 1.38, b = 1.99, e = 0.49)$	510.30	$(a = 1.59, b = 4.45, e = -0.62)$	371.41	$(a = 1.23, b = 2.06, e = -0.59)$	305.05
$(a = 1.01, b = 3.13, e = 0.43)$	510.29	$(a = 2.17, b = 2.22, e = -0.59)$	370.07	$(a = 0.57, b = 0.88, e = -0.19)$	289.54
$(a = 1.40, b = 2.99, e = 0.75)$	510.16	$(a = 3.20, b = 1.31, e = -0.02)$	367.04	$(a = 1.88, b = 0.90, e = -0.51)$	287.81
$(a = 1.39, b = 1.45, e = 0.32)$	510.10	$(a = 2.97, b = 1.48, e = -1.09)$	366.87	$(a = 0.64, b = 1.99, e = -0.48)$	285.15
$(a = 0.91, b = 3.12, e = 0.48)$	508.89	$(a = 4.47, b = 0.85, e = -1.08)$	365.16	$(a = 0.95, b = 2.03, e = -0.49)$	284.63
$(a = 0.86, b = 3.01, e = 0.45)$	508.16	$(a = 4.43, b = 1.41, e = -1.99)$	359.17	$(a = 1.63, b = 1.28, e = -0.81)$	284.39
$(a = 0.99, b = 3.12, e = 0.34)$	508.12	$(a = 4.33, b = 3.00, e = -0.43)$	359.07	$(a = 2.99, b = 0.61, e = -2.01)$	284.39
$(a = 1.50, b = 0.79, e = 0.22)$	507.96	$(a = 4.88, b = 1.07, e = -1.06)$	359.05	$(a = 1.43, b = 3.11, e = -1.33)$	284.35
$(a = 1.17, b = 1.38, e = 0.21)$	507.59	$(a = 1.92, b = 4.36, e = -0.26)$	359.04	$(a = 0.63, b = 0.97, e = -0.30)$	284.29
$(a = 1.39, b = 1.18, e = 0.15)$	507.53	$(a = 4.54, b = 4.46, e = -4.13)$	359.04	$(a = 1.45, b = 0.70, e = -0.71)$	284.28
$(a = 1.19, b = 1.50, e = 0.16)$	507.33	$(a = 4.42, b = 1.94, e = -0.18)$	359.02	$(a = 1.59, b = 1.00, e = -1.22)$	284.24
$(a = 3.05, b = 1.51, e = 0.90)$	502.74	$(a = 4.64, b = 3.21, e = -0.54)$	359.00	$(a = 0.69, b = 0.75, e = -0.59)$	284.21
$(a = 4.28, b = 1.96, e = 0.05)$	502.70	$(a = 3.05, b = 1.57, e = -0.15)$	358.93	$(a = 1.00, b = 1.13, e = -0.73)$	284.20
$(a = 4.42, b = 1.27, e = 0.03)$	502.66	$(a = 4.65, b = 0.99, e = -1.52)$	358.91	$(a = 2.89, b = 0.69, e = -2.37)$	284.20
$(a = 4.78, b = 1.00, e = 1.12)$	502.57	$(a = 1.95, b = 4.46, e = -0.51)$	358.89	$(a = 1.29, b = 0.67, e = -0.50)$	284.17
$(a = 2.90, b = 4.40, e = 0.63)$	502.56	$(a = 3.00, b = 2.96, e = -2.27)$	358.87	$(a = 1.15, b = 3.01, e = -1.07)$	284.09
$(a = 4.55, b = 1.84, e = 0.58)$	502.46	$(a = 4.48, b = 1.37, e = -1.76)$	358.85	$(a = 0.96, b = 0.55, e = -0.33)$	284.09
$(a = 4.51, b = 2.06, e = 2.21)$	502.38	$(a = 4.67, b = 4.47, e = -2.32)$	358.84	$(a = 1.30, b = 0.66, e = -0.82)$	284.08
$(a = 4.49, b = 3.00, e = 0.60)$	502.38	$(a = 4.52, b = 4.50, e = -4.20)$	358.84	$(a = 2.00, b = 0.85, e = -1.38)$	284.08
$(a = 4.63, b = 3.02, e = 1.22)$	502.37	$(a = 4.47, b = 1.18, e = -0.20)$	358.82	$(a = 1.05, b = 1.84, e = -0.98)$	284.06
$(a = 3.02, b = 3.09, e = 0.56)$	502.35	$(a = 4.39, b = 4.63, e = -0.26)$	358.78	$(a = 1.31, b = 1.03, e = -0.74)$	282.14
$(a = 3.17, b = 1.39, e = 1.06)$	502.25	$(a = 3.04, b = 4.45, e = -0.02)$	358.76	$(a = 2.01, b = 0.63, e = -0.47)$	281.68
$(a = 4.68, b = 1.45, e = 0.85)$	502.20	$(a = 3.19, b = 1.49, e = -1.28)$	358.73	$(a = 1.51, b = 1.36, e = -0.87)$	279.93
$(a = 4.44, b = 2.91, e = 0.19)$	502.18	$(a = 4.64, b = 3.00, e = -0.84)$	358.71	$(a = 1.49, b = 0.92, e = -0.55)$	279.72
$(a = 1.61, b = 1.52, e = 0.38)$	501.40	$(a = 3.00, b = 2.95, e = -1.92)$	358.65	$(a = 1.94, b = 2.00, e = -1.51)$	269.54
$(a = 0.77, b = 1.46, e = 0.27)$	500.57	$(a = 3.11, b = 1.04, e = -0.70)$	356.42	$(a = 2.97, b = 0.75, e = -1.20)$	266.76
$(a = 0.71, b = 1.26, e = 0.20)$	499.56	$(a = 1.29, b = 4.47, e = -0.02)$	355.97	$(a = 2.20, b = 1.15, e = -1.53)$	266.73
$(a = 1.19, b = 4.45, e = 0.48)$	495.44	$(a = 1.49, b = 2.85, e = -0.54)$	352.49	$(a = 2.86, b = 0.82, e = -1.82)$	266.21
$(a = 1.26, b = 0.86, e = 0.25)$	492.03	$(a = 4.36, b = 2.01, e = -3.34)$	348.98	$(a = 1.63, b = 1.51, e = -1.24)$	263.31
$(a = 1.12, b = 1.26, e = 0.22)$	491.49	$(a = 4.50, b = 1.76, e = -3.50)$	348.81	$(a = 1.49, b = 1.46, e = -1.07)$	263.13
$(a = 1.06, b = 4.55, e = 0.16)$	490.83	$(a = 2.98, b = 1.98, e = -2.45)$	348.79	$(a = 1.99, b = 1.23, e = -1.58)$	261.23
$(a = 0.91, b = 4.44, e = 0.48)$	490.77	$(a = 2.04, b = 1.36, e = -0.01)$	348.61	$(a = 3.03, b = 0.85, e = -2.31)$	258.13
$(a = 0.79, b = 1.93, e = 0.30)$	489.52	$(a = 3.05, b = 2.10, e = -2.46)$	348.49		