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# **CS 267: Introduction to Data Parallelism Lecture 7**

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<https://sites.google.com/lbl.gov/cs267-spr2019/>

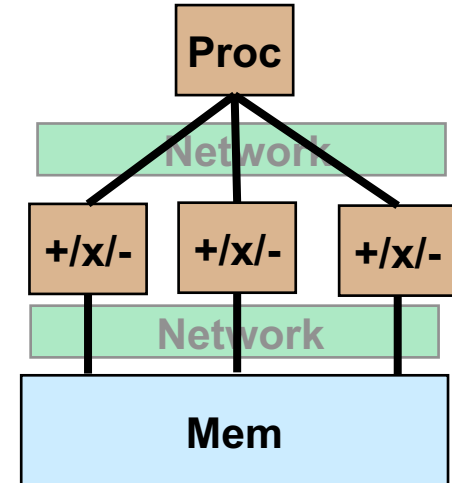
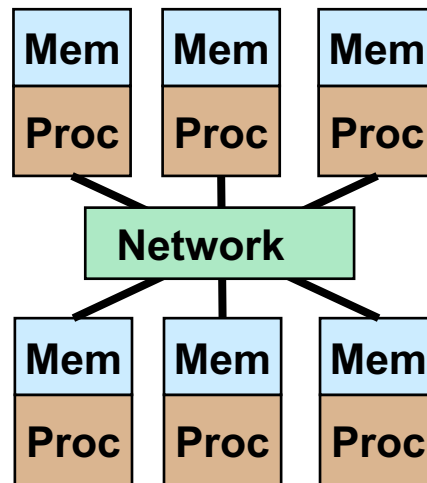
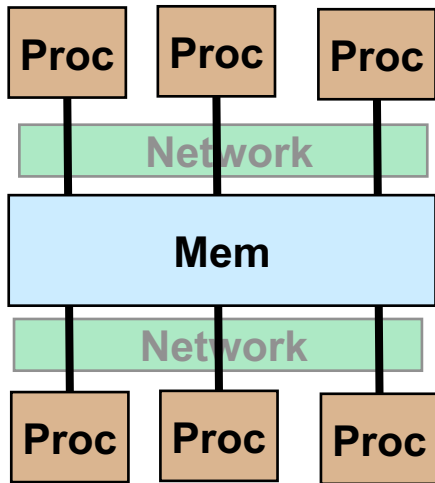
# Lessons from Today's Lecture

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- Data parallelism is beautiful!
- Automatically mapping it to (today's) hardware is hard
- Many parallel programming models use some data parallel features
  - GPUs
  - MPI collectives
  - Cloud MapReduce
- Surprising things you can do with scans
- Useful in designing (nontrivial) parallel algorithms

**“Every nontrivial parallel algorithm uses a prefix scan”**

# Parallel Machines and Programming



## Shared Memory

Processors execute own instruction stream

Communicate by reading/writing memory

Cost of a read/write is constant

## Distributed Memory

Processors execute own instruction stream

Communicate by sending messages

Message time depends on size, but not location

## Single Instruction Multiple Data (SIMD)

One instruction stream (all run same instruction)

Communicate through memory

Assume unbounded # of arithmetic units

- These are the natural “abstract” machine models

# Data Parallel Programming: Unary Operators

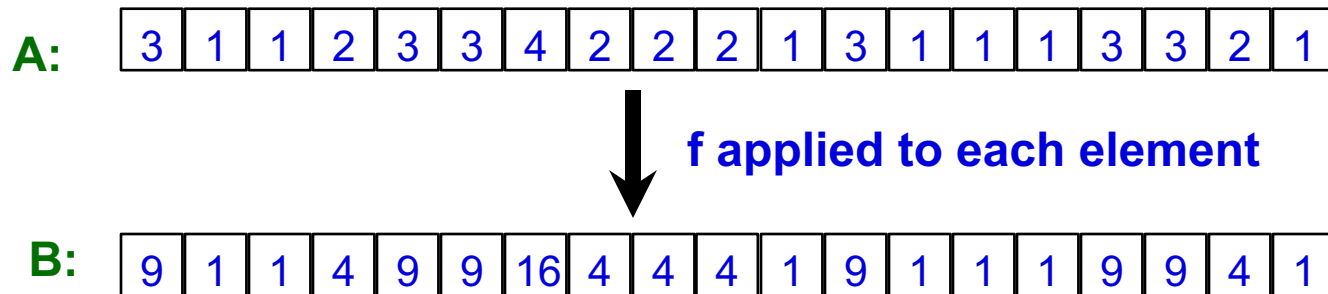
- Unary operations applied to all elements of an array

**A** = array

**B** = array

**f** = square (any unary function, i.e., 1 argument)

**B** = **f**(**A**)



# Data Parallel Programming: Binary Operators

- Binary operations applied to all pairs of elements

A = array

B = array

C = array

- or any other binary operator

C = A - B

A:

3	1	0	2	3	0	4	2	0	2	1	3	0	1	1	0	3	2	1
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

- applied to each pair

B:

0	1	1	4	1	0	2	1	4	3	1	0	1	1	2	3	5	3	2
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---



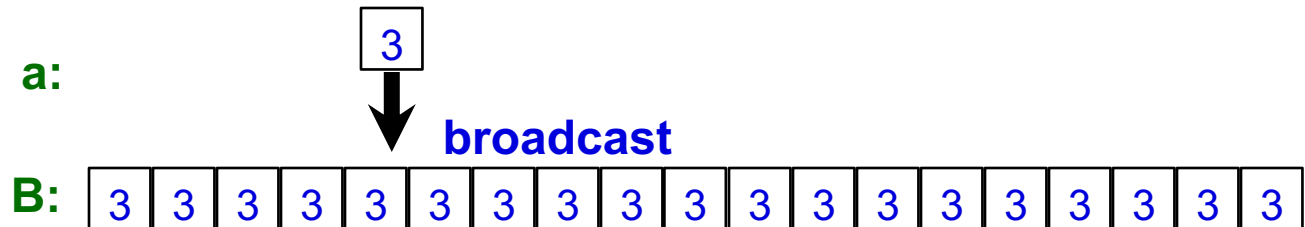
C:

3	0	-1	-2	2	0	2	2	-4	-2	0	3	-1	0	-1	-3	-2	-1	-1
---	---	----	----	---	---	---	---	----	----	---	---	----	---	----	----	----	----	----

# Data Parallel Programming: Broadcast

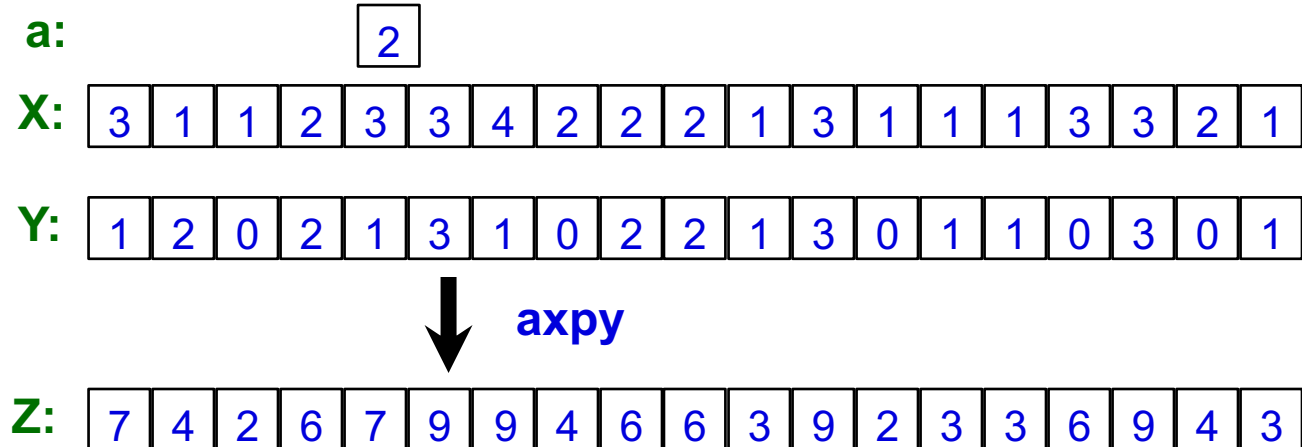
- Broadcast fill a value into all elements of an array

**a = scalar**  
**B = a**



- Useful for  $a \cdot X + Y$  called axpy, saxpy, daxpy

**$a \cdot X + Y$**



# Memory Operations: Strided and Scatter / Gather

- Array assignment work if the arrays are the same shape

A: double [0:4]

B: double [0:4] = [0.0, 1.0, 2.2, 3.3, 4.4]

C: double [0:4, 0:4]

X: int [0:4] = [3, 0, 4, 2, 1]

A = B

- May have a stride, i.e., not be contiguous in memory

A = B [0:4:2] // copy with stride 2 (every other element)

A = C [\*,3] // copy column of C

- Gather (indexed) values from one array

A = B[X] // A now is [3.3, 0.0, 4.4, 2.2, 1.1]

- Scatter (indexed) values from one array

A[X] = B // A now is [1.1, 4.4, 3.3, 0.0, 2.2]

Questions?

What if X = [0,0,0,0,0]

# Data Parallel Programming: Masks

- Can apply operations under a “mask”

**M** = array of 0/1 (True/False)

**A** = array

**B** = array

**A = A + B** under **M**

**A:**

3	1	1	2	3	3	4	2	2	2	1	3	1	1	1	3	3	2	1
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

**B:**

0	1	1	4	1	0	2	1	4	3	1	0	1	1	2	3	5	3	2
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

**M:**

1	0	0	1	1	0	0	0	1	1	1	1	0	0	1	0	0	1	0
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---



**+ under mask**

**A:**

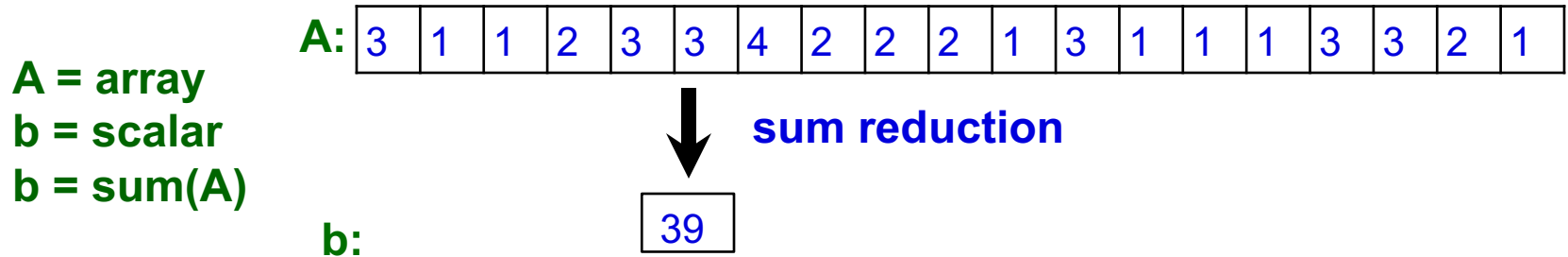
0			4	1				4	3	1	0			2			3	
3	1	1	6	4	3	4	2	6	5	2	3	1	1	3	3	3	5	1

- Related: Segmented scans to be presented later



# Data Parallel Programming: Reduce

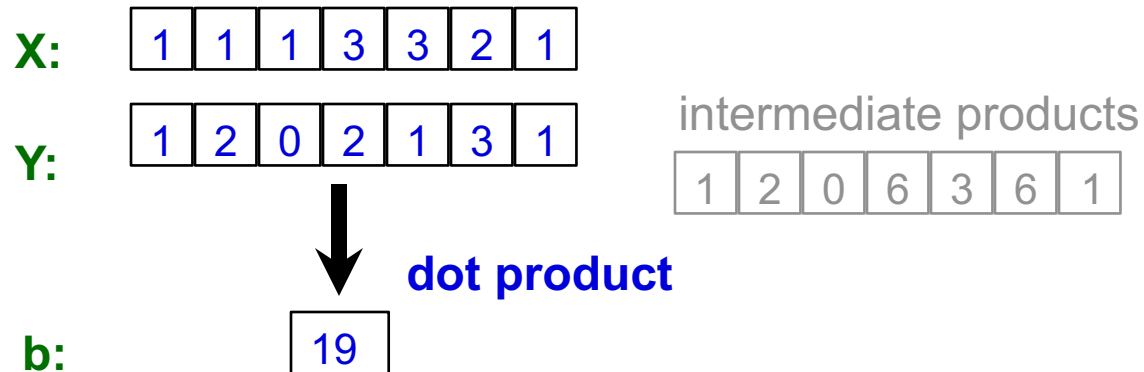
- Reduce an array to a value with + or any associative op



- Associative so we can perform op in different order
- Useful for dot products (ddot, sdot, etc.)

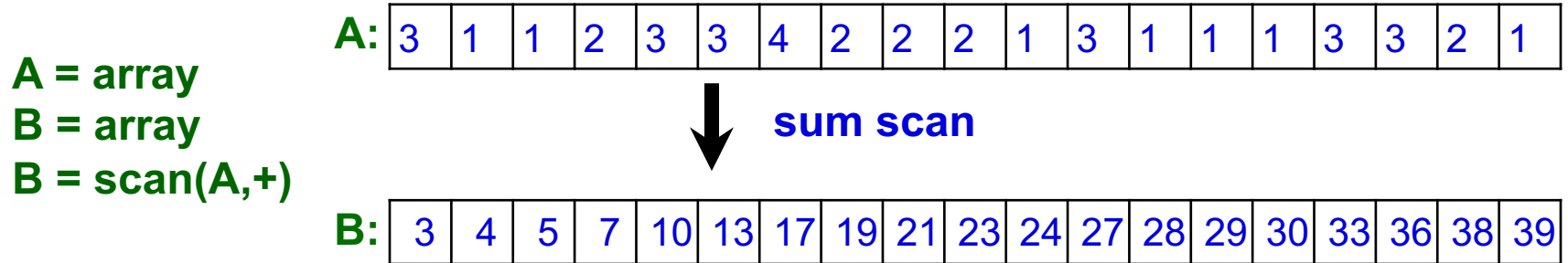
$$\mathbf{b} = \mathbf{X}^T \mathbf{Y} = \sum_j \mathbf{X}[j] * \mathbf{Y}[j]$$

$$\mathbf{b} = \text{dot}(\mathbf{X}, \mathbf{Y}) = \text{sum}(\mathbf{X} .* \mathbf{Y})$$

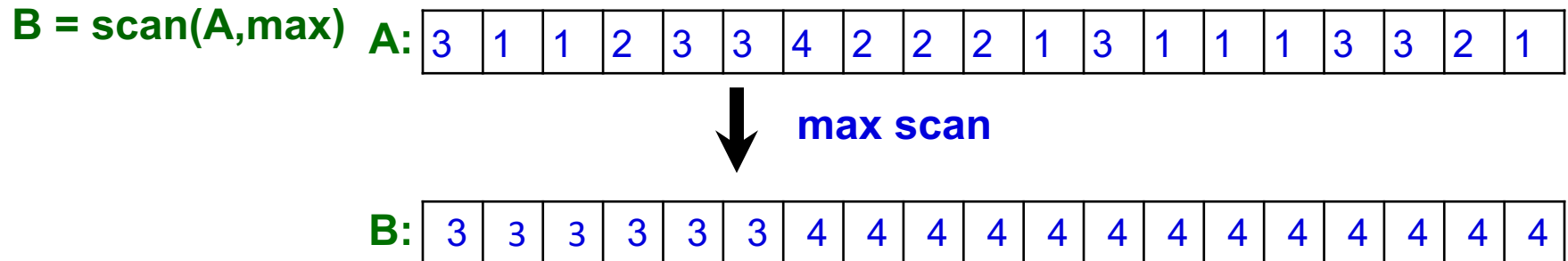


# Data Parallel Programming: Scans

- Fill array with partial reductions any associative op
- Sum scan:



- Max scan:



# Inclusive and Exclusive Scans

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Two variations of a scan, given an input vector  $[x_0, x_1, \dots, x_{n-1}]$ :

- **inclusive** scan includes input  $x_i$  when computing output  $y_i$

$$[a_0, (a_0 \odot a_1), \dots, (a_0 \odot a_1 \dots \odot a_{n-1})]$$

e.g., `add_scan_inclusive([1, 0, 3, 0, 2])`  $\rightarrow$  `[1, 1, 4, 4, 6]`

- **exclusive** scan does *not*  $x_i$  when computing output  $y_i$

$$[I, a_0, (a_0 \odot a_1), \dots, (a_0 \odot a_1 \dots \odot a_{n-2})] \text{ where } I \text{ is the identity for } \odot$$

e.g., `add_scan_exclusive([1, 0, 3, 0, 2])`  $\rightarrow$  `[0, 1, 1, 4, 4]`

Note that inclusive version from the exclusive by applying the operation across vectors: `scan_inclusive(X) = X  $\odot$  scan_exclusive(X)`.

To go the other way you need an inverse for  $\odot$  (**- for +**)

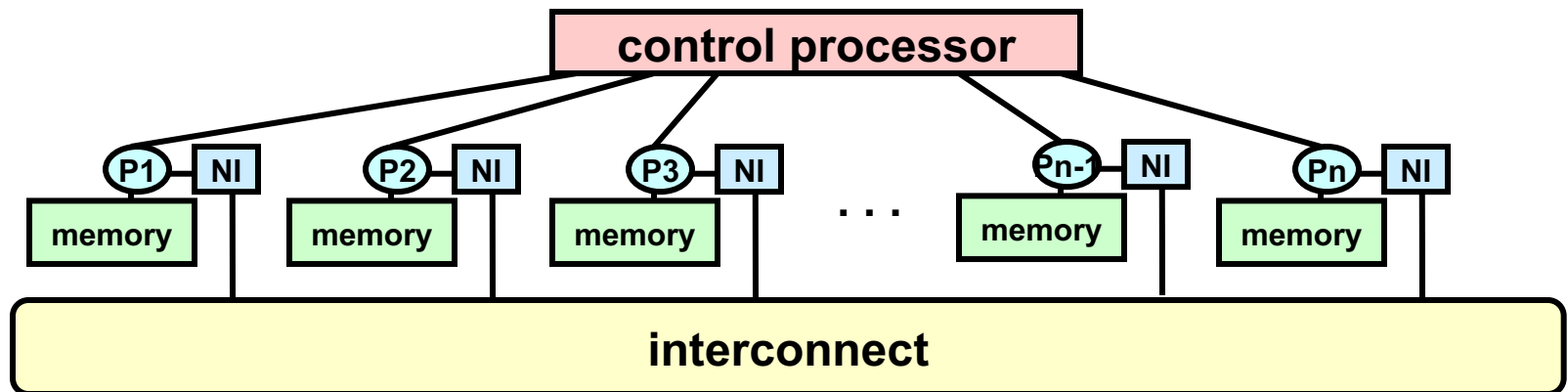
You can convert both directions using vector shifts left or right.

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# **Idealized Hardware and Performance Model**

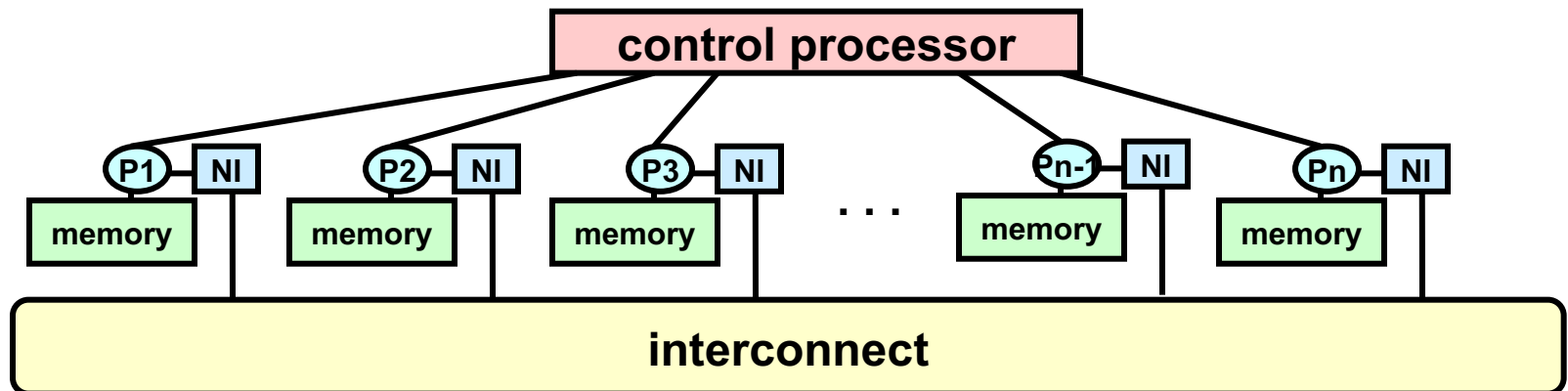
# SIMD Systems Implemented Data Parallelism

- A large number of (usually) tiny processors.
  - **A single “control processor” issues each instruction.**
  - **Each processor executes the same instruction.**
  - **Some processors may be turned off on some instructions.**
- Originally machines were specialized to scientific computing, few made (CM2, Maspar)



# Ideal Cost Model for Data Parallelism

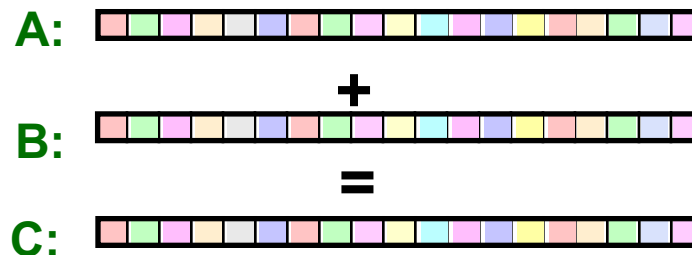
- Machine
  - An unbounded number of processors ( $p$ )
  - Control overhead is free
  - Communication is free
- Shows the inherent parallelism (inherent serialization)
- Called the algorithm's "span"
- Defines a lower bound on real machines



# Cost on Ideal Machine (Span)

- Span for unary or binary operations (pleasingly parallel)

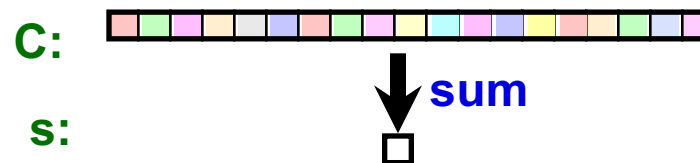
$$C = A + B$$



Cost  $O(1)$   
since  $p$  is unbounded

- Even if arrays are not aligned, communication is “free” here
- Reductions and broadcasts

$$s = \text{sum}(C)$$

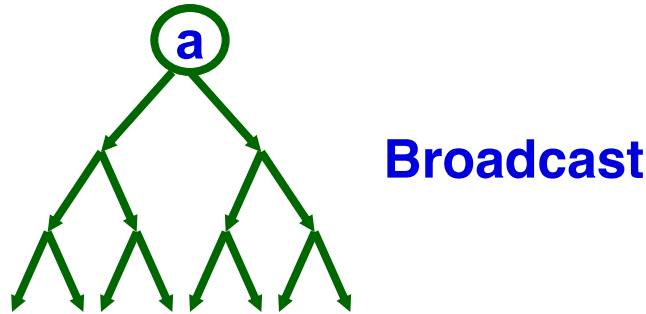


Cost  $O(\log(n))$

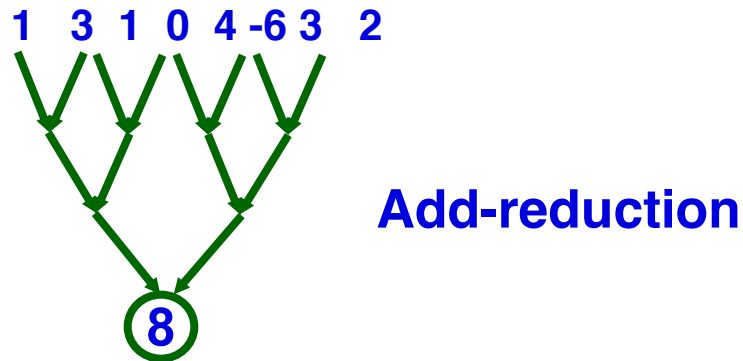
Using a tree of  
processors

# Broadcast and reduction use processor trees

- **Broadcast** of 1 value to  $p$  processors with  $\log n$  span



- **Reduction** of  $n$  values to 1 with  $\log n$  span
- Takes advantage of associativity in  $+$ ,  $*$ ,  $\min$ ,  $\max$ , etc.





# Important of Associativity

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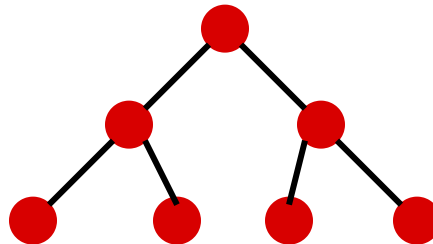
- Is it “OK” to do a reduction on floating point values?
- Neither + nor \* are associative in floating point arithmetic due to rounding

$$(1+10^{20})+ -10^{20} \neq 1+ (10^{20} + -10^{20})$$

- Answer: You won't get the same answer, and may get different ones (up to associativity) on different machines
- Often OK, i.e., floating point +/\* treated as associative
- MPI (coming soon) assumes associativity in reductions
- SPARK assumes associativity and commutativity

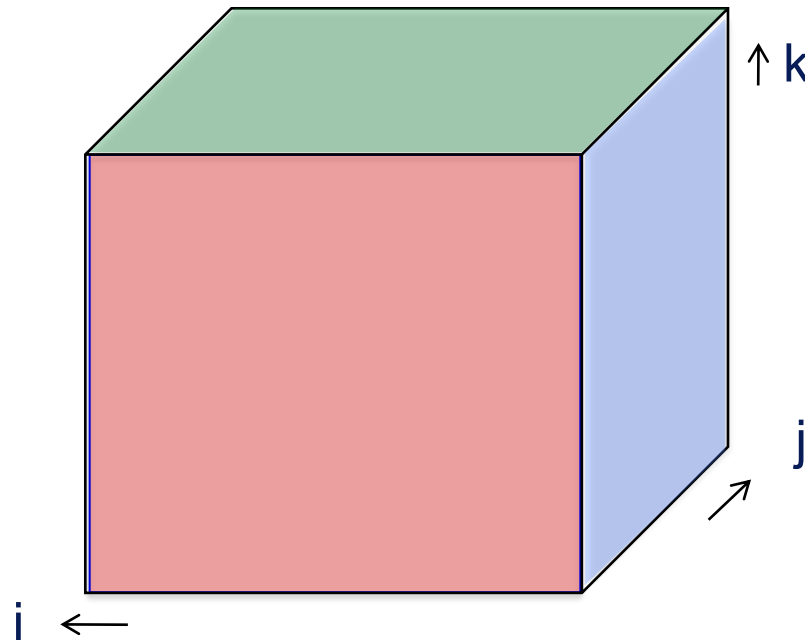
# Can reductions go faster: No, $\log n$ lower bound on any function of $n$ variables!

- Given a function  $f(x_1, \dots, x_n)$  of  $n$  input variables and 1 output variable, how fast can we evaluate it in parallel?
- Assume we only have binary operations, one per time step
- After 1 time step, an output can only depend on two inputs
- Use induction to show that after  $k$  time units, an output can only depend on  $2^k$  inputs
  - After  $\log_2 n$  time units, output depends on at most  $n$  inputs
- A binary tree performs such a computation



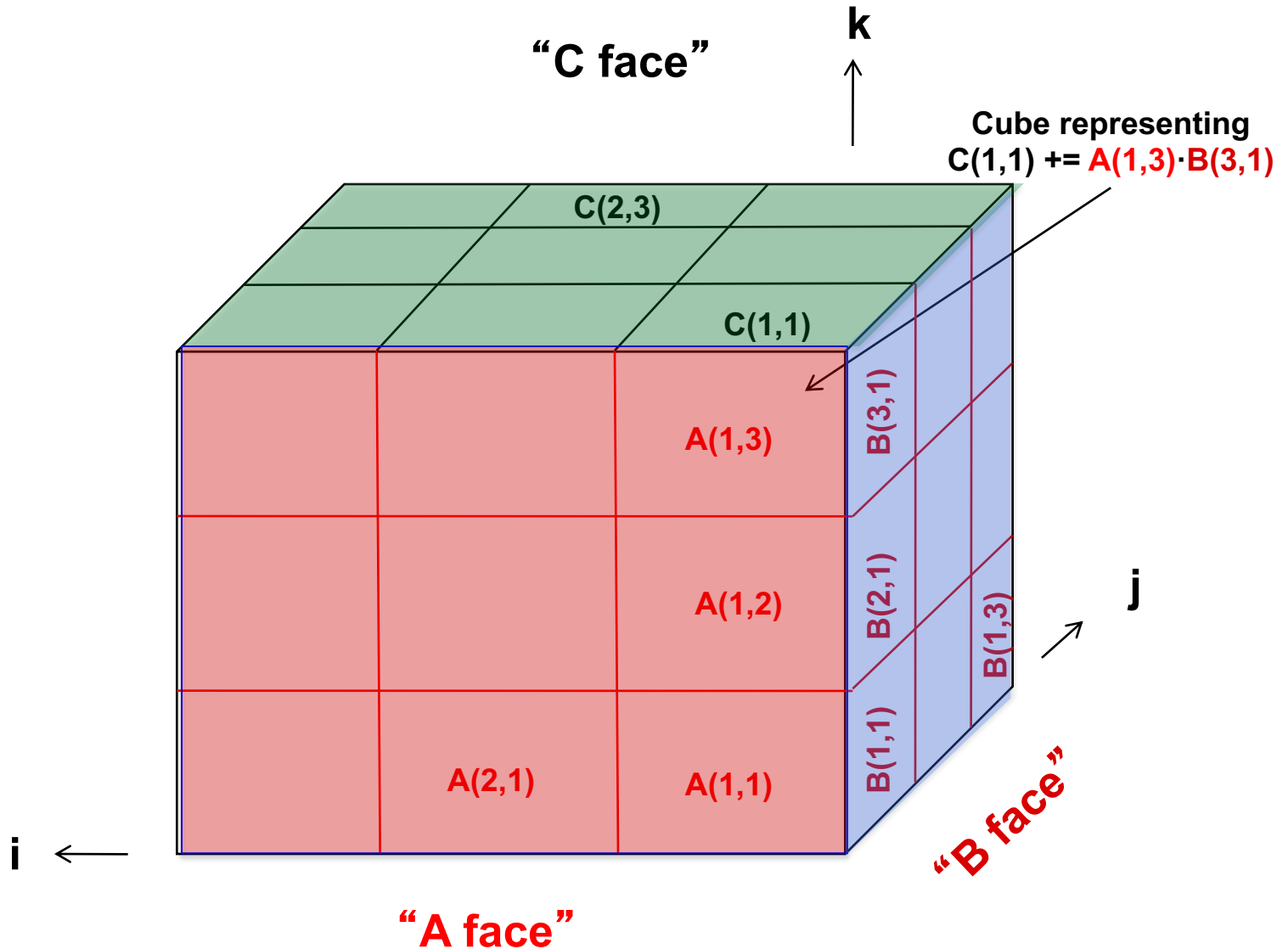
# Multiplying n-by-n matrices in $O(\log n)$ time

- Use  $n^3$  processors
- Step 1: For all  $(1 \leq i, j, k \leq n)$   $P(i, j, k) = A(i, k) * B(k, j)$ 
  - cost = 1 time unit, using  $n^3$  processors
- Step 2: For all  $(1 \leq i, j \leq n)$   $C(i, j) = \sum_{k=1}^n P(i, j, k)$ 
  - cost =  $O(\log n)$  time, using  $n^2$  trees,  $n^3 / 2$  processors each



**Put a processor  
at every point in  
this cube**

# Related to Communication-Optimal “2.5D” MatMul



- Processors execute internal sub-cubes

# What about Scan (aka Parallel Prefix)?

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- Recall: the **scan** operation takes a **binary associative** operator  $\odot$ , and an array of  $n$  elements

$$[a_0, a_1, a_2, \dots a_{n-1}]$$

and produces the array

$$[a_0, (a_0 \odot a_1), \dots (a_0 \odot a_1 \dots \odot a_{n-1})]$$

- Example: **add scan** of

$$[1, 2, 0, 4, 2, 1, 1, 3] \quad \text{is} \quad [1, 3, 3, 7, 9, 10, 11, 14]$$

- Other operators
  - Reals:  $+$ ,  $*$ ,  $\min$ ,  $\max$  (in floating point will assume associative)
  - Booleans:  $\text{and}$ ,  $\text{or}$
  - Matrices:  $\text{mat mul}$

# Can we parallelize a scan?

---

- It looks like this:

```
y(0) = 0;  
for i = 1:n  
    y(i) = y(i-1) + x(i);
```

- Takes  $n-1$  operations (adds) to do in serial
- The  $i^{\text{th}}$  iteration of the loop depends completely on the  $(i-1)^{\text{st}}$  iteration.
- Impossible to parallelize, right?

# A clue

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input = ( 1, 2, 3, 4, 5, 6, 7, 8 )

output = ( 1, 3, 6, 10, 15, 21, 28, 36 )

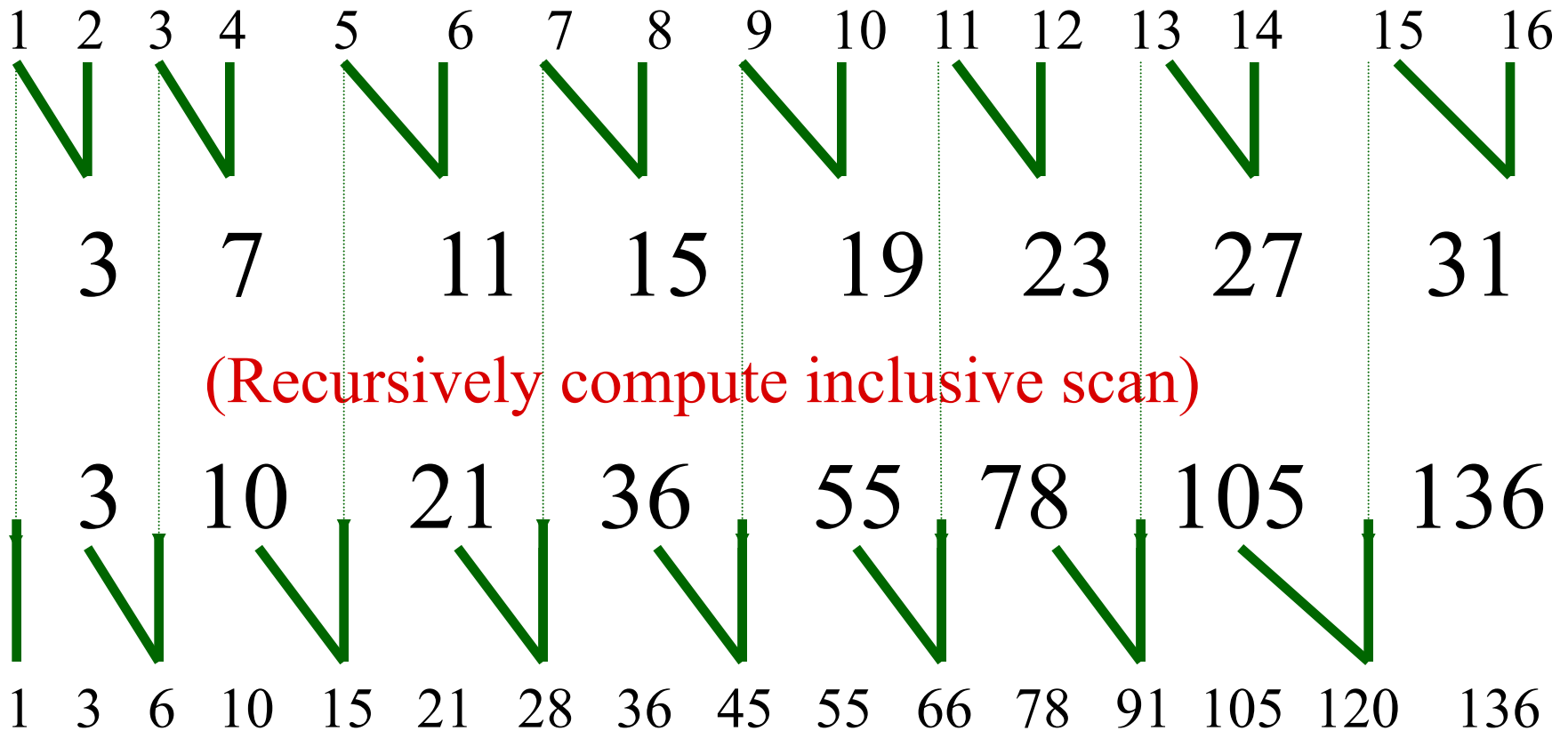
Is there any value in adding, say,  $5+6+7+8$ ?

If we separately have  $1+2+3+4$ , what can we do?

Suppose we added  $1+2$ ,  $3+4$ , etc. pairwise -- what could we do?

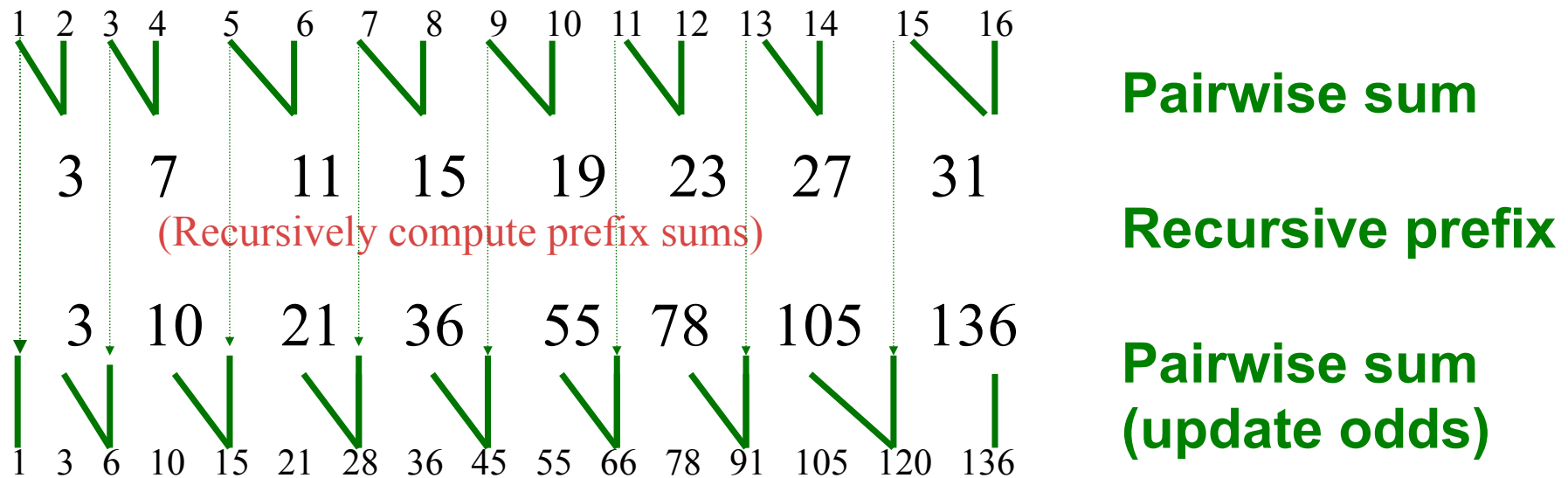
# Sum Scan (aka prefix sum) in parallel

**Algorithm:** 1. Pairwise sum 2. Recursive prefix 3. Pairwise sum





# Scan (parallel prefix) cost



Time for this algorithm on one processor (work)

- $T_1(n) = n/2 + n/2 + T_1(n/2) = n + T_1(n/2) = 2n - 1$

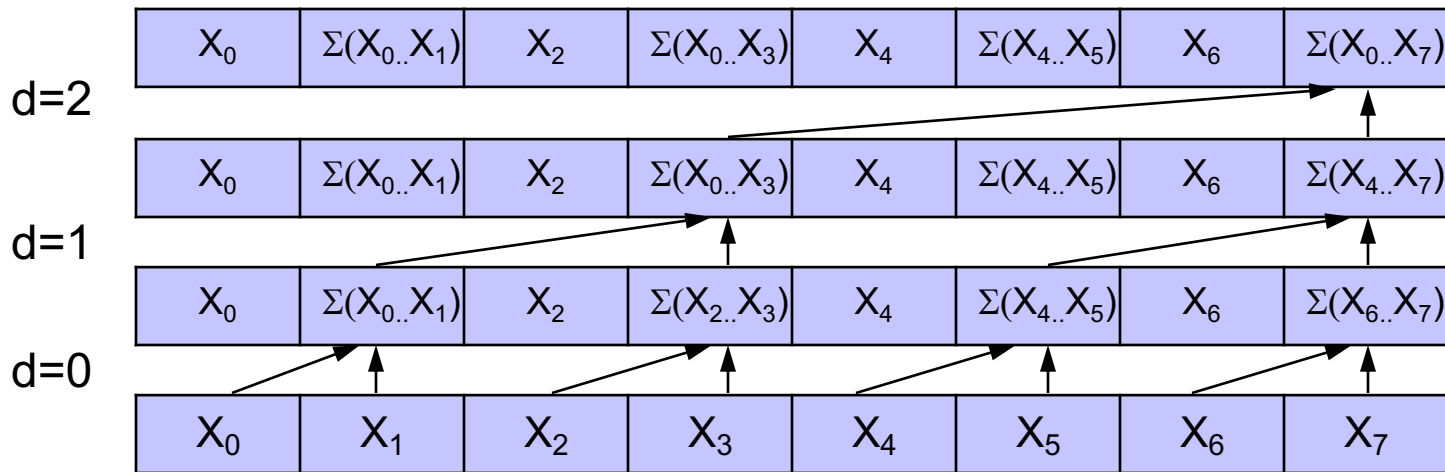
Time on unbounded number of processors (span)

- $T_\infty(n) = 2 \log n$

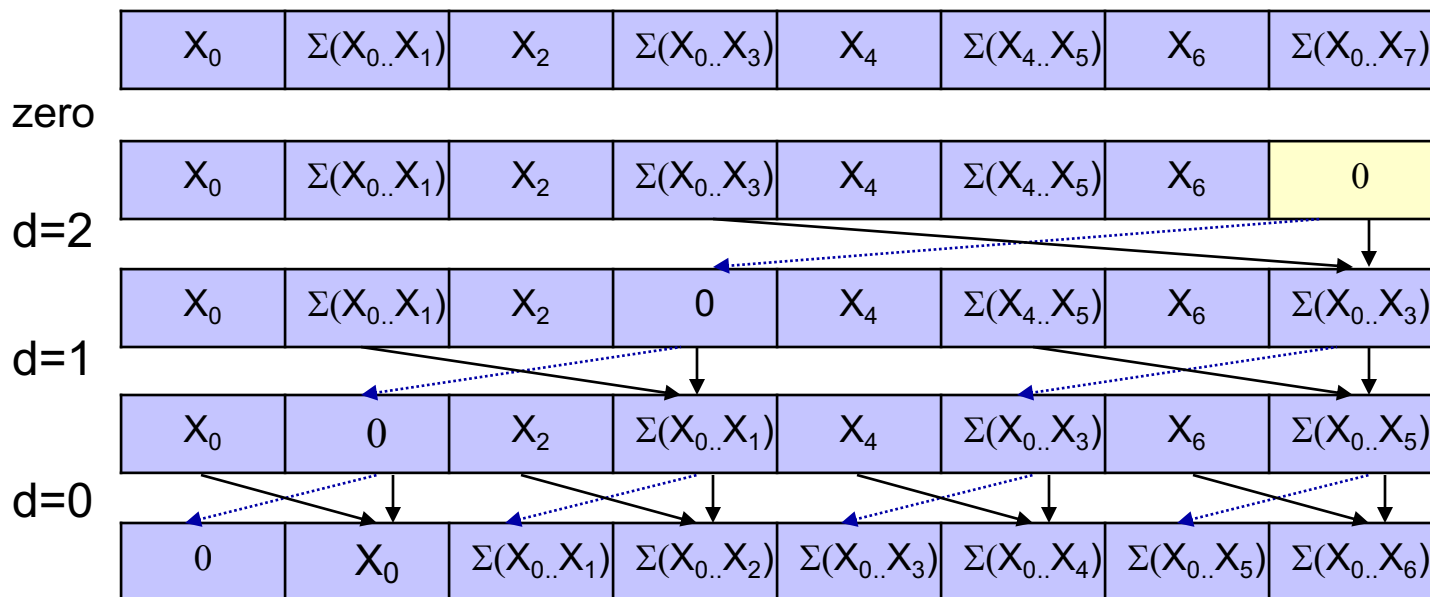
**Parallelism at the cost of more work (2x)!**

# Non-recursive view of parallel prefix scan

Up-sweep



Down-sweep



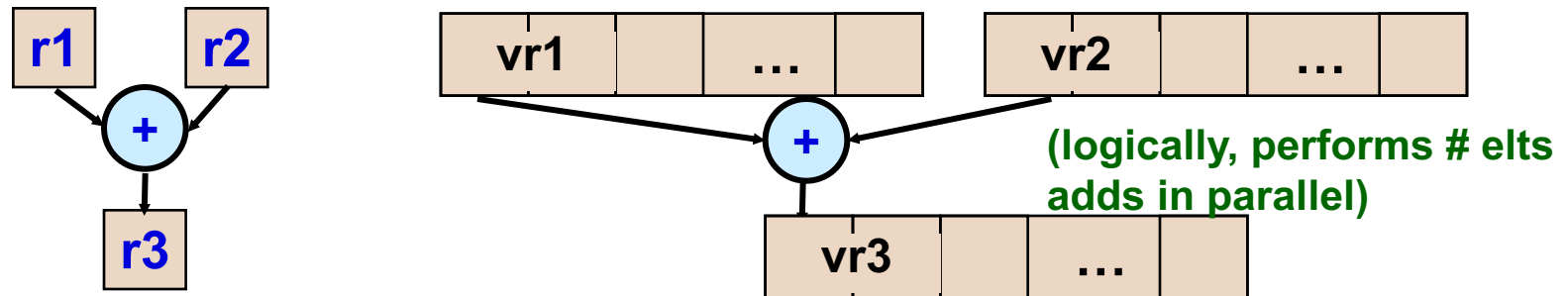
*This is both work-efficient ( $n$  adds) and space-efficient (update in place)*

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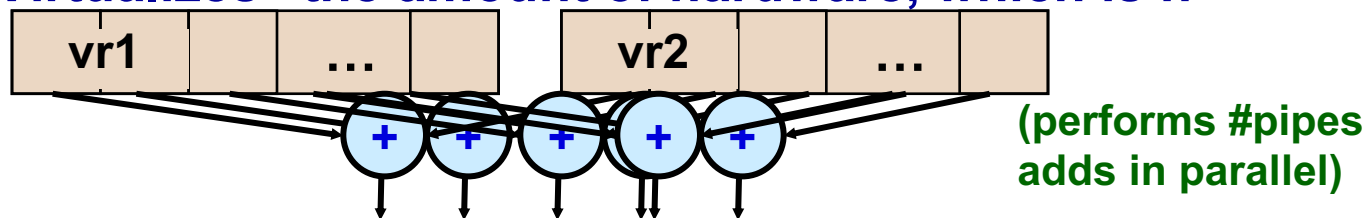
# Real Hardware (Today)

# Vector Machines Use Data Parallelism

- Vector instructions operate on a vector of elements
  - These are specified as operations on vector registers



- Old supercomputer vector register: ~32-64 elts
  - The number of elements is larger than the amount of parallel hardware, called vector **pipes** or **lanes**, say 2-4
- The hardware performs a full vector operation in
  - #elements-per-vector-register / #pipes steps
  - E.g., 64 elements in register, but only 8 fp adders to use
  - “Virtualizes” the amount of hardware, which is  $n$

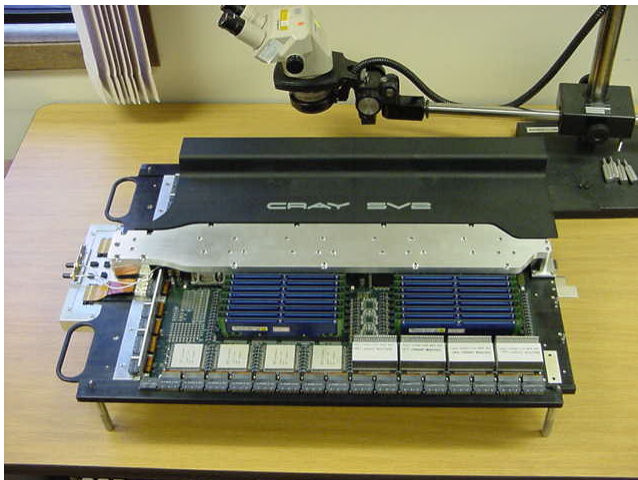


# **Cray X1: *Parallel Vector Architecture***

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Cray combined several technologies in the X1

- **12.8 Gflop/s Vector processors (MSP)**
- **Shared caches (unusual on earlier vector machines)**
- **4 processor nodes sharing up to 64 GB of memory**
- **Single System Image to 4096 Processors**
- **Remote put/get between nodes (faster than MPI)**



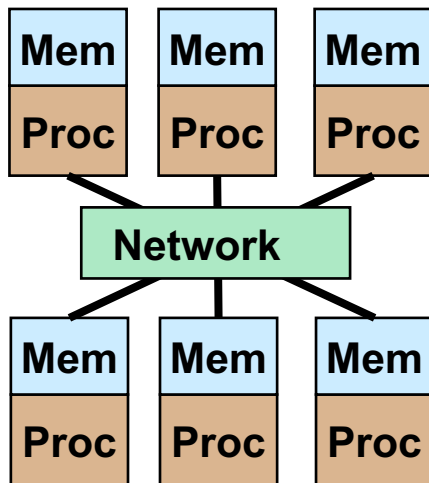
- **Expensive to design and build, market too small**

# **SIMD instructions use Data Parallelism**

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- **SIMD instructions on microprocessors are vectors**
  - Shorter than old vector supercomputers (e.g., 256 bits)
  - They don't virtualize the hardware (arithmetic units), so each processor version may require code rewrites
- **Reductions and broadcasts are in register**
  - Require inside-register data movement
- **Assuming vector length (or SIMD width) are small constants → no theoretical speedup**
  - But in practice this can make a big different (2-16x...)
  - And algorithms may still be useful
- **Revisit these ideas with GPUs**

# Data parallelism on Distributed Memory



## Distributed Memory

Processors execute  
own instruction stream

Communicate by  
sending messages

Message time depends  
on size, but not location

- **Today's parallel machines**
  - Powerful processors
  - Distributed memory (at scale)
  - Clusters or MPPs (Massively Parallel Processors)
- **Need to map n-way parallelism to p-way**
  - Attempts to do this automatically
- **High Performance Fortran**
  - Large effort in the 90s
  - Semi-automatic: Data layout hints were necessary
  - And it was still hard
- **But still useful manually**

# Mapping Data Parallelism to Clusters

- Binary and unary operations on MPPs

$$C = A + B$$

A: 

+

B: 

=

C: 

Cost  $O(n/p)$

$p$  speedup

- If arrays are not “aligned” then communication required
- Reductions and broadcasts

$$s = \text{sum}(C)$$

C: 



local sum



s:



tree reduction

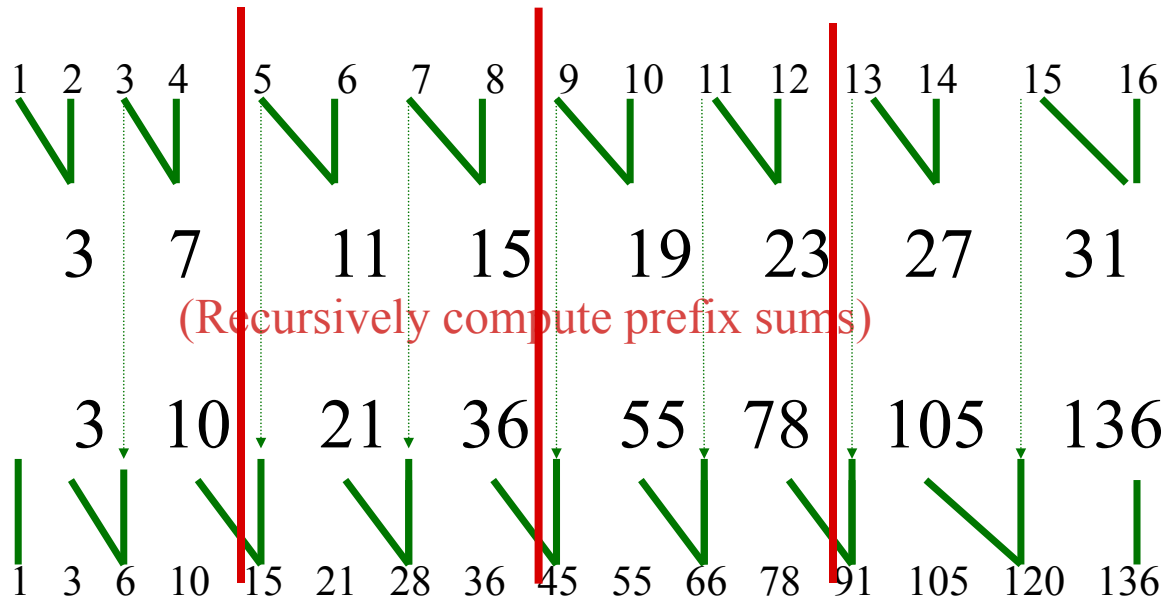
Cost  $O(n/p)$

+  $\log p$ )

Almost  $p$  speedup



# Parallel prefix cost on p processors



**Compute local  
prefix sums in  
 $n/p$  steps**

**Updates across  
processors in  
 $\log p$  steps**

Time for this algorithm in parallel:

- $T_p(n) = O(n/p + \log p)$

**serial time  
on each  
processor**

**communication and  
computation up and  
down the processor tree**

# The myth of $\log n$

- The  $\log_2 n$  span is **not** the main reason for the usefulness of parallel prefix.

- **Say  $n = k \cdot p$**  ( $k = 1,000,000$  elements per proc)

• Cost =  $(k \text{ adds})$  +  $(\log_2 P \text{ steps})$  +  $(k \text{ adds})$

↑  
compute and store  $k$   
values  $a[0]..a[k-1]$

↑  
parallel scan on  
 $a[k-1]$  values

↑  
add 'my' scan result  
to  $a[0]..a[k-1]$

(2,000,000 local adds are serial for each processor, of course)

**Key to implementing data parallel algorithms on clusters, SMPs, MPPs, i.e., modern supercomputers**

# Data Parallelism is an Elegant Programming Model

- Strict data parallelism has serial semantics:
  - E.g., no difference from executing  $A+B$  one element at a time or in parallel
- Reductions also preserve serial semantics for truly associative operations:
  - $+$   $*$   $\min$ , etc. on integers and more;
  - some differences for floating point due to order of evaluation (but can be deterministic, i.e., the same result every time)
- Easy to understand and reason about
- “In spirit” in MPI collectives, CUDA, MapReduce...

## Limitations:

- Some algorithms (e.g., adaptive) don't fit easily
- Non-trivial to implement on some hardware

# Scans are useful for many things (partial list here)

- Reduction and broadcast in  $O(\log n)$  time
- Parallel prefix (scan) in  $O(\log n)$  time
- Adding two  $n$ -bit integers in  $O(\log n)$  time
- Multiplying  $n$ -by- $n$  matrices in  $O(\log n)$  time
- Inverting  $n$ -by- $n$  triangular matrices in  $O(\log^2 n)$  time
- Inverting  $n$ -by- $n$  dense matrices in  $O(\log^2 n)$  time
- Evaluating arbitrary expressions in  $O(\log n)$  time
- Evaluating recurrences in  $O(\log n)$  time
- “2D parallel prefix”, for image segmentation (Catanzaro & Keutzer)
- Sparse-Matrix-Vector-Multiply (SpMV) using Segmented Scan
- Parallel page layout in a browser (Leo Meyerovich, Ras Bodik)
- Solving  $n$ -by- $n$  tridiagonal matrices in  $O(\log n)$  time
- Traversing linked lists
- Computing minimal spanning trees
- Computing convex hulls of point sets...

# Application: Stream Compression

- Given an array of 0/1 flags

flags = 

1	0	1	1	0	0	1	1
---	---	---	---	---	---	---	---

and an array (stream) of values

values = 

3	2	4	1	5	3	3	1
---	---	---	---	---	---	---	---

compress into

result = 

3	4	1	3	1
---	---	---	---	---

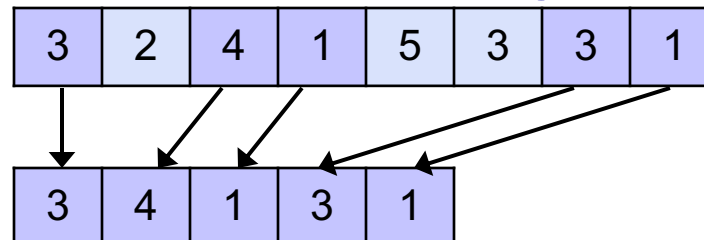
- Step 1: Compute an exclusive add scan of flags:

index = 

0	1	1	2	3	3	3	4
---	---	---	---	---	---	---	---

- Step 2: “Scatter” values into result at index, masked by flags

result[index] = values at flags



## Application: Radix Sort (serial algorithm to start)

---

4	7	2	6	3	5	1	0
---	---	---	---	---	---	---	---

Sort on least significant bit ([Bit<sub>2</sub>, Bit<sub>1</sub>, Bit<sub>0</sub>])

4	2	6	0	7	3	5	1
---	---	---	---	---	---	---	---

XX0 < XX1 (evens before odds)

Bit<sub>0</sub>=0      Bit<sub>0</sub>=1

4	2	6	0	7	3	5	1
---	---	---	---	---	---	---	---

Stably sort entire array on next bit

4	0	5	1	2	6	7	3
---	---	---	---	---	---	---	---

X0X < X1X

Bit<sub>1</sub>=0      Bit<sub>1</sub>=1

4	0	5	1	2	6	7	3
---	---	---	---	---	---	---	---

Stably sort on next bit

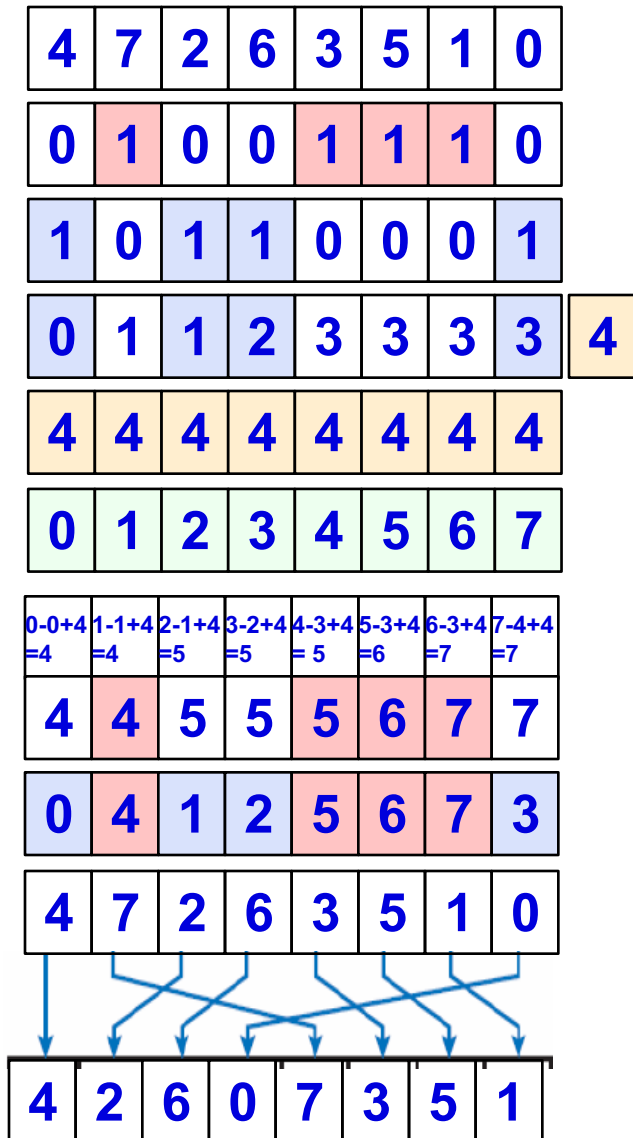
0	1	2	3	4	5	6	7
---	---	---	---	---	---	---	---

0XX < 1XX (<4 before >=4)

Bit<sub>2</sub>=0      Bit<sub>2</sub>=1

*A “stable” sort means it preserves the ordering. unless they have to switch based on the current bit*

# Application: Radix Sort



input

odds = last bit of each element

evens = complement of odds (last bit = 0)

epos = exclusive sum scans of evens

totalEvens = broadcast last element

indx = constant array of 0..n

oddpos = indx + totalEvens – epos

pos = if evens then esum else oddpos

*Using two  
masked  
assignments*

Scatter input using pos as index

Repeat with next bit to left until done

# Application: Adding n-bit integers in $O(\log n)$ time

- Computing sum  $s$  of two  $n$ -bit binary numbers,  $a$  and  $b$ 
  - $a = a[n-1] a[n-2] \dots a[0]$  and  $b = b[n-1] b[n-2] \dots b[0]$
  - $s = a+b = s[n] s[n-1] \dots s[0]$  (use carry-bit array  $c = c[n-1] \dots c[0] c[-1]$ )

- Formula

$c[-1] = 0$  ... rightmost carry bit

for  $i = 0$  to  $n-1$  ... compute right to left

$s[i] = (a[i] \text{ xor } b[i]) \text{ xor } c[i-1]$  ... one or three 1s

$c[i] = ((a[i] \text{ xor } b[i]) \text{ and } c[i-1]) \text{ or } (a[i] \text{ and } b[i])$  ... next carry bit

- Example

- $a = 22$

- $b = 29$

$a = 1\ 0\ 1\ 1\ 0$  (22)

$b = 1\ 1\ 1\ 0\ 1$  (29)

$c = 1\ 1\ 1\ 0\ 0\ 0\ 0$

$s = 1\ 1\ 0\ 0\ 1\ 1$  (51)

- Challenge: compute all  $c[i]$  in  $O(\log n)$  time via parallel prefix



# Application: Adding n-bit integers in $O(\log n)$ time

- Recall carry bit calculation

$c[-1] = 0$  ... rightmost carry bit

for  $i = 0$  to  $n-1$

$c[i] = ( (a[i] \text{ xor } b[i]) \text{ and } c[i-1] ) \text{ or } ( a[i] \text{ and } b[i] )$  ... next carry bit

- Compute all  $c[i]$  in  $O(\log n)$  time via parallel prefix

for all  $(0 \leq i \leq n-1)$   $p[i] = a[i] \text{ xor } b[i]$  ... propagate bit  
for all  $(0 \leq i \leq n-1)$   $g[i] = a[i] \text{ and } b[i]$  ... generate bit

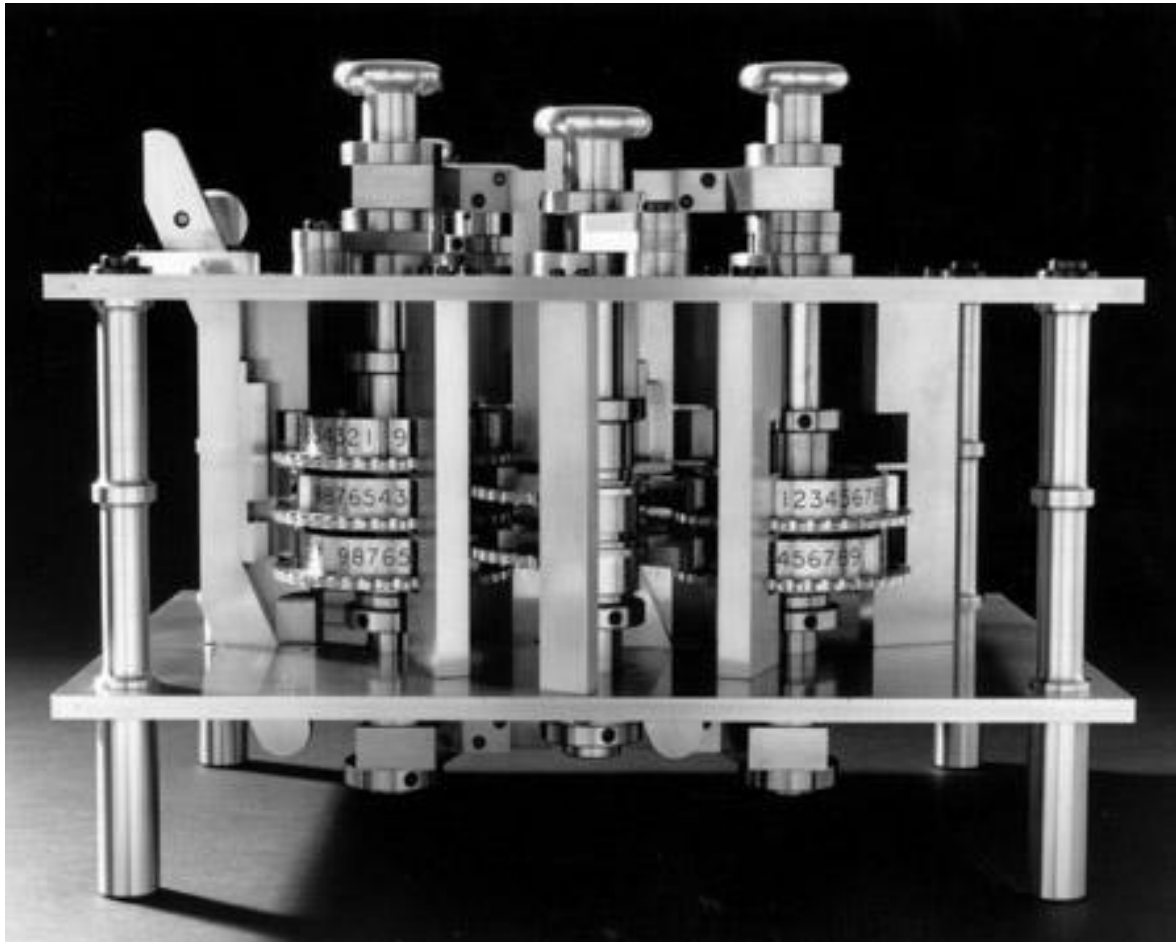
Both  $O(1)$   
on  $n$  procs

$$\begin{aligned} \begin{bmatrix} c[i] \\ 1 \end{bmatrix} &= \begin{bmatrix} (p[i] \text{ and } c[i-1]) \text{ or } g[i] \\ 1 \end{bmatrix} = \begin{bmatrix} p[i] & g[i] \\ 0 & 1 \end{bmatrix} * \begin{bmatrix} c[i-1] \\ 1 \end{bmatrix} = M[i] * \begin{bmatrix} c[i-1] \\ 1 \end{bmatrix} \\ &= M[i] * M[i-1] * \dots * M[0] * \begin{bmatrix} 0 \\ 1 \end{bmatrix} \end{aligned}$$

... evaluate  $M[i] * M[i-1] * \dots * M[0]$  by parallel prefix  
... 2-by-2 Boolean matrix multiplication is associative

- Used in all computers to -- Carry look-ahead addition

# This idea is used in all hardware



- Even going back to Babbage

# Segmented Scans

---

Inputs = value array, flag array,  
associative operator  $\oplus$

**Inclusive segmented sum scan**

1	2	3	4	5	6	7	8
0	0	1	0	0	1	0	1

**Flags are sometimes done with  
Boolean and switch points**

F	F	T	T	T	F	F	T
---	---	---	---	---	---	---	---

**Result**

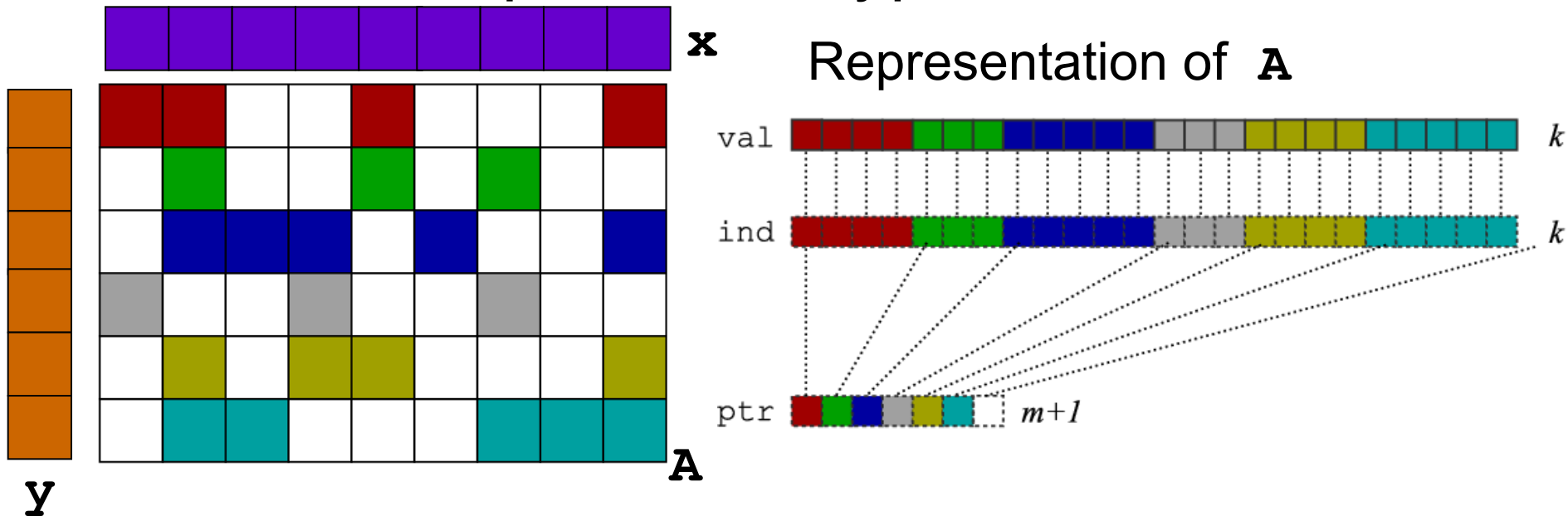
1	3	3	7	12	6	13	8
---	---	---	---	----	---	----	---

# SpMV in Compressed Sparse Row (CSR) Format

SpMV:  $y = y + A \cdot x$

Sparse matrices: only store, do arithmetic, on nonzero entries

CSR format is simplest one of many possible data structures for  $A$



Matrix-vector multiply kernel:  $y(i) \leftarrow y(i) + A(i,j) \times x(j)$

for each row  $i$

for  $k = ptr[i]$  to  $ptr[i+1] - 1$  do

$y[i] = y[i] + val[k] * x[ind[k]]$



# Application: Fibonacci via Matrix Multiply Prefix

---

$$\mathbf{F}_{n+1} = \mathbf{F}_n + \mathbf{F}_{n-1}$$

$$\begin{pmatrix} \mathbf{F}_{n+1} \\ \mathbf{F}_n \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{F}_n \\ \mathbf{F}_{n-1} \end{pmatrix}$$

**Can compute all  $\mathbf{F}_n$  by matmul\_prefix on**

$$\left[ \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \right]$$

**then select the upper left entry**

**Slide source: Alan Edelman**

# Lexical analysis (tokenizing, scanning)

- Given a language of:
  - Identifiers: string of chars
  - Strings: in double quotes
  - Ops: +, -, \*, =, <, >, <=, >=

TABLE I. A Finite-State Automaton for Recognizing Tokens

Old State	Character Read													New line
	A	B	...	Y	Z	+	-	*	<	>	=	"	Space	
N	A	A	...	A	A	*	*	*	<	<	*	Q	N	N
A	Z	Z	...	Z	Z	*	*	*	<	<	*	Q	N	N
Z	Z	Z	...	Z	Z	*	*	*	<	<	*	Q	N	N
*	A	A	...	A	A	*	*	*	<	<	*	Q	N	N
<	A	A	...	A	A	*	*	*	<	<	=	Q	N	N
=	A	A	...	A	A	*	*	*	<	<	*	Q	N	N
Q	S	S	...	S	S	S	S	S	S	S	S	E	S	S
S	S	S	...	S	S	S	S	S	S	S	S	E	S	S
E	E	E	...	E	E	*	*	*	<	<	*	S	N	N

- Lexical analysis
  - Replace every character in the string with the array representation of its state-to-state function (column).
  - Perform a parallel-prefix operation with  $\oplus$  as the array composition. Each character becomes an array representing the state-to-state function for that prefix.
  - Use initial state (row 1) to index into these arrays.

Hillis and Steele, CACM 1986

# Lessons from Data Parallel Languages

- Sequential semantics (or nearly) is very nice
  - Debugging is much easier without non-determinism
  - Correctness easier to reason about
- Cost model is independent of number of processors
  - How much inherent parallelism
- Need to “throttle” parallelism
  - $n \gg p$  can be hard to map, especially with nesting
  - Memory use is a problem

See: Blelloch “NESL Revisited”, Intel Workshop 2006