



Múltiples tiempos de relajación

Miller Mendoza ^{1,2} y José Daniel Muñoz

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¹

Basado grandemente en la presentación de
² Jaider Iván Pulido





Múltiples tiempos de relajación

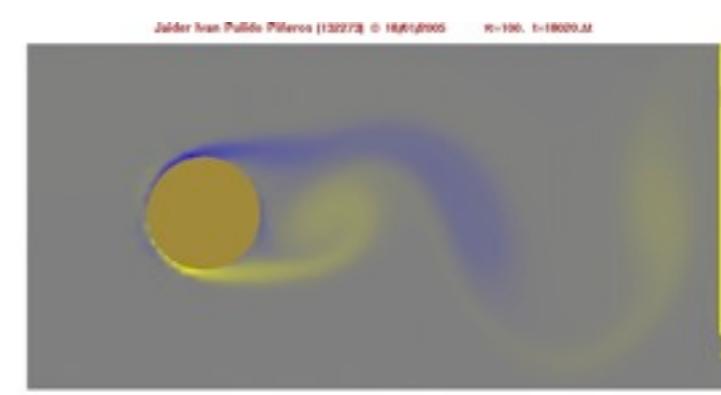
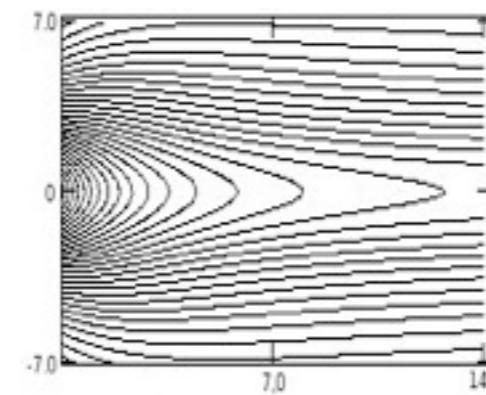
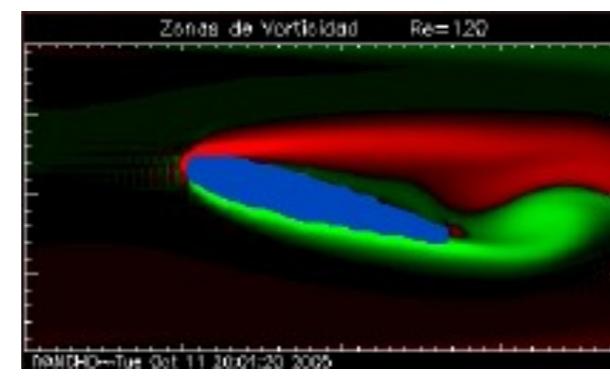
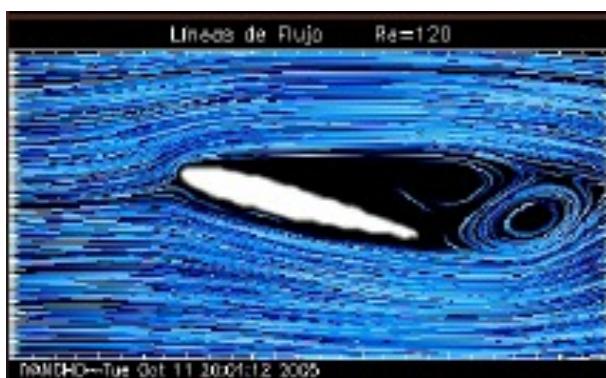
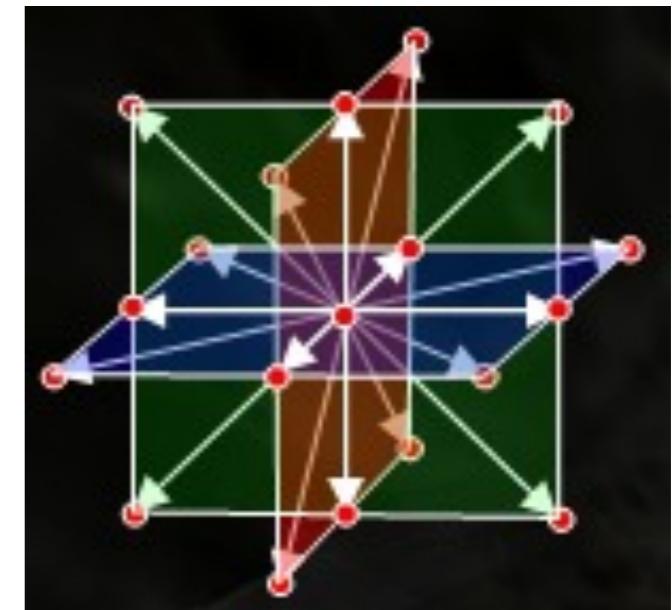
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1

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2

1



COLCIENCIAS



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Recordemos...

LBGK para fluidos

D2Q9

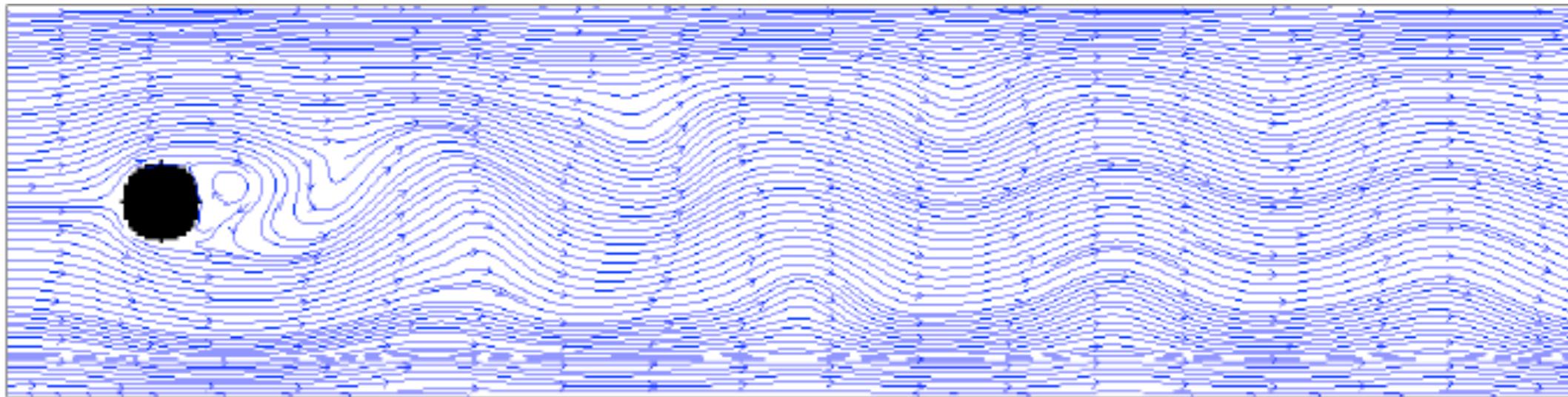
$$\rho = \sum_i f_i \quad , \quad \rho \vec{U} = \sum_i \vec{v}_i f_i$$

$$f_i^{(\text{eq})} = w_i \rho \left[1 + 3(\vec{U} \cdot \vec{v}_i) + \frac{9}{2}(\vec{U} \cdot \vec{v}_i)^2 - \frac{3}{2}U^2 \right]$$

$$P = c_s^2 \rho \quad \nu = \frac{1}{3} \left(\tau - \frac{1}{2} \right)$$

**Todo evoluciona
con un único
tiempo de relajación**

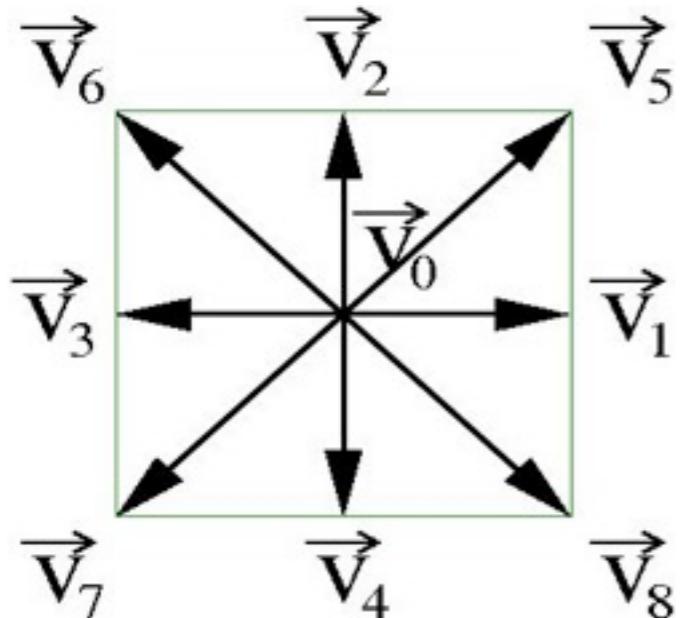
$$f_i(\vec{x} + \delta t, t + \delta t) = f_i(\vec{x}, t) - \frac{1}{\tau} \left[f_i(\vec{x}, t) - f_i^{(\text{eq})} \right]$$



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LBGK para fluidos

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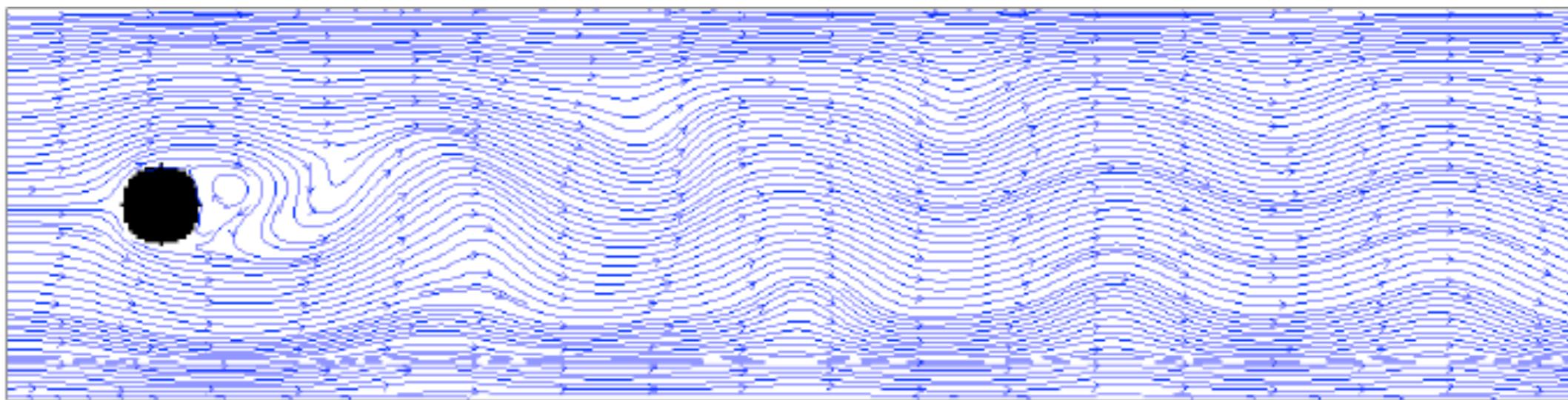
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Múltiples Tiempos de Relajación: LBM-MRT

d'Humieres (1992), P. Lallemand, L-S. Luo (2000)

Las 9 funciones f_i como un vector

$$|f(\mathbf{r}_j)\rangle \equiv (f_0, f_1, \dots, f_8)^\top$$

Se construyen 9 “momentos”

$$|\varrho\rangle \equiv (\rho, e, \varepsilon, j_x, q_x, j_y, q_y, p_{xx}, p_{xy})^\top$$

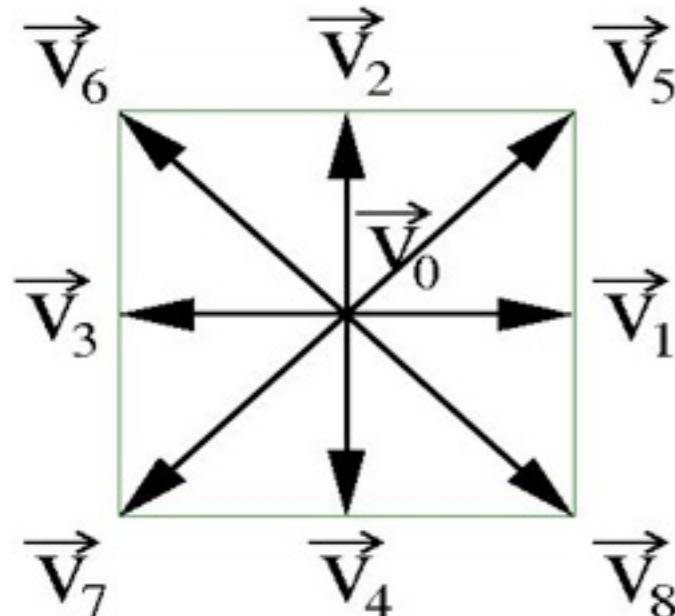
energía  calor   stress

definidos como

$$|\varrho\rangle = \mathbf{M}|f\rangle, \text{ con } \mathbf{M} = \begin{pmatrix} |\rho\rangle \\ |J_x\rangle \\ |J_y\rangle \\ |q_x\rangle \\ |q_y\rangle \\ |\epsilon\rangle \\ |e\rangle \\ |p_{xx}\rangle \\ |p_{xy}\rangle \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & -1 & -1 & -1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & -1 & -1 & -1 \\ 0 & -2 & 1 & 0 & -1 & 2 & -1 & 0 & 1 \\ 0 & 0 & 1 & -2 & 1 & 0 & -1 & 2 & -1 \\ 4 & -2 & 1 & -2 & 1 & -2 & 1 & -2 & 1 \\ -4 & -1 & 2 & -1 & 2 & -1 & 2 & -1 & 2 \\ 0 & 1 & 0 & -1 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 1 & 0 & -1 \end{pmatrix}$$

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energía calor stress

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Múltiples Tiempos de Relajación: LBM-MRT

d'Humieres (1992), P. Lallemand, L-S. Luo (2000)

Se asume que los “momentos” ρ y \vec{J} se conservan en las colisiones

$$\rho^{(eq)} = \rho \quad \vec{J}^{(eq)} = \vec{J}$$

Para cada “momento” no conservado se define un valor de equilibrio en función de ρ y \vec{J} :

$$\left| \begin{array}{l} \rho = \sum_i f_i^{(0)} \\ \vec{J} = \sum_i \vec{v}_i f_i^{(0)} \end{array} \right| \begin{array}{lll} q_x^{(eq)} = -J_x & e^{(eq)} = -2\rho + 3 \frac{(J_x^2 + J_y^2)}{\rho} & p_{xx}^{(eq)} = \frac{(J_x^2 - J_y^2)}{\rho} \\ q_y^{(eq)} = -J_y & \epsilon^{(eq)} = \rho - 3 \frac{(J_x^2 + J_y^2)}{\rho} & p_{xy}^{(eq)} = \frac{(J_x J_y)}{\rho} \end{array}$$

Múltiples Tiempos de Relajación: LBM-MRT

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La idea central es que cada “momento” relaja con un tiempo diferente:

$$|\varrho_{\text{new}}\rangle = |\varrho\rangle - S \left(|\varrho\rangle - |\varrho^{(eq)}\rangle \right)$$

con

$$S = \text{diag}(0, 0, 0, s_3, s_4, s_5, s_6, s_7, s_8)$$

Múltiples Tiempos de Relajación: LBM-MRT

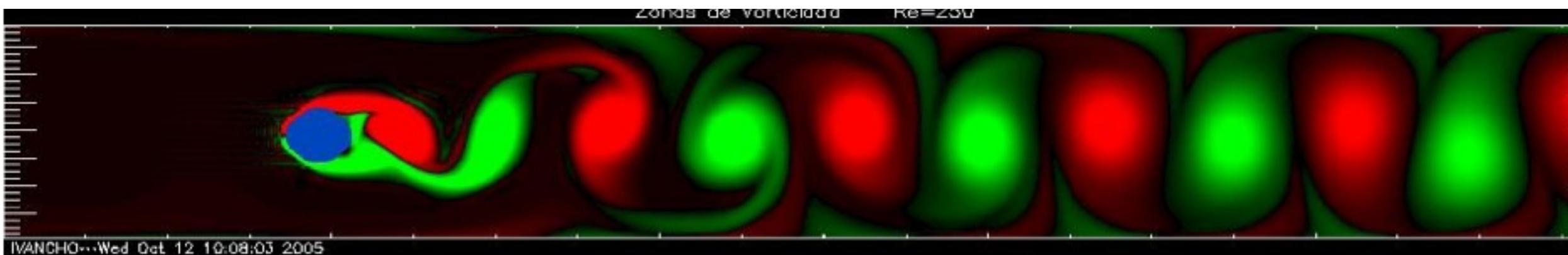
d'Humieres (1992), P. Lallemand, L-S. Luo (2000)

Los valores S se escogen para mantener isotropía y otras propiedades deseadas

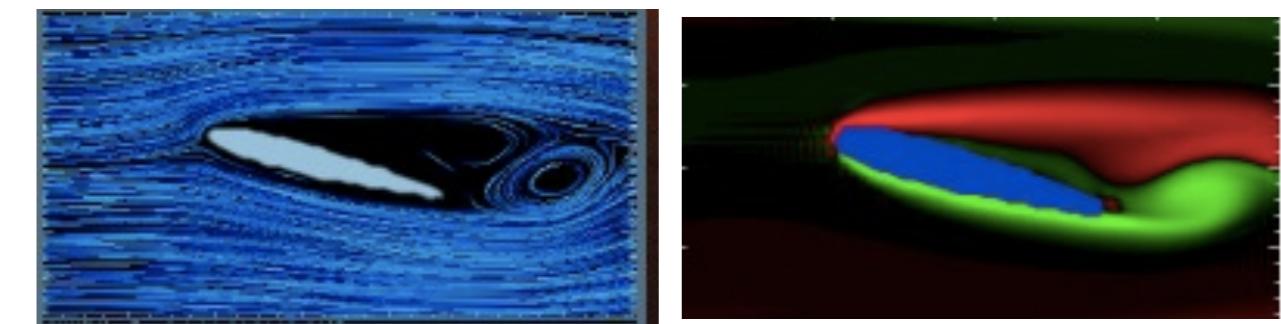
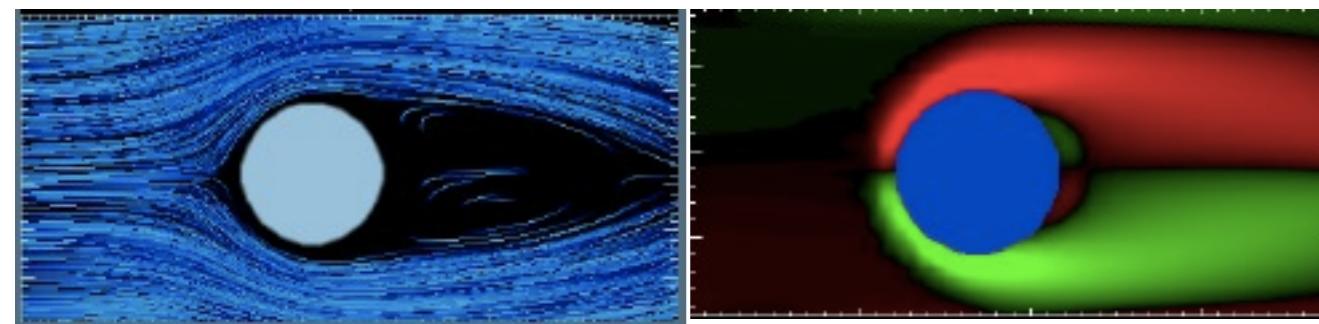
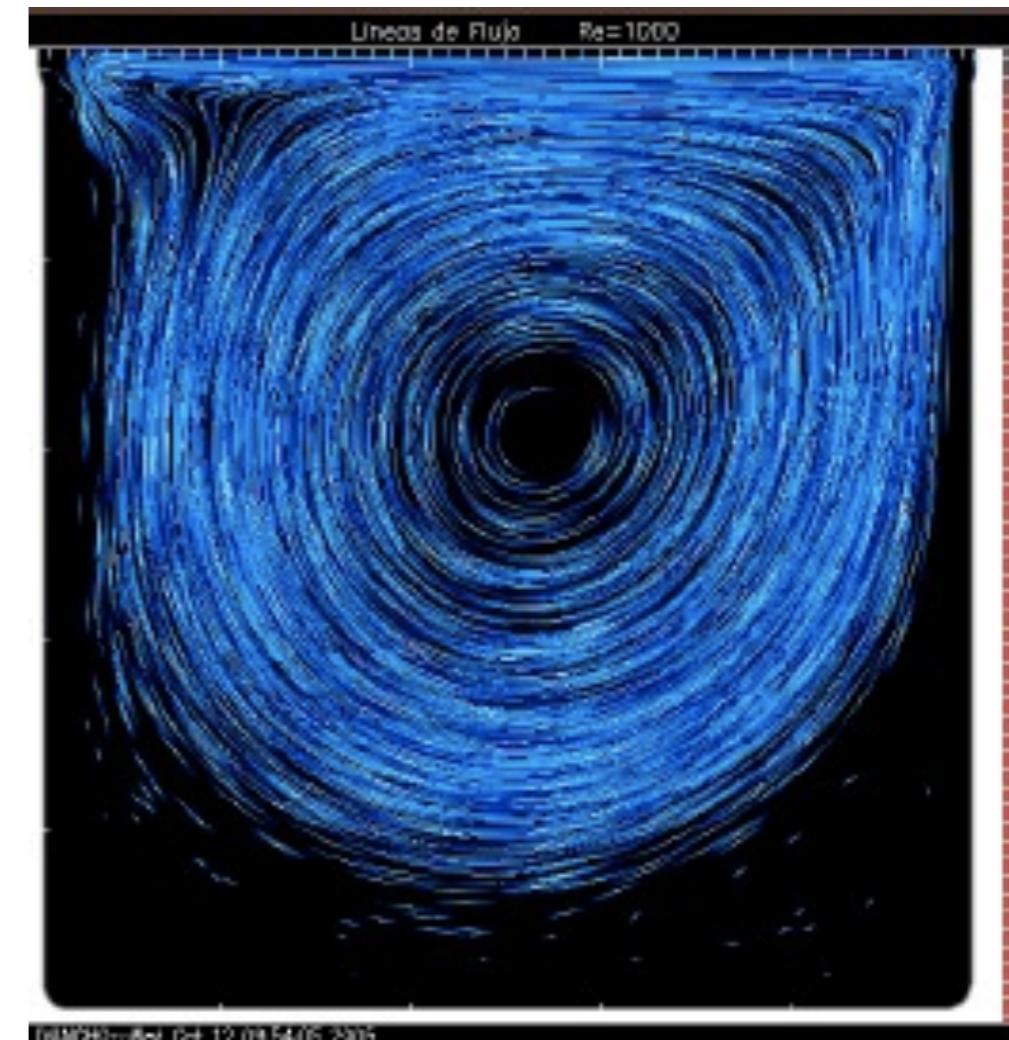
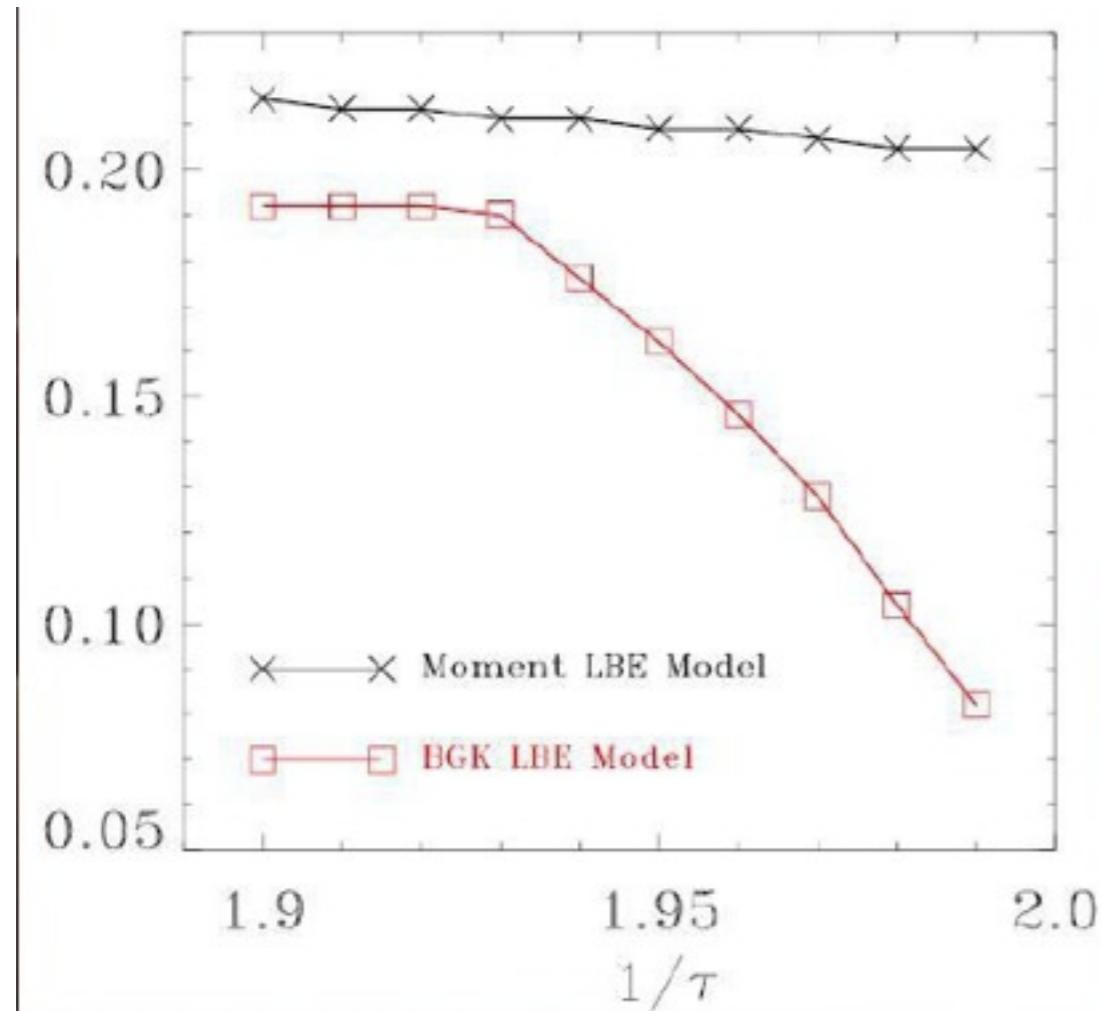
Ej: $S = \text{diag} \left(0, 0, 0, 3 \frac{(2 - s_7)}{(3 - s_7)}, 1.92, 1.14, 1.64, s_7, s_7 \right) \quad s_7 = \frac{1}{\tau}$

Finalmente,

$$|f_{\text{new}}\rangle = |f\rangle - M^{-1}S \left(|\varrho\rangle - |\varrho^{(eq)}\rangle \right)$$



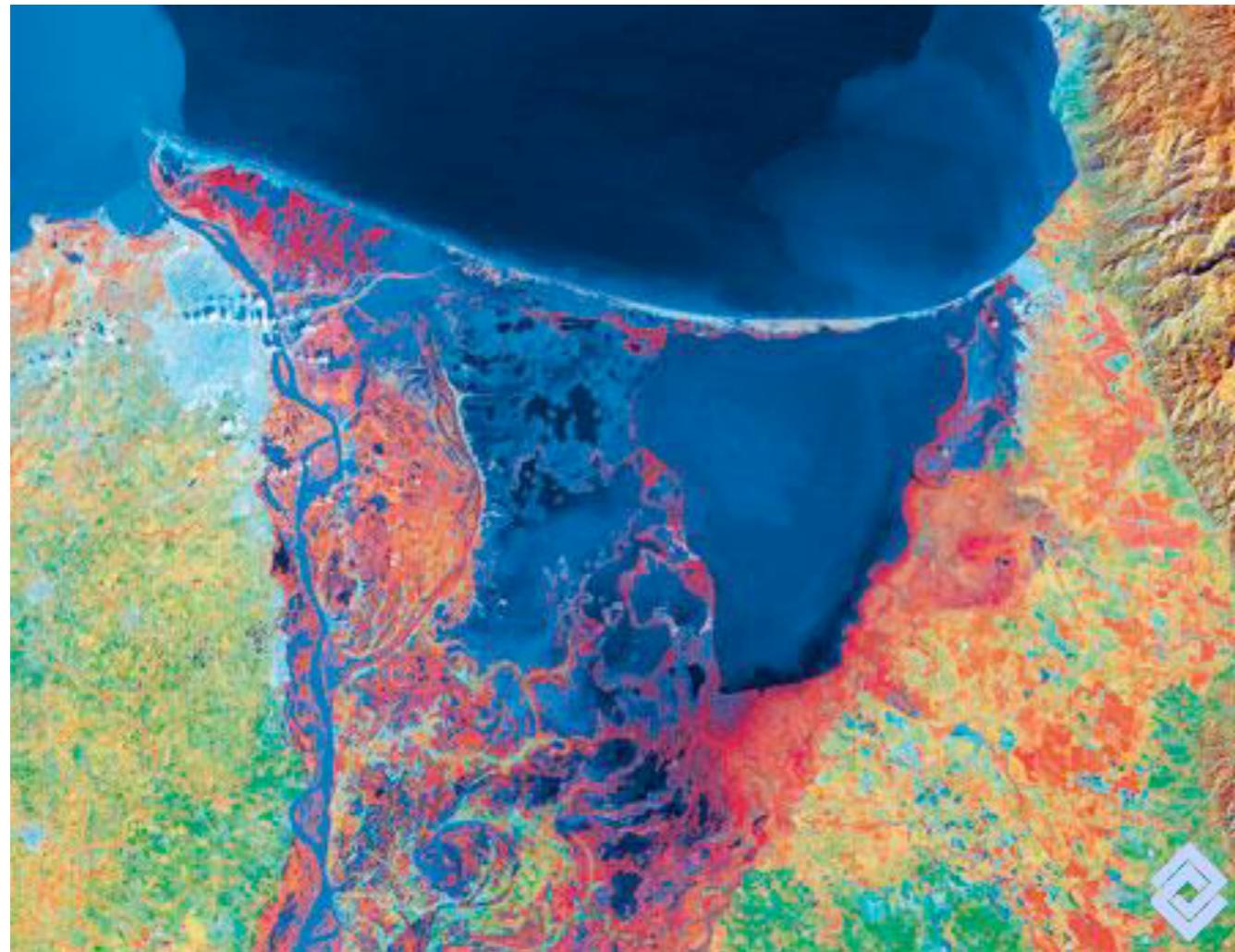
Ejemplos de aplicación (Jaider Iván Pulido, 2005)



La Ciénaga Grande de Santa Marta



Jonathan Florez



Lattice-Boltzmann for Shallow Waters

Zhou,2002-2004; Li and Hwang, 2008 ; Tubbs, 2010

Macroscopic quantities

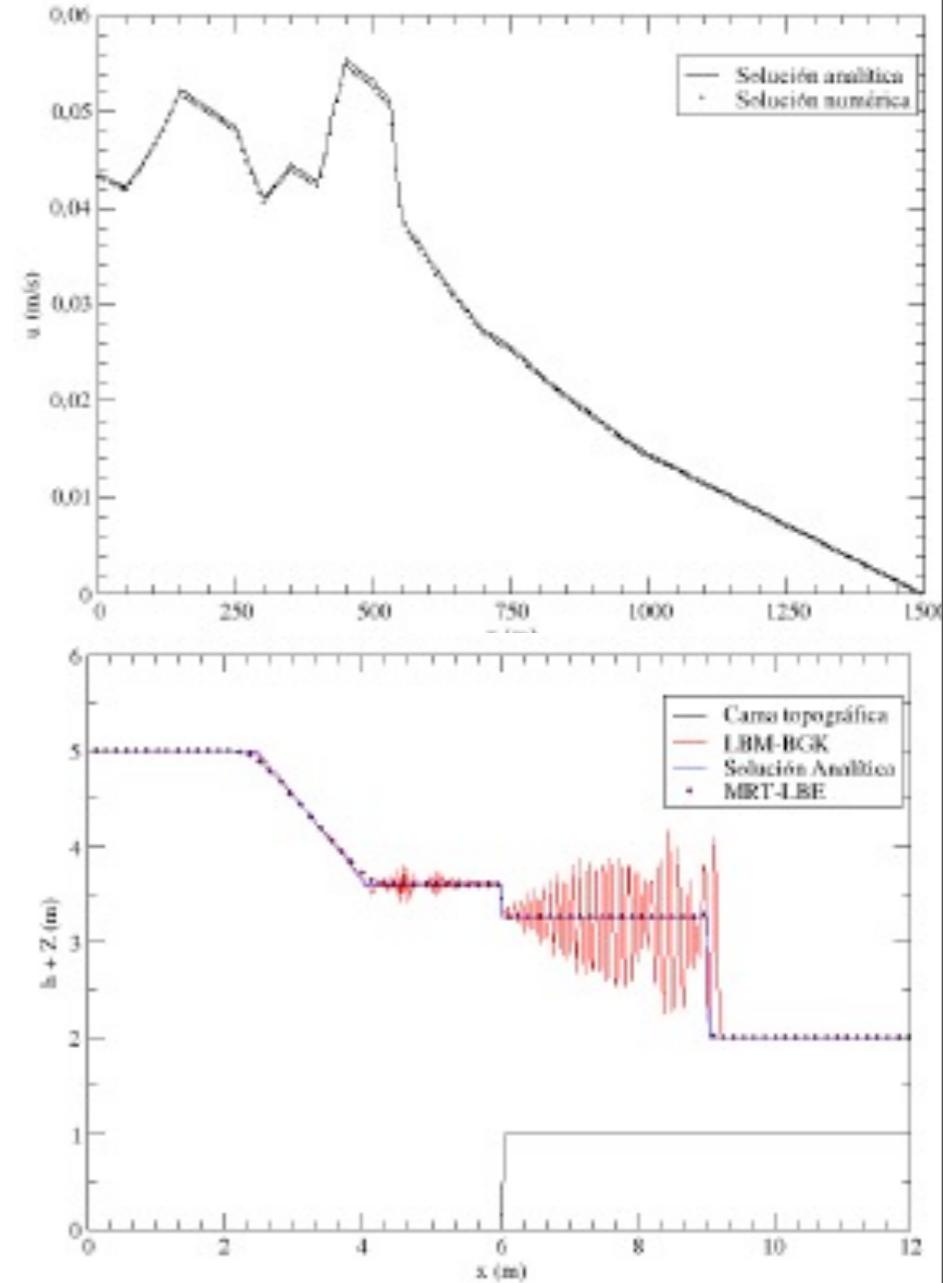
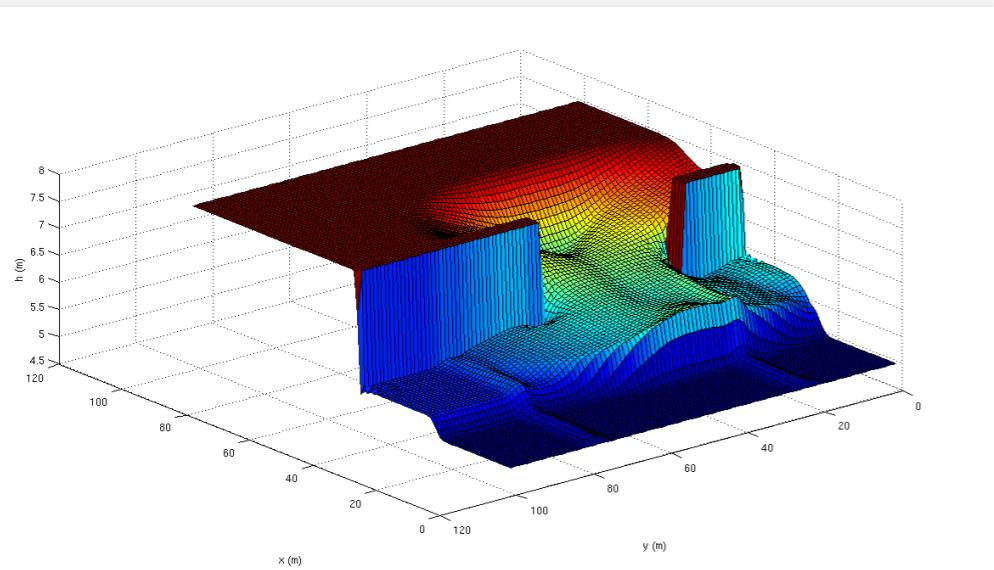
$$h = \sum_i f_i = \sum_i f_i^{(eq)} \quad hU_\alpha = \sum_i v_{i\alpha} f_i$$

Equilibrium functions

$$f_i^{eq} = \begin{cases} h - \frac{5gh^2}{6c^2} - \frac{2h}{3c^2} U_\alpha U_\beta \delta_{\alpha\beta} & i = 0 \\ \frac{gh^2}{6c^2} + \frac{h}{3c^2} v_{i\alpha} U_\alpha + \frac{h}{2c^4} v_{i\alpha} v_{i\beta} U_\alpha U_\beta - \frac{h}{6c^2} U_\alpha U_\beta \delta_{\alpha\beta} & i = 1, 3, 5, 7 \\ \frac{gh^2}{24c^2} + \frac{h}{12c^2} v_{i\alpha} U_\alpha + \frac{h}{8c^4} v_{i\alpha} v_{i\beta} U_\alpha U_\beta - \frac{h}{2c^2} U_\alpha U_\beta \delta_{\alpha\beta} & i = 2, 4, 6, 8 \end{cases}$$

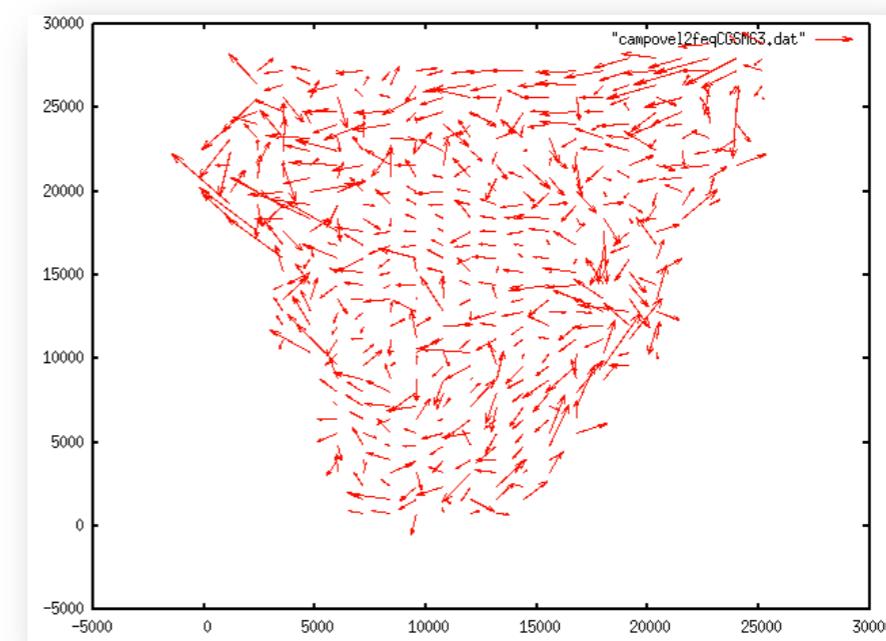
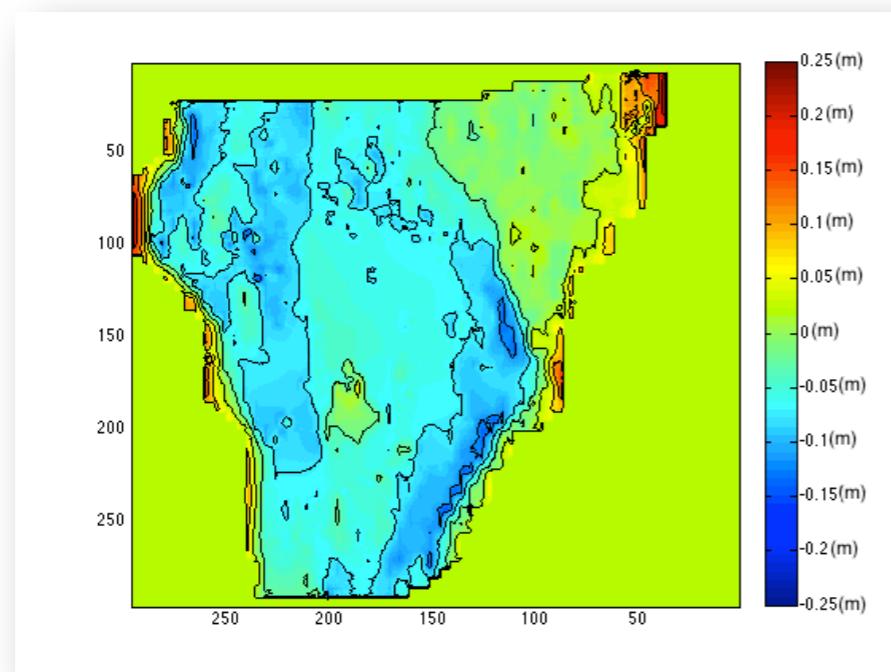
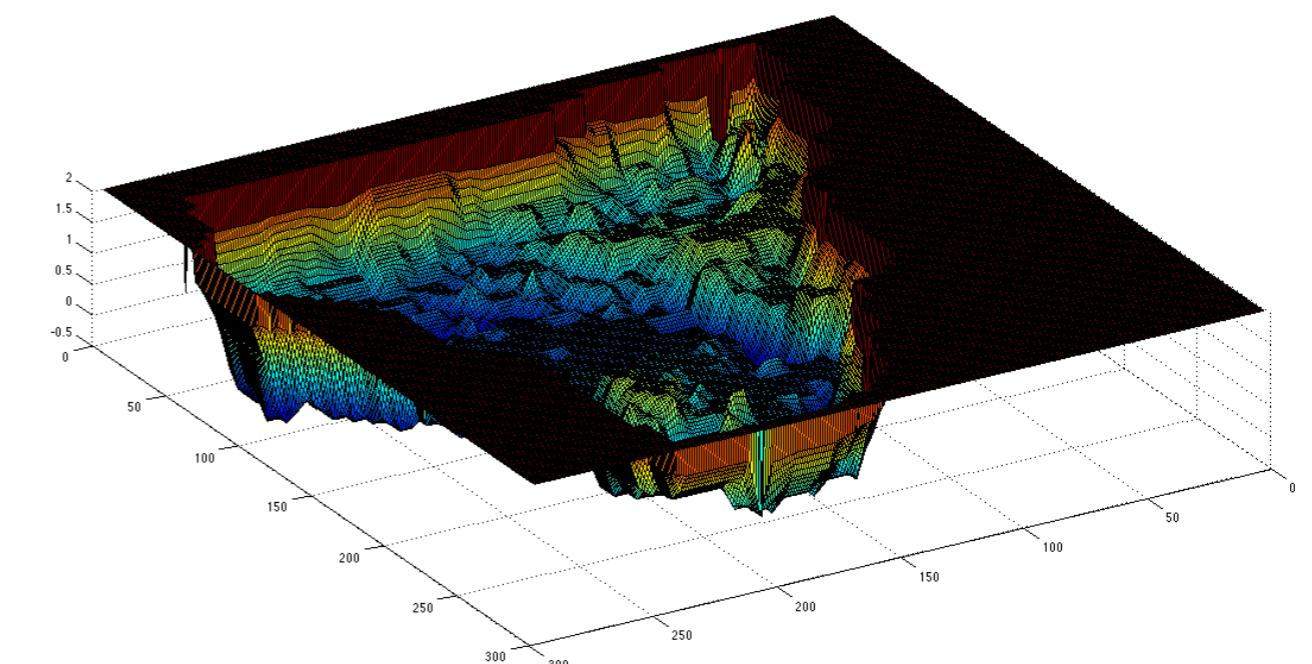
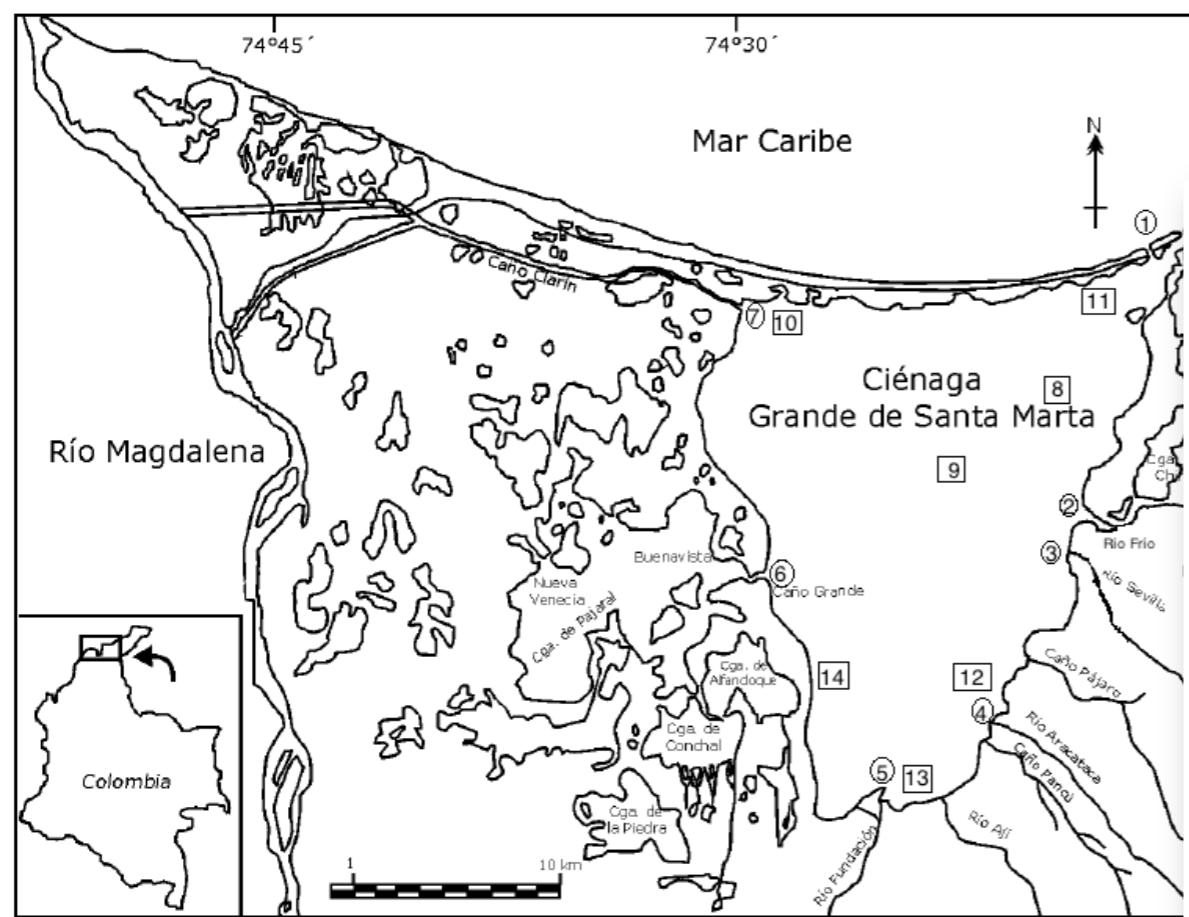
Evolution rule

$$f_i(\vec{x} + \delta t \vec{v}_i, t + \delta t) - f_i(\vec{x}, t) = -\frac{1}{\tau} [f_i(\vec{x}, t) - f_i^{(eq)}(\vec{x}, t)] + \frac{\Delta t}{6c^2} v_{i\alpha} F_\alpha(\vec{x}, t)$$

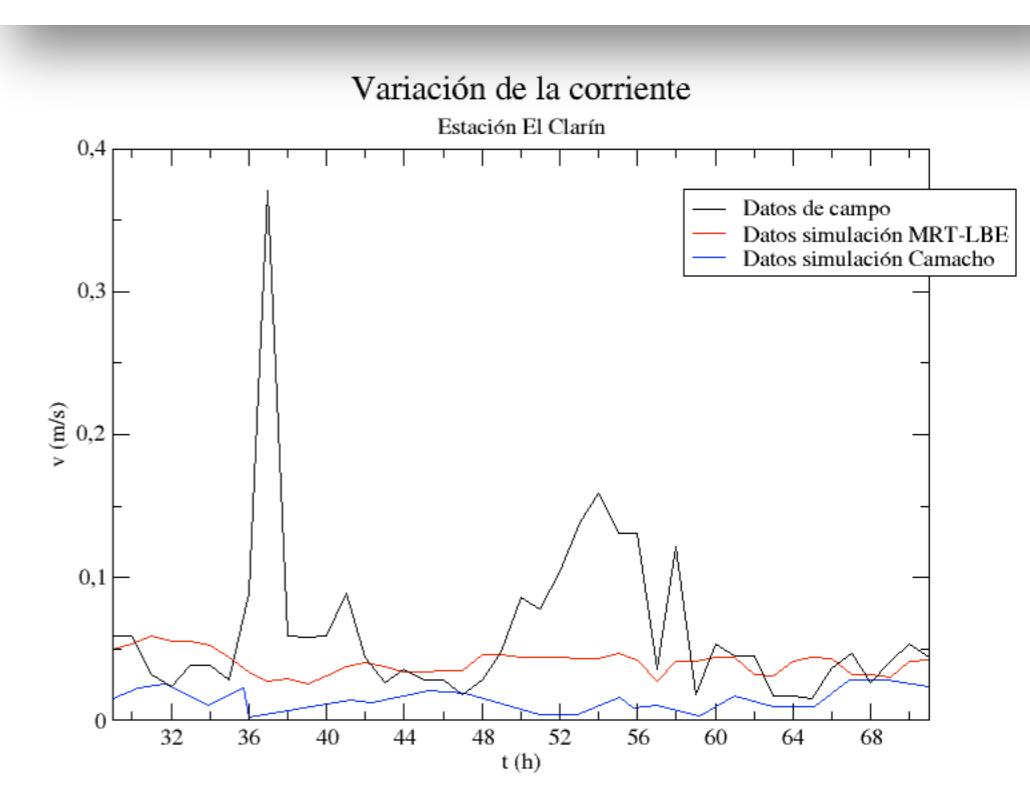
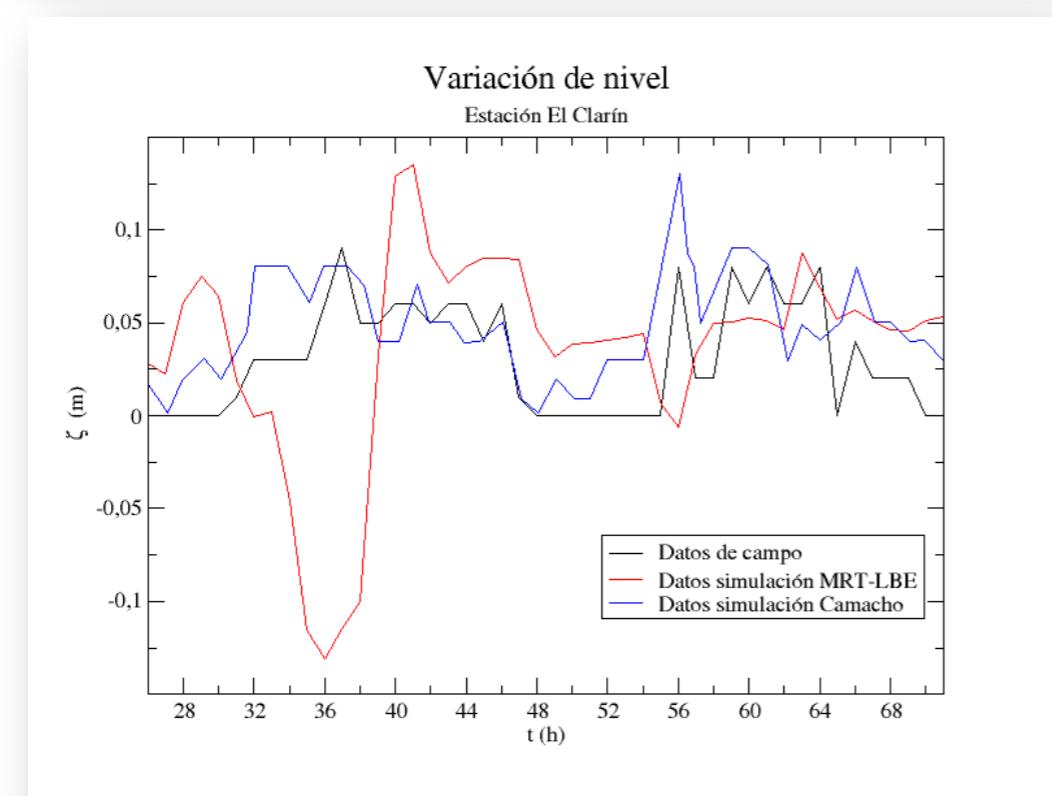
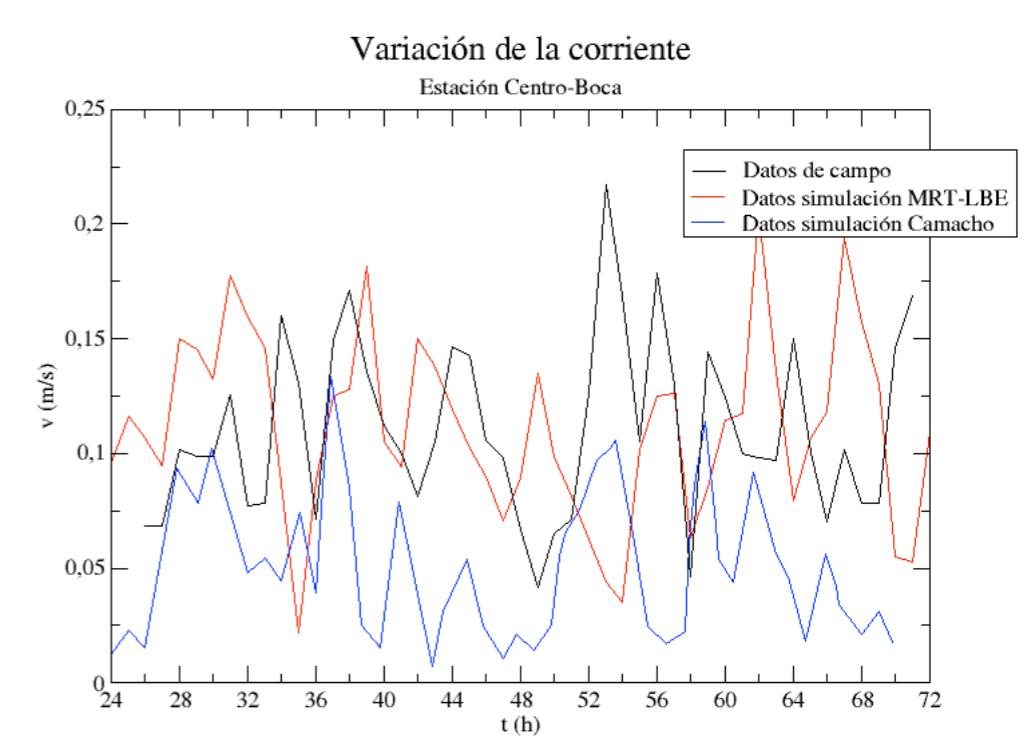
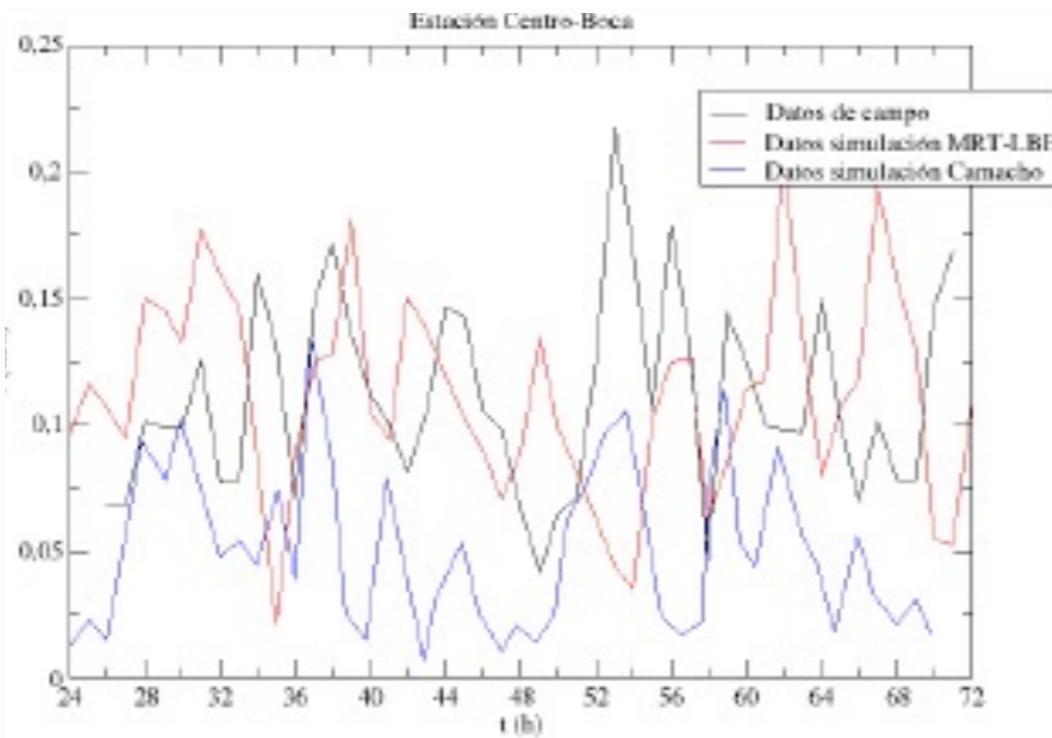


You need MRT-LBM to reach the required Reynold's numbers

Simulating the Ciénaga Grande



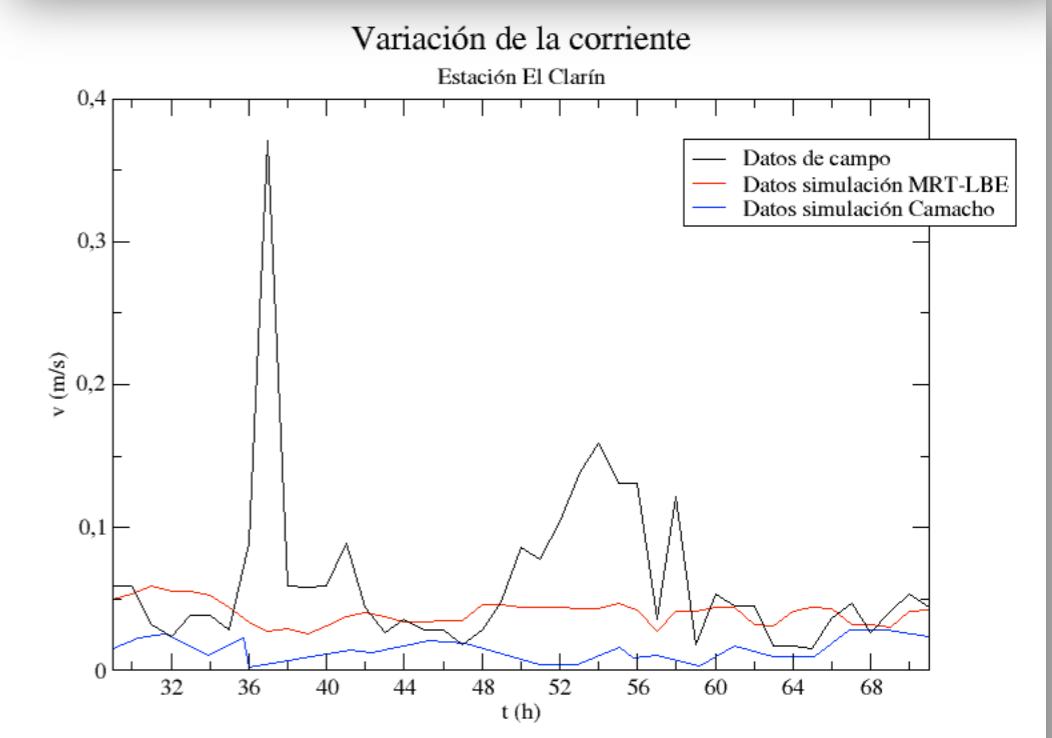
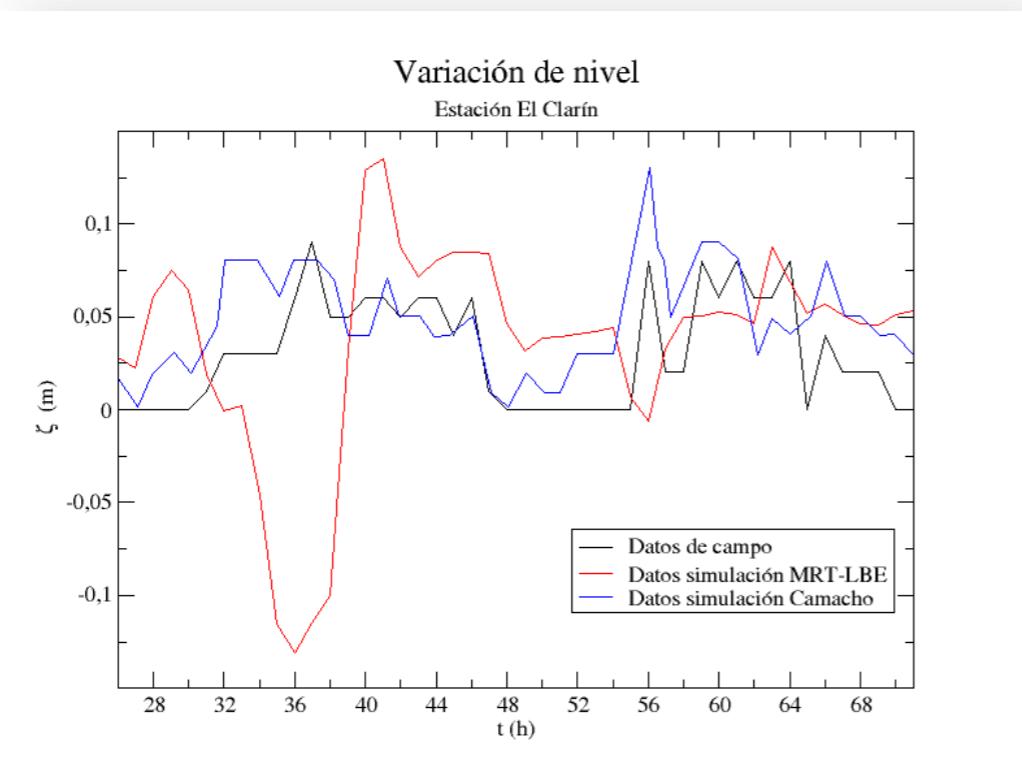
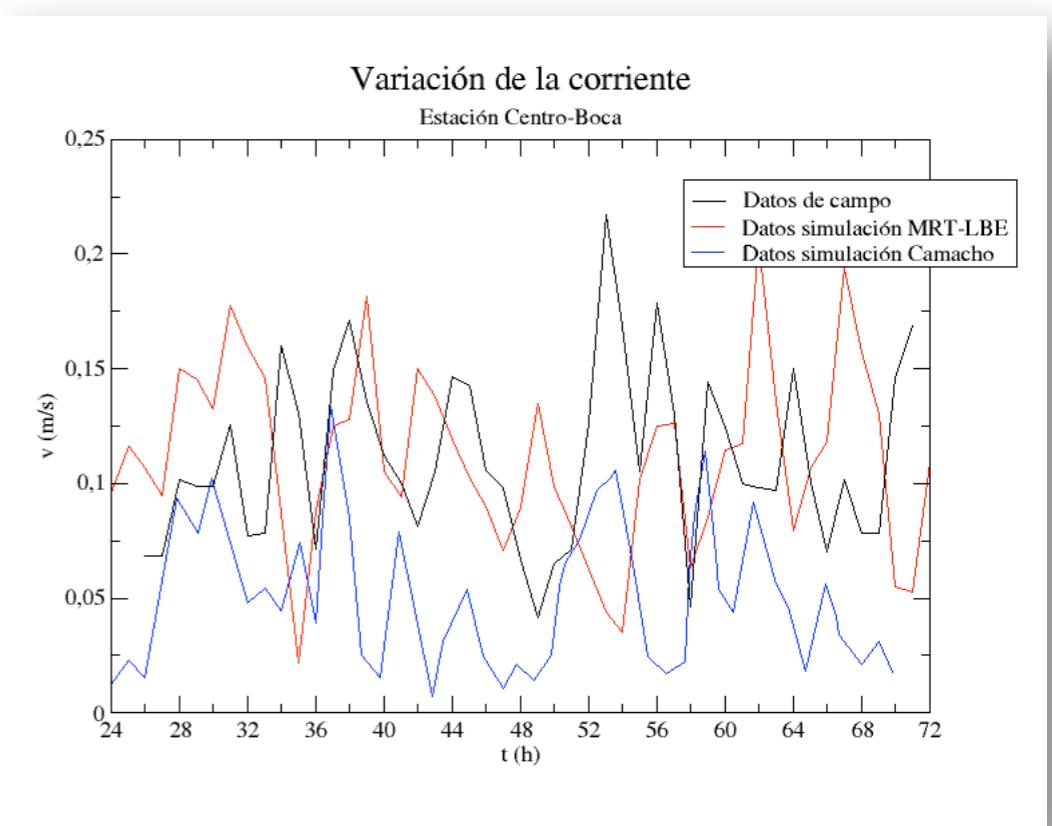
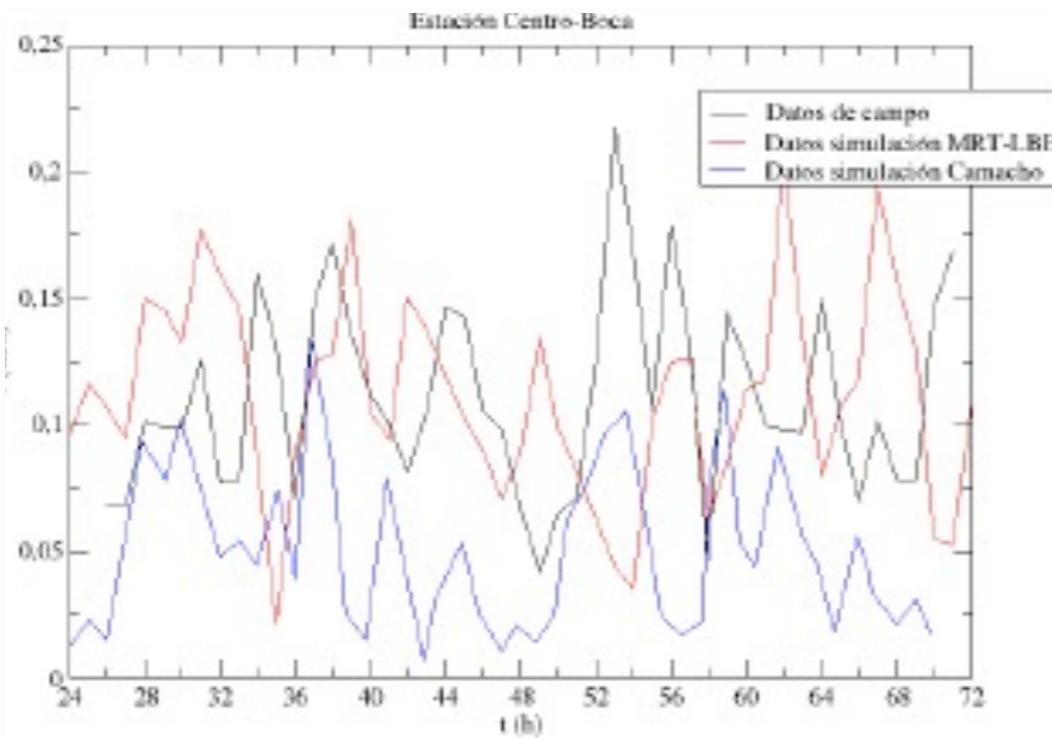
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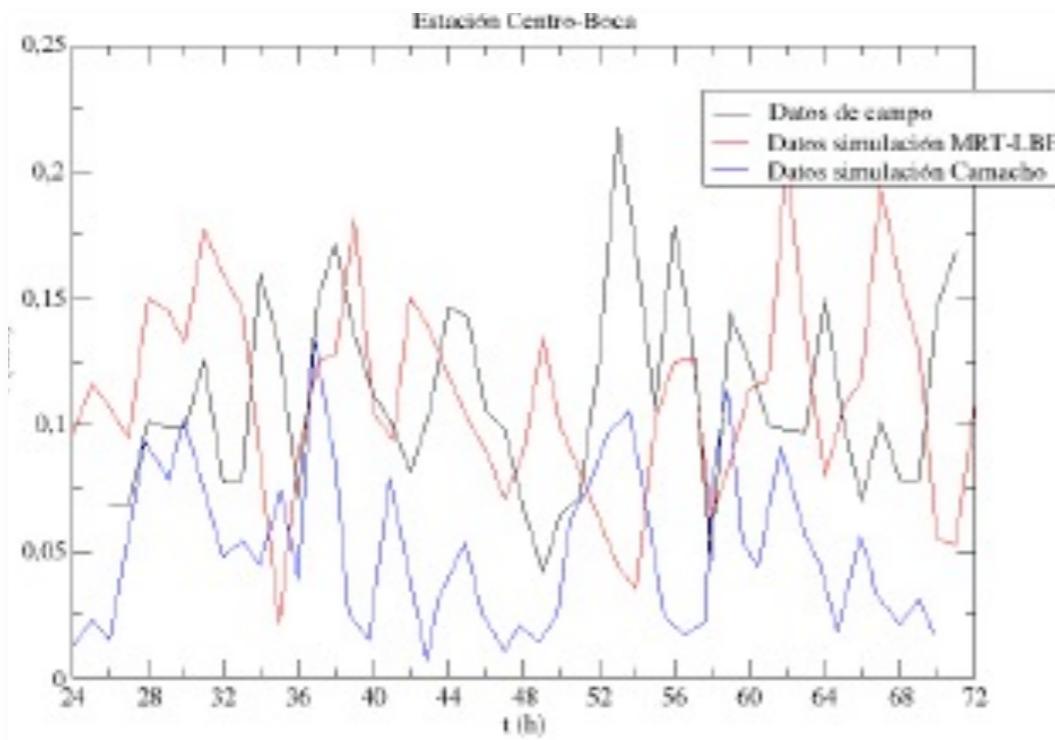
Water level



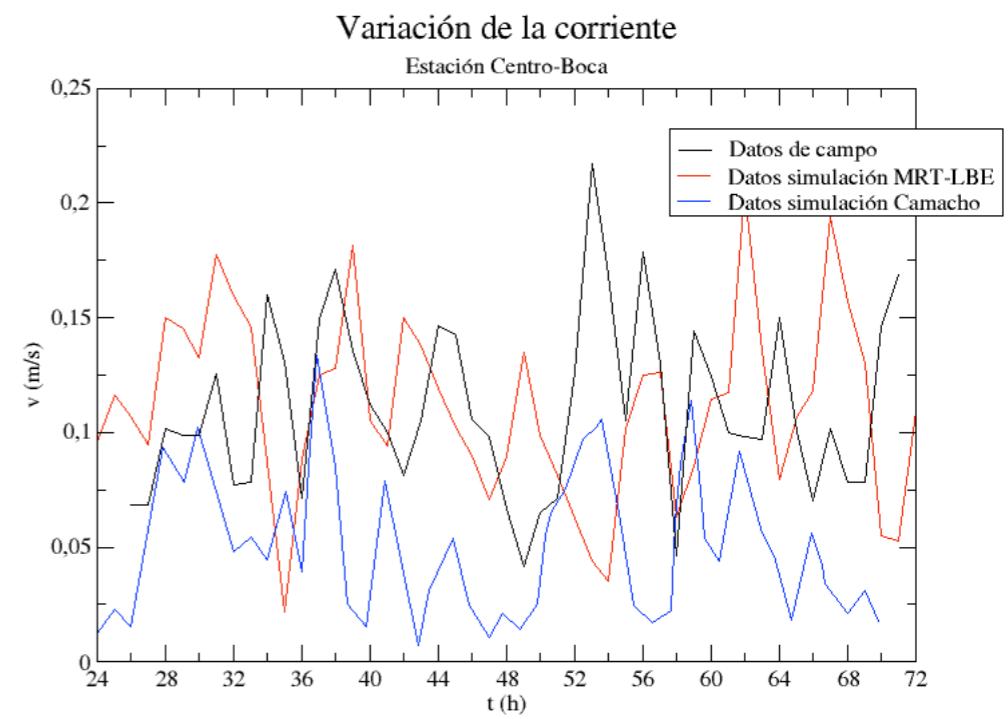
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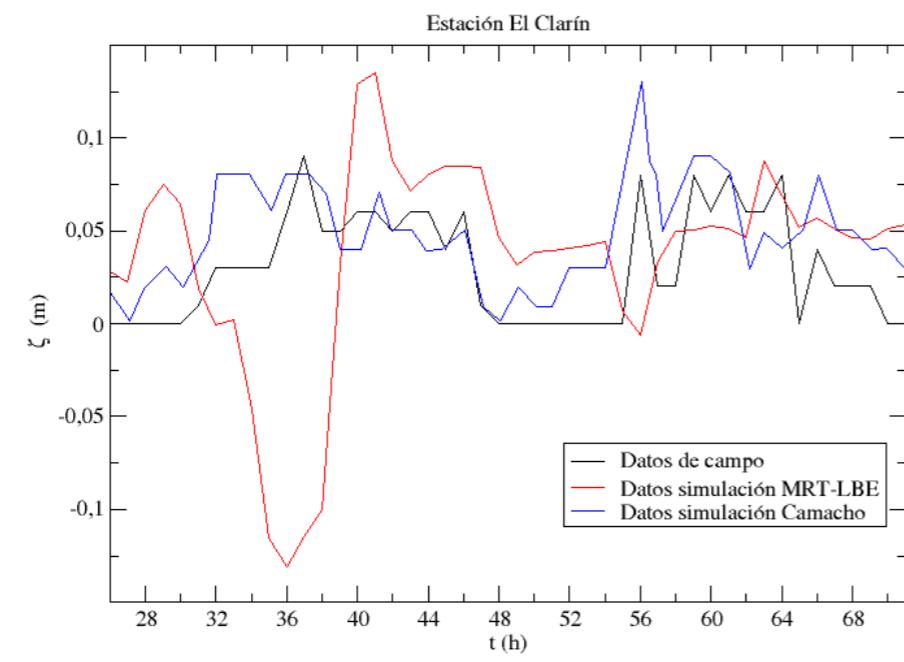
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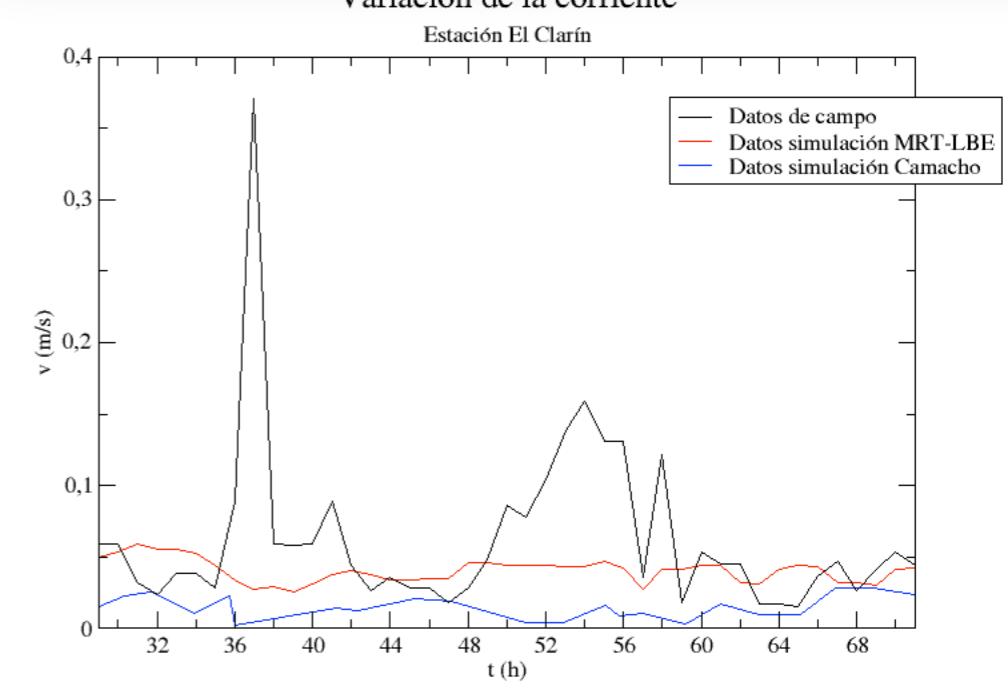
Water horizontal speed



Variación de nivel



Variación de la corriente

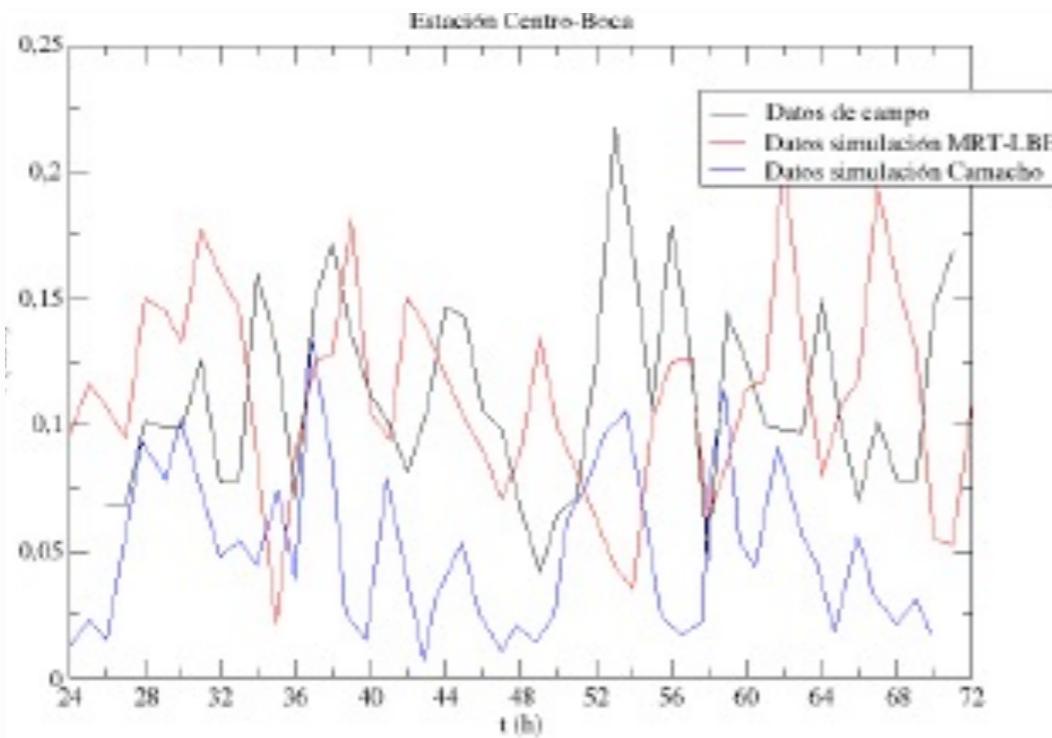


Simulating the Ciénaga Grande

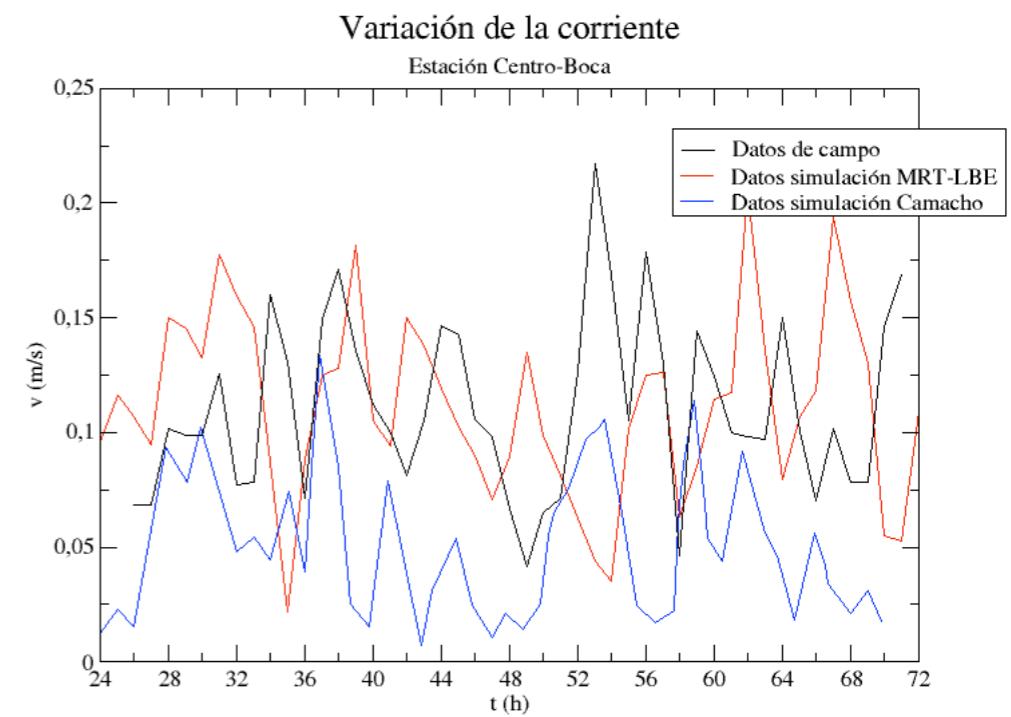


Centro

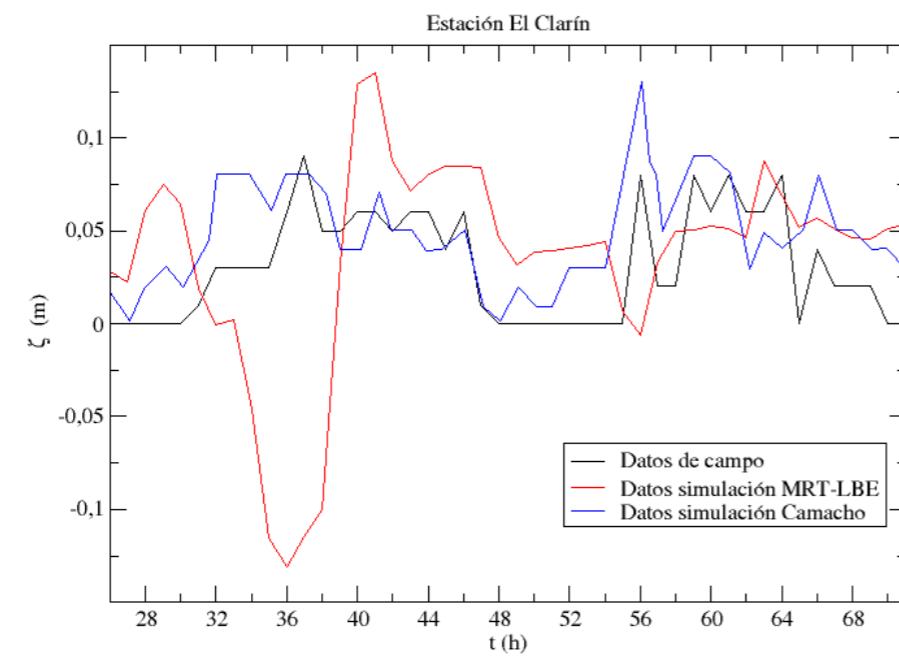
Water level



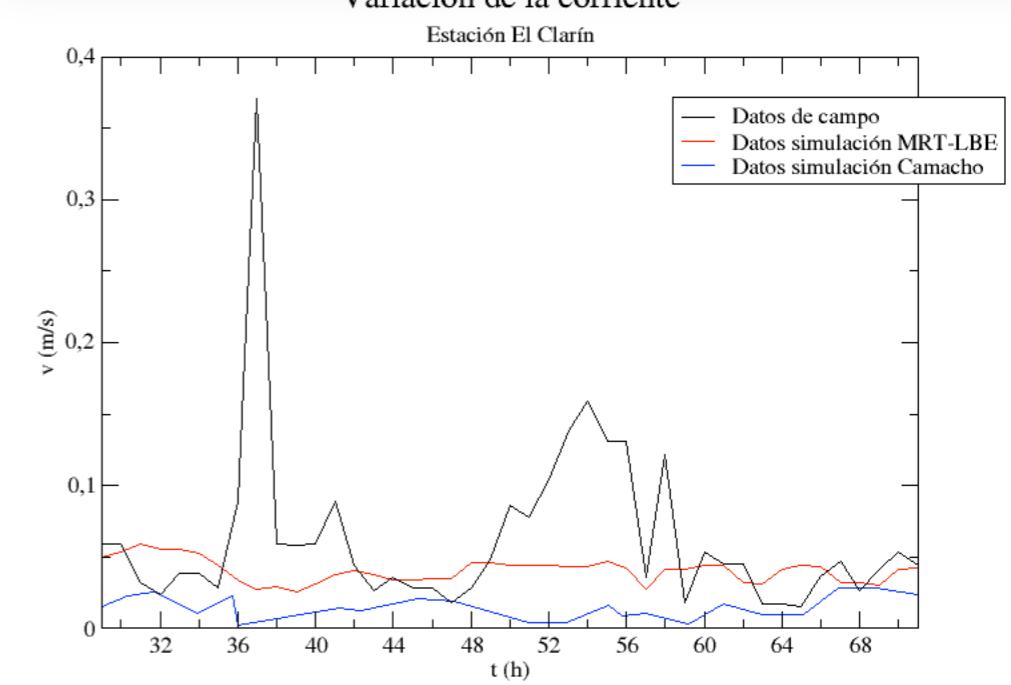
Water horizontal speed



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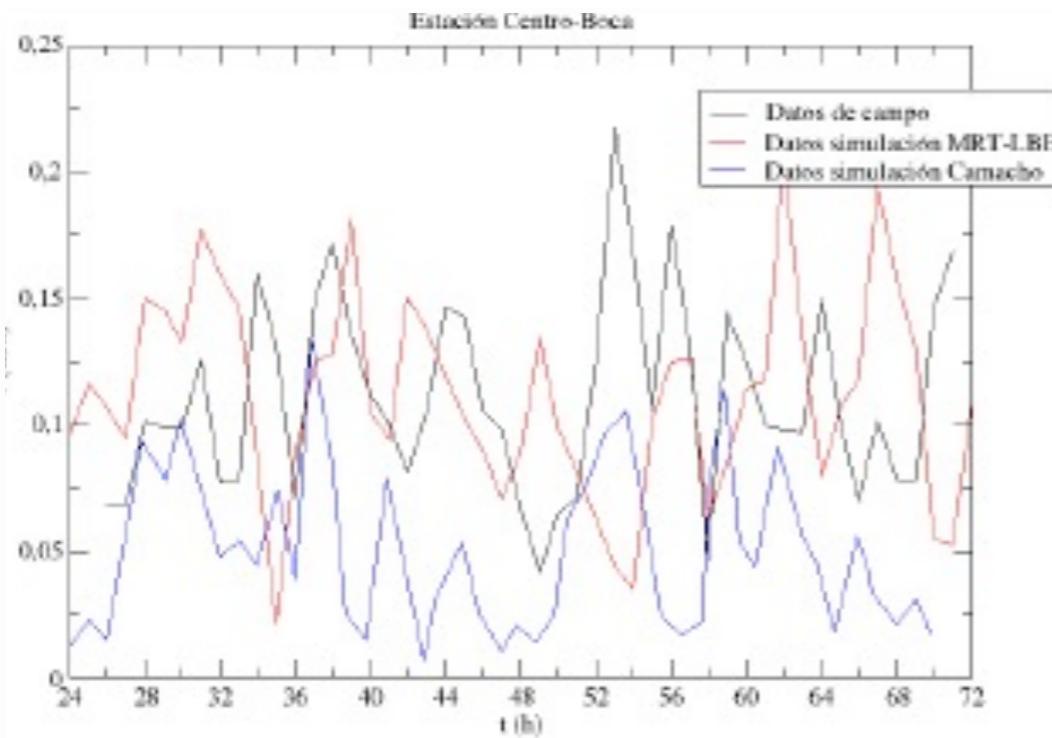


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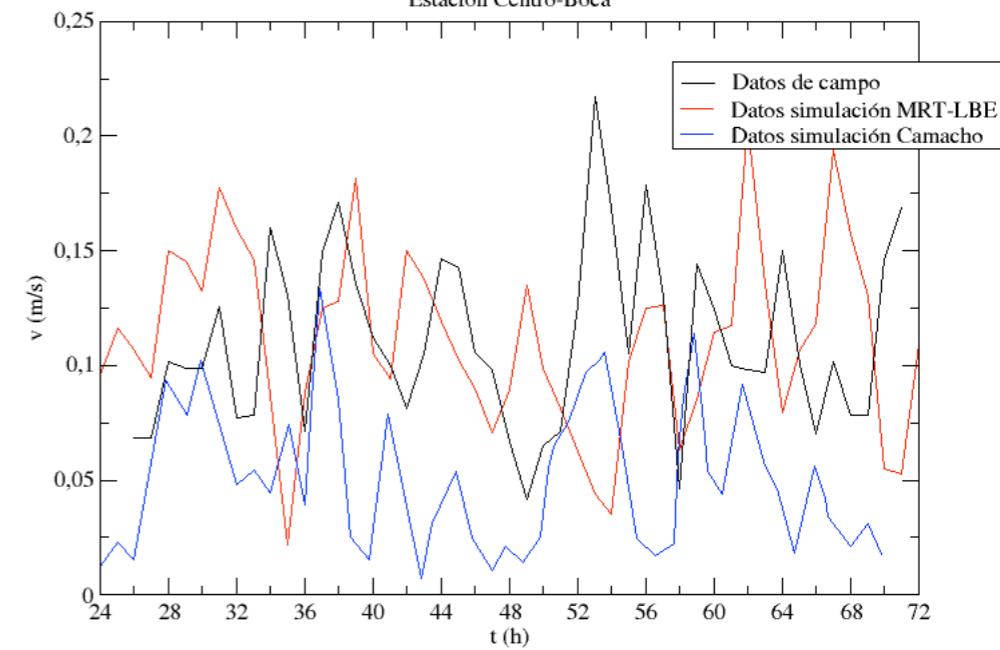
Water level

Centro



Water horizontal speed

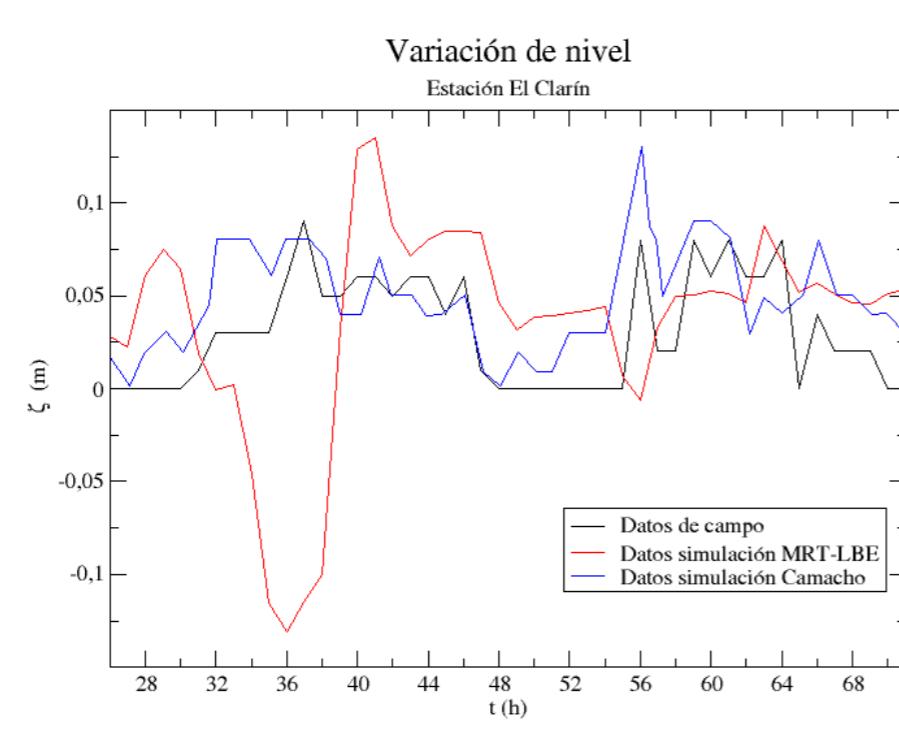
Variación de la corriente
Estación Centro-Boca



El Clarín

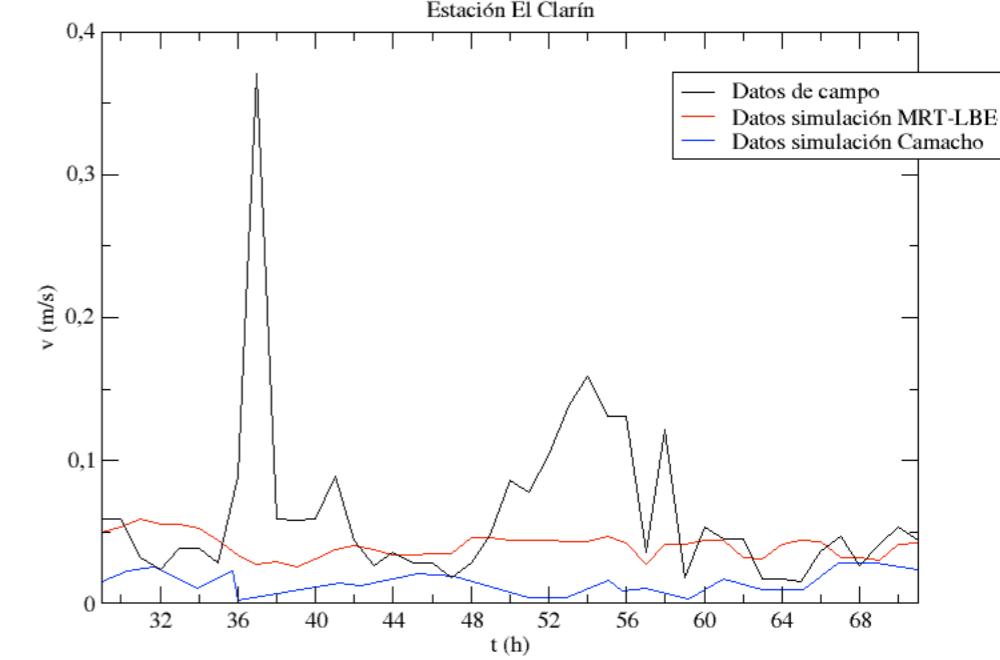
Variación de nivel

Estación El Clarín



Variación de la corriente

Estación El Clarín



Gracias!

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