Construcción de Modelos de Lattice-Boltzmann a partir de la expansión de Chapman-Enskog

Miller Mendoza[1,2] y José Daniel Muñoz[1]

[1] Grupo Simulación de Sistemas Físicos, CEIBA-Complejidad
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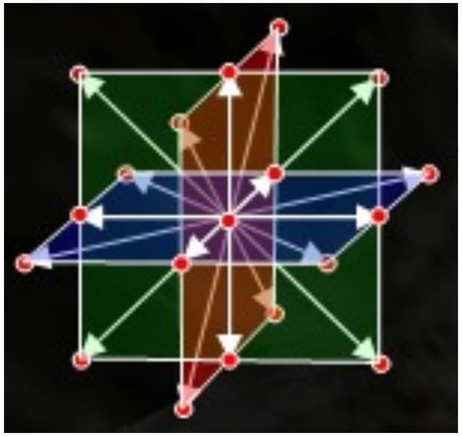

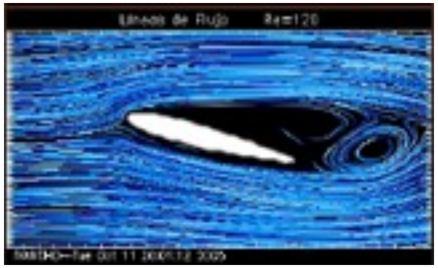

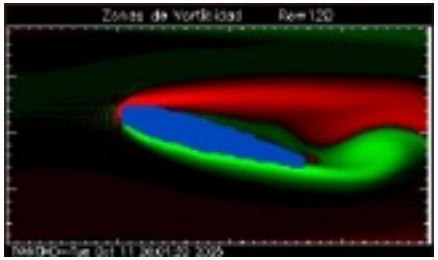

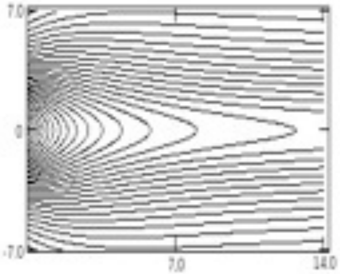

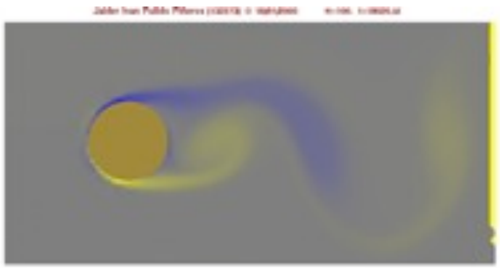

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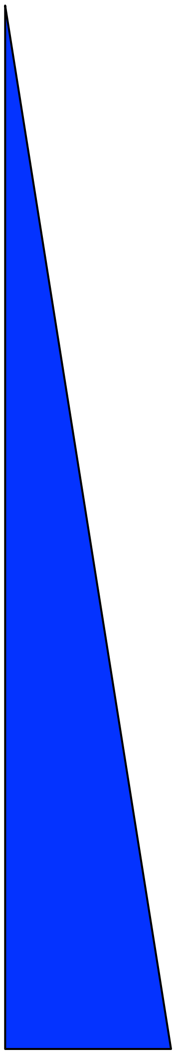
¿Qué es un Lattice-Boltzmann BGK?

(McNamara-Zanetti,1988, Higuera-Jiménez-Succi,1989, Benzi-Succi-Vergassola,1992)



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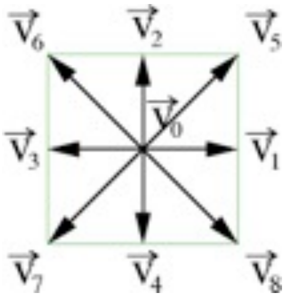

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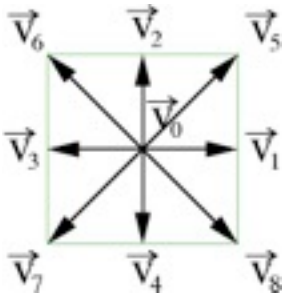

$+\ f_i$

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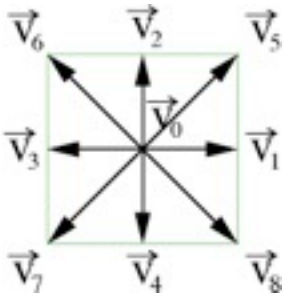

$+ f_i$

Cantidades macroscópicas

$$\rho = \sum_i f_i \quad , \quad \rho \vec{U} = \sum_i \vec{v}_i f_i$$

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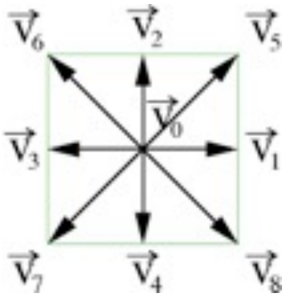

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Funciones de equilibrio

$$f_i^{(\mathrm{eq})} = f_i^{(\mathrm{eq})}(\rho, \vec{U})$$

$$f_i(\vec{x} + \delta t \vec{v}_i, t + \delta t) - f_i(\vec{x}, t) = -\frac{1}{\tau}\left[f_i(\vec{x}, t) - f_i^{(\mathrm{eq})}(\vec{x}, t)\right]$$

Ley de evolución BGK (Bhatnagar-Gross-Krook)

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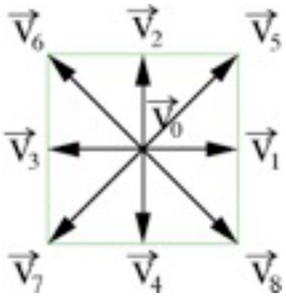

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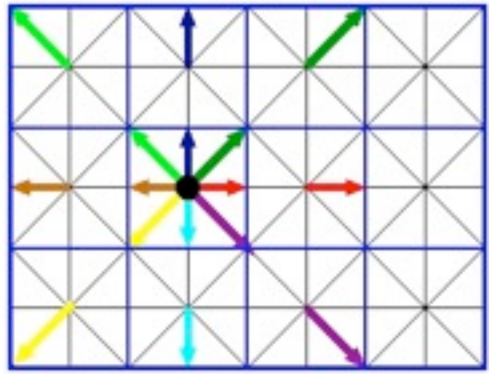

Advección

¿Qué es un Lattice-Boltzmann BGK?

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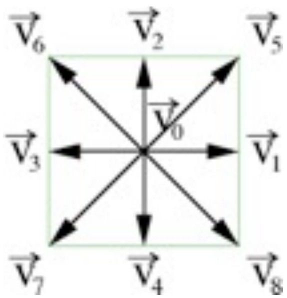

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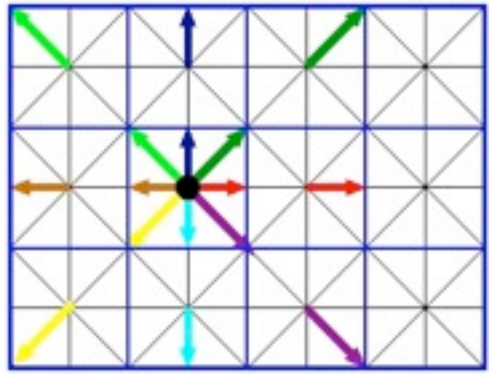
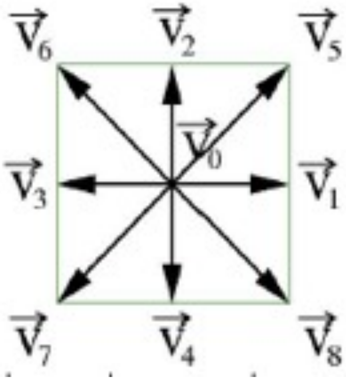


Advección

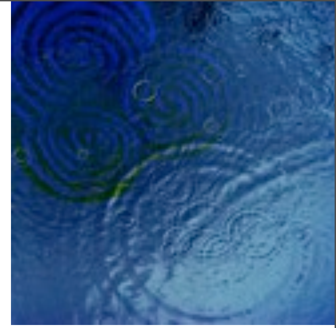
Al escoger la $f_{(eq)}$ adecuada se reproduce la ecuación diferencial parcial que uno desea.

Ejemplo: Ondas



$$w_i = \begin{cases} 4/9 & \text{for } i = 0 \\ 1/9 & \text{for } i = 1,2,3,4 \\ 1/36 & \text{for } i = 5,6,7,8 \end{cases}$$

$$\sum_i w_i v_{i\alpha} v_{i\beta} = \frac{1}{3}\delta_{\alpha\beta}$$



$+f_i$

Cantidades macroscópicas

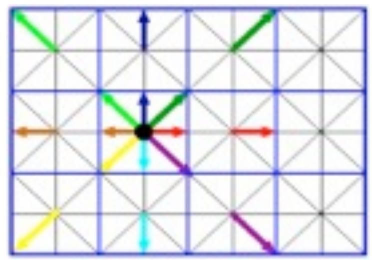
$$\rho = \sum_i f_i \quad , \quad \vec{J} = \sum_i \vec{v}_i f_i$$

Si se escoge

$$f_i^{(\mathrm{eq})} = \begin{cases} 3w_i\left(\vec{v}_i \cdot \vec{J} + \rho c^2\right) & \text{for } i > 0 \\ \frac{9}{4} w_0 \rho \left(1 - \frac{5}{3}c^2\right) & \text{for } i = 0 \end{cases}$$

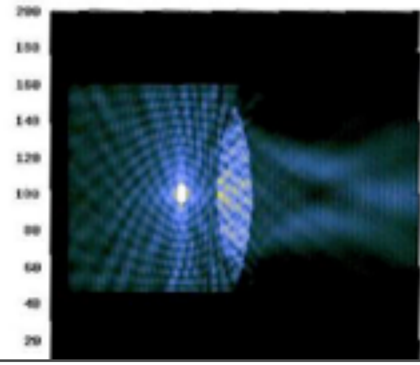
y si las variables evolucionan como

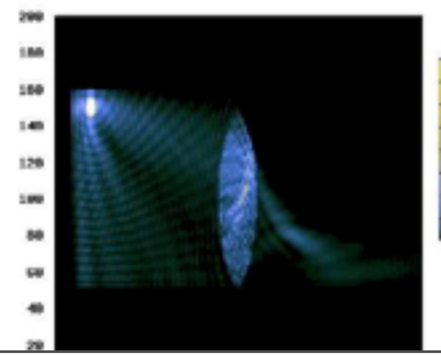
$$f_i(\vec{x} + \delta t, t + \delta t) = f_i(\vec{x}, t) - \frac{1}{\tau}\left[f_i(\vec{x}, t) - f_i^{(\mathrm{eq})}\right]$$



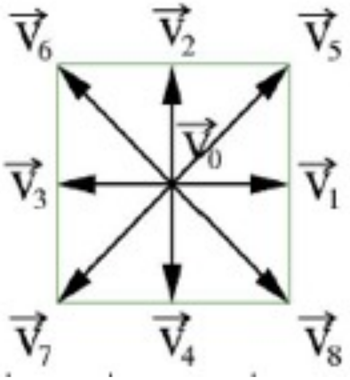
se obtiene

$$\boxed{\frac{\partial^2 \rho}{\partial t^2} = c^2 \nabla^2 \rho}$$






Otro ejemplo:
Mecánica de Fluidos



$$w_i = \begin{cases} 4/9 & \text{for } i = 0 \\ 1/9 & \text{for } i = 1,2,3,4 \\ 1/36 & \text{for } i = 5,6,7,8 \end{cases}$$

$$\sum_i w_i \, v_{i\alpha} v_{i\beta} v_{i\gamma} v_{i\delta} = \frac{1}{9}\left(\delta_{\alpha\beta}\delta_{\gamma\delta} + \delta_{\alpha\gamma}\delta_{\beta\delta} + \delta_{\alpha\delta}\delta_{\beta\gamma}\right)$$

$$+f_i$$

Cantidades macroscópicas

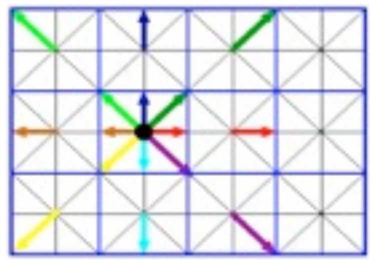
$$\rho = \sum_i f_i \quad , \quad \rho\vec{U} = \sum_i \vec{v}_i f_i$$

Si se escoge

$$f_i^{(\mathrm{eq})} = w_i \rho \left[1 + 3(\vec{U}\cdot\vec{v}_i) + \frac{9}{2}(\vec{U}\cdot\vec{v}_i)^2 - \frac{3}{2}U^2\right]$$

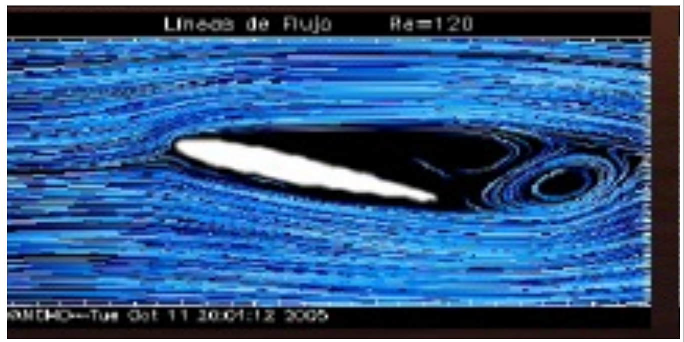
y si las variables evolucionan como

$$f_i(\vec{x}+\delta t, t+\delta t) = f_i(\vec{x},t) - \frac{1}{\tau}\left[f_i(\vec{x},t) - f_i^{(\mathrm{eq})}\right]$$



se obtiene

$$\rho\left[\frac{\partial\vec{U}}{\partial t} + \left(\vec{U}\cdot\nabla\right)\vec{U}\right] = -\nabla p + \rho\nu\nabla^2\vec{U} \qquad 0 = \frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{u})$$



Otro ejemplo: Electrodinámica

Lattice Boltzmann para Electrodinámica

M.Mendoza and J.D.Muñoz, *Phys. Rev. E* **82**, 056708 (2010)

Miller Mendoza

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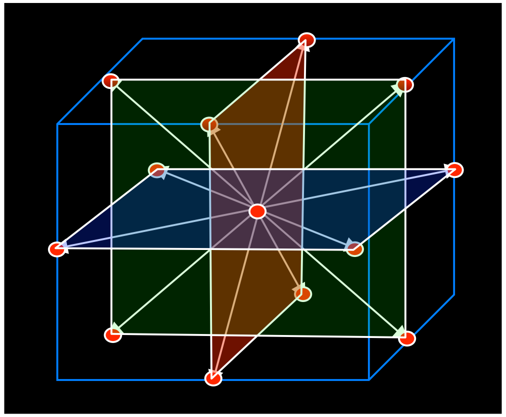
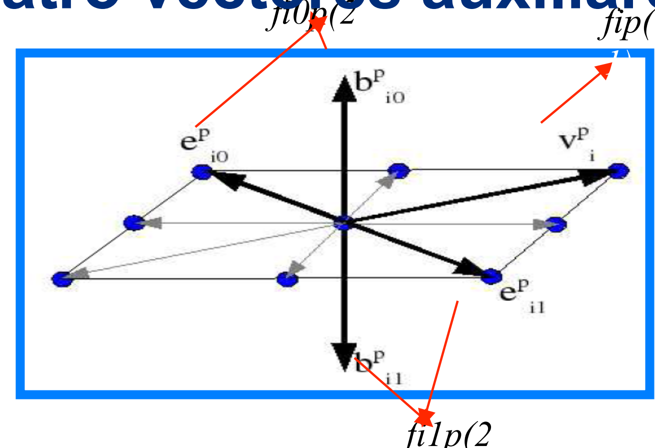


Cuatro vectores auxiliares por $\vec{v}_i$



+ cuatro funciones por $\vec{v}_i$

$$f_{i0}^{p(1)}, f_{i1}^{p(1)}, f_{i0}^{p(2)}, f_{i1}^{p(2)}$$

Cantidades Macroscópicas

$$\vec{D} = \sum_{i=1}^{4} \sum_{p=0}^{2} \sum_{j=0}^{1} f_{ij}^{p(0)} \vec{e}_{ij}^{\,p}$$

$$\vec{B} = \sum_{i=1}^{4} \sum_{p=0}^{2} \sum_{j=0}^{1} f_{ij}^{p(1)} \vec{b}_{ij}^{\,p}$$

$$\rho_c = f_0^{(0)} + \sum_{i=1}^{4} \sum_{p=0}^{2} \sum_{j=0}^{1} f_{ij}^{p(0)}$$

$$\vec{E} = \frac{\vec{D}}{\epsilon_r} \qquad \vec{H} = \frac{\vec{B}}{\mu_r}$$

$$\vec{J} = \sigma \vec{E}$$

Los campos reales se calculan como

$$\vec{J}' = \frac{\sigma}{1+\frac{\mu_0\sigma}{4\epsilon_r}}\vec{E} \qquad \vec{E}' = \vec{E} - \frac{\mu_0}{4\epsilon_r}\vec{J}'$$

Con las funciones de equilibrio

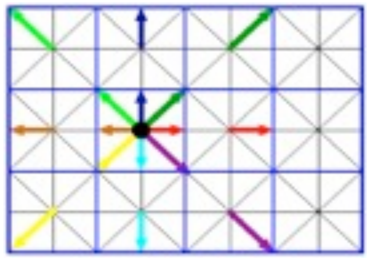
$$f_{ij}^{p(0)\mathrm{eq}}(\vec{x},t) = \frac{1}{16}\vec{v}_i^p\cdot\vec{J}' + \frac{\epsilon}{4}\vec{E}'\cdot\vec{e}_{ij}^p + \frac{1}{8\mu}\vec{B}\cdot\vec{b}_{ij}^p$$

$$f_{ij}^{p(1)\mathrm{eq}}(\vec{x},t) = \frac{1}{16}\vec{v}_i^p\cdot\vec{J}' + \frac{1}{4}\vec{E}'\cdot\vec{e}_{ij}^p + \frac{1}{8}\vec{B}\cdot\vec{b}_{ij}^p$$

$$f_0^{(0)\mathrm{eq}}(\vec{x},t) = f_0^{(0)\mathrm{eq}}(\vec{x},t) = \rho_c$$

y tomando $(\tau = 1/2)$

$$f_{ij}^{p(r)}(\vec{x}+\vec{v}_i^p, t+1) = f_{ij}^{p(r)}(\vec{x},t) - 2\left(f_{ij}^{p(r)}(\vec{x},t) - f_{ij}^{p(r)\mathrm{eq}}(\vec{x},t)\right)$$



se obtienen las ecuaciones de Maxwell en materiales

$$\nabla\times\vec{E}' = -\frac{\partial\vec{B}}{\partial t} \qquad \nabla\times\vec{H} = \mu_0\vec{J}' + \frac{1}{c^2}\frac{\partial\vec{D}'}{\partial t} \qquad \frac{\partial\rho_c}{\partial t} + \nabla\cdot\vec{J}' = 0$$

$$\frac{\partial}{\partial t}\left(\nabla\cdot\vec{D}' - \frac{\rho_c}{\epsilon_0}\right) = 0 \qquad \frac{\partial}{\partial t}\left(\nabla\cdot\vec{B}\right) = 0$$

Resumiendo

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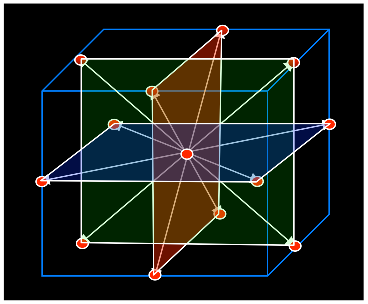
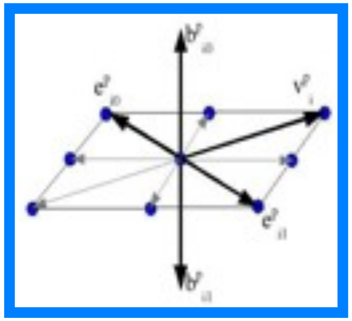
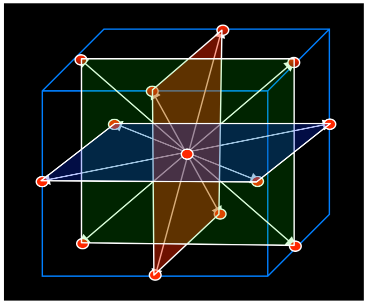
 + 

Cuatro funciones por $\vec{v}_i$

$f_{i0}^{p(1)}, f_{i1}^{p(1)}, f_{i0}^{p(2)}, f_{i1}^{p(2)}$

$$\boxed{\sum_{i,p} v_{i\alpha}^{p} e_{ij\beta}^{p} b_{ij\gamma}^{p} = 4\varepsilon_{\alpha\beta\gamma}}$$

Resumiendo



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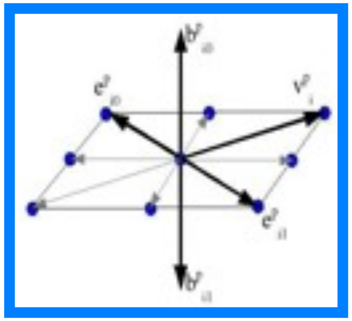

1

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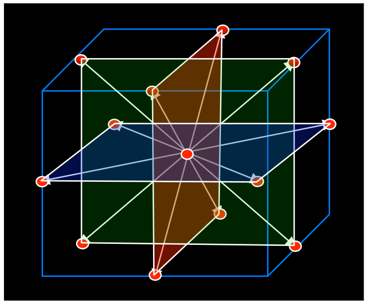

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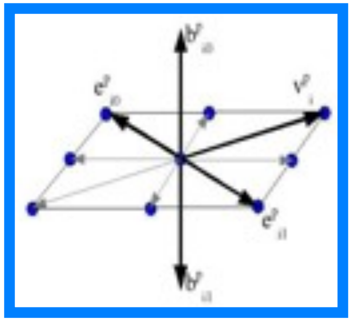

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Macroscopic Quantities

$$\vec{D} = \sum_{i=1}^{4}\sum_{p=0}^{2}\sum_{j=0}^{1} f_{ij}^{p(0)} \vec{e}_{ij}^{\,p}$$

$$\vec{B} = \sum_{i=1}^{4}\sum_{p=0}^{2}\sum_{j=0}^{1} f_{ij}^{p(1)} \vec{b}_{ij}^{\,p}$$

$$\rho_c = f_0^{(0)} + \sum_{i=1}^{4}\sum_{p=0}^{2}\sum_{j=0}^{1} f_{ij}^{p(0)}$$

$$\vec{E} = \frac{\vec{D}}{\epsilon_r} \qquad \vec{H} = \frac{\vec{B}}{\mu_r}$$

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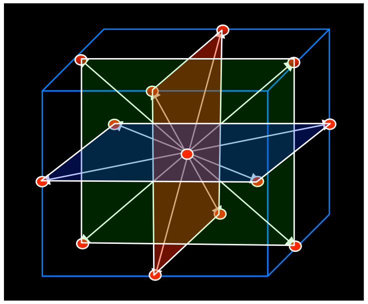

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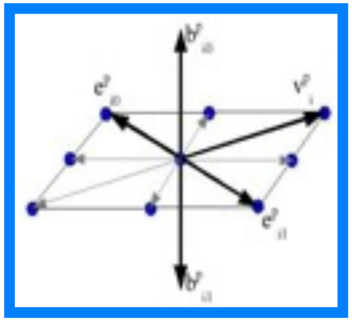

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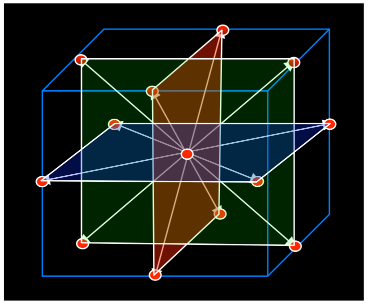
Resumiendo



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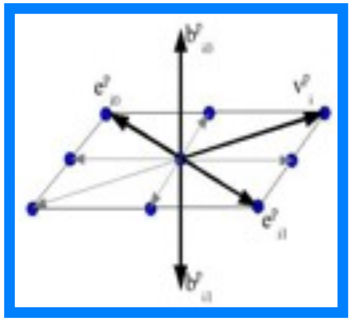

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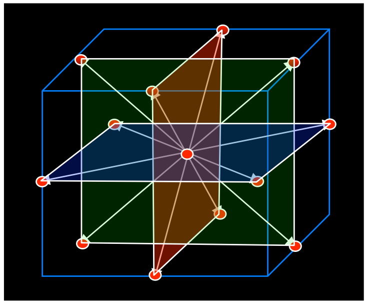
Funciones de Equilibrio

3

$$f_{ij}^{p(0)\mathrm{eq}}(\vec{x}, t) = \frac{1}{16} \vec{v}_i^{\,p} \cdot \vec{J}' + \frac{\epsilon}{4} \vec{E}' \cdot \vec{e}_{ij}^{\,p} + \frac{1}{8\mu} \vec{B} \cdot \vec{b}_{ij}^{\,p}$$

$$f_{ij}^{p(1)\mathrm{eq}}(\vec{x}, t) = \frac{1}{16} \vec{v}_i^{\,p} \cdot \vec{J}' + \frac{1}{4} \vec{E}' \cdot \vec{e}_{ij}^{\,p} + \frac{1}{8} \vec{B} \cdot \vec{b}_{ij}^{\,p}$$

Resumiendo



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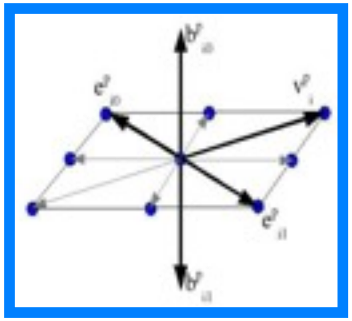

Cuatro funciones por $\vec{v}_i$

$$f_{i0}^{p(1)}, f_{i1}^{p(1)}, f_{i0}^{p(2)}, f_{i1}^{p(2)}$$

$$\boxed{\sum_{i,p} v_{i\alpha}^{p} e_{ij\beta}^{p} b_{ij\gamma}^{p} = 4\varepsilon_{\alpha\beta\gamma}}$$

1

2

Macroscopic Quantities

$$\vec{D} = \sum_{i=1}^{4}\sum_{p=0}^{2}\sum_{j=0}^{1} f_{ij}^{p(0)} \vec{e}_{ij}^{\,p}$$

$$\vec{B} = \sum_{i=1}^{4}\sum_{p=0}^{2}\sum_{j=0}^{1} f_{ij}^{p(1)} \vec{b}_{ij}^{\,p}$$

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Funciones de Equilibrio

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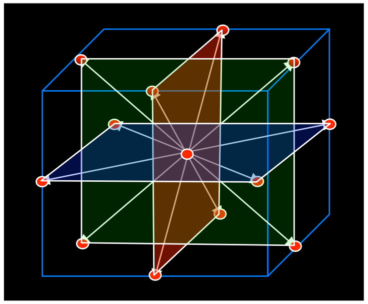
Regla de Evolución

4

$$f_{ij}^{p(r)}(\vec{x} + \vec{v}_i^{\,p}, t + 1) = f_{ij}^{p(r)}(\vec{x}, t) - 2\left[f_{ij}^{p(r)}(\vec{x}, t) - f_{ij}^{p(r)\mathrm{eq}}(\vec{x}, t)\right]$$

Resumiendo



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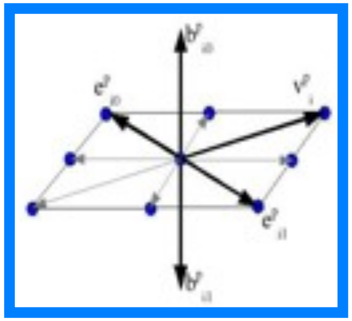

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2

Funciones de Equilibrio

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$$f_{ij}^{p(1)\mathrm{eq}}(\vec{x},t) = \frac{1}{16}\vec{v}_i^{\,p}\cdot\vec{J}' + \frac{1}{4}\vec{E}'\cdot\vec{e}_{ij}^{\,p} + \frac{1}{8}\vec{B}\cdot\vec{b}_{ij}^{\,p}$$

Regla de Evolución

4

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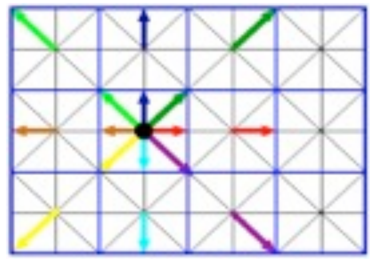
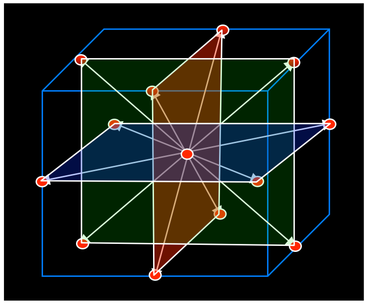
5



Resumiendo

Macroscopic Quantities



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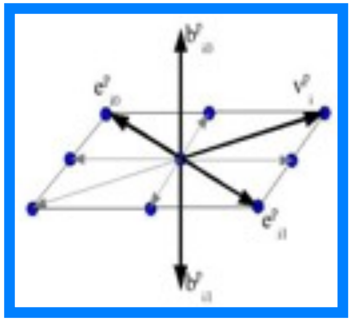

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2

Funciones de Equilibrio

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$$f_{ij}^{p(1)\mathrm{eq}}(\vec{x},t) = \frac{1}{16}\vec{v}_i^{\,p}\cdot\vec{J}' + \frac{1}{4}\vec{E}'\cdot\vec{e}_{ij}^{\,p} + \frac{1}{8}\vec{B}\cdot\vec{b}_{ij}^{\,p}$$

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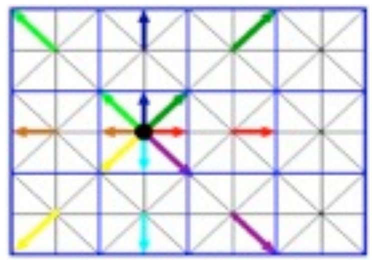
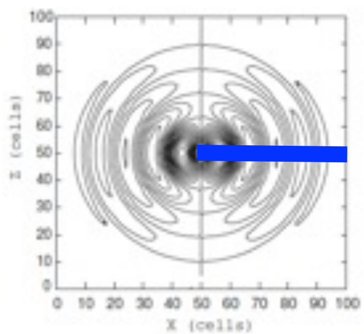
5

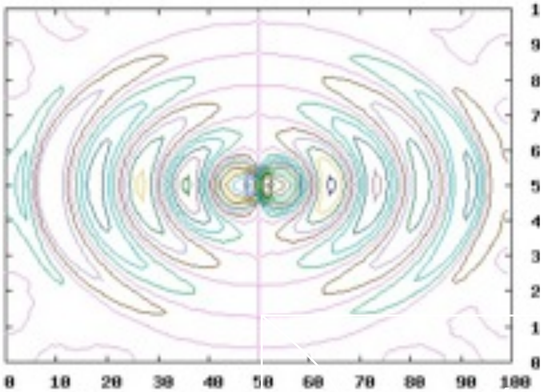


$$\nabla \times \vec{E}' = -\frac{\partial \vec{B}}{\partial t} \qquad \nabla \times \vec{H} = \mu_0 \vec{J}' + \frac{1}{c^2}\frac{\partial \vec{D}'}{\partial t} \qquad \frac{\partial \rho_c}{\partial t} + \nabla\cdot\vec{J}' = 0$$

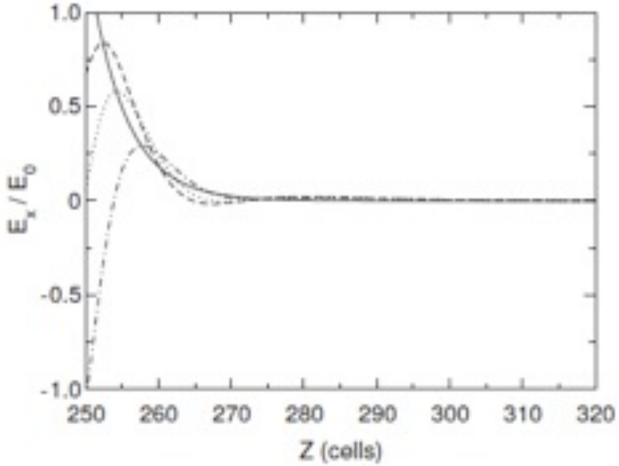
$$\frac{\partial}{\partial t}\left(\nabla\cdot\vec{D}' - \frac{\rho_c}{\epsilon_0}\right) = 0 \qquad \frac{\partial}{\partial t}\left(\nabla\cdot\vec{B}\right) = 0$$



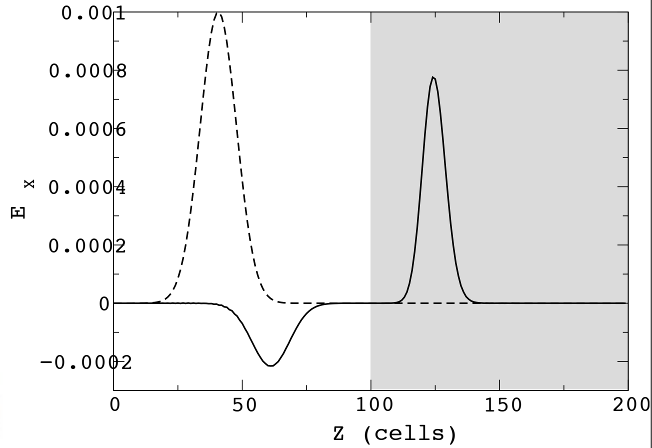
Oscillating dipole



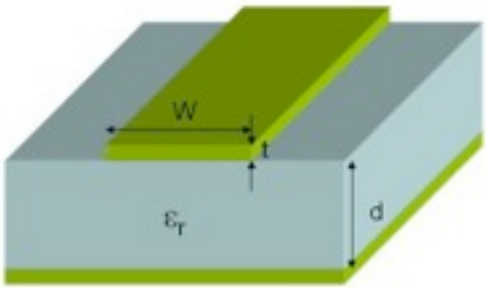
Skin Effect



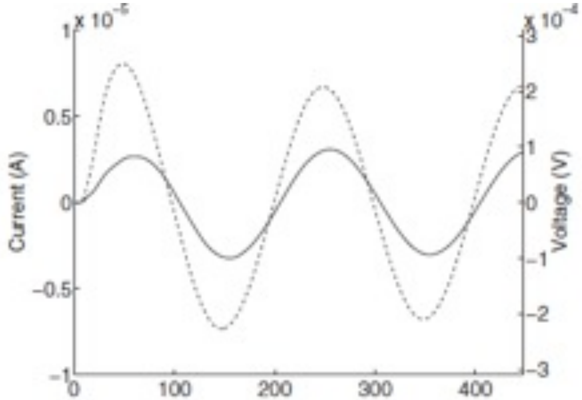
Pulse Reflection



Microstrip



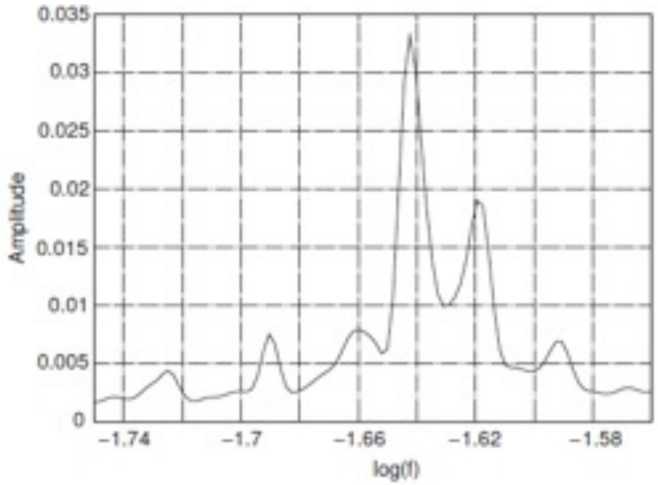
Geometry



Voltage and Current

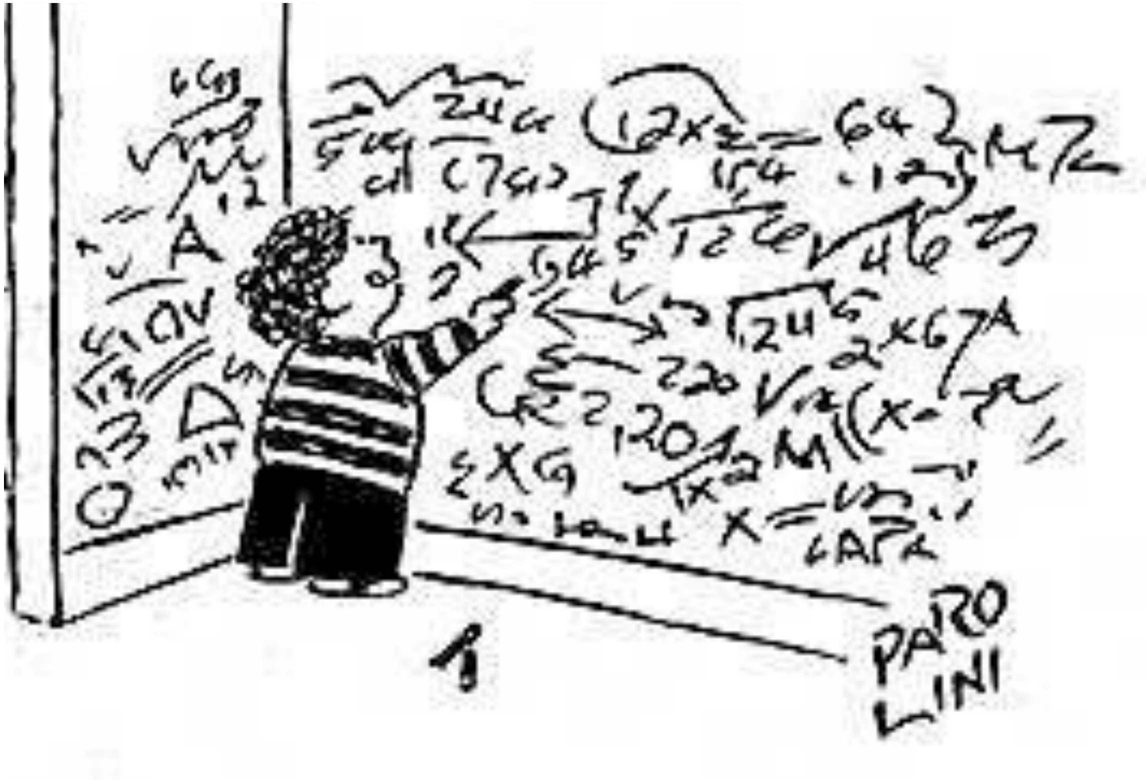
Zo=70.73Ω, error = 3%

Resonant Cavity



ssf+un

¿Por qué funciona?



Necesitamos un poco de trabajo duro…

La expansión de Chapman-Enskog

Tomemos la regla de evolución BGK(Bhatnagar-Gross-Krook)

$$f_i(\vec{x}+\delta t\vec{v}_i, t+\delta t) - f_i(\vec{x},t) = -\frac{1}{\tau}[f_i(\vec{x},t) - f_i^{(\mathrm{eq})}(\vec{x},t)]$$

y hagamos:

•Una serie de Taylor

$$f_i(\vec{x}+\delta t\,\vec{v}_i, t+\delta t) \quad - \quad f_i(\vec{x},t) = \delta t\left[\frac{\partial}{\partial t} + \vec{v}_i\cdot\vec{\nabla}\right] f_i + \frac{\delta t^2}{2}\left[\frac{\partial}{\partial t} + \vec{v}_i\cdot\vec{\nabla}\right]^2 f_i$$

•Una expansión perturbativa en ε (ε->0 es el the límite continuo)

$$\frac{\partial}{\partial t} = \epsilon\frac{\partial}{\partial t_1} + \epsilon^2\frac{\partial}{\partial t_2} \qquad \frac{\partial}{\partial x} = \epsilon\frac{\partial}{\partial x_1} \qquad f_i = f_i^{(0)} + \epsilon f_i^{(1)} + \epsilon^2 f_i^{(2)}$$

•Sólo el orden 0 contribuye a las cantidades macroscópicas

$$\rho = \sum_i f_i^{(0)} \quad , \quad \vec{u} = \sum_i \vec{v}_i f_i^{(0)} \qquad 0 = \sum_i f_i^{(k)} \quad , \quad 0 = \sum_i \vec{v}_i f_i^{(k)}$$

•Una serie de Taylor

$$f_i(t+\delta t + x + v_{ix}\delta t, y + v_{iy}\delta t) - f(t,x,y) = \delta t \left[\frac{\partial}{\partial t} + v_{ix}\frac{\partial}{\partial x} + v_{iy}\frac{\partial}{\partial y}\right] f_i$$

$$+ \quad \frac{\delta t^2}{2}\left[\frac{\partial^2}{\partial t^2} + v_{ix}^2\frac{\partial^2}{\partial x^2} + v_{iy}^2\frac{\partial^2}{\partial y^2} + 2v_{ix}\frac{\partial^2}{\partial t\partial x} + 2v_{iy}\frac{\partial^2}{\partial t\partial y} + 2v_{ix}v_{iy}\frac{\partial^2}{\partial x\partial y}\right] f_i$$

$$= \quad \delta t\left[\frac{\partial}{\partial t} + \vec{v}_i\cdot\vec{\nabla}\right] f_i + \frac{\delta t^2}{2}\left[\frac{\partial}{\partial t} + \vec{v}_i\cdot\vec{\nabla}\right]^2 f_i$$

•Una expansión perturbativa en ε

Número de Knudsen

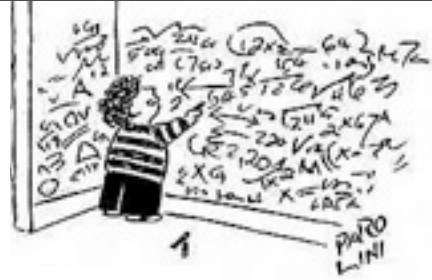
$$\epsilon = \frac{x_{\mathrm{MFP}}}{x}$$

x_{MFP} = tamaño de la celda $= \delta x$

$\epsilon \to 0$ es el límite macroscópico

•Sólo el orden 0 contribuye a las cantidades macroscópicas

Las cantidades macroscópicas no deben depender del tamaño de celda



- Replace into the BGK evolution rule

$$\epsilon\delta t\left[\frac{\partial}{\partial t_1}+\vec{v_i}\cdot\vec{\nabla}_1\right]\left(f_i^{(0)}+\epsilon f_i^{(1)}\right)+\epsilon^2\left\{\delta t\frac{\partial}{\partial t_2}+\frac{\delta t^2}{2}\left[\frac{\partial}{\partial t_1}+\vec{v_i}\cdot\vec{\nabla}_1\right]^2\right\}\left(f_i^{(0)}+\epsilon f_i^{(1)}\right)$$

$$=-\frac{1}{\tau}\left[\left(f^{(0)}-f^{(\mathrm{eq})}\right)+\epsilon f^{(1)}+\epsilon^2 f^{(2)}\right]$$

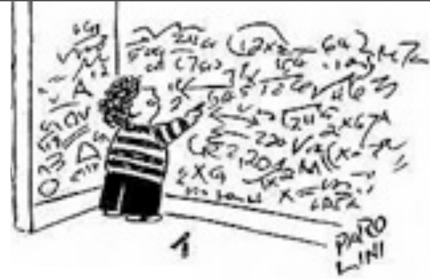
- Take order by order

Order 0: $f_i^{(eq)} = f_i^{(0)}$

Order 1: $-\frac{1}{\tau}f_i^{(1)}=\delta t\left[\frac{\partial}{\partial t_1}+\vec{v_i}\cdot\vec{\nabla}_1\right]f_i^{(0)}$

Order 2: $-\frac{1}{\tau}f_i^{(2)}=\delta t\left[\frac{\partial}{\partial t_1}+\vec{v_i}\cdot\vec{\nabla}_1\right]f_i^{(1)}+\frac{\delta t^2}{2}\left[\frac{\partial}{\partial t_1}+\vec{v_i}\cdot\vec{\nabla}_1\right]^2 f_i^{(0)}+\delta t\left[\frac{\partial}{\partial t_2}\right]f_i^{(0)}$

$$=\delta t\frac{\partial}{\partial t_2}f_i^{(0)}$$ (for $\tau=½$)



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- Take order by order

$$\frac{\delta t^2}{2}\left[\frac{\partial}{\partial t_1}+\vec{v}_i\cdot\vec{\nabla}_1\right]^2 f_i^{(0)}=-\frac{1}{2\tau}\left[\frac{\partial}{\partial t_1}+\vec{v}_i\cdot\vec{\nabla}_1\right]f_i^{(1)}$$

Order 0: $f_i^{(\mathrm{eq})}=f_i^{(0)}$

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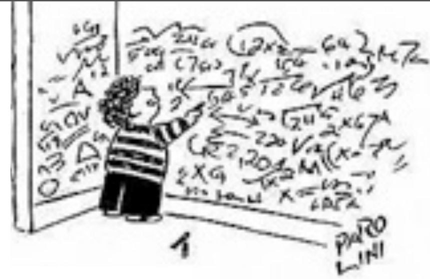

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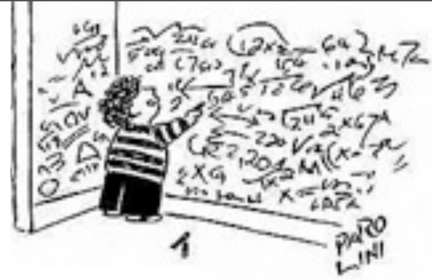

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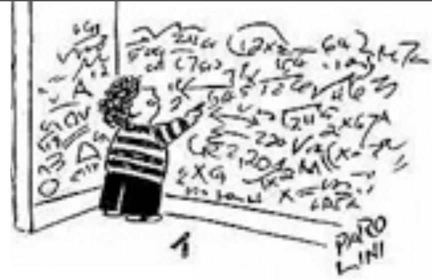

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$$=\delta t\frac{\partial}{\partial t_2}f_i^{(0)}$$ (for $\tau=½$)

- Combine the orders to reconstruct the differential operators,

ε Order 1 + ε^2 Order 2

$$-\frac{1}{\tau\delta t}\left(\epsilon f_i^{(1)}+\epsilon^2 f_i^{(2)}\right)=\left[\frac{\partial}{\partial t_1}+\vec{v_i}\cdot\vec{\nabla}\right]\left[\epsilon f_i^{(0)}+\left(1-\frac{1}{2\tau}\right)\epsilon^2 f_i^{(1)}\right]+\epsilon^2\frac{\partial}{\partial t_2}f_i^{(0)}$$

$$\frac{\partial}{\partial t}=\epsilon\frac{\partial}{\partial t_1}+\epsilon^2\frac{\partial}{\partial t_2}$$

- By multiplying n times by $v_{i\alpha}$, one obtain a family of partial differential equations

$$0=\frac{\partial\rho}{\partial t}+\nabla\cdot\vec{J}$$

$$0=\frac{\partial\vec{u}}{\partial t}+\vec{\nabla}\cdot\left\{\Pi^{(0)}+\left(1-\frac{1}{2\tau}\right)\Pi^{(1)}\right\}$$

$$0=\frac{\partial\Pi^{(0)}}{\partial t}+\nabla\times\Lambda$$

- Reemplazando y recombinando, obtenemos: (con τ=1/2, por simplicidad)

$$-\frac{1}{\tau\delta t}\left(\epsilon f_i^{(1)}+\epsilon^2 f_i^{(2)}\right)=\frac{\partial}{\partial t}f_i^{(0)}+\vec{\nabla}\cdot\left(\vec{v}_i f_i^{(0)}\right)$$

- Multiplicando tensorialmente por $\vec{v}_i$ a ambos lados, sumando sobre i y calculando el límite $\epsilon \longrightarrow 0$ se obtienen las leyes de conservación que el sistema cumple,

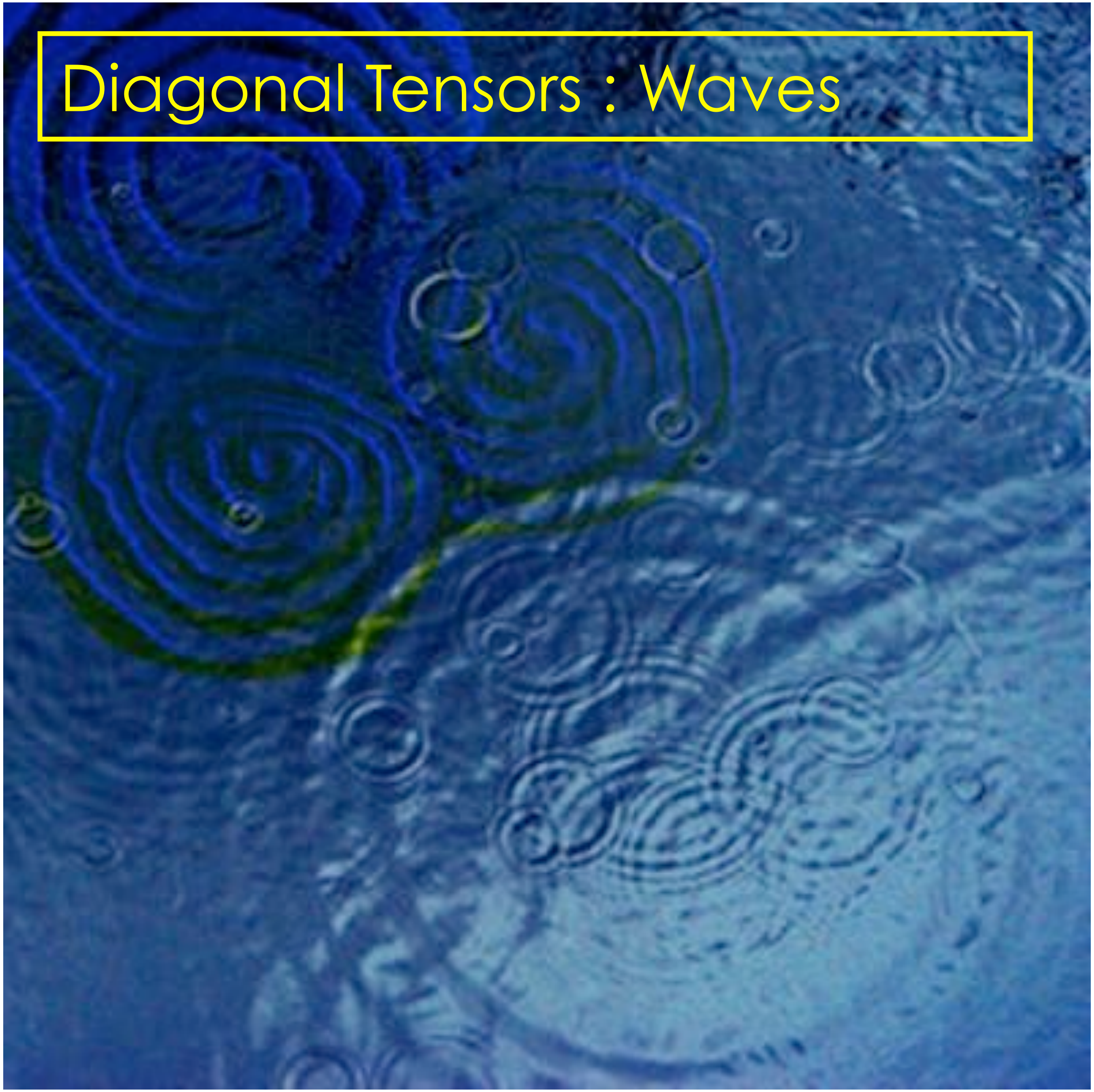
$$0=\frac{\partial\rho}{\partial t}+\nabla\cdot\vec{J} \qquad \rho=\sum_i f_i^{(0)}$$

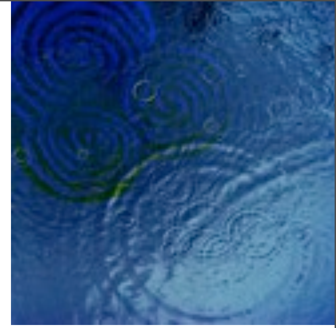
$$0=\frac{\partial\vec{J}}{\partial t}+\nabla\cdot\Pi^{(0)} \qquad \vec{J}=\sum_i \vec{v}_i f_i^{(0)}$$

$$0=\frac{\partial\Pi^{(0)}}{\partial t}+\nabla\cdot\Lambda \qquad \Pi^{(0)}=\sum_i \vec{v}_i\otimes\vec{v}_i f_i^{(0)}$$

Estas son las ecuaciones diferenciales que el sistema cumple

Diagonal Tensors : Waves





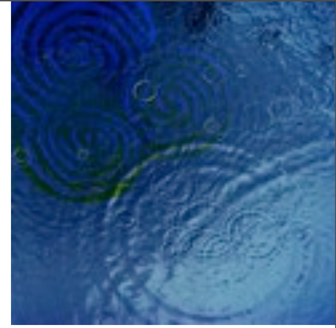
Let us consider the first two conservative laws

$$\frac{\partial \rho}{\partial t} = -\vec{\nabla} \cdot \vec{J} \qquad \frac{\partial \vec{J}}{\partial t} = -\vec{\nabla} \cdot \Pi^{(0)}$$

If we take

$$\Pi^{(0)} = \begin{pmatrix} c^2\rho & 0 \\ 0 & c^2\rho \end{pmatrix} \Rightarrow \vec{\nabla} \cdot \Pi^{(0)} = \vec{\nabla}\left(c^2\rho\right) \Rightarrow \frac{\vec{\partial} J}{\partial t} = -c^2\vec{\nabla}\rho$$

$$\Rightarrow \frac{\vec{\nabla} \cdot \vec{J}}{\partial t} = -c^2\nabla^2\rho \Rightarrow \boxed{\frac{\partial^2 \rho}{\partial t^2} = c^2\nabla^2\rho}$$



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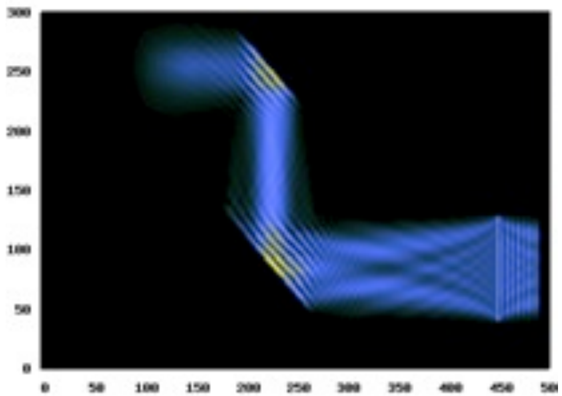
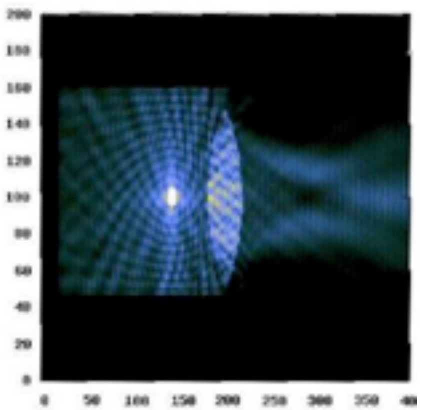
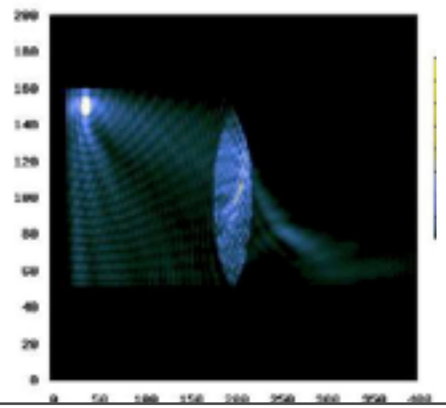
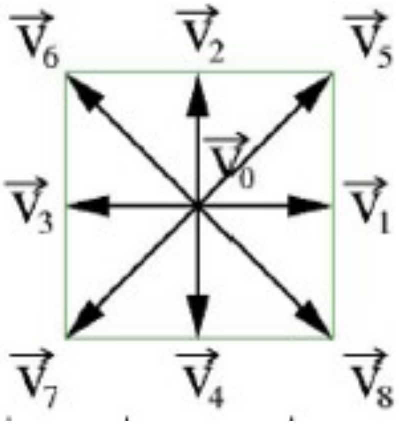






Diagonal Tensors

B. Chopard, S. Succi



$$w_i = \begin{cases} 4/9 & \text{para } i = 0 \\ 1/9 & \text{para } i = 1,2,3,4 \\ 1/36 & \text{para } i = 5,6,7,8 \end{cases}$$

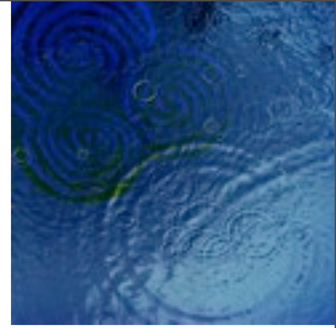
$$\sum_i w_i \, v_{i\alpha} v_{i\beta} = \frac{1}{3}\delta_{\alpha\beta}$$

In order to get

we just take

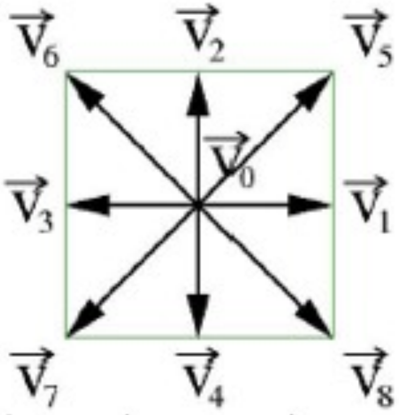
$$\Pi^{(0)} = \begin{pmatrix} c^2\rho & 0 \\ 0 & c^2\rho \end{pmatrix} \quad , \quad f_i^{eq} = \begin{cases} 3c^2\rho w_i & \text{for } i > 0 \\ 3c^2\rho w_0 & \text{for } i = 0 \end{cases}$$

$$\Pi^{(0)}_{\alpha\beta} = \psi\delta_{\alpha\beta} = \sum_i v_{i\alpha} v_{i\beta} f_i^{eq}$$



Diagonal Tensors

B. Chopard, S. Succi



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In order to get

we just take

$$\Pi^{(0)} = \begin{pmatrix} c^2\rho & 0 \\ 0 & c^2\rho \end{pmatrix} \quad , \quad f_i^{eq} = \begin{cases} 3c^2\rho w_i + 3w_i(\vec{v}_i \cdot \vec{J}) & \text{for } i > 0 \\ 3c^2\rho w_0 + \rho - 3\psi & \text{for } i = 0 \end{cases}$$

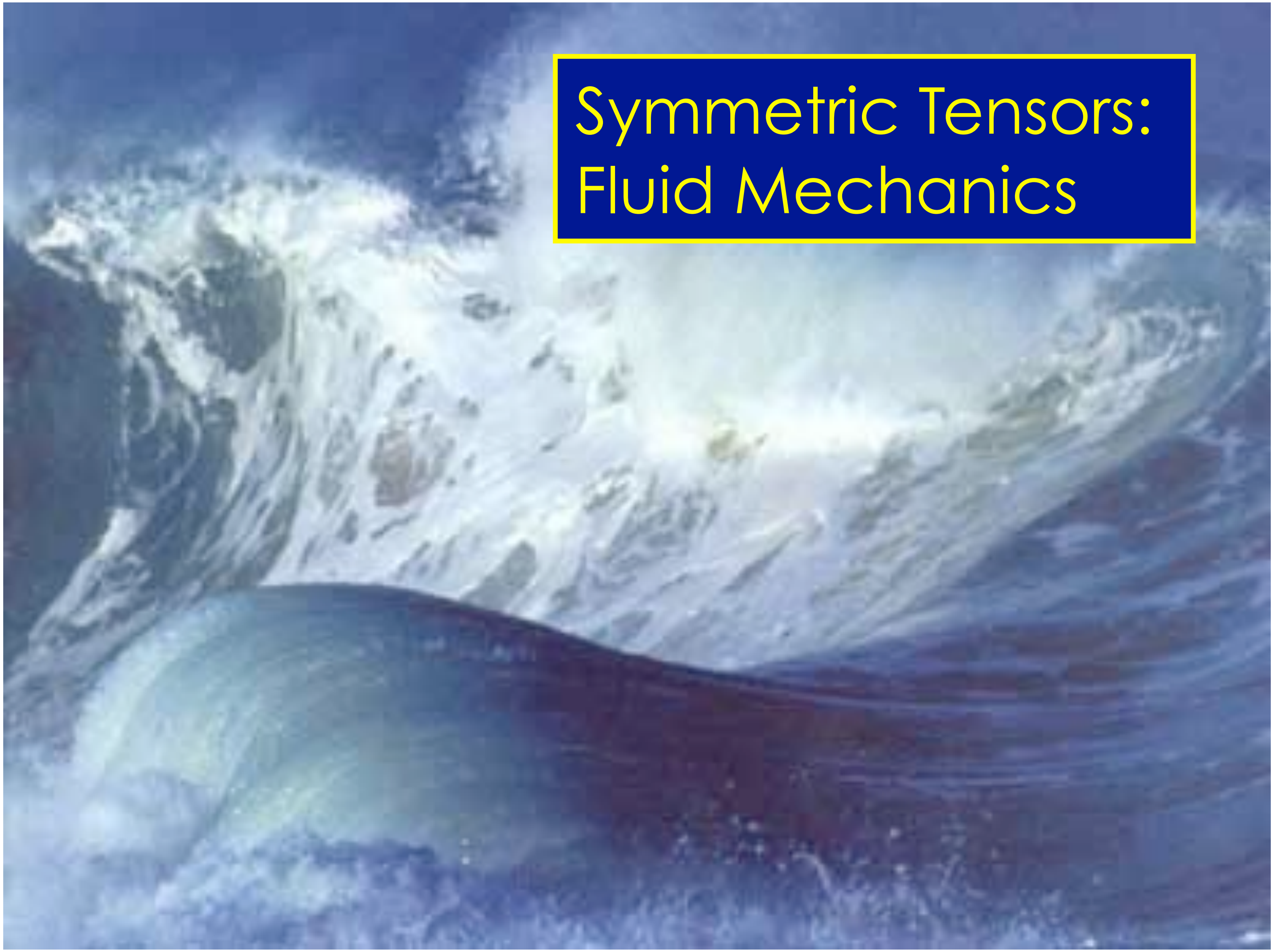
$$\Pi^{(0)}_{\alpha\beta} = \psi\delta_{\alpha\beta} = \sum_i v_{i\alpha} v_{i\beta} f_i^{eq}$$

They are added to get

$$\rho = \sum_i f_i^{eq} \qquad \vec{J} = \sum_i \vec{v}_i f_i^{eq}$$

ssf+un

Symmetric Tensors: Fluid Mechanics



Example: the Euler equation for fluids

$$\rho\left[\frac{\partial \vec{U}}{\partial t}+\left(\vec{U}\cdot\nabla\right)\vec{U}\right]=-\nabla p$$

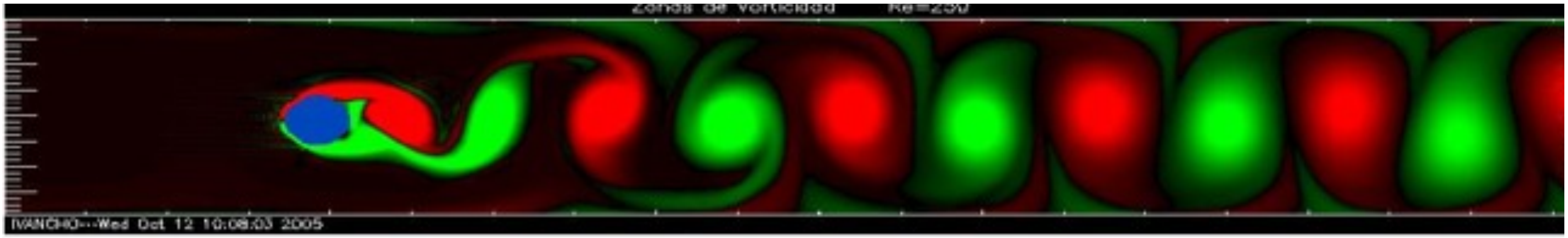
can be written out as a conservative law

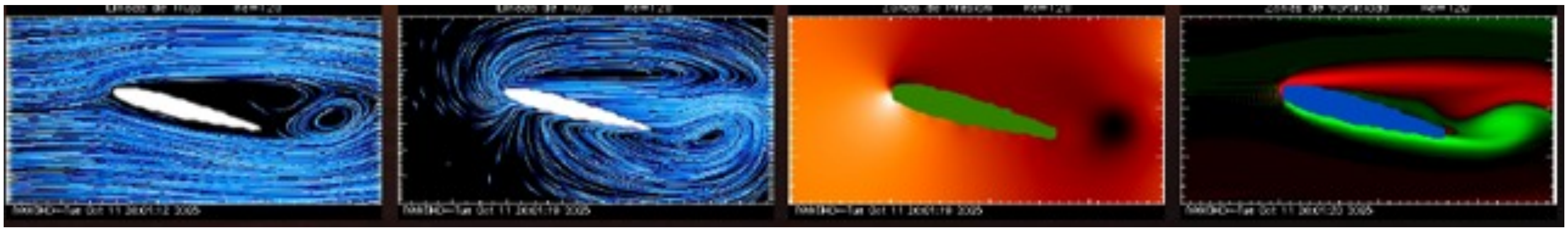
$$\vec{J}=\rho\vec{U} \qquad p=\rho/3$$

$$0=\frac{\partial \vec{J}}{\partial t}+\nabla\cdot\Gamma$$

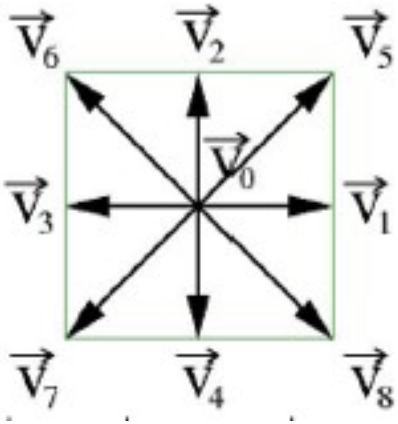
, with

$$\Gamma=\rho\begin{pmatrix} U_xU_x+1/3 & U_xU_y & U_xU_z \\ U_yU_x & U_yU_y+1/3 & U_yU_z \\ U_zU_x & U_zU_x & U_zU_z+1/3 \end{pmatrix}$$



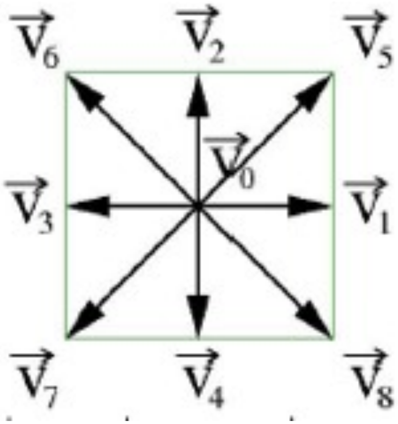


Symmetric tensors Γ



$$\sum_i w_i \, v_{i\alpha} v_{i\beta} v_{i\gamma} v_{i\delta} = \frac{1}{9} \left(\delta_{\alpha\beta}\delta_{\gamma\delta} + \delta_{\alpha\gamma}\delta_{\beta\delta} + \delta_{\alpha\delta}\delta_{\beta\gamma} \right)$$

Symmetric tensors Γ

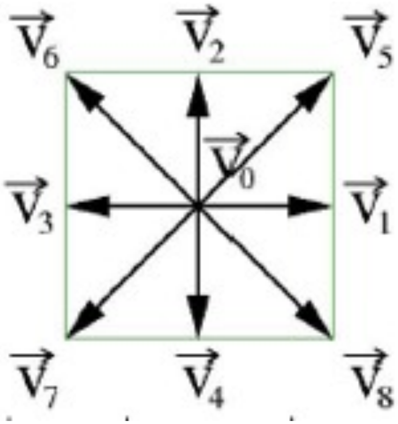


$$\sum_i w_i \, v_{i\alpha} v_{i\beta} v_{i\gamma} v_{i\delta} = \frac{1}{9}(\delta_{\alpha\beta}\delta_{\gamma\delta} + \delta_{\alpha\gamma}\delta_{\beta\delta} + \delta_{\alpha\delta}\delta_{\beta\gamma})$$

Let us take

$$f_i^{(\mathrm{eq})} = w_i(\vec{v}_i \cdot \Gamma \cdot \vec{v}_i)$$

Symmetric tensors Γ



$$\sum_i w_i\, v_{i\alpha} v_{i\beta} v_{i\gamma} v_{i\delta} = \frac{1}{9}\left(\delta_{\alpha\beta}\delta_{\gamma\delta} + \delta_{\alpha\gamma}\delta_{\beta\delta} + \delta_{\alpha\delta}\delta_{\beta\gamma}\right)$$

$$\Pi^{(0)}_{\alpha\beta} = \sum_i v_{i\alpha} v_{i\beta} f_i^{eq} = \sum_\gamma \sum_\delta \Gamma_{\gamma\delta} \left(\sum_i w_i v_{i\alpha} v_{i\beta} v_{i\gamma} v_{i\delta} \right)$$

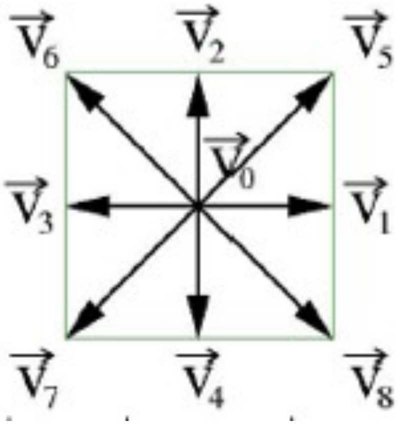
$$= \frac{1}{9} Tr(\Gamma)\delta_{\alpha\beta} + \frac{1}{9}\left(\Gamma_{\alpha\beta} + \Gamma_{\beta\alpha}\right) = \frac{1}{9} Tr(\Gamma)\delta_{\alpha\beta} + \frac{2}{9}\Gamma_{\alpha\beta}$$

Let us take

$$f_i^{(\mathrm{eq})} = w_i(\vec{v}_i \cdot \Gamma \cdot \vec{v}_i)$$

$$\Pi^{(0)} = \frac{1}{9}\begin{pmatrix} Tr(\Gamma) & 0 & 0 \\ 0 & Tr(\Gamma) & 0 \\ 0 & 0 & Tr(\Gamma) \end{pmatrix} + \frac{2}{9}\begin{pmatrix} \Gamma_{xx} & \Gamma_{xy} & \Gamma_{xz} \\ \Gamma_{yx} & \Gamma_{yy} & \Gamma_{yz} \\ \Gamma_{zx} & \Gamma_{zy} & \Gamma_{zz} \end{pmatrix}$$

Symmetric tensors Γ



$$\sum_i w_i \, v_{i\alpha} v_{i\beta} v_{i\gamma} v_{i\delta} = \frac{1}{9}\left(\delta_{\alpha\beta}\delta_{\gamma\delta} + \delta_{\alpha\gamma}\delta_{\beta\delta} + \delta_{\alpha\delta}\delta_{\beta\gamma}\right)$$

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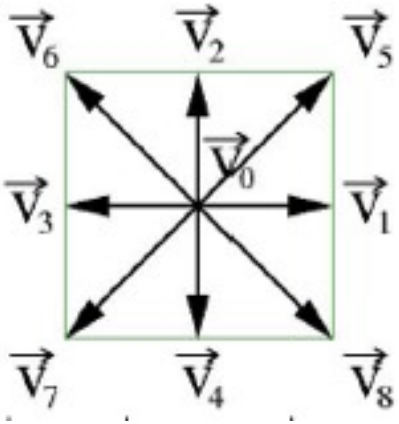
Let us take

$$f_i^{(\mathrm{eq})} = w_i(\vec{v}_i \cdot \Gamma \cdot \vec{v}_i)$$

$$\Pi^{(0)} = \frac{1}{9}\begin{pmatrix} Tr(\Gamma) & 0 & 0 \\ 0 & Tr(\Gamma) & 0 \\ 0 & 0 & Tr(\Gamma) \end{pmatrix} + \frac{2}{9}\begin{pmatrix} \Gamma_{xx} & \Gamma_{xy} & \Gamma_{xz} \\ \Gamma_{yx} & \Gamma_{yy} & \Gamma_{yz} \\ \Gamma_{zx} & \Gamma_{zy} & \Gamma_{zz} \end{pmatrix}$$

$$f_i^{(\mathrm{eq})} = \begin{cases} \frac{9}{2} w_i(\vec{v}_i \cdot \Gamma \cdot \vec{v}_i) - \frac{3}{2} w_i Tr(\Gamma) & \text{for } i > 0 \\ & \text{for } i = 0 \end{cases}$$

Symmetric tensors Γ



$$\sum_i w_i\, v_{i\alpha} v_{i\beta} v_{i\gamma} v_{i\delta} = \frac{1}{9}\left(\delta_{\alpha\beta}\delta_{\gamma\delta} + \delta_{\alpha\gamma}\delta_{\beta\delta} + \delta_{\alpha\delta}\delta_{\beta\gamma}\right)$$

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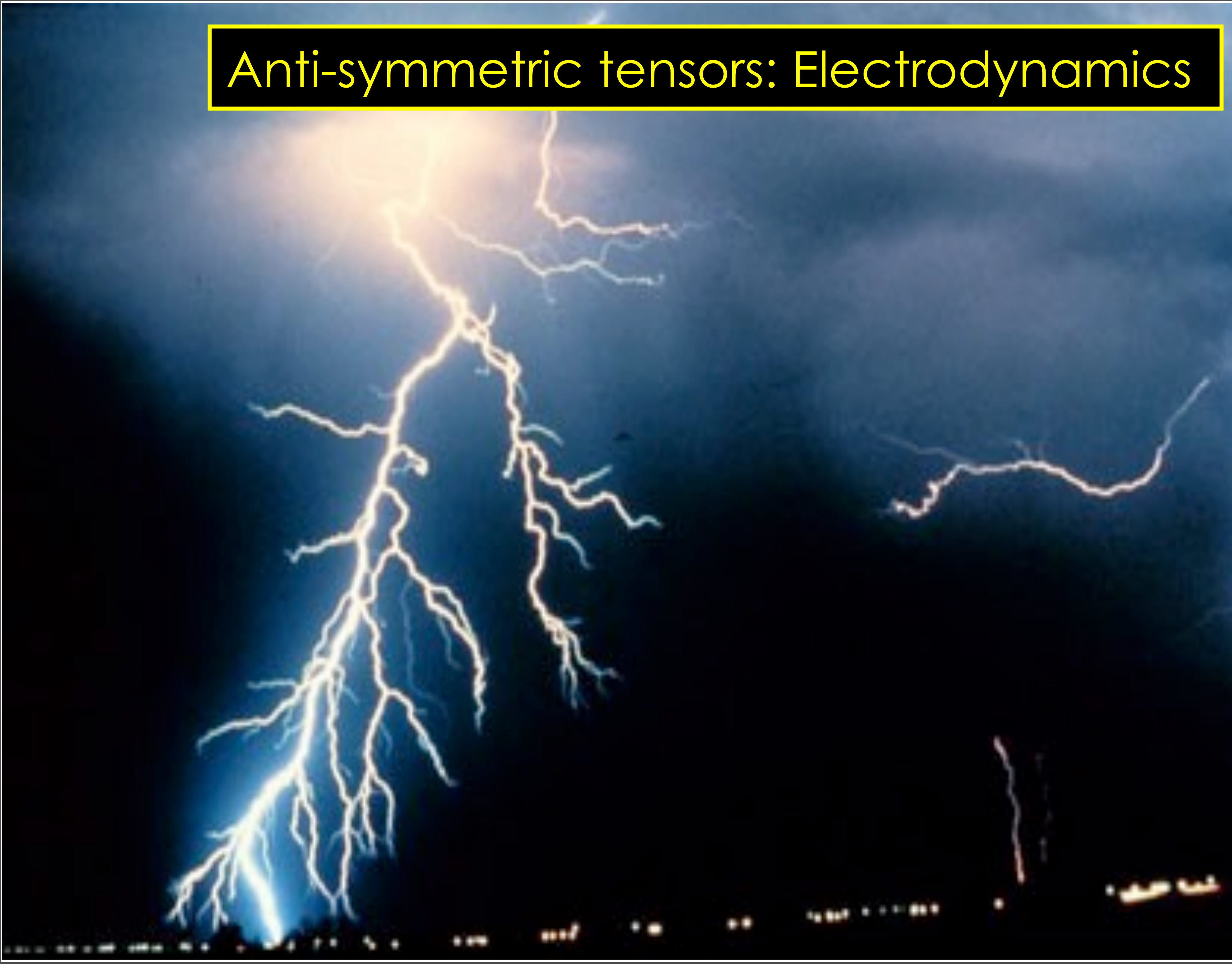
Let us take

$$f_i^{(\mathrm{eq})} = w_i(\vec{v}_i \cdot \Gamma \cdot \vec{v}_i)$$

$$\Pi^{(0)} = \frac{1}{9}\begin{pmatrix} Tr(\Gamma) & 0 & 0 \\ 0 & Tr(\Gamma) & 0 \\ 0 & 0 & Tr(\Gamma) \end{pmatrix} + \frac{2}{9}\begin{pmatrix} \Gamma_{xx} & \Gamma_{xy} & \Gamma_{xz} \\ \Gamma_{yx} & \Gamma_{yy} & \Gamma_{yz} \\ \Gamma_{zx} & \Gamma_{zy} & \Gamma_{zz} \end{pmatrix}$$

$$f_i^{(\mathrm{eq})} = \begin{cases} \frac{9}{2} w_i(\vec{v}_i \cdot \Gamma \cdot \vec{v}_i) - \frac{3}{2} w_i Tr(\Gamma) \; + 3w_i(\vec{v}_i \cdot \vec{J}) & \text{for } i > 0 \\ \rho - \dfrac{3}{2}(1 - w_0) Tr(\Gamma) & \text{for } i = 0 \end{cases}$$

Anti-symmetric tensors: Electrodynamics

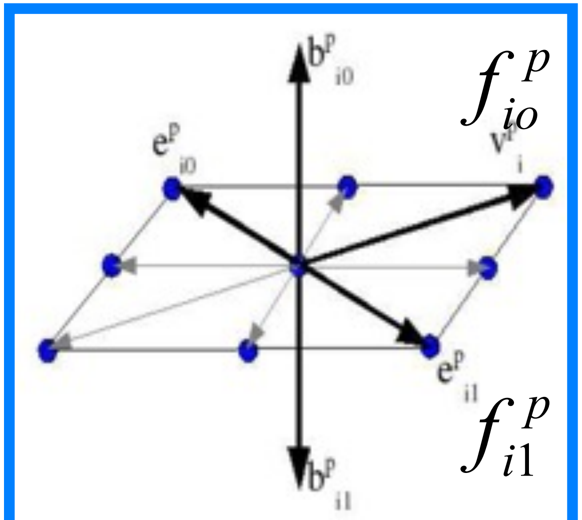


Anti-Symmetric tensors: Faraday's law

$$\frac{\partial \vec{B}}{\partial t} = -\vec{\nabla} \times \vec{E} = -\vec{\nabla} \cdot \begin{pmatrix} 0 & -E_x & E_y \\ E_x & 0 & -E_z \\ -E_y & E_z & 0 \end{pmatrix}$$

Miller Mendoza

Define four auxiliary vectors e_{i0}^{p}, e_{i1}^{p} ; b_{i0}^{p}, b_{i1}^{p} per velocity vector $\vec{v}_i^{\,p}$



, and two distribution functions f_{i0}^{p}, f_{i1}^{p} traveling with $\vec{v}_i^{\,p}$

$$\vec{E} = \sum_{i,j,p} \vec{e_{ij}}^{\,p} f_{ij}^{p} \qquad \vec{B} = \sum_{i,j,p} \vec{b_{ij}}^{\,p} f_{ij}^{p}$$

Choose the following equilibrium function:

$$f_{ij}^{p(eq)} = \frac{1}{4}\left(\vec{e}_{ij}^{\,p} \cdot \vec{E}\right) + \frac{1}{8}\left(\vec{b}_{ij}^{\,p} \cdot \vec{B}\right)$$

$$\boxed{\sum_{i,p} v_{i\alpha}^{p} e_{ij\beta}^{p} b_{ij\gamma}^{p} = 4\varepsilon_{\alpha\beta\gamma}}$$

From the Chapman-Enskog expansion,

$$-\frac{1}{\tau\delta t}\left(\epsilon f_i^{(1)}+\epsilon^2 f_i^{(2)}\right)=\frac{\partial}{\partial t}f_i^{(0)}+\vec{\nabla}\cdot\left(\vec{v}_i f_i^{(0)}\right)$$

Multiply by $\vec{b}_{ij}^{\,p}$ and add on i,j,p

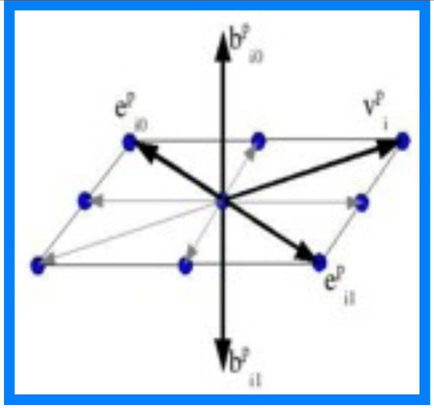
$$\boxed{0=\frac{\partial\vec{B}}{\partial t}+\vec{\nabla}\cdot\Lambda^{(0)} \qquad \Lambda^{(0)}_{\alpha\beta}=\sum_{i,j,p} f_{ij}^{p(eq)} v_{i\,\alpha}^{p} b_{ij\;\beta}^{p}}$$

With the equilibrium function

$$f_{ij}^{p(eq)}=\frac{1}{4}\left(\vec{e}_{ij}^{\,p}\cdot\vec{E}\right)+\frac{1}{8}\left(\vec{b}_{ij}^{\,p}\cdot\vec{B}\right)$$

, we obtain

$$\Lambda^{(0)}_{\alpha\beta}=\frac{1}{4}\sum_\gamma E_\gamma\left(\sum_{i,j,p} v_{i\alpha}^p e_{ij\beta}^p b_{ij\gamma}^p\right)=\sum_\gamma E_\gamma \varepsilon_{\alpha\beta\gamma}$$



$$\boxed{\sum_{i,p} v_{i\alpha}^p e_{ij\beta}^p b_{ij\gamma}^p = 4\varepsilon_{\alpha\beta\gamma}}$$

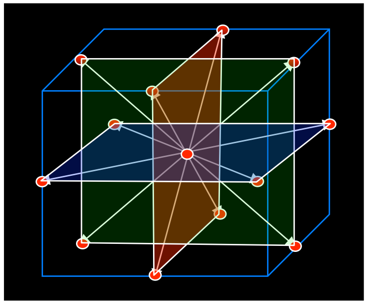
$$\vec{E}=\sum_{i,j,p}\vec{e_{ij}}^{\,p} f_{ij}^p$$

$$\vec{B}=\sum_{i,j,p}\vec{b_{ij}}^{\,p} f_{ij}^p$$

$$\boxed{\Lambda^{(0)}=\begin{pmatrix}0 & -E_x & E_y\\ E_x & 0 & -E_z\\ -E_y & E_z & 0\end{pmatrix}}$$

Resumiendo

Resumiendo



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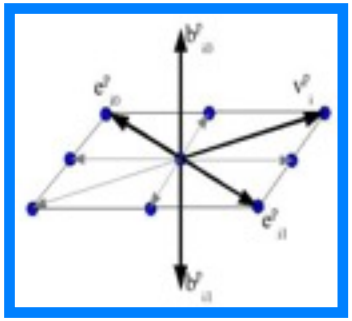
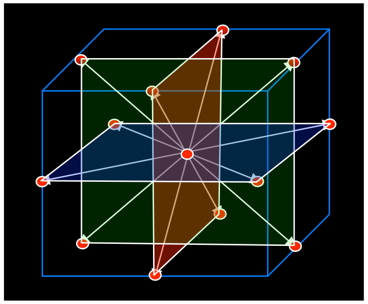


Cuatro funciones por $\vec{v}_i$

$f_{i0}^{p(1)}, f_{i1}^{p(1)}, f_{i0}^{p(2)}, f_{i1}^{p(2)}$

$$\boxed{\sum_{i,p} v_{i\alpha}^{p} e_{ij\beta}^{p} b_{ij\gamma}^{p} = 4\varepsilon_{\alpha\beta\gamma}}$$

Resumiendo



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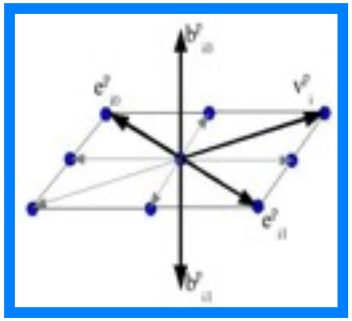

1

Cuatro funciones por $\vec{v}_i$

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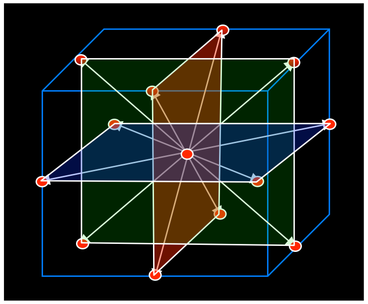
$$\boxed{\sum_{i,p} v_{i\alpha}^{p} e_{ij\beta}^{p} b_{ij\gamma}^{p} = 4\varepsilon_{\alpha\beta\gamma}}$$

Resumiendo



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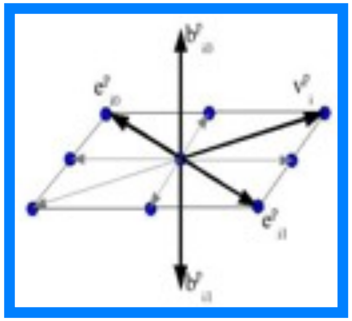


Cuatro funciones por $\vec{v}_i$

$$f_{i0}^{p(1)}, f_{i1}^{p(1)}, f_{i0}^{p(2)}, f_{i1}^{p(2)}$$

$$\boxed{\sum_{i,p} v_{i\alpha}^{p} e_{ij\beta}^{p} b_{ij\gamma}^{p} = 4\varepsilon_{\alpha\beta\gamma}}$$

1

Macroscopic Quantities

$$\vec{D} = \sum_{i=1}^{4}\sum_{p=0}^{2}\sum_{j=0}^{1} f_{ij}^{p(0)} \vec{e}_{ij}^{\,p}$$

$$\vec{B} = \sum_{i=1}^{4}\sum_{p=0}^{2}\sum_{j=0}^{1} f_{ij}^{p(1)} \vec{b}_{ij}^{\,p}$$

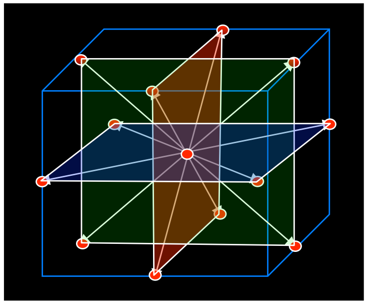
$$\rho_c = f_0^{(0)} + \sum_{i=1}^{4}\sum_{p=0}^{2}\sum_{j=0}^{1} f_{ij}^{p(0)}$$

$$\vec{E} = \frac{\vec{D}}{\epsilon_r} \qquad \vec{H} = \frac{\vec{B}}{\mu_r}$$

$$\vec{J} = \sigma \vec{E}$$

$$\vec{J}' = \frac{\sigma}{1 + \frac{\mu_0 \sigma}{4\epsilon_r}} \vec{E} \qquad \vec{E}' = \vec{E} - \frac{\mu_0}{4\epsilon_r} \vec{J}'$$

Resumiendo



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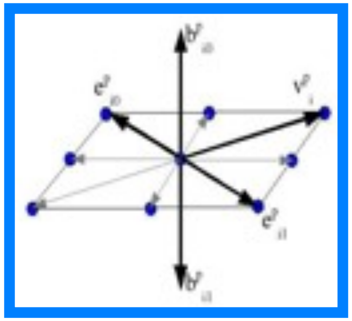


Cuatro funciones por $\vec{v}_i$

$f_{i0}^{p(1)}, f_{i1}^{p(1)}, f_{i0}^{p(2)}, f_{i1}^{p(2)}$

$$\boxed{\sum_{i,p} v_{i\alpha}^{p} e_{ij\beta}^{p} b_{ij\gamma}^{p} = 4\varepsilon_{\alpha\beta\gamma}}$$

1

2

Macroscopic Quantities

$$\vec{D} = \sum_{i=1}^{4}\sum_{p=0}^{2}\sum_{j=0}^{1} f_{ij}^{p(0)} \vec{e}_{ij}^{\,p}$$

$$\vec{B} = \sum_{i=1}^{4}\sum_{p=0}^{2}\sum_{j=0}^{1} f_{ij}^{p(1)} \vec{b}_{ij}^{\,p}$$

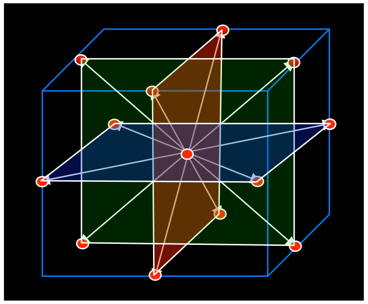
$$\rho_c = f_0^{(0)} + \sum_{i=1}^{4}\sum_{p=0}^{2}\sum_{j=0}^{1} f_{ij}^{p(0)}$$

$$\vec{E} = \frac{\vec{D}}{\epsilon_r} \qquad \vec{H} = \frac{\vec{B}}{\mu_r}$$

$$\vec{J} = \sigma \vec{E}$$

$$\vec{J}' = \frac{\sigma}{1 + \frac{\mu_0 \sigma}{4\epsilon_r}} \vec{E} \qquad \vec{E}' = \vec{E} - \frac{\mu_0}{4\epsilon_r} \vec{J}'$$

Resumiendo



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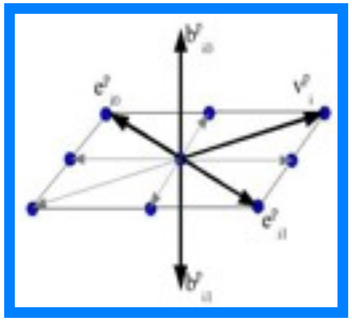


1

2

Cuatro funciones por $\vec{v}_i$

$$f_{i0}^{p(1)}, f_{i1}^{p(1)}, f_{i0}^{p(2)}, f_{i1}^{p(2)}$$

$$\boxed{\sum_{i,p} v_{i\alpha}^{p} e_{ij\beta}^{p} b_{ij\gamma}^{p} = 4\varepsilon_{\alpha\beta\gamma}}$$

Macroscopic Quantities

$$\vec{D} = \sum_{i=1}^{4}\sum_{p=0}^{2}\sum_{j=0}^{1} f_{ij}^{p(0)}\vec{e}_{ij}^{\,p}$$

$$\vec{B} = \sum_{i=1}^{4}\sum_{p=0}^{2}\sum_{j=0}^{1} f_{ij}^{p(1)}\vec{b}_{ij}^{\,p}$$

$$\rho_c = f_0^{(0)} + \sum_{i=1}^{4}\sum_{p=0}^{2}\sum_{j=0}^{1} f_{ij}^{p(0)}$$

$$\vec{E} = \frac{\vec{D}}{\epsilon_r} \qquad \vec{H} = \frac{\vec{B}}{\mu_r}$$

$$\vec{J} = \sigma\vec{E}$$

$$\vec{J}' = \frac{\sigma}{1+\frac{\mu_0\sigma}{4\epsilon_r}}\vec{E} \qquad \vec{E}' = \vec{E} - \frac{\mu_0}{4\epsilon_r}\vec{J}'$$

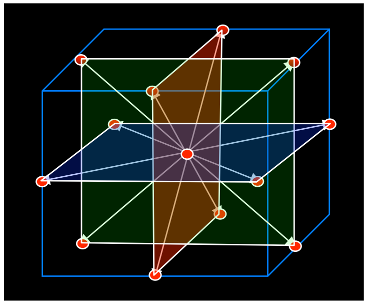
Funciones de Equilibrio

3

$$f_{ij}^{p(0)\mathrm{eq}}(\vec{x},t) = \frac{1}{16}\vec{v}_i^{\,p}\cdot\vec{J}' + \frac{\epsilon}{4}\vec{E}'\cdot\vec{e}_{ij}^{\,p} + \frac{1}{8\mu}\vec{B}\cdot\vec{b}_{ij}^{\,p}$$

$$f_{ij}^{p(1)\mathrm{eq}}(\vec{x},t) = \frac{1}{16}\vec{v}_i^{\,p}\cdot\vec{J}' + \frac{1}{4}\vec{E}'\cdot\vec{e}_{ij}^{\,p} + \frac{1}{8}\vec{B}\cdot\vec{b}_{ij}^{\,p}$$

Resumiendo



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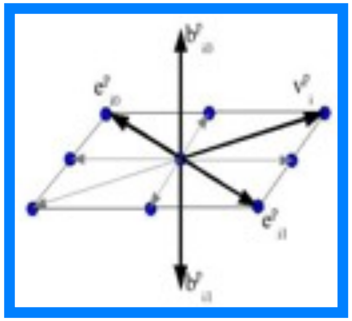

1

2

Macroscopic Quantities

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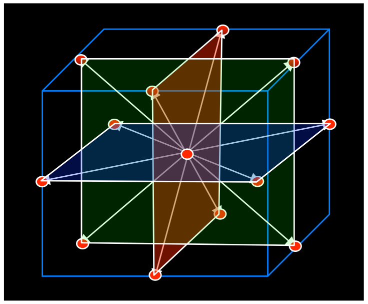
Regla de Evolución

4

$$f_{ij}^{p(r)}(\vec{x}+\vec{v}_i^{\,p}, t+1) = f_{ij}^{p(r)}(\vec{x},t) - 2\left[f_{ij}^{p(r)}(\vec{x},t) - f_{ij}^{p(r)\mathrm{eq}}(\vec{x},t)\right]$$

Resumiendo



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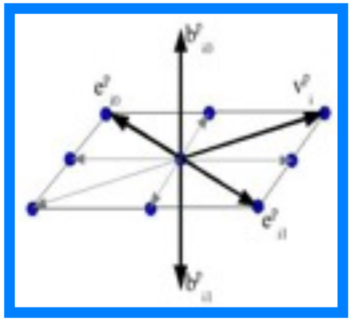

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Funciones de Equilibrio

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$$f_{ij}^{p(0)\mathrm{eq}}(\vec{x},t) = \frac{1}{16} \vec{v}_i^{\,p} \cdot \vec{J}' + \frac{\epsilon}{4} \vec{E}' \cdot \vec{e}_{ij}^{\,p} + \frac{1}{8\mu} \vec{B} \cdot \vec{b}_{ij}^{\,p}$$

$$f_{ij}^{p(1)\mathrm{eq}}(\vec{x},t) = \frac{1}{16} \vec{v}_i^{\,p} \cdot \vec{J}' + \frac{1}{4} \vec{E}' \cdot \vec{e}_{ij}^{\,p} + \frac{1}{8} \vec{B} \cdot \vec{b}_{ij}^{\,p}$$

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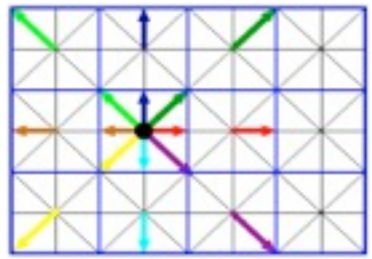
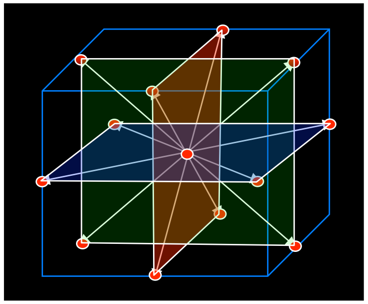
5



Resumiendo



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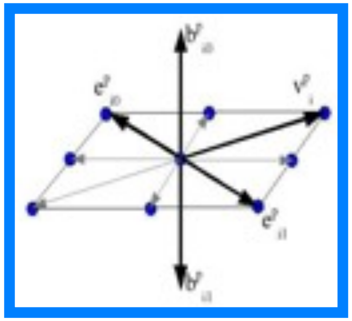

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Funciones de Equilibrio

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$$f_{ij}^{p(0)\mathrm{eq}}(\vec{x},t) = \frac{1}{16}\vec{v}_i^{\,p}\cdot\vec{J}' + \frac{\epsilon}{4}\vec{E}'\cdot\vec{e}_{ij}^{\,p} + \frac{1}{8\mu}\vec{B}\cdot\vec{b}_{ij}^{\,p}$$

$$f_{ij}^{p(1)\mathrm{eq}}(\vec{x},t) = \frac{1}{16}\vec{v}_i^{\,p}\cdot\vec{J}' + \frac{1}{4}\vec{E}'\cdot\vec{e}_{ij}^{\,p} + \frac{1}{8}\vec{B}\cdot\vec{b}_{ij}^{\,p}$$

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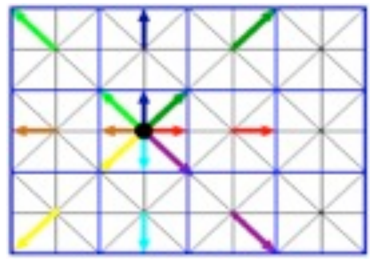
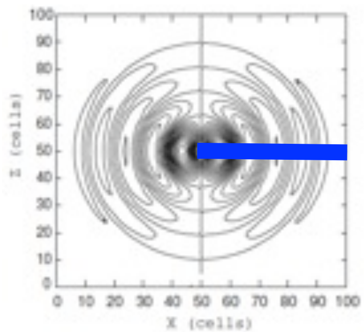
5



$$\nabla\times\vec{E}' = -\frac{\partial\vec{B}}{\partial t} \qquad \nabla\times\vec{H} = \mu_0\vec{J}' + \frac{1}{c^2}\frac{\partial\vec{D}'}{\partial t} \qquad \frac{\partial\rho_c}{\partial t} + \nabla\cdot\vec{J}' = 0$$

$$\frac{\partial}{\partial t}\left(\nabla\cdot\vec{D}' - \frac{\rho_c}{\epsilon_0}\right) = 0 \qquad \frac{\partial}{\partial t}\left(\nabla\cdot\vec{B}\right) = 0$$



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Gracias!

jdmunozc@unal.edu.co