OSCILLATIONS

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- **Oscillatory motion** → To and fro
- **Periodic motion** → Motion is repeated after a fixed period.
- **Simple Harmonic motion (SHM)** :-

Force (acceleration due to gravity) - Brings object from extreme to mean.
Displacement - Always opposite to acceleration
Inertia of motion - Brings object from mean to extreme

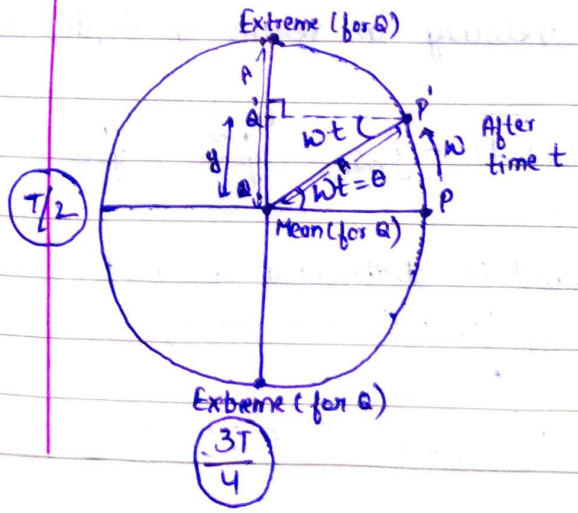
- Oscillatory motion → uniform velocity SHM → non-uniform velocity

θ	sin θ	cos θ	tan θ
0	0	1	0
30	0.5	0.85	
60	0.85	0.5	
90	1	0	∞
120	0.85	-0.5	
150	0.5	-0.85	
180	0	-1	0
210	-0.5	-0.85	
240	-0.85	-0.5	
270	-1	0	∞
300	-0.85	0.5	
330	-0.5	0.85	
360	0	1	0

sin, cos, sec, cosec periodic function hain kyunki unki values ek fixed period ke baad repeat ho rahi hain

tan periodic function nahi hai kyunki usme ∞ aata hai.

Phasor Diagram



When Q is on y-axis

(Maximum displacement Q can do → Amplitude (A))

(Q hamesha P ke perpendicular foot par rehta hai)

(P → circulatory motion
Q → oscillatory (linear motion))

ωt (Angular displacement) is called phase.

$\sin \omega t = \frac{y}{A}$ $\boxed{y = A \sin \omega t}$

After time T/4, $y = A \sin \frac{2\pi}{T} \times \frac{T}{4}$

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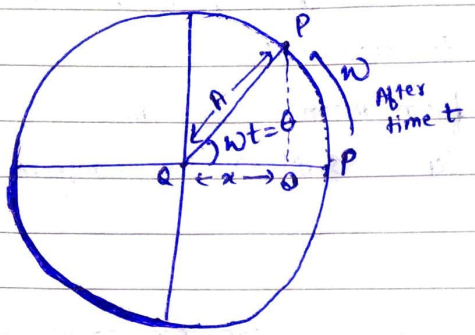
$y = A \sin \frac{\pi}{2}$ $\boxed{y = A}$ (Q ki position A hogi)

When θ is on x-axis



$\cos \omega t = \frac{x}{A}$

$\boxed{x = A \cos \omega t}$

x → Displacement, A → Amplitude, ω → Angular frequency

- When motion starts from mean → $y = A \sin \omega t$
  When motion starts from extreme → $x = A \cos \omega t$
- Putting T = 0 in $y = A \sin \omega t$, then displacement is 0 (mean)
  Putting T = 0 in $x = A \cos \omega t$, then displacement is A (extreme)
- $y = A \sin \omega t$ 30° par Q extreme ki taraf ja raha hai.
  150° par Q mean ki taraf ja raha hai.

90° tak Q extreme ki taraf ja raha hai (If motion starts from mean)

Displacement $y = A \sin \omega t$

For calculation of velocity $= \frac{dy}{dt}$

$v = \frac{d(A \sin \omega t)}{dt} = A \cos \omega t \cdot \omega$

$\boxed{v = A\omega \cos \omega t}$ → velocity in terms of time

$v = A\omega \sqrt{1 - \sin^2 \omega t} = \omega \sqrt{A^2 - A^2 \sin^2 \omega t}$

$\boxed{v = \omega \sqrt{A^2 - y^2}}$ → velocity in terms of displacement

For maximum velocity $v_{max} = \omega \sqrt{A^2 - y_{min}^2}$

$v_{max} = \omega \sqrt{A^2 - 0}$

Extreme ($y = -A$) — Mean ($y = 0$) — Extreme ($y = A$)

To get max. velocity, min. value should be subtracted from A^2.

$\boxed{v_{max} = A\omega}$ → on mean (∵ y = 0 is on mean)

For minimum velocity $v_{min} = \omega \sqrt{A^2 - y_{max}^2} = \omega \sqrt{A^2 - A^2} = 0$

$\boxed{v_{min} = 0}$ → on extreme (∵ y = A is on extreme)

$\theta = \pi$

$t = T/2$

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$t = 3T/4$ | $y = -A$ (E) | $y = 0$ (M) | $y = A$ (E) | $t = T/4$

$V_{min} = 0$ | $t = 0$, $t = T$ | $V_{max} = A\omega$ | $V_{min} = 0$

$\theta = \frac{3\pi}{2}$ | $\theta = 0$, $\theta = 2\pi$ | $\theta = \pi/2$

Acceleration

$$a = \frac{dv}{dt} = \frac{d(A\omega \cos \omega t)}{dt} = -A\omega \sin \omega t \quad \because \omega$$

$$a = -A\omega^2 \sin \omega t$$

-ve sign indicates opposite direction from displacement

$$a = -\omega^2 y$$

a_{max}, when $y = A$ | $a_{max} = -\omega^2 A$ (Extreme)

a_{min}, when $y = 0$ | $a_{min} = 0$ (mean)

$\theta = \pi$

$t = T/2$

$y = -A$ (E) | $y = 0$ (M) | $y = A$ (E)

$t = 3T/4$ | $t = 0$, $t = T$ | $t = T/4$

$a_{max} = -\omega^2 A$ | $a_{min} = 0$ | $a_{max} = -\omega^2 A$

$\theta = \frac{3\pi}{2}$ | $\theta = 0$, $\theta = 2\pi$ | $\theta = \pi/2$

Time Period

$a = \omega^2 y$ $\quad a = \left(\frac{2\pi}{T}\right)^2 y$ $\quad \left(\frac{T}{2\pi}\right) = \sqrt{\frac{y}{a}}$

$$T = 2\pi \sqrt{\frac{y}{a}}$$

$$T = 2\pi \sqrt{\frac{\text{Displacement}}{\text{Acceleration}}}$$

$T \propto$ Displacement $\quad T \propto \frac{1}{\text{Acceleration}}$

$T = 2\pi\sqrt{\frac{my}{ma}} = 2\pi\sqrt{\frac{m}{(F/y)}}$ → spring constant (because of restoring force)

$$T = 2\pi\sqrt{\frac{m}{k}}$$

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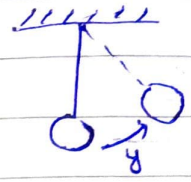
Work & Energy

- $W = \int f.dy = \int ma.dy = \int m\omega^2 y.dy$
  $= m\omega^2 \int y.dy = \frac{m\omega^2 y^2}{2}$



(ii)

$$W = \frac{m\omega^2 y^2}{2}$$

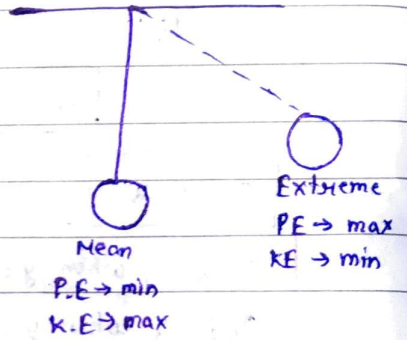
- $$P.E = \frac{1}{2} m\omega^2 y^2$$

P.E (min) ⇒ y = 0 (mean)

$$P.E_{min} = \frac{1}{2} m\omega^2 (0)^2 = 0$$

P.E (max) ⇒ y = A (extreme)

$$P.E_{max} = \frac{1}{2} m\omega^2 A^2$$



(iii)

- $$K.E = \frac{1}{2} mv^2 = \frac{1}{2} m\omega^2 (A^2 - y^2)$$

(iv)

KE (min) ⇒ y = A (extreme)

$$K.E_{min} = \frac{1}{2} m\omega^2 (A^2 - A^2) = 0$$

KE (max) ⇒ y = 0 (mean)

$$K.E_{max} = \frac{1}{2} m\omega^2 (A^2 - 0)$$

$$K.E_{max} = \frac{1}{2} m\omega^2 A^2$$

- Total Energy at mean.

$$T.E = KE + PE = \frac{1}{2} m\omega^2 A^2 + 0 = \frac{1}{2} m\omega^2 A^2$$

Total Energy at extreme

$$TE = KE + PE = 0 + \frac{1}{2} m\omega^2 A^2 = \frac{1}{2} m\omega^2 A^2$$

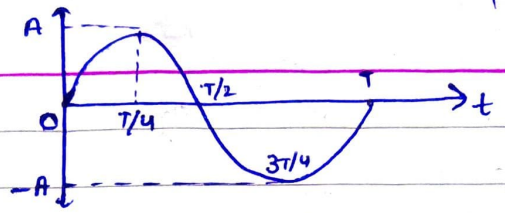
Total Energy at any point

$$TE = \left(\frac{1}{2} m\omega^2 A^2 - \frac{1}{2} m\omega^2 y^2\right) + \frac{1}{2} m\omega^2 y^2 = \frac{1}{2} m\omega^2 A^2$$

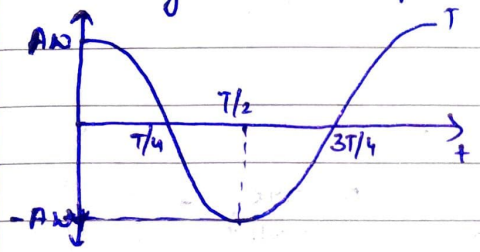
TE remains conserved.

Graphical Representation

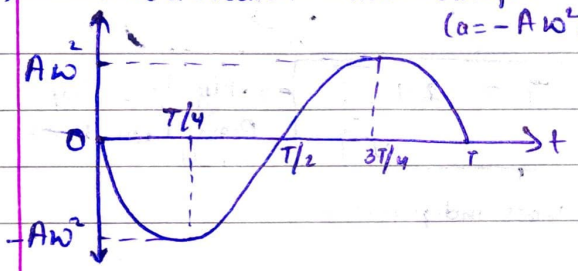
(i) Displacement - Time Graph $(y = A \sin \omega t)$



(ii) Velocity - Time Graph $(v = A\omega \cos \omega t)$

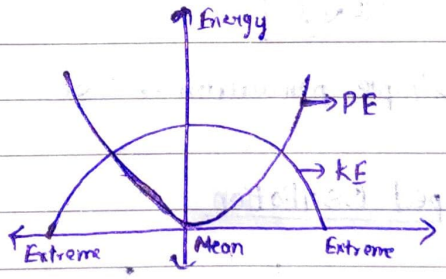


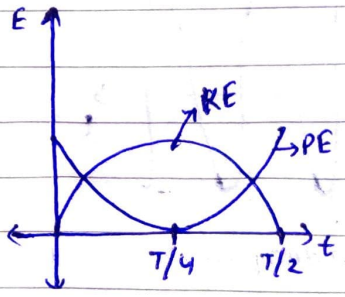
(iii) Acceleration - Time Graph $(a = -A\omega^2 \sin \omega t)$



Acceleration is maximum at a place where the velocity is minimum and vice versa

(iv) Energy





$$KE = \frac{1}{2} m\omega^2 (A^2 - y^2)$$

$(y = A \sin \omega t)$

$$= \frac{1}{2} m\omega^2 (A^2 - A^2 \sin^2 \omega t)$$

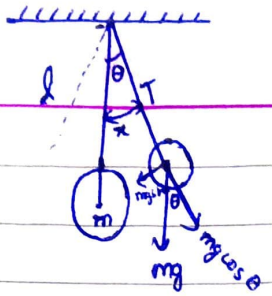
$$= \frac{1}{2} m\omega^2 A^2 (1 - \sin^2 \omega t)$$

$$\boxed{KE = \frac{1}{2} m\omega^2 A^2 \cos^2 \omega t}$$

$$PE = \frac{1}{2} m\omega^2 y^2$$

$$\boxed{PE = \frac{1}{2} m\omega^2 A^2 \sin^2 \omega t}$$

Simple Pendulum



In simple pendulum θ is very small

Tension = $mg\cos\theta$

$mg\sin\theta$ is unbalanced force F

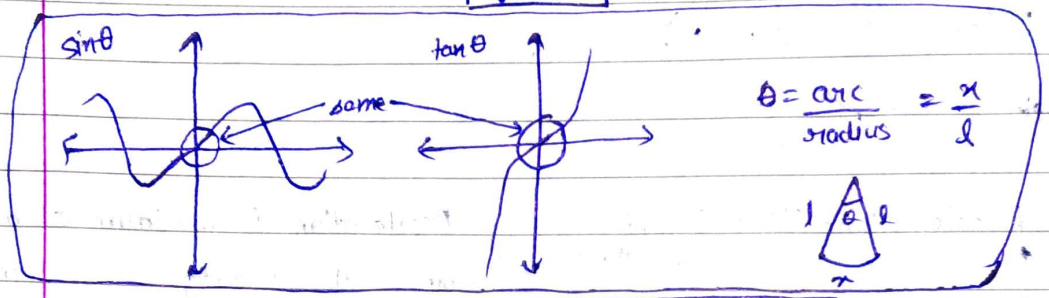
$F = mg\sin\theta$ $\quad ma = mg\sin\theta$

$a = g\sin\theta$

For small angle $\boxed{\theta = \sin\theta = \tan\theta}$

$a = g\theta$ $\quad a = g\frac{x}{l}$

$$\boxed{\frac{l}{g} = \frac{x}{a}}$$



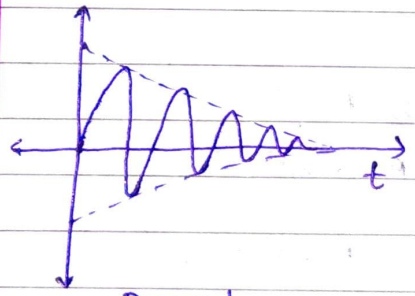
$\left(\because T = 2\pi\sqrt{\frac{y\,(\text{displ.})}{a\,(\text{acc.})}}\right)$ $\quad \boxed{T = 2\pi\sqrt{\frac{l}{g}}}$ → only for simple Pendulum because θ is small

↓ mass independent

$$\boxed{\text{Frequency} = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{g}{l}}}$$

Time period of a simple pendulum → 2 sec

Damped and Undamped Oscillation



Damped

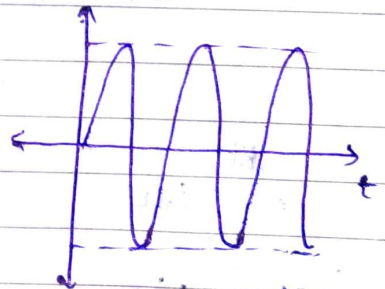
$F_{damping} = -bV$

and $F_{inertia} = -kx$

Total force $F = -bV + (-kx)$

Displacement $x(t) = Ae^{-bt/2m}\cos\omega t$

$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$



Undamping

$y = A\sin\omega t$ $\quad x = A\cos\omega t$

$\omega = \sqrt{\frac{k}{m}}$

$\left(T = 2\pi\sqrt{\frac{m}{K}} \quad \frac{T}{2\pi} = \sqrt{\frac{m}{K}}\right.$

$\left.\frac{2\pi}{T} = \sqrt{\frac{k}{m}} \quad \text{and} \quad \frac{2\pi}{T} = \omega\right)$

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Free and Forced Oscillation

Free Oscillation :- When object (particle) oscillates with its natural frequency.

Forced Oscillation :- When object (particle) oscillation under and external force.

Resonance :- When the natural frequency and applied frequency of a particle match, then resonance happens.

(Resonance) Natural frequency = Applied frequency

$$A = \frac{F_0}{\{m^2(\omega^2 - \omega_d^2) + \omega_d^2 b^2\}^{1/2}}$$

Jab Resonance hoga to oscillation ka amplitude maximum hoga.