Kinetic Theory of Gases

Ideal Gas Assumptions:

1. No. of particles is very large.
2. The volume V is much larger than actual volume of gas.
3. Dynamics of particles is governed by Newtons law of motion.
4. Gas particles interact w/ anything via elastic collisions only.
5. Particles are identical and indistinguishable.

Absolute Temperature:

$$pV = \frac{2}{3} N \left[\frac{1}{2} m\bar{v}^2 \right]$$

$$pV = nRT$$

comparing

$$nRT = \frac{2}{3} N \left[\frac{1}{2} m\bar{v}^2 \right]$$

here; $n = \frac{N}{N_A}$

$$T = \frac{2}{3}\left(\frac{N_A}{R}\right)\left(\frac{1}{2} m\bar{v}^2\right)$$

$$T = \frac{2}{3}k\left(\frac{1}{2} m\bar{v}^2\right)$$

where;

k = boltzmann's constant

$$\boxed{k = R/N_A = 1.38 \times 10^{-23}\ J/k}$$

Root Mean Square speed:

$$v_{rms} = \sqrt{\bar{v}^2} = \sqrt{\frac{3kT}{m}},$$

$$k = \frac{R}{N_A}, \quad mN_A = M$$

$$\frac{RT}{M} = \frac{P}{\rho}$$

$$\boxed{V_{rms} = \sqrt{\frac{3KT}{m}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3P}{\rho}}}$$

Mean or Average Speed :

$$\boxed{V_{RMS} > V_{AV} > V_{MP}}$$

$$V_{av} = \frac{v_1 + v_2 + v_3 - - - v_N}{N}$$

$$\boxed{V_{av} = \sqrt{\frac{8kT}{\pi m}} = \sqrt{\frac{8RT}{\pi M}} = \sqrt{\frac{8P}{\pi \rho}}}$$

Most Probable Speed :

$$\boxed{V_{mp} = \sqrt{\frac{2kT}{m}} = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2P}{\rho}}}$$

Mean Free Path :

↳ The particles of gas undergoe … path of a single gas molecule … paths. The mean free path is t… b/w two successive collisions.

$$\boxed{\lambda = \frac{kT}{\sqrt{2}\pi d^2 \rho}}$$

OR

$$\boxed{\lambda = \frac{RT}{\sqrt{2}\pi d^2 N_A P}}$$

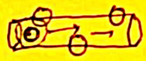
★ Mean free pathway:
↳ Distance b/w two successive collisions.

$$\bar{\lambda} = \frac{\lambda_1 + \lambda_2 + \lambda_3 - \lambda_n}{n}$$

$$\bar{\lambda} = \frac{1}{\sqrt{2}\, n \pi d^2} \Rightarrow \lambda \propto \frac{1}{d^2}$$

★ $\bar{\lambda} = \frac{k_B T}{\sqrt{2}\pi d^2 P}$

k_B = Boltzmann c.
T = temp.
P = pressure



→ Different forms of Ideal Gas eq:

$$\boxed{PV = nRT}$$

→ $PV = \left(\frac{m}{M}\right) RT$

→ $PM = \frac{m}{V} RT$

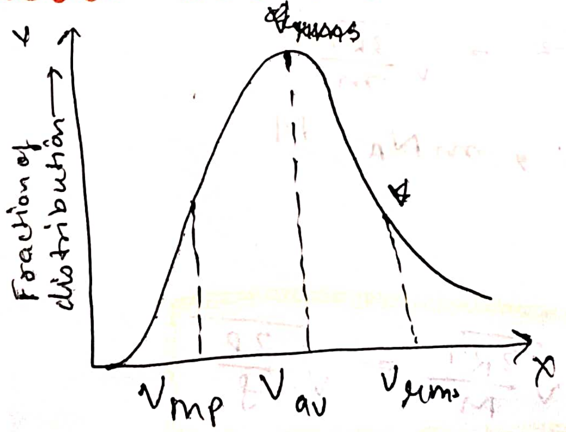
→ $\boxed{PM = \rho RT}$

→ $PV = \left(\frac{N}{N_A}\right) RT$

→ $PV = N\left(\frac{R}{N_A}\right) T$

→ $\boxed{PV = NkT}$

→ MAXWELL - BOLTZMANN DISTRIBUTION of speed:



★ Molecules travelling at very high or very low speed are negligible.

⇒ Probabilistic Distribution Curve.

Pressure Equation:

$[C_{rms} = V_{rms}]$

$$P = \frac{1}{3} \rho (C^2_{rms})$$

$$P = \frac{1}{3} \frac{M}{V} (C^2_{rms})$$

Multiply & divide RHS by 2

$$P = \frac{2}{3} \frac{1}{V} \left(\frac{1}{2} M C^2_{rms}\right)$$

$$P = \frac{2}{3} \frac{KE_{avg}}{V}$$

$$\boxed{KE = \frac{3}{2} PV}$$

$$\boxed{KE = \frac{3}{2} nRT}$$

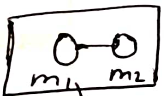
Gas Law:

a) Boyle's law $PV =$ constant

b) Charle's law $\frac{V}{T} =$ constant

c) Gay lussac's law $\frac{P}{T} =$ constant

Internal energy of Gas:



PE neglected

Internal energy = only KE.

$$U = f\left[\frac{n}{2} RT\right]$$

OR

$$U = \frac{f}{2} nRT$$

Mono $U = \frac{3}{2} nRT$

Dia $U = \frac{5}{2} nRT$

Tri $U = \frac{6}{2} nRT$

Gas mixture:

$\boxed{\begin{matrix} n_1 \\ M_1 \end{matrix}}$ $\boxed{\begin{matrix} n_2 \\ M_2 \end{matrix}}$

1. Molecular wt:

$$M_{mix} = \frac{n_1 M_1 + n_2 M_2}{n_1 + n_2}$$

2. Specific heat at constant volume:

$$C_{v\,mix} = \frac{n_1 C_{v_1} + n_2 C_{v_2}}{n_1 + n_2}$$

3. Cp:

$$C_{p\,mix} = \frac{n_1 C_{p_1} + n_2 C_{p_2}}{n_1 + n_2}$$

4. γ mixture (Adiabatic constant):

$$\gamma_{mix} = \frac{C_{p\,mix}}{C_{v\,mix}}$$

$$\boxed{\frac{n_1 + n_2}{\gamma_{mix} - 1} = \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1}}$$

Relaxation Time:

↳ Time between two successive collision.

→ Denoted by 'τ'.

$$\tau \propto \frac{1}{v} \propto \frac{1}{C_{rms}}$$

$$\boxed{\tau = \frac{1}{\sqrt{2}\, n \pi d^2 C_{rms}}}$$

$$\left[\tau = \frac{d}{v}\right]$$

☆ Specific heat capacity of solid = 3R

" " " liquid = 9R (water)

OSCILLATIONS
(SIMPLE HARMONIC MOTION)

Periodic motⁿ:
↳ which repeat itself over & over again after a regular interval of time.

Oscillatory or vibratory motⁿ:
↳ Body moves to & fro or back & forth repeatedly about a fixed point.
→ a.k.a Harmonic motⁿ.

Harmonic motⁿ vs **non-Harmonic motⁿ**

Harmonic motⁿ:
→ can be expressed in single harmonic function (i.e. sine or cosine)

Example:
$$\boxed{\begin{aligned} y &= A \sin \omega t \\ x &= A \cos \omega t \end{aligned}}$$

non-Harmonic motⁿ:
→ cannot be expressed in terms of single harmonic motⁿ.

Example:
$y = A \sin \omega t + b \sin 2\omega t$

Important terms:
(1) **Amplitude**: Maximum displacement from mean positⁿ.

(2) **Frequency**: no. of periodic motⁿ executed by body per second. → ν
→ SI unit: hertz (Hz)
→ $\boxed{\nu = \frac{1}{T}}$ → 1 Hz = 1 cps = $1 s^{-1}$

(3) **Time period**: Least interval of time after which periodic motⁿ repeat itself.
→ Denoted by T
→ SI unit: second.

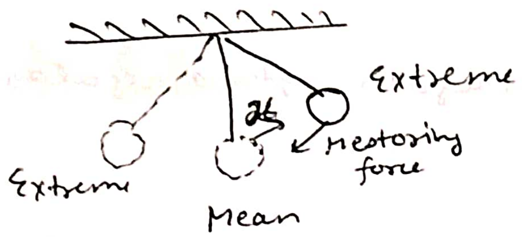
(4) **Angular Frequency**: Equal to product of frequency of the body with factor 2π.
$$\boxed{\omega = 2\pi\nu = \frac{2\pi}{T}}$$
SI unit: Hz or rad/sec

$$\boxed{1 \text{ rad} = \frac{180}{\pi}}$$

$$\boxed{\text{Degree} = \frac{180}{\pi} \times \text{rad}}$$

Simple harmonic mot

↳ special type of periodic motⁿ, in which a particle moves to & fro repeatedly about a mean positⁿ.



Restoring force $\propto$ Displacement of particle from mean positⁿ.

$F \propto -x$

$$\boxed{F = -kx}$$

∴ k = force constant

→ SI unit : $\frac{\text{Newton}}{\text{meter}} = \underline{N/m}$

→ Dimensions : $[MT^{-2}]$

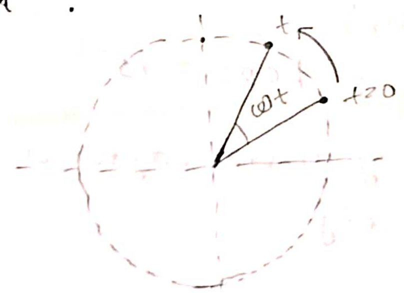
If the particle is oscillating on a small arc of circular path, then for angular SHM :

Restoring torque $\propto$ − Angular displacement

$$\boxed{\tau \propto -\theta}$$

PHASE of SHM :

→ Phase at any instant is a physical qty, which completely expresses the positⁿ and direction of motⁿ.



Displacement when object starts from mean positⁿ: y axis

$$\boxed{y = A \sin \omega t}$$

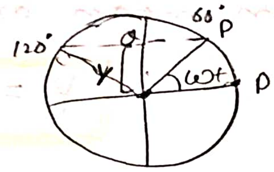
Displacement when object starts from extreme positⁿ: x-axis

$$\boxed{x = A \cos \omega t}$$

* A → amplitude
* ω → angular velocity.
* y / x → displacement.

Phaser / vector diagram:

→ Gives info about direction as well as location of particle in term of phase.



$$\boxed{\text{Phase} = \omega t}$$

Velocity:

$v = \frac{dy}{dt}$; $v = \frac{d(A\sin\omega t)}{dt}$. $\left[\frac{d\sin\theta t}{} = \cos\theta t \cdot \theta\right]$

$$v = A\frac{d(\sin\omega t)}{dt}$$

$$\boxed{v = A\cos\omega t \cdot \omega} \quad --(i)$$

In terms of t

$$\left[\begin{aligned} \sin^2\omega t + \cos^2\omega t &= 1 \\ \cos^2\omega t &= 1-\sin^2\omega t \\ \cos\omega t &= \sqrt{1-\sin^2\omega t} \end{aligned}\right]$$

Putting in eq. (i)

$$v = A\omega\sqrt{1-\sin^2\omega t}$$

$$v = \omega\sqrt{A^2 - A^2\sin^2\omega t}$$

$$\boxed{v = \omega\sqrt{A^2-y^2}}$$

In terms of y

⇒ $V_{max} = \omega\sqrt{A^2-0}$

$V_{max} = \omega . A$ $\left[y=0, \text{ at mean posit}^n\right]$

⇒ $V_{min} = \omega\sqrt{A^2-A^2}$ $\left[y = A(\max), \text{ at extremes}\right]$

$V_{min} = \underline{\underline{0}}$

Acceleration:

$$a = \frac{dv}{dt}$$

$$a = \frac{d(A\omega\cos\omega t)}{dt} \quad \left[\frac{d\cos\theta}{} = -\sin\theta\right]$$

$$a = -A\omega\sin\omega t \cdot \omega$$

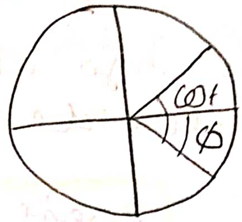
$$\boxed{a = -A\omega^2\sin\omega t}$$

OR

$$\boxed{a = -\omega^2 y}$$

Phase difference:

If object neither starts from mean or extreme.



$$\boxed{Y = A\sin(\omega t - \phi)}$$

[min] $v = 0$	[max] $v = A\omega$	[min] $v = 0$
[E]	[M]	[E]
$y = -A$	$y = 0$	$y = A$
$t = \frac{3T}{4}$	$t = T/2$ or $t = T$ or $t = 0$	$t = T/4$
$\theta = \frac{3\pi}{2}$	$\theta = \pi$ or $\theta = \pi/2$	$\theta = \pi/2$
$a = A\omega^2$ [max]	$a = 0$ [min]	$a = -A\omega^2$ [max]

Time period:

$T \propto$ Displacement

$T \propto \frac{1}{\text{acceleration}}$

$$\left[\omega = \frac{2\pi}{T}\right]$$

$a = -\omega^2 y$ ('-' for direction only)

$$a = \left(\frac{2\pi}{T}\right)^2 \times y$$

$$\frac{a}{y} = \left(\frac{2\pi}{T}\right)^2$$

$$\boxed{T = \frac{2\pi \times \sqrt{y}}{\sqrt{a}} = 2\pi\sqrt{\frac{y}{a}}}$$

OR

$T \propto$ mass / inertia

$$T = 2\pi\sqrt{\frac{y}{a}}$$

$$T = 2\pi\sqrt{\frac{my}{ma}}$$ (multiplies & divided by m)

$$T = 2\pi\sqrt{\frac{\frac{my}{g}}{\frac{ma}{g}}}$$

$$T = 2\pi\sqrt{\frac{m}{\frac{ma}{y}}}$$

$$T = 2\pi\sqrt{\frac{m}{F/y}}$$ [form $\frac{F}{x}$; $F/x = k$]

$$\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$;

k = spring constant

m = mass

Energy:

Kinetic energy:

$$K.E = \frac{1}{2} m v^2$$

$$= \frac{1}{2} m \left[\omega\sqrt{A^2 - y^2}\right]^2$$

$$\boxed{KE = \frac{1}{2} m \omega^2 (A^2 - y^2)}$$

$KE_{max} \Rightarrow y = 0$ [At mean]

$$\boxed{= \frac{1}{2} m \omega^2 (A^2)}$$

$KE_{min} \Rightarrow y = A$

$$= \frac{1}{2} m \omega^2 (A^2 - A^2)$$ [At extreme]

$$= 0$$

Potential energy:

$$P.E = mgh$$ [m.g . h ; F , x]

$$P.E = \text{workdone}$$

$$P.E = \int F \cdot dy$$

$$P.E = \int ma \cdot dy$$

$$PE = \int m\omega^2 y \cdot dy$$

$$\boxed{PE = m\omega^2 \frac{y^2}{2}}$$

PE min : $= \frac{1}{2} m \omega^2 [0]$ $[y = 0]$

$= 0$ [At mean]

PE max : $= \frac{1}{2} m \omega^2 (A)^2$ $[y = A]$

$= \frac{1}{2} m \omega^2 A^2$ [At extreme]

⇒ Conservation of Energy:

$T.E = K.E + P.E.$

$= \frac{1}{2} m \omega (A^2 - y^2) + \frac{1}{2} m \omega^2 y^2$

$= \frac{1}{2} m \omega \cancel{y^2} (A^2 - y^2 + y^2)$

$= \frac{1}{2} m \omega A^2$

TE. at mean :

$= K.E + PE$

$= \frac{1}{2} m \omega^2 A^2 + 0$

$= \frac{1}{2} m \omega^2 A^2$

TE at extreme :

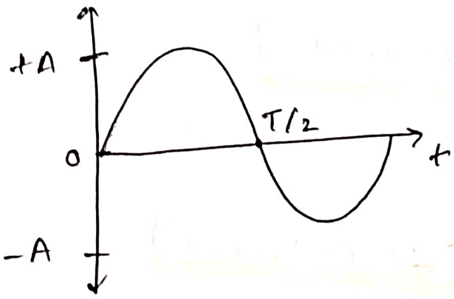
$= KE + PE$

$= 0 + \frac{1}{2} m \omega^2 A^2$

$= \frac{1}{2} m \omega^2 A^2$

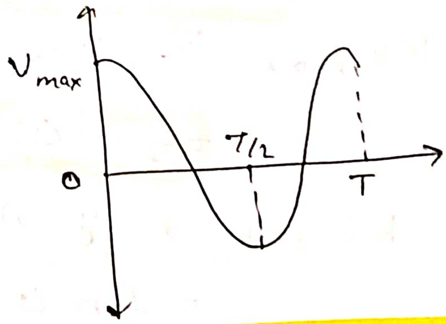
⇒ Important relations:

1. Position :



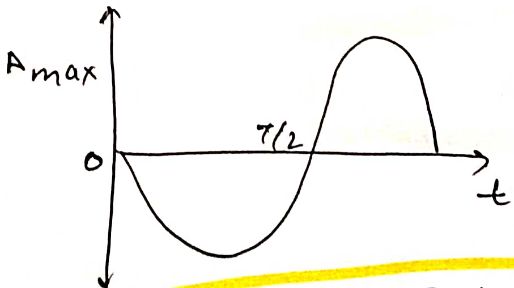
$\boxed{y = A \sin \omega t}$

2. Velocity :



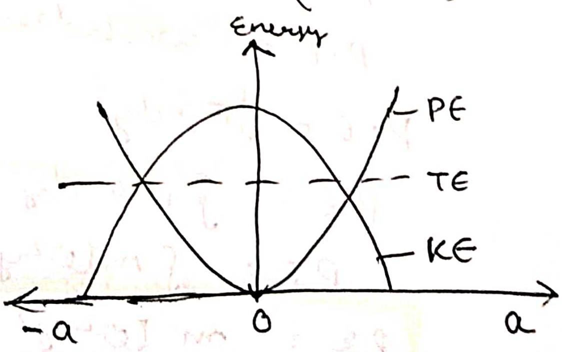
$\boxed{v = A \omega \cos \omega t}$

3. Acceleration :

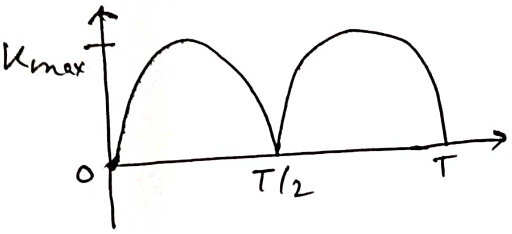


$\boxed{a = - A \omega^2 \sin \omega t}$

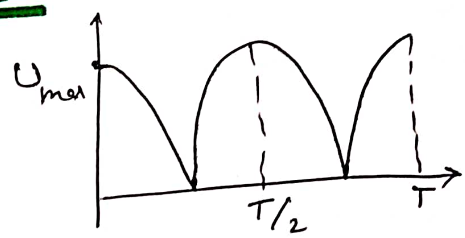
4. Energy :
(scalar)



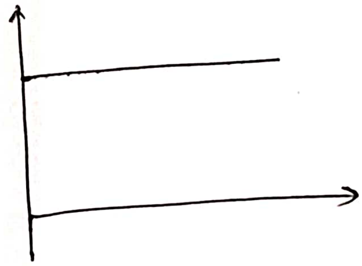
K.E:



P.E.



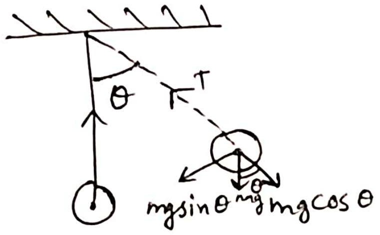
T.E:

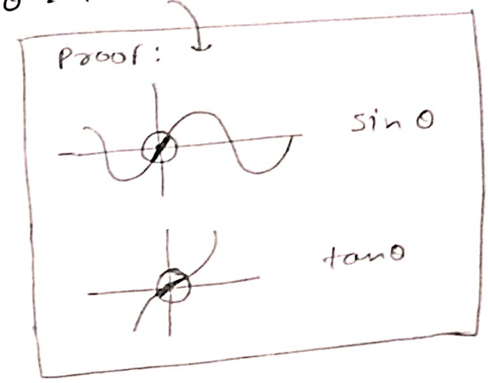


⇒ Simple Pendulum:

θ = very small
that is why;
$\sin\theta = \tan\theta$

Rope:
Inextensible
massless





$$\theta = \frac{arc}{rad} = \frac{x}{l}$$

$$T = mg\cos\theta \quad \text{---(i)}$$

(Restoring force)

$$F_{(R)} = mg\sin\theta$$

$$\cancel{m}a = \cancel{m}g\sin\theta$$

$$a = g\sin\theta$$

$$a = g\tan\theta$$

$$a = g\frac{x}{l}$$

$$\boxed{\frac{l}{g} = \frac{x}{a}}$$

Time period: (T) $= 2\pi\sqrt{\frac{\text{Disp.}}{\text{Acce.}}}$

$$T = 2\pi\sqrt{\frac{x}{a}}$$

OR

$$\boxed{T = 2\pi\sqrt{\frac{l}{g}}}$$

Pendulum in lift:

Downwards

$$\boxed{T = 2\pi\sqrt{\frac{l}{g-a}}}$$

Upwards

$$\boxed{T = 2\pi\sqrt{\frac{l}{g+a}}}$$

★ 1 Time period = 2 seconds