Homework 13.3

September 25, 2025

1. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ for the following function

$$f(x, y) = x^5 - 7xy + 4y^2$$

2. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$

$$f(x, y) = \sqrt{3x^2 + y^2}$$

3. For the function $f(x, y) = \ln(5x + 2y)$, find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$

4. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ for the following function

$$f(x, y) = \mathrm{e}^{6xy} \ln(2y)$$

5. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$

$$f(x, y) = 2x^3 y$$

6. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ for the function $f(x, y)$

$$f(x, y) = \int_x^y g(t)dt$$

7. For the function $f(x, y, z) = \arcsin(xyz)$ find $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, and $\frac{\partial f}{\partial z}$

8. For the function $f(t, \alpha) = \sin(5\pi t + 2\alpha)$, find $\frac{\partial f}{\partial t}$ and $\frac{\partial f}{\partial \alpha}$

9. Find all second order partial derivatives of $g(x, y) = x^4 y + \sin(y) + y\cos(x)$

10. Traveling waves (for example, water waves or electromagnetic waves) exhibit periodic motion in both time and position. In one dimension (for example, a wave on a string), wave motion is governed by the one-dimensional wave equation below, where $u(x,t)$ is the height or displacement of the wave surface at position x and time t, and c is the constant speed of the wave. Show that $u(x,t)$ given below is a solution of the wave equation.

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, u(x,t) = 6\cos(5(x+ct)) + 7\sin(x-ct)$$

(a) What is the first step in showing that $u(x,t)$ is a solution to the wave equation?

A. Calculate the left side of the wave equation, $\frac{\partial^2 u}{\partial t^2}$, by first calculating $\frac{\partial u}{\partial x}$.

B. Multiply $u(x,t)$ by c^2.

C. Calculate the left side of the wave equation, $\frac{\partial^2 u}{\partial t^2}$, by first calculating $\frac{\partial u}{\partial t}$.

D. Factor $u(x,t)$.

(b) Calculate the first partial derivative $\frac{\partial u}{\partial t}$

(c) Calculate the second partial derivative $\frac{\partial^2 u}{\partial t^2}$

(d) What is the next step in showing that $u(x,t)$ is a solution of the wave equation?

A. Calculate the right side of the wave equation, $\frac{\partial^2 u}{\partial t^2}$, by first calculating $\frac{\partial u}{\partial x}$.

B. Multiply $u(x,t)$ by c^2.

C. Calculate the right side of the wave equation, $\frac{\partial^2 u}{\partial t^2}$, by first calculating $\frac{\partial u}{\partial t}$.

D. Factor $u(x,t)$.

(e) Calculate the first partial derivative $\frac{\partial u}{\partial x}$

(f) Calculate the second partial derivative $\frac{\partial^2 u}{\partial x^2}$