Lesson 3: Symmetry, Skewness and Kirtosis in Return Distributions and Arithmetic Versus Geometric Means

A Symmetrical Distribution

- The distribution on either side of the mean is the mirror image of the other.
- For a symmetrical distribution, the mean, median and mode are equal.

A Non-Symmetrical Distribution

- Also knows as a skewed distribution.

Positively skewed (or skewed to the right) distribution

- Has a long tail on the right side.
- Data set contains certain observations that are much larger in value than most of the observations in the data set.
- The mode is *less* than the median, which is *less* than the mean.
- The mean is affected the most by the extreme values (outliers) in the tail on the right side, and is "pulled" towards them.

Negatively skewed (or skewed to the left) distribution

- Has a long tail on the left side.
- Data set contains certain observations that are much smaller in value than most of the observations in the data set.
- The mode is *greater* than the median, which is *greater* than the mean.
- The mean is affected the most by the extreme values (outliers) in the tail on the left side, and is "pulled" towards them.

Sample Skewness

- When the distribution is positively (right) skewed, sample skewness is positive.
- When the distribution is negatively (left) skewed.
- Sample skewness of zero indicates that the data set follows a symmetrical distribution.
- Absolute values of skewness greater than 0.5 suggest that the data set is significantly skewed.

Kurtosis

- Measures the extent to which a distribution is more or less peaked than a normal distribution.
- A normal distribution has a kurtosis of 3.
- Excess kurtosis equals the kurtosis of the distribution minus the kurtosis of the normal distribution (3).
    - A leptokurtic distribution is *more* peaked and has *fatter* tails than a normal distribution and has an excess kurtosis greater than zero.
    - A platykurtic distribution is *less* peaked and has *thinner* tails than a normal distribution and has an excess kurtosis less than zero.
    - A mesokurtic distribution is identical to a normal distribution and has an excess kurtosis of zero.

For reporting historical returns, the geometric mean is more appropriate.

- The geometric mean is an excellent measure of past performance.
- The arithmetic mean can distort evaluation of past performance.

- If we want to gauge performance over a single period, the arithmetic mean should be used because the arithmetic mean is the average of one-period returns.
- If we want to estimate returns over more than one period, we should use the geometric mean as it measures how investment returns are linked over time.

To calculate expected equity risk premiums (in a forward-looking context) use of the arithmetic mean is more appropriate.

- Uncertainty in cash flows or returns causes the arithmetic mean to be larger than the geometric mean.
- The more the uncertainty, the greater the divergence between the two.

Studies have shown that the geometric mean return approximately equals the arithmetic mean minus half the variance of returns.