

Lesson 3: Topics in Probability: Bayes' Formula and Counting Rules

Example

In a particular school, there are 54% boys and 46% girls. The girl students wear blue sweaters or red sweaters in equal numbers, while the boys all wear blue sweaters. An observer sees a (random) student from a distance, and all she can see is that the student is wearing a blue sweater. What is the probability that the student is a girl?

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Multiplication Rule of Counting

Example

There are 5 different musicians who can play any of 5 different instruments in a band. In how many ways can these 5 musicians be allocated different instruments?

Labeling Problems

These refer to situations where there are n items, each of which can receive one of k different labels. We give each object in the group a label to place it in a category. n_1 items can be given the first label, n_2 receive the second label, and so on.

Example

The conductor must determine which instruments to hand to the twelve members of his band. 4 of them will play the drums, 6 of them will play the saxophone, and 2 will play the bagpipes. How many ways can these 12 musicians be organized in the band?

Combinations

- Used when the number of labels that can be assigned, k , equals 2
- Used when the order in which the items are assigned the labels is NOT important.

Example

There are 20 people on a trial for a soccer team out of which only 11 will be chosen. It does not matter who gets chosen first and who is the eleventh one to be chosen – all eleven will be on the team. The remaining 9, unfortunately will not be on the team. How many different ways can the 11-member team be chosen from the 20 aspirants?

Permutations

- Used when the number of labels that can be assigned, k , equals 2
- Used when the order in which the items are assigned the labels IS important.

Example

20 players are trying out for a soccer team when only 11 will be chosen. Further, the eventual list of 11 should rank players from 1 to 11 with a higher rank indicating a more skillful player. How many ways can the eleven players be selected given that the order is important?

TIPS

- Factorials are used when there is only one group i.e., we are simply arranging a given set or group of n items. Given n items, there are $n!$ ways of arranging them.
- The labeling formula is used for three or more groups of predetermined size. Each item must be labeled as a member of one of the groups.
- The combination formula is used when there are only two groups of predetermined size and crucially, the order or rank of labeling is NOT important.
- The permutation formula is used when there are only two groups of predetermined size, and the order or rank IS important.