Lesson 1: Discrete Random Variables, the Discrete Uniform Distribution and the Binomial Distribution

A **random variable** is a variable whose outcome cannot be predicted.

A **discrete random variable** is one that can take on a countable number of values. Each outcome has a specific probability of occurring, which can be measured.

The **probability distribution** of a random variable identifies the probability of each of the possible outcomes of a random variable.

A **probability function, p(x),** expresses the probability that “X”, the random variable, takes on a specific value of “x”.

CONTINUOUS RANDOM VARIABLES

A **continuous random variable** is one for which the number of possible outcomes cannot be counted (there are infinite possible outcomes) and therefore, probabilities cannot be attached to specific outcomes.

- For continuous random variables, the probability of a specific outcome within a range of infinite outcomes is essentially zero.
- A **probability density function (pdf)** is used to interpret their probability structure.

DISCRETE VERSUS CONTINUOUS DISTRIBUTIONS

For a discrete distribution:

- $p(x) = 0$ when x cannot occur; and
- $p(x) > 0$ if x is a possible outcome.
- $p(x)$ is read as "the probability that the random variable, X, equals x."

For a continuous distribution:

- $p(x) = 0$ even though x can occur.
- We can only consider $P(x_1 \leq X \leq x_2)$ where x_1 and x_2 are actual numbers.

CUMULATIVE DISTRIBUTION FUNCTIONS

A **cumulative distribution function (cdf),** also known as a distribution function, expresses the probability that a random variable, X, takes on a value *less than or equal to* a specific value, x.

- It represents the sum of the probabilities of all outcomes that are less than or equal to the specified value of x.

Example: Probability Functions and Cumulative Distribution Functions

- The set of possible values that a random variable, X, can take is given by: X = (5,10,15,20).
- For all other values of X, p(x) = 0.
- The probability function for the random variable is given as: p(x) = x/50.

Calculate the following probabilities:

a. p(5)
b. p(15)
c. p(17)
d. F(10)
e. F(20)

DISCRETE UNIFORM DISTRIBUTION

A **discrete uniform distribution** is one in which the probability of each of the possible outcomes is the same.

Example

For a roll of a fair die:

a) Calculate the probability that the outcome is less than or equal to 2.
b) Calculate the probability that the outcome is greater than 2 but less than or equal to 5.
c) State the probability function, p(x), of this random variable.

THE BINOMIAL DISTRIBUTION

Bernoulli Trial

An experiment that has only 2 possible outcomes which are labeled "success" and "failure". Further, these two outcomes are:

- Mutually exclusive
- Collectively exhaustive

If this experiment is carried out n times, the number of successes, X, is called a **binomial random variable**.

- The distribution that X follows is known as the **binomial distribution**.
  - The probability of success, p, is equal for all trials.
  - The trials are independent.

A random variable that follows the Binomial distribution is defined by n and p.

THE BINOMIAL DISTRIBUTION

THE BINOMIAL DISTRIBUTION

Example

A door-to-door salesman visits 7 houses every day. The probability of making a sale to any customer is 0.4. Compute the probability of making 3 sales on a given day.

BINOMIAL TREES