Lesson 2: Hypothesis Tests Concerning the Mean

Tests Concerning a Single Mean

The **t-test** is used when the variance of the population is unknown and either of the conditions below holds:

- The sample size is large.
- The sample size is small, but the underlying population is normally distributed or approximately normally distributed.

The test statistic (t-statistic) for hypothesis tests concerning the mean of a single population is:

- In a t-test, the sample's t-statistic is compared to the critical t-value with n-1 degrees of freedom, at the desired level of significance.
- Practically speaking, the variance of the population is rarely ever known, so the t-test is very popular.

Tests Concerning a Single Mean

The z-test can be used to conduct hypothesis tests of the population mean when the population is normally distributed and its variance is known.

The z-test can also be used when the population's variance is unknown, but the sample size is large.

- In a z-test, the z-statistic is compared to the critical z-value at the given level of significance.

Tests Concerning a Single Mean

Example

A manufacturer claims that the life of its batteries is normally distributed with a mean of 30 hours. For a random sample of 81 batteries it is observed that the average life of the batteries in the sample is 29 hours with a standard deviation of 5 hours. Using a 5% significance level, determine whether the manufacturer's claims are *inaccurate.*

Tests Relating to the Mean of Two Populations

- In one type of test we assume that the variance of the two populations is *equal.*
- In the other type of test we assume that the population variances are *unequal.*

Note: Both tests require that the populations are normally distributed and that samples are independent.

Hypotheses describing the tests of means of two populations can be structured as:

Tests for Means when Population Variances are Assumed Equal

In tests where it is assumed that the means of the two populations are *equal*, we use the pooled variance (s_p) in the calculation of the t-stat. The test statistic, the pooled variance, and the degrees of freedom for the t-test are calculated as follows:

Tests for Means when Population Variances are Assumed Equal

Example

In this example, we test for the equality of returns on the FTSE across two decades. While there appears to be a substantially higher return during the 1960s, we want to determine if the difference was statistically significant at the 10% level of significance. We will assume that the variances for returns in the two decades are equal.

Decade	No. of Months	Mean Monthly Return	Standard Deviation
1950s	110	0.68%	5.589%
1960s	110	1.57%	5.738%

Tests for Means when Population Variances are Assumed Unequal

In hypothesis tests where it is assumed that the variances of the two populations are unequal, the test statistic and the degrees of freedom for the t-test are calculated as follows:

Tests for Means when Population Variances are Assumed Unequal

Example

The table below details the average recovery rates for senior debt holders in financial and pharmaceutical companies in bankruptcy over the last 2 years. We want to know whether there was a difference between the recovery rates for investors in financial sector and pharmaceutical sector debt at the 10% level of significance.

FINANCIAL SECTOR			PHARMACEUTICAL SECTOR		
Number of Observations	Average Price	Standard Deviation	Number of Observations	Average Price	Standard Deviation
31	$74.42	$24.03	74	$65.75	$35.17

When the samples of the two populations whose means we are comparing are *dependent*, the paired comparisons test is used. Dependence can result from events that affect both populations.

Paired Comparisons Test

Example

A researcher is trying to ascertain whether there is a difference in the annual returns on two investment strategies. Specifically, he is analyzing the return on a buy-and-hold strategy on the FTSE versus the return on the top ten dividend yielding stocks on the FTSE over the last 40 years. Determine whether the returns on the two strategies are different at the 1% level of significance.

Strategy	Mean Return	Standard Deviation
Buy and hold entire FTSE	17.66%	19.50%
Top 10 dividend yielding stocks on FTSE	14.71%	15.54%
Difference	2.95%	6.71% (sample standard deviation of differences)