Level I CFA REVIEW

Quantitative Methods

6. The Time Value of Money
7. Statistical Concepts and Market Returns
8. Probability Concepts
9. Common Probability Distributions
10. Sampling and Estimation
11. Hypothesis Testing

KAPLAN SCHWESER

An investor will make the following deposits:

Today: $1,000

One year from today: $2,000

Three years from today: $3,000

If the account has an effective annual return of 5%, what is the value five years from today?

A. $6,681.

B. $6,847.

C. $7,015.

Elmer has won his state lottery and has been offered 20 annual payments of $200,000 each beginning today or a single payment of $2,267,000. The annual discount rate used to calculate the single-payment amount is *closest to*?

A. 6.15%.

B. 6.75%.

C. 7.00%.

Mean: Average of all observations

Arithmetic $$R_A = \frac{\sum_{t=1}^{T} R_t}{T}$$

Geometric $$R_G = \left[\prod_{t=1}^{T}(1+R_t)\right]^{1/T} - 1$$

Weighted $$\bar{X}_w = \sum_{i=1}^{n} w_i X_i$$

Harmonic $$\bar{X}_H = \frac{N}{\sum_{i=1}^{N}\frac{1}{X_i}}$$

Median

Value of the middle item of an odd-numbered set of items sorted in order

The average of the two middle items of an even-numbered set of items sorted in order

Mode

Most frequently occurring value(s)—may be more than one

Cliff Corporation's dividends the past six years were \$0.31, \$0.12, \$0.40, \$0.50, \$0.60, and \$0.70. The compound annual growth rate of dividends over this period is *closest* to:

A. 14.5%.

B. 17.7%.

C. 46.7%.

A mutual fund had the following returns over 5 years: 6%, 11%, –3%, 8%, 15%. What is the average annual compound rate of return?

A. 7.14%.

B. 7.23%.

C. 7.40%.

Statistical Concepts

Variance

Average of squared deviations around the mean

Standard deviation = $\sqrt{\text{Variance}}$

Mean Absolute Deviation

$$MAD = \frac{\sum_{i=1}^{n} |X_i - \bar{X}|}{n}$$

Range

= maximum – minimum

Population Standard Deviation

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N} (X_i - \mu)^2}{N}}$$

Sample Standard Deviation

$$s = \sqrt{\frac{\sum_{i=1}^{N} (X_i - \bar{X})^2}{n - 1}}$$

Coefficient of Variation

Risk per unit of mean return
Lower is better

$$CV = \frac{\text{Standard deviation}}{\text{Mean}} = \frac{s}{\bar{X}}$$

Chebyshev's Inequality

The proportion of the observations within *k* standard deviations of the arithmetic mean is at least:

$$1 - \left(\frac{1}{k^2}\right)$$

Returns on an index of 100 stocks over four years are 15%, –5%, 12%, and 22%. The estimated annual standard deviation of returns is *closest to:*

A. 9.84%.

B. 11.46%.

C. 12.14%

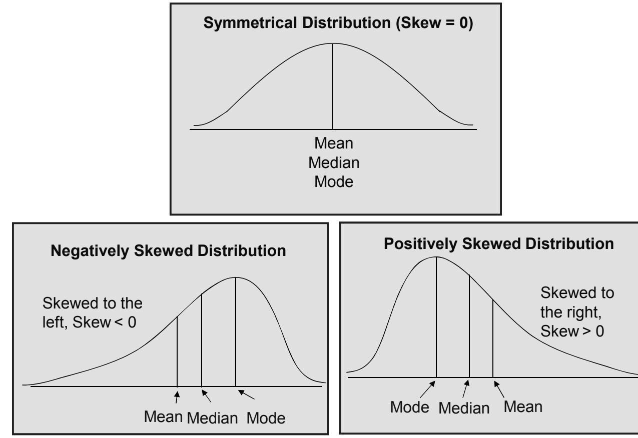
The probability that a random variable will be within:

A. 1.96 standard deviations of the mean is at least 74%.

B. 1.65 standard deviations of the mean is at least 90%.

C. 2.5 standard deviations of the mean is at least 85%.

Statistical Concepts



CFA Curriculum Vol. 1, Reading 7, Question 30

Two portfolios have unimodal return distributions. Portfolio 1 has a skewness of 0.77, and Portfolio 2 has a skewness of –1.11.

Which of the following is correct?

A. For Portfolio 1, the median is less than the mean.

B. For Portfolio 1, the mode is greater than the mean.

C. For Portfolio 2, the mean is greater than the median.

Probability Distributions

Univariate distribution describes a single normal random variable

Multivariate distribution specifies the probabilities for a group of related random variables (influenced by correlations)

Continuously Compounded Return

$$r_{t,t+1} = \ln\left(\frac{S_{t+1}}{S_t}\right) = \ln(1+HPR_{t,t+1})$$

$$HPR_{t,t+1} = e^{r_{t,t+1}} - 1$$

Shortfall Risk

Risk that the portfolio value will fall below some minimum acceptable level, R_L

Roy's safety-first criterion: Optimal portfolio minimizes the probability that portfolio return, R_P, falls below R_L; choose portfolio with highest SFR

$$\text{SFRatio} = \frac{E(R_p) - R_L}{\sigma_p}$$

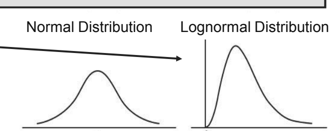
Lognormal Distribution

Generated by the function e^X, where X is normally distributed

Skewed to the right

Bounded by zero from below

Can be used for asset values if the return on the asset is normally distributed



Portfolio	**X**	**Y**	**Z**
E(Rp)	5%	11%	15%
σp	8%	21%	35%

Which portfolio has the lowest probability of returns less than 4%?

A. Portfolio X.
B. Portfolio Y.
C. Portfolio Z.

An investment was purchased 18 months ago for $88. The investment is now worth $165. The stated annual rate of return with continuous compounding is *closest* to:

A. 42%.
B. 63%.
C. 88%.

Probability Concepts

Independent and Dependent Events

Independent events are those events for which the occurrence of one event is not related to the outcomes of others.

Events A and B are independent if: P(A|B) = P(A), or P(B|A) = P(B)

If the events are dependent: P(A|B) is the conditional probability

P(AB) is the joint probability

Joint Probabilities

Multiplication rule for probability: The joint probability of A and B is given by:

P(AB) = P(A) × P(B|A)

If A and B are independent events, this will simplify to:

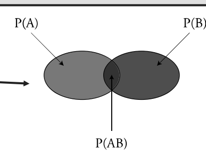
P(AB) = P(A) × P(B)

Addition Rule for Probabilities

Given events A and B:

P(A or B) = P(A) + P(B) – P(AB)

If A and B are mutually exclusive, then P(AB) = 0



A parking lot has 55 blue (B) cars and 45 red (R) cars in it. 25 of the blue cars and 15 of the red cars are electric (E); the rest are gasoline (G).

What is the probability that a car selected at random is blue or electric?

A. 70%.
B. 80%.
C. 95%.

Probability Concepts

Multiplication Rule of Counting

If one task can be done in n_1 ways and a second task, given the first, can be done in n_2 ways and so on for *k* tasks, then the total number of ways the *k* tasks can be done is:

$$n_1 \times n_2 \times n_3 \ldots\ldots \times n_k$$

Permutation Formula

The number of ways that we can choose *r* objects from a total of *n* objects, when **the order matters**:

$$_{n}P_{r} = \frac{n!}{(n-r)!}$$

Combination Formula

The number of ways that we can choose *r* objects from a total of *n* objects, when **the order does not matter**:

$$_{n}C_{r} = \frac{n!}{(n-r)!r!}$$

These functions are on the BAII plus.

You have 5 stocks and want to sell 3, one at a time. How many ways are there to choose the 3 stocks to sell in order?

A. 10.

B. 15.

C. 60.

You have been asked to select 4 of 10 energy stocks in a portfolio to be sold to reduce exposure to that industry. How many different groups of four could you select?

A. 210.

B. 420.

C. 5,040.

Probability Concepts

Total Probability Rule

$P(A) = P(AS_1) + P(AS_2) + \ldots\ldots + P(AS_n)$

$= P(A \mid S_1)P(S_1) + P(A \mid S_2)P(S_2) + \ldots\ldots + P(A \mid S_n)P(S_n)$

where $S_1, S_2, \ldots, S_n$ are mutually exclusive and exhaustive events

Bayes' Formula

$$P(\text{Event} \mid \text{Information}) = \frac{P(\text{Event})P(\text{Information} \mid \text{Event})}{P(\text{Information})}$$

P (Interest rate increase) = 70%
P (Recession | Increase) = 60%
P (Recession | No interest rate increase) = 20%
What is the (unconditional) probability of recession?

A. 80%.
B. 62%.
C. 48%.

The probability that a stock's return will be greater than the return on an index in any given week is 60%. The probability that the stock's return will be greater than the return on the index in four weeks out of the next five weeks is *closest* to:

A. 26%.
B. 47%.
C. 65%.

Probability Distributions

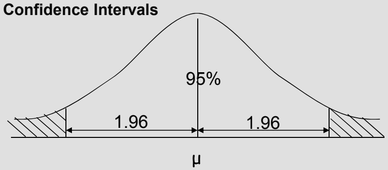
Normal Distribution

Completely described by two parameters – its mean, μ, and variance, σ^2

Has a skewness of 0 (it is symmetric) and a kurtosis of 3

Linear combination of two or more normal random variables is also normally distributed

Confidence Intervals



If a random variable is normally distributed, we can make the following probability statements:

- 68% of observations lie within the interval μ ± 1
- 90% interval = μ ± 1.645
- 95% interval = μ ± 1.96
- 99% interval = μ ± 2.58

Standard Normal Distribution

The standard normal random variable, *Z*, is calculated:

$$Z = \frac{X - \mu}{\sigma} \quad \text{or} \quad Z = \frac{X - \bar{X}}{s}$$

Z is the number of standard deviations from the mean

Look up probability in Standard Normal Distribution table

Returns on an index of 100 stocks are approximately normal, have a mean of 9%, and a std. dev. of 15%. A 99% confidence interval on next year's index return is:

A. –20.4% to 38.4%.
B. –29.7% to 47.7%.
C. 5.1% to 12.9%.

Sampling

Central Limit Theorem

If you repeatedly take large samples (size at least 30) from a population, then the distribution of the means of all of such samples will be normally distributed

The mean of the population, μ, and the mean of all possible sample means are equal

The standard deviation of the distribution of the sample means is called the 'Standard Error of the Sample Mean'

Standard Error of Sample Means

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \qquad s_{\bar{x}} = \frac{s}{\sqrt{n}} \text{ if } \sigma \text{ unknown}$$

Confidence interval:

For large samples, the confidence intervals for the population mean are based on the normal distribution:

90% confidence interval: sample mean ± 1.65 standard errors

95% confidence interval: sample mean ± 1.96 standard errors

99% confidence interval: sample mean ± 2.58 standard errors

Annual returns on energy stocks are approximately normally distributed with a mean of 9% and standard deviation of 6%.

A 90% confidence interval on the mean of the annual returns for a sample of 12 energy stocks is *closest to:*

A. –1% to 18%.

B. 2% to 16%.

C. 6% to 12%.

Sampling

Student's *t*-distribution

- Symmetrical
- Defined by a single parameter, degrees of freedom (df), where df = n – 1
- More probability in the tails ("fatter tails") than a normal distribution
- As degrees of freedom get larger, shape of *t*-distribution approaches a normal distribution

When sampling from a:	and n < 30 use a:	and n ≥ 30 use a:
Normal distribution, known pop. variance	*Z*-statistic	*Z*-statistic
Normal distribution, unknown pop. variance	*t*-statistic	*t*-statistic*
Non-normal dist, known pop. variance	no test available	*Z*-statistic
Non-normal dist, unknown pop. variance	no test available	*t*-statistic*
*use of Z also acceptable, especially for very large samples		

Hypothesis Testing

Steps

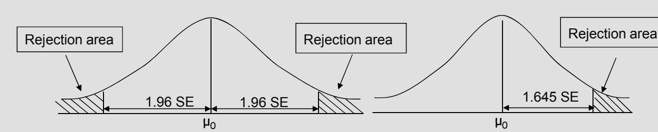
1. State the hypotheses
2. Identify the test statistic and its distribution
3. Specify the significance level
4. State the decision rule
5. Collect the data and perform the calculations
6. Make the statistical decision
7. Make the economic or investment decision

Hypothesis about a Single Mean

$$Z = \frac{\bar{X} - \mu_0}{\sigma_{\bar{x}}} \quad \text{or} \quad t = \frac{\bar{X} - \mu_0}{s_{\bar{X}}}$$

if $\sigma_{\bar{x}}$ unavailable

TWO TAILED		ONE TAILED	
Null hypothesis:	$\mu = \mu_0$	Null hypothesis:	$\mu \le 0$
Alternative hypothesis:	$\mu \ne \mu_0$	Alternative hypothesis:	$\mu > 0$



Quarterly returns on an investment strategy are approximately normally distributed with a standard deviation of 10%. Strategy returns over the most recent 40 quarters have a mean of 2.7%. Based on a 5% significance level, a researcher who wants to show that strategy returns are positive should:

A. Reject the null and conclude that returns ≤ 0.

B. Fail to reject the null and conclude returns > 0.

C. Reject the null and conclude returns are > 0.

In testing the hypothesis that the mean monthly returns on an investment strategy is greater than or equal to zero, a researcher reports a p-value of 5%. The test statistic the researcher found is *closest* to:

A. 1.65.

B. 1.96.

C. 2.55.

Hypothesis Testing

Hypotheses About Two Population Means

Independent samples: Difference-in-means test

Dependent samples: Paired comparisons test

Both of these are *t*-tests

Hypotheses About Variance

Variance of a single population: Chi-square test

Compare variances of two populations: F-test

Hypotheses About Correlation

t-tests with n – 2 degrees of freedom

$$t\text{-statistic} = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$$

Parametric Tests:

- Rely on assumptions regarding the distribution of the population
- Are specific to population parameters

All tests covered on the previous slides are examples of parametric tests

Nonparametric Tests:

- Either do not consider a particular population parameter, or
- Make few assumptions about the population that is sampled.

Used primarily when:

- Data do not meet distributional assumptions
- Data are given in ranks
- Hypothesis does not concern a parameter (e.g., is a sample random or not?)

Which of the following statements about hypothesis tests of the equality of two population means is *least accurate*?

A. The statistics used to test the equality of two population means follow a *t*-distribution.

B. The test statistic for a difference in means test uses a pooled variance if the population variances are unknown.

C. Equality of population means can be tested whether the samples are dependent or independent.

What is the appropriate test statistic for a hypothesis test concerning the variance of a normally distributed population?

A. The t-statistic.
B. The chi-square statistic.
C. The F-statistic.