Lesson 1: Probability, Expected Value and Variance

- A random variable is one whose possible values or results are *uncertain.*
- An outcome is the observed value of a random variable.
- An event could be a single outcome or a set of outcomes.
- Mutually exclusive events are events that cannot happen simultaneously.
- Exhaustive events cover the range of *all possible outcomes* of an event.

A **probability** is a number between 0 and 1 that reflects the chance of a certain event occurring.

- The probability of any event, E, is a number from 0 to 1.
- The sum of the probabilities of mutually exclusive exhaustive events equals 1.

Methods of Estimating Probabilities

- An empirical probability estimates the probability of an event based on the frequency of its occurrence in the past.
- A subjective probability draws on subjective reasoning and personal judgment to estimate probabilities.
- An a priori probability is based on formal analysis and reasoning rather than personal judgment.

The odds *for* an event are stated as the probability of the event occurring to the probability of the event not occurring.

- Odds *for* event, E, are stated as P(E) to [1 - P(E)].

The odds *against* an event are stated as the probability of the event not occurring to the probability of the event occurring.

- Odds *against* event, E, are stated in the form of [1 - P(E)] to P(E).

Given the odds for an outcome, the odds against the outcome are simply the *reciprocal* of the odds for.

Example

John asserts that the odds *for* the market going up tomorrow are 1 to 4, to which Mary responds by saying that the odds *for* the market rising are 1 to 7. What do these statements really mean?

Example

If Susan thinks that the probability of the market rising tomorrow is 0.40, what odds would she offer *for* the market rising, and *against* the market rising?

Unconditional or marginal probabilities estimate the probability of an event irrespective of the occurrence of other events.

- What is probability of a return on a stock above 10%?
- What are the chances of getting a 3 on a roll of the die?

Conditional probabilities express the probability of an event occurring *given* that another event has occurred.

- What is the probability of the return on the stock being above 10% *given* that the return is above the risk free rate?
- What is the probability of rolling a 3, *given* that an odd number is rolled?

JOINT PROBABILITIES

The **joint probability, P(AB),** answers the question, “What is the probability of *both* A **and** B occurring?”

- If A and B are *mutually exclusive* events, the joint probability P(AB) equals zero. This is because mutually exclusive events cannot occur simultaneously.

- If A is contained within the set of possible outcomes for B, P(AB) = P(A).

CONDITIONAL VERSUS UNCONDITIONAL PROBABILITIES

A **conditional probability, P(A|B)** (the probability of event A occurring given that event B has occurred), equals the joint probability, P(AB), divided by the unconditional probability of event B occurring, P(B).

Example

Calculate the unconditional probability of rolling a 3, and the conditional probability of rolling a 3 *given* that an odd number is rolled.

THE MULTIPLICATION RULE FOR PROBABILITY

The **multiplication rule for probability** to calculate joint probabilities can be derived from the conditional probability formula by simply rearranging it.

Example

Probability that the state of economy is good = 0.3
Probability that stock performance is good *given* a good economic environment = 0.8
What is the probability of having a good economic environment *and* good stock performance?

ADDITION RULE FOR PROBABILITIES

The **addition rule for probability** is used when we want to determine the probability of Event A **or** Event B occurring, P(A or B). It calculates the probability of *at least one* of A and B occurring.

Example

Probability that the state of economy is bad = 0.28
Probability that stock performance is good = 0.3
Probability that stock performance is good and the state of economy is bad = 0.12
Determine the probability that either the stock performance is good or the state of economy is bad.

With two dependent events, the occurrence of one is related to the occurrence of the other.

- The probability of doing well on an exam is related to the probability of preparing well for it.
- If we are trying to forecast an event, information about a dependent event may be useful.

Two events are independent if the occurrence of one does *not* have any bearing on the occurrence of the other.

- The probability of doing well on an exam is unrelated to the probability of there being exactly 5 trees in the park nearby.
- If we are trying to forecast an event, information about an independent event will not be useful.

With independent events, the word *and* implies multiplication, and the word *or* implies addition.

THE TOTAL PROBABILITY RULE

The **total probability rule** expresses the unconditional probability of an event in terms of conditional probabilities for mutually exclusive and exhaustive events.

The probability of event A, P(A) is expressed as a weighted average in the total probability rule.

- The weights applied to the conditional probabilities are the probabilities of the scenarios.

THE TOTAL PROBABILITY RULE

Example

Calculate the probability of the currency depreciating given the following information:

- Probability of monetary authorities increasing money supply = 0.36
- Probability of the monetary authorities *not* increasing money supply = 0.64
- Probability of depreciation of currency given that money supply is increased = 0.52
- Probability of depreciation of currency given that money supply is *not* increased = 0.74

THE TOTAL PROBABILITY RULE

EXPECTED VALUE

The **expected value** of a random variable is the probability-weighted average of all possible outcomes for the random variable.

Example

At the upcoming Federal Reserve Board meeting, the probability that the Fed will increase money supply is 0.3, while the probability of no increase in money supply is 0.7. Historically, stocks have risen 4% on the day of the Fed announcement when money supply has been increased and have fallen by 5% on the day when money supply has not been increased. What is the expected value of stock returns on the day of the Fed announcement assuming that no other variables have any impact on stock returns on the day of announcement?

VARIANCE AND STANDARD DEVIATION

The **variance** of a random variable around its expected value is the probability weighted sum of the squared differences between each outcome and the expected value.

- If variance equals zero, there is no dispersion in the distribution.

Standard deviation is the positive square root of the variance.

VARIANCE AND STANDARD DEVIATION

Example

Calculate the variance and standard deviation of coal prices given the following possible outcomes and their associated probabilities:

Coal Price (X)	Probability P(X)
\$40	8%
\$50	15%
\$60	40%
\$70	25%
\$80	12%

The **total probability rule for expected value** is used to refine forecasts of expected values in light of new information or events.

- Uses conditional expected values.
  - Calculated using conditional probabilities.

The goal is to calculate the expected value of a random variable given all the possible scenarios that can occur.

The set of possible scenarios (A and B) must be *mutually exclusive and exhaustive* for the total probability rule for expected value to hold.

Example

The probability of monetary authorities increasing or not increasing the money supply in an economy depends on whether the state of economy is good or bad.

- Given a "good" economy, the conditional probabilities are as follows:
  - P(Money supply increase| good economy) = 0.25
  - P(Money supply not increase| good economy) = 0.75
- Given a "bad" economy, the conditional probabilities are as follows:
  - P(Money supply increase| bad economy) = 0.64
  - P(Money supply not increase| bad economy) = 0.36

Compute the conditional expected value for stock returns for each possible state of the economy. The stock market goes up 4% if money supply is increased, but falls by 5% when money supply is not increased.

Example

Supercar Inc's EPS was $5.76 in 2009. Its 2010 earnings depend on the state of the economy. There is a 0.7 probability of a "good" economy in 2010 and 0.3 probability of the economy being "not good".

- If there is a good economy in 2010, the probability that EPS will be $7.20 is estimated at 0.6, and the probability that EPS will be $6.4 is estimated at 0.4
- If there is a "not good" economy in 2010, the probability of a $5.80 EPS is estimated at 0.25, and the probability of EPS being $4.90 is estimated at 0.75

Use a tree diagram to model this problem and calculate the expected value of Supercar Inc's EPS for the 2010.