

Lesson 2: Covariance and Correlation and Calculating Portfolio Expected Return and Variance

Covariance is a measure of the extent to which two random variables move together.

Properties of Covariance

- Variance measures how a random variable varies with itself, while covariance measures how a random variable varies with another random variable.
- Covariance is symmetric.
- Covariance can range from positive infinity to negative infinity.
- The covariance of a random variable with itself equals its variance.
- When the covariance of returns of two assets is *negative*, it means that when the return on one asset is above its expected value, the return on the other tends to be below its expected value. There is an *inverse* relationship between the two variables.
- When the covariance of returns of two assets is *positive*, it means that when the return on one asset is above its expected value, the return on the other also tends to be above its expected value.
- Covariance of returns is zero if the returns are unrelated.

Limitations of Covariance

- Because the unit that covariance is expressed in depends on the unit that the data is presented in, it is difficult to compare covariance across data sets that have different scales.
- It is difficult to interpret covariance as it can take on extreme large values.
- Covariance does not tell us anything about the strength of the relationship between the two variables.

Example

Calculate and interpret the covariance of the returns for Stock A and Stock B given three possible states of the economy – expansion, normal and recession. The returns of Stock A and Stock B under each state and the probability of each state are listed below.

State	P(S)	R_A	R_B
Expansion	0.25	0.04	0.02
Normal	0.36	0.08	0.01
Recession	0.39	0.01	0.04

The **correlation coefficient** measures the strength and direction of the linear relationship between two random variables.

Properties of the Correlation Coefficient

- It measures the strength of the relationship between two random variables.
- It has no unit.
- It lies between -1 and $+1$
- A correlation coefficient of $+1$ indicates a perfect positive correlation between two random variables.
- A correlation coefficient of -1 indicates a perfect negative correlation between two random variables.
- A correlation coefficient of zero indicates no linear relationship between two random variables.

A **shortcoming** of the correlation coefficient is the fact that it does not specify which factor or variable causes the linear relationship between the two variables.

Example

Using the data in the previous example, compute the correlation coefficient given that the variance of Stock A is 0.123 and the variance of Stock B is 0.325