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Mathematical Model of a Pulse Submerger and the Influence of Manufacturing Tolerances

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Abstract—This article discusses a mathematical model of the process of driving pile structures using an impulse driver, which is a device based on the forces caused by the centrifugal force of imbalances. This model is a composition of mathematical models for the operation of an impulse driver, a model for the interaction of a pile with soil. Also, for the first time, the issue of product operation in the presence of belt drive defects, which can violate the ratio of the angular velocities of the unbalances, and leads to a decrease in the asymmetry coefficient, is considered. Gaussian white noise was taken as a model of the belt drive errors. When setting up a numerical experiment, the influence of instability at high speeds on the nature of the immersion and the immersion depth was studied.

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1. INTRODUCTION

There are two main methods of immersing pile structures in the soil. This is clogging and vibration immersion. The advantage of the second method over a hammer is to work without damaging nearby buildings. This method of immersion is based on the effect of a sharp decrease in the resistance to immersion of a pile element when the latter is imparted with vibration. Such units are called vibrators and are designed to immerse various pile elements in sandy and clayey soils.

The mathematical model of the modified design of the vibratory pile driver proposed in [1] posed an optimization problem for mathematicians. Adding imbalances to the vibratory pile driver rotating at double speeds with respect to the main shafts led to a difference in the absolute values of the maximum and minimum of the driving force created by the pile driver. This asymmetry effect made it possible to create a larger positive impulse with a lower weight of the installation. Thus, the criterion for optimization was a parameter—the coefficient of asymmetry, which is equal to the ratio of the maximum to the minimum value of the function of the driving force. The solution to the problem of constructing the optimal impulse and a detailed description of this problem is covered in [2, 3]. This article describes the mathematical impulse immersion model in Section 2.

A further development of this topic was the application of the impulse driver model to a mathematical model of the process of driving a pile or sheet pile into the soil, taking into account the rheological properties of the medium. This model is a second order differential equation with initial conditions [4].

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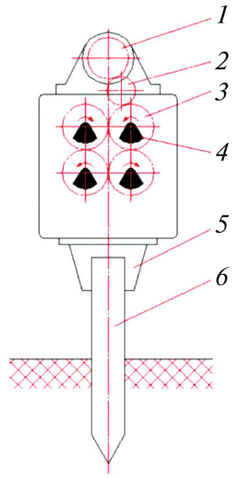

Fig. 1. Vibrating submer. Structural diagram of the vibrator: *1*—electric motor or hydraulic motor; *2*—intermediate gear; *3*—synchronizing gears; *4*—eccentrics; *5*—headgear; *6*—pile.

It takes into account the frontal and lateral resistance of the soil to the movement of the pile, and also includes the parameter for controlling the operation of the impulse driver—rotation frequency of imbalances. More details in Section 4.

In this work, for the first time, the case is considered when the rotation speed of shafts connected by a belt drive occurs in a white noise field. This formulation of the problem comes from the practical implementation of structural details and all kinds of defects in the elements of the immersion installation. It is important to note that the theoretical results obtained for obtaining the optimal impulse depend on the ratio of the angular velocities of the imbalances and, even with a slight desynchronization, can lead to a significant deviation of the optimal value and a change in the important parameter of the immersion process—speed.

To study such a problem, a mathematical model and its computer implementation through the difference approximation in Python are required. In the formulation of a numerical experiment, various orders of deviations of random variables describing the defects of the belt drive elements were considered and the dependence of the immersion process on them was analyzed.

In the final section, graphs of the results of numerical experiments are presented.

2. IMPULSE PLUNGER DRIVING FORCE MODEL

In the theory of immersion devices, a method of immersion is known, based on the effect of a sharp decrease in the resistance to immersion of a pile element when the latter is impressed with vibration. This effect is based on the thixotropy property of two-phase liquids. Such units are called vibratory drivers and are designed to immerse various pile elements in sandy and clayey soils. The structural diagram of the vibrator is shown in Fig. 1.

When the imbalances *4* rotate, centrifugal force acts on their attachment axis and the vibratory driver receives a vibrating motion, which communicates to the pile element *6* through the headgear *5*. Eccentrics are driven into rotation by electric motor *1* (or hydraulic motor) through mechanical transmission *2* and *3* (or directly from the motor shaft). Symmetrically located unbalance—synchronously rotate in different directions to balance radial loads and compensated for horizontal forces.

The mathematical model of the driving force of the vibratory driver can be represented as

$$F_{\text{ind}} = 2nm\omega^2 R\cos(\omega t), \tag{1}$$

where F_{ind} is compelling force of the vibratory driver, n is the number of imbalance pairs, m is imbalance mass, ω is angular speed of rotation of imbalances (in the figure of two), R is radius of displacement of the center of mass of the imbalance relative to the axis of rotation, t is time.

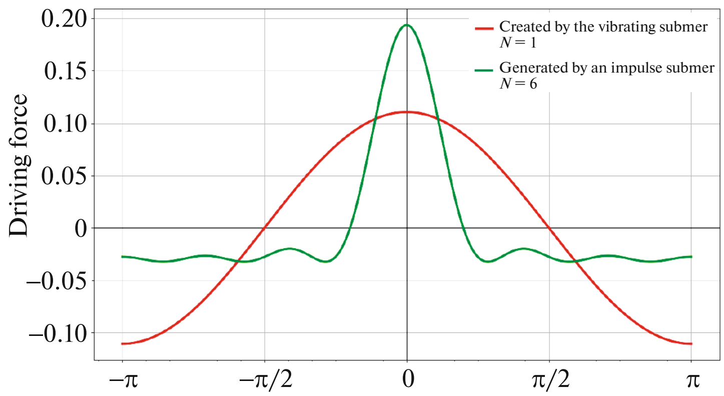

Fig. 2. Graph of the functions of the driving force of the plunger with one pair of unbalances (vibrating submer, $N = 1$, red) and with 6 pairs of unbalances (impulse submer, $N = 6$, green) for one time period.

A mathematical model of the formation of the driving force by an impulse plunger, which was proposed by Ermolenko [1], differs from the classical vibrator in different radii of pairs of imbalances;

$$\sum_{k=0}^{N} m_k \omega_k^2 R_k \cos\left(\omega_k t\right), \tag{2}$$

where m_k is mass of kth pair of imbalances, ω_k is angular velocity of rotation of kth pair of imbalances, R_k is radius of unbalances.

The difference in radii also makes the angular velocity of rotation of the pairs of unbalances different ω_k. In this case, the following relation is satisfied

$$\omega_k = k\omega_0, k = 1 \ldots N. \tag{3}$$

In an impulse immersion device, in addition to the effect of reducing friction by means of vibration, a key property appears, which is the appearance of an asymmetry between useful and harmful driving forces. We will call "useful" the driving force at the moment when the installation plunges the pile into the ground ($F_{\text{ind}}\,(t) > 0$). A "negative" driving force is called at that moment of time when it is directed in the direction opposite to immersion ($F_{\text{ind}}\,(t) < 0$). In the case of a classic vibrator, the useful and negative driving forces are equal in amplitude, which is clearly seen from the graph of function (1) shown in Fig. 2 (red). These amplitudes are different for a pulse immersion (Fig. 2, green).

The maximum useful force is the maximum of the function $F_{\text{ind}}\,(t)$, the minimum value of the function $F_{\text{ind}}\,(t)$ is the greatest absolute value of the negative driving force. The absolute ratio of the maximum value of the function to the minimum value is called the asymmetry coefficient K_n,

$$K_n = \left| \frac{\max\limits_{-\pi<t<\pi} F_{\text{ind}}(t)}{\min\limits_{-\pi<t<\pi} F_{\text{ind}}(t)} \right|, \tag{4}$$

where n is the number of imbalance pairs in the impulse immersion device, t is the operating time in one period of the function $F_{\text{ind}}\,(t)$. The problem of how to choose the radii of imbalances to obtain the best asymmetry coefficient was solved in [2], [3], and [5], where the definition of the optimal impulse is given and the theorem on the choice of the parameters of the optimal impulse immersion is proved.

When constructing a mathematical model of pile driving, described in Section 4 of this work, we used the model of an optimal impulse pile driver, specified up to a constant by the formula

$$f_n(t,\lambda) = \sum_{k=1}^{n} (n-k+1)\cos(k\omega t), t \in [-\pi, \pi], \tag{5}$$

where ω is the speed of rotation of the first pair of eccentrics. The function (5) is called the impulse of Maxwell—Fejer.

3. MODEL OF DEFECTS IN BELT TRANSMISSION OF WHITE NOISE

The theoretical impulse proposed in the previous section, created by the pressing device, has the highest asymmetry coefficient equal to the number of imbalance pairs $K_n = n$ [3]. But in practical implementation, belt or gear drives from an electric motor or hydraulic drive are used to impart rotation to the imbalance shafts. A number of defects are possible in such a design. For example, the main defects in belt drives, together with bearing defects, are: pulley defects: misalignment, uneven wear, misalignment with the shaft lead to a periodic change in belt tension, which is expressed in the modulation of low-frequency transmission vibration and friction forces in bearings with a frequency rotation of the defective pulley. Also, the nonparallelism of the shafts and the axial displacement of the pulleys, leading to the occurrence of periodic shock loads on the belt (chain) with rotation of one of the pulleys (or both), pulse modulating friction forces in the transmission bearings and high-frequency vibration of these bearings. In addition, the weakening of the belt tension, leading to instability of the harmonic amplitudes with the rotation of both shafts. Belt wear, leading to a periodic change in the tension force, which, in turn, causes modulation of low-frequency transmission vibration by the belt rotation frequency and its harmonics, as well as modulation of friction forces in the transmission bearings and high-frequency vibration of these bearings.

All of the above is also an additional condition for the ratio of the rotation frequencies of the imbalance shafts (3), which leads to the model

$$\omega_k = \omega_k \left(1 + \delta_k(t)\right), \tag{6}$$

where $\delta_k(t)$ is the error in the angular speed of rotation of the imbalance shaft at time t, caused by various defects of the submerged units.

To select a model for such an error, this article proposes a model of white noise, or an additive Gaussian distribution—a generalized stationary random process $X(t)$ with a constant spectral density and a normal distribution of the standard deviation.

In a numerical experiment, we will consider various orders of standard deviations of the random variable $\sigma(\delta)$, which describes the noise in the belt drive, and evaluate how much this affects the asymmetry coefficient and the main characteristics of the immersion process as a whole.

4. MODELING AND COMPUTER IMPLEMENTATION OF THE IMMERSION PROCESS

In applications to construction topics, in particular to installations of a pile foundation, to model the processes of driving piles, the following equation is used [7]

$$R = F_{\text{ind}} + F_{\text{grav}} - F_{\text{lr}} - F_{\text{drag}}, \tag{7}$$

where R is resultant force, F_{ind} is the pressing force created by the immersion unit, F_{grav} is gravity, F_{lr} is lateral resistance force, F_{drag} is drag force. The solution to this equation allows to determine the time and depth of immersion, depending on the type of immersion installation, pile dimensions and type of soil.

The stage when the submersible installation pulls the pile up, overcoming the force of gravity and soil resistance along the lateral surface will not be considered in this work. Since, in this case, the destruction of the pile is possible, since the concrete pile tolerates strong compression well, but collapses when trying to stretch. In practice, this is avoided by controlling the angular velocity of rotation of the imbalances, preventing the device from operating at high RPM at the beginning of the dive.

If the useful force of the immersion unit and the force of gravity cannot exceed the resistance of the ground, the dive will stop. After this stage, the speed of revolutions of the imbalance shafts increases, until the driving force becomes sufficient to continue the pile sinking.

In what follows, we will use equation (7) for the numerical model process control.

Let m denote the mass of the entire installation and let $x(t)$ denote the depth of immersion of the pile, and t is the time of immersion of the pile. Then, $R = ma = m\ddot{x}$, where a is acceleration, $F_{\text{ind}} = mg$, where g is the acceleration due to gravity, $F_{\text{drag}} = S_{\text{cs}} h_i(x(t), \xi)$, where S_{cs} is the cross-sectional area of the pile, $h_i(x(t), \xi\ )$ is the specific drag, ξ is the coefficient of soil working conditions under the lower end of the pile, $F_{\text{ls}} = Px(t)f_i(\psi)$ is the lateral resistance force, which is the product of the pile perimeter P, the immersion depth $x(t)$ and the specific lateral resistance force $fi(\psi)$, depending on the type of soil.

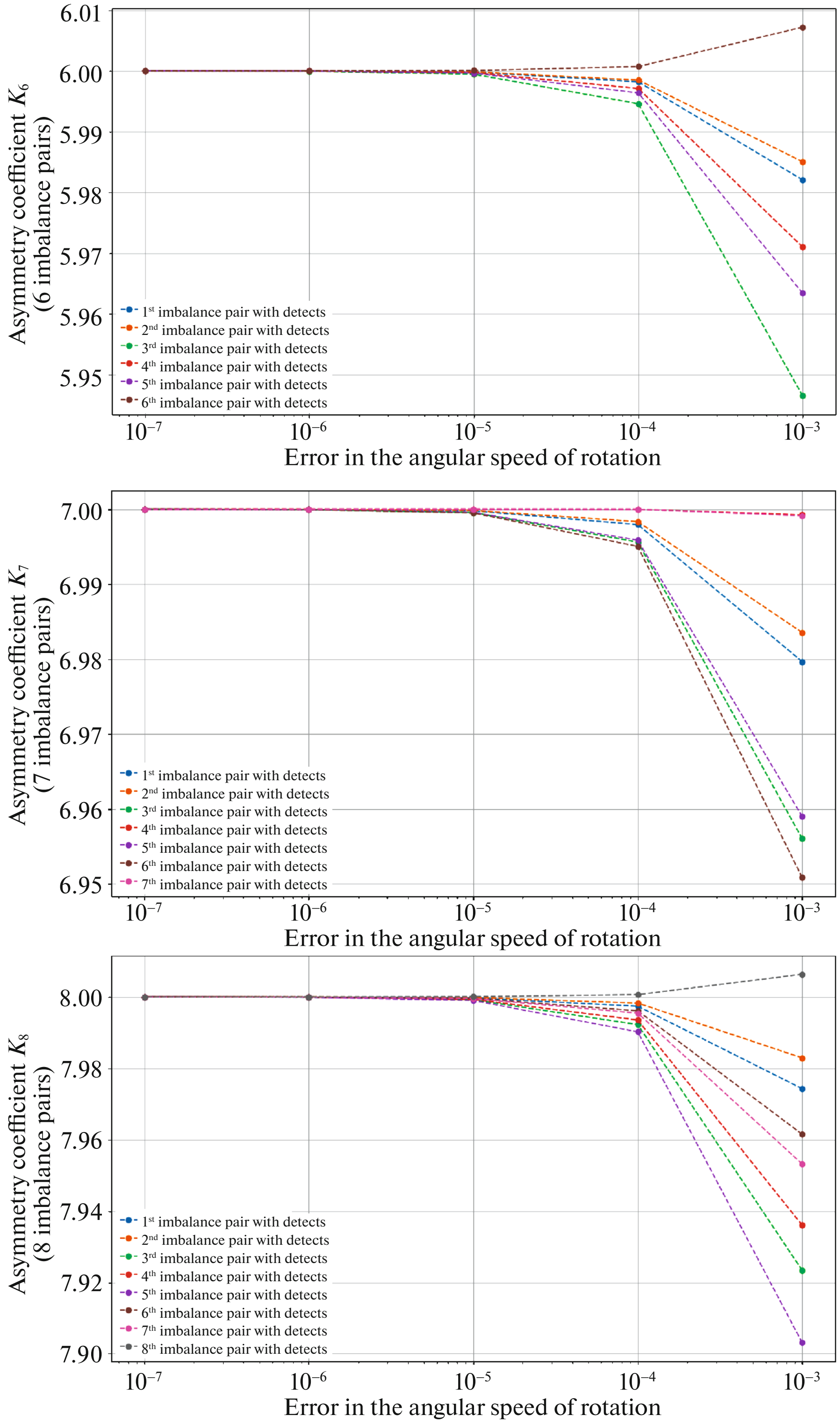

Fig. 3. Graph of the functions of the coefficients for each pair of unbalances and manufacturing errors.

We will assume that at time $t = 0$, the immersion depth is 0 and the pile is motionless. Based on this, we obtain the following second-order differential equation with initial conditions

$$m\ddot{x} = F_{\text{ind}} + mg + S_{\text{cs}}\, h_i(x(t), \xi) + Px(t) f_i(\psi), \tag{8}$$

$$x(0) = \dot{x}(0) = 0. \tag{9}$$

Solving the task (8) and (9) makes it possible to determine the time and depth of immersion depending on the characteristics of the immersion installation, the size and weight of the pile, as well as the type of soil.

For modeling using an explicit difference scheme. We replace $\ddot{x}$ in equation (8) by the difference approximation

$$m\frac{x_{i+1}-2x_i+x_{i-1}}{h^2} = F_{\text{ind}} + mg + S_{\text{cs}}h_i(x(t),\xi) + Px(t)f_i(\psi),$$

$$x_{i+1}-2x_i+x_{i-1} = \frac{h^2}{m}\left(F_{\text{ind}} + mg + S_{\text{cs}}h_i(x(t),\xi) + Px(t)f_i(\psi)\right),$$

$$x_{i+1} = 2x_i - x_{i-1} + \frac{h^2}{m}\left(F_{\text{ind}} + mg + S_{\text{cs}}h_i(x(t),\xi) + Px(t)f_i(\psi)\right). \tag{10}$$

The resulting recurrent equality allows us to numerically calculate the current value of the function x, provided $x_0 = x_1 = 0$.

For software implementation and numerical calculation of the depth and time of pile loading, the Python programming language was chosen. The program calculates the pile insertion depth using equality (10).

On the basis of this program, numerical experiments were carried out to simulate the processes of driving a pile by an impulse pile driver at different values of the standard deviation of the white noise field.

5. ABOUT THE INFLUENCE OF EXTERNAL NOISE ON THE NATURE OF THE DRIVING FORCE AND THE IMMERSION PROCESS

The stop condition in the calculation was carried out when any of the conditions were met

1. Immersion of the pile to a depth of $x(t) = 4$ m. The work is done.
2. Achieving the maximum value of the number of revolutions of the first pair of unbalances $\max(\omega) = 3000$ rpm. However, the work has not been completed.
3. The negative driving force will exceed the sum of the force of gravity and the force of lateral resistance $F_{\text{ind}} > F_{lr} + mg$, which can lead to the destruction of concrete piles, which are fragile to tensile deformation.

In the article [11] we put a random change in the angular velocity and saw how the nature of the dive changes. This work is a direct development of this numerical experiment. The next step was to determine the influence of the manufacturing error for each pair of unbalances separately and determine the error on which pair brings the greatest reduction in the asymmetry coefficient and, consequently, the immersion time.

The figure below shows the values of the asymmetry coefficients for each pair of unbalances (ordinate axis) and manufacturing errors (abscissa axis).

In the course of numerical calculations, it was found that the errors of the first (largest) pair of unbalances lead to the greatest deviation of the asymmetry coefficient and thereby to a decrease in the efficiency of the pulse submerger. This result correlates with the result stated in [11] at the same time, it expands and clarifies it. Since manufacturing tolerances can now be determined for each link of the pulse submerger. The presented formulas can be used to calculate tolerances for any devices of a standard-sized ruler with a different number of links.

The most interesting thing is that in the case of an even number of unbalances and when the manufacturing error is reached on the smallest pair above 10^{-3}, an increase in the asymmetry coefficient is observed. This amazing fact can be interpreted as a transition to the next state, when the number of pairs of unbalances is one more, which means that the asymmetry coefficient is higher. At the same time, for an odd number of unbalances, the traditional pattern of simply reducing the asymmetry coefficient is observed. That is, we get an output to a more general problem when the ratio of the angular velocities of the unbalances may not be an integer. The study of this fact requires careful mathematical study.

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