Attenuation Bias, Measurement Error & Principal Component Analysis

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Abstract

Shorter version of the abstract (I would say 4-5 sentences in a single paragraph max) goes here

Many variables of interest in economics are not directly available as empirical data. Instead, economists often use other variables that are imperfect measurements of the true focus of their analysis. These available variables are known as *proxies* or "variables measured with error", and, if they suffer from classical measurement error, their use causes *attenuation bias* when they are used as independent variables in econometric estimation. Traditionally, instrumental variables are used as a shock of exogeneity to get rid of this bias, but finding truly exogenous variables that satisfy the exclusion restriction is difficult, and so this method can often not be feasibly applied.

As an alternative to dealing with attenuation bias, we propose the use of Principal Component Analysis (PCA) over several variables measured with error. When there are multiple observed variables driven by a single "true" one, we propose to use PCA over these variables to extract the "true" variable. We then use this extracted value and use it in a standard OLS regression, thus providing a solution to attenuation bias that does not require the strong assumptions of instrumental variable analysis.

To show the properties and behaviour of our estimator on large samples under standard assumptions, we present a theoretical framework and a Monte-Carlo analysis. Additionally, we explore a basic empirical application to our method, by estimating the effect of economic development on life expectancy at birth. Since there is no consensus on how to measure economic development, we take a sample of different variables that may measure economic development with error (GDP per capita, GNI per capita, Household Income Per Capita, among others) over which we apply PCA to apply our identification strategy.

Literature

Brief discussion of https://warwick.ac.uk/fac/soc/economics/staff/knagasawa/PartialEffects.pdf, as well as anything else important that comes up on Google Scholar

So it actually kind of seems like we are doing something unique, but it's not clear that what we are doing is better than existing approaches to measurement error. Like averaging or instrumenting variables with each other.

Nagasawa 2020 theoretically develops the use of a proxy variable to deal with unobserved heterogeneity in line with the definitions in the measurement error literature Uses an imperfect measurment of the error, the proxy problem is in nuisance parameters has a single proxy variable nonparametric approach mentioned as having limited usefulness due to curse of dimensionality and restrictive common support kernel first stage new partial effects method of proxy

Schennach 2016 focuses on nonclassical measurement error and nonlinear cases- classical cases 'uninteresting' because IV is very easy to use relaxing common assumptions origins and simple approaches validation data or repeated measurements proxies are related to the variable of interest but maybe nonlinearly... indicators may be instruments time series and panel repetition factor high and low dimensional relations multivariate linear regresison- worse than attenuation bias... and nonlinear models lead IV to fail. repeated measurments- old lemma. nonparametric density estimation moments apporach and polynomials, basis and transforms, etc. symmetric kernel smoothing factor models- x is informed by factor loadings plus noise- latent covariance identification. normalization is needed. SEE HECKMAN 2010A control variables- NO need for normalization with the matrix known construct vectors of repeated measurment and decompose nonlinear exxtension. iv more general and can be biased... polynomial paramteric identificaiton is difficult in this nonlinear situation quantile regression possilbity panel data- future values can give information.

factor analysis can handle measurement errors in multiple ways exploratory factor analysis can be used to explore dimensionality of measurement instrument (like prof mentioned i guess). I think this is like just the PCA step and looking at the Loadings confirmatory factor analysis-test if constructs influence responses some unique observation factors there is discussion of the variance of the true variable and the variance of the measurmenet error. i think there might be misuse of the term measurement errors/this is not clear overall not very related

latent variable models... are really basically mathematically the same as measurement error simple and standard me explanation reminder- standardized coefificetn is very similar to correlation coefficient reminder that there is no impact of measurmeent error in y on unstandardized coeff multiple regression- x1 coeff can indeed be biased, middle of p.2 lower statistical power no bias in the mean of x bad impact on the regression use Structural Equation Modelling to estimate path coefficients among latent variables need three or more measures to estimate a latent variable in a footnote it mentions a case with only two indicators not always me but sometimes uniqueness... really it's just items unaccounted for by the factor latent variance is independent.. classical assumption

Wegge measurment error regression models are factor analysis mdoels, with the correct regressors being the factors. indeed. but no common stat method, because true factors not known- no clear coeffiicent linkage. instead, grouping dependent variables and latent factors uncorrelated with errors counting rules data grouping remedies- structural equations grouped regression model- ivs and weighted averages of ivs if there is very little credible info about the variance of measurment errors or the covariance of equation erros, factor loading restrictions are needed for identification this paper is about identification and IVs really

https://www.ncbi.nlm.nih.gov/pmc/articles/PMC4787301/ schoefield mixed effects structural equations model combine structural equations and item response theory attenuation, or nonclassical bias in any direction clear misestimation consequences solutions in IV or nonparametric bounds. here IRT error structure assumed by IV often violated bayesian structural framework and model this paper is focused on ME in the main regressor

*perhaps note somewhere that governmnet spending is less likely to have measurement error because it's an official statistic... though health spending in the denominator could have error... ow.

Theoretical framework

Consider a model where the outcome is denoted by y_i. This outcome depends on a variable of interest denoted by t_i and a vector of covariates denoted by $X_i = (x_{i,1}, x_{i,2}, \dots x_{i,p})'$. Additionally, consider a vector of variables $X_i^* = (x_{i,1}^*, x_{i,2}^*, \dots x_{i,p}^*)'$ that correspond to the covariates X_i but observed with measurement error, where $x_{i,k}^* = x_{i,k} + \eta_{i,k}$ with $\eta_{i,k} \sim iid(0, \sigma_{\eta_k}^2)$, $\mathrm{E}(x_{i,k}'\eta_{i,k}) = 0, \forall i$, $\mathrm{E}(x_{i,k}'\eta_{j,l}) = 0, \forall i \neq j$ and $k \neq l$, and $\mathrm{E}(\eta_{i,k}'\eta_{j,l}) = 0, \forall i \neq j$ and $k \neq l$. Therefore, each $x_{i,k}^*$ suffers from classical measurement error. Note that $\mathrm{E}(x_{i,k}) = \mathrm{E}(x_{i,k}^*) = \mu_{x_k}$ and that $\mathrm{V}(x_{i,k}) = \sigma_{x_k}^2$ while $\mathrm{V}(x_{i,k}^*) = \sigma_{x_k}^2 + \sigma_{\eta_k}^2 \geq \sigma_{x_k}^2$.

Data Generating Process

Assume that the outcome y_i is determined by the following Data Generation Process (DGP):

$$y_i = \gamma t_i + X_i'\beta + \epsilon_i \tag{1}$$

where γ is the parameter of the variable of interest t_i, $\beta = (\beta_1, \beta_2, \ldots \beta_p)'$ is the vector of the parameters of the covariates X_i including a constant and $\epsilon_i \sim \text{iid}(0, \sigma_\epsilon^2)$. Under this specification, the coefficients are such that:

$$\begin{pmatrix} \gamma \\ \beta \end{pmatrix} = \begin{pmatrix} \sigma_t^2 & \Sigma_{tX} \\ \Sigma_{Xt} & \Sigma_X \end{pmatrix}^{-1} \begin{pmatrix} \Sigma_{yt} \\ \Sigma_{yX} \end{pmatrix} \tag{2}$$

Suppose that the econometrician has access to t_i but, instead of X_i she observes X_i^*. Then, she specifies the following linear model

$$y_i = \gamma^* t_i + X_i^{*\prime}\beta^* + \zeta_i \tag{3}$$

the coefficients would be such that

$$\begin{aligned} \begin{pmatrix} \gamma^* \\ \beta^* \end{pmatrix} &= \begin{pmatrix} \sigma_t^2 & \Sigma_{tX^*} \\ \Sigma_{X^*t} & \Sigma_{X^*} \end{pmatrix}^{-1} \begin{pmatrix} \Sigma_{yt} \\ \Sigma_{yX^*} \end{pmatrix} & (4) \\ &= \begin{pmatrix} \sigma_t^2 & \Sigma_{tX} \\ \Sigma_{Xt} & \Sigma_X + \Sigma_\eta \end{pmatrix}^{-1} \begin{pmatrix} \sigma_t^2 & \Sigma_{tX} \\ \Sigma_{Xt} & \Sigma_X \end{pmatrix} \begin{pmatrix} \gamma \\ \beta \end{pmatrix} & (5) \end{aligned}$$

To the see the implications of the of this measurement error in the covariates, consider a simple case where the DGP depends only of the variable of interest and a covariate such that:

$$\begin{pmatrix} \gamma \\ \beta \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{6}$$

and with $\sigma_t^2 = \Sigma_X = \Sigma_\eta = 1$ while $\Sigma_{Xt} = 0.6$. Then

$$\begin{aligned} \begin{pmatrix} \gamma^* \\ \beta^* \end{pmatrix} &= \begin{pmatrix} 1 & 0.6 \\ 0.6 & 2 \end{pmatrix}^{-1} \begin{pmatrix} 1 & 0.6 \\ 0.6 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \\ \begin{pmatrix} \gamma^* \\ \beta^* \end{pmatrix} &= \begin{pmatrix} 1.37 \\ 0.39 \end{pmatrix} \end{aligned}$$

Clearly, both coefficients shows bias when the econometrician assumes a DGP with X_i^*: while there is attenuation bias on the coefficient of the covariate, the coefficient of the variable of interest is biased upward given that some of the effect of the covariates is "omitted" given this attenuation.

Instrumental Variables Regression as a Bias-Correction Method

The classical solution for the measurement-error induced bias in econometrics has been the usage of instrumental variables. Suppose an instrument Z_i that satisfies the relevance condition $\mathrm{E}(Z_i'X_i) \neq 0$

and $\mathrm{E}(Z_i' t_i) \neq 0$, and also the exclusion restriction $\mathrm{E}(Z_i' \epsilon_i) = \mathrm{E}(Z_i' \zeta_i) = \mathrm{E}(Z_i' \eta_{i,k}) = 0$, for all i and k. Then premultiplying by Z_i we have

$$Z_i' y_i = Z_i' \gamma^* t_i + Z_i' X_i^{*\prime} \beta^* + Z_i' \zeta_i \tag{7}$$

and so

$$\begin{pmatrix} \gamma^{IV} \\ \beta^{IV} \end{pmatrix} = \begin{pmatrix} \Sigma_{Zt} & \Sigma_{ZX,Zt} \\ \Sigma_{Zt,ZX} & \Sigma_{ZX} + \Sigma_{Z\eta} \end{pmatrix}^{-1} \begin{pmatrix} \Sigma_{Zt} & \Sigma_{ZX,Zt} \\ \Sigma_{Zt,ZX} & \Sigma_{ZX} \end{pmatrix} \begin{pmatrix} \gamma \\ \beta \end{pmatrix} \tag{8}$$

$$= \begin{pmatrix} \Sigma_{Zt} & \Sigma_{ZX,Zt} \\ \Sigma_{Zt,ZX} & \Sigma_{ZX} \end{pmatrix}^{-1} \begin{pmatrix} \Sigma_{Zt} & \Sigma_{ZX,Zt} \\ \Sigma_{Zt,ZX} & \Sigma_{ZX} \end{pmatrix} \begin{pmatrix} \gamma \\ \beta \end{pmatrix} \tag{9}$$

$$\begin{pmatrix} \gamma^{IV} \\ \beta^{IV} \end{pmatrix} = \begin{pmatrix} \gamma \\ \beta \end{pmatrix} \tag{10}$$

However, finding a reliable source of exogeneity is difficult, and it is impossible to conclusively prove a suitable exclusion restriction. The use of IV as a bias-correction method is thus often unfeasible.

Principal Component Regression as Bias-Correction Method

Alternatively, we propose an alternative bias-correction method for when there are several mismeasured variables for each covariate; that is, when we have more than one $x_{i,k}^*$ for every $x_{i,k}$. Given that in all the mismeasured variables the underlying value is the real value, one could think of extracting the underlying true $x_{i,k}$ through a linear combination of the different $x_{i,k}^*$. Then, we could treat all the $x_{i,k}^*$ as variables that share components as follows:

$$h_j = \underset{h'h=1, h'h_1=0, \ldots, h'h_{j-1}=0}{\operatorname{argmax}} \operatorname{var}\left[h' X_k^*\right] \tag{11}$$

where h_j is the eigenvector of Σ associated with the j^{th} ordered eigenvalue λ_j of $\Sigma_{X_k^*}$, and the principal components of X_k^* are $U_j = h_j' X_k^*$, where h_j is the eigenvector of Σ associated with the j^{th} ordered eigenvalue λ_j of Σ.

Under our assumptions, the vector of mismeasured values X_k^* of $x_{i,k}$, share only one principal component which is precisely $x_{i,k}$. Then, we only have one principal component, $x_{i,k}$, and so the $x_{i,k}$ is such that

$$x_{i,k} = h_k' X_k^* \tag{12}$$

Finally, we could then retrieve the vector of true variables X_i

$$X_i = H X_i^* \tag{13}$$

where H is a matrix such that

$$H = \begin{pmatrix} h_1 & 0 & 0 & \ldots & 0 \\ 0 & h_2 & 0 & \ldots & 0 \\ \vdots & \ddots & h_3 & \ddots & \vdots \\ 0 & \ldots & \ldots & \ldots \ddots & h_p \end{pmatrix}$$

and h_k is the vector of eigenvalues for the variable $x_{i,k}$.
Our new linear model then becomes

$$y_i = \gamma^{PCR} t_i + HX_i^{*\prime}\beta^{PCR} + \epsilon_i \tag{14}$$

where the coefficients are as follows

$$\begin{pmatrix} \gamma^{PCR} \\ \beta^{PCR} \end{pmatrix} = \begin{pmatrix} \sigma_t^2 & \Sigma_{t,HX^*} \\ \Sigma_{HX^*,t} & \Sigma_{HX^*} \end{pmatrix}^{-1} \begin{pmatrix} \Sigma_{yt} \\ \Sigma_{y,HX^*} \end{pmatrix} \tag{15}$$

$$= \begin{pmatrix} \sigma_t^2 & \Sigma_{t,HX^*} \\ \Sigma_{HX^*,t} & \Sigma_{HX^*} \end{pmatrix}^{-1} \begin{pmatrix} \sigma_t^2 & \Sigma_{tX} \\ \Sigma_{Xt} & \Sigma_X \end{pmatrix} \begin{pmatrix} \gamma \\ \beta \end{pmatrix} \tag{16}$$

$$= \begin{pmatrix} \gamma \\ \beta \end{pmatrix} \tag{17}$$

where the last equality comes from (13).

Properties of the Estimator: Monte Carlo Simulations

We then complement our theoretical analysis by using Monte Carlo Simulation to analyze the effects of using Principal Components Regression as a method of bias correction. For these simulations, we assume that the true DGP for the data is:

$$y_i = \beta_1 x_i + \beta_2 z_i + u_i$$

... where x_i and z_i are single variables drawn from $\mathcal{N}(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix})$, where ρ is some covariance between our main variable of interest (x_i) and the covariate (z_i). The u_i is drawn from a white noise distribution($\mathcal{N}(0,1)$) that is uncorrelated with both x_i and z_i. We then assume (as with the theoretical analysis) that z_i is not directly observable and instead the researchers only have access to p many measurements $z_{i,j}^*$ where $z_{i,j}^* = z_i + \eta_j$ where η_j is drawn from a white noise distribution $\mathcal{N}(\mathbf{0}, \Sigma)$ where $\mathbf{0}$ is a p-vector and Σ is a diagonal p by p matrix with only 1s on the diagonal.

In our simulations, we assume default values of $\rho = 0.5$, $\beta_1 = \beta_2 = 1$, and $p = 5$. We then vary each factor while holding the others fixed, and perform 1,000 simulations of the DGP followed by an OLS regression on either the PCA value from the p measurements of the true z_i, or on a single one of the measurements of z_i. For each simulation, we generate 100 observations of y_i, x_i,etc. Below are the results for different values of p:

We can see that using PCA to extract the latent covariate driving the mismeasured covariates noticeably outperforms using a single mismeasured covariate across several values of p. Both the average coefficient on β_1 obtained when including the PCA output in the regression, and the mean absolute percentage error obtained on the 1,000 simulations are both much closer to the target values with the PCA-based regression than with the single measurement regression. Additionally, we can see that as p increases the estimated β_1^* coefficient in the PCA regression gets steadily closer to the true β_1 value of 1. Appendix 1 contains charts that show that this increase in performance is also true for different values of β_1 and β_2.

		Number of p		
	5	10	20	50
		Coefficient on Main Variable		
PCA	1.105	1.066	1.033	1.022
	(0.121)	(0.122)	(0.119)	(0.117)
Single Measurement	1.280	1.283	1.282	1.292
	(0.124)	(0.129)	(0.131)	(0.167)
		Absolute Percentage Error		
PCA	13.1%	11.1%	10.0%	9.4%
	(9.3 ppts)	(8.3 ppts)	(7.3 ppts)	(7.3 ppts)
Single Measurement	28.2%	28.5%	28.3%	29.3%
	(12.6 ppts)	(12.6 ppts)	(12.6 ppts)	(12.7 ppts)
Observations	1,000	1,000	1,000	1,000

However, there are certain circumstances where the PCA method does not lead to more accurate estimates of β_1^*. Let's now look at the simulation results for different values of ρ (the covariance between the main variable of interest x_i and the true latent covariate z_i):

			ρ *Value*		
	-1	-0.5	0	0.5	1
			Coefficient on Main Variable		
PCA	-0.006	0.900	0.996	1.105	2.009
	(0.238)	(0.120)	(0.111)	(0.121)	(0.242)
Single Measurement	-0.002	0.720	0.998	1.280	2.003
	(0.142)	(0.130)	(0.127)	(0.129)	(0.147)
			Absolute Percentage Error		
PCA	100.6%	12.7%	8.9%	13.1%	100.9%
	(23.8 ppts)	(9.1 ppts)	(6.6 ppts)	(9.3 ppts)	(24.2 ppts)
Single Measurement	100.2%	28.1%	10.2%	28.2%	100.3%
	(14.2 ppts)	(12.7 ppts)	(7.6 ppts)	(12.6 ppts)	(14.7 ppts)
Observations	1,000	1,000	1,000	1,000	

When the covariance between x_i and z_i is equal to $0, -1$, or 1 then there is no notable improvement from using the PCA-extracted latent variable (and notice that since the variances of x_i and z_i are 1, this means that the covariance is equal to the correlation in these simulations). These simulation results suggest that so long as the correlation between x_i and z_i is not close to -1,0, or 1, there are noticeable performance gains from using PCA to extract the true covariate from a collection of observed variables that try to measure that true covariate.

However, the performance advantages that we see from using PCA could be driven by the benefit of having multiple measurements of our true covariate of interest, as opposed to any special advantages from PCA specifically. We test this question by comparing the estimated β_1^* in our PCA regressions with the estimated β_1^* when we include all p measurements as separate covariates in the regression, and the β_1^* obtained when the covariate is the mean of all p measurements of the true covariate. The results from these regressions for different values of p is shown below:

	Number of p			
	5	10	20	50
	Coefficient on Main Variable			
PCA	1.105	1.066	1.033	1.022
	(0.121)	(0.122)	(0.119)	(0.117)
All Measurements	1.100	1.061	1.025	1.010
	(0.124)	(0.129)	(0.131)	(0.167)
Average of Measurements	1.100	1.060	1.026	1.015
	(0.121)	(0.122)	(0.119)	(0.117)
	Absolute Percentage Error			
PCA	13.1%	11.1%	10.0%	9.4%
	(9.3 ppts)	(8.3 ppts)	(7.3 ppts)	(7.3 ppts)
All Measurements	12.9%	11.4%	10.7%	13.2%
	(9.3 ppts)	(8.5 ppts)	(7.9 ppts)	(10.2 ppts)
Average of Measurements	12.8%	10.9%	9.8%	9.3%
	(9.2 ppts)	(8.2 ppts)	(7.2 ppts)	(7.2 ppts)
Observations	1,000	1,000	1,000	1,000

As one can see from these results (and results for different values of β_1, β_2, and ρ in Appendix 2), there does not seem to be a noticeable difference between these three regression methods (across any values of p, β_1, β_2, and ρ. Thus, our simulations suggest that there are major benefits to having multiple measurements of a latent covariate of interest, but that using PCA, taking the average of these measurements, and including all measurements as separate covariates seem to give similar benefits to the performance of the regression.

Application: Government Share of Healthcare Spending and Life Expectancy

Explain economic importance/interesting-ness of the chosen application

Explain how GDP/economic development is measured with error

It is very difficult to find an instrumental variable for economic development which satisfies a reasonable exclusion restriction.

In the left column in the table below I first regress the life expectancy at birth for all individuals in a given country and year on a measure of government spending as a share of total health expenditure.

In the middle column I include the economic controls/covariates of GDP per capita (PPP), GNI per capita (PPP), Survey Mean Income/Consumption Per Capita, ILO GDP per person employed, and Net Foreign Assets Per Capita, all from the World Bank. In the rightmost column I instead use the first principal component combining these covariates.

I standardize all variables by subtracting the mean and dividing by the standard deviation, linearly interpolate data between known observations, and remove country-years with missing values for any of the economic indicators.

	Life Expectancy at Birth (Years)				
	(1)	(2)	(3)	(4)	(5)
Govt. Share of Health Exp.	0.613***	0.308***	-0.016	0.343***	-0.012
	(0.019)	(0.020)	(0.019)	(0.019)	(0.018)
Covariates	None	Econ Indicators	Econ Indicators	PCs	PCs
Fixed Effects	No	No	Yes	No	Yes
Observations	1,799	1,799	1,799	1,799	1,799
R^2	0.376	0.586	0.987	0.536	0.987
Adjusted R^2	0.375	0.582	0.985	0.535	0.985
Residual Std. Error	0.791	0.646	0.122	0.682	0.123
F Statistic	1081.530***	168.157***	922.656***	1036.417***	1114.735***

Note: *p<0.1; **p<0.05; ***p<0.01
All variables are standardized

Conclusion

Appendix 1

	True β_1			
	0.1	1	10	100
	Coefficient on Main Variable			
PCA	0.207	1.105	10.104	100.117
	(0.121)	(0.121)	(0.123)	(0.124)
Single Measurement	0.383	1.280	10.278	100.289
	(0.128)	(0.129)	(0.131)	(0.133)
	Absolute Percentage Error			
PCA	131.1%	13.1%	1.3%	0.1%
	(95.1 ppts)	(9.3 ppts)	(0.9 ppts)	(0.1 ppts)
Single Measurement	283.6%	28.2%	2.8%	0.3%
	(126.6 ppts)	(12.6 ppts)	(1.3 ppts)	(0.1 ppts)
Observations	1,000	1,000	1,000	1,000

	True β_2			
	0.1	1	10	100
	Coefficient on Main Variable			
PCA	1.018	1.105	2.112	12.171
	(0.115)	(0.121)	(0.477)	(4.555)
Single Measurement	1.034	1.280	3.865	29.751
	(0.107)	(0.129)	(0.703)	(7.231)
	Absolute Percentage Error			
PCA	9.4%	13.1%	111.6%	1,119.6%
	(7.0 ppts)	(9.3 ppts)	(47.0 ppts)	(449.4 ppts)
Single Measurement	8.9%	28.2%	286.5%	2,875.1%
	(6.8 ppts)	(12.6 ppts)	(70.3 ppts)	(723.1 ppts)
Observations	1,000	1,000	1,000	1,000

Appendix 2

Appendix 3

		Number of p		
	5	10	20	50
		Coefficient on Main Variable		
PCA	1.105	1.066	1.033	1.022
	(0.121)	(0.122)	(0.119)	(0.117)
Single Measurement	1.280	1.283	1.282	1.292
	(0.124)	(0.129)	(0.131)	(0.167)
		Absolute Percentage Error		
PCA	13.1%	11.1%	10.0%	9.4%
	(9.3 ppts)	(8.3 ppts)	(7.3 ppts)	(7.3 ppts)
Single Measurement	28.2%	28.5%	28.3%	29.3%
	(12.6 ppts)	(12.6 ppts)	(12.6 ppts)	(12.7 ppts)
Observations	1,000	1,000	1,000	1,000

			ρ Value		
	-1	-0.5	0	0.5	1
			Coefficient on Main Variable		
PCA	-0.006	0.900	0.996	1.105	2.009
	(0.238)	(0.120)	(0.111)	(0.121)	(0.242)
Single Measurement	-0.002	0.720	0.998	1.280	2.003
	(0.142)	(0.130)	(0.127)	(0.129)	(0.147)
			Absolute Percentage Error		
PCA	100.6%	12.7%	8.9%	13.1%	100.9%
	(23.8 ppts)	(9.1 ppts)	(6.6 ppts)	(9.3 ppts)	(24.2 ppts)
Single Measurement	100.2%	28.1%	10.2%	28.2%	100.3%
	(14.2 ppts)	(12.7 ppts)	(7.6 ppts)	(12.6 ppts)	(14.7 ppts)
Observations	1,000	1,000	1,000	1,000	

	True β_1			
	0.1	1	10	100
	Coefficient on Main Variable			
PCA	0.207	1.105	10.104	100.117
	(0.121)	(0.121)	(0.123)	(0.124)
All Measurements	0.201	1.100	10.098	100.11
	(0.123)	(0.124)	(0.126)	(0.127)
Average of Measurements	0.202	1.100	10.098	100.111
	(0.121)	(0.121)	(0.123)	(0.124)
	Absolute Percentage Error			
PCA	131.1%	13.1%	1.3%	0.1%
	(95.1 ppts)	(9.3 ppts)	(0.9 ppts)	(0.1 ppts)
All Measurements	128.9%	12.9%	1.3%	0.1%
	(93.6 ppts)	(9.3 ppts)	(1.0 ppts)	(0.1 ppts)
Average of Measurements	127.9%	12.8%	1.3%	0.1%
	(93.1 ppts)	(9.2 ppts)	(0.9 ppts)	(0.1 ppts)
Observations	1,000	1,000	1,000	1,000

	True β_2			
	0.1	1	10	100
	Coefficient on Main Variable			
PCA	1.018	1.105	2.112	12.171
	(0.115)	(0.121)	(0.477)	(4.555)
All Measurements	1.02	1.100	2.067	11.664
	(0.118)	(0.124)	(0.477)	(4.519)
Average of Measurements	1.017	1.100	2.061	11.625
	(0.115)	(0.121)	(0.470)	(4.415)
	Absolute Percentage Error			
PCA	9.4%	13.1%	111.6%	1,119.6%
	(7.0 ppts)	(9.3 ppts)	(47.0 ppts)	(449.4 ppts)
All Measurements	9.7%	12.9%	107.1%	1,069.3%
	(7.0 ppts)	(9.3 ppts)	(46.8 ppts)	(445.0 ppts)
Average of Measurements	9.4%	12.8%	106.5%	1,065.2%
	(6.9 ppts)	(9.2 ppts)	(46.0 ppts)	(435.0 ppts)
Observations	1,000	1,000	1,000	1,000

	Number of p			
	5	10	20	50
	Coefficient on Main Variable			
PCA	1.105	1.066	1.033	1.022
	(0.121)	(0.122)	(0.119)	(0.117)
All Measurements	1.100	1.061	1.025	1.010
	(0.124)	(0.129)	(0.131)	(0.167)
Average of Measurements	1.100	1.060	1.026	1.015
	(0.121)	(0.122)	(0.119)	(0.117)
	Absolute Percentage Error			
PCA	13.1%	11.1%	10.0%	9.4%
	(9.3 ppts)	(8.3 ppts)	(7.3 ppts)	(7.3 ppts)
All Measurements	12.9%	11.4%	10.7%	13.2%
	(9.3 ppts)	(8.5 ppts)	(7.9 ppts)	(10.2 ppts)
Average of Measurements	12.8%	10.9%	9.8%	9.3%
	(9.2 ppts)	(8.2 ppts)	(7.2 ppts)	(7.2 ppts)
Observations	1,000	1,000	1,000	1,000

	ρ Value				
	-1	-0.5	0	0.5	1
	Coefficient on Main Variable				
PCA	-0.006	0.900	0.996	1.105	2.009
	(0.238)	(0.120)	(0.111)	(0.121)	(0.242)
All Measurements	-0.007	0.904	0.996	1.100	2.011
	(0.249)	(0.122)	(0.112)	(0.124)	(0.249)
Average of Measurements	-0.005	0.905	0.996	1.100	2.010
	(0.243)	(0.120)	(0.110)	(0.121)	(0.246)
	Absolute Percentage Error				
PCA	100.6%	12.7%	8.9%	13.1%	100.9%
	(23.8 ppts)	(9.1 ppts)	(6.6 ppts)	(9.3 ppts)	(24.2 ppts)
All Measurements	100.7%	12.6%	9.0%	12.9%	101.1%
	(24.9 ppts)	(9.0 ppts)	(6.7 ppts)	(9.3 ppts)	(24.9 ppts)
Average of Measurements	100.5%	12.4%	8.9%	12.8%	101.0%
	(24.3 ppts)	(8.9 ppts)	(6.6 ppts)	(9.2 ppts)	(24.6 ppts)
Observations	1,000	1,000	1,000	1,000	

Figure 1: Correlations Between Covariates and Life Expectancy

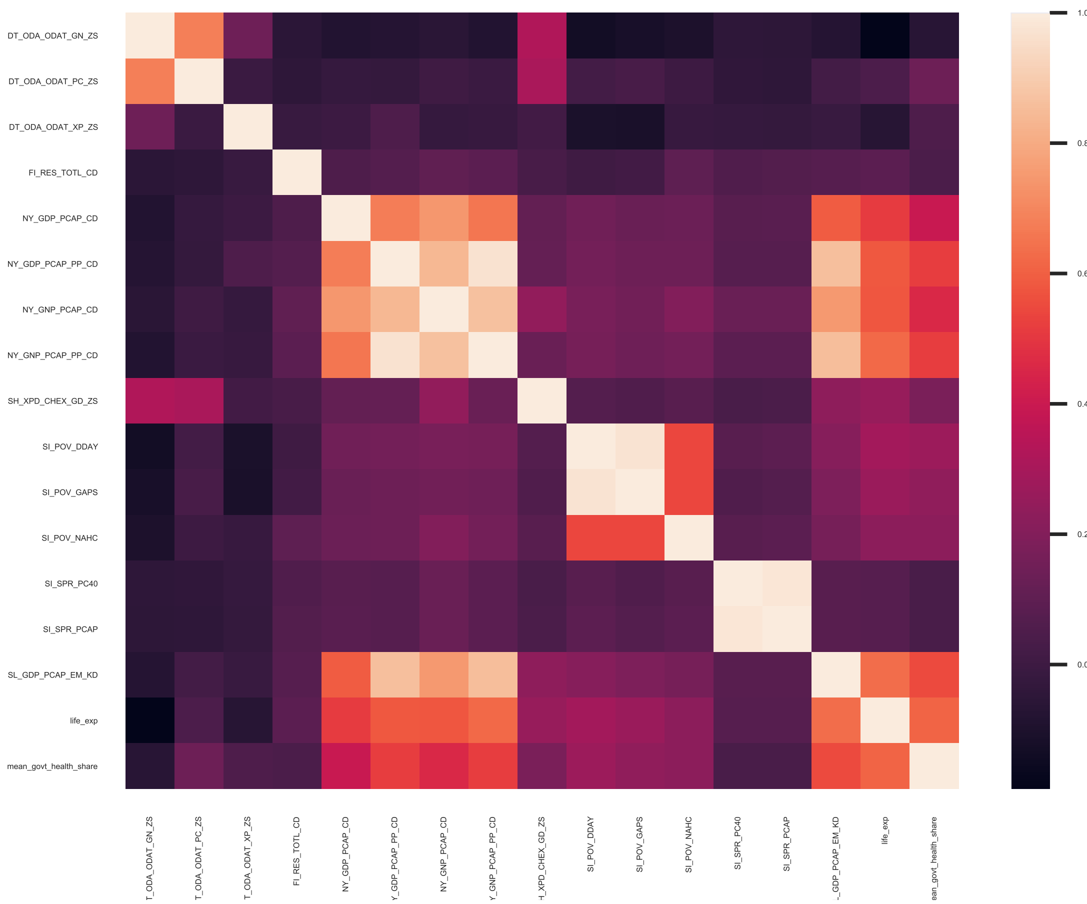

Figure 2: Economic Measures PCA Loadings

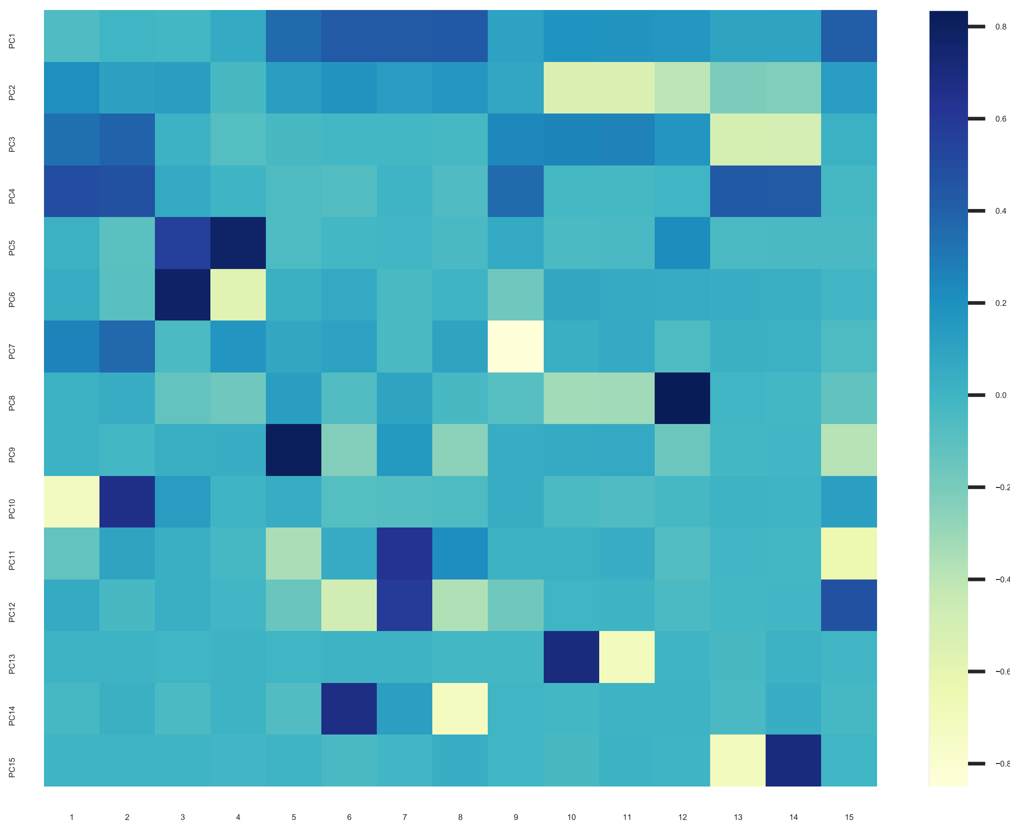

Figure 3: Economic Measures PCA Share of Variance Explained