AVL Trees
Data Structures

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Adopted from
a presentation by Simon Garrett
and the Mark Allen Weiss book

AVL (Adelson-Velskii and Landis) tree

- A balanced binary search tree where the height of the two subtrees (children) of a node differs by at most one. Look-up, insertion, and deletion are O(log n), where n is the number of nodes in the tree.
- https://www.cs.usfca.edu/~galles/visualization/AVLtree.html

Source: nist.gov

Definition of height (reminder)

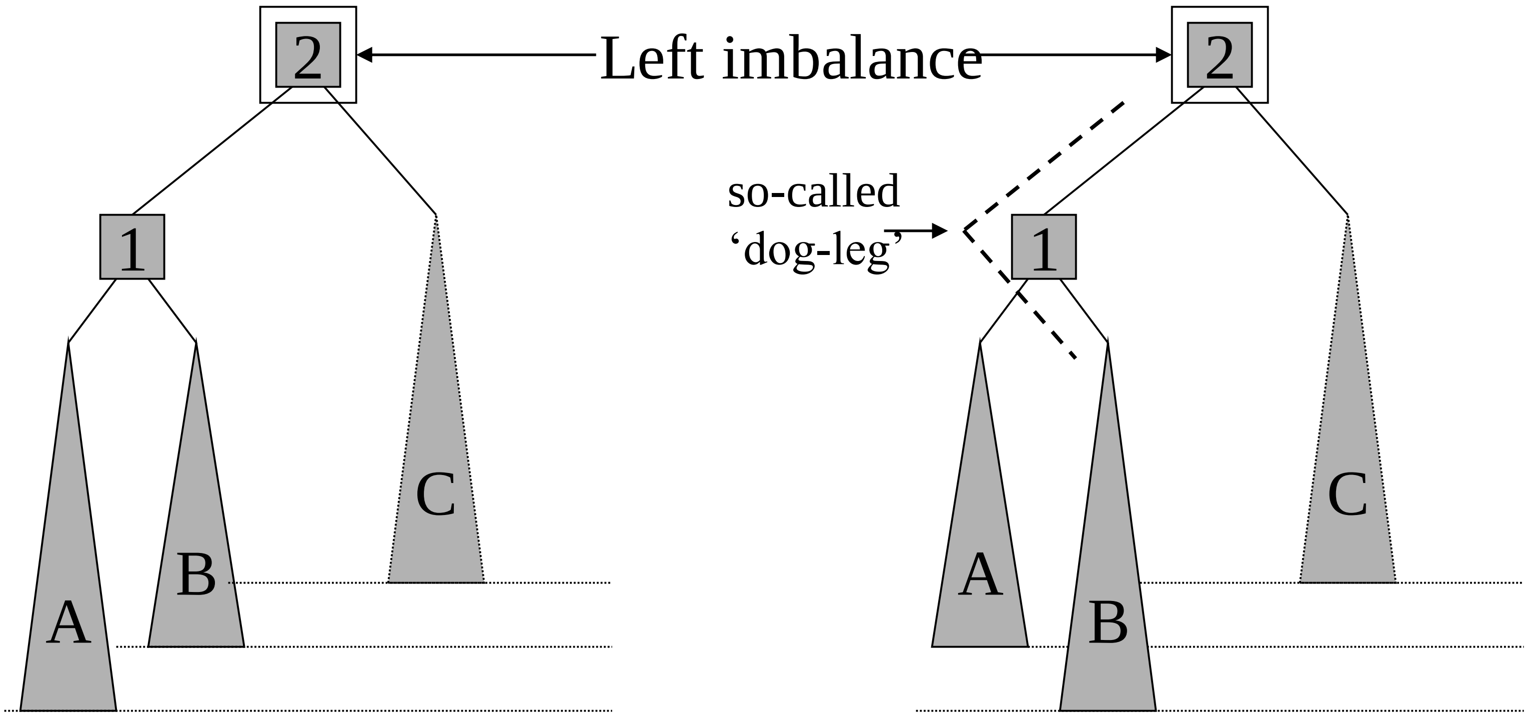
- Height: the length of the longest path from a node to a leaf.
  - All leaves have a height of 0
  - An empty tree has a height of –1

The insertion problem

- Unless keys appear in just the right order, imbalance will occur
- It can be shown that there are only two possible types of imbalance (see next slide):
  - Left-left (or right-right) imbalance
  - Left-right (or right-left) imbalance
  - The right-hand imbalances are the same, by symmetry

The two types of imbalance

- Left-left (right-right)
- Left-right (right-left)



There are no other possibilities for the left (or right) subtree

Localising the problem

Two principles:

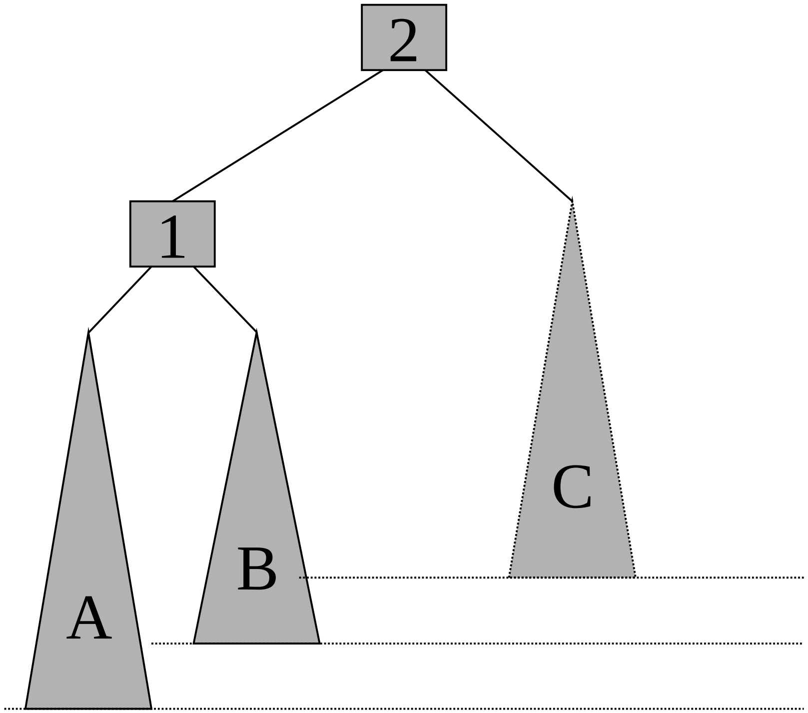
- Imbalance will only occur on the path from the inserted node to the root (only these nodes have had their subtrees altered - local problem)

- Rebalancing should occur at the ***deepest unbalanced node*** (local solution too)

Left(left) imbalance (1)

[and right(right) imbalance, by symmetry]

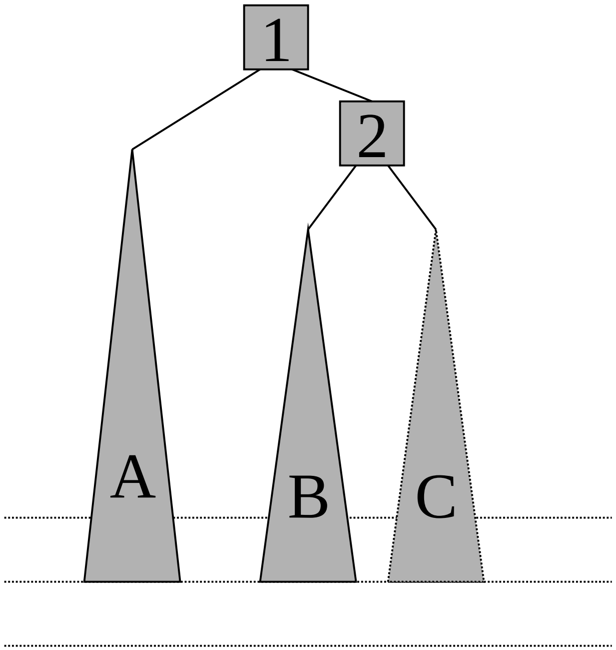
- B and C have the same height
- A is one level higher
- Therefore make 1 the new root, 2 its right child and B and C the subtrees of 2
- Note the levels



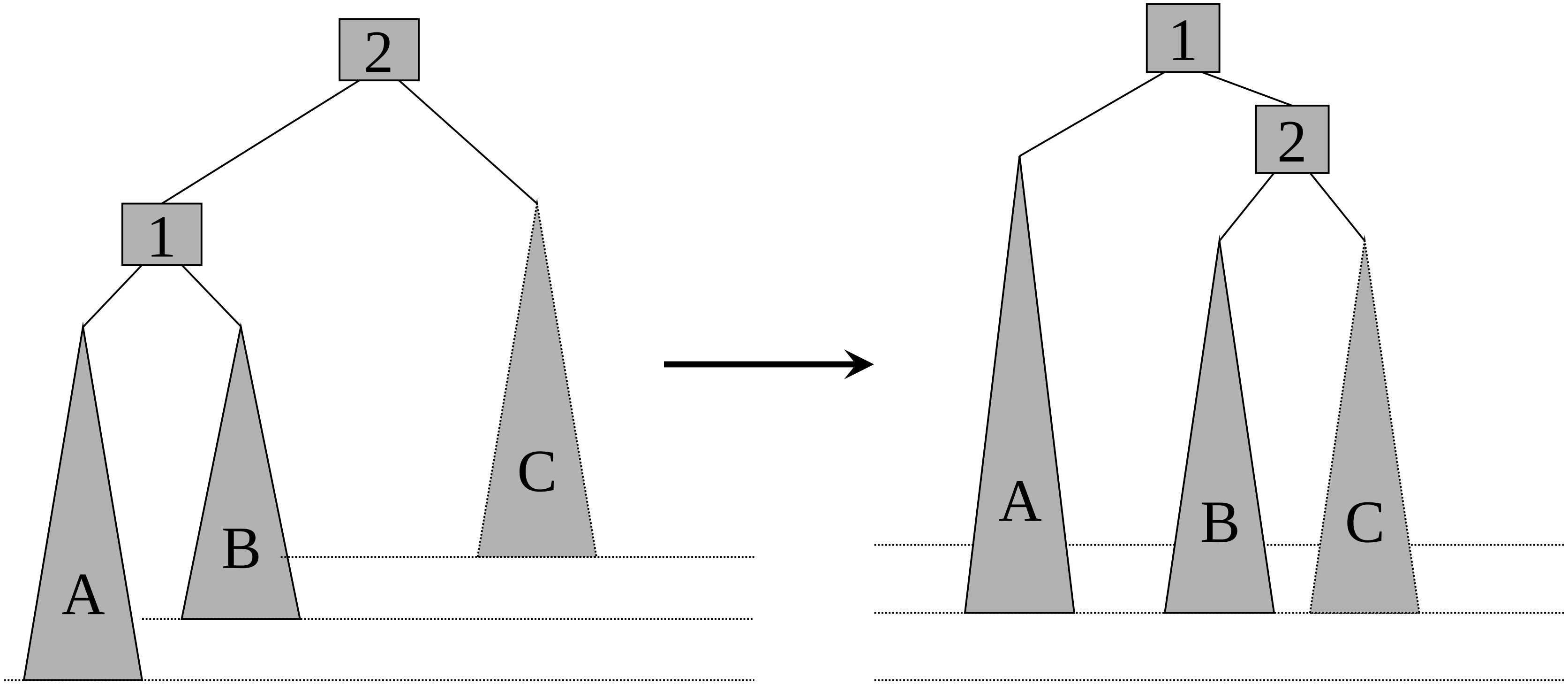
Left(left) imbalance (2)
[and right(right) imbalance, by symmetry]

- B and C have the same height
- A is one level higher
- Therefore make 1 the new root, 2 its right child and B and C the subtrees of 2
- Result: a more balanced and legal AVL tree

- Note the levels



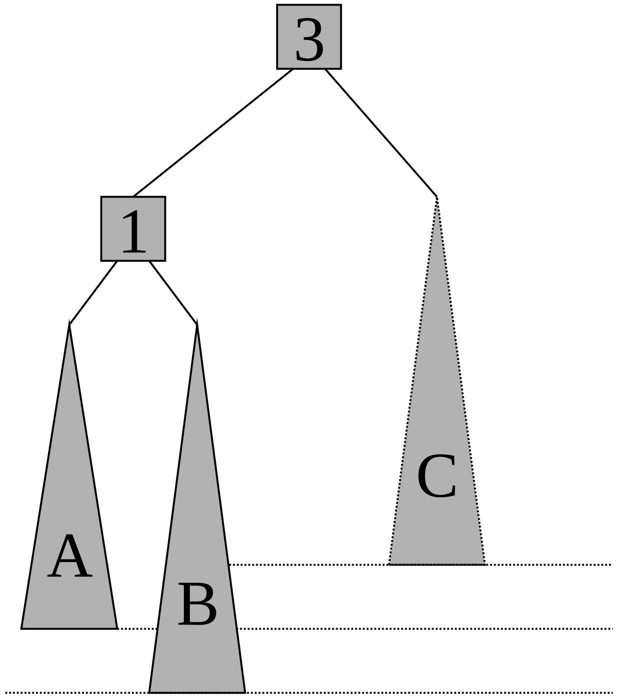
Single rotation



Left(right) imbalance (1)

[and right(left) imbalance by symmetry]

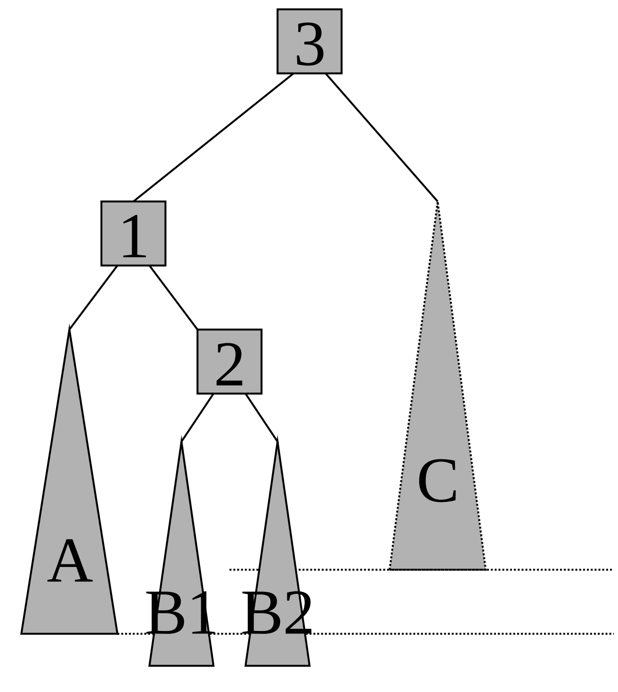
- Can’t use the left-left balance trick - because now it’s the *middle subtree*, i.e. B, that’s too deep.
- Instead consider what’s inside B...



Left(right) imbalance (2)

[and right(left) imbalance by symmetry]

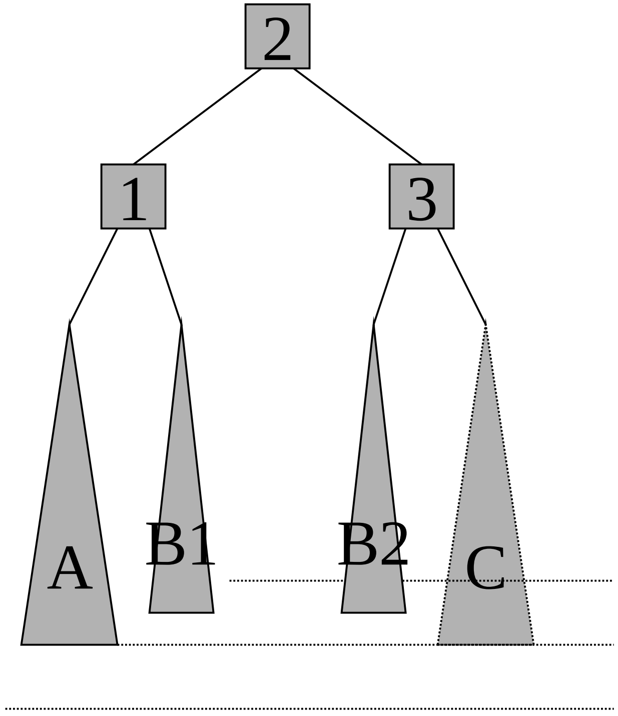
- B can be broken into a root and two subtrees. The 3 parts of B contain at least one item (just added)
- We do not know which is too deep - set them both to 0.5 levels below subtree A



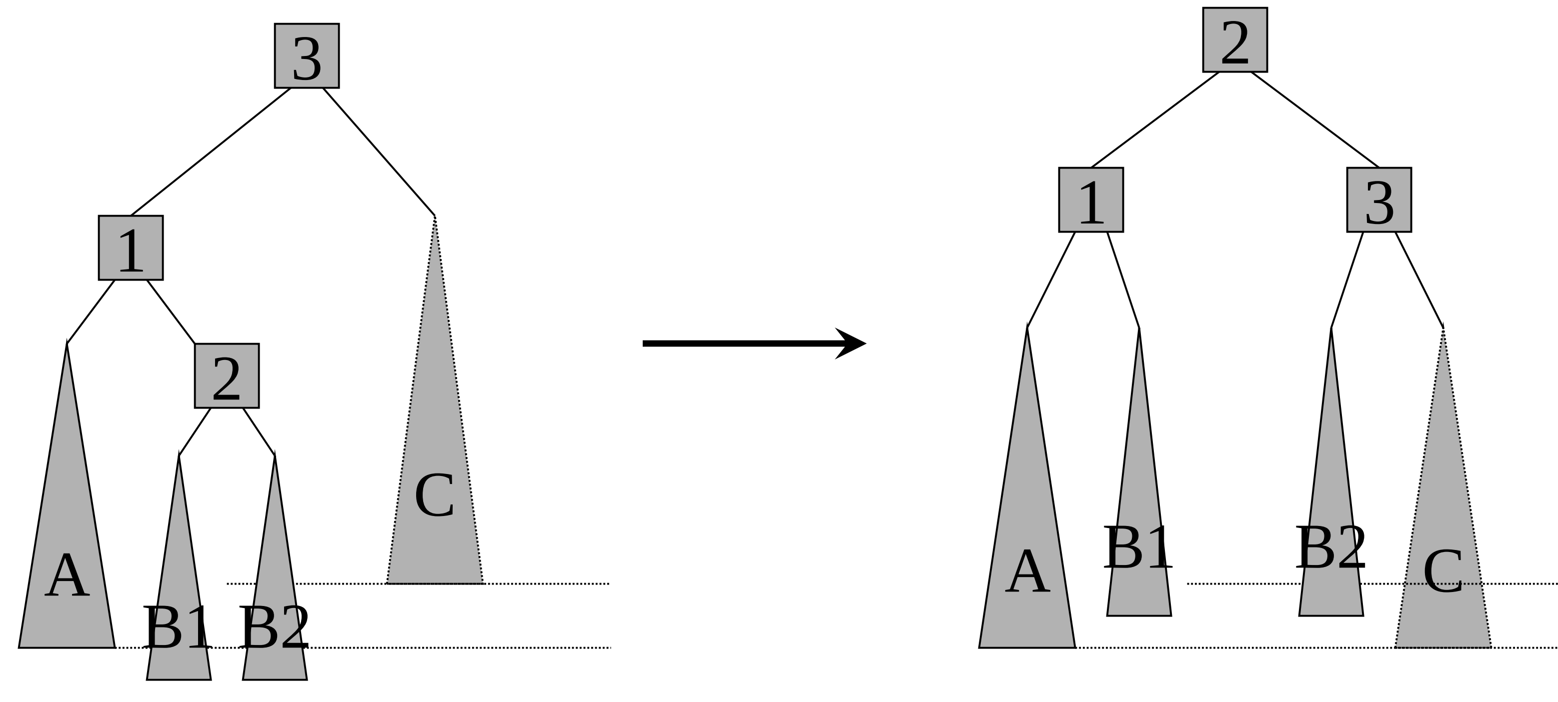
Left(right) imbalance (3)

[and right(left) imbalance by symmetry]

- Neither 1 nor 3 worked as root node so make 2 the root
- Rearrange the subtrees in the correct order
- No matter how deep B1 or B2 (+/- 0.5 levels) we get a legal AVL tree again



double rotation



insert method

```
private AvlNode<Anytype> insert(Anytype x, AvlNode<Anytype> t )
{
/*1*/    if( t == null )
                t = new AvlNode<Anytype>( x, null, null );
/*2*/    else if( x.compareTo( t.element ) < 0 )
         {
                t.left = insert( x, t.left );
                if( height( t.left ) - height( t.right ) == 2 )
                       if( x.compareTo( t.left.element ) < 0 )
                              t = rotateWithLeftChild( t );
                       else
                              t = doubleWithLeftChild( t );
         }
/*3*/    else if( x.compareTo( t.element ) > 0 )
         {
                t.right = insert( x, t.right );
                if( height( t.right ) - height( t.left ) == 2 )
                       if( x.compareTo( t.right.element ) > 0 )
                              t = rotateWithRightChild( t );
                       else
                              t = doubleWithRightChild( t );
         }
/*4*/    else
                ;  // Duplicate; do nothing
         t.height = max( height( t.left ), height( t.right ) ) + 1;
         return t;
}
```

rotateWithLeftChild method

```
private static AvlNode<Anytype> rotateWithLeftChild(
AvlNode<Anytype> k2 )
{
        AvlNode<Anytype> k1 = k2.left;
        k2.left = k1.right;
        k1.right = k2;
        k2.height = max( height( k2.left ), height( k2.right ) ) + 1;
        k1.height = max( height( k1.left ), k2.height ) + 1;
        return k1;
}
```

rotateWithRightChild method

```
private static AvlNode<Anytype> rotateWithRightChild(
    AvlNode<Anytype> k1 )
{
    AvlNode<Anytype> k2 = k1.right;
    k1.right = k2.left;
    k2.left = k1;
    k1.height = max( height( k1.left ), height( k1.right ) ) + 1;
    k2.height = max( height( k2.right ), k1.height ) + 1;
    return k2;
}
```

doubleWithLeftChild method

```
private static AvlNode<Anytype>
doubleWithLeftChild( AvlNode<Anytype>
k3 )
{
    k3.left = rotateWithRightChild( k3.left );
    return rotateWithLeftChild( k3 );
}
```

doubleWithRightChild method

```
private static AvlNode<Anytype>
   doubleWithRightChild(
   AvlNode<Anytype> k1 )
{
   k1.right = rotateWithLeftChild( k1.right );
   return rotateWithRightChild( k1 );
}
```