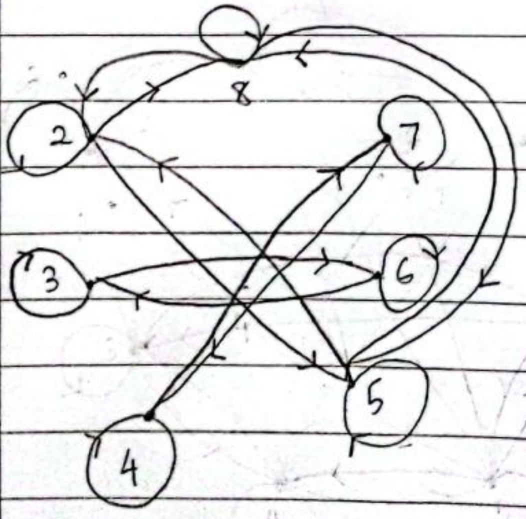
No: .... Date: ....

Assignment 2

Q1

1. $A = \{2, 3, 4, 5, 6, 7, 8\}$

$R = \{(6,3), (7,4), (8,5), (5,2), (8,2), (3,6), (4,7), (5,8), (2,5),$
$(2,8), (2,2), (3,3), (4,4), (5,5), (6,6), (7,7), (8,8)\}$



$$M_R = \begin{array}{c} \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{array} \begin{array}{c} \begin{array}{ccccccc} 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

No: Q1 Date:

1. $$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$M_R \otimes M_R = M_R$

$\therefore$ R is transitive

R is equivalence because it is reflexive, symmetric & transitive

STANDARD

2. $A = \{1, 2, 3\}$

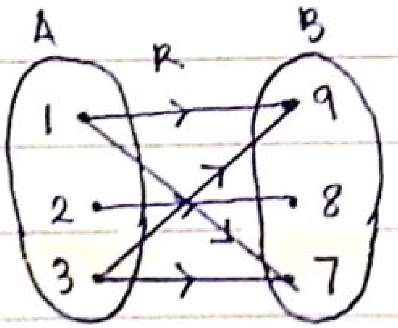
$B = \{9, 8, 7\}$

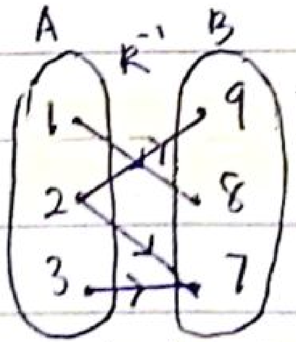
$A \times B = \{(1,9), (1,8), (1,7), (2,9), (2,8), (2,7), (3,9), (3,8), (3,7)\}$

a) $R = \{(1,9), (2,8), (1,7), (3,9), (3,7)\}$

$R^{-1} = \{(1,8), (2,9), (2,7), (3,8)\}$

b)

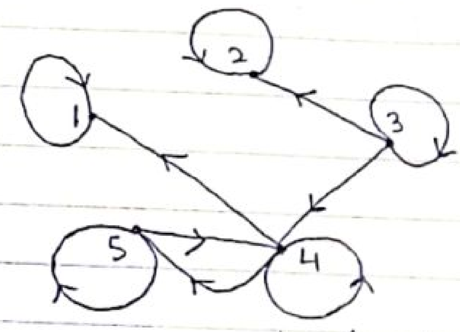




c) For all $(a,b) \in A \times B$, $a R b \leftrightarrow a + b$ is an odd number

3. $A = \{1, 2, 3, 4, 5\}$

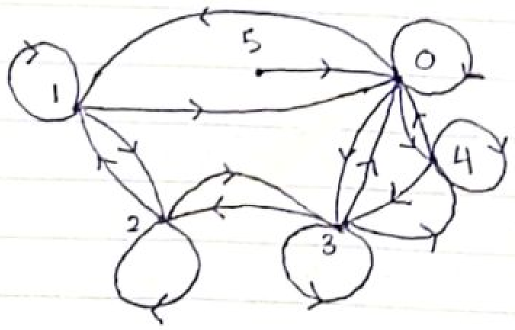
$$\begin{array}{c c}
 & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}
\end{array}$$



	1	2	3	4	5
in-degree	2	2	1	3	2
out-degree	1	1	3	3	2

4. $A = \{0, 1, 2, 3, 4\}$

$$\begin{array}{c c}
 & \begin{array}{ccccc} 0 & 1 & 2 & 3 & 4 \end{array} \\
\begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix}
\end{array}$$



- R is reflexive

$$M_R^T = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix}$$

- R is symmetric

No. Date

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

$M_{13} \neq M_{13}$

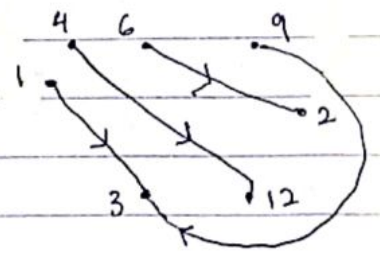
- R is not transitive

5. $R = \{(1,3), (2,6), (3,9), (4,12)\}$

a) R is irreflexive because there is no loop is set R

b) R is not symmetric because $(1,3) \in R$ but $(3,1) \notin R$

c)



R is not transitive because $(1,4), (4,6), (6,9) \notin R$

6.

$$R = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad S = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

a) $$RS = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

b) $$SR = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

No: Date:

Question 2

7. Function has only one output for a given input, whereas a relation can have multiple possible outputs for a given input.

8. $A = \{2, 3, 4, 5\}$

i) Function because one-to-one
ii) Function because one-to-many
iii) Not a function because $f(2) = 3$ $f(2) = 4$ but $3 \neq 4$.
iv) Function because one-to-many
v) Not a function because many-to-one

9. $Z = \{1, 2, 3, 4, 5\}$

$R = \{(1,6), (2,7), (3,8), (4,9), (5,10)\}$

Domain $= \{1, 2, 3, 4, 5\}$

Range $= \{6, 7, 8, 9, 10\}$

10. v) $f(x_1) = f(x_2)$

$1 - 2x_1 = 1 - 2x_2$

$-2x_1 = -2x_2$

$x_1 = x_2$

$\therefore$ $f(x)$ is one-to-one

$f(x) = 1 - 2x$

$y = 1 - 2x$

$x = \dfrac{y-1}{-2}$

$f\left(\dfrac{y-1}{-2}\right) = 1 - \cancel{2}\left(\dfrac{y-1}{-\cancel{2}}\right)$

$= 1 + y - 1$

$= y$ $\therefore$ $f(x)$ is onto y & $f(x)$ is bijection

STANDARD

No. Date

vi) $f = R \to R, f(x) = 5x^2 - 1$

$f(x_2) = f(x_1)$

$5x_2^2 - 1 = 5x_1^2 - 1$

$5x_2^2 = 5x_1^2$

$x_2^2 = x_1^2$

$x_2 = \pm x_1$

$\therefore$ f(x) is not one-to-one

$y = 5x^2 - 1$

$x = \sqrt{\frac{y+1}{5}}$

$f\left(\sqrt{\frac{y+1}{5}}\right) = \cancel{5}\left(\sqrt{\frac{y+1}{\cancel{5}}}\right)^{\cancel{2}} - 1$

$= y + 1 - 1$

$= y$

$\therefore$ f(x) is onto y

f(x) is bijection

vii) $f = R \to R, f(x) = x^4$

$f(x_2) = f(x_1)$

$x_2^4 = x_1^4$

$x_2 = \pm x_1$

$\therefore$ f(x) is not one-to-one

$y = x^4$

$x = \sqrt[4]{y}$

$f(\sqrt[4]{y}) = (\sqrt[4]{y})^4$

$= y$

$\therefore$ f(x) is onto y

STANDARD

No Date

viii) $f = R \to R$, $f(x) = \left(\frac{x-2}{x-3}\right)$

$f(x_1) = f(x_2)$

$\frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3}$

$(x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3)$

$f(4) = \frac{4-2}{4-3} = \frac{2}{1} = 2$

$f(5) = \frac{5-2}{5-3} = \frac{3}{2}$

$f(4) \neq f(5)$

$\therefore f(x)$ is one-to-one

$y = \frac{x-2}{x-3}$

$y(x-3) = x-2$

$yx - 3y = x - 2$

$yx - x = -2 + 3y$

$x(y-1) = -2 + 3y$

$x = \frac{-2+3y}{y-1}$

$f\left(\frac{-2+3y}{y-1}\right) = \frac{\left(\frac{-2+3y}{y-1}\right) - 2}{\left(\frac{-2+3y}{y-1} - 3\right)}$

$= \frac{\frac{-2+3y-2(y-1)}{y-1}}{\frac{-2+3y-3(y-1)}{y-1}}$

$= \frac{-2+3y-2y+2}{\cancel{y-1}} \times \frac{\cancel{y-1}}{-2+3y-3y+3}$

$f\left(\frac{-2+3y}{y-1}\right) = \frac{y}{1}$

$= y$

$\therefore f(x)$ is onto y

$f(x)$ is bijection

STANDARD

No. Date

11. ix) $f(x) = 3x - 1$; $g(x) = x^2 - 1$ $\qquad x = \{0, 1, 2, 3\}$

$fg(x) = f(x^2 - 1)$
$= 3(x^2 - 1) - 1$
$= 3x^2 - 3 - 1$
$= 3x^2 - 4$

$fg(0) = 3(0)^2 - 4$
$= -4$

$fg(1) = 3(1)^2 - 4$
$= -1$

$fg(2) = 3(2)^2 - 4$
$= 8$

$fg(3) = 3(3)^2 - 4$
$= 23$

x) $f(x) = x^2$, $g(x) = 5x - 6$

$fg(x) = f(5x - 6)$
$= (5x - 6)^2$

$fg(0) = (5(0) - 6)^2$
$= 36$

$fg(1) = (5(1) - 6)^2$
$= 1$

$fg(2) = (5(2) - 6)^2$
$= 16$

$fg(3) = (5(3) - 6)^2$
$= 81$

xi) $f(x) = x - 1$; $g(x) = x^2 + 1$

$fg(x) = f(x^2 + 1)$
$= (x^2 + 1) - 1$
$= x^2$

$fg(0) = 0^2$
$= 0$

$fg(1) = 1^2$
$= 1$

$fg(2) = 2^2$
$= 4$

$fg(3) = 3^2$
$= 9$

No Date

Question 3

12. xiii) $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$

$a_0 = 2, a_1 = 5, a_2 = 15$

$n \geq 3$

$a_3 = 6a_{3-1} - 11a_{3-2} + 6a_{3-3}$

$= 6a_2 - 11a_1 + 6a_0$

$= 6(15) - 11(5) + 6(2)$

$= 47$

$a_4 = 6a_{4-1} - 11a_{4-2} + 6a_{4-3}$

$= 6a_3 - 11a_2 + 6a_1$

$= 6(47) - 11(15) + 6(5) = 147$

$\therefore 2, 5, 15, 47, 147, \ldots$

xi) $a_n = 6a_{n-1} - 9a_{n-2}$

$a_0 = 1, a_1 = 6$

$n \geq 2$

$a_2 = 6a_{2-1} - 9a_{2-2}$

$= 6a_1 - 9a_0$

$= 6(6) - 9(1)$

$= 27$

$a_3 = 6a_{3-1} - 9a_{3-2}$

$= 6a_2 - 9a_1$

$= 6(27) - 9(6)$

$= 108$

$a_4 = 6a_{4-1} - 9a_{4-2}$

$= 6a_3 - 9a_2$

$= 6(108) - 9(27)$

$= 405$

$\therefore 1, 6, 27, 108, 405, \ldots$

STANDARD

xiv) $a_n = -3a_{n-1} - 3a_{n-2} + a_{n-3}$

$a_0 = 1, \; a_1 = -2, \; a_2 = -1$

$n \geq 3$

$a_3 = -3a_{3-1} - 3a_{3-2} + a_{3-3}$

$= -3a_2 - 3a_1 + a_0$

$= -3(-1) - 3(-2) + 1$

$= 10$

$a_4 = -3a_{4-1} - 3a_{4-2} + a_{4-3}$

$= -3a_3 - 3a_2 + a_1$

$= -3(10) - 3(-1) + (-2)$

$= -29$

$\therefore 1, -2, -1, 10, -29, \ldots$

13. $a_{n+1} = 5a_n - 3 \; ; \; a_1 = k$

$n \geq 1$

a) $a_{1+1} = 5a_1 - 3$

$a_2 = 5k - 3$

$a_{2+1} = 5a_2 - 3$

$a_3 = 5(5k-3) - 3$

$= 25k - 15 - 3$

$a_3 = 25k - 18$

$a_{3+1} = 5a_3 - 3$

$a_4 = 5(25k - 18) - 3$

$= 125k - 90 - 3$

$a_4 = 125k - 93$

b) $7 = 125k - 93$

$125k = 100$

$k = 0.8$