JACOBS UNIVERSITY BREMEN

Assignment #3

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Problem 3.1: distributive laws for sets

a) Claim: $A \cup (B \cap C) \equiv (A \cup B) \cap (A \cup C)$

Proof: *Proof by Construction*
Assume that $\exists x \in A \cup (B \cap C)$ and $\exists y \in (A \cup B) \cap (A \cup C)$.
Let $L \equiv A \cup (B \cap C)$ and $R \equiv (A \cup B) \cap (A \cup C)$.

$$\begin{array}{ll}
x \in A \cup (B \cap C) & y \in (A \cup B) \cap y \in (A \cup C) \\
x \in A \vee x \in (B \cap C) & y \in (A \cup B) \wedge y \in (A \cup C) \\
x \in A \vee \{x \in B \wedge x \in C\} & \{y \in A \vee y \in B\} \wedge \{y \in A \vee y \in C\} \\
\{x \in A \vee x \in B\} \wedge \{x \in A \vee x \in C\} & y \in A \vee \{y \in B \wedge y \in C\} \\
x \in (A \cup B) \wedge x \in (A \cup C) & y \in A \vee y \in (B \cap C) \\
x \in (A \cup B) \cap x \in (A \cup C) & y \in A \cup (B \cap C) \\
\Rightarrow (\forall x.\ x \in L \Rightarrow x \in R) \Leftrightarrow L \subseteq R & \Rightarrow (\forall y.\ y \in R \Rightarrow y \in L) \Leftrightarrow R \subseteq L
\end{array}$$

$$\therefore (L \subseteq R) \wedge (R \subseteq L) \Leftrightarrow (L \equiv R) \Leftrightarrow (A \cup (B \cap C) \equiv (A \cup B) \cap (A \cup C))$$

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b) Claim: $A \cap (B \cup C) \equiv (A \cap B) \cup (A \cap C)$

Proof: *Proof by Construction*
Assume that $\exists x \in A \cap (B \cup C)$ and $\exists y \in (A \cap B) \cup (A \cap C)$.
Let $L \equiv A \cap (B \cup C)$ and $R \equiv (A \cap B) \cup (A \cap C)$.

$$\begin{array}{ll}
x \in A \cap (B \cup C) & y \in (A \cap B) \cup y \in (A \cap C) \\
x \in A \wedge x \in (B \cup C) & y \in (A \cap B) \vee y \in (A \cap C) \\
x \in A \wedge \{x \in B \vee x \in C\} & \{y \in A \wedge y \in B\} \vee \{y \in A \wedge y \in C\} \\
\{x \in A \wedge x \in B\} \vee \{x \in A \wedge x \in C\} & y \in A \wedge \{y \in B \vee y \in C\} \\
x \in (A \cap B) \vee x \in (A \cap C) & y \in A \wedge y \in (B \cup C) \\
x \in (A \cap B) \cup x \in (A \cap C) & y \in A \cap (B \cup C) \\
\Rightarrow (\forall x.\ x \in L \Rightarrow x \in R) \Leftrightarrow L \subseteq R & \Rightarrow (\forall y.\ y \in R \Rightarrow y \in L) \Leftrightarrow R \subseteq L
\end{array}$$

$$\therefore (L \subseteq R) \wedge (R \subseteq L) \Leftrightarrow (L \equiv R) \Leftrightarrow (A \cap (B \cup C) \equiv (A \cap B) \cup (A \cap C))$$

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Problem 3.2: reflexive, symmetric, transitive

a) $R = \{(a, b)|a, b \in \mathbb{Z} \wedge a \neq b\}$

1. reflexive: $a \neq a$ - a number cannot be different than itself $\Rightarrow (a, a) \notin R \Leftrightarrow a \nsim a$.
2. symmetric: $a \neq b \wedge b \neq a \Rightarrow (b, a) \in R \Leftrightarrow b \sim a$.
3. transitive: Let $(b, c) \in R \Leftrightarrow b \sim c$, such that $a \neq b \wedge b \neq c$. Therefore we are not sure if $a \neq c$, since there is a chance that $a = c \Rightarrow (a, c) \notin R \Leftrightarrow a \nsim c$.

$\therefore$ The relation is symmetric only, so it is not an equivalence relation.

b) $R = \{(a, b)|a, b \in \mathbb{Z} \wedge |a - b| \leq 3\}$

1. reflexive: $|a - a| \leq 3 \Rightarrow 0 \leq 3 \,.\, \forall a \Rightarrow (a, a) \in R \Leftrightarrow a \sim a$.
2. symmetric: $|a - b| = |b - a| \Rightarrow (b, a) \in R \Rightarrow b \sim a$.
3. transitive: Let $(b, c) \in R \Leftrightarrow b \sim c$, such that $c \in \mathbb{Z}$. Therefore we are not sure if $|a - b| + |b - c| = |a - c|$, since it holds if and only if $a > b > c$ or $a < b < c$. Therefore $(a, c) \notin R \Leftrightarrow a \nsim c$.

$\therefore$ The relation is reflexive and symmetric only, so it is not an equivalence relation.

c) $R = \{(a, b)|a, b \in \mathbb{Z} \wedge (a \bmod 10) = (b \bmod 10)\}$

1. reflexive: $a \bmod 10 - a \bmod 10 = 0 \Rightarrow (a, a) \in R \Leftrightarrow a \sim a$.
2. symmetric: $a \bmod 10 - b \bmod 10 = b \bmod 10 - a \bmod 10 \Rightarrow (b, a) \in R \Rightarrow b \sim a$.
3. transitive: Let $(b, c) \in R \Leftrightarrow b \sim c$, such that $c \in \mathbb{Z}$.
   $(a \bmod 10 - b \bmod 10) + (b \bmod 10 - c \bmod 10) = a \bmod 10 - c \bmod 10$.
   Therefore $(a, c) \in R \Leftrightarrow a \sim c$.

$\therefore$ The relation is reflexive, symmetric and transitive, so it is an equivalence relation.

Problem 3.3: circular prime numbers (haskell)

```
prime :: Integer -> Bool
prime x
  |x < 0 = prime (-x) --implementing the condition for negative numbers
  |x < 2 = False
  |length(filter (\n -> if mod x n > 0 then False else True)
          [2..(x-1)]) == 0 = True   -- should be on previous line
  |otherwise = False
-- lambda function which go through the numbers from
-- 0 to (number-1) to check if it is divisible by something
-- for example P(4) = [2, 3] and (4 mod 2)==0 == "Not prime" (False) (True)


rotate :: Int -> [a] -> [a]
```

```haskell
rotate _ [] = []
rotate 0 n = n
rotate n (x:xs) = rotate(n-1) (xs ++ [x])

circle :: [a] -> [[a]]
circle n = map (\x -> rotate x n) [0..length(n)-1]

--functions from the previous assignment

circprime :: Integer -> Bool
circprime x
  |x < 0 = circprime(-x)
  |prime x == False = False
  -- there is not a non-prime number which is a circular prime number
  |length(filter (\n -> prime ((read::String->Integer) n))(circle (show x)))
          == length(show x) = True   -- should be on previous line
  |otherwise = False

-- the lambda function converts the type of the input
-- circular prime: if it is possible to create a circle and remain prime
```