Assignment #2

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Problem 2.1: proof by contrapositive

Claim: Given $n \in \mathbb{N}$. If n is not divisible by 3, thus n is also not divisible by 15.

Proof: *Proof by Contrapositive*
Let us assume that $\exists n = 15l \Leftrightarrow n = 3(5l) \Leftrightarrow n = 3k \Leftrightarrow$ such that $l, k \in \mathbb{N}$. Therefore n is divisible by 3.
$\therefore$ By the principle of contrapositive($\neg Q \Rightarrow \neg P$; $P \Rightarrow Q$), n is not divisible by 3 $\forall n \in \mathbb{N}$.

$\blacksquare$

Problem 2.2: proof by induction

Claim: Given $n \in \mathbb{N}$. Then:

$$1^2 + 3^2 + 5^2 + ...(2n-1)^2 = \sum_{k=1}^{n}(2k-1)^2 = \frac{2n(2n-1)(2n+1)}{6}$$

Proof: *Proof by Induction*
Base case (n = 1): $1^2 = \frac{2\cdot 1(2\cdot 1-1)(2\cdot 1+1)}{6} = 1$
Inductive Step:

$$\begin{aligned}
\sum_{k=1}^{m}(2k-1)^2 + (2(k+1)-1)^2 &= \frac{2m(2m-1)(2m+1)}{6} + (2(m+1)-1)^2 \\
&= \frac{(2m^2-m)(2m+1)}{3} + \frac{3(2m+1)^2}{3} \\
&= \frac{(2m^2-m+6m+3)(2m+1)}{3} \\
&= \frac{(2m^2+2m+3m+3)(2m+1)}{3} \\
&= \frac{(2m(m+1)+3(m+1))(2m+1)}{3} \\
&= \frac{(2m+3)(m+1)(2m+1)}{3} \\
&= \frac{2(m+1)(2(m+1)-1)(2(m+1)+1)}{6}
\end{aligned}$$

$\therefore$ By the principle of induction, the claim holds for all $n \in \mathbb{N}$. $\blacksquare$

Problem 2.3: rotate a list and produce all possible rotations of a list (haskell)

```haskell
rotate :: Int -> [a] -> [a]
rotate _ [] = []
rotate 0 n = n
rotate n (x:xs) = rotate(n-1) (xs ++ [x])
-- separate the head from the tail so that I can move head's position


circle :: [a] -> [[a]]
circle n = map (\x -> rotate x n) [0..length(n)-1]
-- replace every free occurrence of the variable with the parameter value
-- map shows every position of the head
```