Assignment #7

Student name: *Ivan Kabadzhov*

Course: *Introduction to Computer Science*
Date: *November, 2018*

Problem 7.1: quine-mccluskey algorithm

a) As we have given the following minterms in order to find the prime implicants: We classify the minterms by the number of positive literals (PI-1), compare the minterms, as we mark if there is a difference of only one bit position and combine those terms (PI-2). We repeat this step until we arrive at the unmarked implicants (PI-3) and those unmarked implicants we call prime implicants (PI-4).

	minterm	pattern	minterm	pattern	minterm	pattern
0 positive	m_0	0 0 0 0 0	$m_{0,2}$	0 0 0 - 0	$m_{0,2,4,6}$	0 0 - - 0
			$m_{0,4}$	0 0 - 0 0	$m_{0,4,2,6}$	0 0 - - 0
			$m_{0,16}$	- 0 0 0 0		
1 positive	m_2	0 0 0 1 0	$m_{2,6}$	0 0 - 1 0	$m_{2,6,10,14}$	0 - - 1 0
	m_4	0 0 1 0 0	$m_{2,10}$	0 - 0 1 0	$m_{2,10,6,14}$	0 - - 1 0
	m_6	1 0 0 0 0	$m_{4,6}$	0 0 1 - 0		
			$m_{16,17}$	1 0 0 0 -		
2 positive	m_6	0 0 1 1 0	$m_{6,14}$	0 - 1 1 0	$m_{10,14,26,30}$	- 1 - 1 0
	m_9	0 1 0 0 1	$m_{9,13}$	0 1 - 0 1	$m_{10,26,14,30}$	- 1 - 1 0
	m_{10}	0 1 0 1 0	$m_{10,14}$	0 1 - 1 0		
	m_{17}	1 0 0 0 1	$m_{10,26}$	- 1 0 1 0		
			$m_{17,21}$	1 0 - 0 1		
3 positive	m_{13}	0 1 1 0 1	$m_{13,15}$	0 1 1 - 1	$m_{14,30,15,31}$	- 1 1 1 -
	m_{14}	0 1 1 1 0	$m_{14,15}$	0 1 1 1 -	$m_{14,15,30,31}$	- 1 1 1 -
	m_{21}	1 0 1 0 1	$m_{14,30}$	- 1 1 1 0		
	m_{26}	1 1 0 1 0	$m_{26,30}$	1 1 - 1 0		
	m_{28}	1 1 1 0 0	$m_{28,30}$	1 1 1 - 0		
4 positive	m_{15}	0 1 1 1 1	$m_{15,31}$	- 1 1 1 1		
	m_{30}	1 1 1 1 0	$m_{30,31}$	1 1 1 1 -		
5 positive	m_{31}	1 1 1 1 1				

I coloured in green the matching monomials, and marked one bit difference with a dash. After repeating this steps I got my prime monomials and I see that 4 of them are repeating, so I just coloured them in gray, as it is pointless to rewrite the same expression twice (from equivalence laws $X \lor X = X$).

Therefore there are 10 prime implicants:

1. $m_{0,16} = (\neg B \land \neg C \land \neg D \land \neg E)$

2. $m_{16,17} = (A \land \neg B \land \neg C \land \neg D)$
3. $m_{9,13} = (\neg A \land B \land \neg D \land E)$
4. $m_{17,21} = (A \land \neg B \land \neg D \land E)$
5. $m_{13,15} = (\neg A \land B \land C \land E)$
6. $m_{28,30} = (A \land B \land C \land \neg E)$
7. $m_{0,2,4,6} = (\neg A \land \neg B \land \neg E)$
8. $m_{2,6,10,14} = (\neg A \land D \land \neg E)$
9. $m_{10,14,26,30} = (B \land D \land \neg E)$
10. $m_{14,15,30,31} = (B \land C \land D)$

b) Finding the essential prime implicants is expressing the minimum coverage of the remaining prime implicants in the special table (MS-1, MS-2). Columns that have only

	m_0	m_2	m_4	m_6	m_9	m_{10}	m_{13}	m_{14}	m_{15}	m_{16}	m_{17}	m_{21}	m_{26}	m_{28}	m_{30}	m_{31}
$m_{0,16}$	✓									✓						
$m_{16,17}$										✓	✓					
$m_{9,13}$					✓		✓									
$m_{17,21}$											✓	✓				
$m_{13,15}$							✓		✓							
$m_{28,30}$														✓	✓	
$m_{0,2,4,6}$	✓	✓	✓	✓												
$m_{2,6,10,14}$		✓		✓		✓		✓								
$m_{10,14,26,30}$						✓		✓					✓		✓	
$m_{14,15,30,31}$								✓	✓						✓	✓

one marked cell, marked with dark green, indicate essential prime implicants. The non-essential prime implicants are shown in red, as they are not needed to express the Boolean function. And we can choose one of the blue prime implicants because there is no other way to express m_{16}. But it makes no sense to express both of them as prime implicants at the same time.

Consequently, there are 7 essential prime implicants, which can be either:

1. $m_{0,16} = (\neg B \land \neg C \land \neg D \land \neg E)$
2. $m_{9,13} = (\neg A \land B \land \neg D \land E)$
3. $m_{17,21} = (A \land \neg B \land \neg D \land E)$
4. $m_{28,30} = (A \land B \land C \land \neg E)$
5. $m_{0,2,4,6} = (\neg A \land \neg B \land \neg E)$
6. $m_{10,14,26,30} = (B \land D \land \neg E)$
7. $m_{14,15,30,31} = (B \land C \land D)$

or

1. $m_{16,17} = (A \land \neg B \land \neg C \land \neg D)$
2. $m_{9,13} = (\neg A \land B \land \neg D \land E)$
3. $m_{17,21} = (A \land \neg B \land \neg D \land E)$
4. $m_{28,30} = (A \land B \land C \land \neg E)$
5. $m_{0,2,4,6} = (\neg A \land \neg B \land \neg E)$
6. $m_{10,14,26,30} = (B \land D \land \neg E)$
7. $m_{14,15,30,31} = (B \land C \land D)$

c) We can write now the minimal expression in one of these two different forms:

$$\begin{aligned}\varphi'(A,B,C,D,E) = &(\neg B \land \neg C \land \neg D \land \neg E) \lor (\neg A \land B \land \neg D \land E) \lor (A \land \neg B \land \neg D \land E) \lor \\ &\lor (A \land B \land C \land \neg E) \lor (\neg A \land \neg B \land \neg E) \lor (B \land D \land \neg E) \lor (B \land C \land D)\end{aligned}$$

$$\begin{aligned}\varphi''(A,B,C,D,E) = &(A \land \neg B \land \neg C \land \neg D) \lor (\neg A \land B \land \neg D \land E) \lor (A \land \neg B \land \neg D \land E) \lor \\ &\lor (A \land B \land C \land \neg E) \lor (\neg A \land \neg B \land \neg E) \lor (B \land D \land \neg E) \lor (B \land C \land D)\end{aligned}$$

We conclude that both of those expressions contain 24 operations($\land, \lor$). Whereas the initial expression φ used $16 \cdot 4 + 15 = 79$ operations.