Lecture 30

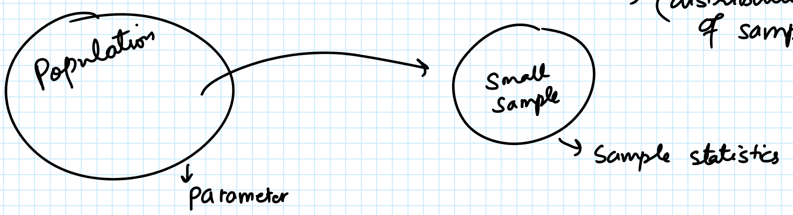
02 November 2022 11:20

Chapter 8

Sampling Distribution

→ (distribution of sample statistics)

Aim :-



Example A cold drink dispensing machine is suppose to dispense 240 ml of drink per dispense.

We may collect samples to see if machine is functioning well or not.

Let us fix the sample size say 40.

		mean
1^{st} trial :-	$x_{1,1}$ $x_{1,2}$ $x_{1,3}$ . . . $x_{1,40}$	$\bar{x}_1$
2^{nd} trial :-	x_{21} x_{22} x_{23} . . . x_{240}	$\bar{x}_2$
⋮		⋮
m^{th} trial :-	x_{m1} x_{m2} x_{m3} . . . x_{m40}	$\bar{x}_{40}$

→ $\bar{X}$ → sample mean & it is a r.v.

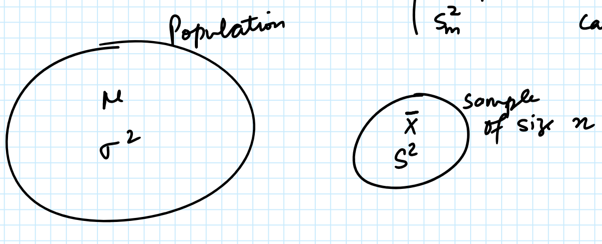
→ These are random numbers. The distribution is called sampling distribution of mean.

variance
s_1^2
s_2^2
⋮
s_m^2

We can also have Sampling distribution of Variance.

→ s^2 is a r.v. and its distribution is called Sampling distribution of variance.

Sim



Next task is

Find distribution of $\bar{X}$ if you have some information about the population.

We have the following results:-

If the original population is normally distributed ($\to N(\mu,\sigma)$) with mean $= \mu$ & Variance $= \sigma^2$ then $\bar{X}$ which is sampling distribution of mean with sample size n has $N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$.

Que The annual income of employees in an industry follows normal distribution with $\mu = 4$ lakhs & variance $= 1$ lakh. A random sample of 49 is taken. What is the probability that sample mean is greater than 4.25 lakhs?

Soln:- Population — $N(4, 1)$

than 4.25 lakhs?

Sol^n Population $- N(4, 1)$

$\therefore \bar{X} \sim N\left(4, \frac{1}{\sqrt{49}}\right)$

Now we are looking for

$P(\bar{x} > 4.25)$

$= 1 - P(\bar{x} \le 4.25)$

Normal

$\bar{z} = \frac{\bar{x} - 4}{1/\sqrt{49}}$

$= 1 - P(\bar{z} \le 1.75)$

From table

$= 1 - 0.9599$

$= 0.0401.$

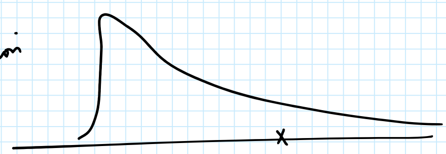
If the original population is not normally distributed, then we have the following result (Central limit theorem CLT)

CLT:- If $\bar{X}$ is the mean of a random sample of size n taken from a population with mean μ & variance σ^2, then $\bar{X}$ follows $N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$ as $n \to \infty$.

Conclusion For population normally distribution every n (sample size) will work.

For population not normally distributed it has been observed that $n \ge 30$ works provided the population is not skewed badly.

population



Que An electrical firm manufactures light bulbs that have a length of life approximately normally distributed with $\mu = 800$ hours & S.D $= 40$ hours. Find the probability that a random sample of 16 bulbs have mean ≤ 775 hours.

Sol^n Population $\sim N(800, 40)$

Sample mean $\bar{x} \sim N\left(800, \frac{40}{\sqrt{16}}\right) = N(800, 10)$

$P(\bar{x} \le 775) \longrightarrow 0.0062.$

Que A manufacturing unit manufactures a cylinderical part which is supposed to have diameter = 5 mm.

An engineer claims that the mean of population is 5 mm & S.D. = 0.1 mm

Q An experiment is then performed by choosing 100 parts randomly & their mean was = 5.027.

Does this experiment support or refute engineer's claim?

Sol^n <u>Idea</u> Assume that engineer's hypothesis is true then we will find the probability of getting $\bar{X} = 5.027$ (or something like that)

If this probability is high then experiment supports engineer

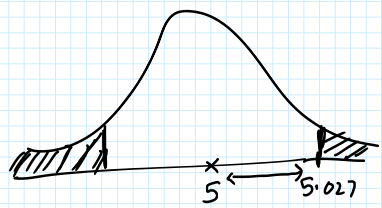
& if it is low " rejects "

Now according to engineer population $\sim N(5, 0.1)$

$$\downarrow \text{CLT}$$

$$\bar{X} \sim N\left(5, \frac{0.1}{\sqrt{100}}\right)$$

$$N(5, \underline{0.01})$$



What is the probability that experiments mean is 0.027 away from 5

= Shaded area

$= 2\,P(\bar{X} \geq 5.027) \rightarrow 2(1 - P(\bar{X} \leq 5.027))$

$= 0.007$

very low

$\therefore$ Experiment is refuting engineer's claim