Chapter 5

Some discrete Probability distributions

	Situation	pdf	Mean	Variance (with Proofs)	Parameters
① Binomial	Do write	Do write	np	$np(1-p)$	n, p, x
② Hypergeometric	Do write	Do write	$\frac{nK}{N}$	$\frac{N-n}{N-1} \cdot n \cdot \frac{K}{N}\left(1-\frac{K}{N}\right)$	n, N, K

without proofs.

③ negative Binomial

④ Geometric

⑤ Poisson

Note :- A Hypergeometric situation with large N and very small n, can be approximated by a binomial situation (whenever $\frac{n}{N} < 0.05$, you can always approximate the Hypergeometric situation with binomial)

Negative Binomial

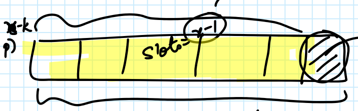
Situation

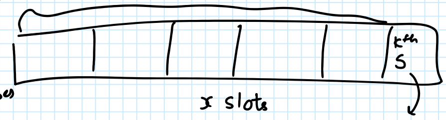
- Independent Trials
- Each trials results in a success or a failure → S → p, F → $1-p$
- p remains constant.
- What is the probability that k^{th} success occurs at x^{th} trial.

This kind of distribution is called a negative binomial distribution.

pdf $= b^*(x, k, p) =$

$^{x-1}C_{k-1}\, p^{k-1}\, (p)\, (1-p)^{x-k}$





x-1 slots I want k-1 successes

$^{x-1}C_{k-1}\, p^{k-1}\, \underbrace{(1-p)^{x-k}}$ (p)

Probability of failure at x-k slots.

$$\boxed{b^*(x, k, p) = {}^{x-1}C_{k-1}\, p^{k}\, (1-p)^{x-k}}$$

Que In a series of 7 matches of cricket, probability that team A wins → 0.55
B win → 0.45

→ To win this series, a team has to win 4 matches

What is the probability that team A wins in 5 matches.

Sol^n I am looking for team A's 4^{th} win in 5^{th} match.

$$b^*(5, 4, 0.55) = {}^4C_3\, (0.55)^4\, (1-0.55)^1$$

Ans.

What is mean & variance?

Mean: $\frac{k(1-p)}{p}$ Variance: $\frac{k(1-p)}{p^2}$ (without proofs)

Geometric distribution

If $k=1$ in negative binomial : We want 1^{st} success to occur at x^{th} trial.

$$b^*(x,k,p) = b^*(x,1,p)$$

$\downarrow k=1$

$$^{x-1}C_0\, p^{k=1} (1-p)^{x-1}$$

$1-p=q$

pdf for geometric distribution

$$\boxed{g(x,p) = p q^{x-1}}$$

$p,\ pq,\ pq^2,\ pq^3 \ldots$ forms a geometric sequence.

$x = 1,2,3,\ldots$

Que For a certain manufacturing process, it is known that, on the average, 1 in every 100 items is defective. What is the probability that the fifth item inspected is the first defective to be found?

Sol^n

item → defective $p = 0.01$
item → non defective $1-p = 1-0.01$

$$p(1-p)^{5-1} = g(5,p)$$

We want our first defective to be at 5^{th} trial.

$0.01\,(0.99)^4 = 0.0096$ Ans.

Mean $\frac{1-p}{p}$ Variance $\frac{1-p}{p^2}$

Binomial $(p+q)^n$

Negative binomial ? Why the name

$$b^*(x,k,p) = {}^{x-1}C_{k-1}\, p^k\, q^{x-k}$$

$q = 1-p$ } Think about it

$$p^k(1-q)^{-k}$$

$$p^k\left(1 + qk + \frac{k(k+1)}{2!}q^2 \ldots\right)$$

Look for a series whose expansion gives you these terms.

$$(1-x)^{-n} = 1 + nx + \frac{n(n+1)x^2}{2!} + \ldots$$

Remember that in negative binomial $x = k, k+1, k+2, \ldots$

Que At a "busy time" a telephone exchange is very near its capacity, so the callers have difficulty placing their calls. It may be of interest to know number of attempts necessary in order to make a connection. Suppose that we let $p = 0.05$ be the probability of connection during such an hour. We are interested in knowing the probability that 5 attempts are required for a successful call.

our first to occur at 5th trial

Sol^n

$$g(5, 0.05) = 0.05(1-0.05)^4 \text{ Ans.}$$

Poisson distribution

MST

Negative Binomial pdf

$$b^*(x,k,p) = {}^{x-1}C_{k-1}\, p^k\, q^{x-k}$$

$x = k, k+1, k+2, \ldots$

$b^*(k,k,p) = p^k$

$b^*(k+1,k,p) = k p^k q$

$\vdots$

$$p^k(1-q)^{-k} \rightarrow p^k\left(1 + kq + \frac{k(k+1)}{2}q^2 + \text{------}\right)$$

$x = k, k+1, \ldots$

$$p^k (1-q)^{-k} \longrightarrow p^k \left(1 + kq + \frac{k(k+1)}{2} q^2 + \cdots\cdots \right)$$