Recap

Binomial process :- n trials (independent)

Trial $<$ S (p), F $(1-p)$

$X =$ no. of success is a Binomial r.v.

$X = 0, 1, 2, \ldots n$

pdf $P(X = x) = {}^nC_x \, p^x \, (1-p)^{n-x} = b(x; n, p)$

cdf $\boxed{B(r; n, p)} = \sum_{x=0}^{r} b(x; n, p)$

Tables

Que A large chain retailer purchases a certain kind of electronic device from a manufacturer. The manufacturer indicates that the defective rate of the device is 3%.

(a) The quality inspector randomly picks 20 items from a shipment. What is the probability that there will be at least one defective item in these 20 picked items?

Sol^n

$n = 20$
$p = 0.03$

$P(X \geq 1) = 1 - P(X = 0)$

$= 1 - {}^{20}C_0 (0.03)^0 (1-0.03)^{20}$

$= 0.4562$ Ans.

$B(20, 20, 0.03) - B(0, 20, 0.03)$

H.W.
(b) Suppose that the retailer receives 10 shipments in a month and the inspector randomly tests 20 devices per shipment. What is the probability that there are exactly 3 shipments containing at least 1 defective piece in the 20 selected pieces.

(Ans. 0.1602)

Mean and Variance of Binomial r.v.

① $E(X) = np$ & ② $\sigma^2 = npq$

Where $n \to$ no. of trials

$p \to$ probability of success

$q \to 1-p$

Proof $E(X) = \sum_{x=0}^{n} x\, b(x,n,p) = \sum_{x=0}^{n} x \; {}^nC_x\, p^x (1-p)^{n-x}$

$= np$

Simpler way

write $X = \underbrace{I_1 + I_2 + I_3 + \cdots + I_n}$

where I_1 is result of first trial, it will take value 1 for success, 0 for failure

$I_j = \begin{cases} 1 & \text{if } j^{th} \text{ trial is success with probability } p \\ 0 & \text{if '' '' '' failure with '' } 1-p \end{cases}$

$E(I_j) = 1 * p + 0 * (1-p) = p$

$E(X) = E(I_1 + I_2 + \cdots + I_n) = E(I_1) + E(I_2) + \cdots + E(I_n)$

$= (1\cdot p + 0(1-p)) + (1\cdot p + 0(1-p)) + \cdots + (1\cdot p + 0(1-p))$

$= \underbrace{p + p + \cdots + p}_{n \text{ times}}$

$\boxed{E(X) = np}$

Next

$\boxed{\sigma^2_{aX+bY} = a^2\sigma_X^2 + b^2\sigma_Y^2 + 2ab\,\sigma_{XY}}$

$X = I_1 + I_2 + \cdots + I_n$

$\sigma_X^2 = \sigma^2_{1I_1 + 1I_2 + \cdots + I_n}$

$= \underbrace{1^2\sigma^2_{I_1} + 1^2\sigma^2_{I_2} + \cdots + 1^2\sigma^2_{I_n}}$ ($\because I_1, I_2 - I_n$ are independent)

$I_j = \begin{cases} 1 & p \\ 0 & 1-p \end{cases}$ $\quad E(I_j) = p$

using *

$\sigma^2_{I_j} = E(I_j^2) - (E(I_j))^2$

$= (1^2 p + 0^2(1-p)) - (p)^2$

$= p - p^2 = p(1-p)$ *

$\boxed{\sigma_X^2 = np(1-p)}$ ✓

Que The probability that a patient recovers from a blood disease is 0.4. If 15 people are known to have contracted this disease &

$X =$ no. of people recovered

then find mean & variance of X. Use chebyshev theorem to interpret the interval $\mu \pm 2\sigma$.

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Soln This is a binomial random process with $n = 15$ & $p = 0.4$

$$\therefore \mu = np = 15 * 0.4 = 6$$

$$\sigma^2 = np(1-p) = 15 * 0.4 * 0.6 = 3.6$$

$$\sigma = \sqrt{3.6} = 1.897$$

Chebyshev Given a r.v. X with mean μ & s.d. σ

$$P(\mu - k\sigma \le X \le \mu + k\sigma) \geq 1 - \frac{1}{k^2}$$

$$P(\mu - 2\sigma \le X \le \mu + 2\sigma) \geq 1 - \frac{1}{2^2} = 3/4$$

$$P(6 - 2 * 1.897 \le X \le 6 + 2 * 1.897) \geq 3/4$$

$$P(2.206 \le X \le 9.794) \geq 3/4$$

Probability that 3 to 9 people will recover $\geq 3/4$

Hypergeometric r.v.

Situation

- N items
- K are good items/desirable items K Bad items N - K
- You are choosing n items
- $X \rightarrow$ no. of good items chosen in these n no. of items. $\rightarrow$ a Hypergeometric random variable (why this name)
- What are possible values for ~~X~~ $= 0, 1, 2, \ldots n$

& pdf $P(X = x) = \dfrac{\text{Favourable no. of ways}}{\text{Total no. of ways}}$

pdf $P(X = x) = \dfrac{{}^{K}C_{x} * {}^{N-K}C_{n-x}}{{}^{N}C_{n}}$ — x good items

Result Mean $\mu = \dfrac{nK}{N}$ Variance $= \dfrac{N-n}{N-1} * n * \dfrac{K}{N}\left(1 - \dfrac{K}{N}\right)$

(without proof)

Problems Next class