Recap

Population

$\mu \quad \sigma$

Sample of size n

$\left.\begin{array}{l} \bar{x}_1 \\ \bar{x}_2 \\ \bar{x}_3 \\ \vdots \end{array}\right\} \rightarrow$ Sampling distribution of mean

$\text{CLT} \searrow N\left(\mu, \dfrac{\sigma}{\sqrt{n}}\right)$

$\left.\begin{array}{l} S_1^2 \\ S_2^2 \\ \vdots \end{array}\right\} \rightarrow$ Sampling distribution of variance.
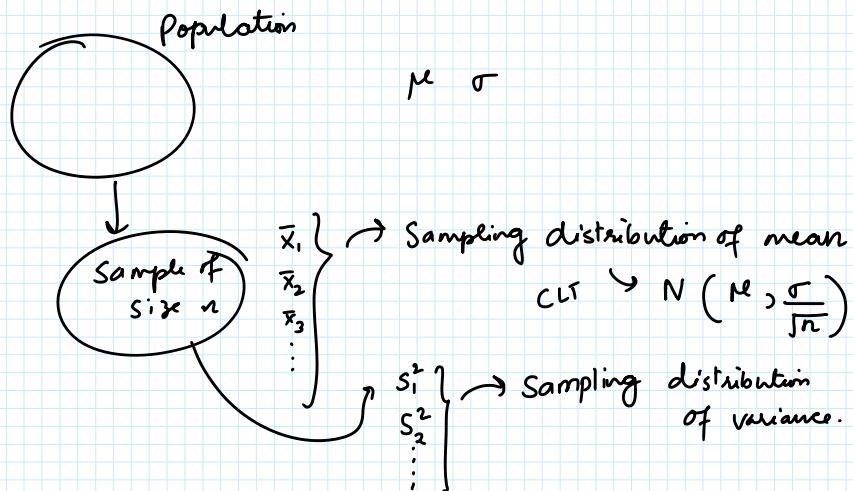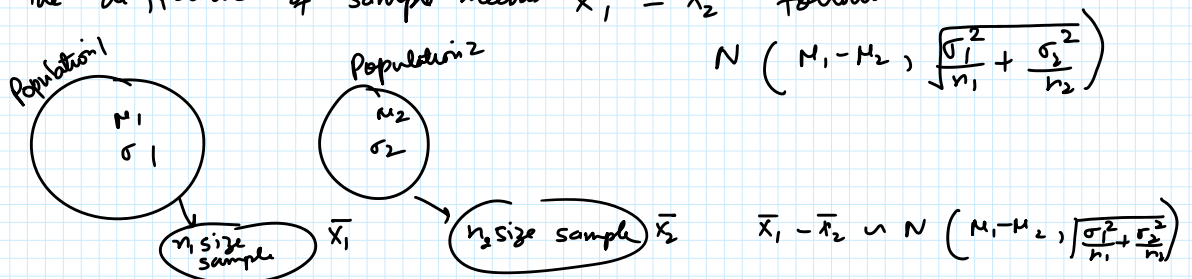
Today's lecture

Sampling distribution of difference b/w two means

If independent samples of sizes n_1 & n_2 are drawn at random from two populations (discrete or continuous) with mean μ_1 & μ_2

S.D. σ_1 & σ_2

then the difference of sample means $\bar{X}_1 - \bar{X}_2$ follows

$$N\left(\mu_1 - \mu_2, \sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}\right)$$

Population 1

μ_1
σ_1

n_1 size sample $\bar{X}_1$

Population 2

μ_2
σ_2

n_2 size sample $\bar{X}_2$

$\bar{X}_1 - \bar{X}_2 \sim N\left(\mu_1 - \mu_2, \sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}\right)$

Que The TV picture tubes are manufactured by two companies A & B

The data is as follows :-

life time	A	B
mean	6.5 years	6 years
S.D.	0.9 years	0.8 year

What is the probability that a random sample of ⟨36⟩ tubes from A will have a mean life time that is at least 1 year more than the mean life time of ⟨49⟩ tubes from B.

Sol^n

$\mu_1 = 6.5 \quad \sigma_1 = 0.9 \quad n_1 = 36$

$\mu_2 = 6 \quad \sigma_2 = 0.8 \quad n_2 = 49$

$\bar{X}_1 - \bar{X}_2 \sim N\left(6.5 - 6, \sqrt{\dfrac{0.9^2}{36} + \dfrac{0.8^2}{49}}\right) = N(0.5, 0.189)$

$$P(\bar{X}_1 - \bar{X}_2 \geq 1) = 1 - P(\underbrace{\bar{X}_1 - \bar{X}_2}_{} \leq 1)$$

$\left\{ \begin{array}{l} \\ \\ = 1 - P(w \leq 2.65) = 1 - 0.9960 = 0.0040 \end{array} \right.$

$w = \dfrac{z - 0.5}{0.189}$

Sampling distribution of variance

Some initial things :-

Chi Square distribution $\longrightarrow$ Special case of Gramma distribution

Can also be seen as $\longrightarrow$

$$\chi^2 = \sum_{i=1}^{v} Z_i^2$$

$Z_1, Z_2 \cdots Z_v$ are Standard normal distribution

Which has pdf

$$f(x; v) = \begin{cases} \dfrac{x^{v/2 - 1} \, e^{-x/2}}{2^{v/2} \, \sqrt{v/2}} & x > 0 \\ \\ 0 & \text{(Otherwise)} \end{cases}$$
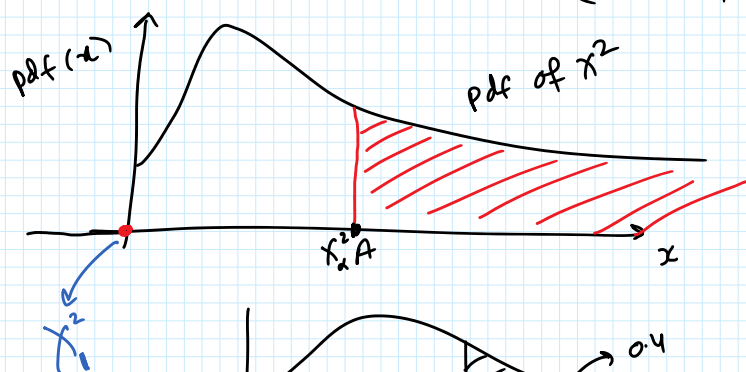
Here v is called degrees of freedom of Chi square distribution

Result If $\boxed{S^2}$ is the variance of a random sample of size $\boxed{n}$ taken from a normal population with variance $\boxed{\sigma^2}$ then the statistic $\chi^2 = \dfrac{(n-1)\, S^2}{\sigma^2}$ has a <u>Chi-square distribution</u> with $v = n-1$
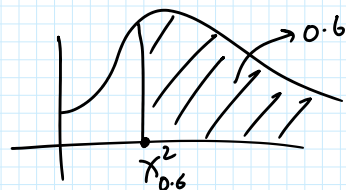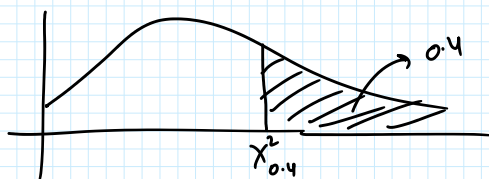
$v \to$ degrees of freedom (dof, df)

Example If a sample of size 10 is taken from a normal population with $\boxed{\text{variance} = 30}$ then $\dfrac{9 * S^2}{30}$ has Chi square distribution with $v = 9$.
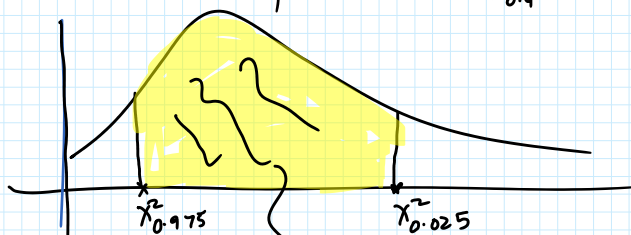

Next Let us see how to use χ^2 tables.

(End of chapter)

If red shaded is α, then the point is denoted by χ^2_α

pdf (x)

pdf of χ^2

$\chi^2_\alpha A$

x

χ^2_1

$\to 0.4$

$\chi^2_{0.4}$

$\to 0.6$

$\chi^2_{0.6}$

Next

$\chi^2_{0.975}$

$\chi^2_{0.025}$

$\chi^2_{0.975}$

$\chi^2_{0.025}$

This area = $0.975 - 0.025$

$$\overline{95\%}$$

That mean when a r.v. has Chi square distribution

95% of data lies b/w $\chi^2_{0.975}$ & $\chi^2_{0.025}$

<u>Home work</u>

df = 10   where   90% of data lies.
                          95% of data lies
                          10% of data lies

df = 9   Repeat

To pratice
Chi square
tables.