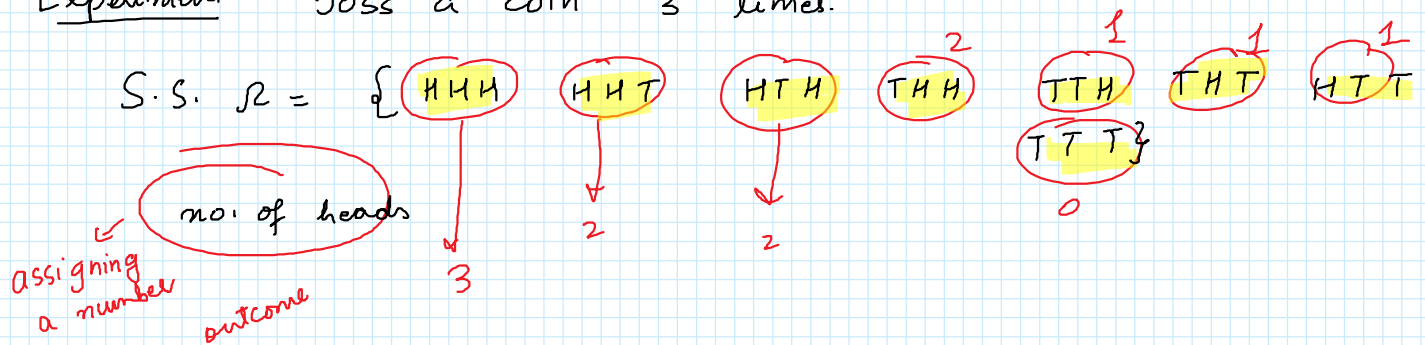


Chapter 3

Random variables & probability distributions

Random variable:- A variable with uncertain values.

Experiment Toss a coin 3 times.



$X = \text{no. of heads.}$

is a variable whose value is uncertain till we perform our experiment.

$\therefore X$ is a random variable.

Definition:- A random variable ^(R.V.) is a function whose domain is sample space of an uncertain experiment & whose range is set of real numbers.

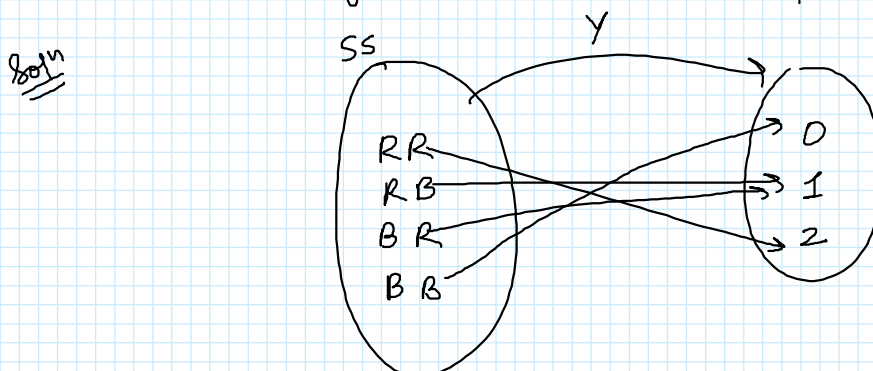
$$X: \underset{\text{Sample space}}{\Omega} \longrightarrow \underset{\substack{\text{set of real numbers} \\ (\text{mathbb notation})}}{\mathbb{R}}$$

Notation :- Use capital letters for denoting r.v.

Que Two balls are drawn in succession without replacement from a bag containing 4 red balls & 3 black balls.

$Y \rightarrow$ Number of red balls.

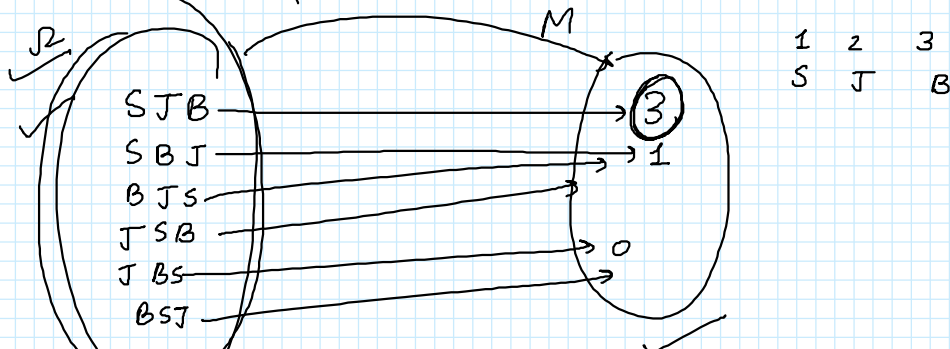
Can you show what Y is?



Que Three workers forgot their helmets in the order Smith, Jones, Brown. The guard returns the helmets in random orders.

$M \rightarrow$ number of correct matches. Show M .

Soln



Till now, the random variable was assuming finite number of values.

Que When a dice is rolled until a 5 occurs, then what $X \rightarrow$ number of rolls.

Soln

$N \rightarrow$ not a 5
 $F \rightarrow$ a five

$\Omega \rightarrow \{ F, NF, NNF, NNNF, \dots \}$

$\downarrow X$

1 2 3 4 5 - - - - -

Random variable is assuming ∞ number of values.

• What is the difference b/w a countable & uncountable set?

- all finite sets
- All sets similar
 $N \rightarrow$ set of natural numbers

\downarrow
 $[0, 1]$
 any interval is uncountable.

Definition

If your random variable assumes countable number of values, we call it discrete r.v.

All the above examples were discrete r.v.

Continuous R.v.

o- Any r.v. which assumes uncountable values.

X :- defining the waiting time

- Train arrival time (Suppose the train can be late max by 1 hour)

then X :- delay

$$X \in [0, 1]$$

Discrete r.v.

Coin Toss
Dice roll
⋮

Continuous r.v.

Temperature
Time
Heights

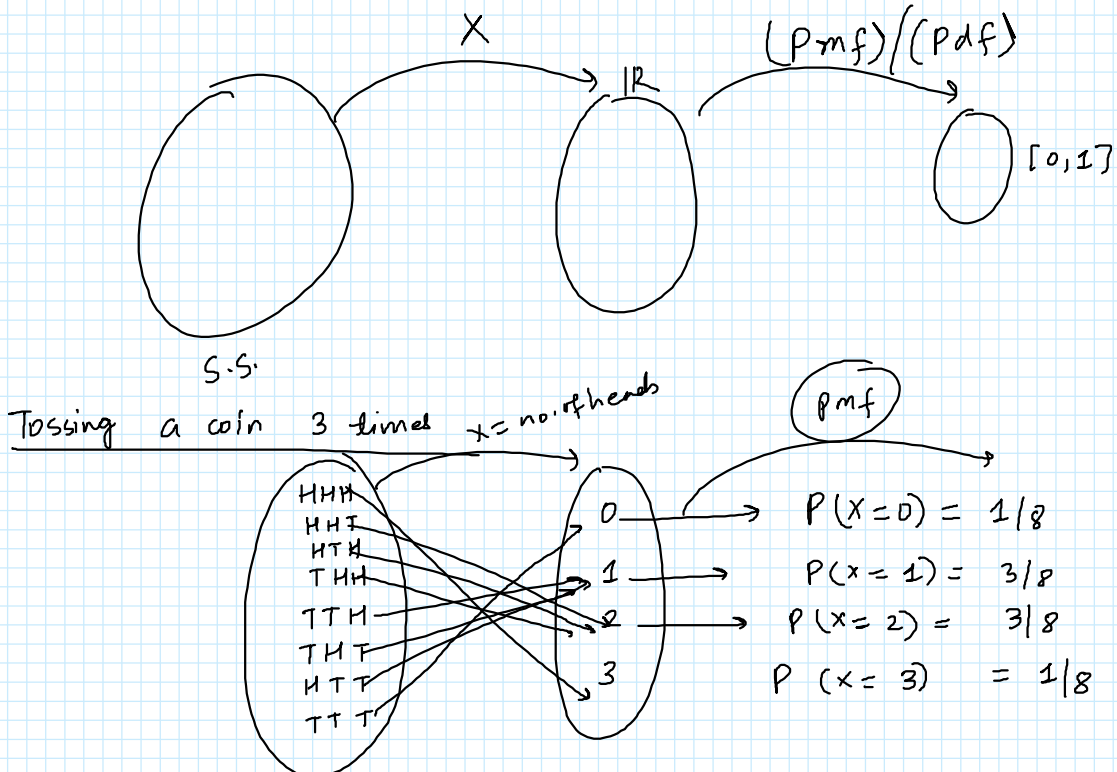
Discrete r.v.

① Probability mass function / probability function / probability distribution function (pdf)

$X \rightarrow$ r.v.

and $x \rightarrow$ possible value it can assume.

then $P(X=x)$, is a function subset of $\mathbb{R} \rightarrow [0, 1]$



Defⁿ

The set of pairs $(x, f(x))$ is a pmf of a discrete r.v. X if for each x

① $f(x) \geq 0$ ✓

③

$P(X=x) = f(x)$

n.v. 1 if for each x

① $f(x) \geq 0$ ✓

③

$P(X=x) = f(x)$

② $\sum_x f(x) = 1$ ✓

Que

A batch of 20 similar laptops contains 3 defective pieces. If a school makes a random purchase of two laptops. Find the pmf for number of defective laptops?

Solⁿ

$X \rightarrow$ no. of defective laptops

Possible values $\rightarrow 0, 1, 2$

$$P(X=0) \rightarrow \frac{{}^{17}C_2}{{}^{20}C_2}$$

$$P(X=1) \rightarrow \frac{{}^3C_1 * {}^{17}C_1}{{}^{20}C_2}$$

$$P(X=2) \rightarrow \frac{{}^3C_2}{{}^{20}C_2}$$

pmf.