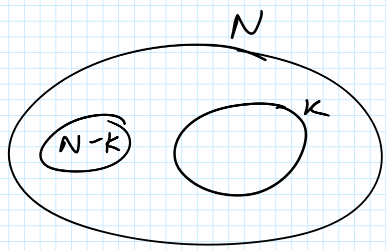
Chapter 5

Some discrete probability distribution

Name	What it is?	pdf	mean	variance
① Binomial	n trials independent; each trial: S (p), F ($1-p$); $X \to$ no. of success; p remains constant	$P(X=x) = {}^{n}C_{x}\, p^{x}(1-p)^{n-x}$	np	$np(1-p)$
② Hypergeometric	N objects, K are good, n are chosen; $X \to$ no. of good items chosen in n	$P(X=x) = \dfrac{{}^{K}C_{x}\ {}^{N-K}C_{n-x}}{{}^{N}C_{n}}$	$\dfrac{nK}{N}$	$\dfrac{N-n}{N-1}\, n\, \dfrac{K}{N}\left(1-\dfrac{K}{N}\right)$
③ Negative Binomial				
④ Geometric				
⑤ Poisson				

N very large, n very small: variance $\to np(1-p)$

N is large, n small: mean $\to np$



$$P(X=x) = {}^{K}C_{x}\ {}^{N-K}C_{n-x}$$

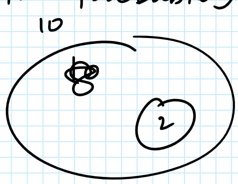
n object (Any)

$X \to$ what are good no. of object in these n objects.

Que A particular part that is used as an injection device is sold in a lots of 10. The producer deems a lot acceptable if no more than one defective is in the lot. A sampling plan involves random sampling and testing 3 of the parts out of 10. If none of the 3 is defective, the lot is accepted. Comment on the utility of this plan.

Sol^n Let us take a lot of 10 devices having 2 bad products (This lot will be rejected by producer)

Now What is the probability that the sampling plan will accept it.



we are choosing 3

$$P(X=0) = \frac{{}^{2}C_{0}\ {}^{8}C_{3}}{{}^{10}C_{3}}$$

$= 0.467$

46.7% chance of selection.

Note That

Note That

Hypergeometric case will be approximated with Binomial when

N is very large & n is very small.

$K \to$ no. of bad products

$p \to \frac{K}{N}$ probability of success

$1-p \to 1-\frac{K}{N}$ " " failure

N 2000 balls

n 3 balls select

1500 red

K 500 blue

If $\frac{n}{N} \le 0.05$, we always approximate hypergeometric with binomial

Que A manufacturer of automobile tyres reports that among a shipment of 5000 sent to a local distributor, 1000 are slightly blemished. If one purchases 10 of these tyres at random from the distributor; what is the probability that exactly 3 are bad?

Sol^n

$$P(X=3) \quad \frac{{}^{1000}C_3 \; {}^{4000}C_7}{{}^{5000}C_{10}}$$

$\frac{n}{N} = \frac{10}{5000}$

$= 0.02$

< 0.05

$\therefore$ approximate with binomial

~~b(3)~~

$b\left(3, 10, \frac{1000}{5000}\right)$

$= 0.2013$

Negative Binomial

Situation

① n trials independent $\to$ Trials can go on.

② each trial $\to$ ~~S~~ S $\to p$; $\to$ F $\to 1-p$

③ p remains constant

$\}$ $\to$ Same as Binomial Situation

In binomial you looked for x a no. of successes.

For example

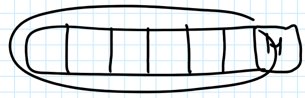
You toss a coin and I want that ~~5th head~~ 5th heads

In binomial you looked for x a no. of successes.

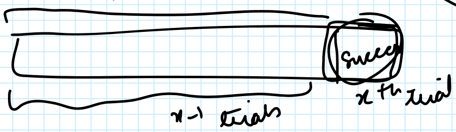
In negative binomial you are looking for the process where k^{th} success occurs ~~at~~ in x trials.

and I want that ~~5th head~~ 5^{th} heads at 7 trials

4 heads



$$P(k^{th} \text{ success occurs at } x^{th} \text{ trial}) = {}^{x-1}C_{k-1} (p^{k-1}) (p) \, q^{x-k}$$



$\boxed{x} \to \boxed{k}$ success

$\to x-k$ failure

pdf for negative binomial

$$b^*(x, k, p) = {}^{x-1}C_{k-1} \, p^k \, q^{x-k}$$

Probability that k^{th} success occurs at x^{th} trial.

$b(x, n, p)$ → probabilities of success; x → successes; n → trials

Mean $= \dfrac{k(1-p)}{p}$

Variance $\dfrac{k(1-p)}{p^2}$

Que In a basket ball championship series, the team that win (7 matches) four games out of seven is the winner. Suppose that teams A & B face each other in the game & $P(A \text{ winning}) = 0.55$

$P(B \text{ winning}) = 0.45$

(a) What is the probability that team A will win the series in 6 games.

$$b^*(6, 4, 0.55) = {}^{5}C_{3} (0.55)^4 (1-0.55)^{6-4}$$

(b) What is the probability that team A will win the series

$$b^*(4,4,0.55) + b^*(5,4,0.55) + b^*(6,4,0.55) + b^*(7,4,0.55)$$

(c) Now a team will win if it wins 3 out of 5 matches. What is the probability that team B will win?

Sol^n

$$b^*(3,3,0.45) + b^*(4,3,0.45) + b^*(5,3,0.45)$$