Chapter 6

Some continuous probability distributions.

(1) Uniform distribution / Rectangular

Experiment :- Waiting for a train which can be late for maximum 1 hour.

$X \to$ waiting time

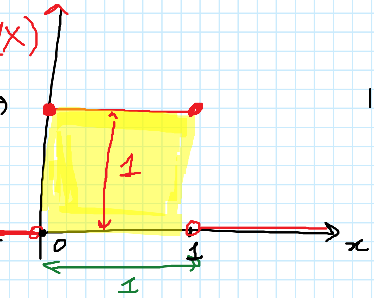
- What values X can assume?

  $[0, 1]$

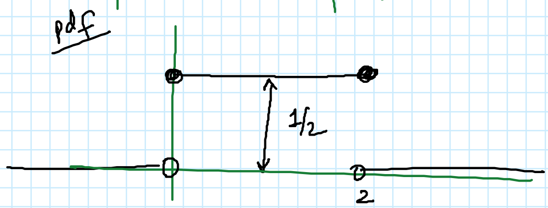
- with equal probabilities

$\Rightarrow$ pdf (X) will be constant function.

It looks like



- Train can be late for max. of two hours.

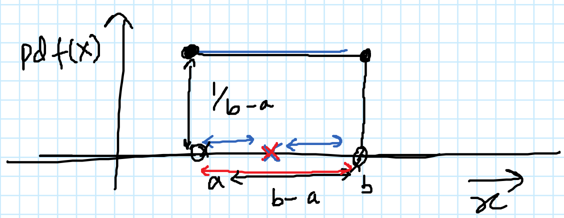


(Note area under pdf = 1 that gives us height in case of uniform distribution)

Definition

A r.v. X is said to be uniformly distributed on $[a, b]$ if its pdf looks like

$$\text{pdf}(x) = \begin{cases} \frac{1}{b-a} & a \le x \le b \\ 0 & \text{otherwise.} \end{cases}$$



- Task :- Think about real life situations where you find such a distribution.

- Mean for a uniform distribution $\rightarrow \left(\frac{a+b}{2}\right)$

punif
dunif

$$E(X) = \int_{-\infty}^{\infty} x \, \text{pdf}(x) dx = \int_a^b x \frac{1}{b-a} dx$$

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$$\frac{1}{b-a}\left[\frac{x^2}{2}\right]_a^b$$

$$\frac{1}{b-a}\left[\frac{b^2-a^2}{2}\right] = \frac{1}{\cancel{b-a}}\,\frac{\cancel{(b-a)}(b+a)}{2}$$

$$\boxed{\frac{a+b}{2}}$$

Variance of uniform distribution

$$= E(X^2) - (E(X))^2$$

$$= \int_a^b x^2 \frac{1}{b-a}\,dx - \left(\frac{a+b}{2}\right)^2 = \boxed{\frac{(b-a)^2}{12}}$$

Que A conference room can be booked for a maximum of 4 hours where booking hours are uniformly distributed.

If X = the booking hours

find ① pdf (X)? →
$$\begin{cases} \frac{1}{4-0} & 0 \le x \le 4 \\ 0 & \text{otherwise} \end{cases}$$

② what is the probability that on a given day the room is booked for at least 3 hours. → $P(X \geq 3) = 1 - P(X \leq 3)$

$$\int_3^4 pdf(x)\,dx$$

$$\int_3^4 \frac{1}{4}\,dx$$

$$= \boxed{1/4}$$

② Normal distribution (Most used distribution)

or Gaussian distribution

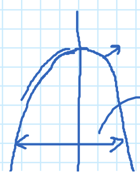
X

If I tell you that mean of a data set is μ

and variance is σ^2.

$\int_{-\infty}^{\infty}$

pdf of X should look like



This width will be relative to σ.

mean of a data set is 50

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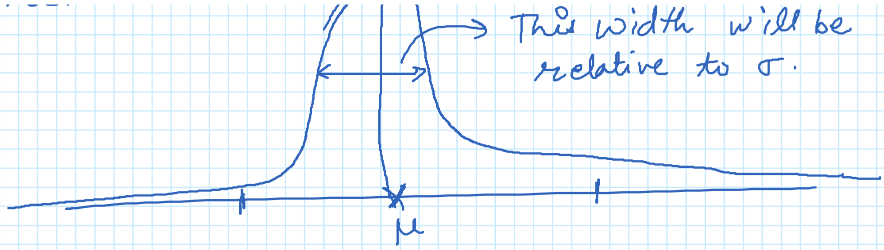
$P(X = 10^{10}) =$



Given a r.v. X with mean μ and S.D. σ, it is said to be normal if its pdf is the above bell shaped curve/ Gaussian curve.

$\boxed{n(x;\mu,\sigma)}$

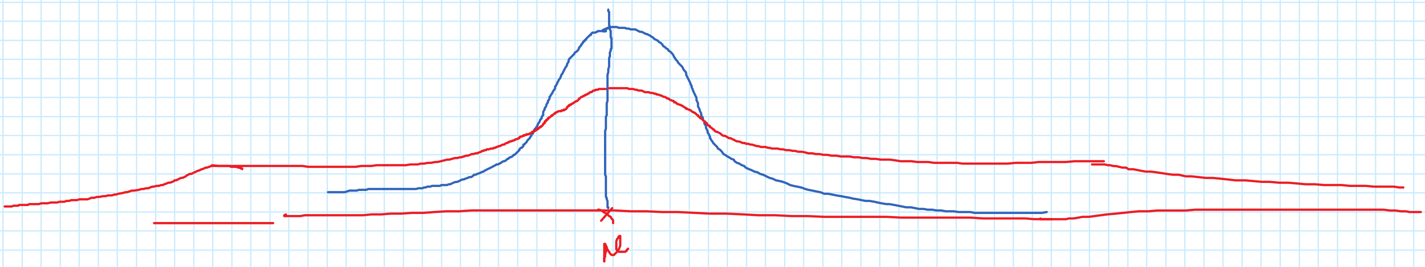
$\sum \frac{1}{n^2}$

It turns out that this curve has or pdf is following equation

$$\boxed{\frac{1}{\sqrt{2\pi}(\sigma)} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}} \rightarrow$$

If I have X_1 (normal distribution) with μ & S.D σ

X_2 (" ") with μ & S.D. 2σ



we will continue from here

$$\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx = \lim_{b\to\infty} \tan^{-1}x \Big]_{-b}^{b} = \lim_{b\to\infty} \tan^{-1}b - \tan^{-1}(-b)$$

$$\lim_{b\to\infty} 2\tan^{-1}b = 2\pi/4 = \pi/2$$

$\frac{1}{1+x^2}$