# Type I and Type II errors and Power of a Test

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Null Hypothesis $H_0$
Alternative Hypothesis $H_1$

| | $H_0$ is true | $H_1$ is true |
|---|---|---|
| Do not reject $H_0$ | Correct decision | Type II error |
| Reject $H_0$ | Type I error | correct decision |

Type I error :- Rejection of null Hypothesis when it is true

Type II error :- Non rejection of null Hypothesis when it is false.

- Prob. of committing Type I error is also the level of significance $\alpha$.
  $P(\text{Type I error}) = \alpha$.
- Prob. of Type II error is denoted by $\beta$.
- $1-\beta$ is called Power of the test.

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Example

- A certain vaccine is known to be only $\boxed{25\%}$ effective after a period of two years.
- To test a new vaccine, 20 people are given the vaccination.
- If more than 8 of those people surpass 2 years without catching the disease, we will consider new vaccine superior.

$H_0: p = 1/4$

$H_1: \boxed{p > 1/4}$

no of people with disease after 2 years who were given the new vaccine

$$\alpha = P(\text{Type I error}) = P(X > 8 \;\&\; p = 1/4)$$

$$= \sum_{x=9}^{20} b(x; 20, 1/4)$$

$$= 1 - \sum_{x=0}^{8} b(x; 20, 1/4)$$

cdf

$$= 1 - 0.9591$$

$$= 0.0409$$

Calculate $\beta$ if $\boxed{H_0: p = 1/4}$ & $H_1: p = 1/2$ ✓

Calculate $\beta$ if $\boxed{H_0: p = 1/4}$
& $H_1: p = 1/2$

$\beta = P(\text{Type II error}) = P(X \le 8 \ \& \ p = 1/2)$

$= \sum_{x=0}^{8} b(x, 20, 1/2) = 0.1316$

$\therefore$ power of test $= 1 - 0.1316 =$

Note
- For a fixed sample size, if $\alpha \uparrow$ $\beta$ will $\downarrow$ & vice versa.
- But you can $\downarrow$ both $\alpha$ & $\beta$ by $\uparrow$ sample size.

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Example: Consider the null hypothesis that the average weight of male students of a certain college is 68 Kg against the alternative hypothesis that it is not.

$H_0: \mu = 68$
$H_1: \mu \ne 68$

Let us assume critical region $= \bar{X} < 67$ or $\bar{X} > 69$
Acceptance " $= 67 \le \bar{X} \le 69$

$n = 36$
$\sigma = 3.6$

Calculate $\alpha$

$\alpha = P(\text{type I error}) = P(\bar{X} < 67 \ \& \ \mu = 68) + P(\bar{X} > 69 \ \& \ \mu = 68)$

$$= P\left(\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} < \frac{67 - 68}{3.6/\sqrt{36}}\right) + P\left(\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} > \frac{69 - 68}{3.6/\sqrt{36}}\right)$$

$= P(Z < -1.67) + P(Z > 1.67)$

$= 2P(Z < -1.67)$

$\alpha = \boxed{0.0950}$

Note with $\boxed{n = 64} \rightarrow \alpha = 0.0264$

$n = 64$ **Calculate $\beta$**

$H_0: \mu = 68$

$H_1: \mu = 70$

$\beta$ = Type II error = $P(67 \le \bar{X} \le 69$ when $\mu = 70)$

$$= P\left(\frac{67-70}{3.6/\sqrt{64}} \le Z \le \frac{69-70}{3.6/\sqrt{64}}\right)$$

$$= 0.0132$$