Lecture 23 (Gr 25-27)

10 October 2022 10:33

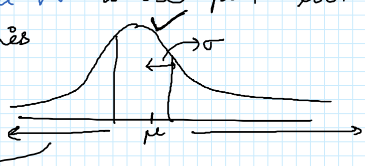
**Recap**

Chapter 6 (Some continuous distributions)

→ uniform distribution

→ Normal distribution

Given a r.v. whose pdf looks like this graph




is called a normal distribution

$$pdf(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

→ $n(x; \mu, \sigma)$

Today's work

**Result** :- Prove that for a r.v. X whose $pdf(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$, the

Mean $= \mu$ and S.D. $= \sigma$

**Proof**

$$E(X-\mu) = \int_{-\infty}^{\infty} (x-\mu)\, pdf(x)\, dx = \int_{-\infty}^{\infty} (x-\mu) \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

$E(X)$

$E(X) - E(\mu)$

$E(X) - \mu$

Put

$$\frac{x-\mu}{\sigma} = z$$

$$dx = \sigma\, dz$$

$$\int_{-\infty}^{\infty} (\sigma z) \frac{1}{\sqrt{2\pi}\,\cancel{\sigma}} e^{-\frac{1}{2}z^2} \cancel{\sigma} dz$$

$$\frac{\sigma}{\sqrt{2\pi}} \int_{-\infty}^{\infty} z e^{-\frac{1}{2}z^2} dz$$

odd function

$f(-z) = -z e^{-\frac{1}{2}z^2} = -f(z)$

$= 0$

$\int_{-a}^{a} f(x)\,dx$ — $0$ if $f$ is odd, $f(-x) = -f(x)$

— $2\int_0^a f(x)\,dx$ if $f$ is even, $f(-x) = f(x)$

$\therefore E(X) - \mu = 0$

$$\boxed{\Rightarrow E(X) = \mu}$$

For variance

$$E\left((X-\mu)^2\right) = \int_{-\infty}^{\infty} (x-\mu)^2 \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

Integration by parts

$\int (I)(II)\, dx$

$I \int II\, dx - \int \left(\frac{d}{dx} I \int II\, dx\right) dx$

I
L
A
T E

$$\frac{x-\mu}{\sigma} = z \Rightarrow dx = \sigma\, dz$$

$$\int_{-\infty}^{\infty} \sigma^2 z^2 \frac{1}{\sqrt{2\pi}\,\cancel{\sigma}} e^{-\frac{1}{2}z^2} (\cancel{\sigma} dz)$$

$$\frac{\sigma^2}{\sqrt{2\pi}} \int_{-\infty}^{\infty} z^2 e^{-\frac{1}{2}z^2} dz = \frac{2\sigma^2}{\sqrt{2\pi}} \int_0^{\infty} z^2 e^{-\frac{1}{2}z^2} dz$$

$$\frac{\cancel{2}\sigma^2}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \underset{I}{z} \left(\underset{II}{z e^{-z^2/2}}\right) dz$$

Integration by parts

→ Home work $\frac{\sqrt{2\pi}}{2}$

$$\boxed{E\left((X-\mu)^2\right) = \sigma^2}$$

$\Rightarrow$ Variance $= \sigma^2$

$$E((X-\mu)^2) = \sigma^2$$

$\Rightarrow$ Variance $= \sigma^2$

$\Rightarrow$ S.D. $= \sqrt{\text{variance}} = \sigma$

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- $X \rightarrow n(x; \mu, \sigma) \rightarrow$ continuous r.v.
- If I want to compute $P(X=a) = 0$

" " " " " $P(a \le X \le b) = \int_a^b pdf(x)\,dx$

$$\int_a^b \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

computing this integral is hard. $\leftarrow$ we seek help of softwares (numerical techniques) $\leftarrow$ & make tables.

$\downarrow$

You can use these tables.

$\downarrow$

But making tables for every value of $\mu$ & $\sigma$ is not possible.

$\downarrow$

What we do, we take help of the following transformation:

$$\underset{\text{r.v.}}{X} \quad n(x;\mu,\sigma) \xrightarrow{Z = \frac{X-\mu}{\sigma}} \underset{\text{r.v.}}{Z} \quad n(z, \underset{\text{Mean}}{0}, \underset{\text{S.D.}}{1})$$

$$\sigma_Z^2 = \sigma^2_{\frac{X-\mu}{\sigma}} = \sigma^2_{\frac{X}{\sigma} - \frac{\mu}{\sigma}}$$

$$= \left(\frac{1}{\sigma}\right)^2 \sigma_X^2 = \frac{1}{\sigma^2} * \sigma^2 = \boxed{1}$$

$\Rightarrow$ S.D. $= 1$

$$\boxed{E(Z)} = E\left(\frac{X-\mu}{\sigma}\right)$$

$$= \frac{1}{\sigma}(E(X) - E(\mu))$$

$$= \frac{1}{\sigma}(\mu - \mu) = \boxed{0}$$

That means you can make tables only for $n(z, 0, 1)$

Standard normal distribution.

$\downarrow$

A normal distribution with mean $= 0$ & S.D. $= 1$

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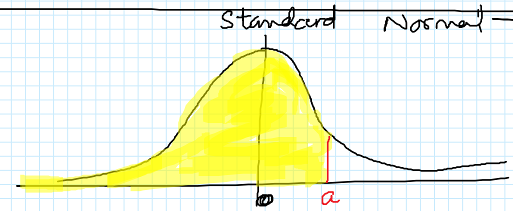
Something about tables $\rightarrow$

Standard Normal distribution




$\downarrow$

$P(X \le a) \Rightarrow$ yellow area value will be given in the table

**Que** Find the probability that a standard normally distributed data lies b/w $-0.5$ & $0.5$

**Sol^n**

$$P(-0.5 \le X \le 0.5) = P(X \le 0.5) - P(X \le -0.5)$$

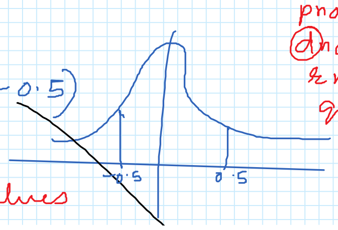
look for these values

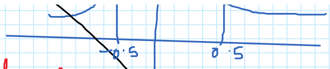
pnorm $\rightarrow$ cdf

dnorm $\rightarrow$ pdf

rnorm

qnorm






look for these values in the table.

0.6915

0.3085

$= 0.6915 - 0.3085 =$