Probability

Probabilistic model

Sample space : Set of all possible outcomes.

Example ① Toss a coin
$$S = \{H, T\}$$

② roll a dice
$$S = \{1, 2, 3, 4, 5, 6\}$$
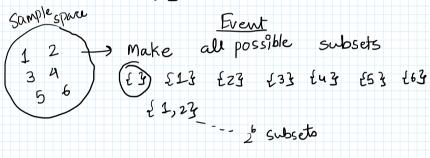
③ Outcomes for UCS 410 course
$$\{E \; \bar{C} \; C \; \bar{B} \; B \; \bar{A} \; A \; A+\}$$

Note :- These are finite sample spaces.

④ Experiment :- choosing a point randomly from [0,1]
$$S = [0, 1]$$

Experiment :-

Sample space

1   2
3   4
5   6

→ Make all possible subsets

$\{\}$ $\{1\}$ $\{2\}$ $\{3\}$ $\{4\}$ $\{5\}$ $\{6\}$
$\{1, 2\}$
- - - - 2^6 subsets

Event : Any subset of a sample space is an event.

Note   Consider the following two games involving 10 successive coin tosses.

Game 1 :- You will get $1 for each Head.

Game 2 :- You will get $1 upto the toss when you get 1st head & then you will get $2 upto when you get 2nd head :

Sample space in Game 1 :-
$$S = \{0, 1, 2, 3, - \cdot, 10\}$$

Sample space in Game 2

$\frac{10(10+1)}{2} = 55$

$\{$ <u>HHH - - - - H</u>
HTTT - - - - T

2	2	2	2					-	-	-	2

$2^{10} = 1024$

:
$\}$

TT - - - - T = 10

$TT \text{ - - - } T - \boxed{10}$
$HTTT \text{ - - - } T$

$\overbrace{1 + 2 + 2 + \text{ - - - } + 2}$

Uncertain Situation

Sample space

Probability law

Sample Space

$P(A)$
Probability
law

You want to assign a number to each event that tells you the chances of happening for that event

Expectations

① $P(A) \geq 0$ (positivity)

② $P(A \cup B) = P(A) + P(B)$ (additivity) (If $A \cap B = \phi$)

③ $0 \leq P(A) \leq 1$ (normalization)

$\searrow$ axiomatic definition of probability

(You can choose any P satisfying ①, ② & ③)

Note: P : Power set of Sample space $\longrightarrow [0, 1]$

Experiment:- Tossing a coin

S.S. $\rightarrow \{H, T\}$

power set $\rightarrow \{ \{\}, \{H\}, \{T\}, \{H, T\} \}$

$\downarrow P$

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

$0 \quad 1/2 \quad 1/2 \quad 1$

$\rightarrow$ This P is satisfying all ①, ② & ③ ∴ is a valid probability law.

Most widely used P :-

① When all the outcomes are equally likely & S.S. is finite

then you can define

$\swarrow$ Classical definition of probability

$P(A) = \dfrac{n(A)}{n(S.S.)} = \dfrac{\text{No. of favourable outcomes}}{\text{Total no. of outcomes.}}$

② Probability of an event as relative frequency

$\downarrow$

Shador: Relative frequency definition of Probability

you get your favourable outcome
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you performed the experiment

Que Roll a four sided dice twice

$P\{$ the sum of the rolls is even$\} = \dfrac{8}{16} = \dfrac{1}{2}$

$P\{$ " " " " " odd $\} = 8/16 = 1/2$

P { " first roll = the second roll $\} = \dfrac{4}{16} = 1$

(1,1) (2,2) (3,3)
{ (1,3) (3,1) (4,4)
(2,4) (4,2) }

$\sqrt{6}$ $\sqrt[3]{4}$

$\sqrt{6}$ $\sqrt[3]{4}$