# Lecture 27 (Gr 25-27)

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## Recap

- Uniform
- Normal
- Gamma $\Gamma(x; \alpha, \beta)$
- Exponential $\alpha = 1$

Exponential distributions models the time to first occurence in a Poison process. (with proof)

Today's work

If you want to model the time passed for $\alpha$ occurences,

In that case $T$ is modeled with gamma distribution. (without proof)

$\rightarrow \Gamma(x; \alpha, \beta)$ $\beta = \frac{1}{\lambda}$

**Que** Suppose that telephone calls arriving at a particular switchboard follows a Poisson process with an average of 5 calls coming per minute. What is the probability that upto a minute will pass, by the time 2 calls have come into the switchboard?

**Sol$^n$** Define $T =$ time for 2 calls

we know this is gamma distribution $\alpha = 2$ & $\beta = \frac{1}{\lambda} = \frac{1}{5}$

$\Gamma(x; 2, \frac{1}{5})$

and we are looking $P(T \le 1) = \int_0^1 \text{pdf}(x)\,dx = \int_0^1 \frac{1}{\beta^2 \Gamma 2} x^{2-1} e^{-x/\beta} dx$ $\quad \Gamma 2 = 1$

$= 25 \int_0^1 x e^{-5x} dx \rightarrow$ Integrate by parts

$= 0.96$ Ans.

$\Gamma n = (n-1)!$

$\Gamma 2 = 1! = 1$

$\Gamma \alpha \quad \alpha > 0$

$\Gamma \frac{1}{2} = \sqrt{\pi}$

$\Gamma n = (n-1)\Gamma n-1$

$\Gamma \frac{3}{2} = \frac{1}{2}\Gamma\frac{1}{2} = \frac{1}{2}\sqrt{\pi}$

**Note** We have table for some integrals at the end of the chapter

the values of $F(x; \alpha) = \int_0^x \frac{1}{\Gamma \alpha} y^{\alpha - 1} e^{-y} dy$ $\leftarrow$ incomplete gamma integral.

**Que** For a certain dose of a toxicant, a study determines that the survival time of rats in weeks, has a gamma distribution with $\alpha = 5$ & $\beta = 10$. What is the probability that a rat survives no longer than 60 weeks?

**Sol$^n$** $T \rightarrow \Gamma(x; 5, 10)$

$P(T \le 60) = \int_0^{60} \text{pdf}\,dx = \int_0^6 \frac{1}{\beta^5 \Gamma 5} x^{5-1} e^{-x/\beta} dx$

Put $\frac{x}{\beta} = y$, $dx = \beta\,dy$

$\int_0^6 \frac{1}{\beta^5 \Gamma 5} (\beta y)^4 e^{-y} \beta\,dy = \int_0^6 \frac{1}{\Gamma 5} y^4 e^{-y} dy = F(6, 5) = 0.715$

## Chi square distribution

In gamma distribution if you put $\alpha = \frac{\nu}{2}$ & $\beta = 2$

then it becomes chi square distribution.

& $\beta = 2$

then it becomes chi square distribution.

pdf

$$\Gamma(x;\alpha,\beta) = \begin{cases} \frac{1}{\beta^{\alpha}\Gamma\alpha} x^{\alpha-1} e^{-x/\beta} & x \geq 0 \\ 0 & x < 0 \end{cases} \xrightarrow[\beta = 2]{\alpha = \frac{\nu}{2}} f(x;\nu) = \begin{cases} \frac{1}{2^{\nu/2}\Gamma\nu/2} x^{\nu/2-1} e^{-x/2} & x > 0 \\ 0 & x < 0 \end{cases}$$

(We will study in detail later)

## Lognormal distribution :-

If $X$ is a r.v. with $\log(X)$ following normal distribution, then we say that $X$ has log normal distribution.

(Obviously, in this case $X$ should assume only $>0$ values)

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**Que** Suppose it is assumed that concentration of certain pollutant, in parts per million has lognormal distribution with the parameters $\mu = 3.2$ & $\sigma = 1$. What is the probability that the concentration exceeds 8 parts per million?

**Sol$^n$** $X \rightarrow$ parts per million of the pollutant

then $\log(X) \sim n(x; 3.2, 1)$

$W \rightarrow$ ND

$Z \rightarrow$ SND

we are looking for $P(X > 8) = 1 - P(X \leq 8)$

$$= 1 - P(\underbrace{\log(X)}_{W} \leq \log 8)$$

$W \sim n(x; 3.2, 1)$

$Z = \frac{W - 3.2}{1}$

$$= 1 - P\left(Z \leq \frac{\log 8 - 3.2}{1}\right)$$

$$= 1 - \underbrace{P(Z \leq -1.12)}$$

$$= 1 - 0.1314$$

$$= 0.8686$$

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You can also note pdf & mean & $\sigma$ of lognormal distribution

$$\text{pdf} = \begin{cases} \frac{1}{\sqrt{2\pi}\,\sigma x} e^{-\frac{1}{2\sigma^2}\left[\log(x) - \mu\right]^2} & x > 0 \\ 0 & x < 0 \end{cases}$$

$$\text{mean} = e^{\left(\mu + \sigma^2/2\right)}$$

$$\text{Variance} = e^{\left(2\mu + \sigma^2\right)} * \left(e^{\sigma^2} - 1\right)$$

END OF THE CHAPTER