<u>Recap</u>

Normal distribution $\rightarrow n(x; \mu, \sigma)$

$Z = \dfrac{X - \mu}{\sigma}$ $X = \mu + \sigma Z$

Standard normal distribution $\rightarrow n(z; 0, 1)$

We have tables for cdf of S.N.D.

Today's lecture

<u>Que</u> Given a r.v. X having a normal distribution with $\mu = 50$ & $\sigma = 10$, find the probability that X assumes a value b/w 45 & 62.

<u>Solⁿ</u>

X $Z \rightarrow$ S.N.D.

$n(x; 50, 10)$ $Z = \dfrac{X - 50}{10}$ $n(z; 0, 1)$

$P(45 < X < 62)$

$P\left(\dfrac{45-50}{10} < Z < \dfrac{62-50}{10}\right)$

$= P(-0.5 < z < 1.2)$
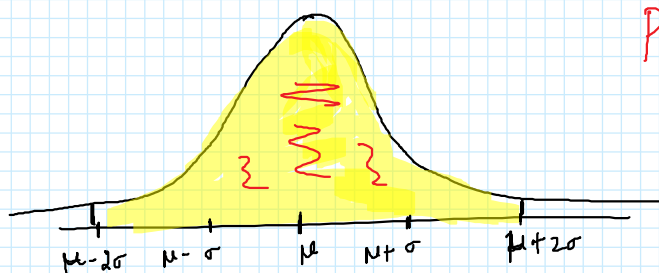
$= P(z < 1.2) - P(z < -0.5)$

$= 0.8849 - 0.3085$

$= \boxed{0.5764}$ Ans.

<u>Que</u> Given a normal distribution with mean $= \mu$ & S.D. $= \sigma$ in the region what is the probability that X lies 2 S.D. from mean.

<u>Solⁿ</u>

$P(\mu - 2\sigma \leq X \leq \mu + 2\sigma)$

$Z = \dfrac{X - \mu}{\sigma}$

$P(-2 \leq Z \leq 2)$

$P(z \leq 2) - P(z \leq -2)$

$= 0.9772 - 0.0228$

$= 0.9544$

For a normal distribution

$P(\mu - \sigma \leq X \leq \mu + \sigma) = 68\%$

$P(\mu - 2\sigma \leq X \leq \mu + 2\sigma) \approx 95\%$

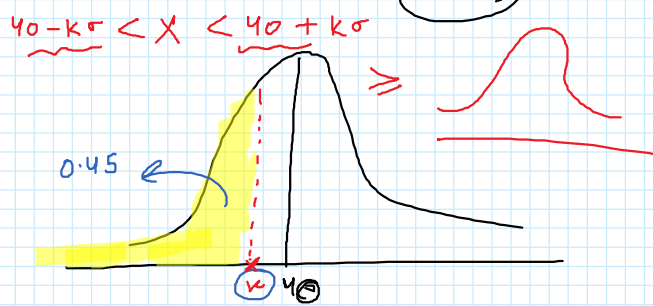$P(\mu - 3\sigma \leq X \leq \mu + 3\sigma) = 99.7\%$

<u>Que</u> Given a normal distribution with $\mu = 40$ & $\boxed{\sigma = 6,}$ find the value of x that has $40 - k\sigma < x < 40 + k\sigma$

Que Given a normal distribution with $\mu = 40$ & $\boxed{\sigma = 6}$

find the value of x that has $\quad 40 - k\sigma < X < 40 + k\sigma$

(a) $\boxed{45\%}$ of the area to its left.

(b) 14% of the area to its right.

0.45

Sol^n $P(X < x) = \boxed{0.45} \longrightarrow 1 - \frac{1}{k^2}$

$\downarrow ?$

$Z = \dfrac{X - 40}{6}$

$X = 40 + 6 * z$

look for z in the table

$P(Z < z) = \boxed{0.45} \longrightarrow \boxed{z = -0.13} \longrightarrow x = 40 + 6 * (-0.13)$

$= \boxed{39.22}$

(b) $P(X > x) = 0.14$

$1 - P(X \leq x) = 0.14 \Rightarrow \boxed{P(X \leq x) = 0.86}$

$Z = \dfrac{X - 40}{6}$

$\longrightarrow P(Z \leq z) = 0.86$

$\downarrow$ Table
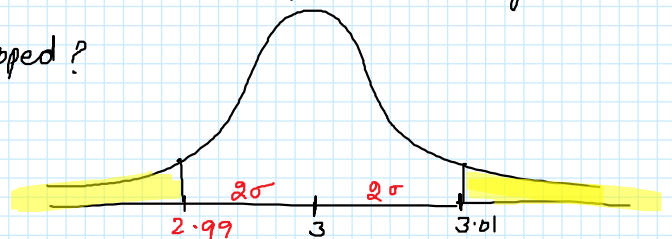
$z = 1.08$

$X = 40 + 6 * 1.08$

$x = 46.48$

Que In an industrial process, the diameter of ball bearing is an important measurement. The buyer sets specifications for the diameter to be 3.0 ± 0.01 cm. Anything outside it will not be accepted. It is know that the diameters are normally distributed with $\mu = 3$ & $\sigma = \boxed{0.005.}$ On an average, how many ball bearings will be scrapped?

Sol^n ∴ 5% will be scrapped.

$2\sigma \qquad 2\sigma$

$2.99 \qquad 3 \qquad 3.01$

Que The average grade for an exam is 74 and the S.D. is 7. It is normally distributed. If 12% of the class is given an A. What is the lowest marks that will get you an A.
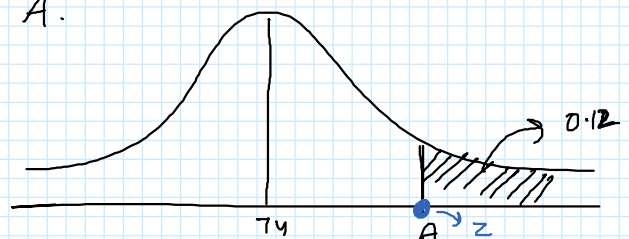
Sol^n $P(Z \geqslant z) = 0.12$

$\downarrow$

$1 - P(Z < z) = 0.12$

$\rightarrow P(Z < z) = 0.88$

0.12

$74 \qquad A \qquad z$

$$\Rightarrow P(Z < z) = 0.88$$

$\downarrow$ Table

$z = 1.18$

$\downarrow$

$x = \mu + 1.18\,\sigma$

$74 + 1.18 * 7 = \boxed{82.26}$

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(1) **Quartiles :-** Divide your data into four equal parts.

Quantiles :- Divide your data into n equal parts.

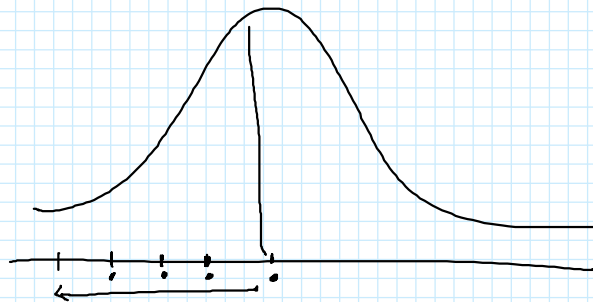**Que** for a normal distribution with $\mu = 50$ S.D. $= 6$ find 4th decile ?

$$P(Z < z) = 0.40$$

$\downarrow$

Table

$z = -0.25$

$$X = 50 + (-0.25)(6)$$

$$= 50 - 1.5 = 48.5$$

Que 3rd pentile $n(x; 50, 6)$

Soln $$P(Z < z) = 0.60 \longrightarrow z = 0.25 \longrightarrow x = 50 + 6 * 0.25$$

$$= 50 + 1.5 = 51.5$$

H.W Que An electrical firm manufactures light bulbs that have life which is normally distributed with $\mu = 800$ hours & $\sigma = 40$ hours. Find the probability that a randomly selected bulb burns b/w 778 & 834 hours.

Ans 0.5111