# Sampling distribution of mean when standard deviation of population is not known

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**t distribution :-** Let $Z$ be a standard normal r.v. and $V$ a Chi-squared random variable with $\nu$ degrees of freedom. If $Z$ and $V$ are independent, then the distribution of the random variable $T = \dfrac{Z}{\sqrt{V/\nu}}$ is known as the t-distribution with $\nu$ degree of freedom.

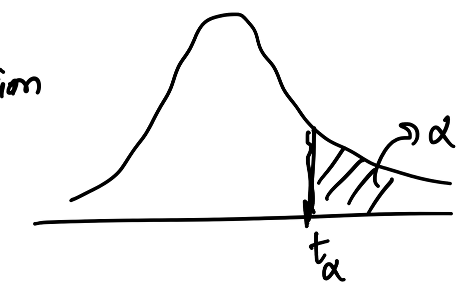
- The Central limit tells us that $\dfrac{\bar{X}-\mu}{\sigma/\sqrt{n}}$ is normally distributed.

→ Here we assume that the standard deviation $(\sigma)$ of population is known.

→ But if $\sigma$ is not known, then we work on $\dfrac{\bar{X}-\mu}{S/\sqrt{n}}$

Note that t distribution is symmetric about origin

- Notation




- as $n \to \infty$, T and Z distribution becomes same.

- Table of t distribution is similar in nature as of $\chi^2$ distribution.

$$\frac{\bar{X}-\mu}{S/\sqrt{n}} = \frac{(\bar{X}-\mu)/\sigma/\sqrt{n}}{S/\sqrt{n}/\sigma/\sqrt{n}} = \frac{\frac{\bar{X}-\mu}{\sigma/\sqrt{n}}}{\sqrt{\frac{S^2}{\sigma^2}}} = \frac{\frac{\bar{X}-\mu}{\sigma/\sqrt{n}}}{\sqrt{\frac{\chi^2}{(n-1)}}}$$

This is a t distribution with $(n-1)$ dof.

Therefore we have the result

$\dfrac{\bar{X}-\mu}{S/\sqrt{n}}$ is t distribution with $(n-1)$ degrees of freedom.

**Que** A chemical engineer claims that the population mean yield of a certain batch process is 500 grams per millilitre of raw material. To check this claim he 25 batches each month. If the computed t-value falls between $-t_{0.05}$ and $t_{0.05}$, he is satisfied with his claim. What conclusion should he draw from a sample that has a mean $\bar{x} = 518$ grams per millilitre and a sample S.D. $s = 40$ grams. Assume that the distributions of means is approximately normal.

**Soln** From table

$t_{0.05} = 1.711$ for dof $= 24$

$-t_{0.05} = -1.711$

Between these two values 90% of t data lies for dof $= 24$

Now the t statistics is $\dfrac{\bar{x}-\mu}{s/\sqrt{n}} = \dfrac{518-500}{40/\sqrt{25}} = 2.25$

$2.25 \notin [-1.711, 1.711]$

$\therefore$ $\mu = 500$ is not a valid claim.