Lecture 11

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Recap

r.v. (X) → discrete
→ Continuous

discrete r.v.	Continuous r.v.
d → distribution	d → density
pdf → P(X=x)	pdf
cdf → P(X≤x)	cdf

Today's work

- If X and Y are two discrete random variables, then we define joint probability distribution function pdf $f(x,y)$ as

  ① $f(x,y) \geq 0$ for all x & y

  ② $\sum_x \sum_y f(x,y) = 1$

  ③ $P(X=x \text{ and } Y=y) = \boxed{f(x,y)}$

- If X and Y are cts. r.v., $f(x,y)$ will be a joint pdf if

  ① $f(x,y) \geq 0$ for all x & y

  ② $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y)\,dx\,dy = 1$

  ③ $P(a<X<b,\ c<Y<d) = \int_c^d \int_a^b f(x,y)\,dx\,dy$

- X → discrete Y cts.

  ① $f(x,y) \geq 0$

  ② $\sum_x \left(\int_y f(x,y)\,dy \right)$

  ③ $P(X=a,\ c<Y<d) = \int_c^d f(a,y)\,dy$

Que Two pens are selected at random from a box that contains

3 Blue pens
2 Red pens
3 Green pens

X → no. of blue pens selected → 0, 1, 2,
Y → no. of red pens selected → 0 1 2

Find (a) Joint pdf (b) $P\{(X,Y) \in A\}$ where $A = \{(x,y) \mid x+y \leq 1\}$

Soln

- Without replacement

$\frac{{}^3C_2}{{}^8C_2}$ (we are picking two pens together)

3×2

$\frac{{}^3C_1}{{}^8C_1} * \frac{{}^2C_1}{{}^7C_1}$ + $\frac{{}^2C_1}{{}^8C_1} \frac{{}^3C_1}{{}^7C_1}$

$\boxed{f(0,0)}$
X=0
Y=0

f(0,1)

$\frac{3C_1}{8C_1} * \frac{2C_1}{7C_1}$ ~~+~~ $\frac{2C_1}{8C_1} \frac{3C_1}{7C_1}$

$Y = 0$

$f(0,1)$

$f(1,1)$

- with replacement — $\frac{3C_1}{8C_1} * \frac{3C_1}{8C_1}$ $\quad C_1\ C_2\ C_3$

Joint pdf

$f(x,y)$	x = 0	1	2
0	$f(0,0) = 3/28$	$f(1,0) = 9/28$	$f(2,0) = 3/28$
1	$f(0,1) = 3/14$	$f(1,1) = 3/14$	$f(2,1) = 0$
2	$f(0,2) = 1/28$	$f(1,2) = 0$	$f(2,2) = 0$

$f(x,y)$	x = 0	1	2
0	3/28	9/28	3/28
1	3/14	3/14	0
2	1/28	0	0

$f(0,0) = P(X=0, Y=0) = \frac{3C_2}{8C_2} = 3/28$

$f(1,0) = P(X=1, Y=0) = \frac{3C_1 * 3C_1}{8C_2} = 9/28$

$f(2,0) = \frac{3C_2}{8C_2}$

(b) $= P\{(X,Y) \in A\}$

$A = \{(x,y) \mid x+y \le 1\}$

$f(0,0) + f(0,1) + f(1,0)$

$= \frac{18}{28} \neq \frac{9}{14}$ Ans.

Que

$X \rightarrow$ Proportion of time for using drive in facility

$Y \rightarrow$ Proportion of time for using walk in facility

their joint pdf is

$$f(x,y) = \begin{cases} \frac{2}{5}(2x+3y) & 0 \le x \le 1,\ 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Find $P\{(X,Y) \in A\}$ where $A = \{(x,y) \mid 0 < x < \frac{1}{2},\ \frac{1}{4} < y < \frac{1}{2}\}$

Soln

$$= \int_{y=1/4}^{1/2} \int_{x=0}^{1/2} f(x,y)\,dx\,dy$$

$$= \int_{1/4}^{1/2} \left(\int_{0}^{1/2} \frac{2}{5}(2x+3y)\,dx \right) dy = \frac{13}{160}$$

Next

joint pdf

$f(x,y)$	x = 0	1	2	$h(y)$
0	3/28	9/28	3/28	15/28 → $h(0)$
1	6/28	6/28	0	12/28 = → $h(1)$
2	1/28	0	0	1/28 = → $h(2)$

$(f(x,y)\,dx$

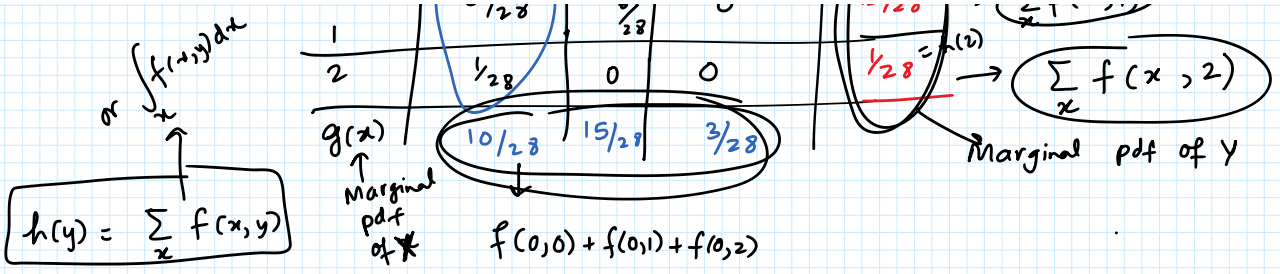
distribution of Y alone.

$\sum_x f(x,0)$

$f(0,0) + f(1,0) + f(2,0)$

$\sum_x f(x,1)$

$\sum f(x,2)$



$g(x) = \sum_y g(x,y)$ or $\int_y g(x,y)\,dy$

Que

$$f(x,y) = \begin{cases} \frac{2}{5}(2x+3y) & 0 \le x \le 1, \quad 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$ joint pdf

Find $g(x)$ & $h(y)$ marginal pdfs.

Soln

$$g(x) = \int_y f(x,y)\,dy = \int_0^1 \frac{2}{5}(2x+3y)\,dy = \frac{2}{5}\left(2xy + \frac{3y^2}{2}\right)_0^1$$

$$= \frac{2}{5}\left(2x + \frac{3}{2}\right)$$

$$h(y) = \int_x f(x,y)\,dx = \int_0^1 \frac{2}{5}(2x+3y)\,dx = \frac{2}{5}\left(x^2 + 3xy\right)_0^1$$

$$= \frac{2}{5}(1+3y)$$ ✓