Lecture 10 (Gr 25-27)

17 August 2022 08:16

Recap:-

Random variable (r.v.) → X, Y,

discrete r.v. & continuous r.v.

pmf / pdf → Tells us the pointwise probability

$P(X=x) \rightarrow f(x)$

cdf → Cumulative Probability

$P(X \le x) \rightarrow F(x)$

Example roll a dice ~~thrice~~ twice

X → sum of the no.s on the dice

pdf ?

cdf ?

pdf & cdf graphs.

Sol^n X

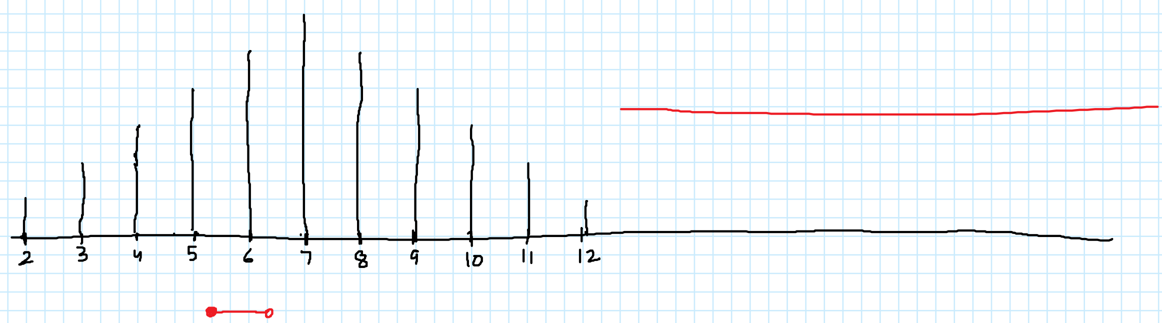
Total no. of outcomes = 36

$x \rightarrow$	2	3	4	5	6	7	8	9	10	11	12
	(1,1)	(2,1) (1,2)	(2,2) (3,1) (1,3)	(2,3) (3,2) (4,1) (1,4)	(3,3) (4,2) (2,4) (5,1) (1,5)	(4,3) (3,4) (5,2) (2,5) (6,1) (1,6)	(4,4) (5,3) (3,5) (6,2) (2,6)				
pdf $f(x)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

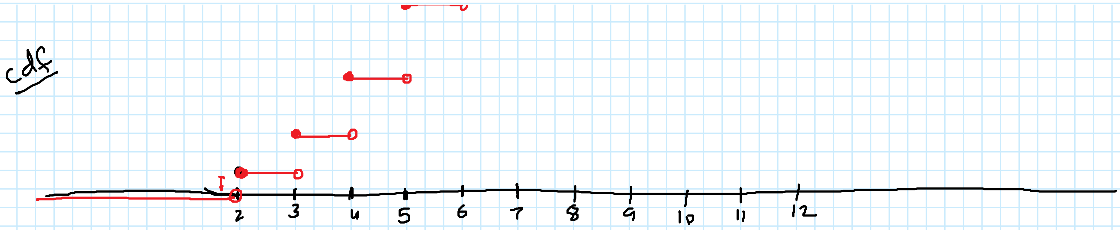
cdf $F(x)$

$$F(m) = P(X \le m) = \begin{cases} 0 & m < 2 \\ 1/36 & 2 \le m < 3 \\ \frac{1}{36}+\frac{2}{36} = 3/36 & 3 \le m < 4 \\ 6/36 & 4 \le m < 5 \\ 10/36 & 5 \le m < 6 \\ 15/36 & 6 \le m < 7 \\ 21/36 & 7 \le m < 8 \\ 26/36 & 8 \le m < 9 \\ \frac{30}{36} & 9 \le m < 10 \\ \frac{33}{36} & 10 \le m < 11 \\ \frac{35}{36} & 11 \le m < 12 \\ 1 & m \ge 12 \end{cases}$$

pdf



cdf



Continuous r.v.

pdf ↓ probability density function

cdf ↓ cumulative density function

Example X – Waiting time for a train which can have a delay of max 1 hour

$x \in [0, 1]$

$P(X = 0.5) = 0$

For continuous r.v. probability of a single point is always zero.

$\therefore$ In this case $P(X=x)$ does not make much sense for pdf.

We define pdf as a function $f(x)$ such that

(1) $f(x) \geq 0$ for all x

(2) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

(3) $P(a \leq X \leq b) = \int_a^b f(x)\,dx$.

Note $P(a \leq X < b) = \underbrace{P(X=a)}_{=0} + P(a < X < b)$

$$\boxed{P(a \leq X < b) = P(a < X < b)}$$

$\therefore$ You can always exclude/include the end points in continuous r.v. case.

|||ly $P(a < X \leq b) = P(a < X < b)$

$P(a \leq X \leq b) = P(a < X < b)$

Que Suppose that error in the reaction temperature (in °C) for a controlled lab experiment is a continuous r.v. X having the following pdf

$$f(x) = \begin{cases} \frac{x^2}{3} & -1 < x < 2 \\ 0 & \text{elsewhere.} \end{cases}$$

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① Verify that $f(x)$ is valid pdf. ② Find $P(0 < X < 1)$

(a) $f(x) \geq 0$ clearly seen from the definition

(b) $\int_{-\infty}^{\infty} f(x)\,dx = \int_{-1}^{2} \frac{x^2}{3}\,dx = \frac{1}{3}\left.\frac{x^3}{3}\right]_{-1}^{2}$

$\frac{1}{9}(8+1) = 1$

$\therefore$ It is a valid pdf.

$P(0 < X < 1)$

$= \int_0^1 f(x)\,dx$

$= \int_0^1 \frac{x^2}{3}\,dx$

$\left.\frac{x^3}{9}\right]_0^1 = \boxed{1/9}$

For continuous r.v. X, we define cdf (cumulative density function)

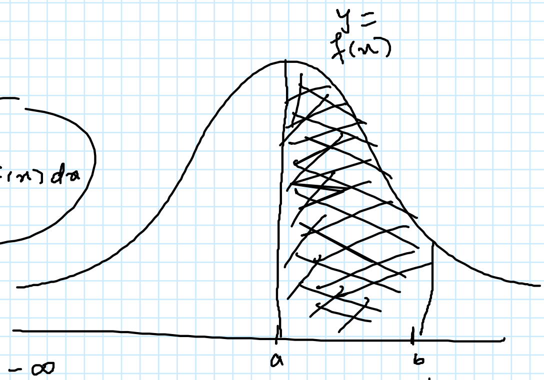
$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(t)\,dt \quad \text{where } f(x) \to \text{pdf}$$

Note

In terms of cdf $F(x)$ what will be

$$P(a < X < b) = \int_{-\infty}^{b} f(x)\,dx - \int_{-\infty}^{a} f(x)\,dx$$

$$= F(b) - F(a)$$



$\boxed{P(a < X < b)} \to \int_a^b f(x)\,dx \to \int_{-\infty}^{b} f(x)\,dx - \int_{-\infty}^{a} f(x)\,dx$

$\to F(b) - F(a)$

Note

$$\boxed{\begin{array}{ll} F(x) = \int_{-\infty}^{x} f(t)\,dt & \text{cdf from pdf} \\ f(x) = \frac{dF(x)}{dx} & \text{pdf from cdf} \end{array}}$$

Que For the pdf $f(x) = \begin{cases} \frac{x^2}{3} & -1 \leq x < 2 \\ 0 & \text{otherwise} \end{cases}$ compute cdf. & Use this cdf to find $P(0 < X < 1)$

Sol^n

$$F(x) = \int_{-\infty}^{x} f(t)\,dt$$

$$= \begin{cases} 0 & \text{if } x < -1 \\ \int_{-1}^{x} \frac{t^2}{3}\,dt = \frac{x^3+1}{9} & \text{if } -1 \leq x < 2 \end{cases} = \begin{cases} 0 & x < -1 \\ \frac{x^3+1}{9} & -1 \leq x < 2 \end{cases}$$

$$= \begin{cases} \int_{-1}^{x} \frac{t^2}{3}\,dt = \frac{x^3+1}{9} & \text{if } -1 \le x < 2 \\ \int_{-\infty}^{x} f(t)\,dt & \text{if } x \geq 2 \\ = \int_{-1}^{2} f(t)\,dt = 1 \end{cases} = \begin{cases} \frac{x^3+1}{9} & -1 \le x < 2 \\ 1 & x \geq 2 \end{cases}$$

$$P(0 < X < 1) = F(1) - F(0) = \frac{1^3+1}{9} - \frac{0^3+1}{9} = \frac{2}{9} - \frac{1}{9} = \frac{1}{9}$$

H.W.

A company puts projects out on bid & generally estimates what a reasonable bid should be (call this estimate b). The company has determined that the pdf of lowest bid (winning) is

$$f(y) = \begin{cases} \frac{5}{8b} & \frac{2}{5}b \le y \le 2b \\ 0 & \text{elsewhere.} \end{cases}$$

Find cdf $F(y)$ & use ~~it to determine~~ the probability that the winning bid is less that the company's estimate b.