Practice Questions

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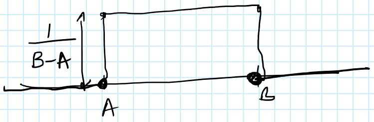
Chapter 6

Some continuous distribution

① Uniform continuous distribution

let X be a r.v. taking values in the interval [A, B] with pdf

$$f(x) = \begin{cases} \frac{1}{B-A} & A \le x \le B \\ 0 & \text{otherwise.} \end{cases}$$



Mean of the r.v. X = $\frac{A+B}{2}$

Proof $E(X) = \int_{-\infty}^{\infty} x f(x)\,dx = \int_A^B x \frac{1}{B-A}\,dx = \frac{1}{B-A}\left[\frac{x^2}{2}\right]_A^B = \frac{1}{2}\frac{(B^2-A^2)}{\cancel{(B-A)}}$

$= \frac{A+B}{2}$

$var = E(x^2) - (E(x))^2$

$E(x^2) \rightarrow \int_A^B x^2 \frac{1}{B-A}\,dx = \frac{1}{B-A}\left[\frac{x^3}{3}\right]_A^B = \frac{1}{3}\left[\frac{B^3-A^3}{B-A}\right] = \frac{B^2+BA+A^2}{3}$

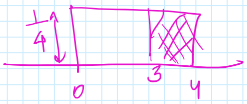
$= \frac{B^2+BA+A^2}{3} - \left(\frac{A+B}{2}\right)^2 = \frac{(B-A)^2}{12}$

Que X is a uniformally distributed r.v. taking values in [0, 4].

What is its pdf? what is the prob that X is at least 3.

Sol^n $pdf = \begin{cases} \frac{1}{4-0} & 0 \le x \le 4 \\ 0 & \text{otherwise} \end{cases} = \begin{cases} \frac{1}{4} & 0 \le x \le 4 \\ 0 & \text{otherwise} \end{cases}$

$P(X \geq 3) = \int_3^{\infty} f(x)\,dx =$



$= \int_3^4 \frac{1}{4}\,dx = \frac{1}{4}(4-3) = \boxed{1/4}$

Practice Questions

$P(H) = 3/4$
$P(T) = 1/4$

Que A coin is biased such that a head is three times as likely to occur as a tail. Find the expected no. of tails when the coin is tossed twice?

$X = \text{no. of tails} = \begin{cases} 0 \rightarrow (3/4)(3/4) = \frac{9}{16} \\ 1 \rightarrow (3/4)(1/4) + (1/4)(3/4) = \frac{6}{16} \\ 2 \rightarrow (1/4)(1/4) = \frac{1}{16} \end{cases}$

$E(X) = 0 * \frac{9}{16} + 1 * \frac{6}{16} + 2 * \frac{1}{16} =$

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Que For a certain airport with three runways, it is known that in the ideal setting following are the probabilities that the individual runways are accessed by a randomly arriving plane

Runway 1 : $\boxed{P_1 = 2/9}$
Runway 2 : $P_2 = 1/6$
Runway 3 : $P_3 = 11/18$

What is the prob. that 6 randomly arriving planes are distributed in the following fashion

Runway 1 — ② planes
" 2 — ① plane
" 3 — ③ planes.

Soln

$$\frac{6!}{2!\,1!\,3!} \left(\frac{2}{9}\right)^2 \left(\frac{1}{6}\right)^1 \left(\frac{11}{18}\right)^3 = 0.1127$$

Que A physician claims that 70% of people with lung cancer are chain smokers. Let us assume that he is true.

(a) Find the prob that out of 10 patients, fewer than 5 are chain smokers.

Soln

$X \to$ no. of chain smokers out of 10 $\to p = 0.7$
$X \to$ Binomial

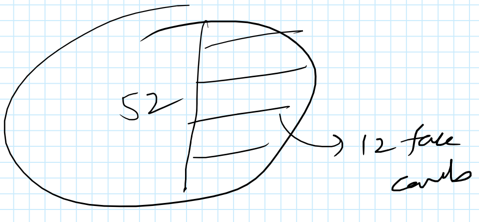
$$P(X<5) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4)$$

$P(X \le 4)$

$B(4, 10, 0.7) \leftarrow \sum_{r=0}^{4} {}^{n}C_{r}\, p^{r} (1-p)^{n-r}$

Que If 7 cards are taken out from an ordinary deck of 52 cards, what is the prob that

(a) exactly ② of them will be face cards.



$$h(2; \overset{N,\ K,\ n}{52, 12, 7})$$

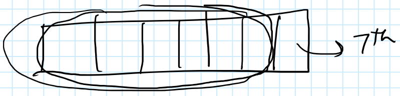
$$= \frac{{}^{12}C_2\ {}^{40}C_5}{{}^{52}C_7}$$

Biased → H → 2/3, T → 1/3

Que Find the prob. that a person flipping a coin gets the third head

1 Que Find the prob. that a person flipping a coin gets the third head on seventh flip.

→ negative Binomial



$b^*($

$${}^6C_2 \left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right)$$

Que No. of customers arriving per hour at a certain car facility is assumed to follow Poisson distribution with $\lambda = 7$.

Compute the prob. that more than 10 customers will arrive in a 2 hour period? What is the mean no. of arrivals during a 2 hour period.

Sol^n

~~$e^{-\lambda t}$~~

$$P(X > 10) = 1 - P(X \le 10)$$
$$= 1 - \sum_{x=0}^{10} \frac{e^{-\lambda t} (\lambda t)^x}{x!}$$

$$\frac{e^{-\lambda t} (\lambda t)^x}{x!}$$

$\lambda = 7$
$t = 2$
$x = 10$

$$\text{mean} = \lambda t = \boxed{14}$$

pdf mean variance

- Binomial
- Multinomial
- Hypergeometric
- Negative Bino
- geometric
- Poisson
- Discrete uniform
- continuous uniform

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