F distribution

Saturday, 2 December 2023 11:48 AM

F distribution The random variable F is a ratio of two independent Chi-squared random variables each divided by its number of degrees of freedom.

Hence we can write

$$F = \frac{U/\nu_1}{V/\nu_2}$$

$U, V \to$ independent Chi squared r.v. with ν_1 and ν_2 d.o.f.

- F distribution is not symmetric about origin.
- For tables we look at (ν_1, ν_2) dof.

F - Distribution ($\alpha = 0.01$ in the Right Tail)

Denominator Degrees of Freedom df_2 \ df_1 (Numerator Degrees of Freedom)	1	2	3	4	5	6	7	8	9
1	4052.2	4999.5	5403.4	5624.6	5763.6	5859.0	5928.4	5981.1	6022.5
2	98.503	99.000	99.166	99.249	99.299	99.333	99.356	99.374	99.388
3	34.116	30.817	29.457	28.710	28.237	27.911	27.672	27.489	27.345
4	21.198	18.000	16.694	15.977	15.522	15.207	14.976	14.799	14.659
5	16.258	13.274	12.060	11.392	10.967	10.672	10.456	10.289	10.158
6	13.745	10.925	9.7795	9.1483	8.7459	8.4661	8.2600	8.1017	7.9761
7	12.246	9.5466	8.4513	7.8466	7.4604	7.1914	6.9928	6.8400	6.7188
8	11.259	8.6491	7.5910	7.0061	6.6318	6.3707	6.1776	6.0289	5.9106
9	10.561	8.0215	6.9919	6.4221	6.0569	5.8018	5.6129	5.4671	5.3511
10	10.044	7.5594	6.5523	5.9943	5.6363	5.3858	5.2001	5.0567	4.9424
11	9.6460	7.2057	6.2167	5.6683	5.3160	5.0692	4.8861	4.7445	4.6315
12	9.3302	6.9266	5.9525	5.4120	5.0643	4.8206	4.6395	4.4994	4.3875
13	9.0738	6.7010	5.7394	5.2053	4.8616	4.6204	4.4410	4.3021	4.1911
14	8.8616	6.5149	5.5639	5.0354	4.6950	4.4558	4.2779	4.1399	4.0297
15	8.6831	6.3589	5.4170	4.8932	4.5556	4.3183	4.1415	4.0045	3.8948
16	8.5310	6.2262	5.2922	4.7726	4.4374	4.2016	4.0259	3.8896	3.7804
17	8.3997	6.1121	5.1850	4.6690	4.3359	4.1015	3.9267	3.7910	3.6822
18	8.2854	6.0129	5.0919	4.5790	4.2479	4.0146	3.8406	3.7054	3.5971
19	8.1849	5.9259	5.0103	4.5003	4.1708	3.9386	3.7653	3.6305	3.5225
20	8.0960	5.8489	4.9382	4.4307	4.1027	3.8714	3.6987	3.5644	3.4567
21	8.0166	5.7804	4.8740	4.3688	4.0421	3.8117	3.6396	3.5056	3.3981
22	7.9454	5.7190	4.8166	4.3134	3.9880	3.7583	3.5867	3.4530	3.3458
23	7.8811	5.6637	4.7649	4.2636	3.9392	3.7102	3.5390	3.4057	3.2986
24	7.8229	5.6136	4.7181	4.2184	3.8951	3.6667	3.4959	3.3629	3.2560
25	7.7698	5.5680	4.6755	4.1774	3.8550	3.6272	3.4568	3.3239	3.2172
26	7.7213	5.5263	4.6366	4.1400	3.8183	3.5911	3.4210	3.2884	3.1818
27	7.6767	5.4881	4.6009	4.1056	3.7848	3.5580	3.3882	3.2558	3.1494
28	7.6356	5.4529	4.5681	4.0740	3.7539	3.5276	3.3581	3.2259	3.1195
29	7.5977	5.4204	4.5378	4.0449	3.7254	3.4995	3.3303	3.1982	3.0920
30	7.5625	5.3903	4.5097	4.0179	3.6990	3.4735	3.3045	3.1726	3.0665
40	7.3141	5.1785	4.3126	3.8283	3.5138	3.2910	3.1238	2.9930	2.8876
60	7.0771	4.9774	4.1259	3.6490	3.3389	3.1187	2.9530	2.8233	2.7185
120	6.8509	4.7865	3.9491	3.4795	3.1735	2.9559	2.7918	2.6629	2.5586
∞	6.6349	4.6052	3.7816	3.3192	3.0173	2.8020	2.6393	2.5113	2.4073

- For different values of α, we will have different pages.
- One of the useful result:-

$$f_{1-\alpha}(\nu_1, \nu_2) = \frac{1}{f_\alpha(\nu_2, \nu_1)}$$

$$f_{1-0.99}(\nu_1, \nu_2) = \frac{1}{f_{0.99}(\nu_2, \nu_1)}$$

i.e. $\boxed{f_{0.01}(\nu_1 \nu_2) = \frac{1}{f_{0.99}(\nu_2, \nu_1)}}$

Note the interchange in degree of freedoms

Now if we have two normal populations and we take two independent samples of size n_1 & n_2 respectively from pop1 & pop2.

Let S_1^2 & S_2^2 be the sample variances, then

$\dfrac{(n_1-1)S_1^2}{\sigma_1^2}$ is a chi square r.v. with (n_1-1) dof.

$\dfrac{(n_2-1)S_2^2}{\sigma_2^2}$ is also a chi-square r.v. with (n_2-1) dof.

Hence

$$\frac{\dfrac{(n_1-1)s_1^2}{\sigma_1^2} \Big/ (n_1-1)}{\dfrac{(n_2-1)s_2^2}{\sigma_2^2} \Big/ (n_2-1)}$$

is F distribution.

i.e. $\dfrac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2}$ or $\dfrac{\sigma_2^2 S_1^2}{\sigma_1^2 S_2^2}$ is F distribution.

Que Two independent samples of students of a programme under distance education are taken from normal population with same variances. The size and variances of marks of the 1st sample are 8 & 100 respectively. The size & variances of marks of the 2nd sample are 20 & 40 respectively. What is the calculated F statistics.

Soln

$n_1 = 8 \qquad S_1^2 = 100$

$n_2 = 20 \qquad S_2^2 = 40$

$$F = \frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} = \frac{100}{40} \quad (\because \sigma_1 = \sigma_2)$$

$= 2.5$ with $(7, 19)$ dof