Lecture 6 (Gr 25-27)

08 August 2022 10:26

Recap A & B are two events

$$P(A|B) = \frac{P(A\cap B)}{P(B)} \quad \text{or} \quad P(B|A) = \frac{P(B\cap A)}{P(A)}$$

Today's class

$$P(A\cap B) = P(A|B)\,P(B) \qquad P(B\cap A) = P(B|A)P(A)$$

Product rule / multiplication rule

$$\boxed{P(A\cap B) = P(A|B)\,P(B) = P(B|A)\cdot P(A)}$$

Que Box containing 20 bulbs, out of which 5 are defective. If two bulbs are selected at random and removed from the box in succession without replacing the first. What is the probability that both the bulbs are defective.

Soln
$A \to$ Bulb 1 is defective
$B \to$ Bulb 2 " "

$$P(A\cap B) = P(A)\,P(B|A)$$
$$= \frac{{}^5C_1}{{}^{20}C_1} * \frac{{}^4C_1}{{}^{19}C_1} = \frac{1}{4} * \frac{4}{19} = \frac{1}{19}$$

Next If $P(A|B) = P(A) \Rightarrow A$ and B are independent events.

Defn:- We say that A and B are independent if
$$P(A|B) = P(A)$$
$$\text{Or } P(A\cap B) = P(A|B) * P(B) = P(A)\,P(B)$$

$$\boxed{P(A\cap B) = P(A)\,P(B)}$$

Generalization

If in an experiments, we have events

multiplication rule: $A_1, A_2 \ldots A_n$

$$\boxed{P(A_1\cap A_2 \ldots \cap A_n) = P(A_1)\,P(A_2|A_1)\,P(A_3|A_1\cap A_2)\ldots P(A_n|A_1\cap \ldots \cap A_{n-1})}$$

* $n=3$

R.H.S.

$$P(A_1)\,P(A_2|A_1)\,P(A_3|A_1\cap A_2)$$

$$\cancel{P(A_1)} * \frac{P(A_1\cap A_2)}{\cancel{P(A_1)}} \cdot \frac{P(A_1\cap A_2\cap A_3)}{\cancel{P(A_1\cap A_2)}} = \text{L.H.S.}$$

Que:- Three cards are drawn in succession without replacement from an ordinary deck of playing cards. Find the probability of $A_1\cap A_2\cap A_3$, where

A_1:- first card is a red ace
A_2:- second " " a 10 or a jack
A_3 third " is > 3 but < 7.

Soln:-
$$P(A_1\cap A_2\cap A_3) = P(A_1)\cdot P(A_2|A_1)\cdot P(A_3|A_1\cap A_2)$$
$$= \frac{2}{52}\cdot\frac{8}{51}\cdot\frac{12}{50} = \frac{8}{5525}$$

Mutually independent events:- A set of events $\{A_1, \ldots A_n\}$ is said to be mutually independent if for any subset $\{A_{i_1}, A_{i_2}, \ldots A_{i_k}\}$

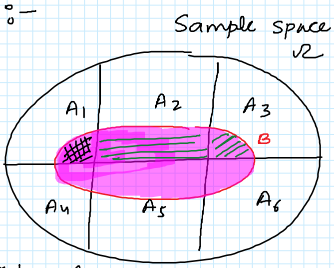
$P(A_{i_1} \cap A_{i_2} \cap \dots A_{i_k}) = P(A_{i_1}) * P(A_{i_2}) * \dots * P(A_{i_k})$

Home work — Monty Hall Problem (Gambling)

Summary :-
discrete
continuous
conditional

Total Probability theorem :-

Statement



Let Ω be the sample space.
Let $A_1, A_2 \dots A_n$ be disjoints events that form a partition of Ω (i.e. $A_1 \cup A_2 \dots \cup A_n = \Omega$)
and assume $P(A_i) > 0$ for $i = 1, \dots n$
then for any event B, $P(B) = \sum_{i=1}^{n} P(A_i)\, P(B/A_i)$

Proof

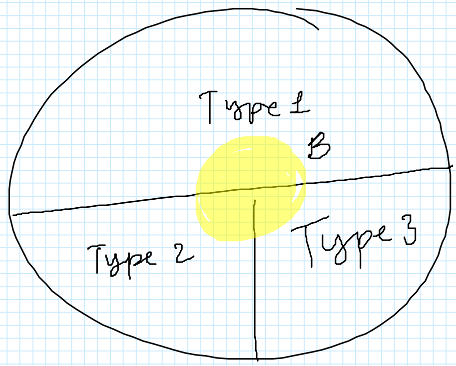
$B = (B \cap A_1) \cup (B \cap A_2) \cup (B \cap A_3) \dots \cup (B \cap A_6)$ ∵ $B \cap A_i$ are disjoint

$\Rightarrow P(B) = \sum_{i=1}^{n} P(B \cap A_i)$

$$\boxed{P(B) = \sum_{i=1}^{n} P(A_i)\, P(B/A_i)}$$

$$\boxed{P(B) = P(A_1) P(B/A_1) + P(A_2) P(B/A_2) + P(A_3) P(B/A_3) + \dots}$$

Que You enter a chess tournament where your probability of winning a game is 0.3 against half of the players (call them type 1), 0.4 against a quarter of the players (call them type 2), and 0.5 against the remaining quarter of players (call them type 3).
You play a game against a randomly chosen opponent. What is the probability that you will win.



Sol^n

$P(A_1) = 1/2 \qquad P(B/A_1) = 0.3$

$P(A_2) = 1/4 \qquad P(B/A_2) = 0.4$

$P(A_3) = 1/4 \qquad P(B/A_3) = 0.5$

$$P(B) = P(A_1) P(B/A_1) + P(A_2) \cdot P(B/A_2) + P(A_3) \cdot P(B/A_3)$$
(TPT)

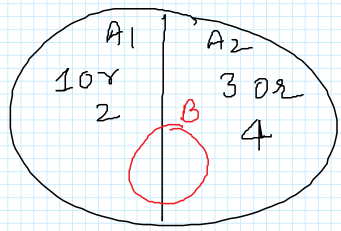
$$= 1/2 * 0.3 + 1/4 * 0.4 + 1/4 * 0.5 = 0.375$$

Que You roll a fair four sided dice. If the result is 1 or 2, you roll once more but otherwise you stop (you ~~will~~ have two rolls at most). What is the probability the sum total of your rolls is at least 4?

Sol^n

$P(A_1) = 1/2$

$P(A_2) = 1/2$



A_1

(1, 1) (2, 1)

(1, 2) (2, 2)

(1, 3) (2, 3)

(1, 4) (2, 4)

→ favourable

$P(B/A_1) = 5/8$

A_2

$\{3, 4\}$

$P(B/A_2) = 1/2$

$P(B) = P(A_1)\,P(B/A_1) + P(A_2) * P(B/A_2) = \frac{1}{2} * \frac{5}{8} + \frac{1}{2} * \frac{1}{2} = \frac{9}{16}$

Ans