Lecture 5 (Gr 25-27)

03 August 2022 09:44

Properties of probability laws :-

(1) $A \subseteq B$ then $P(A) \leq P(B)$

(2) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

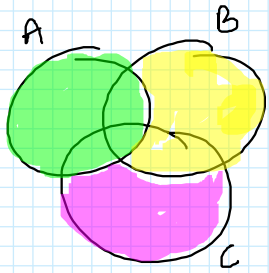
(3) $P(A \cup B) \leq P(A) + P(B)$

(4) $P(A_1 \cup A_2 \cup \dots \cup A_n) = \sum_{i=1}^{n} P(A_i)$ provided A_is are mutually disjoint

(5) $P(A \cup B \cup C)$

$= P(A) + P(A^c \cap B)$

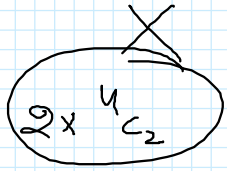
$+ P(A^c \cap B \cap C)$



Que In how many ways can 7 graduate students be assigned 1 triple and 2 double rooms in a hostels?

Sol:

$${}^{7}C_3 * {}^{4}C_2 * {}^{2}C_2$$



(7) → partition 3, 2, 2 → 210

$$\frac{7!}{3!\,2!\,2!}$$

Note n things

Number of ways to partition into r cells with $n_1\ n_2\ \dots\ n_r$ elements s.t. $n_1 + n_2 + \dots + n_r = n$

$$= \frac{n!}{n_1!\ n_2!\ \dots\ n_r!}$$

Que STATISTICS → no. of words

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$$\frac{10!}{3!\,3!\,2!}$$

Que In a pocker hand consisting of 5 cards, find the probability of holding 2 aces and 3 jacks.

Soln

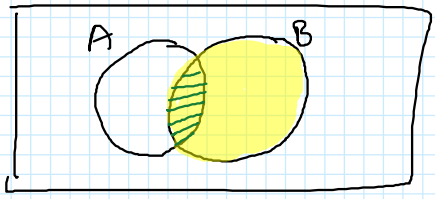
$$\frac{{}^{4}C_{2} * {}^{4}C_{3}}{{}^{52}C_{5}} = 0.9 \times 10^{-5}$$

Que If the probabilities are, respectively 0.09, 0.15, 0.21, & 0.23 that a person purchasing a new car will choose the color green, white, red or blue?
What is the probability that a new buyer will purchase one of these colors?

Soln $0.09 + 0.15 + 0.21 + 0.23 = 0.68$ (∵ Events are disjoint)

Conditional Probability

Suppose there are two events A and B
B has already occured with $P(B) > 0$
then what is probability of A



$$= \frac{P(A \cap B)}{P(B)}$$

$P(A|B)$

$$\frac{n(A \cap B)}{n(S)} \Big/ \frac{n(B)}{n(S)}$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

Que We toss a fair coin three times. What is $P(A|B)$ where

A = more heads than tails

B = 1st toss is a head.

Soln S.S. $\Omega \longrightarrow 8$

$B \rightarrow$ { HHH HHT HTH HTT }

$A \rightarrow$ { HHH HHT HTH (THH) }

$A \cap B \rightarrow$ { HHH HHT HTH }

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{3/8}{4/8} = \boxed{3/4} \text{ Ans.}$$

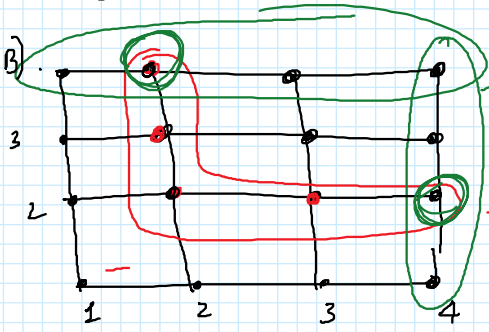
Que A fair 4 sided die is rolled twice.

Let X and Y be the result of 1st & 2nd roll.

$A = \{ \max\{X,Y\} = 4 \}$ $B = \{ \min\{X,Y\} = 2 \}$

what is $P(A|B)$?

Soln Ω



$\{(2,2)\ (2,3)\ (2,4)\ (3,2)\ (4,2)\}$

$n(A \cap B) = 5$

$\therefore P(A|B) = \frac{2/16}{5/16} = 2/5$ Ans.

Que The probability that a regularly scheduled flight departs on time $P(D) = 0.83$

$P(A) \rightarrow$ arrives on time $= 0.82$

$P(A \cap B) \rightarrow$ departs & arrives on time $= 0.78$

What is the probability that it arrives on time given it departed late.

Soln :- $P(A|D') = \frac{P(A \cap D')}{} = \frac{P(A) - P(A \cap B)}{}$

Both:- $P(A \mid D') = \frac{P(A \cap D')}{P(D')} = \frac{P(A) - P(A \cap B)}{1 - P(D)}$

$= \frac{0.82 - 0.78}{1 - 0.83} = 0.24.$