Recap

- X & Y two r.v.

  Joint pdf $f(x,y) = P(X=x, Y=y)$ in discrete case

  $$P(a < X < b,\ c < X \leq d) = \int_c^d \int_a^b f(x,y)\,dx\,dy$$

- Marginal pdfs

  X & Y

  $f(x,y) \rightarrow$ Joint pdf

  $P(X=x)$

  Marginal pdf of x $\leftarrow$ $g(x) = \sum_y f(x,y)$ discrete r.v.

  $X = 0, 1, 2$

  $g(x) = \int f(x,y)\,dy$ cts. r.v.

$X \rightarrow 0, 1, 2$

$Y = 0, 1, 2$

$P(X=0) = f(0,0) + f(0,1) + f(0,2)$

$P(X=1) = f(1,0) + f(1,1) + f(1,2)$

$P(X=2) = f(2,0) + f(2,1) + f(2,2)$

Marginal pdf of y $h(y) = \sum_x f(x,y)$ or $\int f(x,y)\,dx$

Que If $f(x,y) = \begin{cases} \frac{2}{5}(2x+3y) & 0 \leq x \leq 1,\ 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$ is joint pdf of X & Y. (cts r.v.)

Find marginal pdfs $g(x)$ & $h(y)$ of X & Y.

Soln

$$g(x) = \int_y f(x,y)\,dy = \int_0^1 \frac{2}{5}(2x+3y)\,dy = \frac{2}{5}\left(2xy + \frac{3y^2}{2}\right)\Big]_0^1$$

$$= \frac{2}{5}\left(2x + \frac{3}{2}\right)$$

$$= \frac{4x}{5} + \frac{3}{5}$$

$$h(y) = \int_x f(x,y)\,dx = \int_0^1 \frac{2}{5}(2x+3y)\,dx = \frac{2}{5}\left(x^2 + 3xy\right)_0^1 = \frac{2}{5}(1+3y)$$

Conditional pdfs

$X \rightarrow$ r.v.

$X = x \rightarrow$ is an event.

$$P(\underbrace{Y=y}_{\text{call it event A}} \mid \underbrace{X=x}_{\text{call it event B}}) = \frac{P(Y=y, X=x)}{P(X=x)}$$

$P(A|B) = \frac{P(A \cap B)}{P(B)}$

$$= \frac{f(x,y) \rightarrow \text{Joint pdf}}{g(x) \rightarrow \text{Marginal pdf of } X}$$

$$\boxed{\begin{aligned} \underbrace{P(Y=y \mid X=x)}_{f(y|x)} &= \frac{f(x,y)}{g(x)} \\ \text{Illy } \underbrace{P(X=x \mid Y=y)}_{f(x|y)} &= \frac{f(x,y)}{h(y)} \end{aligned}}$$

$f(x|y)$ or $f(y|x)$ are conditional pdfs.

Toss a coin twice

$S = \{HH\ HT\ TH\ TT\}$

$X \rightarrow$ no. of heads

$X \rightarrow 0, 1, 2$

$X = 0$ $\{TT\}$

$X = 1$ $\{HT, TH\}$

$X = 2$ $\{HH\}$

Que For disrete r.v. X & Y, the joint pdf is given below

$f(x,y)$	$x=0$	$x=1$	$x=2$	$h(y)$
$y=0$	3/28	9/28	3/28	15/28 $\rightarrow$ add the row
$y=1$	3/14	3/14	0	3/7 $\rightarrow$ $h(1)$
$y=2$	1/28	0	0	1/28
$g(x)$	5/14	15/28	3/28	

Y \ X				
1	3/14	3/14	0	3/7 → h(1)
2	1/28	0	0	1/28
g(x)	5/14	15/28	3/28	

Find conditional distribution of X given that Y = 1.

Sol^n

$$f(x|y) = f(x|1) = \frac{f(x,1)}{h(1)} = \frac{f(x,1)}{3/7} = \frac{7}{3} f(x,1)$$

$$\boxed{f(x|y) = \frac{7}{3} f(x,1)}$$

All the possible values of x are 0 1 & 2

x	0	1	2
f(x|1)	1/2	1/2	0

→ This is your answer.

$f(0|1) = \frac{7}{3} f(0,1) = \frac{7}{3} * \frac{3}{14} = \frac{1}{2}$

$f(1|1) = \frac{7}{3} f(1,1) = \frac{7}{3} * \frac{3}{14} = \frac{1}{2}$

$f(2|1) = \frac{7}{3} f(2,1) = \frac{7}{3} * 0 = 0$

What is $P(X=0 | Y=1)$

$= f(0|1)$

$= \frac{1}{2}$

Que The Joint pdf for r.v. X & Y

X → unit temperature change
Y → proportion of Spectrum shift
} which an atomic particle produces

$$f(x,y) = \begin{cases} 10xy^2 & 0 < x < y < 1 \\ 0 & \text{elsewhere} \end{cases}$$

$0 < x < 1$
$0 < y < 1$

Find the probability that the spectrum shifts more than half, given that the temperature is increased by 0.25 units.

Sol^n

$$P\left(Y > \frac{1}{2} \,\Big|\, X = 0.25\right) = \int_{1/2}^{\infty} f(y | x = 0.25)\, dy$$

$$= \int_{1/2}^{1} f(y | x = 0.25)\, dy$$

$$\int_{1/2}^{1} \frac{f(0.25, y)}{g(0.25)}\, dy = \int_{1/2}^{1} \frac{f(0.25, y)}{\frac{10 * 0.25}{3}(1 - 0.25^3)}\, dy = \int_{1/2}^{1} \frac{3y^2}{1-(0.25)^3}\, dy$$

$10 * 0.25 * y^2$

$$= \frac{8}{9} \text{ Ans.}$$

$$g(x) = \int_x^1 f(x,y)\,dy = \int_x^1 10xy^2\,dy = 10x\left(\frac{y^3}{3}\right)_x^1$$

$$= \frac{10x}{3}(1 - x^3) \to g(x)$$

Independent events

$$P(A|B) = P(A)$$

$$P(A \cap B) = P(A)\,P(B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\boxed{P(A \cap B) = P(A)\,P(B)}$$

X and Y are said to be independent if

$$f(x,y) = g(x)\,h(y)$$

f(x,y) ↓ Joint pdf; g(x) h(y) ↓ Marginal pdfs

Proof If X & Y are independent then $f(x|y) = \frac{f(x,y)}{h(y)}$

$$g(x) = \int_{-\infty}^{\infty} f(x,y)\,dy = \int_{-\infty}^{\infty} h(y)\, \underbrace{f(x|y)}_{\text{This is a fn of } x \text{ only}}\, dy$$

$$= f(x|y) \int_{-\infty}^{\infty} h(y)\,dy$$

$$\boxed{g(x) = f(x|y)}$$

$$g(x) = \frac{f(x,y)}{h(y)}$$

$\ast$ $\boxed{f(x,y) = g(x)\,h(y)}$ Definition for statistical independence of r.v.

Note In case of continuous r.v. it is very easy to check $\ast$.

In case of discrete r.v. , we have to check each entry.

One problem