

$X$  &  $Y$ Recap $f(x,y) \rightarrow$  joint pdf $g(x) \rightarrow$  Marginal pdfs of  $X$  $h(y) \rightarrow$  " " "  $Y$ If  $X$  &  $Y$  will be statistically independent

$$f(x,y) = g(x)h(y) \quad \text{for all values of } x \text{ & } y.$$

$$P(A \cap B) = P(A) P(B)$$

Today's work

QueLet pdf for  $X$  &  $Y$  is as follows:-

		$x$			
		0	1	2	$h(y)$
$y$	0	$3/28$	$9/28$	$3/28$	$15/28$
	1	$3/14$	$3/14$	0	$3/7$
	2	$1/28$	0	0	$1/28$
		$g(x)$	$5/14$	$15/28$	$3/28$

Are  $X$  &  $Y$  independent?

Soln For  $X$  &  $Y$  to be independent  
 $f(x,y) = g(x)h(y)$  for all  $x$  &  $y$

$$f(0,1) = \frac{3}{14} \quad g(0) = \frac{5}{14} \quad h(1) = \frac{3}{7}$$

$$\frac{5}{14} \times \frac{3}{7} = \frac{15}{98} \neq \frac{3}{14}$$

 $\therefore$  Not independent.

You can also have

Note $X_1, X_2, X_3, \dots, X_n \rightarrow n \text{ i.i.d.}$  $f(x_1, x_2, \dots, x_n)$  joint pdf

Marginal

$$g_1(x_1) = \sum_{x_2} \sum_{x_3} \dots \sum_{x_n} g(x_1, x_2, \dots, x_n)$$

Chapter 4

Mathematical expectation

 $X \rightarrow$  r.v.pdf  $\rightarrow$  probability

} Model your uncertain situation

 $\downarrow$  Next part is analysis

What is mean?

What is variance/sd?

Mean is nothing but mathematical expectation?How to compute expectation of a r.v.  $X$ .

Data      1   1   2   2   2      3   3   3

mean

$$\frac{1+1+2+2+2+3+3+3}{8}$$

$$= \frac{18}{8}$$

$$\frac{1(2) + 2(3) + 3(3)}{8}$$

$$1\left(\frac{2}{8}\right) + 2\left(\frac{3}{8}\right) + 3\left(\frac{3}{8}\right)$$

relative frequencies = probability

$$1 \underbrace{P(X=1)}_{\text{pdf}} + 2 P(X=2) + 3 P(X=3)$$

$$\text{Expected / mean} = \sum_x x \text{pdf}(x)$$

Given a r.v.  $X$  with  $\text{pdf} = f(x)$ , the expected value/mean is defined as

$$E(X) = \sum_x x f(x) \rightarrow \text{when } X \text{ is discrete}$$

$$= \int x f(x) dx \rightarrow \text{when } X \text{ is continuous}$$

Que:- A lot containing 7 components is sampled by a quality inspector.

The lot contains 4 good components & 3 defective components.

A sample of 3 is taken for inspection. find the expected value of number of good components.

Soln

$X \rightarrow$  no. of good components in a sample of 3

$$X = 0, 1, 2, 3$$

$$E(X) = \sum_x x f(x)$$

$X$	0	1	2	3
$\text{pdf}$	$f(x)$			
	$\frac{{}^3C_3}{{}^7C_3} = \frac{1}{35}$	$\frac{{}^4C_1 \cdot {}^3C_2}{{}^7C_3} = \frac{12}{35}$	$\frac{{}^4C_2 \cdot {}^3C_1}{{}^7C_3} = \frac{18}{35}$	$\frac{{}^4C_3}{{}^7C_3} = \frac{4}{35}$

$$E(X) = 0 \cdot \frac{1}{35} + 1 \cdot \frac{12}{35} + 2 \cdot \frac{18}{35} + 3 \cdot \frac{4}{35} = \frac{12}{7} = 1.7$$

Que A salesperson has two appointments in a day.

appointment 1  $\rightarrow$  He has 70% chances of making a deal

$\rightarrow$  He gets \$1000 commission

appointment 2  $\rightarrow$  He has 40% <sup>chances</sup> of making a deal

$\rightarrow$  He gets \$1500 commission

What is the mean commission.

Soln

$$\begin{array}{cc} X & 1000 & 1500 \\ \text{pdf} & 0.7 & 0.4 \end{array}$$

$$\text{mean } E(X) = 1000 \cdot 0.7 + 1500 \cdot 0.4 = \$1300$$

Commission  
 $\uparrow$   
 $\boxed{X}$

$$\begin{array}{ccccccc} & 0 & 1000 & 1500 & 2500 \\ & \downarrow & \downarrow & \downarrow & \downarrow \\ & (1-0.7) \cdot (1-0.4) & (1-0.7) \cdot 0.4 & (1-0.7) \cdot 0.4 & (0.7) \cdot (0.4) \end{array}$$

$\boxed{X}$

	0	1000	1500	2500
		↓	↓	↓
pdf	$(1-0.7) * (1-0.4)$	$0.7 * (1-0.4)$	$(1-0.7) * 0.4$	$(0.7)(0.4)$
	0.18	0.42	0.12	0.28

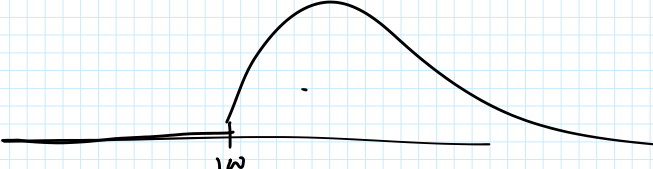
$$0 * 0.18 + 1000 * 0.42 + 1500 * 0.12 + 2500 * 0.28 = \$1700.$$

Que A r.v.  $(X)$  has pdf

$f(x) = \begin{cases} \frac{20000}{x^3} & x > 100 \\ 0 & \text{otherwise} \end{cases}$  find  $E(X)$ ?

Soln

$$E(X) = \int_{-\infty}^{\infty} x f(x) dx = \int_{100}^{\infty} x * \frac{20000}{x^3} dx = 20000 \left[ -\frac{1}{x} \right]_{100}^{\infty}$$

$$= 20000 \left[ 0 + \frac{1}{100} \right] = \boxed{200}$$


$X \rightarrow$  a r.v.

pdf  $\rightarrow f(x)$

then you have  $g(x)$   $g$  is any mathematical function

$$E[g(x)] = \sum_x g(x) f(x) \text{ or } \int g(x) f(x) dx.$$

Que For a r.v.  $X \rightarrow$  no. of cars washed b/w 4 PM to 5 PM in a car wash is

$X$	4	5	6	7	8	9
pdf	$1/12$	$1/12$	$1/4$	$1/4$	$1/6$	$1/6$

Let  $g(x) = (2x-1)$  represents their earning.

What is expected earning.

Soln

$X$	4	5	6	7	8	9	
$g(x)$	$2x-1$	7	9	11	13	15	17
$f(x)$	$1/12$	$1/12$	$1/4$	$1/4$	$1/6$	$1/6$	

$$E(X) = \sum (2x-1) f(x)$$

$$= 7 * \frac{1}{12} + 9 * \frac{1}{12} + 11 * \frac{1}{4} + 13 * \frac{1}{4} + 15 * \frac{1}{8} + 17 * \frac{1}{8} = 12.67$$

H.W X is a k.v. with pdf  $f(x) = \begin{cases} \frac{x^2}{3} & -1 < x < 2 \\ 0 & \text{otherwise} \end{cases}$

Find expected value of  $g(x) = 4x+3$ .

Ans. 8