NUMBER SYSTEMS

1. Natural Numbers

Numbers from 1 one) onward are known as Natural numbers,

denoted by ‘N’.

N = {1, 2, 3, 4,......}

2. Whole Numbers:

Numbers from 0 (zero) onward are known as Whole numbers,

denoted by ‘W’.

W = {0, 1, 2, 3, 4, ...}

3. Integers:

The collection of all whole numbers and negative of natural

numbers are called Integers, denoted by ‘Z’ or ‘I’.

Z or I = {...... –3, –2, –1,0,1,2,3 ......}

4. Rational Number:

A number which can be expressed $\frac{p}{q}$ as where q ≠ 0 and q, q ε Z is know as rational number, denoted by ‘Q’.

5. Irrational Number:

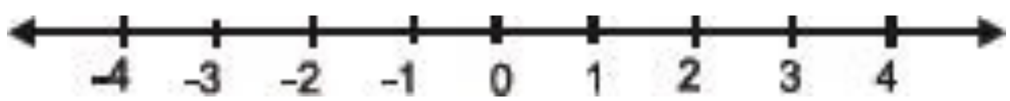
A number which can’t be expressed in the form of p/q and its

decimal representation is non-terminating and non-repeating is

known as irrational number.

e.g., $\sqrt{2}$, $\sqrt{3}$, π,… 1.732105, etc.

6. Number Line:



NUMBER SYSTEMS

Basic Formulae: (Must Remember)

1) $(a - b)^2 = (a^2 + b^2 - 2ab)$
2) $(a + b)^2 = (a^2 + b^2 + 2ab)$
3) $(a + b)(a – b) = (a^2 – b^2)$
4) $(a^3 + b^3) = (a + b)(a^2 – ab + b^2)$
5) $(a^3 - b^3) = (a - b)(a^2 – ab + b^2)$
6) $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
7) $(a^3 + b^3 + c^3 – 3abc) = (a + b + c)(a^2 + b^2 + c^2 – ab – bc – ac)$

1.

If the sum two numbers is 31 and their product is 240, then find the absolute difference between the numbers.

a. 1
b. 3
c. 4
d. 5

Correct Option: (a)

Let two numbers be x and y

We are given that, sum of two numbers $x + y = 31$ and product $= xy = 240$

Therefore,

$x – y = \sqrt{(x + y)^2 – 4xy}$

Substituting the values, we get

$x – y = \sqrt{(31)^2 – 4 \times y}$

$= \sqrt{961 – 960}$

$= \sqrt{1}$

$= 1$

The required difference between the numbers is 1.

2.

Find the three consecutive odd numbers whose sum of the squares is 2531.

a. 19, 21, 23
b. 23, 25, 27
c. 27, 29, 31
d. 31, 33, 35

Correct Option: (c)

Let three consecutive odd numbers be x, x+2, x+4.

$x^2+(x+2)^2+(x+4)^2=2531$

Simplifying we get,

$x^2+4x-837=0$

27 x 31 = 837 and also the difference between 27 and 31 is 4

Therefore,

$x^2+31x-27x-837=0$

$(x + 31) (x - 27)$

X = 27 or x = –31

Hence, the value of

x = 27

(x+2) = 27 + 2 = 29

(x+4) = 27 + 4 = 31

3.

Find a positive number which when increased by 11 is equal to 60 times the reciprocal of the number

a. 3
b. 4
c. 6
d. 9

Correct Option: (b)

Let the number be x.

A positive number (x) increased by 11 is equal to 60 times the reciprocal of the number (1/x)

$$x + 11 = \frac{60}{x}$$

$x^2+11x-60=0$

$x^2+15x-4x-60=0$

$x(x+15)-4(x+15)=0$

$(x+15)(x-4)=0$

x = 4

The positive number (x) = 4

4.

Find the unit digit of $(4137)^{754}$
a. 9
b. 7
c. 3
d. 1

Correct Option:(a)

Hint: Divide b by 4, if it is not divisible then find the remainder of b when divided by 4.

Units digit = a^r, r is the remainder

Number is in the form a^b i.e $(4137)^{754}$

$4 \times 188 = 752$, therefore we get remainder as 2

Units digit = $(4137)^2 = 17114769$

9 is the digit in units place

5.

Find the unit digit in the product $(3^{65} \times 6^{59} \times 7^{71})$

a. 1
b. 4
c. 5
d. 9

Correct Option: (b)

Hint:

If b is multiple of 4

Units digit is 6 : When even numbers 2, 4, 6, 8 are raised to multiple of 4.

Units digit is 1 : When odd numbers 3, 7 and 9 are raised to multiple of 4.

Using the hint given, we can easily solve product of large numbers.

$[3^{(4)16} \times 3] = (1 \times 3) = 3$

$[6^{59}] = 6$

$[7^{71}] = [7^{(4)17} \times 7^3] = [1 \times 3] = 3$

Therefore, $(3 \times 6 \times 3) = 54$

Required unit digit is 4.

6.

1 + 2 + 3 + ---- 50 = ?

a. 1275
b. 1350
c. 1575
d. 1455

Correct Option: (a)

This is an Arithmetic Progression, here a = 1, last term = 50 and n = 50

Sum of n terms = $\frac{n}{2}(a + l)$

Substituting the given values, we get

Sum of n terms = $\frac{50}{2}(1 + 50)$

Sum of n terms = $25 \times 51 = 1275$

7.

Find which of the following number is divisible by 11?

a. 246542
b. 415624
c. 146532
d. 426513

Hint:

Number divisible by 11: If the difference between the sums of the digits at even places and the sum of digits at odd places is either 0 or divisible by 11.

Using the hint, we can easily find the number which is divisible by 11.

Option a) 246542

Sum of digits at even places = (2 + 6 + 4) =12

Sum of digits at odd places = (4 + 5 + 2) = 11

Sum of digits at even places - Sum of digits at odd places = (12 – 11) = 1

Hence, the number is not divisible by 11

Option b) 415624

Sum of digits at even places = (4 + 5 + 2) =11

Sum of digits at odd places = (1 + 6 + 4) = 11

Sum of digits at even places - Sum of digits at odd places = (11 – 11) = 0

Hence, the number is divisible by 11

Option c) 146532

Sum of digits at even places = (1 + 6 + 3) =10

Sum of digits at odd places = (4 + 5 + 2) = 11

Sum of digits at even places - Sum of digits at odd places = (10 – 11) = -1

Hence, the number is not divisible by 11

Therefore, correct answer is option (b) 415624

8.

Find the largest 4 digit number which is divisible by 88.

a. 8844
b. 9999
c. 9944
d. 9930

Correct Option: (c)

We know that the largest 4 digit number is 9999.

Simply divide 9999 by 88.

After dividing 9999 by 88 we get, 55 as remainder.

The number is said to be completely divisible, only if the remainder is zero.

Hence, we can find the required answer by subtracting the remainder obtained from the 4 digit number.

Therefore, required number = 9999 – 55 = 9944

9.

The remainder is 29, when a number is divided 56. If the same number is divided by 8, then what is the remainder?

a. 3
b. 4
c. 7
d. 5

Correct Option : (d)

We know that,

Dividend = [(Divisor × Quotient)] + Remainder

It is given that, the remainder is 29, when a number (dividend) is divided 56(divisor).

Dividend and quotient are unknown, hence assume dividend as X and quotient as Y.

Therefore,

$X = 56 \times Y + 29$

56 is completely divisible by 8, but 29 is not completely divisible and we get remainder as 5, which is the required answer.

OR

$X = 56 \times Y + 29$

$= (8 \times 7Y) + (8 \times 3) + 5$

5 is the required remainder.

10.

Find the largest number of 4-digits divisible by 12, 15 and 18.

a. 9900
b. 9750
c. 9450
d. 9000

Correct Option: (a)

Largest 4-digit number is 9999.

Remember: The question may be asked in a tricky way. Here, largest number does not mean H.C.F.. We have to find a number which is divisible by 12, 15 and 18

Required largest number must be divisible by the L.C.M. of 12, 15 and 18

L.C.M. of 12, 15 and 18

$12 = 2 \times 2 \times 3$

$15 = 5 \times 3$

$18 = 2 \times 3 \times 3$

L.C.M. = 180

Now divide 9999 by 180, we get remainder as 99

The required largest number = (9999 – 99) =9900

Number 9900 is exactly divisible by 180.

11.

Find L.C.M. of 1.05 and 2.1

a. 1.3
b. 1.25
c. 2.1
d. 4.30

Correct Option: (c)

Hint: If numbers are in decimal form, convert them without decimal places. Therefore, the numbers are 105 and 210.

5	105	210
7	21	42
3	3	6
	1	2

L.C.M. = 5 x 7 x 3 x 2 = 210

L.C.M. of 105 and 210 = 210

In decimal form: L.C.M. = 2.1

12. Calculate H.C.F. of $\frac{2}{3}, \frac{16}{81}, \frac{8}{9}$

a. $\frac{2}{9}$

b. $\frac{8}{3}$

c. $\frac{2}{81}$

d. $\frac{3}{16}$

Correct Option: (c)

Hint:

H.C.F. = $\frac{\text{H.C.F. of Numerator}}{\text{L.C.M. of Denominator}}$

H.C.F. = $\frac{\text{H.C.F. of 2, 16, 8}}{\text{L.C.M. of 3, 81, 9}}$

H.C.F. of 2, 16, 8

$2 = 2^1$

$8 = 2^3$

$16 = 2^4$

Number with least index = $2^1 = 2$
H.C.F. of 2, 16, 8 = 2

L.C.M. of 3, 81, 9

$3 = 3^1$

$9 = 3^2$

$81 = 3^4$

Number with highest index = $3^4 = 81$
L.C.M. of 3, 81, 9 = 81

H.C.F. = $\frac{2}{81}$

13.

Find L.C.M. of $\frac{2}{3}, \frac{8}{9}, \frac{64}{81}, \frac{10}{27}$

a. $\frac{250}{9}$

b. $\frac{160}{3}$

c. $\frac{128}{9}$

d. $\frac{320}{3}$

Correct Option: (d)

Hint:

L.C.M. = $\frac{\text{L.C.M. of Numerator}}{\text{H.C.F. of Denominator}}$

L.C.M. of numerators = 2, 8, 64, 10

$2 = 2^1$

$8 = 2^3$

$64 = 2^6$

$10 = 2 \times 5$

L.C.M of 2, 8, 64, 10 = $2^6 \times 5 = 320$

H.C.F. of denominators = 3, 9, 81, 27

$3 = 3^1$

$9 = 3^2$

$81 = 3^4$

$27 = 3^3$

H.C.F. of 3, 9, 81, 27 = 3

L.C.M. of $\frac{2}{3}, \frac{8}{9}, \frac{64}{81}, \frac{10}{27} = \frac{320}{3}$

14
.
Find the greatest number, which on dividing 1657 and 2037 leaves remainders 6 and 5 respectively.

a. 127
b. 132
c. 114
d. 108

The number on dividing 1657 and 2037 leaves remainders 6 and 5 respectively.

Hence, make the dividend completely divisible by the divisor. This is possible, if we subtract remainder from the dividend.

Therefore,

1657 – 6 = 1651

2037 – 5 = 2032

H.C.F. of 1651 and 2032 is 127. 127 is the common factor.

127 × 13 = 1651

Thus by adding 6, we get 1651 + 6 = 1657

127 is the correct answer.

15 .

Find the least number, which when divided by 12, 15, 20 and 54 leaves a remainder of 8 in each case.

a. 548
b. 540
c. 532
d. 524

We are given that, the least number, when divided by 12, 15, 20 and 54 leaves a remainder of 8 in each case.

Therefore, add remainder 8 to the L.C.M. of divisors.

The required least number = (L.C.M. of 12, 15, 20 and 54) + remainder (8)

L.C.M. of 12, 15, 20 and 54

2	12	15	20	54
3	6	15	10	27
2	2	5	10	9
5	1	5	5	9
9	1	1	1	9
	1	1	1	1

L.C.M. = 2 × 3 × 2 × 5 × 9 × 1 = 540

The required least number = (540) + (8) = 548

16 .

5 bells commence tolling together and toll at intervals 2, 4, 6, 8 and 10 seconds respectively. Find in 40 minutes, how many times do they toll together?

a. 8 times
b. 19 times
c. 21 times
d. 30 times

Correct Option: (c)

Given: 5 bells commence tolling together and toll at intervals 2, 4, 6, 8 and 10 seconds respectively
Hence, find the L.C.M. of 2, 4, 6, 8 and 10 seconds.

2	2	4	6	8	10
2	1	2	3	4	5
2	1	1	3	2	5
3	1	1	3	1	5
5	1	1	1	1	5
	1	1	1	1	1

L.C.M. of 2, 4, 6, 8 and 10 = 2 × 2 × 2 × 3 × 5 × 1 = 120
Hence, the bells toll together after 120 seconds = 2 min
We are asked to find, how many times they will toll together in 40 min.

In 40 min, they toll together for = $\left[\frac{40}{} + 1 \right]$ = 21 times

17.

If 2x + 3y = 34 and $\frac{x + y}{y} = \frac{13}{8}$, then find the value of 6x + 4y.

a. 62
b. 58
c. 108
d. 122

Correct Option: (a)

Step 1: Find the value of x and y

$$\frac{x+y}{y} = \frac{13}{8}$$

8x+8y=13y

8x-5y=0-----(1)

Multiply 2x+3y=34 by 4

8x+12y=136----(2)

Adding (1) and (2) we get,

-17y=-136

y=8

Substitute value of y = 8, in (1)

2x+3 × 8=34

x=5

Step 2: Find the value of 6x + 4y

The value of 6x+4y=(6 × 5)+(4 × 8)= 62

18.

A contractor pays Rs. 20 to a worker for each day and the worker forfeits Rs. 10 for each day if he is idle. At the end of 60 days, the worker gets Rs. 300. Find for how many days the worker was idle?

a. 28 days
b. 30 days
c. 34 days
d. 40 days

Correct Option: (b)

Step 1: Number of days worked by the worker = 60 and he remained idle for x days. Therefore, number of days worked = (60 – x)

Step 2: Each day he was getting paid Rs. 20. Therefore, the payment received for working days = (60 – x) 20

Step 3: After subtracting the amount which he forfeited, he receives Rs. 300.

Therefore,

(60 – x) 20 – 10x = 300

1200 – 20x – 10x = 300

900 = 30x

x = 30 days

19.

It is being given that (232 + 1) is completely divisible by a whole number. Which of the following numbers is completely divisible by this number?

A. (216 + 1)

B. (216 - 1)

C. (7 x 223)

D. (296 + 1)

Answer: Option **D**

Explanation:

Let $2^{32} = x$. Then, $(2^{32} + 1) = (x + 1)$.

Let $(x + 1)$ be completely divisible by the natural number N. Then,

$(2^{96} + 1) = [(2^{32})^3 + 1] = (x^3 + 1) = (x + 1)(x^2 - x + 1)$, which is completely divisible by N, since $(x + 1)$ is divisible by N.

20.

What is the unit digit in $\{(6374)^{1793} \times (625)^{317} \times (341^{491})\}$?

A. 0

B. 2

C. 3

D. 5

Answer: Option **A**

Explanation:

Unit digit in $(6374)^{1793}$ = Unit digit in $(4)^{1793}$

= Unit digit in $[(4^2)^{896} \times 4]$

= Unit digit in (6 x 4) = 4

Unit digit in $(625)^{317}$ = Unit digit in $(5)^{317}$ = 5

Unit digit in $(341)^{491}$ = Unit digit in $(1)^{491}$ = 1

Required digit = Unit digit in (4 x 5 x 1) = 0.

21.

Which one of the following numbers is exactly divisible by 11?

A. 235641

B. 245642

C. 315624

D. 415624

Answer: Option **D**

Explanation:

(4 + 5 + 2) - (1 + 6 + 3) = 1, not divisible by 11.

(2 + 6 + 4) - (4 + 5 + 2) = 1, not divisible by 11.

(4 + 6 + 1) - (2 + 5 + 3) = 1, not divisible by 11.

(4 + 6 + 1) - (2 + 5 + 4) = 0, So, 415624 is divisible by 11.

22.

$$\frac{753 \times 753 + 247 \times 247 - 753 \times 247}{753 \times 753 \times 753 + 247 \times 247 \times 247} = ?$$

A. $\frac{1}{1000}$

B. $\frac{1}{506}$

C. $\frac{253}{500}$

D. None of these

Answer: Option **A**

Explanation:

Given Exp. $= \frac{(a^2 + b^2 - ab)}{(a^3 + b^3)} = \frac{1}{(a + b)} = \frac{1}{(753 + 247)} = \frac{1}{1000}$

23.

If the number 481 * 673 is completely divisible by 9, then the smallest whole number in place of * will be:

A. 2

B. 5

C. 6

D. 7

Answer: Option **D**

Explanation:

Sum of digits = $(4 + 8 + 1 + x + 6 + 7 + 3) = (29 + x)$, which must be divisible by 9.

$\therefore \quad x = 7$.

24..

If n is a natural number, then $(6n^2 + 6n)$ is always divisible by:

A. 6 only

B. 6 and 12 both

C. 12 only

D. by 18 only

Answer: Option **B**

Explanation:

$(6n^2 + 6n) = 6n(n + 1)$, which is always divisible by 6 and 12 both, since $n(n + 1)$ is always even.

25.
On dividing a number by 5, we get 3 as remainder. What will the remainder when the square of the this number is divided by 5 ?

A. 0

B. 1

C. 2

D. 4

Answer: Option **D**

Explanation:

Let the number be x and on dividing x by 5, we get k as quotient and 3 as remainder.

$\therefore \quad x = 5k + 3$

$\Rightarrow \quad x^2 = (5k + 3)^2$

$= (25k^2 + 30k + 9)$

$= 5(5k^2 + 6k + 1) + 4$

$\therefore$ On dividing x^2 by 5, we get 4 as remainder.

26.

$$\frac{(963 + 476)^2 + (963 - 476)^2}{(963 \times 963 + 476 \times 476)} = ?$$

A. 1449

B. 497

C. 2

D. 4

Answer: Option **C**

Explanation:

Given Exp. $= \dfrac{(a + b)^2 + (a - b)^2}{(a^2 + b^2)} = \dfrac{2(a^2 + b^2)}{(a^2 + b^2)} = 2$

27.

On dividing a number by 357, we get 39 as remainder. On dividing the same number 17, what will be the remainder ?

A. 0

B. 3

C. 5

D. 11

Answer: Option **C**

Explanation:

Let x be the number and y be the quotient. Then,

$x = 357 \times y + 39$

$= (17 \times 21 \times y) + (17 \times 2) + 5$

$= 17 \times (21y + 2) + 5)$

$\therefore$ Required remainder = 5.

28.

$(11^2 + 12^2 + 13^2 + ... + 20^2) =$?

A. 385

B. 2485

C. 2870

D. 3255

Answer: Option **B**

Explanation:

$(11^2 + 12^2 + 13^2 + ... + 20^2) = (1^2 + 2^2 + 3^2 + ... + 20^2) - (1^2 + 2^2 + 3^2 + ... + 10^2)$

$$\left[\textbf{Ref: } (1^2 + 2^2 + 3^2 + ... + n^2) = \frac{1}{6} n(n+1)(2n+1)\right]$$

$= \left(\frac{20 \times 21 \times 41}{6} - \frac{10 \times 11 \times 21}{6}\right)$

$= (2870 - 385)$

$= 2485.$

29.

On dividing 2272 as well as 875 by 3-digit number N, we get the same remainder. The sum of the digits of N is:

A. 10

B. 11

C. 12

D. 13

Answer: Option **A**

Explanation:

Clearly, (2272 - 875) = 1397, is exactly divisible by N.

Now, 1397 = 11 x 127

∴ The required 3-digit number is 127, the sum of whose digits is 10.

30.
n is a whole number which when divided by 4 gives 3 as remainder. What will be the remainder when 2*n* is divided by 4 ?

A. 3

B. 2

C. 1

D. 0

Answer: Option **B**

Explanation:

Let $n = 4q + 3$. Then $2n = 8q + 6 \;\; = 4(2q + 1) + 2$.

Thus, when $2n$ is divided by 4, the remainder is 2.