Chapter 8
Sampling distributions

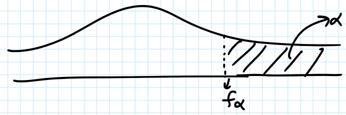
last topic

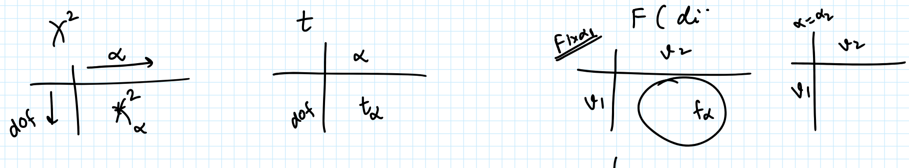
F- distribution

If U & V are two independent r.v. with chi- square distribution with dof $= \nu_1$ & ν_2 respectively, then

$$F = \frac{U/\nu_1}{V/\nu_2}$$ is said to have F- distribution with dof (ν_1, ν_2).

Again the pdf for F distribution is not very friendly.
$\therefore$ we rely on tables.





We have a result

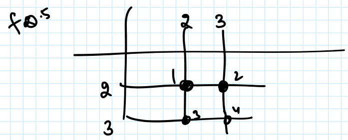
$$f_{0.99}(3,1) = f_{1-0.1}(3,1) = \frac{1}{f_{0.1}(1,3)}$$

Important result

$$f_{1-\alpha}(\nu_1, \nu_2) = \frac{1}{f_{\alpha}(\nu_2, \nu_1)}$$

without proof

We have tables for $\alpha = 0.05$ & $\alpha = 0.01$
$\alpha = 0.95$ $\alpha = 0.99$



$f_{0.5}(2,2) = 1$

$f_{0.5}(2,3) = 2$

$f_{0.5}(3,2) = 3$

$f_{0.95}(2,3) = \frac{1}{f_{0.5}(3,2)} = \frac{1}{3}$

$[F_{0.05} \quad F_{0.95}]$
90 % of data

$[F_{0.01} \quad F_{0.99}]$
98%

We have the following result

Suppose the random samples of size n_1 & n_2 are selected from two normal populations with variances σ_1^2 & σ_2^2 respectively, then

$$F = \frac{s_1^2/\sigma_1^2}{s_2^2/\sigma_2^2} = \frac{\sigma_2^2 s_1^2}{\sigma_1^2 s_2^2}$$ has F distribution with dof $(n_1 - 1, n_2 - 1)$.

Note For $n \geq 30$, we can omit the assumption that population is normally distributed.

Que Two independent samples of items are taken from two normal populations having same variances. The size of 1^{st} sample is 21 & " " 2^{nd} " is 20.

What is the probability that variance of first sample is more than 3 times that of second sample.

$\sigma_1^2 = \sigma_2^2$ $\qquad n_1 = 21$ $\qquad n_2 = 20$

3 times that of second sample.

Soln $\sigma_1^2 = \sigma_2^2$ $n_1 = 21$ $n_2 = 20$

The statistic $F = \dfrac{s_1^2/\sigma_1^2}{s_2^2/\sigma_2^2}$ has F distribution with dof (21-1, 20-1)

($\because \sigma_1^2 = \sigma_2^2$)

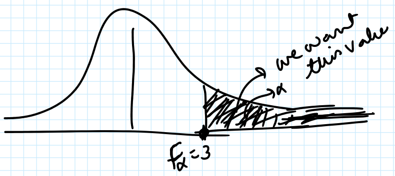
$\dfrac{s_1^2}{s_2^2} \sim F$ dof (20, 19)

Prob. ($s_1^2 \geq 3 s_2^2$) or $P\left(\dfrac{s_1^2}{s_2^2} \geq 3 \right)$

$P(F \geq 3)$ → F_α

This is the val

See from table dof



H.W.

Que Two independent samples are chosen from normal populations having same variances.

I^{st} sample	2^{nd} sample
size → 8	size = 20
$s_1^2 = 100$	$s_2^2 = 40$

$\dfrac{s_1^2/\sigma^2}{s_2^2/\sigma^2} = \dfrac{100}{40} = 2.5$

① Calculate F statistics

② What is the probability that the F value is more than the calculated F-statistics.

$P(F \geq 2.5) = 0.05$

Summary for chapter 8

①	$\dfrac{\bar{X} - \mu}{\sigma/\sqrt{n}}$ → $N(0,1)$	(CLT)	For distribution of sample mean (when σ is known)
②	$\dfrac{(n-1)s^2}{\sigma^2}$ → Chi square distribution with dof $n-1$		sample variance distribution
③	$\dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$ → $N(0,1)$		difference of means
④	$\dfrac{\bar{X} - \mu}{s/\sqrt{n}}$ → t distribution with dof $(n-1)$		For sample mean (when σ is unknown)
⑤	$\dfrac{s_1^2/\sigma_1^2}{s_2^2/\sigma_2^2}$ → F distribution with dof (n_1-1, n_2-1)		For comparing variances of two independent samples.

Note All the five results are for normally distributed population.

But if S.S. n & n_1 & $n_2 \geq 30$ we can always use those results.

10 Quiz I	25 MST	20 Lab	5 marks Quiz (17) NOV chapter 6 & 7	EST 40 marks 1 - 5 chapters 20-30% 6 - 10 " 70-80%

Chapter 9

Estimation problems

Statistical inference (making prediction about population using sample data)

It is of two

Statistical Inference (Making prediction about population using sample data)

It is of two types

Classical method

You rely purely on sample data

Bayesian method.

You rely on sample data + Some knowledge about population parameters.

For next two chapters we are going to do only classical method.

Next There are two kinds of problems

(Chapter 9) Estimation problems

- We would like to estimate something.
- For example

Voting — BJP / Congress

~~Candidate~~ BJP → % of votes.

Here you are estimating % of votes for BJP.

(Chapter 10) Tests of Hypothesis

- Somebody will claim something
- An engineer can claim that the mean life time of his product = 500 days

Hypothesis

- collect sample
- Accept or reject engineer's hypothesis