Interval estimations of mean when population standard deviation is known

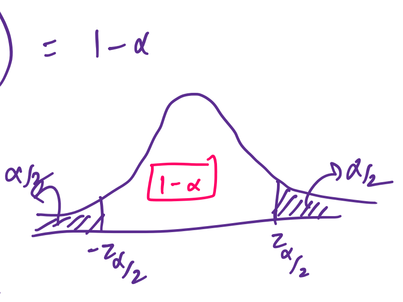
Saturday, 2 December 2023 2:24 PM

- We know that

$$\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \text{ is standard normal distribution } Z(0,1)$$

- So if we are looking for a $(1-\alpha)100\%$ confidence interval then we use the fact that

$$P\left(-z_{\alpha/2} < \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} < z_{\alpha/2}\right) = 1-\alpha$$



multiply with -1 & note the change in inequality signs

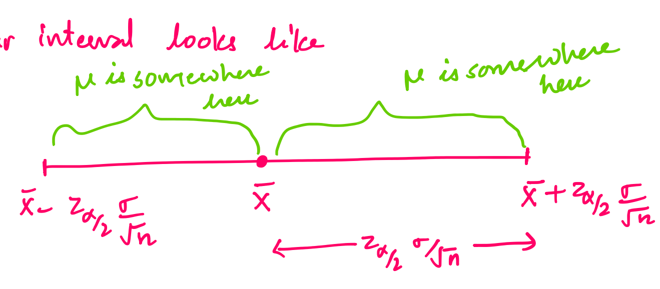
So we have

$$P\left(-z_{\alpha/2} < \frac{\mu-\bar{X}}{\sigma/\sqrt{n}} < z_{\alpha/2}\right) = 1-\alpha$$

$$P\left(\bar{X}-\frac{\sigma}{\sqrt{n}}z_{\alpha/2} < \mu < \bar{X}+\frac{\sigma}{\sqrt{n}}z_{\alpha/2}\right) = 1-\alpha$$

$\therefore$ we are $(1-\alpha)100\%$ confident that $\mu \in \left(\bar{X}-\frac{\sigma}{\sqrt{n}}z_{\alpha/2},\ \bar{X}+\frac{\sigma}{\sqrt{n}}z_{\alpha/2}\right)$

Note that our interval looks like



$\therefore$ the error between $\bar{X}$ & μ is at the maximum $= z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$

- If we denote error with e.
- Then $e \le z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$

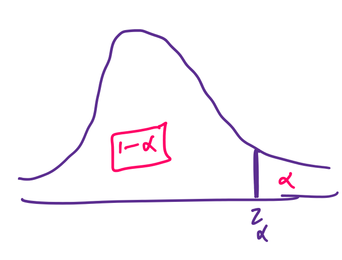
$$\Rightarrow \boxed{n \geq \frac{z_{\alpha/2}^2\,\sigma^2}{e^2}} \quad \text{Result.}$$

Sometimes we are interested in one sided bounds, then we proceed as follows

$$P\left(\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} < z_\alpha\right) = 1-\alpha$$

multiply with -1

$$P\left(\frac{\mu-\bar{X}}{\sigma/\sqrt{n}} > -z_\alpha\right) = 1-\alpha$$



$$P\left(\mu > \bar{X}-z_\alpha\frac{\sigma}{\sqrt{n}}\right) = 1-\alpha$$

This is one sided lower bound.

Note here we use z_α and not $z_{\alpha/2}$

Similarly we have

$$P\left(\mu < \bar{X}+z_\alpha\frac{\sigma}{\sqrt{n}}\right) = 1-\alpha$$

one sided upper bound

Que The average zinc concentration recovered from a sample of measurements taken in 36 different locations in a river is found to be 2.6 grams per millilitre. Find the 95% and 99% confidence interval for the mean zinc concentration in the river. Assume that the population standard deviation is 0.3 gram per milliliter.

Sol^n

$\bar{x} = 2.6$

95% interval

$1-\alpha = 0.95$

$\alpha = 0.05$

$\frac{\alpha}{2} = 0.025$

$z_{\alpha/2} = z_{0.025} = 1.96$ (from table)

95% confidence interval is

$$\left(\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}},\ \bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right)$$

$$\left(2.6-1.96\left(\frac{0.3}{\sqrt{36}}\right),\ 2.6+1.96\left(\frac{0.3}{\sqrt{36}}\right)\right)$$

$$(2.50,\ 2.70)$$

99% confidence interval

$1-\alpha = 0.99$

$\alpha = 0.01$

$\frac{\alpha}{2} = 0.005$

$z_{\alpha/2} = z_{0.005} = 2.575$ from table

Confidence interval is

$$\left(\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}},\ \bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right)$$

$$\left(2.6-(2.575)\left(\frac{0.3}{\sqrt{36}}\right),\ 2.6+2.575\left(\frac{0.3}{\sqrt{36}}\right)\right)$$

$$= (2.47,\ 2.73)$$

Que How large a sample is required if we want to be 95% confident that our estimate of μ in the last question is off by less than 0.05?

Sol^n

$\sigma = 0.3$

$$n \geq \left(\frac{1.96 * 0.3}{0.05}\right)^2 \quad \left[\because n \geq \left(\frac{z_{\alpha/2}\,\sigma}{e}\right)^2\right]$$

$n \geq 138.3$

$$\boxed{n = 139}$$

Que In a psychological testing experiment, 25 subjects are selected randomly and their reaction time, in seconds is measured. Assume $\sigma^2 = 4$ sec and the distribution of reaction is normally distributed. Further $\bar{x} = 6.2$ seconds. Give an upper 99% Bound for mean reaction time.

$1-\alpha = 0.99 \Rightarrow \alpha = 0.01$

Sol^n

$$\text{Upper bound} = \bar{x} + z_\alpha\frac{\sigma}{\sqrt{n}} = 6.2 + (1.645)\sqrt{\frac{4}{25}}$$

$$= 6.2 + 0.658$$

$$= 6.858 \text{ seconds.}$$

So we are 95% confident that the mean reaction time is less than 6.858 seconds.