Hypothesis testing for difference of means.

- Two independent random samples of sizes n_1 and n_2 respectively, are drawn from two populations with means μ_1 & μ_2 and variances σ_1^2 and σ_2^2. We know that the random variable

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

has a standard normal distribution.

- Here we are assuming that n_1 & n_2 are sufficiently large that the CLT applies.

- Of course, if the two populations are normal, the statistics above has standard normal distribution for any n_1 & n_2.

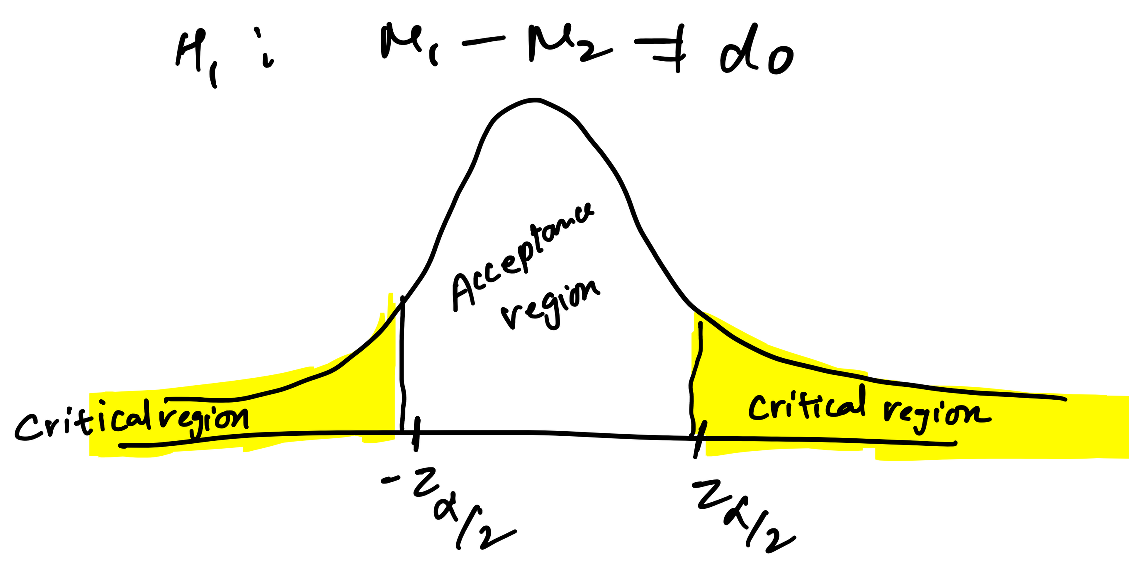
- If $\sigma_1 = \sigma_2$, then the statistics become

$$z = \frac{\bar{x}_1 - \bar{x}_2 - (\mu_1 - \mu_2)}{\sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

- Two sided hypothesis on two means can be generally written as

$H_0: \quad \mu_1 - \mu_2 = d_0$

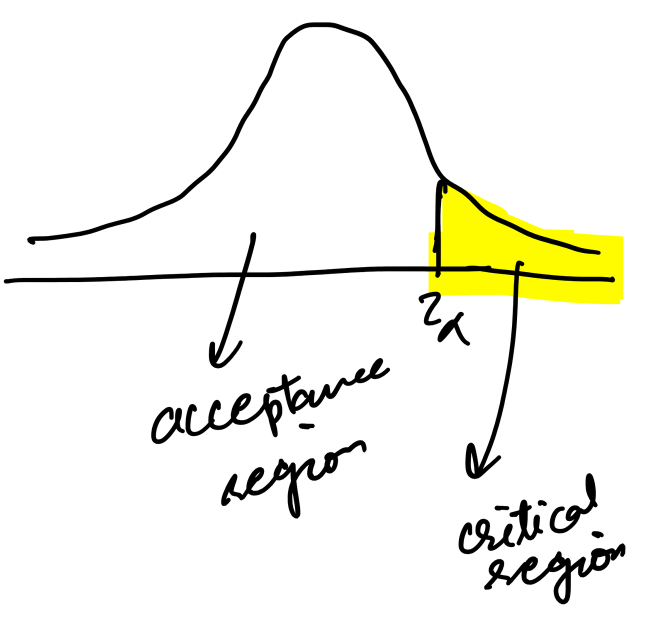
$H_1: \quad \mu_1 - \mu_2 \neq d_0$



- One sided tests

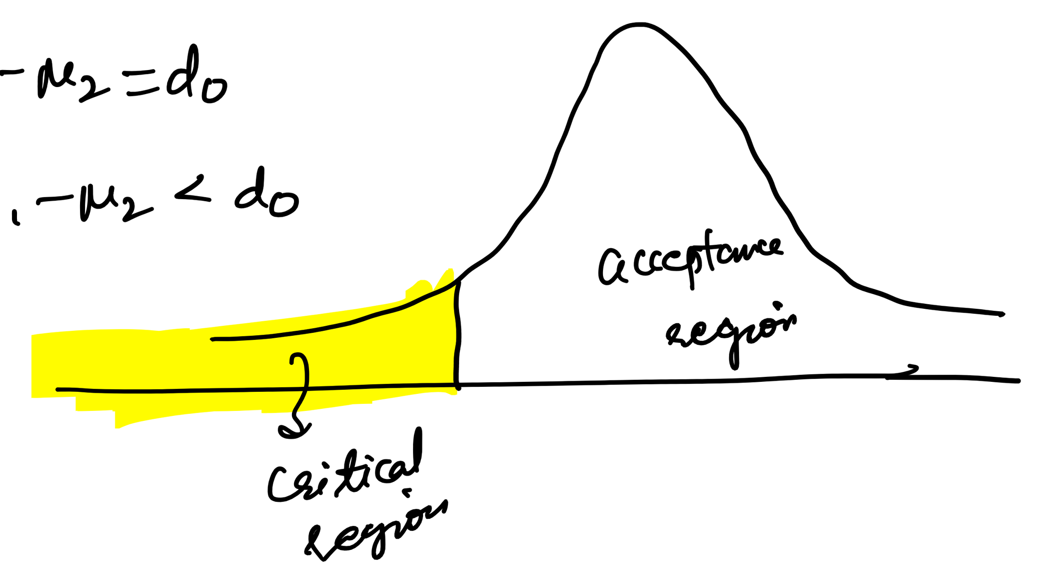
$H_0: \quad \mu_1 - \mu_2 = d_0$

$H_1: \quad \mu_1 - \mu_2 > d_0$



$H_0: \mu_1 - \mu_2 = d_0$

$H_1 \quad \mu_1 - \mu_2 < d_0$



- When the variances are unknown but $\sigma_1 = \sigma_2 = \sigma$ and distribution is normal.

- Two sample pooled t-test

$H_0: \mu_1 = \mu_2 \qquad \mu_1 - \mu_2 = d_0$

$H_1: \mu_1 \neq \mu_2$

we reject H_0 at the significance level α when

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - d_0}{s_p\sqrt{1/n_1 + 1/n_2}}$$

where $s_p^2 = \dfrac{s_1^2(n_1-1) + s_2^2(n_2-1)}{n_1+n_2-2}$

exceeds $t_{\alpha/2,\ \underbrace{n_1+n_2-2}_{\text{dof}}}$ or

$< -t_{\alpha/2,\ n_1+n_2-2}$

Similarly, we can go for one sided tests.

Que An experiment was performed to compare the abrasive wear of two different laminated materials. Twelve pieces of material I were tested by exposing each piece to a machine measuring wear.
Ten pieces of material II were similarly tested.

Sample I – gives $\mu_1 = 85$ units $\quad s_1 = 4$

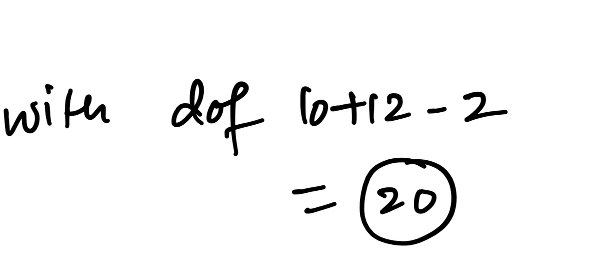
Sample II – $\quad \mu_2 = 81 \quad s_2 = 5$

Can we conclude at the 0.05 level of significance that the abrasive wear of material I exceeds that of material II by more than 2 units?
Assume population to be approximately normal with equal variances.

Sol^n $H_0: \quad \mu_1 - \mu_2 = 2$

$H_1: \quad \mu_1 - \mu_2 > 2$

$\alpha = 0.05$



critical region is $t > t_\alpha \to t_{0.05} = 1.725$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - d_0}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$ with dof $10+12-2 = 20$

$$s_p = \sqrt{\frac{(11)(16) + (9)(25)}{12+10-2}} = 4.478,$$

$$t = \frac{(85-81) - 2}{4.478\sqrt{\frac{1}{12} + \frac{1}{10}}} = 1.04$$

less than critical value
∴ Do not reject H_0

or $P = P(T > 1.04)$

$\approx 0.16 \to$ more than 0.05

∴ Do not reject H_0.

Hence we are unable to conclude that the abrasive wear of the material I exceeds that of material II by more than 2 units.