Lecture 32

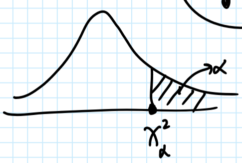
09 November 2022 10:49

Chapter 8

Recap

If you have $S^2 \rightarrow$ sample variance

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} \sim \text{chi square distribution with dof } (n-1)$$



- How to see tables.
- We saw that 95% of data lies b/w

$$\chi^2_{0.975} \ \& \ \chi^2_{0.025}$$

$\therefore \left[\chi^2_{0.975} \quad \chi^2_{0.025}\right] \rightarrow$ 95% of data lie.

How we shall proceed with Questions.

1. Engineer makes a claim $\rightarrow$ Hypothesis
2. You compute sample statistics $\frac{X-\mu}{\sigma/\sqrt{n}}$; $\chi^2 = \frac{(n-1)s^2}{\sigma^2} \rightarrow$
3. Check the interval $\chi^2 \rightarrow \left[\chi^2_{0.975} \quad \chi^2_{0.025}\right]$
4. If sample statistics lies in this interval $\Rightarrow$ data supports engineer

   Otherwise data refutes engineer.

Que The score on a placement test given to college students is having a normal distribution with mean μ = 74 & $\sigma^2 = 8$.

Would you still consider $\sigma^2 = 8$ to be a valid value of variance if a random sample of 20 students' scores has $s^2 = 20$?

Sol^n $\sigma^2 = 8$ $\quad s^2 = 20$ $\quad n = 20$

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{19 * 20}{8} = 47.5 \rightarrow \text{value of Sample statistics } \chi^2$$

If the claim is true 95% of data should lie in the interval $\left[\chi^2_{0.975} \quad \chi^2_{0.025}\right]$ look at the table of chi square dof = 19

$$[8.91 \ , \ 32.85]$$

Since 47.5 is outside the interval. ∴ the claim about variance of population is false.

Que A manufacturer of car batteries guarantees that the life of his batteries has a mean = 3 years & S.D. = 1 year.
If five of these batteries have lifetimes = 1.9, 2.4, 3.0, 3.5 & 4.2 years Comment about manufacturer's claim about population variance.
(Assume original population is normally distributed).

Sol^n $\sigma^2 = 1^2 = 1$

$n \rightarrow 5$ sample size

$s^2 = ?$

For $s^2 = \frac{1}{n-1} \sum_{i=1}^{5} (x_i - \bar{x})^2$

$\bar{X} = \frac{\sum x_i}{5} = \frac{1.9+2.4+3.0+3.5+4.2}{5}$

$s^2 = 0.815$ → sample variance

Now $\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(5-1) * 0.815}{1} = 3.26$ → sample statistics.

Next the interval $[\chi^2_{0.975} \quad \chi^2_{0.025}] \longrightarrow [0.484 \quad 11.143]$

Table dof = 4

Since sample statistics $\chi^2 \in [0.484, 11.143]$

∴ Sample Data supports manufacturer's claim.

Population parameter	μ	σ^2	σ			
Sample Statistics	$\bar{X}$	S^2	S	χ^2	t	F

Next

t-distribution

If $Z \rightarrow$ standard normal distribution

& $V \rightarrow$ chi square distribution with dof $= \nu$.

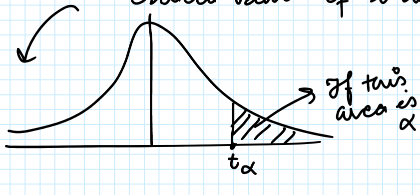
then define a new r.v. $t = \frac{Z}{\sqrt{V/\nu}}$ → new r.v. is said to have t distribution with dof $= \nu$.

Note t-distribution is also called Student's t-distribution

We can write its pdf but that is not very friendly.

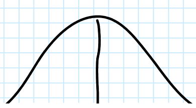
↓

we rely on tables of critical value of t-distribution



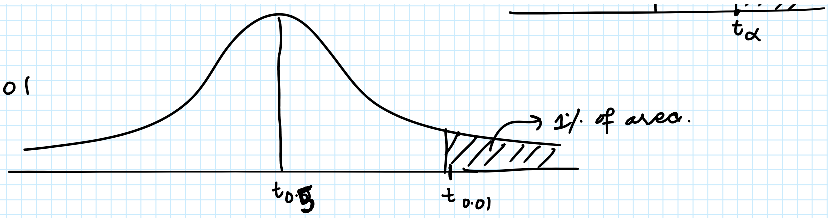
Notation

$t_{0.01}$



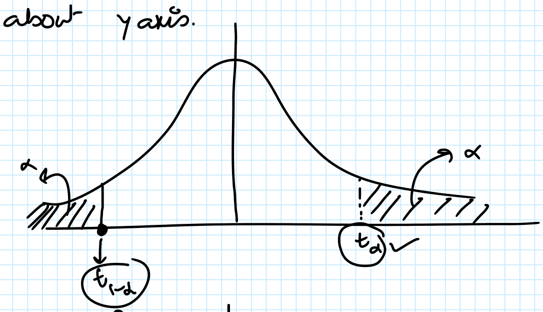
$t_{0.01}$

$t_{0.5}$



Note :- χ^2 was not symmetric about Y axis

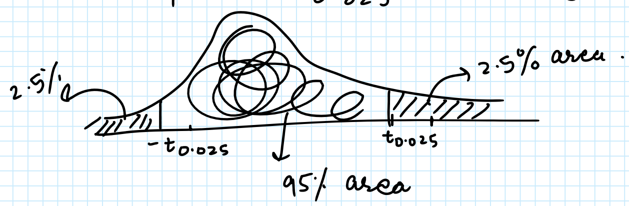
t is symmetric about y axis.



By symmetry

$$\boxed{t_{1-\alpha} = -t_\alpha}$$

Now Can you tell me what is the area b/w $-t_{0.025}$ & $t_{0.025}$



$\therefore$ For t distribution 95% of data lies in $[-t_{0.025} \quad t_{0.025}]$

Result If population variance is not known, then CLT cannot be applied.

$$t = \frac{\bar{X} - \mu}{S/\sqrt{n}}$$ has t distribution with dof $= n-1$ when original population is normally distributed.

Que A chemical engineer claims that the population mean yield of a certain batch process is 500 grams $\rightarrow \mu$ per milliliter of raw material.

He is satisfied if the sample yield, lies in $[-t_{0.05} \quad t_{0.05}]$.

He collects sample of size 25 & found $\bar{X} = 518$ & $s = 40$.

Should he be satisfied with his claim about μ?

Sol^n

$$t = \frac{\bar{X} - \mu}{s/\sqrt{n}} = \frac{518 - 500}{40/\sqrt{25}} = 2.25 \leftarrow \text{Sample statistics}$$

Interval is $[-t_{0.05}, t_{0.05}]$ $\rightarrow$ From table dof $= 24$

$[-1.711, 1.711]$

$2.25 \notin [-1.711, 1.711]$ $\quad \therefore$ His claim is false.