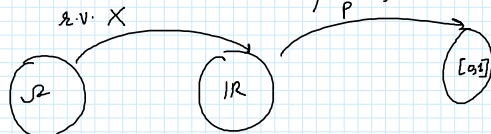


Recap

- Random Variables (r.v.)
- Discrete r.v.
- Continuous r.v.
- probability distribution function pdf / pmf



Que If a car company sells 50% of cars with side airbags & 50% without side airbags depending on customer's choice. What will be the pdf / pmf for the r.v.

X = number of cars with side airbags among the next 4 cars sold by the company.

Soln

X can assume 0, 1, 2, 3, 4

pmf

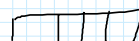
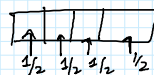
$$P(X=0) = \frac{1}{16}$$

$$P(X=3) = \frac{{}^4C_3}{16}$$

$$P(X=1) = \frac{{}^4C_1}{16}$$

$$P(X=4) = \frac{{}^4C_4}{16}$$

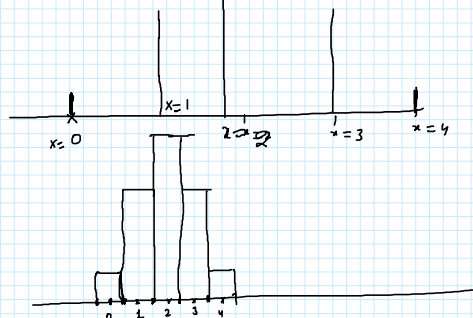
$$P(X=2) = \frac{{}^4C_2}{16}$$



x	0	1	2	3	4
$P(X=x)$	$\frac{1}{16}$	$\frac{4}{16}$	$\frac{6}{16}$	$\frac{4}{16}$	$\frac{1}{16}$

Plot
pmf plot

Probability histogram



Sometimes we are interested in $f(x) \rightarrow$ pmf

$$P(X \leq x) = \sum_{y \leq x} f(y)$$

For example

Tossing a coin twice

 $X \rightarrow$ no. of heads X can assume 0, 1, 2

$$f(x): P(X=x) \quad \begin{array}{ccc} 0 & 1 & 2 \\ \downarrow & \downarrow & \downarrow \\ \frac{1}{4} & \frac{2}{4} & \frac{1}{4} \end{array}$$

Cumulative distribution function

$$P(X \leq 0) = 0$$

$$P(X \leq 1) = \frac{1}{4}$$

$$P(X \leq 1.5) = P(X=0) + P(X=1) = \frac{1}{4} + \frac{2}{4} = \frac{3}{4}$$

$$P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) = \frac{1}{4} + \frac{2}{4} + \frac{1}{4} = 1$$

(cdf)

Definition The cumulative distribution function $F(x)$ of a discrete r.v. X

$$P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3) = 1/4 + 1/4 + 1/4 + 1/4 = 1.$$

Definition The cumulative distribution function $F(x)$ of a discrete r.v. X with pmf $f(x)$ is

$$F(x) = P(X \leq x) = \sum_{y \leq x} f(y)$$

Example on the tosses of three coins
 $X \rightarrow$ no. of heads.

$x =$	0	1	2	3
$P(X=x)$	$1/8$	$3/8$	$3/8$	$1/8$

$$F(m) = P(X \leq m)$$

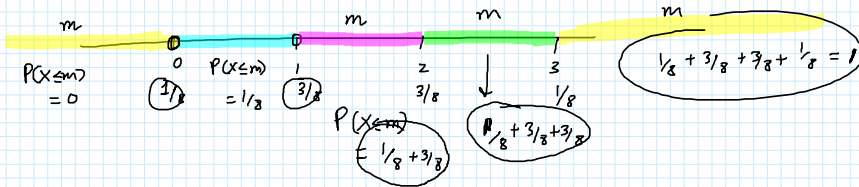
$$F(m) = \begin{cases} 0 & m < 0 \\ f(0) = 1/8 & 0 \leq m < 1 \\ f(0) + f(1) & 1 \leq m < 2 \\ 1 & m \geq 2 \end{cases}$$

$$P(X \leq 0.5) = P(X=0) = f(0)$$

$$P(X \leq 1.5) = f(0) + f(1)$$

$$F(m) = \begin{cases} 0 & m < 0 \\ 1/8 & 0 \leq m < 1 \\ 4/8 & 1 \leq m < 2 \\ 1 & m \geq 2 \end{cases}$$

$X \rightarrow$ no. of heads



Que

Car company 50% with side air bags
 50% without " "

$X \rightarrow$ no. of cars with side airbags in the next 4 sellings.

$X =$ 0 1 2 3 4

$P(X=x)$	$1/16$	$4/16$	$6/16$	$4/16$	$1/16$
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cdf ~~$F(x)$~~

$$F(m) = \begin{cases} 0 & m < 0 \\ 1/16 & 0 \leq m < 1 \\ 1/16 + 4/16 = 5/16 & 1 \leq m < 2 \\ 1/16 + 4/16 + 6/16 = 11/16 & 2 \leq m < 3 \\ 1/16 + 4/16 + 6/16 + 4/16 = 15/16 & 3 \leq m < 4 \\ 1 & m \geq 4 \end{cases}$$

$$P(X \leq 5)$$

