Chapter 9 Estimation problems

- population parameter → Point estimation
  → Interval estimation

Another way is to approximate using Maximum likelihood estimation

- Binomial process → p is the parameter associated
  n
  You perform 100 trials & you get a sample

  D N D D N - - - - - - D ✓

  On the basis of this sample you will try to find p.

- Suppose my binomial experiment 3 trials
  - trial → D (p)
    → N ($1-p$)

  (we want to find out p)

When I actually performed the experiment, suppose I get

D D N

$\begin{cases} p \\ 1-p \end{cases}$

probability of getting this outcome $= \underbrace{p \cdot p\ (1-p)}_{\text{maximize it}}$ Likelihood function

MLE gives you a point estimation

$$L(p) = p^2(1-p)$$

$$L'(p) = 0 \Rightarrow 2p - 3p^2 = 0$$

$$\Rightarrow p = 0 \text{ or } \underbrace{p = 2/3}_{\text{Estimation for } p \text{ by MLE}} \text{ Ans.}$$

Formally MLE (maximum likelihood estimation)

Given independent observations $x_1, x_2 \ldots x_n$ from a pdf / pmf $f(x, \theta)$

$$\underbrace{L(x_1, x_2 \ldots x_n, \theta)}_{\text{Likelihood function}} = f(x_1, \theta) * f(x_2, \theta) \cdots * f(x_n, \theta)$$

maximize this function by using $L'(\theta) = 0$, that gives you an estimation for θ.

Que Consider samples $x_1, x_2 \ldots x_n$ from a normal distribution $N(\mu, \sigma)$. Find the maximum likelihood estimators of μ & σ.

$$\text{pdf} = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

Sol^n Likelihood function

$$L(x_1, x_2 \ldots x_n, \mu, \sigma^2) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x_1-\mu}{\sigma}\right)^2} \cdot \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x_2-\mu}{\sigma}\right)^2} \cdots\cdots \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x_n-\mu}{\sigma}\right)^2}$$

$$L = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left(\frac{-1}{2}\left(\frac{x_i-\mu}{\sigma}\right)^2\right)$$

$$L = \frac{1}{(2\pi)^{n/2}\,\sigma^n} \exp\left(\frac{-1}{2}\sum_{i=1}^{n}\left(\frac{x_i-\mu}{\sigma}\right)^2\right)$$

$$\ln \underset{(\mu,\sigma)}{L} = -\frac{n}{2}\ln(2\pi) - \frac{n}{2}\log\sigma^2$$

$$\ln L_{(\mu,\sigma)} = -\frac{n}{2}\ln(2\pi) - \frac{n}{2}\log\sigma^2 - \frac{1}{2}\sum_{i=1}^{n}\left(\frac{x_i-\mu}{\sigma}\right)^2$$

differentiate with respect to μ

$$\frac{\partial \ln L}{\partial \mu} = -\frac{1}{\cancel{2}}\sum_{i=1}^{n}\cancel{2}\left(\frac{x_i-\mu}{\sigma}\right)(-1) = 0$$

$$\sum_{i=1}^{n}(x_i-\mu) = 0$$

$$\sum_{i=1}^{n}x_i - n\mu = 0$$

$$\Rightarrow \boxed{\mu = \frac{1}{n}\sum_{i=1}^{n}x_i}$$

MLE estimator of μ.

For σ^2 differentiate w.r.t σ^2

$$\frac{\partial \ln L}{\partial \sigma^2} = \frac{-n}{2\sigma^2} + \frac{1}{2\sigma^4}\sum_{i=1}^{n}(x_i-\mu)^2 = 0$$

$$\frac{n}{2\sigma^2} = \frac{1}{2\sigma^4}\sum_{i=1}^{n}(x_i-\mu)^2$$

$$\boxed{\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\mu)^2}$$

MLE estimator for σ^2 is not unbiased.

Que Suppose 10 rats are used in a biomedical study where they are injected with cancer cells and then given a cancer drug that is designed to increase their survival rate. The survival times, in months, are 14, 17, 27, 18, 12, 8, 22, 13, 19 & 12. Assume that the exponential distribution applies. Give a maximum likelihood estimate of the mean survival rate.

Sol^n exponential distribution

$$f(x,\beta) = \begin{cases} \frac{1}{\beta}e^{-x/\beta} & x>0 \\ 0 & \text{otherwise} \end{cases}$$

$\beta \to$ mean value.

$$L(\beta) = \left(\frac{1}{\beta}e^{-\frac{x_1}{\beta}}\right)\left(\frac{1}{\beta}e^{-\frac{x_2}{\beta}}\right)\cdots\left(\frac{1}{\beta}e^{-\frac{x_{10}}{\beta}}\right)$$

$$L(\beta) = \frac{1}{\beta^{10}}e^{-\frac{1}{\beta}\sum_{i=1}^{10}x_i} \longrightarrow \ln L = -10\ln\beta - \frac{1}{\beta}\sum_{i=1}^{10}x_i$$

$$\frac{\partial L}{\partial \beta} = -\frac{10}{\beta} + \frac{1}{\beta^2}\sum_{i=1}^{10}x_i \doteq 0 \Rightarrow \beta = \frac{1}{10}\sum_{i=1}^{10}x_i = \boxed{16.2}$$

$$\frac{d^2L}{d\beta^2} = \frac{10}{\beta^2} - \frac{2}{\beta^3}\sum_{i=1}^{10}x_i$$

this no. is -ve for $\beta = 16.2$

H.W

Que It is known that a sample consisting of values 12, 11.2, 13.5, 12.3, 13.8, & 11.9 comes from a population with pdf

$$f(x,\theta) = \begin{cases} \frac{\theta}{x^{\theta+1}} & x>1 \\ 0 & \text{otherwise} \end{cases}$$

where $\theta > 0$.

$$f(x;\theta) = \begin{cases} \dfrac{\theta}{x^{\theta+1}} & x > 1 \\ 0 & \text{otherwise} \end{cases} \quad \text{where } \theta > 0.$$

Find the MLE of θ.

Sol^n Ans is 0.3970 (Do it yourself). End of chapter 9

Chapter 10 (last chapter)

Test of Hypothesis

- You make a claim ← Hypothesis
- A statistical procedure to reject or fail to reject the hypothesis.

Hypothesis : A person is guilty.

Steps for Hypothesis testing:-

STEP I We will always state the given Hypothesis in the following way:-

Null Hypothesis H_0: $\mu = 68$ ⇝ what you are claiming. Claim population mean $\mu = 68$

Alternative " H_1: $\mu \neq 68$ ⇝ contraction to null hypothesis.

4 more steps ←

Decision Step $\left\{ \begin{array}{l} \text{We reject } H_0 \text{ in favor of } H_1 \\ \text{We fail to reject } H_0 \end{array} \right\}$ These will be our final statements.
