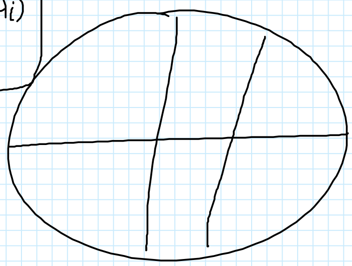
Lecture 7(Gr 25-27)

09 August 2022 13:44

TBT

$$P(B) = \sum_{i=1}^{n} P(A_i)\, P(B|A_i)$$



Que Alice is taking a probability class & at the end of each week, she can either be upto date or behind.

If she is upto date in a given week, then

the probability that she is upto date the following week → 0.8

" " " " " behind " " " → 0.2

If she is behind in a given week, then

the probability that she will be upto date the following week = 0.4

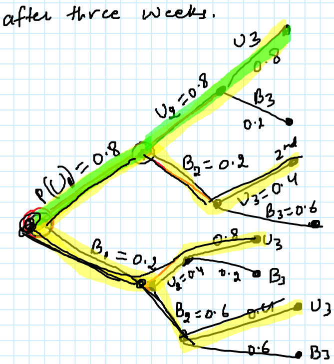
" " " " " " behind " " " = 0.6

By default she starts upto date. What is the probability that she is upto date after three weeks.

Soln

U_i → uptodate after i^{th} week

B_i → Behind after i^{th} week



$0.8 * 0.8 * 0.8$

$+ 0.8 * 0.2 * 0.4$

$+ 0.2 * 0.4 * 0.8$

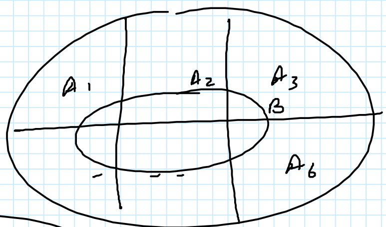
$+ 0.2 * 0.6 * 0.4$

$= \cancel{0.375}$ 0.688

Baye's theorem

TPT

$$P(B) = \sum_{i=1}^{n} P(A_i) P(B|A_i)$$



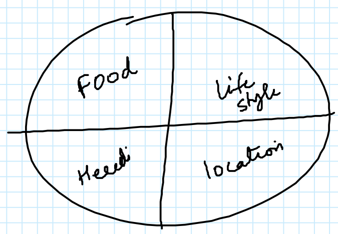
A_1

A_2

A_3

A_4

Out of all the possible causes, which one is most responsible.



$A_1, A_2 - - - A_n$ → causes

B → disease

$P(A_1|B)$

$P(A_2|B)$

$P(A_i|B)$

$\rightarrow P(A_1|B)$

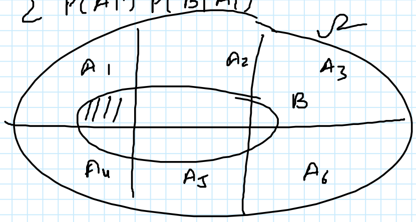
$P(A_2|B)$

$\vdots$ $\quad P(A_i|B)$

Statement :- If the disjoint events $A_1, A_2 \ldots A_n$ consititute a partition of Ω s.t. $P(A_i) > 0$ for all i, then for any event B

$$P(A_i|B) = \frac{P(A_i)\,P(B|A_i)}{\sum P(A_i)\,P(B|A_i)}$$

Proof $B = (B \cap A_1) \cup (B \cap A_2) \cup \cdots \cup (B \cap A_n)$



$$P(B) = \sum_{i=1}^{n} P(B \cap A_i)$$

$$\downarrow$$

$$P(B) = \sum_{i=1}^{n} P(A_i)\,P(B|A_i)$$

$$P(A_i|B) = \frac{P(A_i \cap B)}{P(B)} = \frac{P(A_i)\,P(B|A_i)}{\sum_{i=1}^{n} P(A_i)\,P(B|A_i)}$$

$$\boxed{P(\underset{\text{cause}}{A_i}|\underset{\text{disease}}{B}) = \frac{P(A_i) * P(B|A_i)}{\sum_{i=1}^{n} P(A_i)\,P(B|A_i)}} \quad \star$$

Que In a company there are 3 manufacturing plans

plan 1 $\rightarrow$ used for 30% of production

plan 2 $\rightarrow$ " " 20% " "

plan 3 $\rightarrow$ " " 50% " " ✓

$D \rightarrow$ final product is defective.

$P(D|P_1) = 0.01 \quad * \quad 0.3 \;/\; (0.01 * 0.3 + 0.03 * 0.2 + 0.02 * 0.5)$

$P(D|P_2) = 0.03 \quad * \quad 0.2$

$P(D|P_3) = 0.02 \quad * \quad 0.5$

If a randomly chosen product is defective, which plan was most likely used?

$P(P_1|D) = 0.158$

$P(P_2|D) = 0.316$

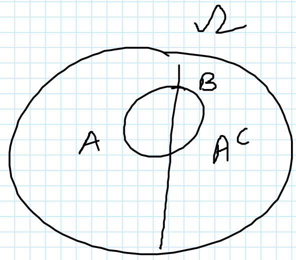
$P(P_3|D) = 0.526$

Que A test for a certain disease is assumed to be correct 95% of the time. A random person drawn from certain population has probability 0.001 of having the disease. Given that a person just tested positive, what is probability of him having the disease?

Sol^n

$A \rightarrow$ person has the disease

$B \rightarrow$ Test is positive



$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

$$= \frac{0.95 * 0.001}{\underset{0.001}{P(A)}\,\underset{0.95}{P(B|A)} + \underset{0.999}{P(A^C)}\,P(B|A^C)}$$

$0.999 * 0.05$

$$= 0.0187$$

Ans.