Confidence interval for proportion

Saturday, 2 December 2023 4:06 PM

Here we use the fact that

$$\frac{\hat{p}-p}{\sqrt{\frac{pq}{n}}} \text{ is a } z \text{ distribution.} \quad (q = 1-p)$$

and we can further replace p & q in denominator by $\hat{p}$ & $\hat{q} = 1-\hat{p}$

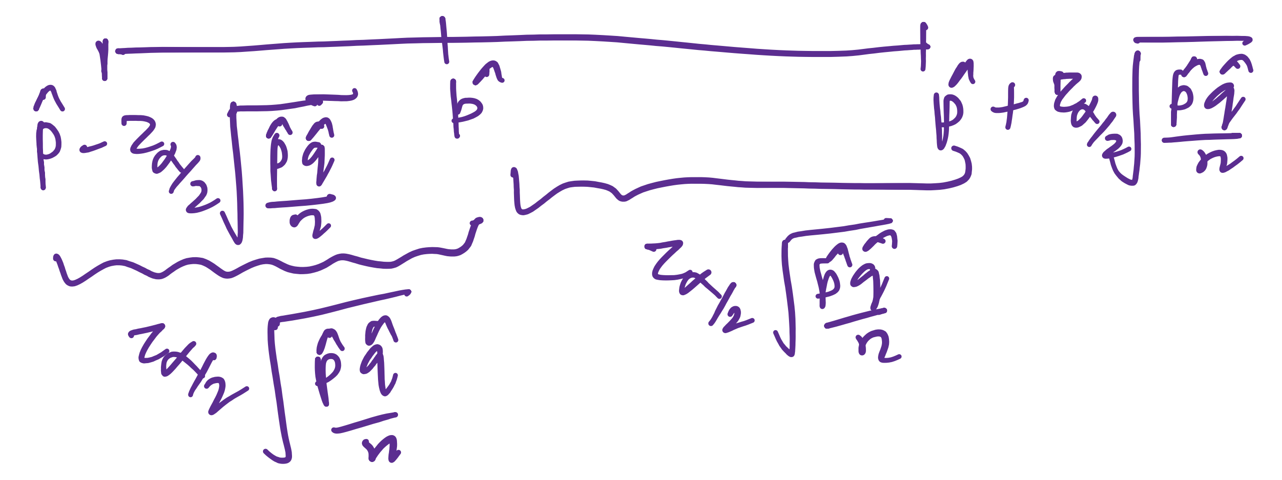
$$P\left(-z_{\alpha/2} < \frac{p-\hat{p}}{\sqrt{\frac{\hat{p}\hat{q}}{n}}} < z_{\alpha/2}\right) = 1-\alpha$$

$$P\left(\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}\right) = 1-\alpha$$

$\therefore$ $(1-\alpha)100\%$ confidence interval is

$$\left(\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}},\ \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}\right)$$

Note Interval looks like



maximum error $= z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$

$$e = z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$\boxed{n = \frac{z_{\alpha/2}^2\,\hat{p}\hat{q}}{e^2}}$$

Que In a random sample of $n=500$ families owing TV, it is found that $x=340$ subscribe to Netflix. Find a 95% confidence interval for the actual proportion of families having Netflix.

Sol^n 95% confidence interval

$1-\alpha = 0.95$

$\alpha = 0.05$

$\hat{p} = \frac{340}{500} = 0.68$

$\hat{q} = 1-0.68 = 0.32$

Interval is

$$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$0.68 - 1.96\sqrt{\frac{(0.68)(0.32)}{500}} < p < 0.68 + 1.96\sqrt{\frac{(0.68)(0.32)}{500}}$$

$$\boxed{0.6391 < p < 0.7209}$$

Que:- How large a sample is required if we want to be 95% confident that our estimate of p is within 0.02 of the true value?

Sol^n

$$n = \frac{z_{\alpha/2}^2\,\hat{p}\hat{q}}{e^2}$$

$$= \frac{(1.96)^2\,(0.68)\,(0.32)}{(0.02)^2}$$

$$= 2089.8$$

$$\therefore \boxed{n = 2090}$$