

Confidence interval for σ^2

Saturday, 2 December 2023

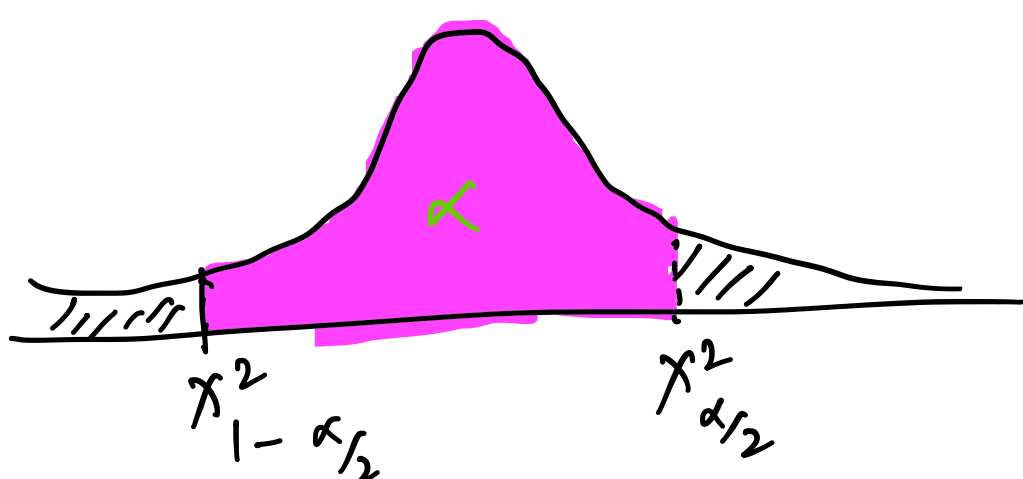
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We know that

$\frac{(n-1)s^2}{\sigma^2}$ is a chi square distribution.
With $(n-1)$ dof.

• Now here we have to note, one thing
since χ^2 is not symmetric, so we don't
use $\chi^2_{\alpha/2}$ & $-\chi^2_{\alpha/2}$. Instead we

use $\chi^2_{\frac{\alpha}{2}}$ & $\chi^2_{1-\frac{\alpha}{2}}$



$$P\left(\chi^2_{1-\frac{\alpha}{2}} < (n-1)\frac{s^2}{\sigma^2} < \chi^2_{\alpha/2}\right) = 1-\alpha$$

↓ Take reciprocal

$$P\left(\frac{(n-1)s^2}{\chi^2_{\alpha/2}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{1-\frac{\alpha}{2}}}\right) = 1-\alpha$$

So the confidence interval is

$$\left(\frac{(n-1)s^2}{\chi^2_{\alpha/2}}, \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}}\right)$$

Que The following are the weights, in
decagrams, of 10 packages of grass
seed distributed by a certain company:-
46.4, 46.1, 45.8, 47.0, 46.1, 45.9
45.8, 46.9, 45.2 and 46.

Find a 95% confidence interval for the
variance of the weights of all such
packages of grass seed distributed by
the company assuming a normal population.

Solⁿ $s^2 = 0.286$ (doing calculations)

95% confidence interval

$$\chi^2_{0.025} = 19.023 \quad (n-1 = 10-1 \text{ dof} = 9)$$

$$\chi^2_{0.975} = 2.700$$

$$\frac{(9)(0.286)}{19.023} < \sigma^2 < \frac{(9)(0.286)}{2.7}$$

$$0.135 < \sigma^2 < 0.953$$