Sampling distribution of mean if the original population is normally distributed

Sunday, 26 November 2023 11:54 AM

$n = 30$

10000 times.

First group of 30 people	x_1^1	x_2^1	x_3^1	...	x_{30}^1	$\frac{\Sigma x_i^1}{30}$
Second group of 30 people	x_1^2	x_2^2	x_3^2		x_{30}^2	$\frac{\Sigma x_i^2}{30}$
⋮						
10000 group of 30 people	x_1^{10000}	x_2^{10000}	x_3^{10000}	...	x_{30}^{10000}	$\frac{\Sigma x_i^{10000}}{30}$
	X_1	X_2	X_3	...	X_{30}	

$\frac{X_1 + X_2 + \dots + X_{30}}{30}$

$\bar{X} = \frac{X_1 + X_2 \dots + X_{30}}{30}$

If your original population is normally distributed $\sim N(\mu, \sigma^2)$

$X_1 \sim N(\mu, \sigma^2)$

$X_2 \sim N(\mu, \sigma^2)$

⋮

$X_{30} \sim N(\mu, \sigma^2)$

$$\bar{X} = \frac{X_1 + X_2 + \dots + X_{30}}{30}$$

$$E(\bar{X}) = E\left(\frac{X_1 + X_2 + \dots + X_{30}}{30}\right)$$

$$= \frac{1}{30}\left[E(X_1) + E(X_2) + \dots + E(X_{30})\right]$$

$$= \frac{1}{30}\left[\mu + \mu + \dots + \mu\right]$$

$$= \frac{1}{30}(30\mu) = \mu$$

$$\boxed{E(\bar{X}) = \mu}$$

$\sigma^2_{aX+bY} = a^2\sigma_x^2 + b^2\sigma_y^2$ (if X & Y are independent)

$$\sigma_{\bar{X}}^2 = \sigma^2_{\frac{X_1 + X_2 + \dots + X_n}{n}}$$

$$= \frac{1}{n^2}\underbrace{\sigma^2_{X_1}}_{\sigma^2} + \frac{1}{n^2}\underbrace{\sigma^2_{X_2}}_{\sigma^2} + \dots + \frac{1}{n^2}\underbrace{\sigma^2_{X_n}}_{\sigma^2}$$

$$\sigma_{\bar{X}}^2 = \frac{1}{n^2}(n\sigma^2) = \frac{1}{n}\sigma^2$$

$$\boxed{\sigma_{\bar{X}} = \frac{1}{\sqrt{n}}\sigma}$$

$\bar{X}$ has mean $= \mu$ (population mean)

& S.D. $= \frac{\sigma}{\sqrt{n}}$ (population S.D.)

Reproductive property of normal distribution

which says if

$X_1, X_2 \dots X_n$ are independent normal r.v. then

$\frac{X_1 + X_2 + \dots + X_n}{n}$ is also a normal r.v.

S.D. $= \sigma$

If the original population is $N(\mu, \sigma^2)$, then

$\bar{X} = \frac{X_1 + X_2 + \dots + X_n}{n}$ is normally distribution

with mean $= \mu$

S.D. $= \frac{\sigma}{\sqrt{n}}$

Sampling distribution of mean