~~Chapter~~ 5

Some discrete Probability distribution

Summary

un uncertain situation

Aim:- To predict something about this uncertain situation

Sample space + Probability law (mathematical model)

$\downarrow$

r.v. + pdf

$\downarrow$

$E(X)$ σ_X^2 σ_{XY} (analysis part)

Luckily, we have some important random variables (~20) and everything around us can be modeled using these r.v.

- Next aim is to talk about these important r.v.

Discrete | Continuous

1. Binomial r.v.
2. Hyper geometric r.v.
3. negative binomial
4. Geometric r.v.
5. Poisson R.V.

Binomial r.v.

Situation:-

(Each trial is called a Bernoulli trial)

1. There are n number of repeated trials.
2. Each trial results in success with probability p & failure with probability $1-p$.
3. The trials are independent.
4. The probability of success i.e. p does not change with trials.

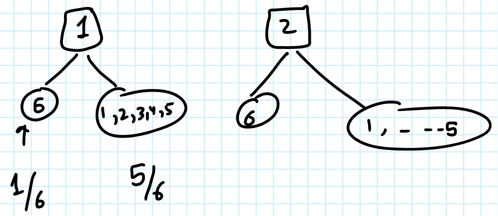
} Bernoulli Experiment

Example of any such situation:- I toss a coin 100 time, Let us say getting a head is success.

Define $\{X \rightarrow$ no. of successes$\}$ is a discrete r.v.

possible values of $X \to 0, 1, 2, 3, \dots, n$

Experiment :- You play a game, roll a dice 100 times, if you get a 6 you get \$1



Binomial r.v :- X :- It is number of successes in a Bernoulli experiment. (n trials)

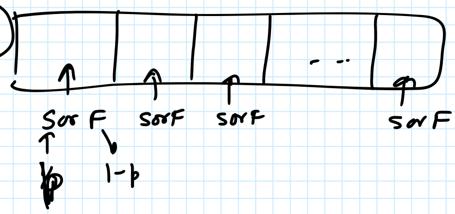
$X = 0, 1, 2, 3, \dots n$

We define pdf $f(k) = P(X = k)$

$P(X = k)$

$= {}^nC_k \, p^k (1-p)^{n-k}$

$\boxed{f(x) = b(k; n, p)}$



Que Consider the set of Bernoulli trials where three items are selected at random from a manufacturing process & classified as Defective or Non defective. (the probability of d is $1/4$ & $N \to 3/4$)

Getting a nondefective piece is called a success

Then what will be the pdf for $X \to$ no. of non defective.

Sol^n Is it a binomial r.v.

$n \to 3$

$X \to 0, 1, 2$ or 3

$p = 3/4$

$b(k; n, p) = {}^nC_k \, p^k (1-p)^{n-k}$

pdf

X	0	1	2	3
	${}^3C_0 (3/4)^0 (1/4)^{3-0}$	${}^3C_1 (3/4)^1 (1/4)^{3-1}$	${}^3C_2 (3/4)^2 (1/4)^{3-2}$	${}^3C_3 (3/4)^3 (1/4)^0$

Que The probability that a certain kind of component will survive a shock test is $3/4$. Find the probability that exactly 2 of next 4 components tested survive.

Sol^n $P(X = 2) = {}^4C_2 \, p^2 (1-p)^2$

pbinom cdf

dbinom pdf

$= \frac{4 \times 3}{2 \times 1} (3/4)^2 (1/4)^2 = \frac{27}{128}$

binomial? pdf

2×1 $\qquad\qquad$ 128

Why the name binomial?

pdf
$$f(k) = {}^{n}C_{k}\, p^{k} (q)^{n-k} \qquad q = 1-p$$

Now look at $(p+q)^n = {}^{n}C_0\, p^{n} + {}^{n}C_1\, q^{n-1} p^{\cancel{1}} + {}^{n}C_2\, q^{n-2} p^{\cancel{2}} + \dots$

$$(p+q)^n = b(0,n,p) + b(1,n,p) + b(2,n,p) + \dots + b(n,n,p)$$

$$\boxed{1 = \sum_{k=0}^{n} b(k,n,p)}$$

Next For Binomial tables:-

Binomial sums (cdf $\qquad B(2, 3, 0.15)$

$$B(0; n,p) = b(0; n,p)$$

$$B(1; n,p) = b(0; n,p) + b(1; n,p)$$

$$B(2; n,p) = b(0; n,p) + b(1; n,p) + b(2; n,p)$$

$$\boxed{B(r, n,p) = \sum_{k=0}^{r} b(k, n,p)}$$

Que The probability that a patient recovers from a blood disease is 0.4. If 15 people are known to have contracted this disease, what is the probability that

(a) at least 10 survive.

(b) from 3 to 8 survive $\rightarrow P(3 \le X \le 8) = B(8, 15, 0.4) - B(2, 15, 0.4)$ $\quad = 0.8779$

(c) exactly 5 survive $\rightarrow P(X=5) = B(5, 15, 0.4) - B(4, 15, 0.4)$

$\rightarrow {}^{15}C_5 (0.4)^5 (0.6)^{10}$

Note that the process is binomial.

Sol^n (a) $P(X \ge 10) = 1 - P(X < 10)$

$$= 1 - \sum_{k=0}^{9} b(k; 15, 0.4)$$

$B(10)$

$$= 1 - B(9; 15, 0.4) \longrightarrow \text{From table}$$

$$= 1 - 0.9662 = 0.0338$$

$P(X \ge 10) =$
$+ P(X=10)$
$+ P(X=11)$
$+ P(X=12)$
$+ P(X=13)$
$+ P(X=14)$
$+ P(X=15)$

$= B(15, 15, p) - B(9, 15, p)$

$= 1 - B(9, 15, p)$