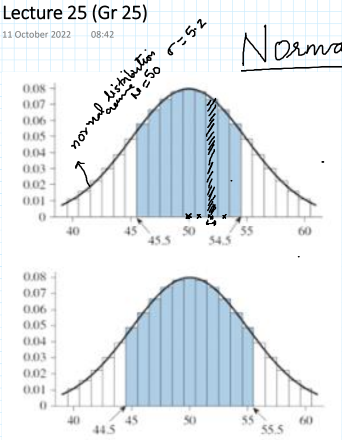
Normal approximation to Binomial



Binomial

$P(X=50) = {}^{100}C_{50} \left(\frac{1}{2}\right)^{50}\left(\frac{1}{2}\right)^{50}$ $n \to$ independent trials

S: p F: 1−p

$X \to$ no. of successes.

• It was discrete

Now solid black curve is normal distribution with

mean $= np$
$= 100 \times \frac{1}{2}$
$= 50$

$\sigma = \sqrt{np(1-p)} = \sqrt{100 \times \frac{1}{2} \times \frac{1}{2}} = \left(\frac{10}{2}\right)$

Experiment :- Tossing an unbiased coin 100 times.

when $p = \frac{1}{2}$, we can very easily approximate binomial distribution $b(x; n, p)$ with normal distribution $n(x; np, \sqrt{np(1-p)})$

$b(x, n, p)$

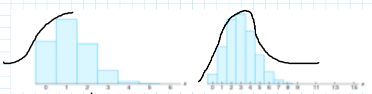
p is close to 1/2

$n(x, np, \sqrt{np(1-p)})$

$X \to$ binomial

$P(X = 52)$
= area of the rectangle

$P(51.5 \le Z \le 52.5)$



$n = 5$ $n = 20$

$p = \frac{1}{4}$

when $p \ne 1/2$

as n increases, you will have a good normal approximation

$b(x; n, p) \xrightarrow[\text{very large}]{\text{when } n \text{ is}} n(x; np, \sqrt{np(1-p)})$

p should not be very close to 0 or 1.

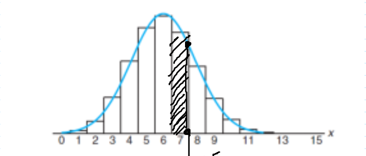
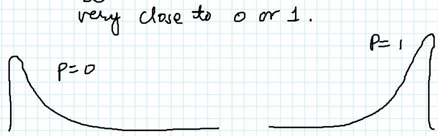
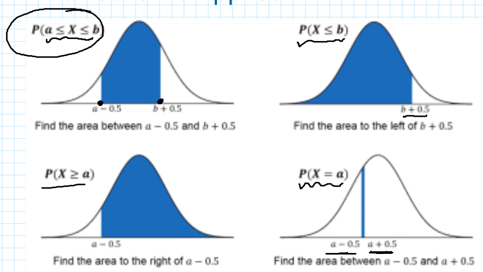


Figure 6.22: Normal approximation of $b(x; 15, 0.4)$.

$P(X \le 7)$



Areas to Use When the Continuity Correction is Applied



If you have approximated binomial with normal

$X \to$ binomial

addition & subtraction of 0.5 is called continuity correction.

Binomial distribution
15 trials

S → 0.4
F → 0.6

The corresponding Histogram is here.

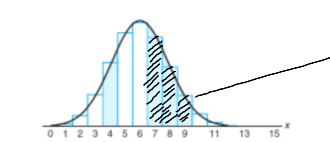


Figure 6.23: Normal approximation of $b(x; 15, 0.4)$ and $\sum_{x=7}^{9} b(x; 15, 0.4)$.

$P(7 \le X \le 9) = \sum_{x=7}^{9} b(x; 15, 0.4)$

$\sum_{x=0}^{9} b(x; 15, 0.4) - \sum_{x=0}^{6} b(x, 15, 0.4)$

$0.9662 - 0.6098$

$= 0.3564$

If I approximate with $n(\cancel{z}, 15 \times 0.4, \sqrt{15 \times 0.4 \times 0.6})$

$n(\cancel{z}, 6, 1.897)$

$P(7 \le X \le 9) \to P(6.5 \le Z \le 9.5)$

$W = \frac{Z - 6}{1.897}$

$P(0.26 \le W \le 1.85)$ $\quad W \to S.N.D.$

$P(W \le 1.85) - P(W \le 0.26)$

$0.9678 - 0.06026$

$= 0.3652$

Que The probability that a patient recovers from a rare blood disease is 0.4. If 100 people are known to have the disease, what is the probability that fewer than 30 survives.

Sol^n $X \to$ binomial $\quad b(x, 100, 0.4)$ $\quad$ given

-2.14

0.0162 for S.N.D.

$Z \to$ Normal $\quad n(x, 40, \sqrt{100 * 0.4 * 0.6})$

$= n(x, 40, 4.899)$

$\downarrow \quad W = \dfrac{Z - 40}{4.899}$ $\qquad \dfrac{29.5 - 40}{4.899} = -2.14$

$W \to$ S.N.D

$P(X < 30) = P(Z \le 29.5) = P(W \le -2.14) = \underline{0.0162}$

$P(X < 30) \to P(X \le 29) \to P(Z \le 29.5)$

Que An mcq has 200 questions each with 4 possible answers of which only 1 is correct. What is the probability that sheer guesswork yields from 25 to 30 correct answers for the 80 of the 200 problems about which a student has no knowledge?

Sol^n Binomial $X \to n \to 80$ $\quad S = 1/4$, $\quad F = 3/4$ $\qquad$ (Homework)

Ans $\underline{0.1196}$

$P(25 \le X \le 30)$

$\downarrow$

$Z \to n(x, 80 * \frac{1}{4}, \sqrt{80 * \frac{1}{4} * 3/4}) = n(x, 20, 3.873)$

$P(24.5 \le Z \le 30.5)$

$\downarrow \quad W = \dfrac{Z - 20}{3.873}$

$P(1.16 \le W \le 2.71) = P(W \le 2.71) - P(W \le 1.16)$

$= 0.9966 - 0.8770$

$= 0.1196$