Sampling distribution of difference of means

Saturday, 2 December 2023 9:54 AM

Let we have two populations, the

Pop1) first one with mean $= \mu_1$ & variance σ_1^2

pop2) second one with mean $= \mu_2$ & variance σ_2^2

let n_1 be the size of first sample from population 1 & n_2 be the size of second sample from population 2.

These two are independent samples

let $\bar{x}_1$ represents the mean of random sample of size n_1 from pop 1

$\bar{x}_2$ represents the mean of random sample of size n_2 from pop 2.

Then $\bar{x}_1$ is normally distributed with mean $= \mu_1$ & variance $= \dfrac{\sigma_1^2}{n_1}$

(if the original pop is normally distributed then this is true for all values of n.
Otherwise we use CLT & this result is true for $n \geq 30$)

Similarly $\bar{x}_2$ is normally distributed with mean $= \mu_2$ & variance $= \dfrac{\sigma_2^2}{n_2}$

Now if we consider $\bar{x}_1 - \bar{x}_2$, then by reproductive property, this is also normally distributed with mean $= \mu_1 - \mu_2$

and variance $= \sigma^2_{\bar{x}_1 - \bar{x}_2} = (1)^2 \sigma^2_{\bar{x}_1} + (-1)^2 \sigma^2_{\bar{x}_2}$

$$\sigma^2_{\bar{x}_1 - \bar{x}_2} = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$$

$\therefore$ $\dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$ is Z distribution.

Que Two independent experiments are run in which two different types of paints are compared. Eighteen specimens are painted using type A, and the drying time, in hours, is recorded for each. The same is done with type B. The population S.D. are both known to be 1. Assuming that mean drying time is equal for two types of paints, find $P(\bar{X}_A - \bar{X}_B > 1.0)$. (Assume that original populations are normally distributed).

Soln

$$\mu_{\bar{X}_A - \bar{X}_B} = \mu_A - \mu_B = 0$$

$$\sigma^2_{\bar{X}_A - \bar{X}_B} = \frac{\sigma_A^2}{n_A} + \frac{\sigma_B^2}{n_B} = \frac{1}{18} + \frac{1}{18} = \frac{1}{9}$$

Now $\dfrac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{1/9}}$ is $Z(0,1)$

$$P\left((\bar{x}_1 - \bar{x}_2) > 1\right) = P\left(\frac{\bar{x}_1 - \bar{x}_2}{1/\sqrt{9}} > \frac{1}{1/\sqrt{9}}\right)$$

$$= P(Z > 3)$$

$$= 1 - P(Z \leq 3)$$

$\downarrow$ From table

$$= 1 - 0.9987$$

$$= 0.0013.$$