

**Algorithm of Graphical Method to solve LPP**

**Step 1:** Enter the provided data (coefficients of variables in objective function, coefficients of variables in constraints, right hand side elements of the constraints) in array's (namely C, A, B).

**Step 2:** Select the range of  $x_1$  in which graph to be plotted.

**Step 3:** Find non-negative value of  $x_2$  from the ith constraint (say  $x_2^i$ ) in terms of  $x_1$

**Step 4:** Plot the graph between  $x_{2i}$  and  $x_1$

**Step 5:** Assume an empty array (say, solution) matrix to store the obtained solution

**Step 6:** Store the  $i^{\text{th}}$  row of A in an array (say, A1)

**Step 7:** Store the ith row of B in an array (say, B1)

**Step 7:** Store the  $(i+1)^{\text{th}}$  row of A in an array (say, A2)

**Step 8:** Store the  $(i+1)^{\text{th}}$  row of B in an array (say, B2)

**Step 9:** Combine A1 and A2 (say, A3)

**Step 10:** Combine B1 and B2 (say, B3)

**Step 11:** Find solution of system of equations  $A3X=B3$

**Step 12:** Store solution in the considered empty array

**Step 13:** Assume first column of solution as  $x_1$

**Step 14:** Assume second column of solution as  $x_2$

**Step 15:** Find such rows of solution which does not satisfy the first constraint

**Step 16:** Delete such rows from Solution

**Step 17:** Assume first column of solution as  $x_1$

**Step 18:** Assume second column of solution as  $x_2$

**Step 19:** Find such rows of solution which does not satisfy the first constraint

**Step 20:** Repeat for all the constraints

**Step 21:** Find value of objective function (say, OBJ) corresponding to  $i^{\text{th}}$  row of solution

**Step 22:** Find maximum or minimum value of objective function

**Step 23:** Find that row corresponding to which OBJ is max or min

**Step 24:** The row represents an optimal solution

**Write a MATLAB code to solve the following LPPs by graphical method:**

1. Maximize/Minimize  $(3x_1 + 2x_2)$  Subject to  $2x_1 + 4x_2 \leq 8$ ,  $3x_1 + 5x_2 \geq 15$ ,

$x_1 \geq 0$ ,  $x_2 \geq 0$ .

2. Maximize/Minimize  $(3x_1 + 2x_2)$  Subject to  $2x_1 + 4x_2 \geq 8$ ,  $3x_1 + 5x_2 \geq 15$ ,

$$x_1 \geq 0, x_2 \geq 0.$$

3. Maximize/Minimize  $(3x_1 + 2x_2)$  Subject to  $2x_1 + 4x_2 \leq 8$ ,  $3x_1 + 5x_2 \leq 15$ ,

$$x_1 \geq 0, x_2 \geq 0$$

**Thapar Institute of Engineering and Technology, Patiala**  
**Department of Mathematics**

Optimization Techniques (UMA-035)  
Lab Experiment- 2 (Basic feasible solutions)

---

**Algorithm to find BFS**

Consider the LPP:

$$\begin{array}{ll} \text{Max } z = C^t X \\ \text{subject to } AX = b, \quad X \geq 0 \end{array}$$

Initially define Input parameters:

1. Number of constraints as  $m$ ,
2. Number of unknowns  $n$ ,
3. Entries  $b_i$  of the R.H.S. vector  $b$  and entries  $a_{ij}$  of matrix  $A$ ,
4. Position of basic variables as  $p_i$  where  $1 \leq i \leq m$ .

**Step 1:** Construct the basis matrix  $B$  from the already defined basic variables  $p_i$ 's.

**Step 2:** If  $\det(B) \neq 0$ , then find  $X_B = B^{-1}b$ , otherwise display (' Not a Basic solution') and STOP.

**Step 3:** If  $X_{B_i} < 0$  for some  $i$  then display (' Not a B.F.S') and STOP.  
If  $X_{B_i} = 0$  for some  $i$  then display (' Degenerate B.F.S' ) and STOP.  
If  $X_{B_i} > 0$  for all  $i$  then display (' Non-degenerate B.F.S' ) and STOP.

Write a MATLAB code to compute the basic feasible solutions of an LPP and test your program on the following set of examples:

1. Convert the following linear programming problem in standard form and find all bfs.

$$\text{Max. } z = x_1 + 2x_2, \quad \text{subject to } -x_1 + x_2 \leq 1, \quad x_1 + x_2 \leq 2, \quad x_1, x_2 \geq 0.$$

2. Check if the the variables (1)  $(x_1, x_4)$  (2)  $(x_3, x_2)$  can be in the basis for the following problem

$$\text{Max. } z = x_1 + 2x_2 - x_3 + x_4,$$

$$\text{subject to } x_1 + x_2 - x_3 + 3x_4 = 15, \quad 5x_1 + x_2 + 4x_3 + 15x_4 = 12, \quad x_1, x_2, x_3, x_4 \geq 0.$$

3. Solve the following LPP by using finding all its BFS.

$$\text{Max. } z = -x_1 + 2x_2 - x_3, \quad \text{subject to } x_1 \leq 4, \quad x_2 \leq 4, \quad -x_1 + x_2 \leq 6, \quad -x_1 + 2x_3 \leq 4, \quad x_1, x_2, x_3 \geq 0$$

4. Check if the following LPP has a degenerate BFS, Find all basis corresponding to this solution.

$$\text{Max. } z = x_1 + x_2 + x_3, \quad \text{subject to } x_1 + x_2 \leq 1, \quad -x_2 + x_3 \leq 0, \quad x_1, x_2, x_3 \geq 0.$$

**Thapar Institute of Engineering and Technology, Patiala**  
**Department of Mathematics**

Optimization Techniques (UMA-035)  
Lab Experiment- 3 (The simplex method)

---

**The Simplex Algorithm ( with slack variables only)**

Consider an LPP with all constraints of ( $\leq$ ) type given as

$$\begin{array}{ll} \text{Max } z = C^t X \\ \text{subject to} & AX \leq b, \quad X \geq 0 \end{array}$$

Convert the above problem to standard form by adding slack variables:

$$\begin{array}{ll} \text{Max } z = C^t X \\ \text{subject to} & AX + IX_s = b, \quad X \geq 0 \end{array}$$

Where  $X_s = (s_1, s_2, \dots, s_m)$  is a vector of slack variables.

Initially definethe following Input paramenters:

1. Enter the Matrix  $A = [A \ I]$ , where  $I$  is an identity matrix of order m.
2. Entries  $b_i$  of the R.H.S. vector  $b$  and entries  $a_{ij}$  of matrix  $A$
3. Define  $[m,n]=\text{size}(A)$
4. Input the variables  $s_1, s_2, \dots, s_m$  as basic variables.

**Step 1:** Construct the basis matrix  $B$  from the already defined basic variables.

**Step 2:** Calculate  $Z_j - c_j = C_B^t(B^{-1}A_j) - c_j$  for each  $j \in 1, 2 \dots n$

Find  $[val, k] = \min\{(Z_j - c_j), j \in 1, 2 \dots n\}$

While  $val < 0$  calculate  $\theta_k = \min_{i \in 1, 2, \dots, m} \left\{ \frac{X_{B_i}}{\alpha_i^k} \mid \alpha_i^k > 0 \right\} = \frac{x_{B_r}}{\alpha_r^k}$  and go to next step

**Step 3** Update the basis by leaving the variable  $x_k$  and entering the variable  $x_l = (x_{B_r})$  and go to Step 1.

**Note:** The variables in the basis are named as  $(x_{B_1}, x_{B_2}, \dots, x_{B_m})$  and in the present case we have initial basic variables as slack variables taken in the order  $s_1, s_2, \dots, s_m$ . So one can understand that initially  $x_{B_1} = s_1, x_{B_2} = s_2, \dots, x_{B_m} = s_m$ . Similarly when the basis is updated by adding a variable  $x_k$  to the basis by replacing a variable  $x_l = x_{B_r}$ , then the new  $r^{th}$  basic variable will be  $x_{B_r} = x_l$ .

**Write a MATLAB code for the simplex method and test your program on the following examples:**

- 1 Max.  $z = x_1 + 2x_2$ , subject to  $-x_1 + x_2 \leq 1, x_1 + x_2 \leq 2, x_1, x_2 \geq 0$ .
- 2 Max.  $z = 4x_1 + 6x_2 + 3x_3 + x_4$ , (Ans:  $x_1 = 1/3, x_3 = 4/3, s_2 = 3; z = 16/3$ )  
subject to  
 $x_1 + 4x_2 + 8x_3 + 6x_4 \leq 11, 4x_1 + x_2 + 2x_3 + x_4 \leq 7, 2x_1 + 3x_2 + x_3 + 2x_4 \leq 2, x_1, x_2, x_3 \geq 0$ .
- 3 Min.  $z = -3/4x_4 + 20x_5 - 1/2x_6 + 6x_7$ , (Ans:  $x_1 = 3/4, x_4 = 1, x_6 = 1; z = -5/4$ )  
subject to  
 $x_1 + 1/4x_4 - 8x_5 - x_6 + 9x_7 = 0, x_2 + 1/2x_4 - 12x_5 - 1/6x_6 + 3x_7 = 0, x_3 + x_6 = 1, x_i \geq 0, i = 1, 2 \dots 7$

**Thapar Institute of Engineering and Technology, Patiala**  
**Department of Mathematics**

Optimization Techniques (UMA-035)  
Lab Experiment- 4 (The Big-M method)

---

**The Big M method**

Consider an LPP with mixed type of constraints ( $\leq$ ,  $\geq$ ,  $=$ ) and then convert the problem into standard form as given below.

$$(P) \quad \begin{array}{l} \text{Max } z = C^t X \\ \text{subject to} \\ \quad AX = b, \quad X \geq 0 \end{array}$$

Assume that the matrix  $A$  does not have an identity submatrix in it. Now, add artificial variables  $R_i$  for  $i \in \{1, 2, \dots, m\}$  so that the new problem has a identity submatrix in it, and the matrix  $A$  is now updated as  $[AI]$ . Assign a value  $M > 10^5 \max(c_i)$ , for convenience, the value of  $M$  can be taken as  $M = 10^6$ .

Now the new problem is of the following form:

$$(P_s) \quad \begin{array}{l} \text{Max } z = C^t X + 10^6 \cdot e^t R \\ \text{subject to} \\ \quad AX + IR_i = b, \quad X, R \geq 0 \end{array}$$

Where  $R = (R_1, R_2, \dots, R_m)$  is a vector of artificial variables and  $e = (1, 1, \dots, 1)^t$  is a vector of one's.

Initially define the following Input paramenters:

1. Enter the Matrix  $A = [A \ I]$ , where  $I$  is an identity matrix of order  $m$ .
2. Entet the the R.H.S. vector  $b$  and the cost matrix  $C = [C \ 10^6 e]$ .
3. Define  $[m,n]=\text{size}(A)$
4. Input the variables  $R_1, R_2, \dots, R_m$  as basic variables.

Now apply simplex method to solve the problem  $(P_s)$ . Suppose an optimal basic feasible solution of the problem  $(P_s)$  so obtained is  $X_B$ .

If  $(X_{B_i} = R_j > 0, \text{ for some } i, j \in \{1, 2, \dots, m\})$       %i.e. *artificial variable appear in basis*  
display( The problem  $(P)$  is infeasible)

Else  $(X_{B_i} \neq R_j, \text{ for all } i, j \in \{1, 2, \dots, m\})$   
display (  $X_B$  is an optimal basic feasible solution of problem  $(P)$  )

**Note:** The variables in the basis are named as  $(x_{B_1}, x_{B_2}, \dots, x_{B_m})$  and in the present case we have initial basic variables as artificial variables taken in the order  $R_1, R_2, \dots, R_m$ . So one can understand that initially  $x_{B_1} = R_1, x_{B_2} = R_2, \dots, x_{B_m} = R_m$ . The idea behind Big M method is to remove artifical variables from the basis

Similarly when the basis is updated by adding a variable  $x_k$  to the basis by replacing a variable  $x_l = x_{B_r}$ , then the new  $r^{th}$  basic variable will be  $x_{B_r} = x_l$ .

**Write a MATLAB code for the simplex method and test your program on the following examples:**

1. Min.  $z = 3x_1 + 5x_2, \text{ S.T. } x_1 + 3x_2 \geq 3, x_1 + x_2 \geq 2, x_1, x_2 \geq 0.$
2. Min.  $z = 12x_1 + 10x_2, \text{ S.T. } 5x_1 + x_2 \geq 10, 6x_1 + 5x_2 \geq 30, x_1 + 4x_2 \geq 8, x_1, x_2 \geq 0.$
3. Max.  $z = 3x_1 + 2x_2, \text{ S.T. } x_1 + x_2 \leq 2, x_1 + 3x_2 \leq 3, x_1 - x_2 = 1, x_1, x_2 \geq 0.$

**Thapar Institute of Engineering and Technology, Patiala**  
**Department of Mathematics**

Optimization Techniques (UMA-035)  
Lab Experiment- 5 (The Two phase method)

---

**The Two phase method**

Consider an LPP with mixed type of constraints ( $\leq$ ,  $\geq$ ,  $=$ ) and then convert the problem into standard form as given below.

$$(P) \quad \begin{array}{l} \text{Max } z = C^t X \\ \text{subject to} \\ \quad AX = b, \quad X \geq 0 \end{array}$$

Assume that the matrix  $A$  does not have an identity submatrix in it. Now, add artificial variables  $R_i$  for  $i \in \{1, 2, \dots, m\}$  so that the new problem has a identity submatrix in it, and the matrix  $A$  is now updated as  $[A \ I]$ . Now construct the Phase-I problem as

$$(PH_1) \quad \begin{array}{l} \min z = O^t X + e^t R \\ \text{subject to} \\ \quad AX + IR_i = b, \quad X, R \geq 0 \end{array}$$

Where  $R = (R_1, R_2, \dots, R_m)^t$  is a vector of artificial variables and  $e = (1, 1, \dots, 1)_{m \times 1}$  is a vector of one's and  $O = (0, 0 \dots, 0)_{n \times 1}$  is a vector of zeros.

Initially define the following Input paramenters:

1. Enter the Matrix  $A = [A \ I]$ , where  $I$  is an identity matrix of order  $m$ .
2. Entet the the R.H.S. vector  $b$  and the cost matrix  $C = [O \ e]_{(n+m) \times 1}$ .
3. Define  $[m,n]=\text{size}(A)$
4. Input the variables  $R_1, R_2, \dots, R_m$  as initial basic variables.

Now apply simplex method to solve the problem  $(PH_1)$ . Suppose an optimal basic feasible solution of the problem  $(P_s)$  so obtained is  $X_B$ .

If  $(X_{B_i} = R_j > 0, \text{ for some } i, j \in \{1, 2, \dots, m\})$  %i.e. artificial variable appear in basis  
display( The problem  $(P)$  is infeasible)

Else  $(X_{B_i} \neq R_j, \text{ for all } i, j \in \{1, 2, \dots, m\})$   
display (  $X_B$  is an optimal basic feasible solution of problem  $(PH_1)$  and goto Phase II )

**Phase-II**

Treat the optimal basic feasible solution of Phase-I as initial basic feasible solution of Phase-II, while incorporating the folloiwng:

1. Update the cost matrix  $C = (c_1, c_2 \dots c_n)^t$  from original variables.
2. Update the matrix Alpha =Alpha(1:n,: ) ( where Alpha=inv(B)\*A)  
( means ignore the columns of artifal variables and update Alpha matrix by considering only first n columns of original variables)
3. Calculate  $Z_j - c_j$  and follow the simplex procedure to obtain an optimal basic feasible solution of the problem  $(P)$ .

**Note:** The variables in the basis are named as  $(x_{B_1}, x_{B_2}, \dots x_{B_m})$  and in the present case we have initial basic variables as artificial variables taken in the order  $R_1, R_2, \dots R_m$ . So one can understand that initially  $x_{B_1} = R_1, x_{B_2} = R_2, \dots x_{B_m} = R_m$ . The idea behind Phase-I method is to remove artifical variables from the basis and obtain an initial basic feasible solution of the problem  $(P)$ . Then Phase-II uses the BFS obtained in Phase-I to get an optimal BFS of problem  $(P)$ .

**Write a MATLAB code for the two phase method and test your program on the following examples:**

1. Min.  $z = 3x_1 + 5x_2$ , S.T.  $x_1 + 3x_2 \geq 3$ ,  $x_1 + x_2 \geq 2$ ,  $x_1, x_2 \geq 0$ .
2. Min.  $z = 12x_1 + 10x_2$ , S.T.  $5x_1 + x_2 \geq 10$ ,  $6x_1 + 5x_2 \geq 30$ ,  $x_1 + 4x_2 \geq 8$ ,  $x_1, x_2 \geq 0$ .
3. Max.  $z = 3x_1 + 2x_2$ , S.T.  $x_1 + x_2 \leq 2$ ,  $x_1 + 3x_2 \leq 3$ ,  $x_1 - x_2 = 1$   $x_1, x_2 \geq 0$ .

### The dual simplex method

Convert the given linear programming problem in the following form:

$$(P) \quad \begin{array}{l} \min / \max z = C^t X + O^t s \\ \text{subject to} \quad AX + Is = b, \quad X, s \geq 0 \end{array}$$

Where  $s = (s_1, s_2, \dots, s_m)^t$  is a vector of slack variables and  $O = (0, 0 \dots, 0)_{n \times 1}$  is a vector of zeros.  
Also Assume that atleast one of the component  $b_i$  of the RHS vector  $b = (b_1, b_2, \dots, b_m)$  is negative

Initially define the following Input paramenters:

1. Enter the Matrix  $A = [A \ I]$ , where  $I$  is an identity matrix of order m.
2. Entet the the R.H.S. vector  $b$  and the cost matrix  $C = [c \ O]_{(n+m) \times 1}$ .
3. Define  $[m,n]=\text{size}(A)$
4. Input the variables  $s_1, s_2, \dots, s_m$  as initial basic variables.

Now construct the simplex table using  $s_1, s_2, \dots, s_m$  as initial basic variables. If the simplex table depicts **an optimal but Infeasible solution**, then dual simplex method is applicable. So apply the following procedure.

1. Select the leaving variable as  $X_{B_r} = \min_i \{X_{B_i} \mid X_{B_i} < 0\}$
2. Select the entering variable  $x_k$  using the formula  $\frac{z_k - c_k}{y_{rk}} = \min_j \left\{ \frac{|z_j - c_j|}{|y_{rj}|} : y_{rj} < 0 \right\}$
3. Now update the basis as by removing  $r^{th}$  basic variable with  $k^{th}$  nonbasic variable. Again construct the simplex table and repeat the above procedure until an optimal basic feasible soluiton is not obtained.

**Write a MATLAB code for the dual simplex method and test your program on the following examples:**

1. Min.  $z = 3x_1 + 5x_2$ , S.T.  $x_1 + 3x_2 \geq 3$ ,  $x_1 + x_2 \geq 2$ ,  $x_1, x_2 \geq 0$ .
2. Min.  $z = 12x_1 + 10x_2$ , S.T.  $5x_1 + x_2 \geq 10$ ,  $6x_1 + 5x_2 \geq 30$ ,  $x_1 + 4x_2 \geq 8$ ,  $x_1, x_2 \geq 0$ .
3. min.  $z = 3x_1 + 2x_2$ , S.T.  $x_1 + x_2 \leq 1$ ,  $x_1 + 2x_2 \geq 3$ ,  $x_1, x_2 \geq 0$ .

**Thapar Institute of Engineering and Technology, Patiala**  
**Department of Mathematics**

Optimization Techniques (UMA-035)  
Lab Experiment- 7 (Least cost method)

**Least cost method of Transportation problem**

Consider a cost matrix representation of a Transportation problem:

				$u_i \downarrow$
$x_{11}$	$c_{11}$	$x_{12}$	$c_{12}$	
$x_{21}$	$c_{21}$	$x_{22}$	$c_{22}$	
.	.	.	.	
$x_{m1}$	$c_{m1}$	$x_{m2}$	$c_{m2}$	
	$b_1$	$b_2$	$\dots$	$b_n$
				$a_1$
				$a_2$
				.
				.
				$a_m$

Here  $a_i$  is the availability of the product at source  $S_i$  and  $b_j$  is the requirement of the same at destination  $D_j$ ,  $c_{ij}$  represents the cost of trasporting a unit product from source  $S_i$  to destination  $D_j$ . The variable  $x_{ij}$  is the quantity to be transported from the source  $i$  to destination  $j$ .

Initially define Input paramenters:

1. Enter the number of sources as  $m$ , and destinations as  $n$ .
2. Enter the cost coefficients  $c_{ij}$ , the availability at  $i^{th}$  source as  $a_i$  and demand at  $j^{th}$  destination as  $b_j$  for each  $i = 1, 2 \dots m$ ,  $j = 1, 2, \dots n$ .

Intially take  $k = 1$

**Step 1:** Define  $c_{pq} = \min(c_{ij})$ , and assign  $x_{pq} = \min(a_p, b_q)$ , go to Step 2.

**Step 2:** If  $\min(a_p, b_q) = a_p$ , then update  $b_q = b_q - a_p$ ,  $a_p = a_p - x_{pq}$  else  $\min(a_i, b_j) = b_q$ , then update  $a_p = a_p - b_q$ ,  $b_q = b_q - x_{pq}$ .

**Step 3** Assigne  $c_{pq} = 10^5$  (a very large no.) , Set  $k = k + 1$ , if  $k = m + n - 1$  go to Step 4 else go to Step 1.

**Step 4:** Stop and note the BFS and calculate the objective function value  $z = \sum_{i,j} c_{ij}x_{ij}$

Write a MATLAB code to compute the basic feasible solutions of a Transportation problem using Northwest corner rule and test your program on the following set of examples:

1. Consider the cost matrix of the following transportation roblem

	$D_1$	$D_2$	$D_3$	$D_4$	$a_i$
$S_1$	2	10	4	5	12
$S_2$	6	12	8	11	25
$S_3$	3	9	5	7	20
$b_j$	25	10	15	5	

2. Consider the cost matrix of the following transportation problem

	$D_1$	$D_2$	$D_3$	$D_4$	$D_5$	$a_i$
$S_1$	3	11	4	14	15	15
$S_2$	6	16	18	2	28	25
$S_3$	10	13	15	19	17	10
$S_4$	7	12	5	8	9	15
$b_j$	20	10	15	15	5	

**Thapar Institute of Engineering and Technology, Patiala**  
**Department of Mathematics**

Optimization Techniques (UMA-035)  
Lab Experiment - 8 (Multi-objective LPP)

---

**Algorithm of Weighted Sum Method to multi-objective LPP**

**Step 1:** Transform the multi-objective linear programming problem (P1) into its equivalent single objective linear programming problem (P2).

(P1)

Maximize/Minimize ( $C_i X$ ),  $i=1,2,\dots,m$

Subject to

$AX \leq$  or  $=$  or  $\geq b$

$X \geq 0$ .

(P1)

Maximize/Minimize ( $C_1 X + C_2 X + \dots + C_m X$ )/m

Subject to

$AX \leq$  or  $=$  or  $\geq b$

$X \geq 0$ .

**Step 2:** Find an optimal solution of the problem i.e., an efficient solution of the problem (P1) by an appropriate method (Simplex method or Big-M method or Two-Phase method).

**Write a MATLAB code to solve the following multi-objective LPPs by weighted sum method:**

1. Maximize  $(3x_1 + 2x_2 + 4x_3)$

Maximize  $(x_1 + 5x_2 + 3x_3)$

Subject to

$2x_1 + 4x_2 + x_3 \leq 8$ ,

$3x_1 + 5x_2 + 4x_3 \geq 15$ ,

$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$

2. Maximize  $(x_1 + 4x_2 + x_3)$

Maximize  $(2x_1 + 7x_2 + 5x_3)$

Subject to

$x_1 + x_2 + x_3 \leq 8$ ,

$x_1 + 5x_2 + 4x_3 \geq 15$ ,

$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$

**Thapar Institute of Engineering and Technology, Patiala**  
**Department of Mathematics**

Optimization Techniques (UMA-035)  
Lab Experiment - 9 (Fibonacci Search Technique)

---

**Algorithm of Fibonacci Search Technique**

Fibonacci numbers

$F_0$	$F_1$	$F_2$	$F_3$	$F_4$	$F_5$	$F_6$	$F_7$	$F_8$
1	1	2	3	5	8	13	21	34

$$F_0 = 1, F_1 = 1, F_n = F_{n-1} + F_{n-2}, n > 1.$$

**Step 1:** Using the relation, Measure of effectiveness =  $\frac{\text{Interval of uncertainty}}{L_0}$ , find the value of Measure of effectiveness.

**Step 2:** Using the relation  $\frac{1}{F_n} \leq \text{Obtained value of measure of effectiveness}$ , find the smallest natural number  $n$ .

**Step 3:** Store the given interval  $[a, b]$

**Step 3:** Find  $L_0 = b - a$

**Step 4:** for  $i = n$ , find

$$x_1 = a + \frac{F_{i-2}}{F_i} L_0, \text{ and } x_2 = a + \frac{F_{i-1}}{F_i} L_0,$$

**Step 5:** If  $f(x_1) > f(x_2)$  and the problem is of minimum. Then, repeat Step 3 with  $n = n - 1$ ,  $a = x_1$  and  $b = b$ .

If  $f(x_1) < f(x_2)$  and the problem is of minimum. Then, repeat Step 3 with  $n = n - 1$ ,  $a = a$  and  $b = x_2$ .

If  $f(x_1) > f(x_2)$  and the problem is of maximum. Then, repeat Step 3 with  $n = n - 1$ ,  $a = a$  and  $b = x_2$ .

If  $f(x_1) < f(x_2)$  and the problem is of maximum. Then, repeat Step 3 with  $n = n - 1$ ,  $a = x_1$  and  $b = b$ .

**Step 6:** Repeat Step 3 to upto  $i = 2$ .

**Write a MATLAB code to solve the following problem**

**Example:**

Minimize the function  $x(x - 2)$ ,  $0 \leq x \leq 1.5$  within the interval of uncertainty  $0.25L_0$ .