Module 6: Overview

Overview

In this module, we will derive an asymptotic lower bound of comparison-based sorting algorithms. A comparison-based sorting algorithm is a sorting algorithm that makes decisions by element-wise comparisons. Examples of comparison-based sorting algorithms include Insertion Sort, Quicksort, and Mergesort. We will show that no comparison-based sorting algorithm can be asymptotically faster than merge sort in the worst-case. In order to achieve this, we study decision tree for a comparison-based sorting algorithm on a given input length (the number of elements to be sorted). After showing how to draw the decision tree of a comparison-based sorting algorithm for a given length, we study the following facts:

(1) There are n! leaf nodes in a decision tree of a comparison-based sorting algorithm operating on a sequence of n elements.

(2) The worst-case time complexity of a sorting algorithm is proportional to the height of the decision tree of that algorithm on a sequence of n elements.

(3) The height of a binary tree with N nodes is lower-bounded by \Omega(log N).

(4) The height of a decision tree of any comparison-based sorting algorithm operating on a sequence of n elements is lower-bounded by \Omega(log(n!)), which is lower-bounded by \Omega(n log(n)).

Learning Objectives

By the end of this module, you will be able to:

1. Study the concept of algebraic decision trees
2. Apply decision trees to derive sorting lower-bounds

Readings

Read the following:

- Section 8.1