Activation Records, Divide and Conquer, Recursive Algorithms

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Sorting

- Sorting is the process of arranging a sequence of objects into order (either increasing or decreasing)

- Mergesort is a sorting algorithm

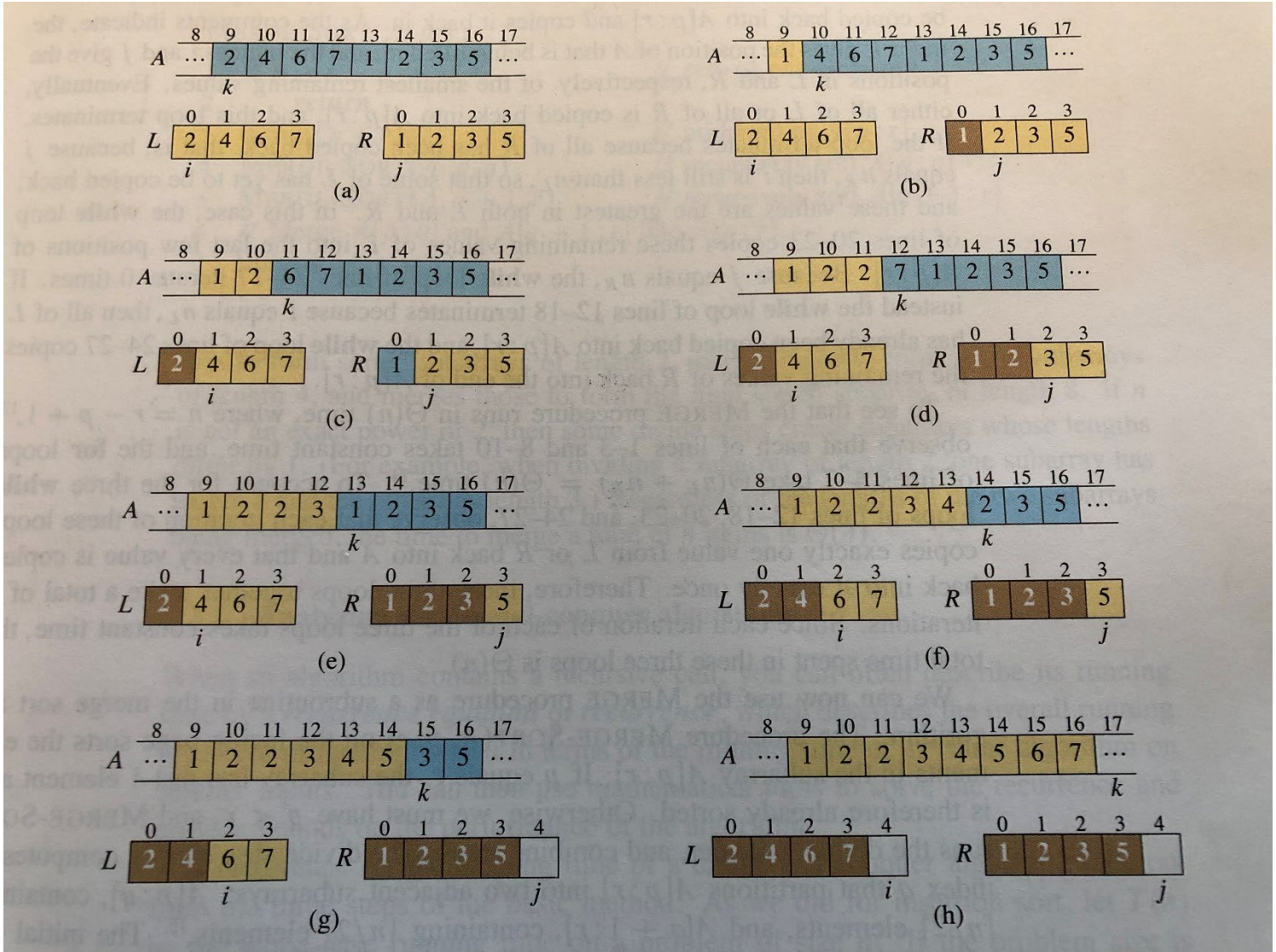
Divide and Conquer, Merge Sort, Recursion

- **Divide and Conquer**
  - **Divide**: break an instance of a problem into several smaller instances of the same problem
  - **Conquer**: Compute solutions for the smaller instances
  - **Combine**: Use the solutions to the smaller instances to obtain the solution of the original instance
- **Merge Sort**: A sorting algorithm that uses divide and conquer.
- **Recursive Algorithm**: An algorithm that calls itself.

Merge

```
Merge(A, p, q, r)                    // A[p:q]and A[q+1:r] are already sorted
01: nL = q - p + 1;                  // nL is the length of A[p:q]
02: nR = r - q;                      // nR is the length of A[q+1:r]
03: new L[0:nL-1]; new R[0:nR-1];    // two new arrays
04: for i=0 to nL-1                  //
05:   L[i] = A[p+i];                // L[0:nL-1] = A[p:q]
06: for j=0 to nR-1                  //
07:   R[j] = A[q+j+1];               // R[0:nR-1] = A[q+1:r]
08: i = 0;                           // i points to the start of L[]
09: j = 0;                           // j points to the start of R[]
10: k = p;                           // k points to the start of A[]
12: while i < nL and j < nR          //
13:   if L[i] <= R[j];               //
14:     A[k] = L[i];                 // L[i] is k-th smallest
15:     i = i+1;                     //
16:   else A[k] = R[j];              // R[j] is k-th smallest
17:     j = j+1;                     //
18:   k = k+1;                       //
20: while i < nL                     // 20-23 and 24-27 are mutually exclusive
21:   A[k] = L[i];                   // copy rest of L[] to A[]
22:   i = i+1;                       //
23:   k = k+1;                       //
24: while j < nR                     //
25:   A[k] = R[j];                   // copy rest of R[] to A[]
26:   j = j+1;                       //
27:   k = k+1;                       //
```

Example of Merge



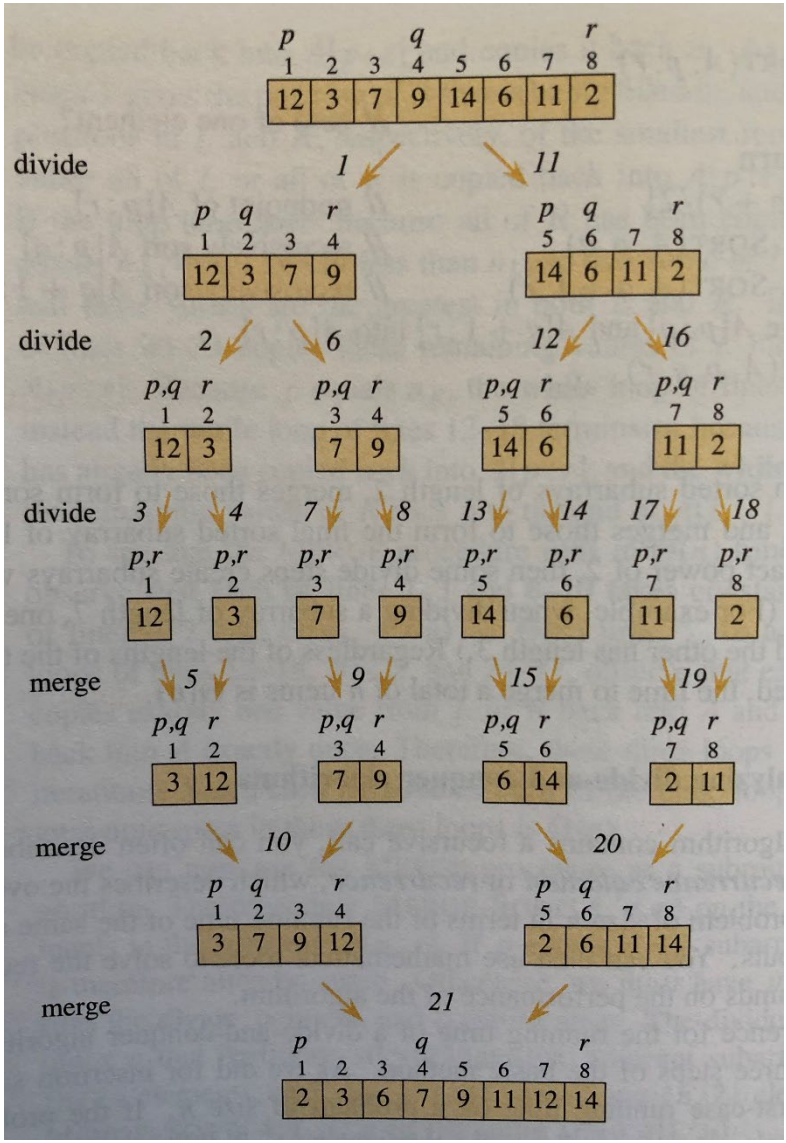
Mergesort

```
Merge-Sort(A, p, r)
1: if p >= r
2:    return;
3: q = (p+r)/2;
4: Merge-Sort(A, p, q);
5: Merge-Sort(A, q+1, r);
7: Merge(A, p, q, r);
```

Note for line 1:
When p=r, we have 1 element.
When p>r, we have 0 element.
We may call Merge-Sort with p>r.

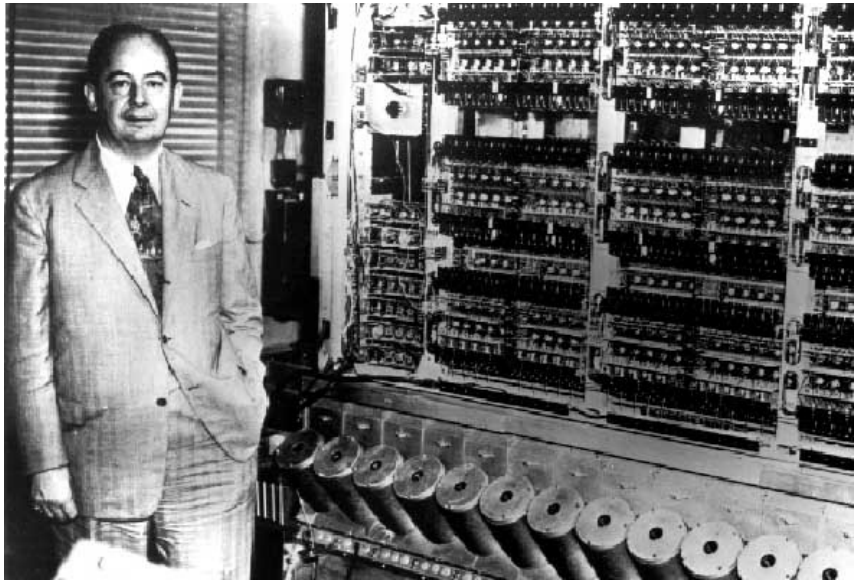
Note for line 3:
q is the floor of (p+r)/2.
Integer division always the floor.

Note for lines 4-5:
Conquer left, then conquer right.



Mergesort

- Merge sort was invented by John von Neumann in 1945
- Divide: break A[p:r] into A[p:q] and A[q+1:r], where q=(p+r)/2
- Conquer: sort A[p:q] and A[q+1:r], using merge sort
- Combine: merge A[p:q] and A[q+1:r]



John von Neumann

Activation Records and Recursive Calls

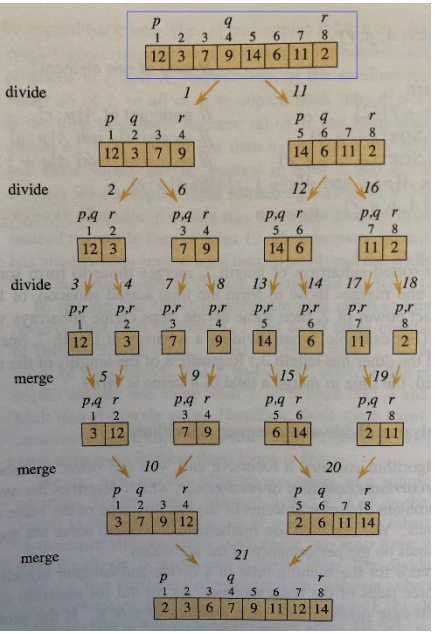
- Run-time stack
- Activation record (AR)
  - One activation record per function call
  - The activation records are managed via a stack
  - The AR of the callee is on top of the AR of the caller
  - When a function completes, its AR is popped off
- Recursive vs iterative algorithms

Activation Records and Recursive Calls

- For each function call, there is an activation record
- The activation record reserves space for the program, the variables, and intermediate results

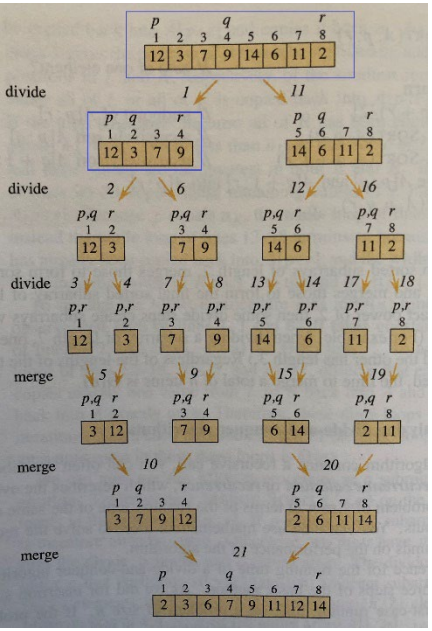
- The activation records form the run-time stack
- When a function is called, we push an AR to the stack.
- When a function is completed, we pop the AR off.

Activation Records and Recursive Calls

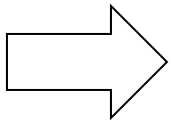


Merge-Sort(A, 1, 8)

Activation Records and Recursive Calls



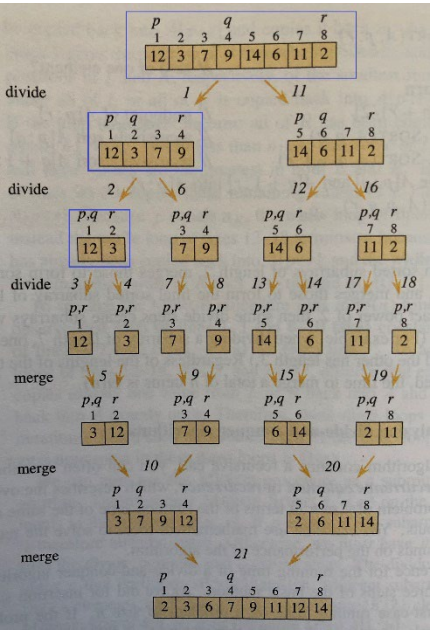
Merge-Sort(A, 1, 8)



Merge-Sort(A, 1, 4)

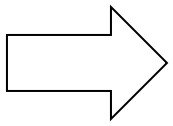
Merge-Sort(A, 1, 8)

Activation Records and Recursive Calls



Merge-Sort(A, 1, 4)

Merge-Sort(A, 1, 8)

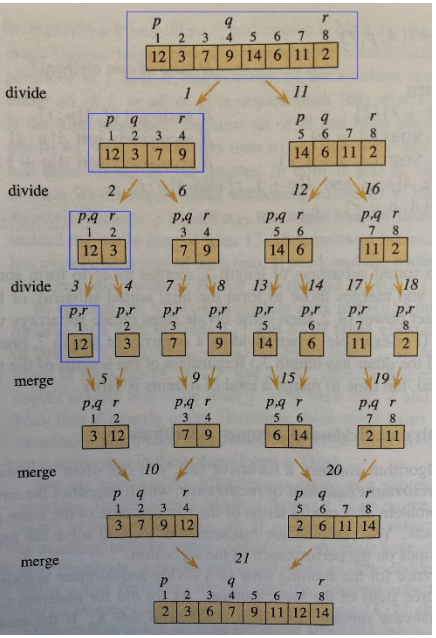


Merge-Sort(A, 1, 2)

Merge-Sort(A, 1, 4)

Merge-Sort(A, 1, 8)

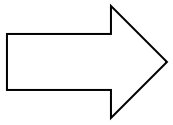
Activation Records and Recursive Calls



Merge-Sort(A, 1, 2)

Merge-Sort(A, 1, 4)

Merge-Sort(A, 1, 8)



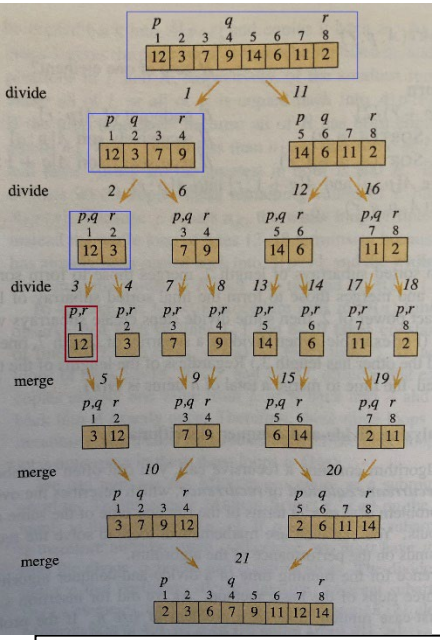
Merge-Sort(A, 1, 1)

Merge-Sort(A, 1, 2)

Merge-Sort(A, 1, 4)

Merge-Sort(A, 1, 8)

Activation Records and Recursive Calls

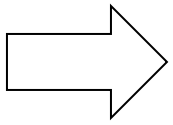


Merge-Sort(A, 1, 1)

Merge-Sort(A, 1, 2)

Merge-Sort(A, 1, 4)

Merge-Sort(A, 1, 8)

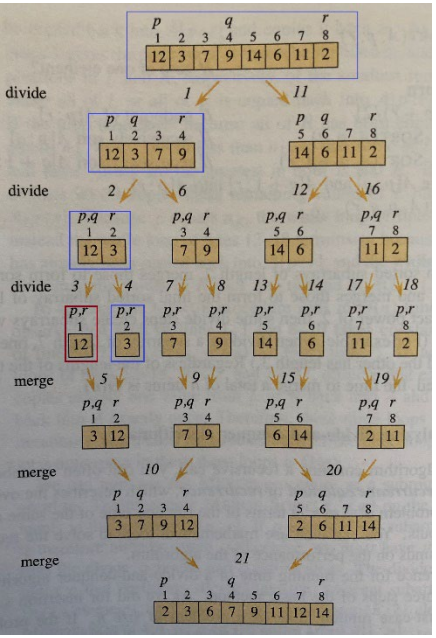


Merge-Sort(A, 1, 2)

Merge-Sort(A, 1, 4)

Merge-Sort(A, 1, 8)

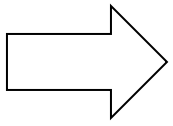
Activation Records and Recursive Calls



Merge-Sort(A, 1, 2)

Merge-Sort(A, 1, 4)

Merge-Sort(A, 1, 8)



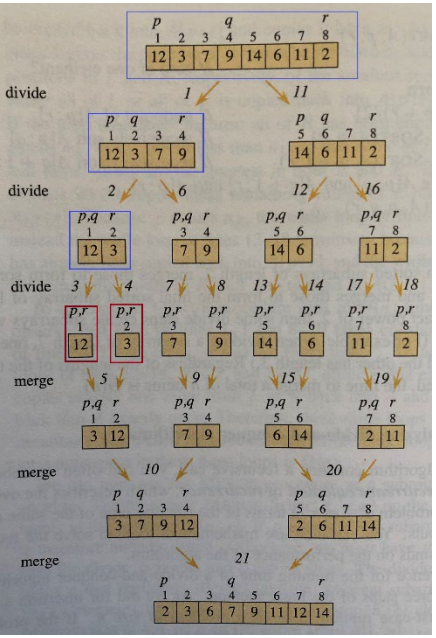
Merge-Sort(A, 2, 2)

Merge-Sort(A, 1, 2)

Merge-Sort(A, 1, 4)

Merge-Sort(A, 1, 8)

Activation Records and Recursive Calls

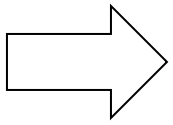


Merge-Sort(A, 2, 2)

Merge-Sort(A, 1, 2)

Merge-Sort(A, 1, 4)

Merge-Sort(A, 1, 8)

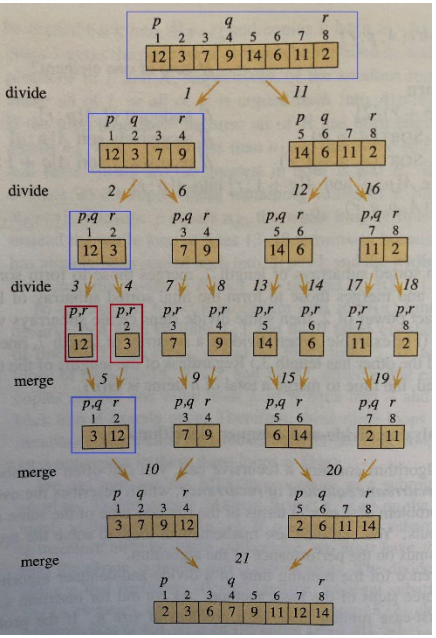


Merge-Sort(A, 1, 2)

Merge-Sort(A, 1, 4)

Merge-Sort(A, 1, 8)

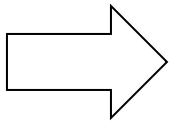
Activation Records and Recursive Calls



Merge-Sort(A, 1, 2)

Merge-Sort(A, 1, 4)

Merge-Sort(A, 1, 8)



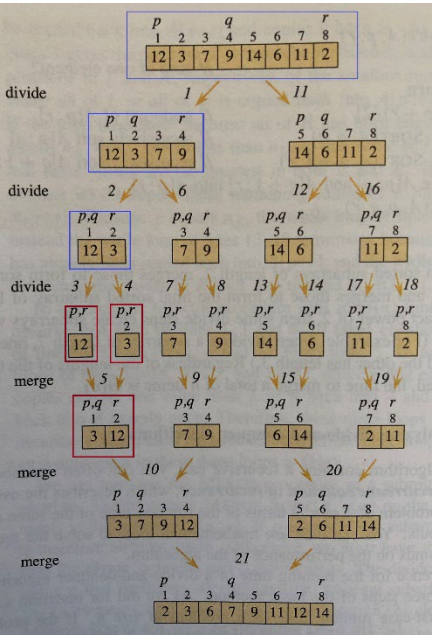
Merge(A, 1, 1, 2)

Merge-Sort(A, 1, 2)

Merge-Sort(A, 1, 4)

Merge-Sort(A, 1, 8)

Activation Records and Recursive Calls

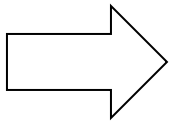


Merge(A, 1, 1, 2)

Merge-Sort(A, 1, 2)

Merge-Sort(A, 1, 4)

Merge-Sort(A, 1, 8)

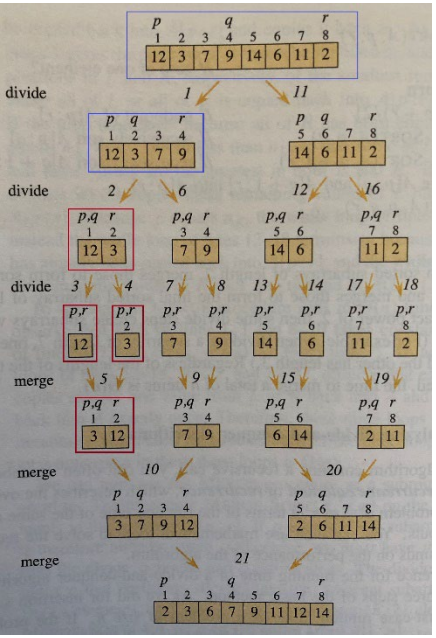


Merge-Sort(A, 1, 2)

Merge-Sort(A, 1, 4)

Merge-Sort(A, 1, 8)

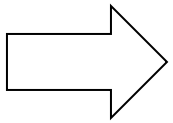
Activation Records and Recursive Calls



Merge-Sort(A, 1, 2)

Merge-Sort(A, 1, 4)

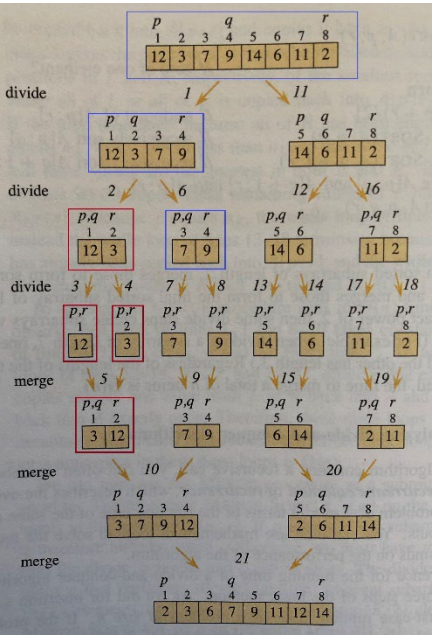
Merge-Sort(A, 1, 8)



Merge-Sort(A, 1, 4)

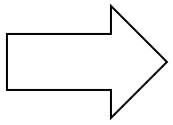
Merge-Sort(A, 1, 8)

Activation Records and Recursive Calls



Merge-Sort(A, 1, 4)

Merge-Sort(A, 1, 8)

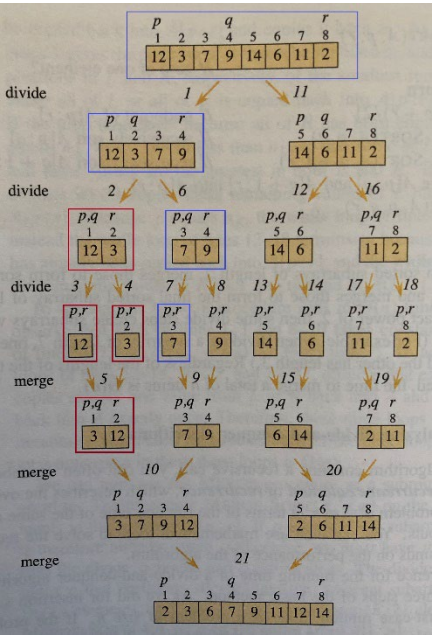


Merge-Sort(A, 3, 4)

Merge-Sort(A, 1, 4)

Merge-Sort(A, 1, 8)

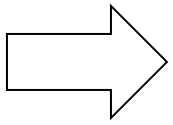
Activation Records and Recursive Calls



Merge-Sort(A, 3, 4)

Merge-Sort(A, 1, 4)

Merge-Sort(A, 1, 8)



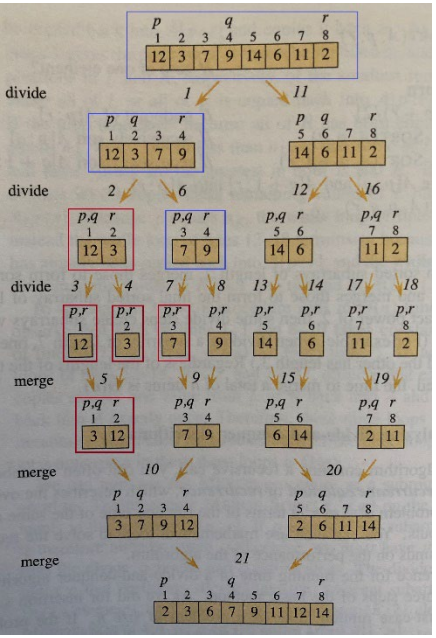
Merge-Sort(A, 3, 3)

Merge-Sort(A, 3, 4)

Merge-Sort(A, 1, 4)

Merge-Sort(A, 1, 8)

Activation Records and Recursive Calls

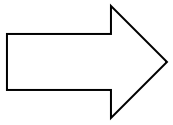


Merge-Sort(A, 3, 3)

Merge-Sort(A, 3, 4)

Merge-Sort(A, 1, 4)

Merge-Sort(A, 1, 8)

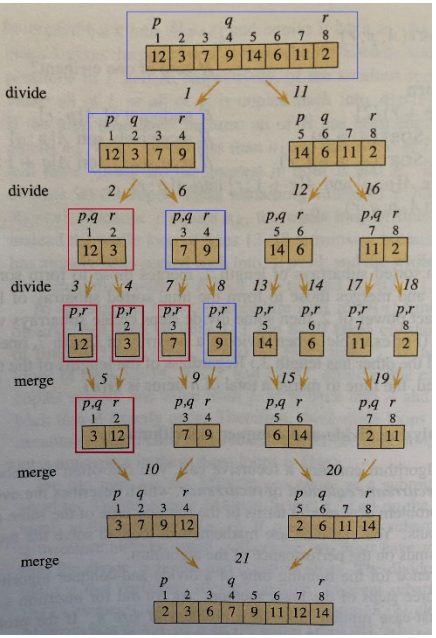


Merge-Sort(A, 3, 4)

Merge-Sort(A, 1, 4)

Merge-Sort(A, 1, 8)

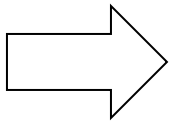
Activation Records and Recursive Calls



Merge-Sort(A, 3, 4)

Merge-Sort(A, 1, 4)

Merge-Sort(A, 1, 8)



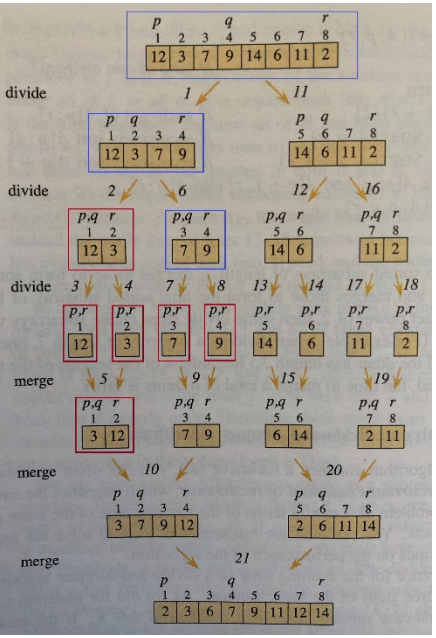
Merge-Sort(A, 4, 4)

Merge-Sort(A, 3, 4)

Merge-Sort(A, 1, 4)

Merge-Sort(A, 1, 8)

Activation Records and Recursive Calls

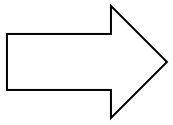


Merge-Sort(A, 4, 4)

Merge-Sort(A, 3, 4)

Merge-Sort(A, 1, 4)

Merge-Sort(A, 1, 8)

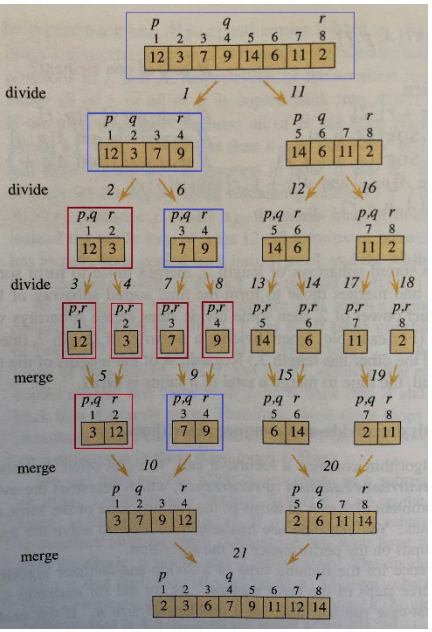


Merge-Sort(A, 3, 4)

Merge-Sort(A, 1, 4)

Merge-Sort(A, 1, 8)

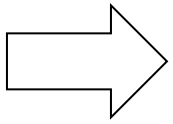
Activation Records and Recursive Calls



Merge-Sort(A, 3, 4)

Merge-Sort(A, 1, 4)

Merge-Sort(A, 1, 8)



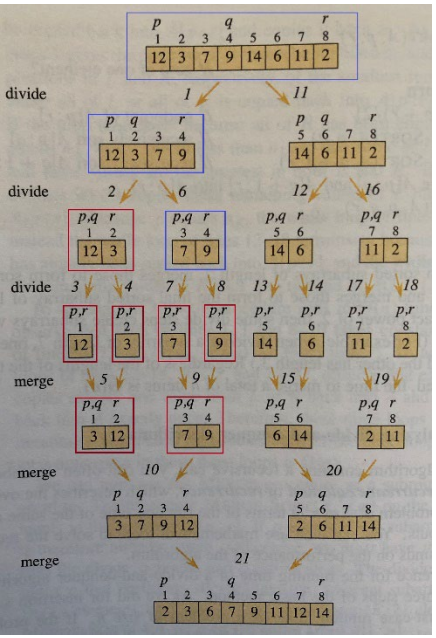
Merge(A, 3, 3, 4)

Merge-Sort(A, 3, 4)

Merge-Sort(A, 1, 4)

Merge-Sort(A, 1, 8)

Activation Records and Recursive Calls

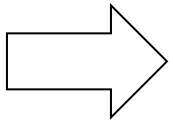


Merge(A, 3, 3, 4)

Merge-Sort(A, 3, 4)

Merge-Sort(A, 1, 4)

Merge-Sort(A, 1, 8)

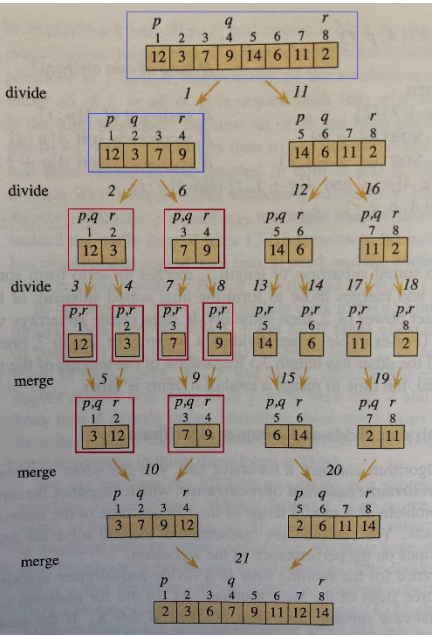


Merge-Sort(A, 3, 4)

Merge-Sort(A, 1, 4)

Merge-Sort(A, 1, 8)

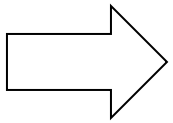
Activation Records and Recursive Calls



Merge-Sort(A, 3, 4)

Merge-Sort(A, 1, 4)

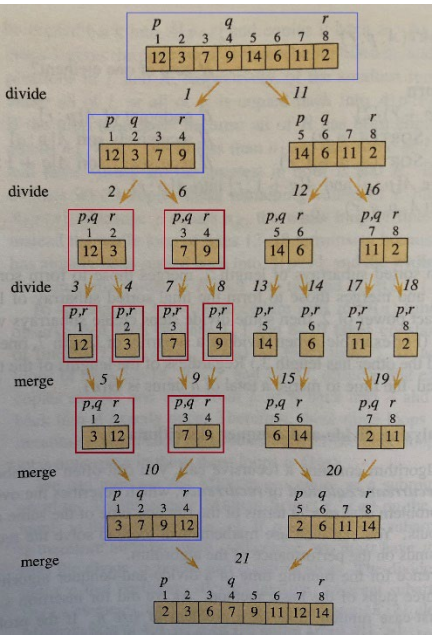
Merge-Sort(A, 1, 8)



Merge-Sort(A, 1, 4)

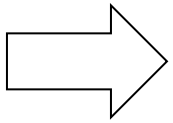
Merge-Sort(A, 1, 8)

Activation Records and Recursive Calls



Merge-Sort(A, 1, 4)

Merge-Sort(A, 1, 8)

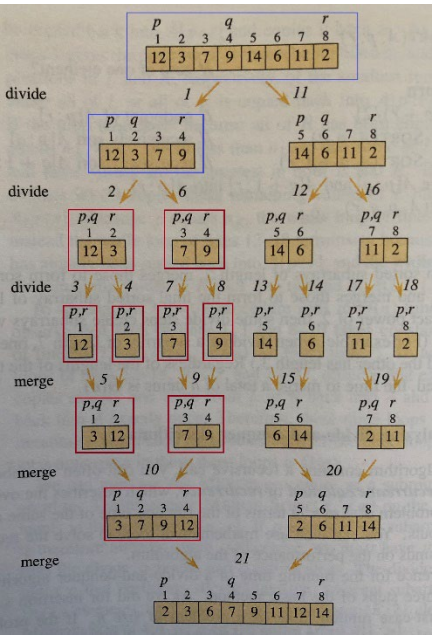


Merge(A, 1, 2, 4)

Merge-Sort(A, 1, 4)

Merge-Sort(A, 1, 8)

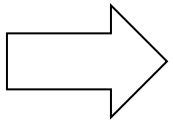
Activation Records and Recursive Calls



Merge(A, 1, 2, 4)

Merge-Sort(A, 1, 4)

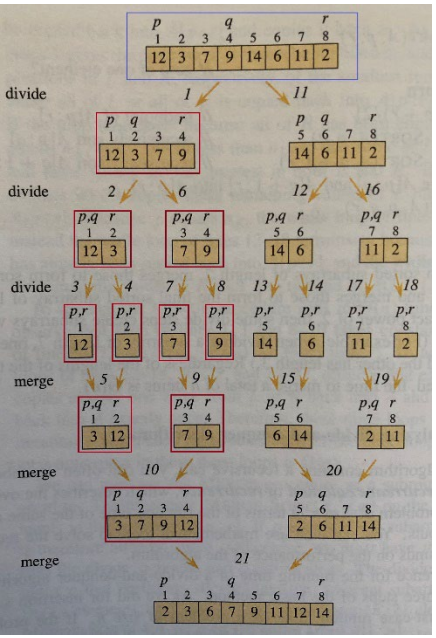
Merge-Sort(A, 1, 8)



Merge-Sort(A, 1, 4)

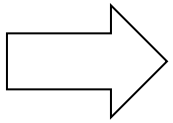
Merge-Sort(A, 1, 8)

Activation Records and Recursive Calls



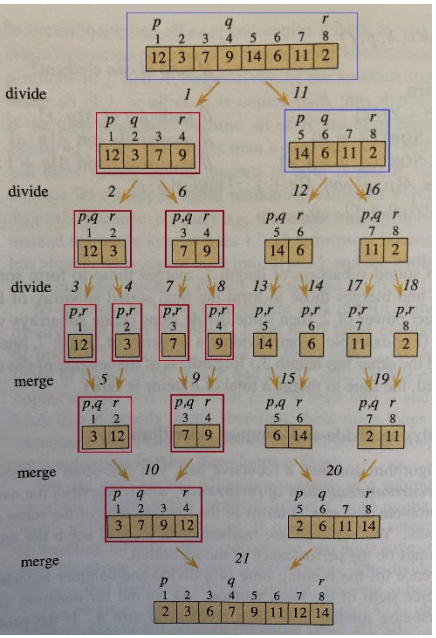
Merge-Sort(A, 1, 4)

Merge-Sort(A, 1, 8)

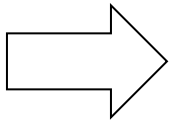


Merge-Sort(A, 1, 8)

Activation Records and Recursive Calls



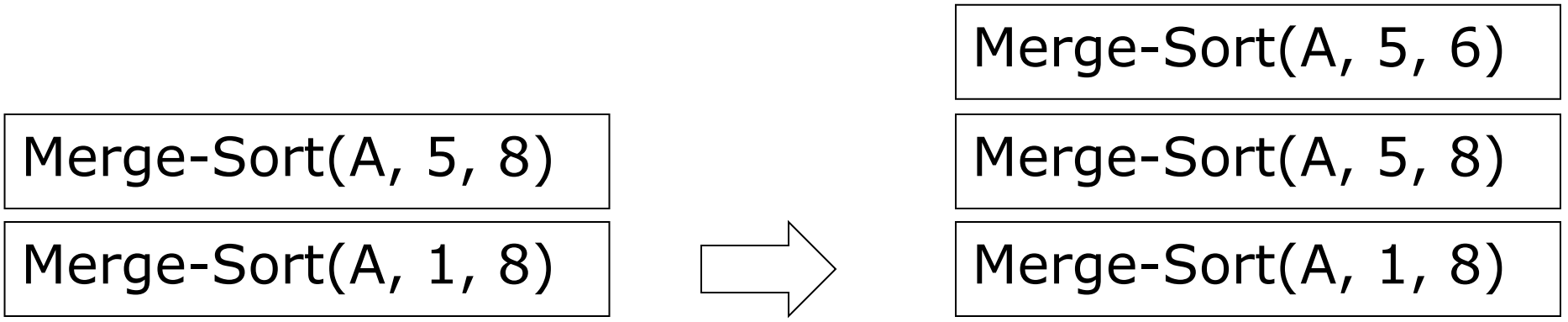
Merge-Sort(A, 1, 8)



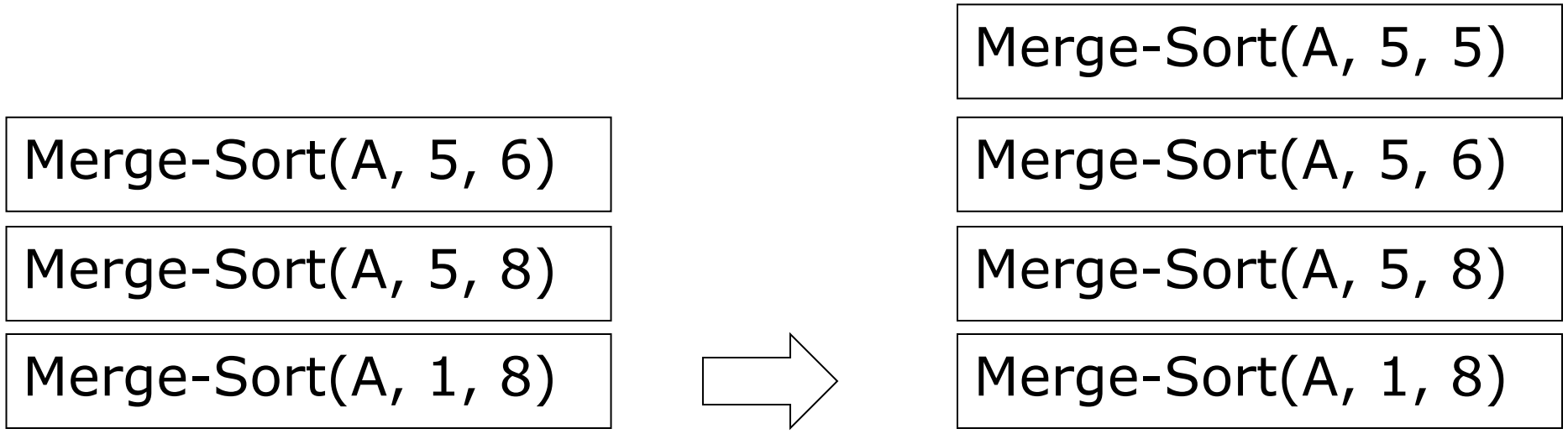
Merge-Sort(A, 5, 8)

Merge-Sort(A, 1, 8)

Activation Records and Recursive Calls



Activation Records and Recursive Calls



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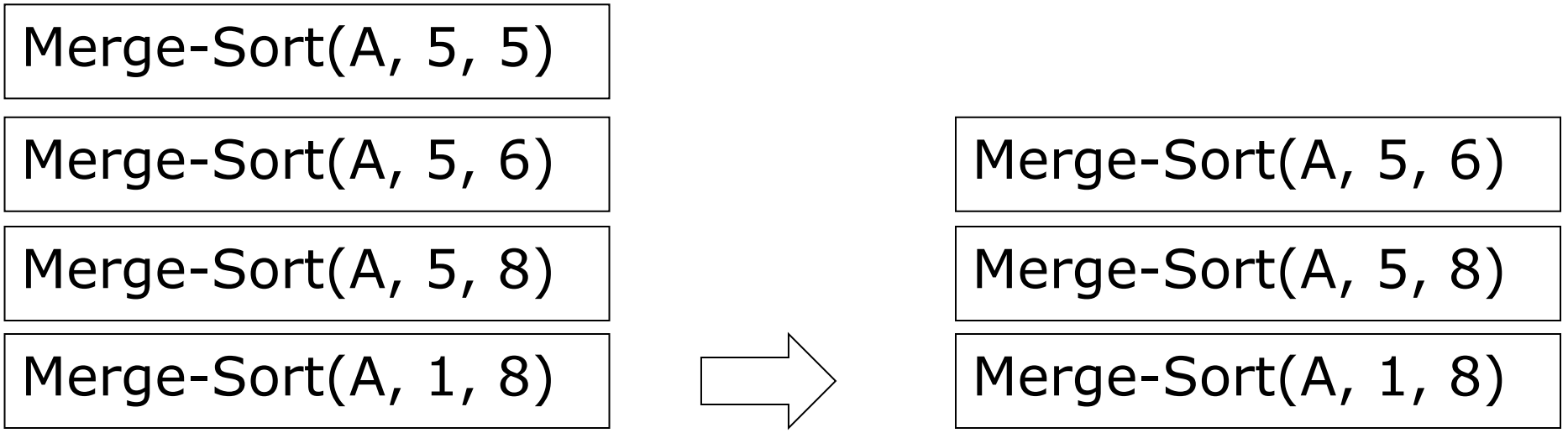

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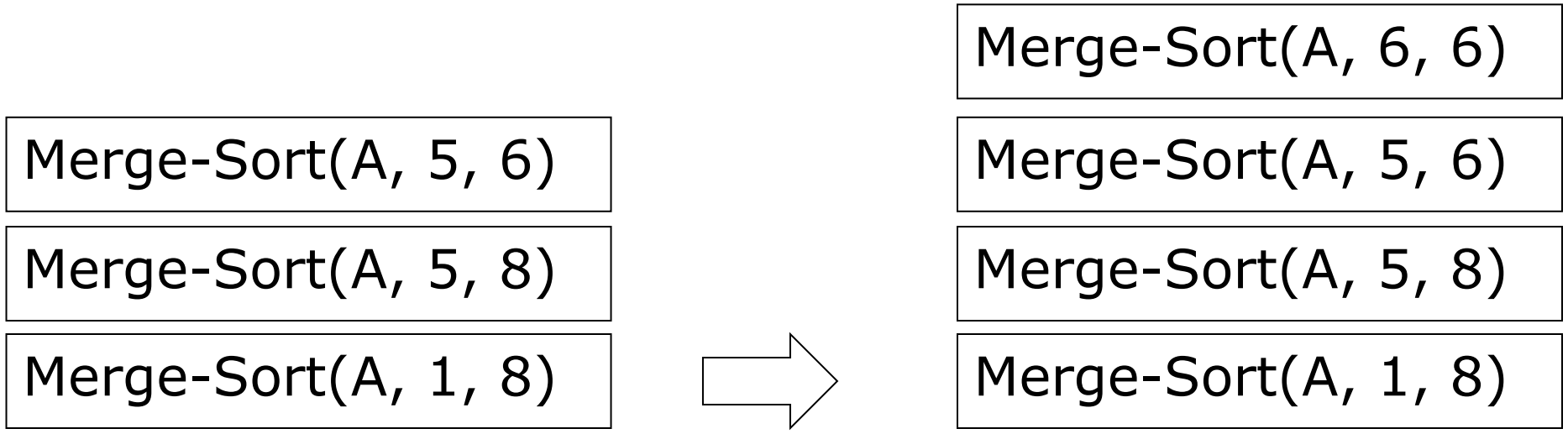

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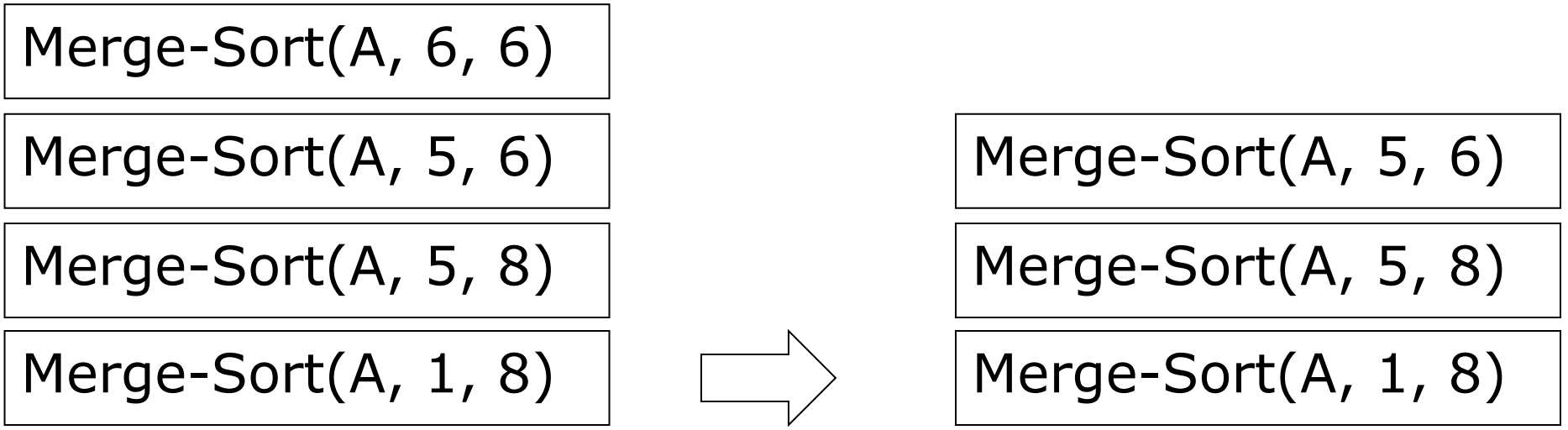

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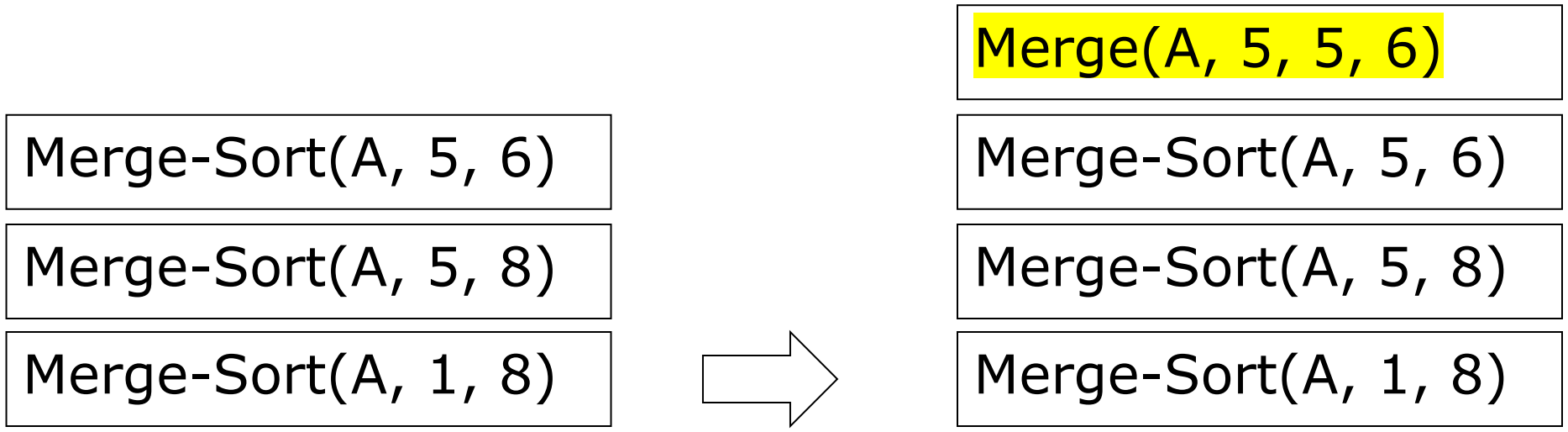
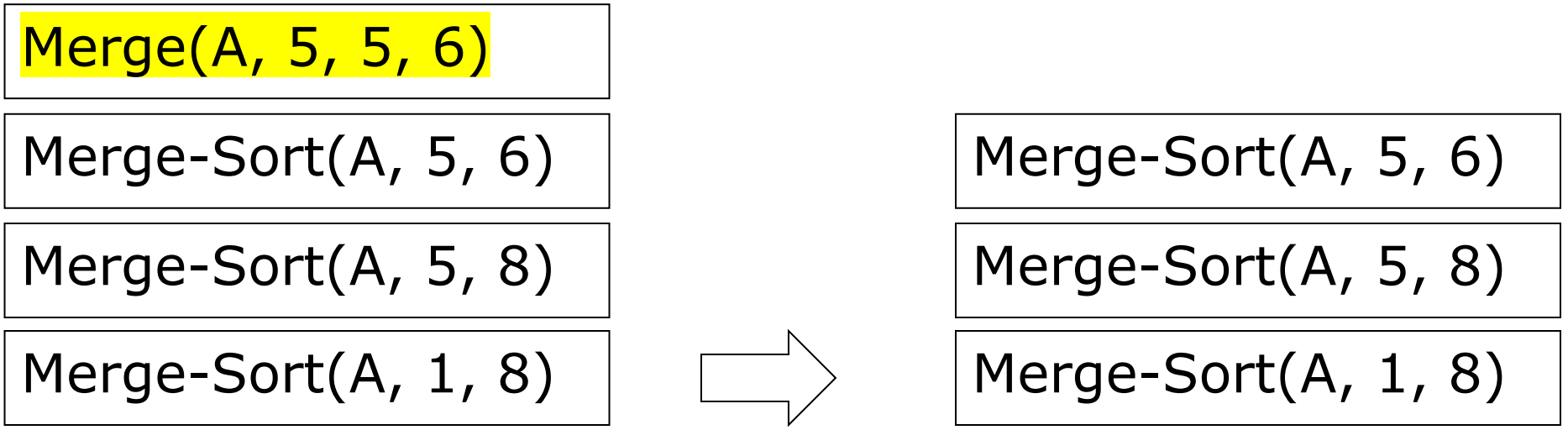


Activation Records and Recursive Calls



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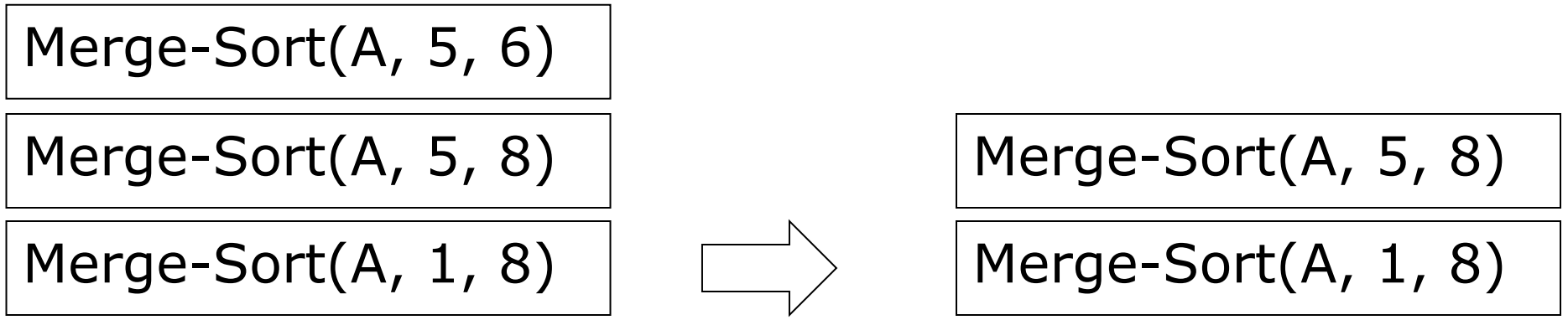

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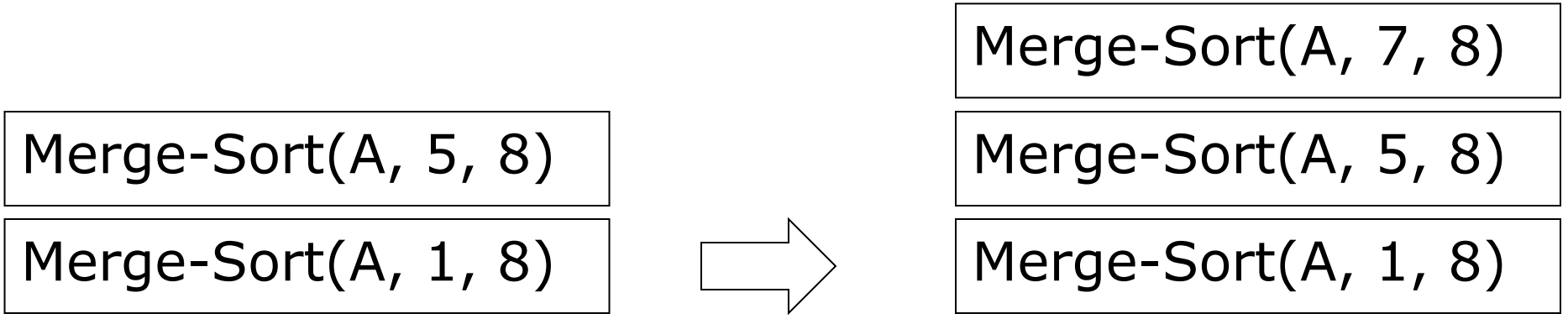

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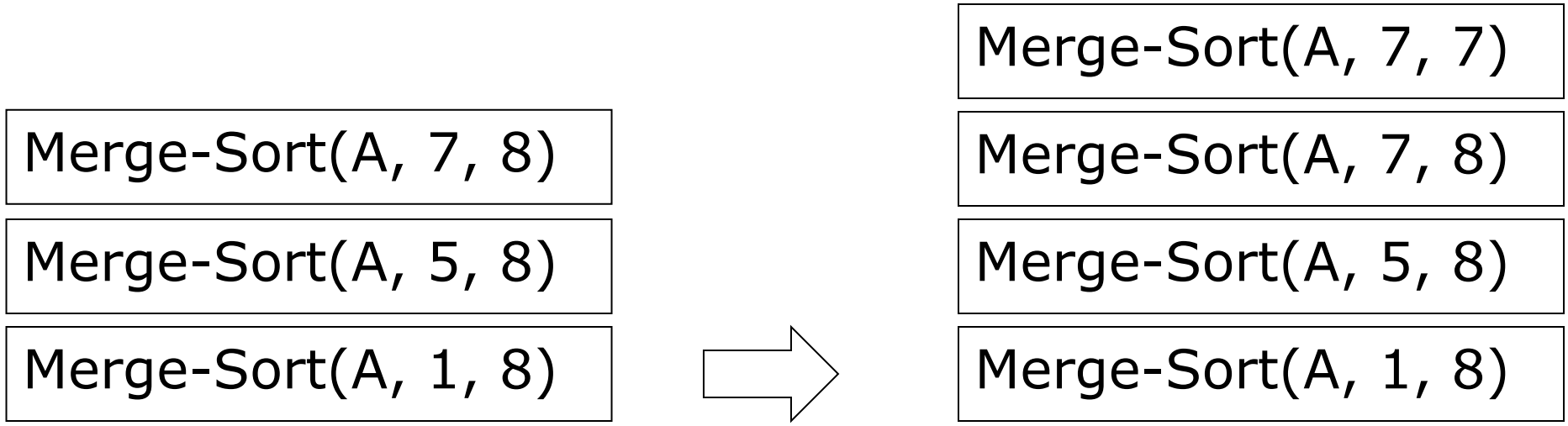

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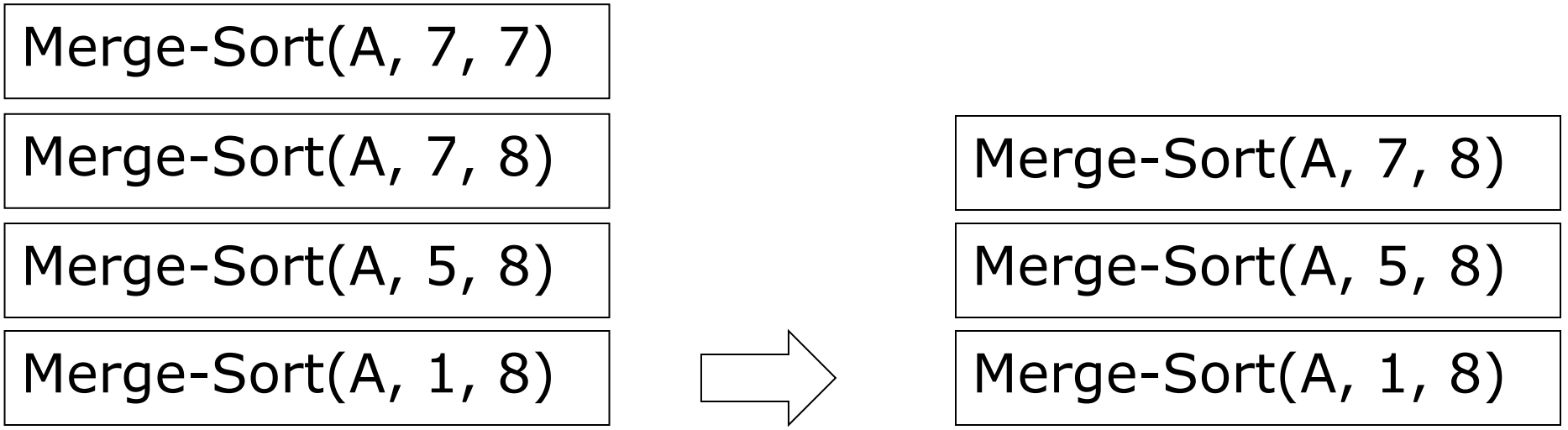

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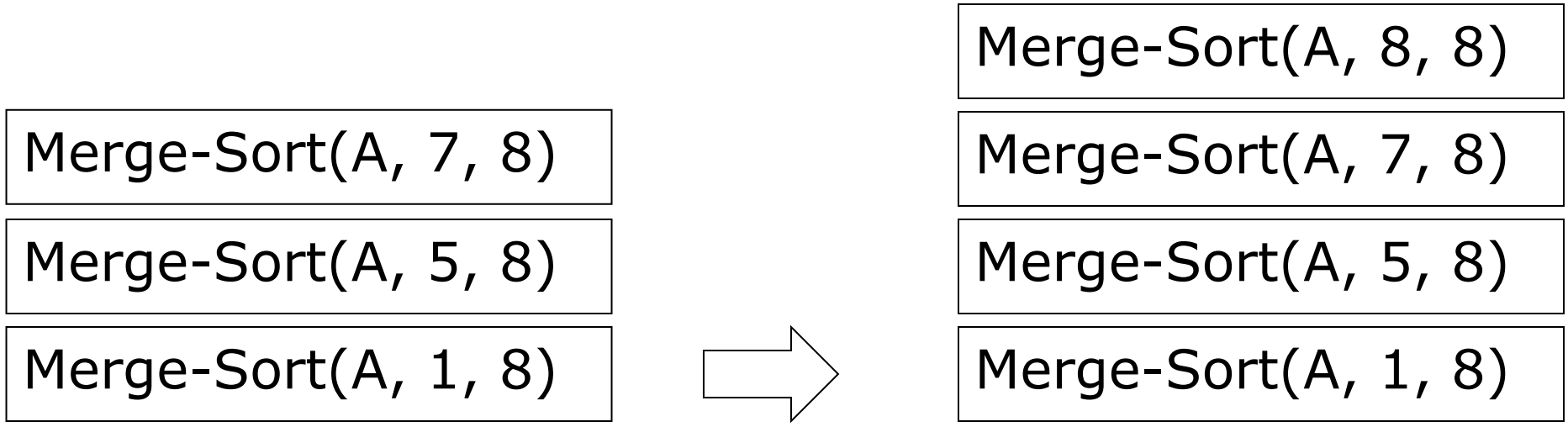

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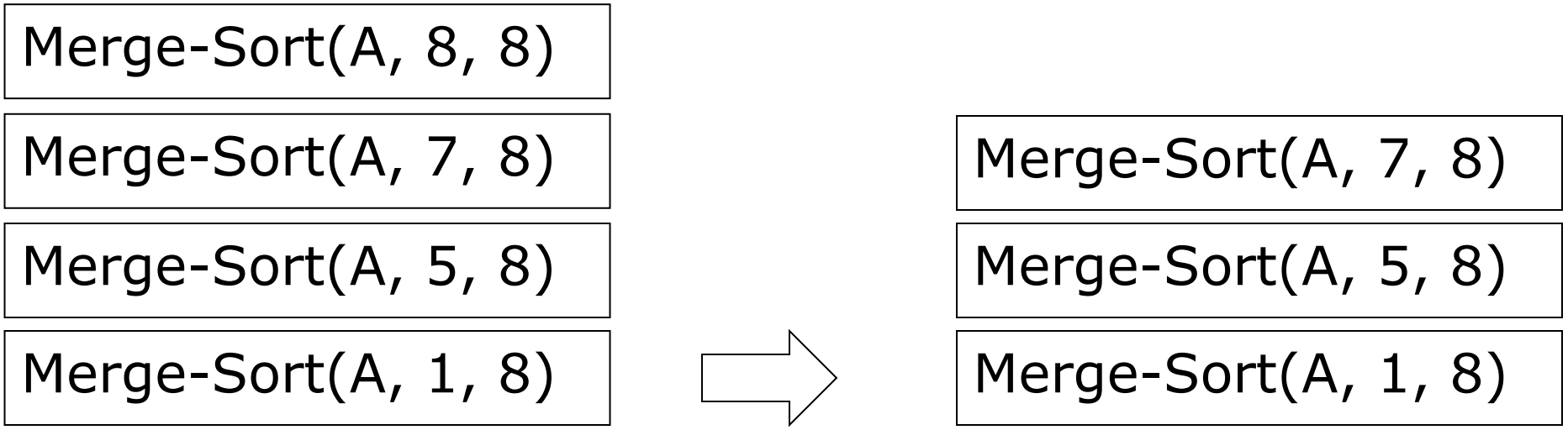
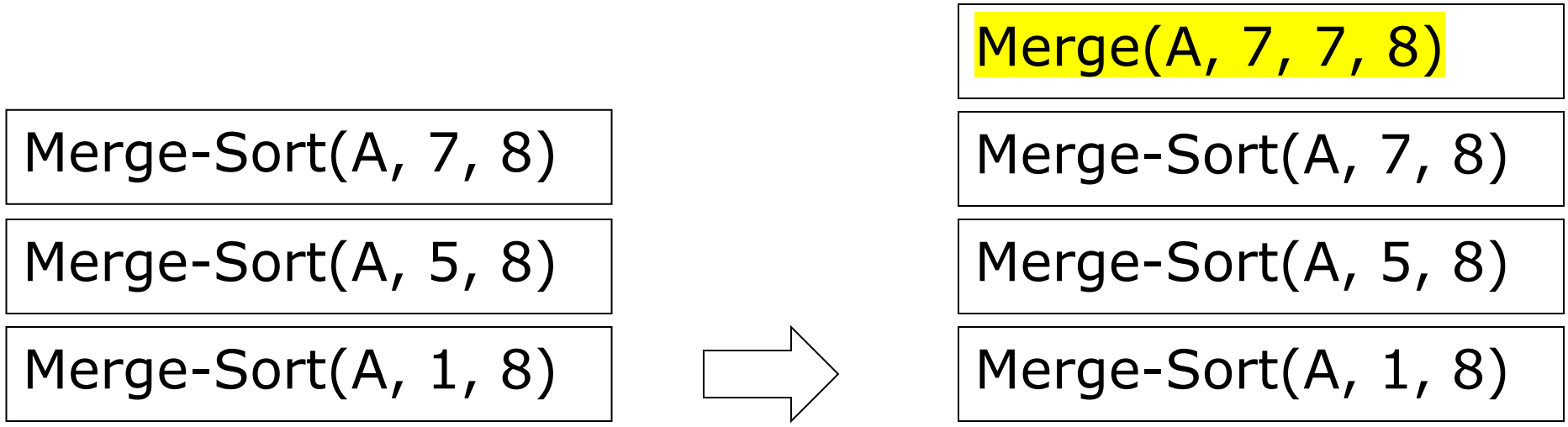
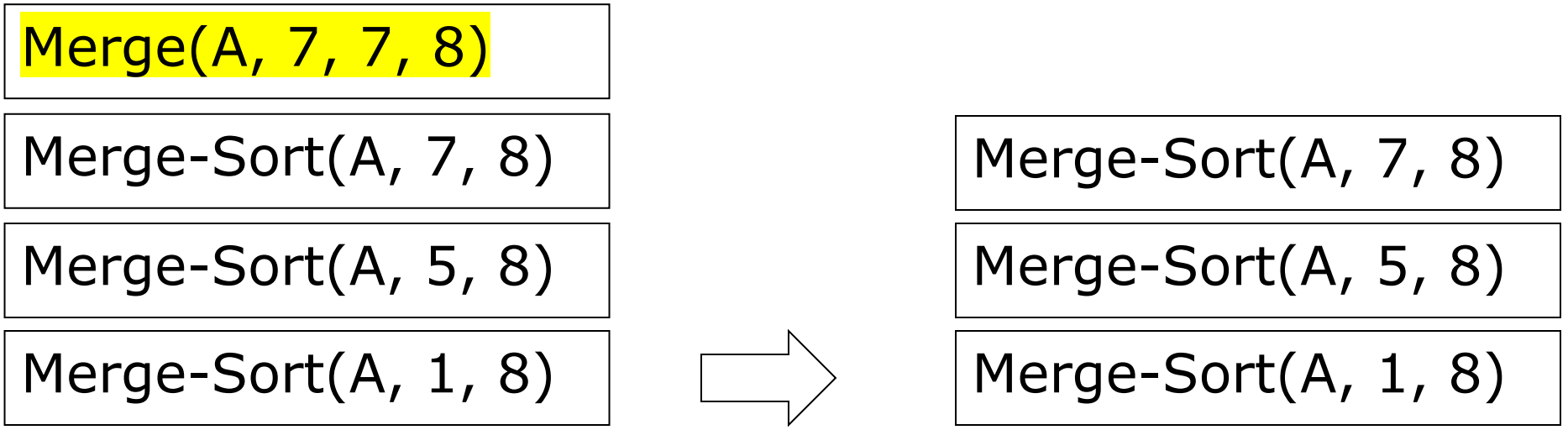


Activation Records and Recursive Calls



Activation Records and Recursive Calls



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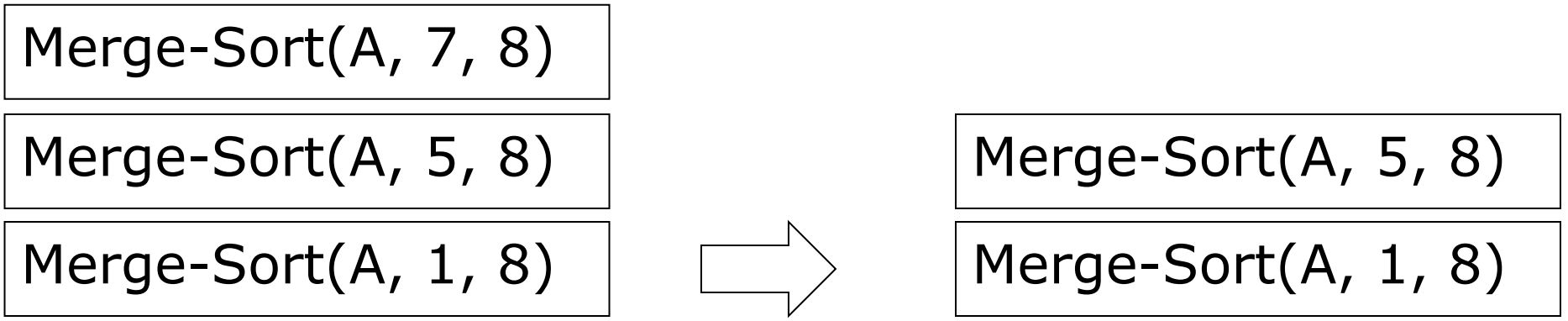
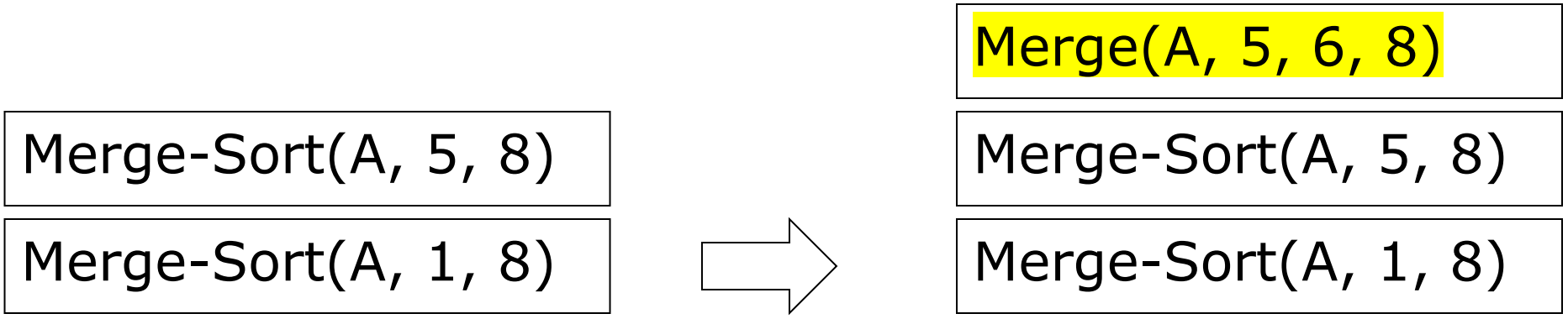


Activation Records and Recursive Calls



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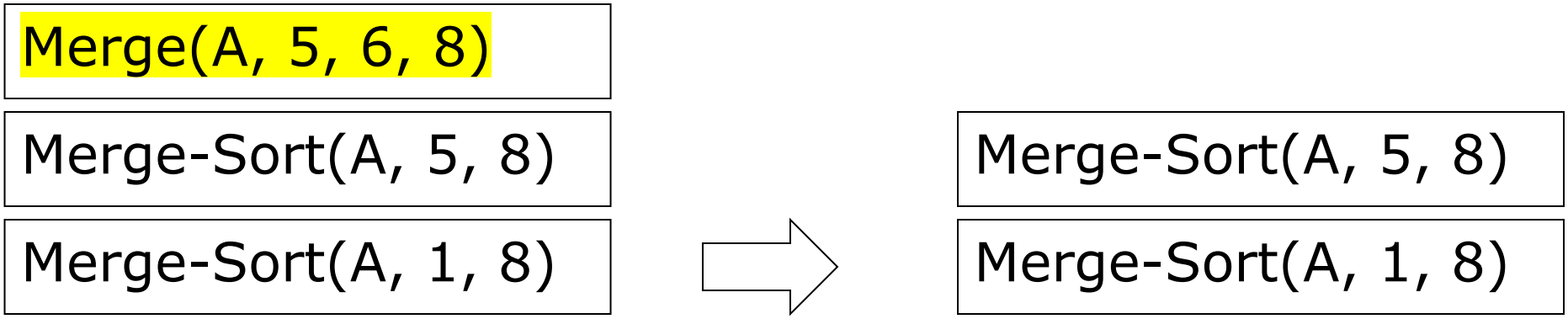

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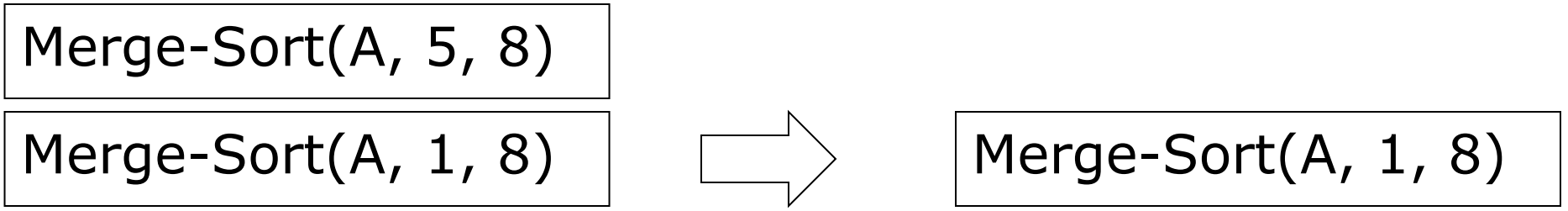

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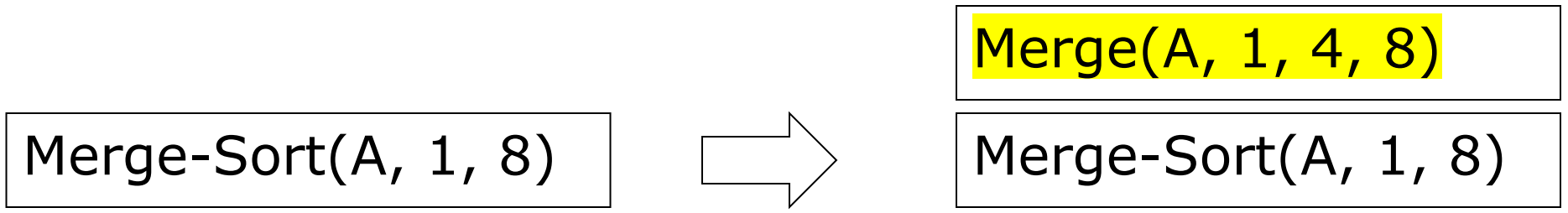
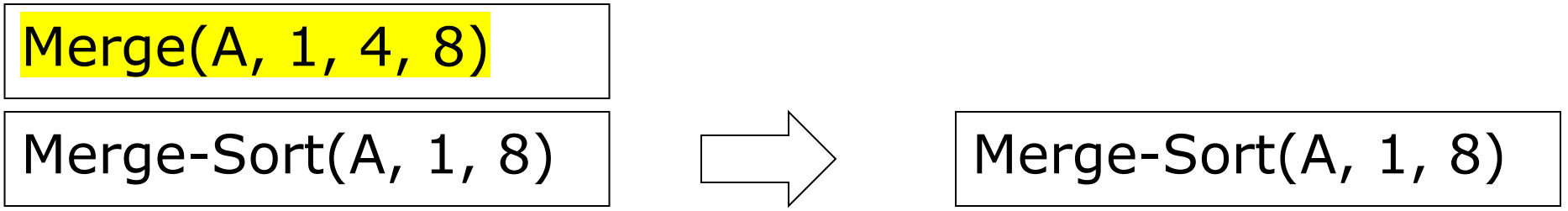
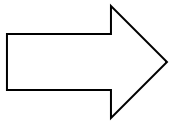


Activation Records and Recursive Calls



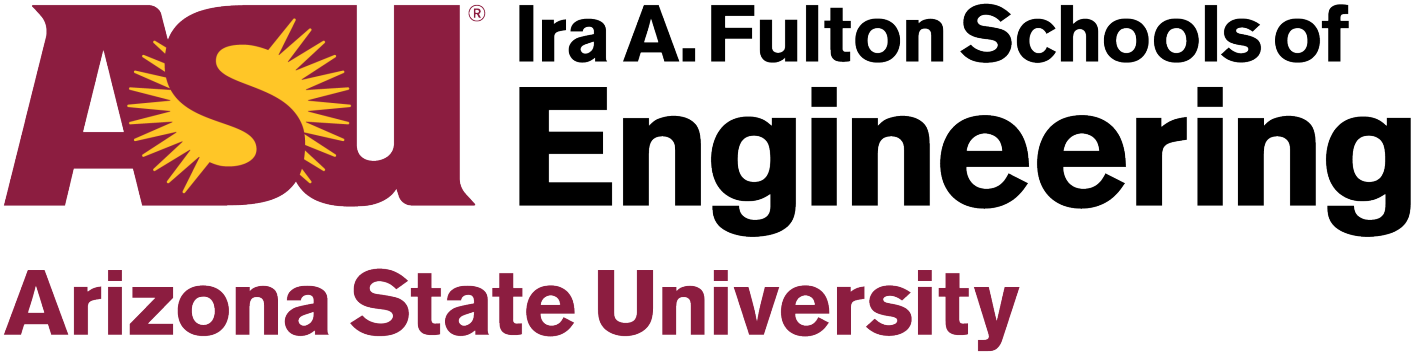
Activation Records and Recursive Calls

Merge-Sort(A, 1, 8)



Running Time of Mergesort

- Time needed for size n instance is T(n)
- The base case (n = 1): $\Theta(1)$
- Time needed to divide the instance: $D(n) = \Theta(1)$
- Time needed for 2 sub-instances is $2T(n/2)$
- $T(n) = 2T(n/2) + n$
- $T(n) = \Theta(n \log(n))$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