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a) $\sum_{i=1}^{n}(y_i-\hat{\beta}_0-\sum_{j=1}^{p}\hat{\beta}_j x_j)^2+\lambda\sum_{i=1}^{p}\hat{\beta}_i^2$

$= (y_1-\hat{\beta}_1 x_{11}-\hat{\beta}_2 x_{12})^2+(y_2-\hat{\beta}_1 x_{21}-\hat{\beta}_2 x_{22})^2+\lambda\sum_{i=1}^{p}\hat{\beta}_i^2$

$=\lambda(\hat{\beta}_1^2+\hat{\beta}_2^2)$

Since $x_{11}=x_{12}=x_1$, $x_{21}=x_{22}=x_2$

$=(y_1-\hat{\beta}_1 x_1-\hat{\beta}_2 x_1)^2+(y_2-\hat{\beta}_1 x_2-\hat{\beta}_2 x_2)^2+\lambda(\hat{\beta}_1^2+\hat{\beta}_2^2)$

b) $\frac{df}{d\beta_1}=-2(y_1-\beta_1 x_1-\beta_2 x_1)x_1-2(y_2-\beta_1 x_2-\beta_2 x_2)x_2+2\lambda\beta_1$

$-2y_1x_1-2y_2x_2+2\beta_1 x_1^2+2\beta_2 x_2^2+2\beta_2 x_1^2+2\beta_2 x_2^2+2\beta_1 x_2^2+$
$2\lambda\beta_1=0$

$\beta_1 x_1^2+\beta_2 x_1^2+\beta_1 x_2^2+\lambda\beta_1+\beta_2 x_2^2=x_1y_1+x_2y_2$ (1)

$\frac{df}{d\beta_2}=-2(y_1-\beta_1 x_1-\beta_2 x_1)x_1-2(y_2-\beta_1 x_2-\beta_2 x_2)x_2+2\beta_2=0$

$-x_1y_1+2\beta_1 x_1^2+2\beta_2 x_1^2-y_2x_2+2\beta_1 x_2^2+2\beta_2 x^2+2\beta_2=0$

$\beta_1 x_1^2+\beta_2 x_1^2+\beta_1 x_2^2+\beta_2 x^2+2\beta_2=x_1y_1+x_2y_2$ (2)

from (1) and (2) we get

$\beta_1=\beta_2$

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c) Lasso minimizing equation,

$$\sum_{i=1}^{n} y_i - \sum_{j=j}^{p} \beta_j x_{ij} + \lambda \sum_{i=1}^{p} |\hat{\beta}_i|$$

d) Alternate lasso ~~equation~~ constraints

$$= |\hat{\beta}_1| + |\hat{\beta}_2| < s$$

Squared optimisation constraints =>

$$(y_1 - \beta_1 x_{11} - \beta_2 x_{12})^2 + (y_2 - \beta_1 x_{21} - \beta_2 x_{22})^2 \quad ①$$

Given $x_{11} = x_{12}$, $x_{21} = x_{22}$, $x_{11} + x_{21} = 0$, $x_{12} + x_{22} = 0$ and $y_1 + y_2 = 0$

Eq ① simplify to $2 \cdot (y_1 - (\hat{\beta}_1 + \hat{\beta}_2) x_{11})^2 = 0$

we get $\frac{y_1}{x_1} = \hat{\beta}_1 + \hat{\beta}_2 \quad - ②$

Eq ② represents the line parallel to the edge of lasso-diamond $\hat{\beta}_1 + \hat{\beta}_2 = s$,

Solution lies of $\hat{\beta}_1 + \hat{\beta}_2 = s$ and $\hat{\beta}_1 + \hat{\beta}_2 = -s$

Hence, Lasso have many solution given by

$$\left.\begin{array}{l} \beta_1 + \beta_2 = s \\ \beta_1 + \beta_2 = -s \end{array}\right\} \begin{array}{l} \text{when } \beta_1, \beta_2 \geq 0 \\ \text{when } \beta_1, \beta_2 \leq 0 \end{array}$$

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