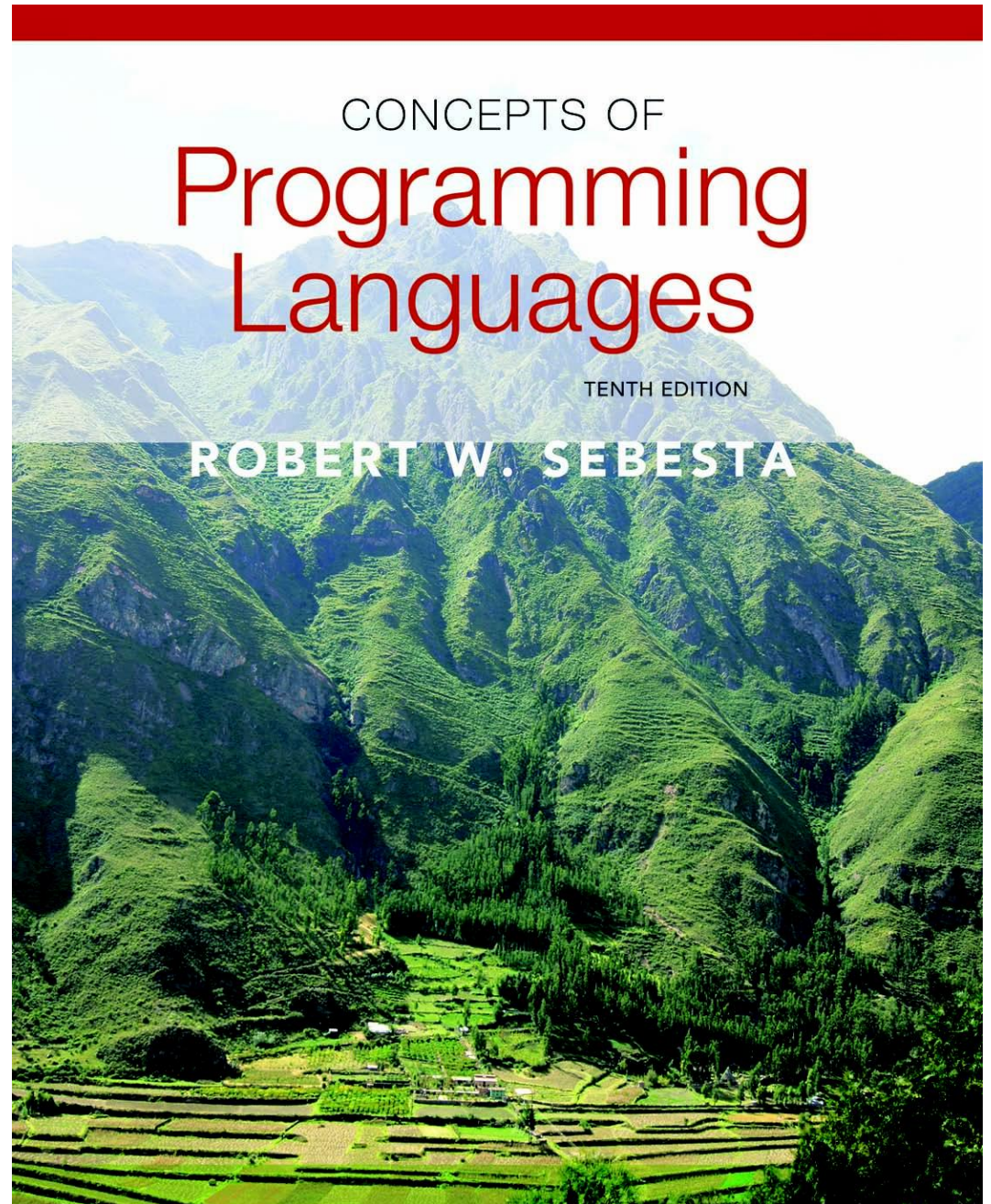


Chapter 16

Logic Programming Languages



Chapter 16 Topics

- Introduction
- A Brief Introduction to Predicate Calculus
- Predicate Calculus and Proving Theorems
- An Overview of Logic Programming
- The Origins of Prolog
- The Basic Elements of Prolog
- Deficiencies of Prolog
- Applications of Logic Programming

Introduction

- Programs in logic languages are expressed in a form of **symbolic** logic
- Use a logical inferencing process to produce results
- ***Declarative*** rather than *procedural*:
 - Only specification of *results* are stated (not detailed *procedures* for producing them)

Proposition

- A logical statement that may or may not be true
 - Consists of objects and relationships of objects to each other

Symbolic Logic

- Logic which can be used for the basic needs of formal logic:
 - Express **propositions**
 - Express **relationships** between propositions
 - Describe how **new propositions** can be inferred from other propositions
- Particular form of symbolic logic used for logic programming called ***predicate calculus***

Object Representation

- Objects in propositions are represented by simple terms: either constants or variables
- *Constant*: a **symbol** that represents an **object**
- *Variable*: a **symbol** that can represent **different objects at different times**
 - Different from variables in imperative languages

Compound Terms

- *Atomic propositions* consist of compound terms
- *Compound term*: one element of a mathematical relation, written like a mathematical function
 - Mathematical function is a **mapping**
 - Can be written as a **table**

Parts of a Compound Term

- Compound term composed of two parts
 - **Functor:** function symbol that names the relationship
 - Ordered list of parameters (tuple)
- **Examples:**

```
student(jon)
```

```
like(seth, OSX)
```

```
like(nick, windows)
```

```
like(jim, linux)
```


Forms of a Proposition

- Propositions can be stated in two forms:
 - *Fact*: proposition is assumed to be true
 - *Query*: truth of proposition is to be determined
- Compound proposition:
 - Have two or more atomic propositions
 - Propositions are **connected** by operators

Logical Operators

Name	Symbol	Example	Meaning
negation	\neg	$\neg a$	not a
conjunction	\cap	$a \cap b$	a and b
disjunction	\cup	$a \cup b$	a or b
equivalence	\equiv	$a \equiv b$	a is equivalent to b
implication	\supset	$a \supset b$	a implies b
	\subset	$a \subset b$	b implies a

Quantifiers

Name	Example	Meaning
universal	$\forall X.P$	For all X, P is true
existential	$\exists X.P$	There exists a value of X such that P is true

Clausal Form

- Too many ways to state the same thing
- Use a standard form for propositions
- *Clausal form*:
 - $B_1 \cup B_2 \cup \dots \cup B_n \subset A_1 \cap A_2 \cap \dots \cap A_m$
 - means if all the As are true, then at least one B is true
- *Antecedent*: right side
- *Consequent*: left side

Predicate Calculus and Proving Theorems

- A use of propositions is to **discover new theorems** that can be **inferred from known axioms and theorems**
- ***Resolution***: an inference principle that allows inferred propositions to be **computed from given propositions**

Resolution

- *Unification*: finding values for variables in propositions that allows **matching** process to succeed
- *Instantiation*: assigning temporary values to variables to allow unification to succeed
- After instantiating a variable with a value, if matching fails, may need to *backtrack* and instantiate with a different value

Proof by Contradiction

- *Hypotheses*: a set of pertinent propositions
- *Goal*: negation of theorem stated as a proposition
- Theorem is proved by finding an inconsistency

Theorem Proving

- Basis for logic programming
- When propositions used for resolution, only restricted form can be used
- *Horn clause* – can have only two forms
 - *Headed*: single atomic proposition on left side
 - *Headless*: empty left side (used to state facts)
- Most propositions can be stated as Horn clauses

Overview of Logic Programming

- Declarative semantics
 - There is a simple way to determine the meaning of each statement
 - Simpler than the semantics of imperative languages
- Programming is nonprocedural
 - Programs do not state now a result is to be computed, but rather the form of the **result**

Example: Sorting a List

- Describe the **characteristics** of a sorted list, **not the process** of rearranging a list

$\text{sort}(\text{old_list}, \text{new_list}) \subset \text{permute}(\text{old_list}, \text{new_list}) \cap \text{sorted}(\text{new_list})$

$\text{sorted}(\text{list}) \subset \forall_j \text{ such that } 1 \leq j < n, \text{list}(j) \leq \text{list}(j+1)$

The Origins of Prolog

- University of Aix–Marseille (Calmerauer & Roussel) – French
 - Natural language processing
- University of Edinburgh – UK (Kowalski)
 - Automated theorem proving

Terms

- This book uses the Edinburgh syntax of Prolog
- *Term*: a constant, variable, or structure
- *Constant*: an atom or an integer
- *Atom*: symbolic value of Prolog
- Atom consists of either:
 - a string of letters, digits, and underscores beginning with a lowercase letter
 - a string of printable ASCII characters delimited by apostrophes

Terms: Variables and Structures

- *Variable*: any string of letters, digits, and underscores beginning with an uppercase letter
- *Instantiation*: binding of a variable to a value
 - Lasts only as long as it takes to satisfy one complete goal
- *Structure*: represents atomic proposition
functor (*parameter list*)

Fact Statements

- Used for the hypotheses
- Headless Horn clauses

```
female(shelley) .
```

```
male(bill) .
```

```
father(bill, jake) .
```

Rule Statements

- Used for the hypotheses
- Headed Horn clause
- Right side: *antecedent* (*if* part)
 - May be single term or conjunction
- Left side: *consequent* (*then* part)
 - Must be single term
- *Conjunction*: multiple terms separated by logical AND operations (implied)

Example Rules

```
ancestor(mary,shelley):- mother(mary,shelley).
```

- Can use variables (*universal objects*) to generalize meaning:

```
parent(X,Y):- mother(X,Y).
```

```
parent(X,Y):- father(X,Y).
```

```
grandparent(X,Z):- parent(X,Y), parent(Y,Z).
```


Goal Statements

- For theorem proving, theorem is in form of proposition that we want system to prove or disprove – *goal statement*
- Same format as headless Horn

`man(fred)`

- Conjunctive propositions and propositions with variables also legal goals

`father(X, mike)`

Inferencing Process of Prolog

- Queries are called goals
- If a goal is a compound proposition, each of the facts is a subgoal
- To prove a goal is true, must find a chain of inference rules and/or facts. For goal Q:

$P_2 \text{ :- } P_1$

$P_3 \text{ :- } P_2$

...

$Q \text{ :- } P_n$

- Process of proving a subgoal called matching, satisfying, or resolution

Approaches

- *Matching* is the process of proving a proposition
- Proving a subgoal is called *satisfying* the subgoal
- *Bottom-up resolution, forward chaining*
 - Begin with facts and rules of database and attempt to find sequence that leads to goal
 - Works well with a large set of possibly correct answers
- *Top-down resolution, backward chaining*
 - Begin with goal and attempt to find sequence that leads to set of facts in database
 - Works well with a small set of possibly correct answers
- Prolog implementations use backward chaining

Subgoal Strategies

- When goal has more than one subgoal, can use either
 - Depth-first search: find a complete proof for the first subgoal before working on others
 - Breadth-first search: work on all subgoals in parallel
- Prolog uses depth-first search
 - Can be done with fewer computer resources

Backtracking

- With a goal with multiple subgoals, if fail to show truth of one of subgoals, reconsider previous subgoal to find an alternative solution: *backtracking*
- Begin search where previous search left off
- Can take lots of time and space because may find all possible proofs to every subgoal

Simple Arithmetic

- Prolog supports integer variables and integer arithmetic
- `is` operator: takes an arithmetic expression as right operand and variable as left operand

`A is B / 17 + C`

- Not the same as an assignment statement!
 - The following is illegal:

`Sum is Sum + Number.`

Example

```
speed(ford,100) .
speed(chevy,105) .
speed(dodge,95) .
speed(volvo,80) .
time(ford,20) .
time(chevy,21) .
time(dodge,24) .
time(volvo,24) .
distance(X,Y) :-    speed(X,Speed) ,
                    time(X,Time) ,
                    Y is Speed * Time.
```

A query: `distance(chevy, Chevy_Distance) .`

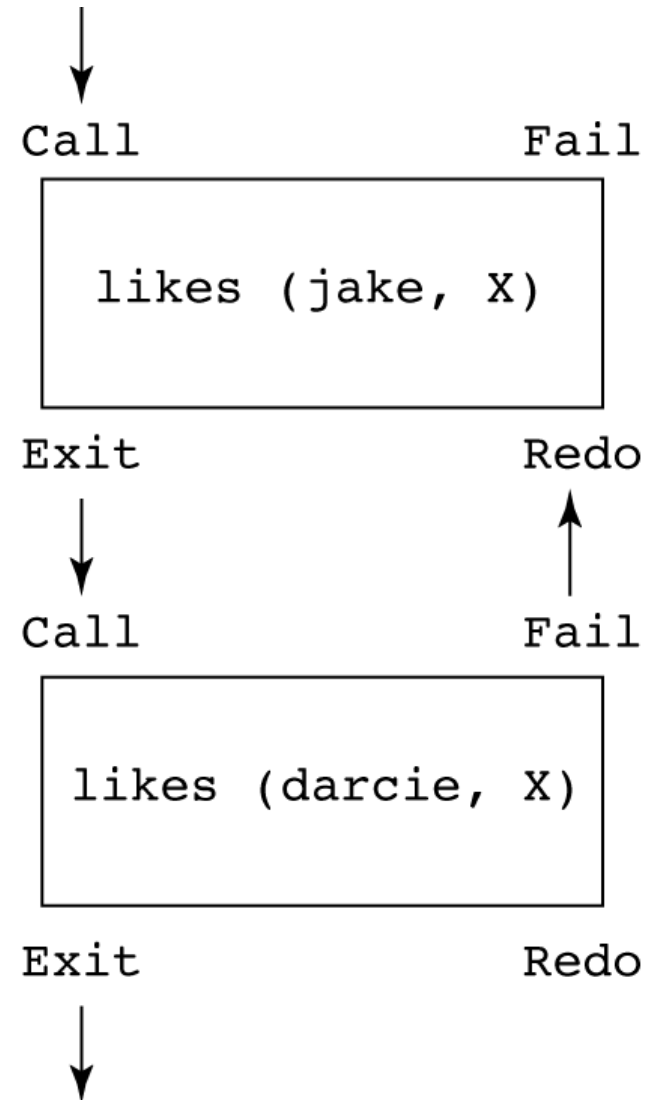
Trace

- Built-in structure that displays instantiations at each step
- *Tracing model* of execution – four events:
 - *Call* (beginning of attempt to satisfy goal)
 - *Exit* (when a goal has been satisfied)
 - *Redo* (when backtrack occurs)
 - *Fail* (when goal fails)

Example

```
likes(jake, chocolate).
likes(jake, apricots).
likes(darcie, licorice).
likes(darcie, apricots).

trace.
likes(jake, X), likes(darcie, X).
(1) 1 Call: likes(jake, _0)?
(1) 1 Exit: likes(jake, chocolate)
(2) 1 Call: likes(darcie, chocolate)?
(2) 1 Fail: likes(darcie, chocolate)
(1) 1 Redo: likes(jake, _0)?
(1) 1 Exit: likes(jake, apricots)
(3) 1 Call: likes(darcie, apricots)?
(3) 1 Exit: likes(darcie, apricots)
X = apricots
```



List Structures

- Other basic data structure (besides atomic propositions we have already seen): list
- *List* is a sequence of any number of elements
- Elements can be atoms, atomic propositions, or other terms (including other lists)

[apple, prune, grape, kumquat]

[] (*empty list*)

[X | Y] (*head X and tail Y*)

Append Example

```
append([], List, List).  
append([Head | List_1], List_2, [Head | List_3]) :-  
    append (List_1, List_2, List_3).
```

More Examples

```
reverse([], []).  
reverse([Head | Tail], List) :-  
    reverse (Tail, Result),  
        append (Result, [Head], List).
```

```
member(Element, [Element | _]).  
member(Element, [_ | List]) :-  
    member(Element, List).
```

The underscore character means an anonymous variable—it means we do not care what instantiation it might get from unification

Deficiencies of Prolog

- Resolution order control
 - In a pure logic programming environment, the order of attempted matches is nondeterministic and all matches would be attempted concurrently
- The closed-world assumption
 - The only knowledge is what is in the database
- The negation problem
 - Anything not stated in the database is assumed to be false
- Intrinsic limitations
 - It is easy to state a sort process in logic, but difficult to actually do—it doesn't know how to sort

Applications of Logic Programming

- Relational database management systems
- Expert systems
- Natural language processing

Summary

- Symbolic logic provides basis for logic programming
- Logic programs should be nonprocedural
- Prolog statements are facts, rules, or goals
- Resolution is the primary activity of a Prolog interpreter
- Although there are a number of drawbacks with the current state of logic programming it has been used in a number of areas