1 Week 1

1.1 Screening for HIV

This exercise, with many alternative stories and numerical values, is a classical example of the importance of prior information and of taking into account the baseline prevalence. This specific example is taken from David Spiegelhalter's Understanding Uncertainty blog.

Common HIV blood tests are very accurate – estimates vary, but it is estimated that using current techniques (ELISA and Western blot) around 99.8% of people with HIV test positive, and 99.99% of people without the virus test negative. In the UK, the prevalence of HIV in adults with no risk factors is around 1 in 10000. What is the probability that a person (with no risk factors) with positive test result has HIV?

1.2 About uncertainty and subjective probability

- Read the first Chapter of BD3 and Anthony O'Hagan (2004). *Dicing with the uncertainty*. Significance. (can be found from the course home page)
- Reflect your thoughts about the above texts. For example, does the aleatory and epistemic uncertainties make sense? Describe how you understand the term uncertainty. Where does uncertainty arise from, how does it relate to data analysis?
- Discuss the following statement. "The probability of event E is considered *subjective* if two rational persons A and B can assign unequal probabilities to E, $P_A(E)$ and $P_B(E)$. These probabilities can also be interpreted as *conditional*: $P_A(E) = P(E|I_A)$ and $P_B(E) = P(E|I_B)$, where I_A and I_B represent the knowledge available to persons A and B, respectively." Apply this idea to the following examples.
  - The probability that a 6 appears when a fair die is rolled, where A observes the outcome of the die roll and B does not.
  - The probability that Brazil wins the next World Cup, where A is ignorant of soccer and B is a knowledgeable sports fan.

1.3 Conditional probability

Suppose that if $\theta = 1$, then y has a normal distribution with mean 1 and standard deviation σ, and if $\theta = 2$, then y has a normal distribution with mean 2 and standard deviation σ . Also, suppose $Pr(\theta = 1) = 0.5$ and $Pr(\theta = 2) = 0.5$.

a) For $\sigma = 2$, write the formula for the marginal probability density for y and sketch it.

b) What is $Pr(\theta = 1|y = 1)$, again supposing $\sigma = 2$

c) Describe how the posterior probability of θ, $p(\theta|y)$, changes as σ is increased and as it is decreased.