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Results

In this section, we give an example of the solution of the optimization model we derived in the previous section. In this example, we consider an instant where $N = 6$ and $I = 51$, so that we have 2508 decision variables.

Parameters values

The following tables show the matrix of raw-material flexibility of all items.

Next, we present the instant of the total raw material demands during a planning horizon.

Table 1: the flexibility matrix of items 1-11

item	raw -material demand (in kg)			
	week 1	week 2	week 3	week 4
1	2,555.1	2,555.1	0.0	10,220.4
2	0.0	4,300.0	8,600.0	0.0
3	0.0	0.0	0.0	1,800.0
4	0.0	495.0	0.0	0.0
5	0.0	0.0	0.0	0.0
6	3,150.0	0.0	6,300.0	0.0
7	0.0	1,050.0	0.0	350.0
8	1,650.0	0.0	0.0	2,200.0
9	2,080.0	1,560.0	1,040.0	0.0
10	0.0	1,280.0	0.0	0.0
11	0.0	0.0	4,200.0	0.0
12	0.0	2,700.0	0.0	0.0
13	350.0	350.0	0.0	0.0
14	0.0	300.0	500.0	200.0
15	85.0	0.0	0.0	0.0
16	110.0	165.0	0.0	0.0
17	0.0	0.0	17,400.0	17,400.0
18	74,050.0	222,150.0	14,800.0	0.0
19	0.0	11,605.0	0.0	0.0
20	12,750.0	0.0	0.0	0.0

Table 2: the flexibility matrix of items 12-51

	raw-material demand (in kg)			
item	week 1	week 2	week 3	week 4
21	3,500.0	0.0	14,000.0	14,000.0
22	42,100.0	0.0	10,550.0	0.0
23	0.0	0.0	5,600.0	11,200.0
24	5,400.0	5,400.0	0.0	5,400.0
25	1,350.0	2,250.0	0.0	0.0
26	0.0	32,400.0	43,200.0	0.0
27	10,350.0	31,050.0	0.0	0.0
28	118,200.0	0.0	0.0	0.0
29	3,330.0	4,995.0	0.0	0.0
30	0.0	1,950.0	0.0	5,850.0
31	0.0	6,300.0	6,300.0	0.0
32	2,310.0	3,465.0	0.0	0.0
33	0.0	2,500.0	0.0	0.0
34	0.0	0.0	20,850.0	0.0
35	12,150.0	0.0	0.0	0.0
36	0.0	0.0	0.0	0.0
37	12,750.0	12,750.0	8,500.0	8,500.0
38	1,550.0	3,100.0	0.0	0.0
39	0.0	10,600.0	5,300.0	21,200.0
40	0.0	0.0	10,450.0	20,900.0
41	9,625.0	0.0	1,375.0	1,375.0
42	0.0	0.0	3,080.0	0.0
43	0.0	2,250.0	450.0	2,250.0
44	0.0	0.0	14,100.0	0.0
45	6,700.0	13,400.0	10,050.0	6,700.0
46	6,500.0	13,000.0	0.0	0.0
47	26,200.0	13,100.0	0.0	0.0
48	12,000.0	8,000.0	0.0	12,000.0
49	0.0	126.0	0.0	0.0
50	1,323.0	0.0	3,969.0	0.0
51	1,170.0	1,170.0	0.0	780.0

We can see that items 5 and 36 do not have to be produced during this planning horizon. We also can see that most items have to be produced only in to or three weeks of this planning horizon, with varying demand.

The others parameters values are given in the following table.

Table 3: the other parameter's values

	raw-material					
	1	2	3	4	5	6
price	16.6	15.5	18.1	14.7	19.8	15.2
σ_k (in kg)	15,000.0	10,000.0	11,500.0	9,000.0	8,000.0	12,500.0
z_{0k} (in kg)	3,250.0	2,850.0	3,300.0	2,500.0	3,500.0	3,050.0
m_{0k} (in kg)	7,956,000.0	19,944,000.0	648,000.0	2,952,000.0	288,000.0	4,212,000.0
ss (in kg)	2,500.0	2,500.0	2,500.0	2,500.0	2,500.0	2,500.0

Note:

maxcap (in kg) = 1,427,000

From Table 2 we know that raw material 4 must be purchased since item 49 and 50 can be produced just by using raw material 4. The price of raw material 4 is the lowest one. But the minimal one-year order quantity is the second smallest. So that we may guess that on the optimal solution, x_4 will have a big value but is not the biggest one among others.

Optimal solution

We solve the optimization problem by creating computer codes using R language (version 4) where the optimization problem formulation and the solution technique used are referred to dplyr (Wickham et al. 2023) and ompr (Schumacher 2022) libraries. This program runs on a computer with the Linux Ubuntu 20 LTS operating system with an Intel i7 8 Cores processor and 16 GB RAM.

The values of $x_k, k \in \mathfrak{N}$, are given in the second column of Table 6 in the following. The weekly deliveries are given on columns 4 up to 6. We see that the total one-month order quantity exceeds the minimum one-month order quantity σ_k, so that set of Constraints I is satisfied.

Table 4: Total order quantity and weekly deliveries

		weekly deliveries (in kg)			
Raw Material	Order Qty (in kg)	Week 1	Week 2	Week 3	Week 4
1	192,649	6,553	65,295	81,722	39,079
2	543,385	354,839	23,595	96,122	68,829
3	90,742	41,092	26,550	14,400	8,700
4	56,508	33,264	209	18,369	4,666
5	52,958	42,608	10,350	0	0
6	202,853	70,728	111,075	0	21,050
Total	1,139,094	6,553	65,295	81,722	39,079

Besides the values x_k above, we also get $\hat{x}_{ijk}, a_{ijk}, b_{ijk}$ and values z_{jk} in the optimal solution. We have checked that set of **Constraints II − V** are satisfied by this optimal solution. For brevity, we do not present those values here. We just present and discuss some of them, to show that the remaining constraints are satisfied.

Most values of b_{ijk} are 0.5 which mean most of items are produced by using a composition of two raw materials. The values of b_{ijk} may vary from one week to another week, as illustrated in Table 7. The composition of raw materials for item 46 in week 1is different from the composition in week 2. Furthermore, Table 7 shows us that the proportions of raw materials used for producing an item are the same. We have checked this property through all for all and for all so that we are sure that set of **Constraints VI** are satisfied.

Table 5: the values of b_{ijk} for $i = 45$ and $i = 46$

item	week	raw-material 1	2	3	4	5	6
45	1	0.00	0.50	0.00	0.50	0.00	0.00
45	2	0.50	0.50	0.00	0.00	0.00	0.00
45	3	0.50	0.50	0.00	0.00	0.00	0.00
45	4	0.50	0.50	0.00	0.00	0.00	0.00
46	1	0.33	0.33	0.00	0.33	0.00	0.00
46	2	0.50	0.50	0.00	0.00	0.00	0.00
46	3	0.00	0.00	0.00	0.00	0.00	0.00
46	4	0.00	0.00	0.00	0.00	0.00	0.00

The following table shows the composition of raw materials for items 49 and 50. We can see that these items are produced by using raw material 4, which confirm the flexibility matrix in Table 3.

Table 6: the values of b_{ijk} for $i = 49$ and $i = 50$

item	week	raw-material 1	2	3	4	5	6
49	1	0	0	0	0	0	0
49	2	0	0	0	1	0	0
49	3	0	0	0	0	0	0
49	4	0	0	0	0	0	0
50	1	0	0	0	1	0	0
50	2	0	0	0	0	0	0
50	3	0	0	0	1	0	0
50	4	0	0	0	0	0	0

The fulfillment of the safety stock constraint and the maximum capacity constraint (set of **Constraints VII and VIII**) can be seen in the following table.

Table 7: The fulfillment of the safety stock constraint and the maximum capacity constraint (set of Constraints VII and VIII)

Raw material	end-of-week stock (in kg) week 1	week 2	week 3	week 4
1	2,501	2,501	2,500	2,500
2	181,743	2,501	2,500	2,500
3	2,500	2,500	2,500	2,500
4	2,500	2,501	2,501	2,500
5	2,500	2,500	2,500	2,500
6	2,500	2,500	2,500	2,500

Schumacher, Dirk. 2022. *Ompr: Model and Solve Mixed Integer Linear Programs.* https://github.com/dirkschumacher/ompr.

Wickham, Hadley, Romain François, Lionel Henry, Kirill Müller, and Davis Vaughan. 2023. *Dplyr: A Grammar of Data Manipulation.*