Abstract

This paper concerns the optimization model for supplier selection, order allocation, and raw-material composition in a beverage company that produces a large number of drink powders. There are a number of suppliers that can provide the same key raw material of the drink powders, but the colour or some physical characteristics are slightly different so that we may assume those raw materials are different. The drink powders produced by this company, which in the remainder of this paper are called items, can be classified into two classes of items.

Optimization Model for Raw Material Selection and Composition, and Order Allocation in a Beverage Company

S. Uttunggadewa, M. R. Fadhli, R. Hadianti, S. R. Pudjaprasetya

1 Introduction

This paper concerns the optimization model for supplier selection, order allocation, and raw-material composition in a beverage company that produces a large number of drink powders. There are a number of suppliers that can provide the same key raw material of the drink powders, but the colour or some physical characteristics are slightly different so that we may assume those raw materials are different. The drink powders produced by this company, which in the remainder of this paper are called items, can be classified into two classes of items.

1. The first class consists of items that can be produced by using exactly a single type of raw material.
2. The second class consists of more flexible items, where each item in this class can be produced by using one raw material or by using a composition of a number of raw materials. For each item in this class we then have a set of possible raw materials. The sets of possible materials may vary one to each other.

In order to avoid supply disruption, the company decides to use multiple sources for these raw materials. The company has established selection criteria for each raw material, which are based on the estimated total one-year demand for items and a subjective assessment of whether the raw material cannot be substituted, price, service, and the minimum order for each purchase. After determining the score for each raw material, the company decided to make contracts or agreements with six suppliers. Each contract stated the unit price and the minimum order quantity within a year. Based on these contracts, production planning and inventory control of raw materials is carried out.

The estimated total one-year demand for items is obtained from the forecasting process performed yearly. This forecasting process yields the monthly total demand for items, which is time varying. But at the production level, the company refines the monthly total demand on a monthly basis as a response to some disruptions such as sudden additional requests due to flash sales practices in e-commerce, and others.

Once the demand for items for a month is issued, the company must perform the decision for purchasing the raw materials from some suppliers. This purchase decision from a supplier includes purchase for four serial deliveries one week apart. The first delivery must be no later than 17 days before the following month's start. The period of 17 days here is the total time required for the company's internal inspection and preparation of raw materials.

This decision process is a complex one since there are a large number of items that has to produced which mostly belong to the second class, and the monthly demand may vary. Additionally, the company imposes a production regulation for the second class of items as a result of the multiple-sources policy, which states that each item in the second class must be produced using a composition of at least two types of the corresponding possible raw materials. The decision process must be performed carefully in order to obtain results in the form of:

- which raw materials are purchased along with the delivery size for every four corresponding weeks,
- the composition of raw materials for every item in the second class which has to be produced,

while minimizing the total inventory cost.

The company developed a decision support system for this monthly decision process, which is developed based on an optimization model. This paper concerns the derivation of the optimization model, that can be categorized as a multi-product multi-period raw material selection and composition, and order quantity problem. This problem has been addressed in a number of articles such as Sambatt, Woarawichai, and Naenna(24).

The supplier selection problem initially focused on determining suppliers with only one criterion, namely the price of the materials offered. Scientific papers on supplier selection with usingriterion include Reck Long(23), Monckza Trecha[16], Porter[23], and Harding[11] papers. But then it was realized that using only one criterion, namely price, did not answer the problem comprehensively. So supplier selection research has developed into a problem with multiple criteria, such as criteria for quality of goods, on time delivery, after-sales service, as well as environmental and socio-political criteria for suppliers (see Smytka Clemens[27], Gray, Helper Osborn[9]). . What is interesting is that in general these criteria contradict each other, for example, goods offered at low prices (positive values for the price criteria) may have negative values for on time delivery criteria. The complexity of this issue is compounded by the fact that some criteria are quantitative (price, timeliness of delivery, specification/quality of goods, etc.), but other criteria are qualitative (after sales service, environmental and socio-political criteria of suppliers).

Weber's paper, Current Benton[29] is a paper at the beginning of this research on multi-criteria supplier selection, which presents research results with four criteria, namely Price, Quality, Delivery and Service (PDQS). This paper together with Hurkens, van der Valk, Wynstra [12] introduces the supplier selection problem under the concept of Total Cost Ownership (TCO), a financial analysis tool to examine the direct and indirect costs of a product's production. These direct and indirect costs then become the criteria in the supplier selection process. These papers on TCO include Ferrin Plank[8], Degraeve Roodhooft[5],

After the rise of conceptual research on supplier selection with multi-criteria, then we quite easily find a proposal to use the Analytic Hierarchy Process (AHP), a decision-making method when it comes to ranking of many criteria (see Dyer[6]), as a method of solving supplier problems. selection. AHP provides a framework for addressing various criteria involving intuitive, rational, qualitative and quantitative aspects. Other papers that discuss the AHP approach to supplier selection solutions include Bard[2], Belton[3], Bhutta Huq[4], Nydick Hill[20].

Another method proposed as a solution to the supplier selection problem is an optimization method or mathematical programming as proposed by Degraeve and Roodhooft [5], Khalifa Mohammed Al-Shabi [10], and Nispeling [19]. A special optimization method, namely multi-objective goal programming, was proposed by Weber Ellram [30]. Multi-objective programming is very suitable to be used to resolve conflicts between existing criteria and the existence of just-in-time scenarios. Meanwhile, Masella A. Rangone [15] offer a dynamic programming method as a method of completing this supplier selection, where input variables are set as controls and environmental variables and status variables are set as the internal workings of the organization, and output variables are seen as company performance. Another optimization method used as a solution method is Data Envelopment Analysis (DEA), as proposed in the paper of Pitchipoo, et al.[22] and Shahrzad, et al.[26].

Apart from these methods, we get the combined use of the two methods above (hybrid method), such as the one proposed by Li, Wong, Kwong [13] which combines the AHP method and multi-objective programming. Another approach is the metaheuristic method proposed by Alejo-Reyes, et al.[1]. This heuristic method is likely to be a method that is widely used as a method of solving supplier selection problems considering that the heuristic method has proven to be an effective and efficient method as a method of solving optimization problems, although there is no guarantee this method will provide an optimal solution but it is guaranteed to provide a solution. near optimal'. For supplier selection problems where the procurement division must be able to make decisions quickly, this near optimal solution is often considered sufficient because sometimes decision making has to be done many times due to disturbances from outside the company such as in-time delivery by one supplier. .

2 Production Planning and Inventory Control for Raw Materials

As mentioned above, the company deals with six raw material suppliers. The decision process for supplier selection and order allocation is carried out every month based on the results of demand forecasting at the beginning of the year and production performance in the previous month. From this demand forecasting process, the company then makes the purchase and sales agreements with all six suppliers concerning on one-year minimum purchase, unit price, and minimum one month delivery. The serial process in one calendar year can be seen in the following figure. At the beginning of a month, the company forecasts the demand for items in the following month.

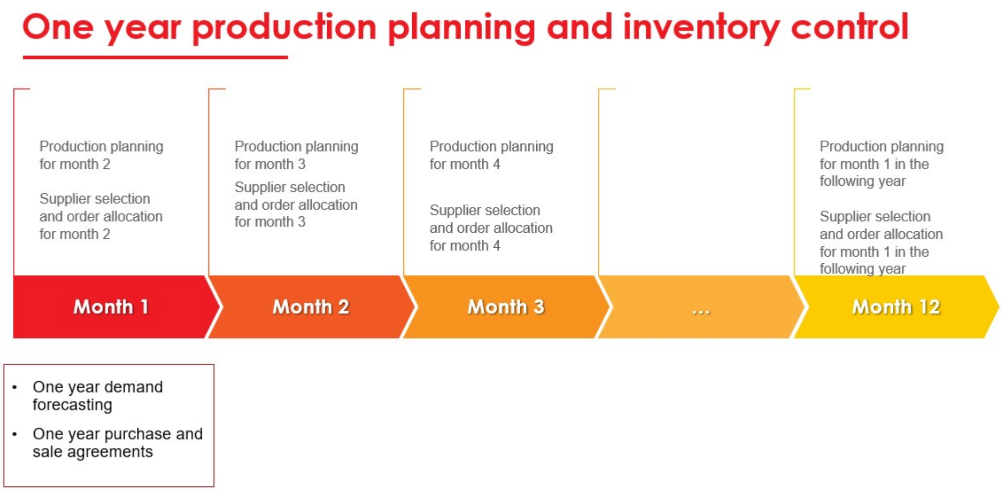

Figure 1: One year production planning and inventory control

From the estimated monthly demand for items obtained from the yearly forecasting process, we directly can find out the estimated monthly total demand for raw materials in one year. In practice, this one-month estimated demand must be reviewed due to several things, such as production in the previous month experiencing disruptions, sudden additional requests due to flash sales practices in e-commerce, and others. Reviewing the one-month demand and determining the production schedule we call production planning for one month. As soon as the production planning is performed, the company performs the decision process for purchasing the raw materials from some suppliers. In the following, we assume that one month can be divided into four weeks (the fourth week may be longer than seven days). This purchase decision covers purchases for four serial deliveries one week apart. The first delivery must be no later than 17 days before the following month's start since the internal inspection and the preparation for the raw materials delivered takes 17 days. Figure 2 in the following illustrates one-month production planning and raw material selection, and four consecutive delivery points follow it.

The decision process considers some parameters for decision making such as:

- raw materials prices,
- existing stock of raw materials in the warehouse,
- minimum one-month delivery of raw materials(if ordered),
- minimum one-year purchase of each raw material,
- raw-material flexibility of each item, which is known by the number of raw materials that can be used to produce the item. The larger this number for an item, the more flexible the item.

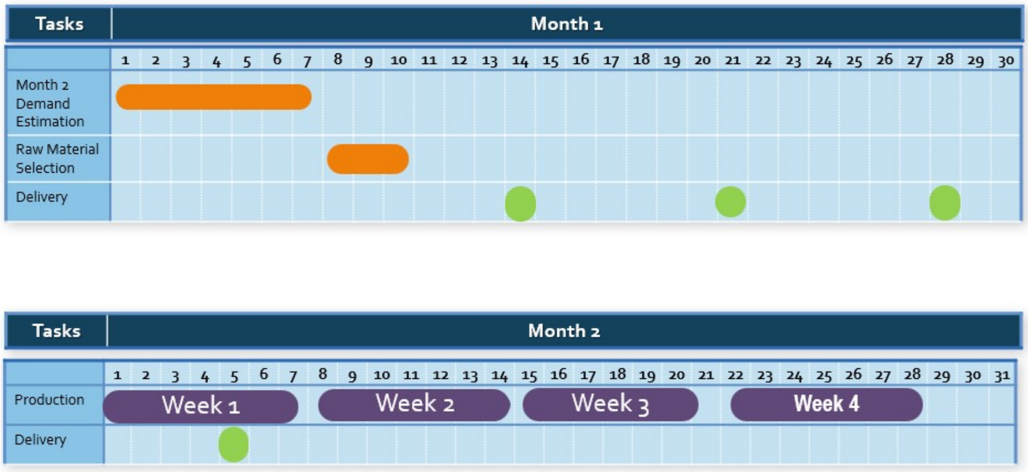

Figure 2: Supply Cycle

- and others.

The purchase must also comply with the company's internal policies in the following.

Policy I purchase raw materials from at least two suppliers in order to maintain supply security,

Policy II if an item must be produced by using more than one raw material, the proportions of raw materials used are the same.

In the following section, we will accommodate all these policies into some constraints of an optimization model that can be regarded as the main engine of the Decision Support System developed by the company in order to achieve an optimal decisions in inventory control.

3 Mathematical Model of the problem

In this section, we formulate a mathematical model of the decision problem. As described before, we need to make a decision for raw material selection, delivery quantities, and raw material compositions on the four consecutive weeks covered. In the following, we derive a mixed integer linear programming that represents the decision problem. We first present the sets, the parameters, and the decision variables used in the mathematical model. We then present the constraints which represent the production rules and capacity, followed by the discussion concerning on the objective function of the optimization model. The integer linear programming is written after that.

3.1 Sets and parameters

Denote by:

- $\mathcal{M} = \{1, 2, 3, 4\}$ as the set of weeks on the supply cycle,
- N as the number of raw-material types,
- $\mathcal{N} = \{1, 2, \cdots, N\}$ as the set of material types,
- I as the number of items,
- $\mathcal{I} = \{1, 2, \cdots, I\}$ as the set of items,
- $P = P_1 \bigcup P_2 \bigcup P_3 \bigcup P_4$, as the set of items to be produced on the planning horizon, where P_j as the set of items to be produced on week $j \in M$. The raw-material flexibility property makes us may assume that $P_i = P_i^1 \bigcup P_i^2$, for $i \in M$, where P_i^1 as the sets of items that must be produced by one certain type of raw material, and P_i^2 as the sets of items that have to be produced by composing at least two types of raw materials.

- for $i \in \mathcal{I}$, $k \in \mathcal{N}$,

$$f_{ik} = \begin{cases} 1 & \text{, if item } i \text{ can be produced by using raw material } k \\ 0 & \text{, otherwise,} \end{cases}$$

- $\forall j \in \mathcal{M}$, D_j as the total demand of raw materials on week j, and let $D = \sum_{j \in \mathcal{M}} D_j$,
- $maxcap$ as the maximum capacity of the raw-material warehouse
- ss as the safety stock of the raw materials at the warehouse
- $\forall j \in \mathcal{M}$, $i \in P_j, g_{ik}$ as the tonnage of raw material k needed if item i produced on the week j,
- $\forall k \in \mathcal{N}$, mo_k is the minimum one-year order quantity of raw material k,
- $\forall k \in \mathcal{N}, c_k$ as the price of raw material k per ton.
- $\forall k \in \mathcal{N}, \sigma_k$ as the minimum order quantity of raw material k, if it is purchased,
- $\forall k \in \mathcal{N}, z_{0k}$ as the level of inventory of raw material k, just before the first delivery on the first week.

3.2 Decision variables

Define:

- $\forall k \in \mathcal{N}$, x_k as the amount of raw material k purchased. $x_k = 0$ if raw material k is not purchased, and $\sigma_k \leq x_k \leq D$ otherwise.
- $\forall k \in \mathcal{N}$,

$$y_k = \begin{cases} 0, & x_k = 0 \\ 1, & \sigma_k \leq x_k \leq D \end{cases}$$

  The variables y_k are defined to handle the discontinuity property of the variables x_k.

- $\forall j \in \mathcal{M}, \forall k \in \mathcal{N}$,

  $\hat{x}_{jk}$ as the amount of raw material k delivered at the beginning of week j.

- $\forall j \in \mathcal{M}, \forall i \in P_j, \forall k \in \mathcal{N}$,

$$a_{ijk} = \begin{cases} 1, & \text{if item } i \text{ on the week } j \text{ produced by using raw material } k \\ 0, & \text{otherwise} \end{cases}$$

- $\forall j \in \mathcal{M}, \forall i \in P_j, \forall k \in \mathcal{N}$,

  b_{ijk} as the proportion of raw material k used to produce item i on the week j if it uses raw material k.

- $\forall j \in \mathcal{M}, \forall k \in \mathcal{N}$,

  z_{jk} as the level of inventory raw material k at the end of week j.

The following figure illustrates a supply cycle and the parameters and the decision variables associated in each week in the cycle.

<table>
<tr><td></td><td colspan="28">Planned Cycle</td></tr>
<tr><td></td><td colspan="7">Week 1</td><td colspan="7">Week II</td><td colspan="7">Week III</td><td colspan="7">Week IV</td></tr>
<tr><td></td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td><td>21</td><td>22</td><td>23</td><td>24</td><td>25</td><td>26</td><td>27</td><td>28</td></tr>
<tr><td></td><td colspan="7">1st delivery</td><td colspan="7">2nd delivery</td><td colspan="7">3rd delivery</td><td colspan="7">4th delivery</td></tr>
<tr><td>Parameter</td><td colspan="7">$z_{0k}, maxcap, d_{i1}$</td><td colspan="7">$maxcap, d_{i2}$</td><td colspan="7">$maxcap, d_{i3}$</td><td colspan="7">$maxcap, d_{i4}$</td></tr>
<tr><td>Decision Variables</td><td colspan="7">$\hat{x}_{1k}, a_{i1k}, b_{i1k}, z_{1k}$</td><td colspan="7">$\hat{x}_{2k}, a_{i2k}, b_{i2k}, z_{2k}$</td><td colspan="7">$\hat{x}_{3k}, a_{i3k}, b_{i3k}, z_{3k}$</td><td colspan="7">$\hat{x}_{4k}, a_{i4k}, b_{i4k}, z_{4k}$</td></tr>
<tr><td></td><td colspan="28">x_k</td></tr>
</table>

Figure 3: Parameters and Decision Variables Associated per Week

3.2.1 Constraints

The following mathematical expressions are the constraints for our mathematical model. We write these constraints in groups where we give a short explanation in each group the purpose of creating the constraints.

Constraints I are set to handle the discontinuity value of x_k.
$\forall k \in \mathcal{N}$,

$$x_k \leq y_k D \tag{1}$$

$$x_k \geq \sigma_k y_k. \tag{2}$$

Constraints II are set to fulfill the weekly allocation of each type of raw material. $\forall k \in \mathcal{N}$,

$$x_k = \sum_{j \in \mathcal{M}} \hat{x}_{jk}. \tag{3}$$

Constraints III are set to fulfill the raw-material demand each week.

$\forall j \in \mathcal{M}$,

$$\sum_{k=1}^{N} \hat{x}_{jk} + \sum_{k=1}^{N} z_{(j-1)k} \geq D_j. \tag{4}$$

Constraints IV are set to ensure each item in P^2 is produced by using at least two raw materials.
$\forall j \in \mathcal{M}, \forall i \in P_j^2$,

$$\sum_{k \in \mathcal{N}} a_{ijk} \geq 2. \tag{5}$$

Constraints V concern on the relation among f_{ik}, a_{ijk}, b_{ijk}, and x_{jk}.
$\forall j \in \mathcal{M}, i \in P_j,\ k \in \mathcal{N}$,

$$a_{ijk} \leq f_{ik}. \tag{6}$$

$\forall j \in \mathcal{M}, \forall i \in P_j, \forall k \in \mathcal{N}$,

$$b_{ijk} \leq f_{ik} a_{ijk}, \tag{7}$$

$$\mu a_{ijk} \leq b_{ijk}, \tag{8}$$

for a small value of μ.

$\forall j \in \hat{M}, \forall i \in P_j$,

$$\sum_{k \in \mathcal{N}} b_{ijk} = 1. \tag{9}$$

Constraints VI are set to fulfill the Policy III.

$\forall j \in \hat{M}, \forall i \in P_j^2, k_1,\ k_2 \in \mathcal{N},\ k_1 \neq k_2$,

$$(1 - a_{ijk_1}) + (1 - a_{ijk_2}) \geq b_{ijk_1} - b_{ijk_2}, \tag{10}$$

$$(1 - a_{ijk_1}) + (1 - a_{ijk_2}) \geq b_{ijk_2} - b_{ijk_1}, \tag{11}$$

Constraints VII are set to ensure that the level of inventory just after raw material delivery does not exceed the maximum capacity.

On the beginning of week 1,

$$\sum_{k\in\mathcal{N}} (z_{0k} + \hat{x}_{1k} - +z_{1k}) - D_1 \leq maxcap \tag{12}$$

$\forall j \in \{2, 3, 4\}$,

$$\sum_{k\in\mathcal{N}} (z_{(j-1)k} + \hat{x}_{(j-1)k}) - \sum_{i\in P_j} b_{ijk} g_{ik} + z_{jk} \leq maxcap \tag{13}$$

$\forall j \in \mathcal{M}$,

$$\sum_{k\in\mathcal{N}} (z_{(j-1)k} + \hat{x}_{jk}) - \sum_{i\in P_j} b_{ijk} g_{ik} + z_{jk} \leq maxcap \tag{14}$$

Constraints VIII are set to ensure that the level of inventory at the end of each week must be greater than or equal to the safety stock.

$\forall j \in \mathcal{M}$, $\forall k \in P_j$,

$$z_{jk} \geq ss \tag{15}$$

3.3 Objective function

We define the objective function as the sum of the inventory cost, the purchase cost, and a function for accommodating the minimum one-year order quantity contracts.

The level of inventory of raw-material k during one week can be seen in the following figure.

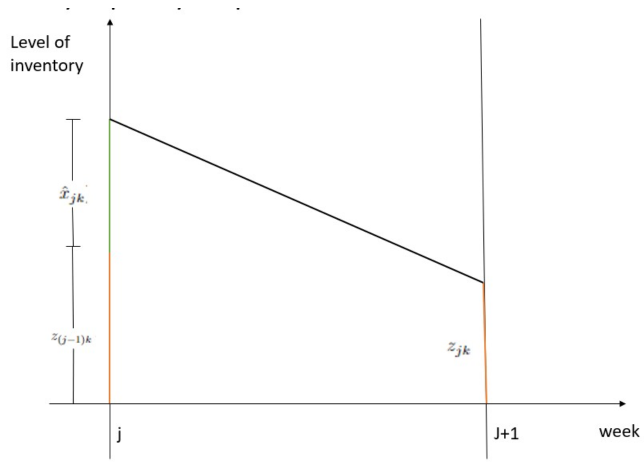

Figure 4: Level of inventory of raw material k in one week.

So that the inventory cost can be given as

$$\frac{1}{2} ic \sum_{j\in\mathcal{M}} \sum_{k\in\mathcal{N}} (z_{(j-1)k} + z_{jk} + \hat{x}_{jk}), \tag{16}$$

Meanwhile the purchase cost can be given as

$$\sum_{k\in\mathcal{N}} c_k x_k. \tag{17}$$

We consider another function in the objective function, which is created to accommodate the one-year minimum order quantity contracts. Constraints I - VIII concern fulfillment of the minimum purchase agreements, the monthly demand, warehouse capacity constraint, safety stock constraint, and raw material composition requirements. Meanwhile, the one-year minimum order quantity contract is quite difficult to be expressed as a constraint in the optimization model that has a one-month-long planning horizon. Therefore, we accommodate the yearly purchase contract which we represent as a part of the objective function of our optimization problem. To fulfill the one-year minimum order quantity contracts, we define a penalty function

$$\sum_{k \in \mathcal{N}} \alpha_k mo_k x_k, \tag{18}$$

where for $k \in \mathcal{N}$, α_k is multiplier constants that will be discussed later.

The objective function of our optimization model is then can be written as

$$\frac{1}{2} ic \sum_{j \in \mathcal{M}} \sum_{k \in \mathcal{N}} (z_{(j-1)k} + z_{jk} + \hat{x}_{jk}) + \sum_{k \in \mathcal{N}} c_k x_k - \sum_{k \in \mathcal{N}} \alpha_k mo_k x_k. \tag{19}$$

3.4 Optimization model

The optimization model for supplier selection, order allocation, and raw material composition can be written as a mixed integer linear programming

$$\begin{aligned}
&\textbf{minimize } \frac{1}{2} ic \sum_{j \in \mathcal{M}} \sum_{k \in \mathcal{N}} (z_{(j-1)k} + z_{jk} + \hat{x}_{jk}) + \sum_{k \in \mathcal{N}} c_k x_k - \sum_{k \in \mathcal{N}} \alpha_k mo_k x_k \\
&\textbf{subject to Constraints I - VIII} \\
&\qquad x_k \in \mathbb{Z}^+, \hat{x}_{jk} \geq 0, \\
&\qquad y_k, a_{ijk} \in \{0, 1\}, \\
&\qquad 0 \leq b_{ijk} \leq 1,\ 0 \leq z_{jk} \leq maxcap.
\end{aligned} \tag{20}$$

4 Numerical solution

In this section, we give an example of the solution of the MIP (20). In this example, we consider an instant where $N = 6$, and $I = 51$ so that we have 2508 decision variables.

4.1 Some of parameters

The following figure shows the matrix of raw-material flexibility of all items.

item	raw material					
	1	2	3	4	5	6
1	1	1	0	1	0	0
2	1	1	1	1	1	1
3	1	1	1	1	1	1
4	1	1	1	1	1	1
5	1	1	0	1	0	0
6	1	1	1	1	1	1
7	1	1	1	1	1	1
8	1	1	1	1	1	1
9	1	1	0	1	0	0
10	1	1	0	1	0	0
11	1	1	1	1	1	1
12	1	1	1	1	1	1
13	1	1	1	1	1	1
14	1	1	1	1	1	1
15	1	0	0	1	0	0
16	1	0	0	1	0	0
17	1	1	1	1	1	1
18	1	1	1	1	1	1
19	1	1	1	1	1	1
20	1	1	0	1	0	0
21	1	1	0	1	0	0
22	1	1	1	1	1	1
23	1	1	1	1	1	1
24	1	1	1	1	1	1
25	1	1	1	1	1	1
26	1	1	1	1	1	1
27	1	1	1	1	1	1
28	1	1	1	1	1	1
29	1	1	1	1	1	1
30	1	1	1	1	1	1
31	1	1	1	1	1	1
32	1	1	1	1	1	1
33	1	1	1	1	1	1
34	1	1	1	1	1	1
35	1	1	0	1	0	1
36	1	1	1	1	1	1
37	1	1	1	1	1	1
38	1	1	1	1	1	1
39	1	1	1	1	1	1
40	1	1	1	1	1	1
41	1	1	0	1	0	0
42	1	1	0	1	0	0
43	1	1	1	1	0	1
44	1	1	0	1	0	0
45	1	1	0	1	0	0
46	1	1	0	1	0	1
47	1	1	1	1	1	1
48	1	1	1	1	1	1
49	0	0	0	1	0	0
50	0	0	0	1	0	0
51	1	1	0	1	0	0

Figure 5: The Matrix of Raw Material Flexibility

Next, we present the instant of the total raw material demands during a supply cycle.

item	raw -material demand (in kg)			
	week 1	week 2	week 3	week 4
1	2555,1	2555,1	0	10220,4
2	0	4300	8600	0
3	0	0	0	1800
4	0	495	0	0
5	0	0	0	0
6	3150	0	6300	0
7	0	1050	0	350
8	1650	0	0	2200
9	2080	1560	1040	0
10	0	1280	0	0
11	0	0	4200	0
12	0	2700	0	0
13	350	350	0	0
14	0	300	500	200
15	85	0	0	0
16	110	165	0	0
17	0	0	17400	17400
18	74050	222150	14800	0
19	0	11605	0	0
20	12750	0	0	0
21	3500	0	14000	14000
22	42100	0	10550	0
23	0	0	5600	11200
24	5400	5400	0	5400
25	1350	2250	0	0
26	0	32400	43200	0
27	10350	31050	0	0
28	118200	0	0	0
29	3330	4995	0	0
30	0	1950	0	5850
31	0	6300	6300	0
32	2310	3465	0	0
33	0	2500	0	0
34	0	0	20850	0
35	12150	0	0	0
36	0	0	0	0
37	12750	12750	8500	8500
38	1550	3100	0	0
39	0	10600	5300	21200
40	0	0	10450	20900
41	9625	0	1375	1375
42	0	0	3080	0
43	0	2250	450	2250
44	0	0	14100	0
45	6700	13400	10050	6700
46	6500	13000	0	0
47	26200	13100	0	0
48	12000	8000	0	12000
49	0	126	0	0
50	1323	0	3969	0
51	1170	1170	0	780

Figure 6: The Total Raw Material Demands

and the purchase prices and the ratios of the one-year purchase caps to the total one-year purchase caps.

	Raw Material					
	1	2	3	4	5	6
Price per tons	16.55	15.5	18.11	14.72	19.8	15.19
r_k	22.1	55.4	1.8	8.2	0.8	11.7
Minimum order qty (in tons)	15	10	11.5	9	8	12.5
Z_0k (in tons)	3.25	2.85	3.3	2.5	3.5	3.05
maxcap (in tons)	1427					
safety stock (in tons)	2.5					

Figure 7: Some other parameters

From Figures 5 and 7 we know that raw material 4 must be selected since item 49 and item 50 can be produced just by using raw material 4. Furthermore, the purchase price of raw material 4 is the smallest one. But since r_4 is smaller compare to r_1, r_2, and r_6, we may guess that on the optimal solution, x_4 will have a big value but is not the biggest one among others.

4.2 Solution

We solve the optimization problem by creating a program using R language (version 4) where the optimization problem formulation and the solution technique used are referred to dplyr [27] and ompr [25] libraries. This program runs on a computer with the Linux Ubuntu 20 LTS operating system with an Intel i7 8 Cores processor and 16 GB RAM.

Using the data in figures 4, 5, and 6, we get the optimal solution for x_k as follows:

Table 1: Optimal Solution

Raw material	Total order qty (in tons)
1	192.6
2	543.4
3	90.7
4	56.5
5	53.0
6	202.9
Total	1139.1

Let us take a look at the following results from $\hat{x}_{jk}$:

Table 2: Raw Material order per week (in tons)

Raw material	Week 1	Week 2	Week 3	Week 4
1	6.6	65.3	81.7	39.1
2	354.8	23.6	96.1	68.8
3	41.1	26.6	14.4	8.7
4	33.3	0.2	18.4	4.7
5	42.6	10.4	0.0	0.0
6	70.7	111.1	0.0	21.0

From this raw material ordering configuration, we calculate the stock at the end of week j by adding $\hat{x}_{jk}$ and the stock for week $j-1$ and then subtracting the raw material requirement for that week.

Table 3: Stock at the end of the week (in tons)

Raw material	Week 1	Week 2	Week 3	Week 4
1	2.5	2.5	2.5	2.5
2	181.7	2.5	2.5	2.5
3	2.5	2.5	2.5	2.5
4	2.5	2.5	2.5	2.5
5	2.5	2.5	2.5	2.5
6	2.5	2.5	2.5	2.5

4.3 Development of Decision Support System

We develop a Decision support system for this optimization problem by creating program in the R language. The process is as follows:

1. Converting several raw data into matrix and vector forms. Several data are used as input in this problem, namely:

    - Parameters data for raw materials, such as price, minimum order quantity, current stock, and one-year purchase cap. Global parameters data such as *maxcap* and safety stock. As is Figure 7.
    - Data on the composition of raw materials per product produced. As in Figure 5.
    - Weekly demand data for each product, including raw material requirements. As in Figure 6.

2. Finding the optimal solution numerically.
3. Create output worksheets that suit business processes.

5 Discussion

Seperti didiskusikan sebelumnya, solusi dari masalah optimisasi (20) akan bergantung pada koefisien dari fungsi objektif $\hat{c}_k = \frac{\alpha_k}{r_k}$. Dari ekspresi $\hat{c}_k$ ini jelas bahwa kita mempertimbangkan faktor perjanjian minimal pembelian satu tahun. Faktor-faktor lainnya, seperti purchase price, dapat kita akomodir di multipicator constant α_k.

Di awal tahun, ketika kita optimis bahwa total pembelian tiap jenis raw material dapat memenuhi minimum purchase agreement, maka kita dapat memilih $\alpha_k = c_k$ agar total purchase cost can be minimized. Tetapi ketika seiring waktu berjalan kita melihat total pembelian suatu jenis raw material masih memiliki shortage dari minimum purchase agreement yang cukup signifikan maka kita dapat mengatur $r_k =$, $k \in \mathcal{N}$,

References

[1] A. Alejo-Reyes, E. Cuevas, A. Rodriguez, A. Mendoza, & E. Olivares-Benitez, An Improved GreyWolf Optimizer for a Supplier Selection and Order Quantity Allocation Problem, Mathematics, 8, 2020

[2] J. F. Bard, A comparison of the AHP process with Multi-Attribute Utility Theory: A case study, IIE transaction. 24(5), -121, 1992.

[3] V. Belton, A comparison of the analytic hierarchy process and simple multi-attribute value function, European Journal of operational Research, 26, 7-21, 1986.

[4] K. S Bhutta F. Huq , Supplier selection problem: a comparison of the total cost of ownership and analytic hierarchy process, Supply Chain Management: An International journal, Vol 17(3), 2002.

[5] Z. Degraeve, F. Roodhooft, A mathematical programming approach for procurement using activity based costing. Journal of Business Finance and Accounting 27 (1 - 2), 69 – 98, 2000.

[6] J. S. Dyer, Remarks on the analytic hierarchy Process, Management Science, 36, 249-258, 1990.

[7] L. M. Ellram, Total cost of ownership: elements and implementation, International Journal of Purchasing and Materials Management, 29(2), 3-11, 1993.

[8] B. G. Ferrin R. E. Plank, Total Cost of Ownership Models: An Exploratory Study, Journal of Supply Management, Volume 8, Issue 2, 18 – 29, 2002.

[9] J. V. Gray, S. Helper, B. Osborn, Value first, cost later: Total value contribution as a new approach to sourcing decisions, Journal of Operations Management, Volume 66, Issue 6, 735 – 750, 2020.

[10] H. Abd El- Wahed Khalifa M. A. Al-Shabi, Solving the inexact rough intervals vendor selection problems, European Journal of Scientific Research, Vol 150(3):265-272, 2018.

[11] M. L. Harding, How to Calculate total purchase cost, Hospital Management Quarterly, Volume 19(4):9-13, 1998.

[12] K. Hurkens, W. van der Valk, F. Wynstra, Total Cost of Ownership in the Services Sector: A Case Study, Journal of Supply Chain Management, Volume 29, Issue 3, 2 – 11, 1993.

[13] Z. Li, W. K. Wong, and C. K. Kwong, An Integrated Model of Material Supplier Selection and Order Allocation Using Fuzzy Extended AHP and Multi-objective Programming, Mathematical Problems in Engineering, Volume 2013, Article ID 362718, Hindawi Publishing Corporation, 2013.

[14] C. Masella A. Rangone, A Contingent Approach to the Design of Vendor Selection Systems for Different Types of Co-Operative Customer/Supplier Relationships. International Journal of Operations Production Management, Vol 20(1), 70-84, 2000.

[15] R.M. Monckza S. J. Trecha, Cost-based supplier performance evaluation, Journal of Purchasing and Materials Management, 24(1), 2-7, 1988.

[16] Mouli, K. V. V. C , Subbaiah, K. V., Rao, K. M. and Acharyulu, S. G.(2006), "Particle Swarm Optimization Approach for Vendors Selection", IE(I) Journal-PR, Vol. 87,pp.3-6.

[17] K. Mukherjee, B. Sarkar, and A. Bhattacharyya, Supplier selection by F-compromise method: a case study of cement industry of NE India, Int. J. Computational Systems Engineering, Vol. 1, No. 3, 2013.

[18] Nispeling, T., Multi-Criteria Supplier Selection in the Edible Oil Industry: The Case of a New Oils Fats Plant in China , Master Thesis, TU Delf, 2015

[19] R. L. Nydick & R. P. Hill, Using the Analytic Hierarchy Process to Structure the Supplier Selection Procedure, International Journal of Purchasing and Materials Management, Volume 28, Issue2, 31-36, 1992.

[20] O. Pal, A. K. Gupta, R. K. Garg, Supplier Selection Criteria and Methods in Supply Chains: A Review, World Academy of Science, Engineering and Technology International Journal of Economics and Management Engineering Vol:7, No:10, 2013.

[21] P. Pitchipoo, R. Ragavan, P. Venkumar, R. Sivaprakasam, Development of DEA Decision Model for Supplier Selection, Proceeding of IEEE – International Conference on Research and Development Prospects on Engineering and Technology, EGS Pillay Engineering College, Nagapattinam, India, 2013.

[22] A. M. Porter, Supplier Evaluation Revisited. Purchasing, 111(6), 58-68, 1991.

[23] Reck, R. F., Long, B. G. (1988). Purchasing: A competitiveweapon.Journal of Purchasing and Materials Management,24(3), 2–8.

[24] M. Sambatt, C. Woarawichai, T. Naenna, Inventory lot sizing and supplier selection for multiple products, multiple suppliers, multiple periods with storage space using lingo program, MATEC Web of Conferences 259, 04004 (2019), 2018 6th International Conference on Traffic and Logistic Engineering (ICTLE 2018), https://doi.org/10.1051/matecconf/201925904004

[25] Schumacher D (2022). ompr: Model and Solve Mixed Integer Linear Programs. R package version 1.0.3.9000, https://github.com/dirkschumacher/ompr.

[26] T. Shahrzad, E. Mohammad, R. S. M. Reza, Integration of DEA (Data Envelopment Analysis) Approach for Supplier Selection with Hierarchical Analysis Process and Risk Considerations, presented at INTERNATIONAL CONFERENCE ON MANAGEMENT, TOURISM AND TECHNOLOGY, Malaysia, November 2021.

[27] D. L. Smytka & M. W. Clemens, Total Cost Supplier Selection Model: A Case Study, International Journal of Purchasing and Materials Management, Volume 29, Issue 4, 42 – 49, 1993.

[28] Wickham H, François R, Henry L, Müller K (2022). dplyr: A Grammar of Data Manipulation. https://dplyr.tidyverse.org, https://github.com/tidyverse/dplyr.

[29] C.A. Weber, J.R. Current JR W. C. Benton, Vendor Selection Criteria and Methods, European Journal of operational research, 50 (1), 2-18, 1991.

[30] C.A. Weber L. M. Ellram , Supplier selection using multi-objective programming: A decision support system approach, International Journal of Physical Distribution & Logistics Management. 23(2), 3-14, 1993.