Mortality & life tables

Ilya Kashnitsky

SDU 2019

Overview:

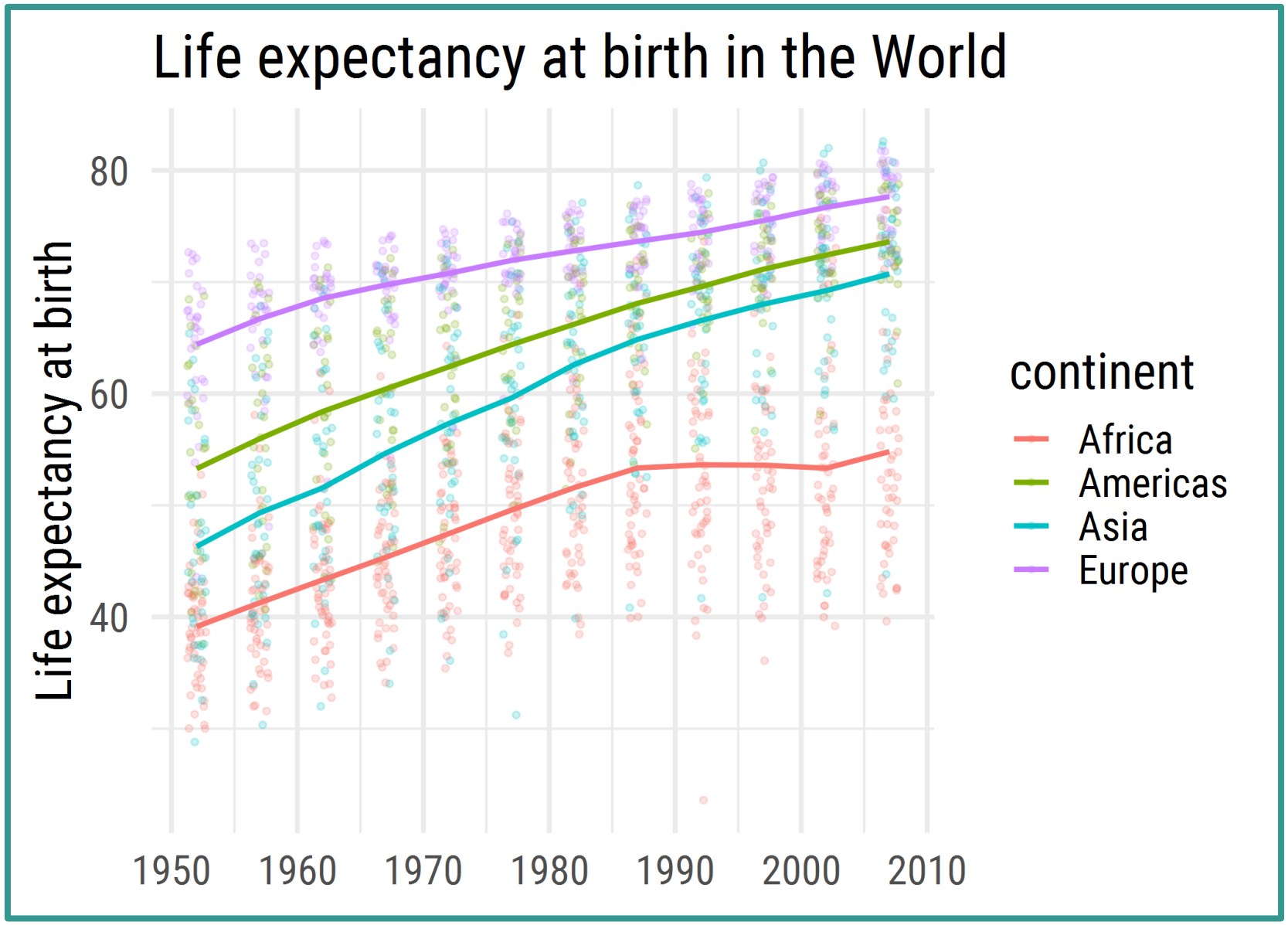
Part 1
Introduction
Age-pattern of mortality
Sex-differences in mortality
Life expectancy

Part 2
Life Table
Excel exercise

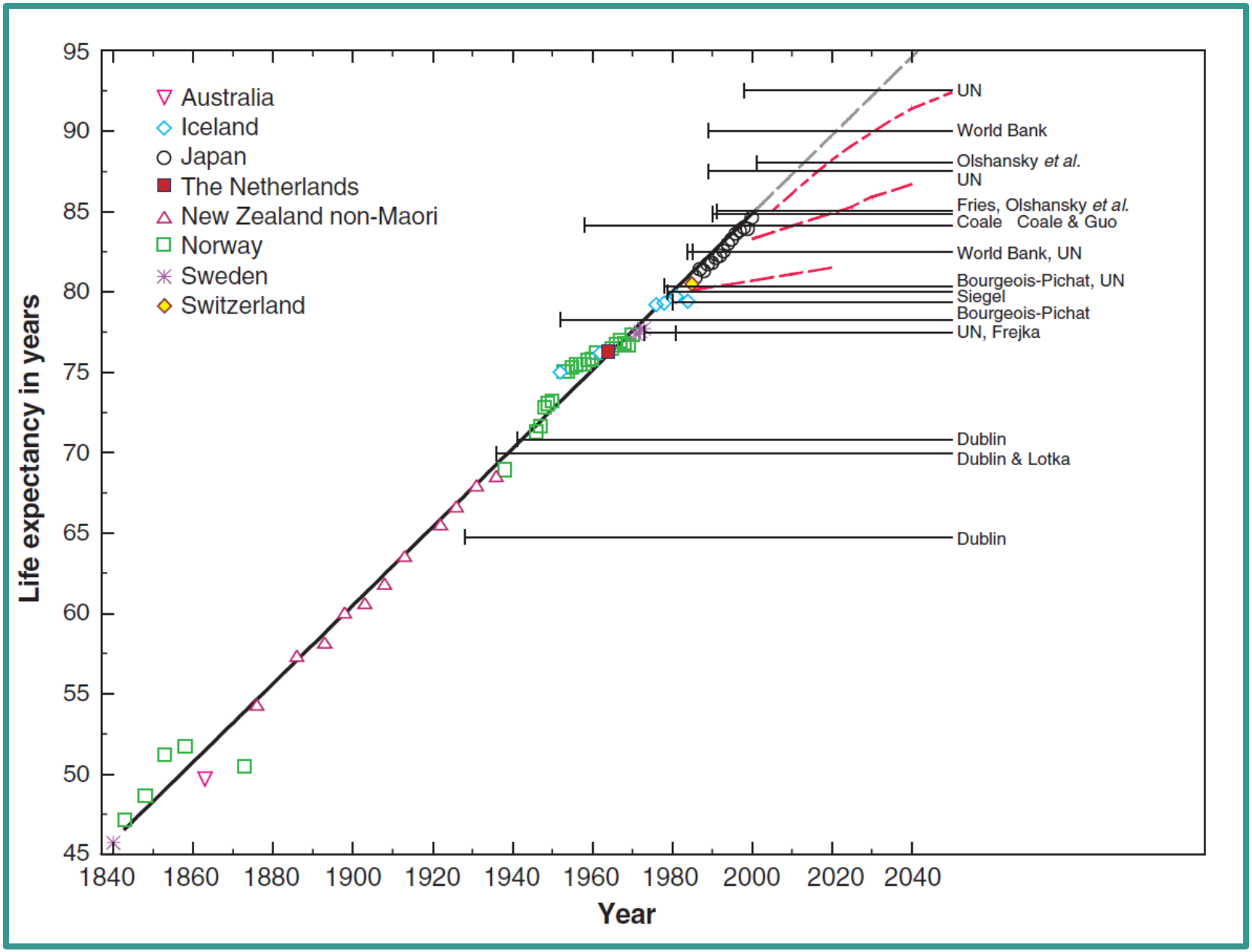
Part 3
R exercise

Readings Preston 2001
chapters 3.0--3.6 + 3.8 + appendix 3.1

Introduction

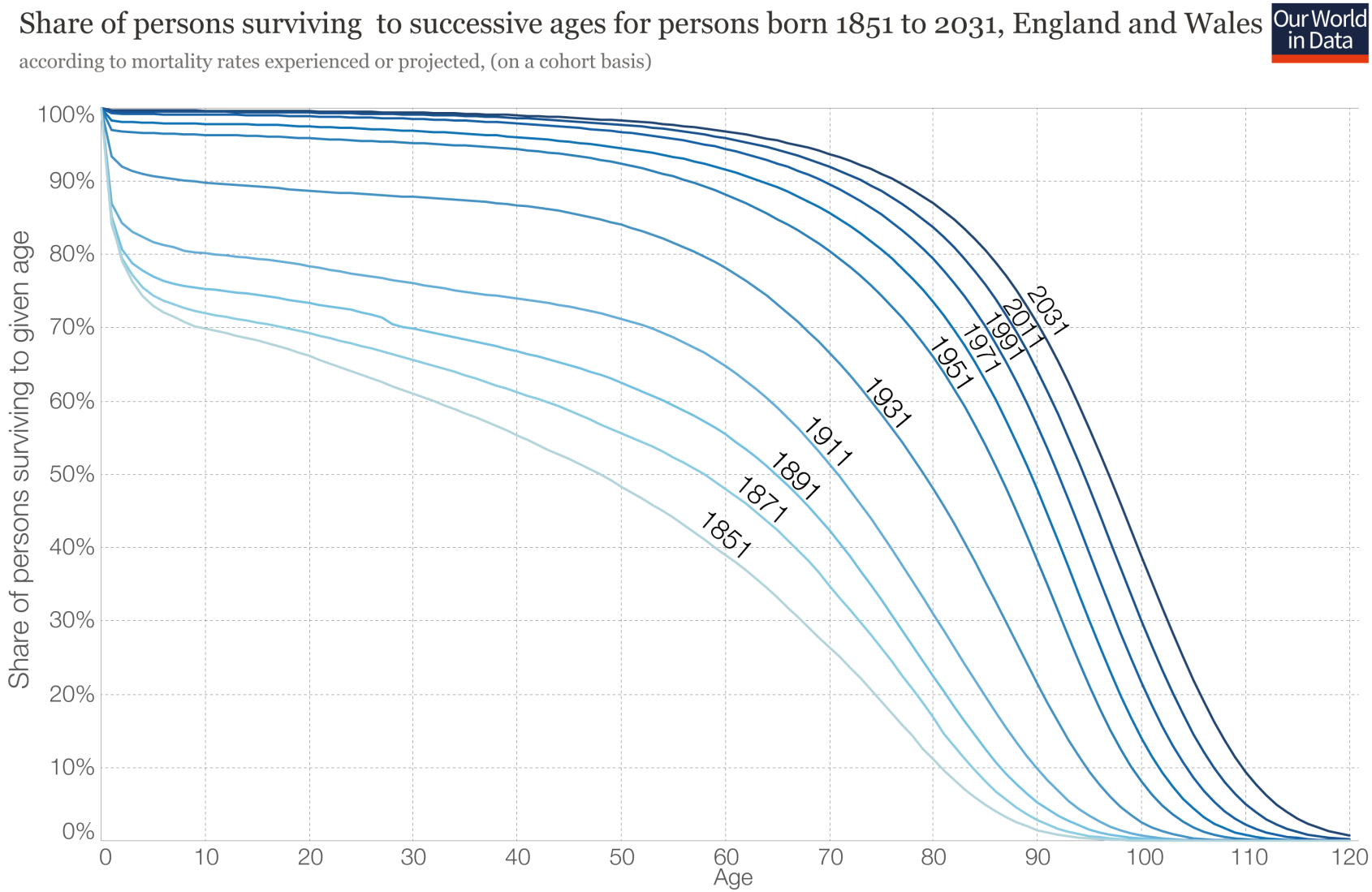


https://www.gapminder.org



Oeppen, J., & Vaupel, J. W. (2002). Broken limits to life expectancy. *Science*, 296(5570), 1029–1031. https://doi.org/10.1126/science.1069675

Rectangulatization of survival



Data source: Office for National Statistics (ONS). Note: Life expectancy figures are not available for the UK before 1951; for long historic trends England and Wales data are used
The interactive data visualization is available at OurWorldinData.org. There you find the raw data and more visualizations on this topic. Licensed under CC-BY-SA by the author Max Roser.

https://ourworldindata.org/life-expectancy

Age-pattern of mortality

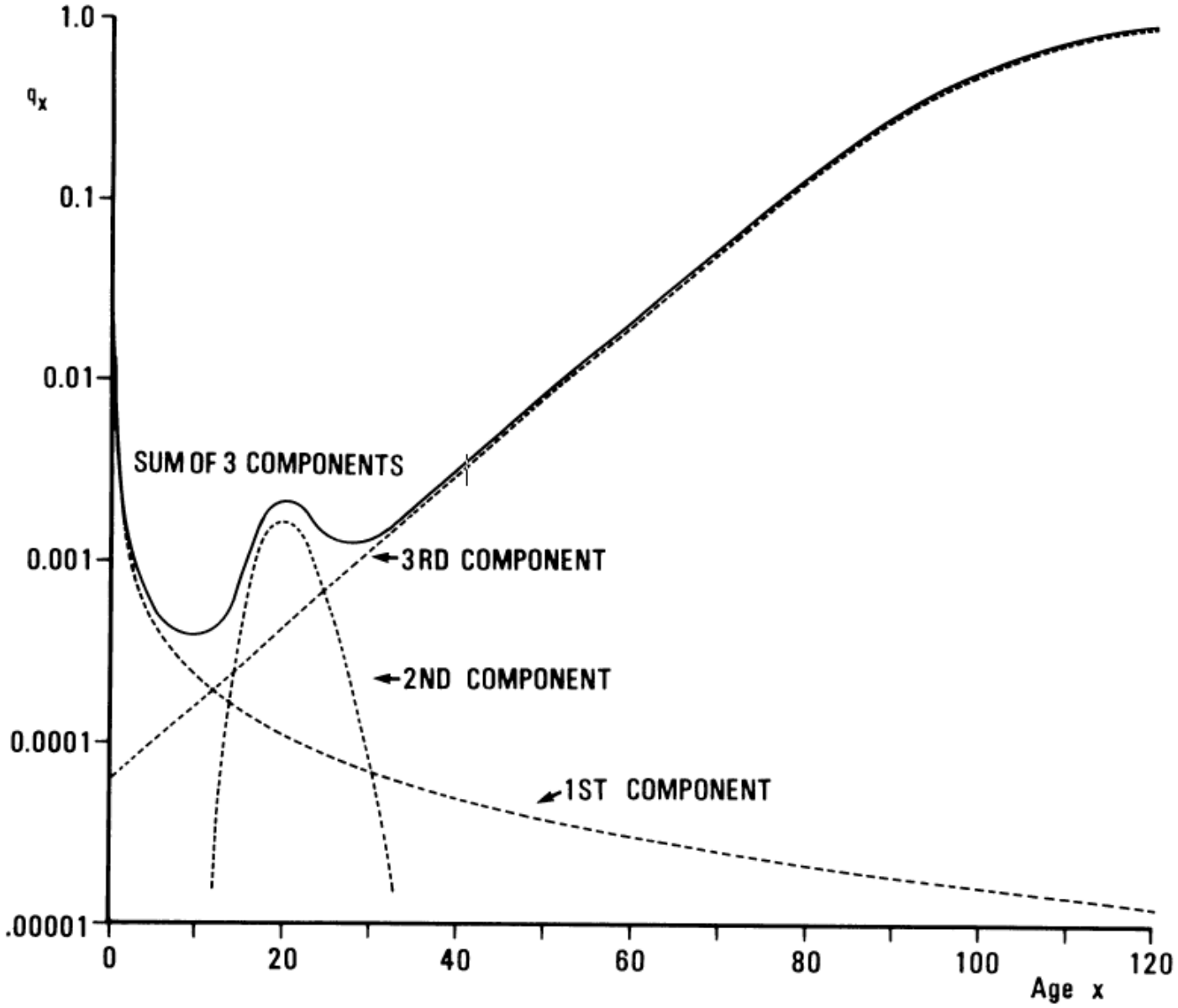
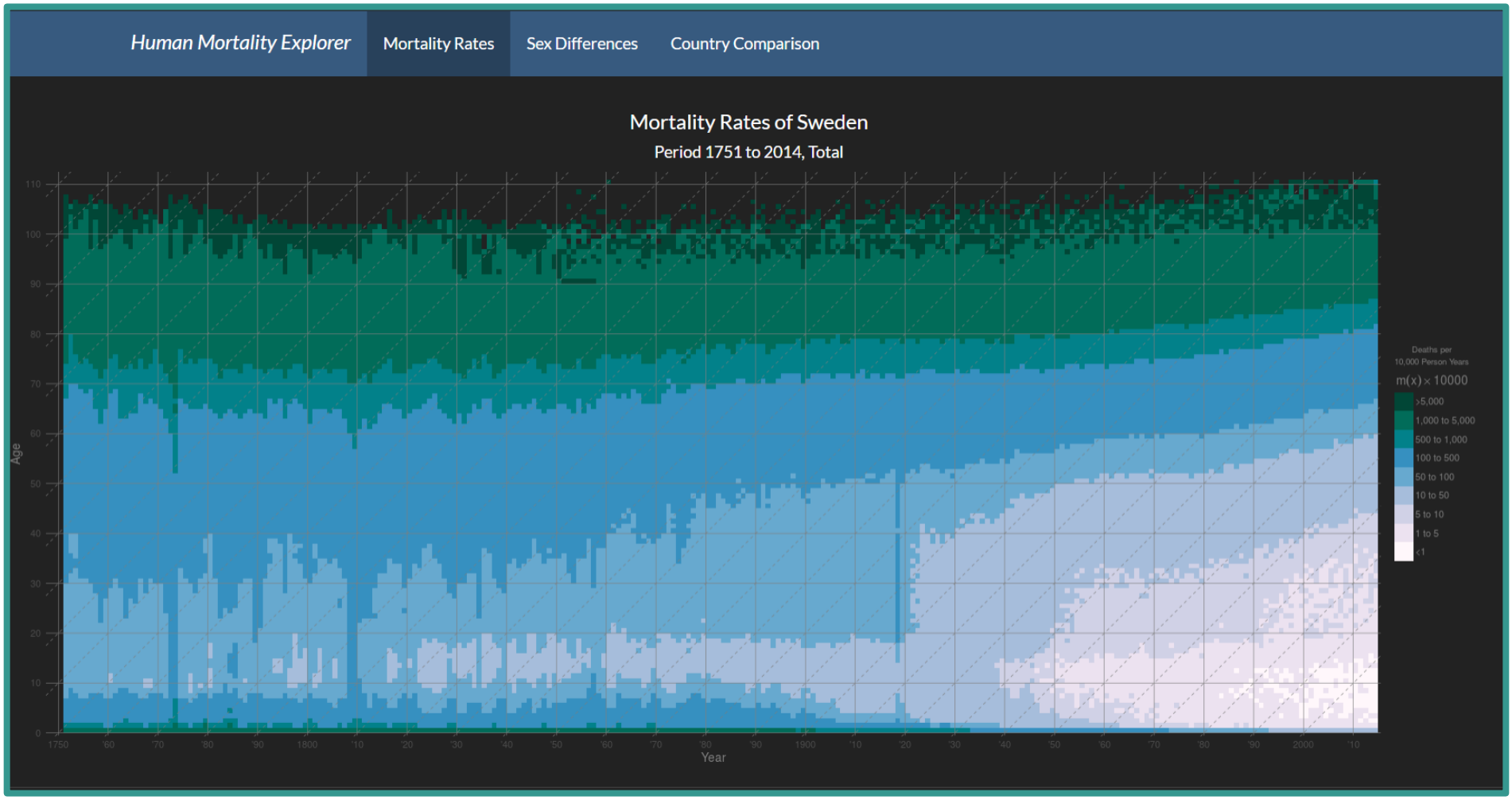


Figure 1. The graduated q_x curve and its three components: Australian national mortality, 1970–72 (males).

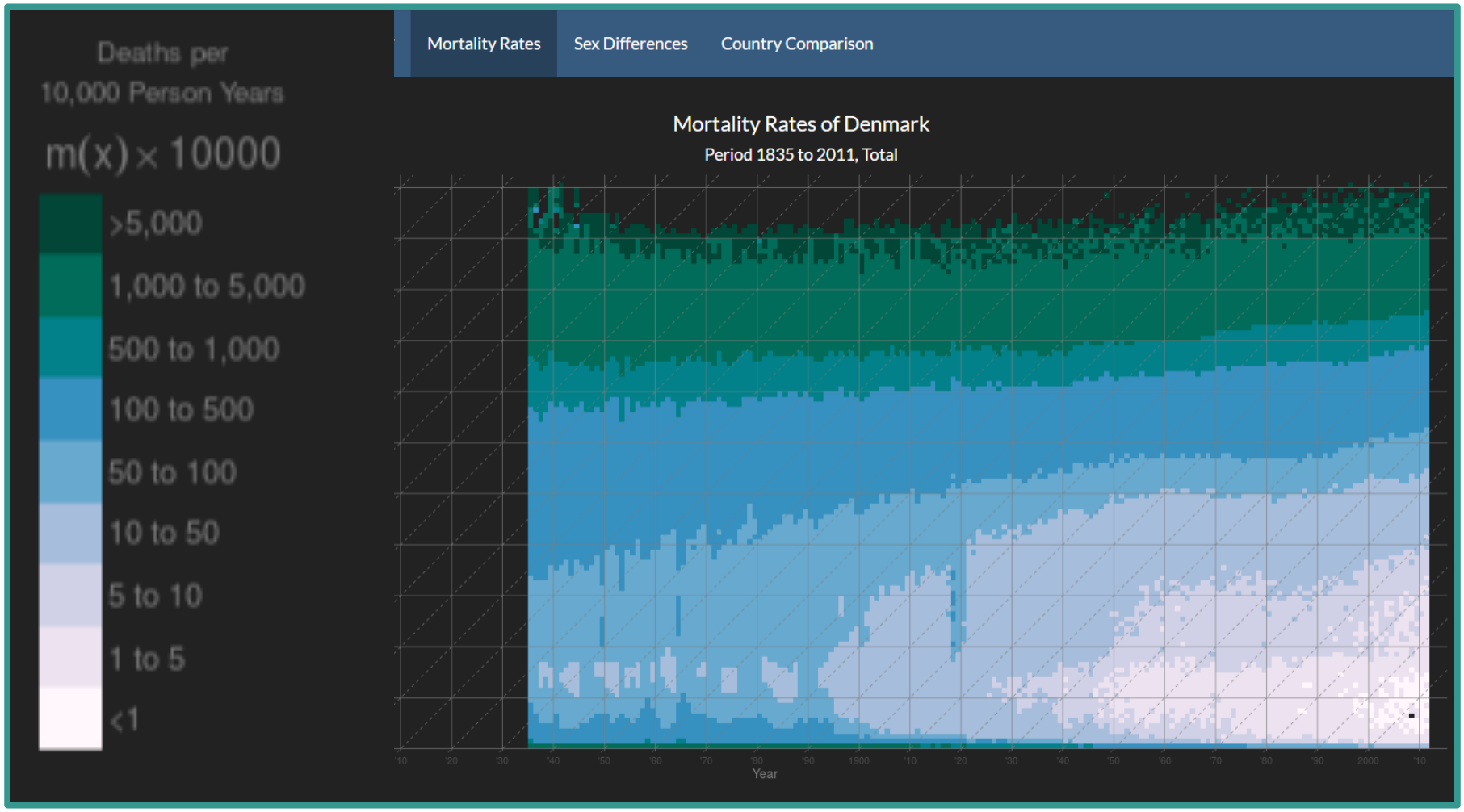
Heligman, L., & Pollard, J. H. (1980). The age pattern of mortality. *Journal of the Institute of Actuaries*, 107(1), 49–80.

Lexis heatmaps, Sweden



https://jschoeley.shinyapps.io/hmdexp/

Lexis heatmaps, HMD explorer



https://jschoeley.shinyapps.io/hmdexp/

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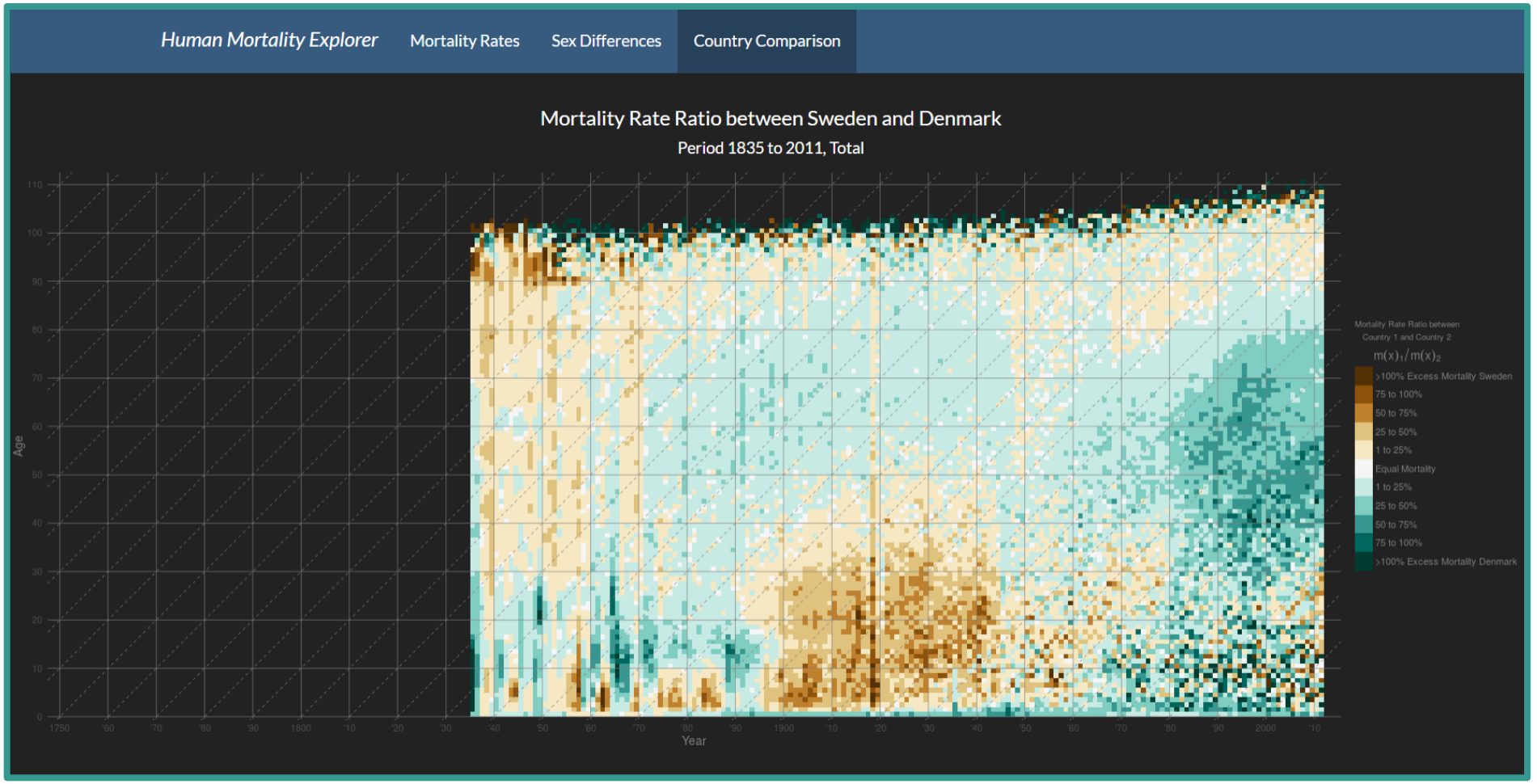
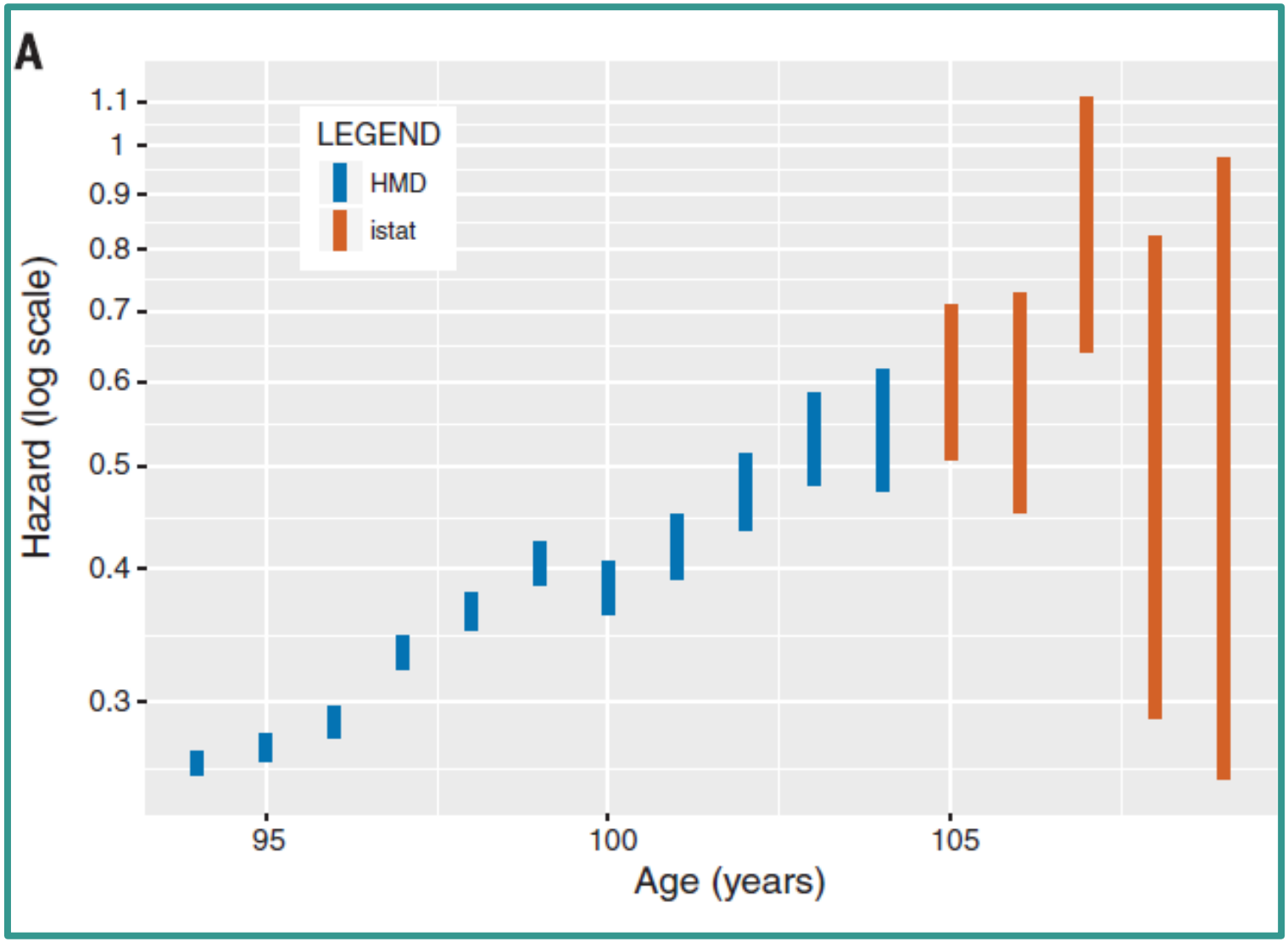


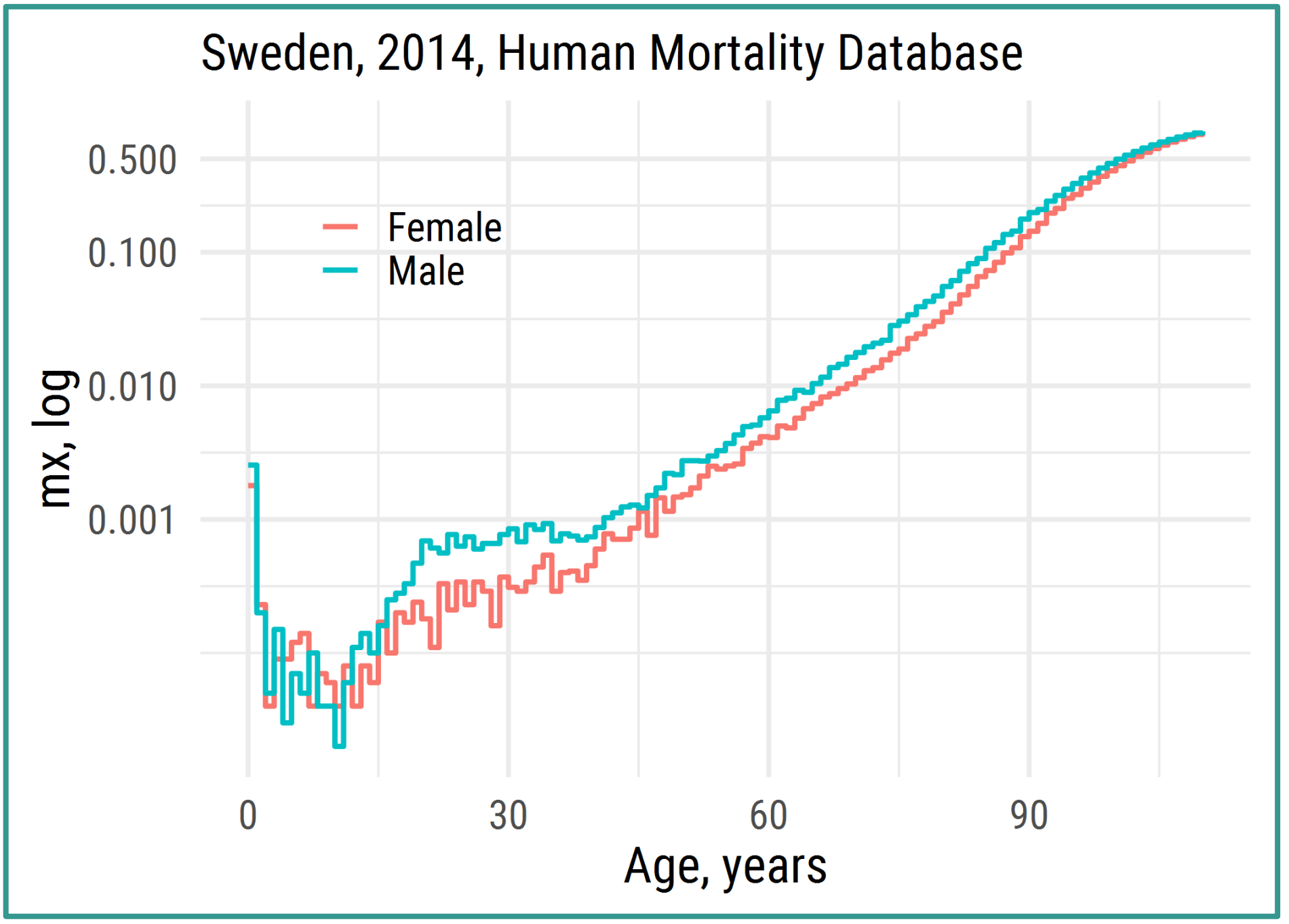
https://jschoeley.shinyapps.io/hmdexp/

Old age plateau

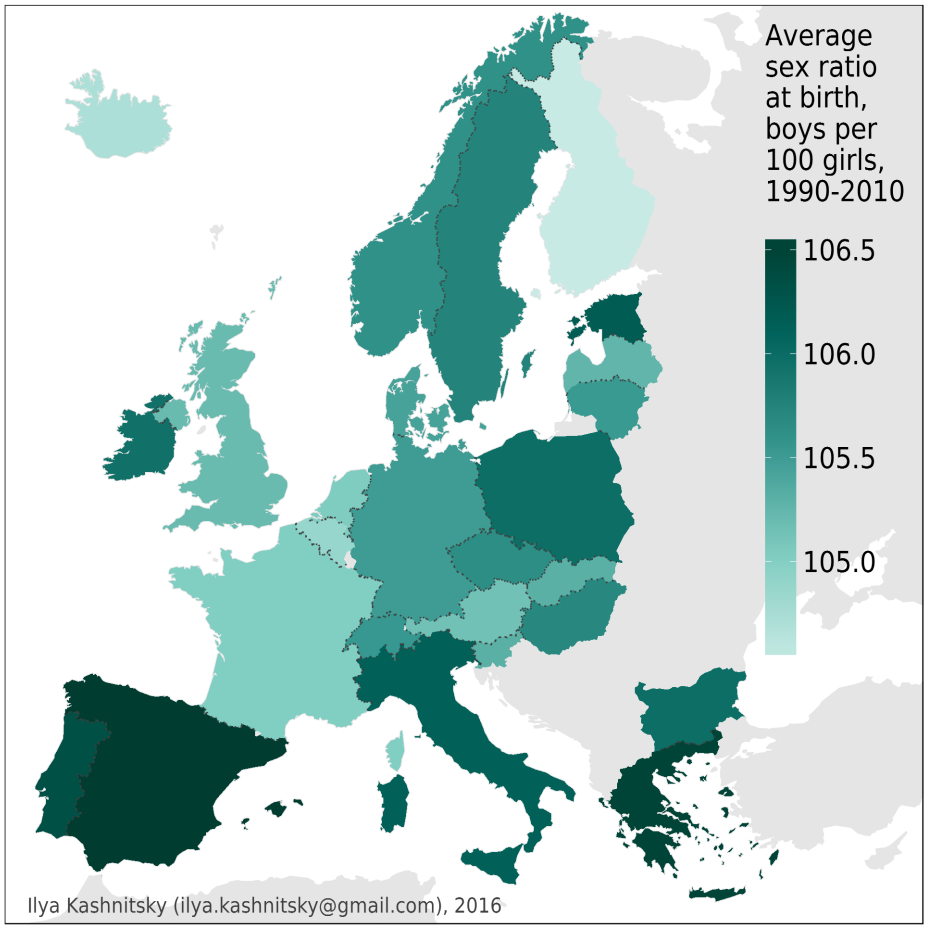


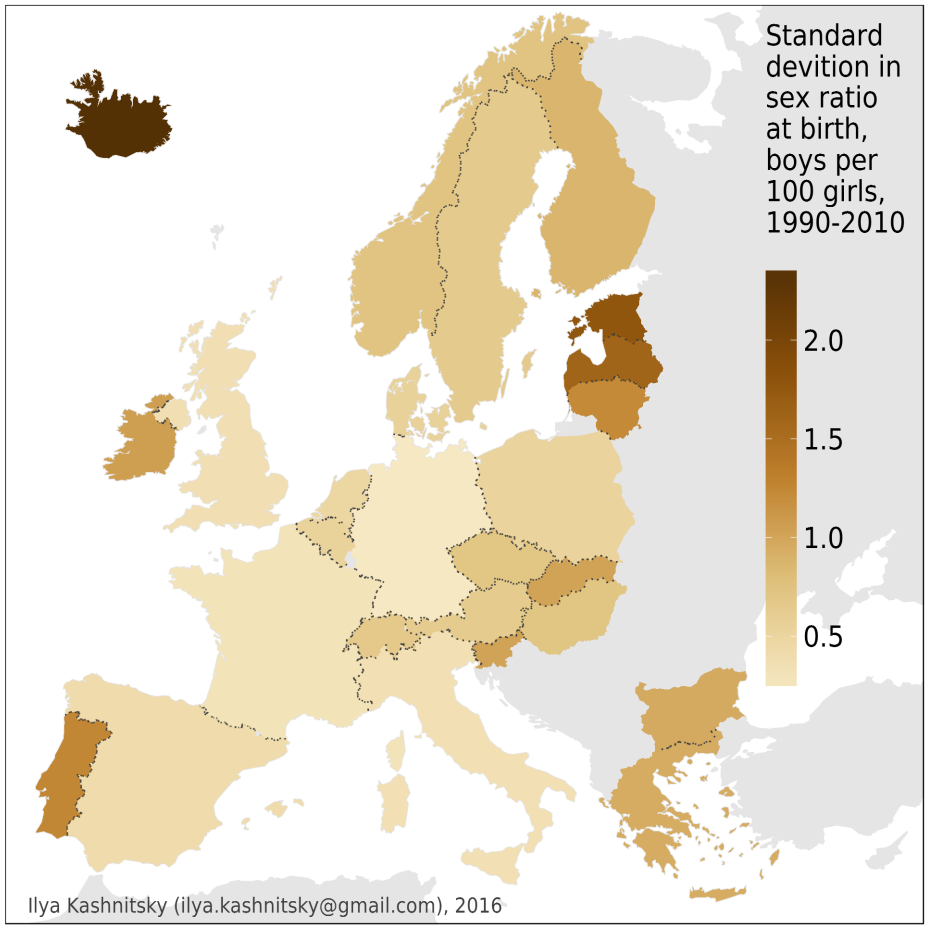
Barbi, E., Lagona, F., Marsili, M., Vaupel, J. W., & Wachter, K. W. (2018). The plateau of human mortality: Demography of longevity pioneers. Science, 360(6396), 1459–1461. https://doi.org/10.1126/science.aat3119

Sex-differences in mortality

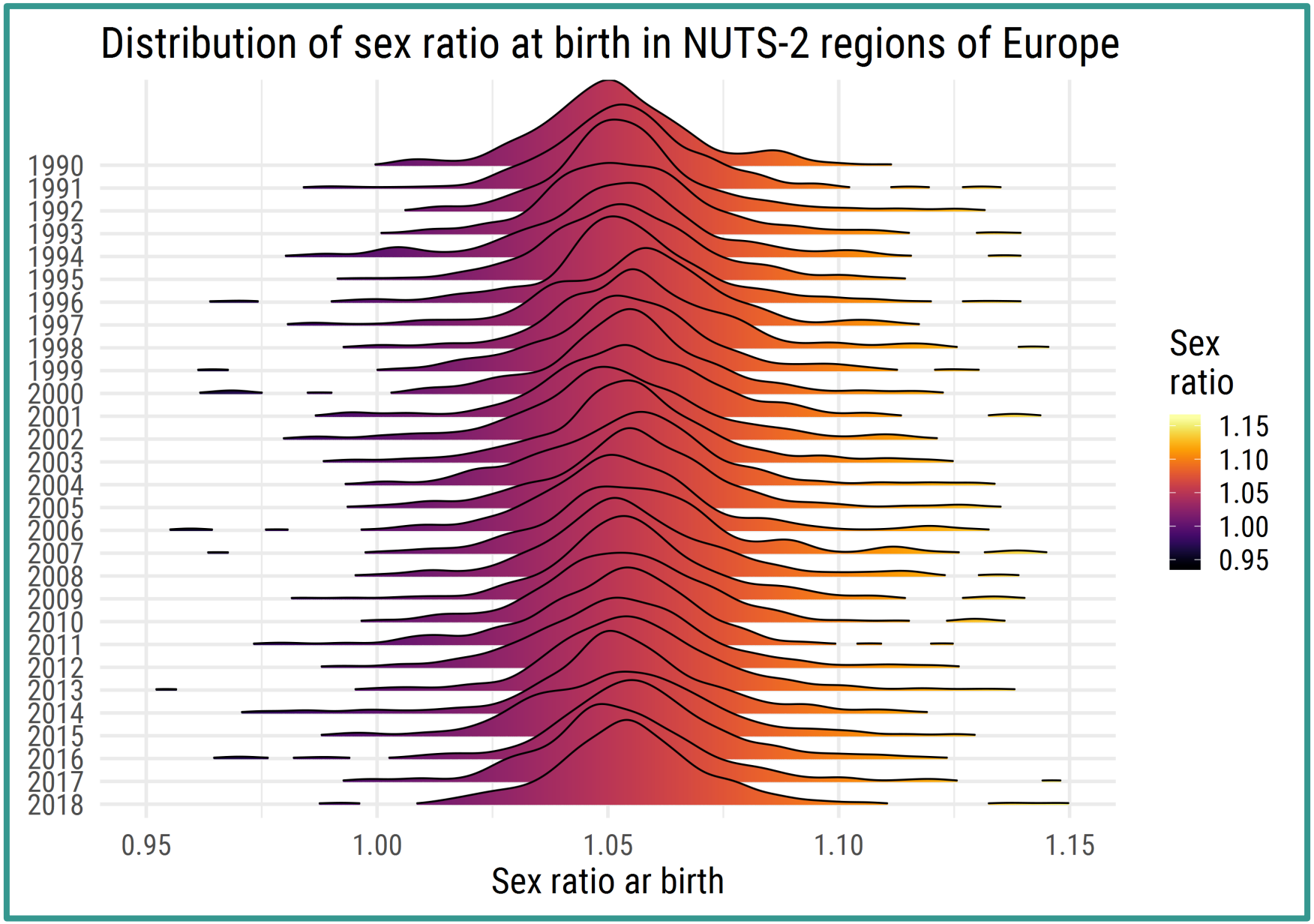

Sweden, 2014, Human Mortality Database
0.500
0.100
0.010
0.001
mx, log
Female
Male
0
30
60
90
Age, years

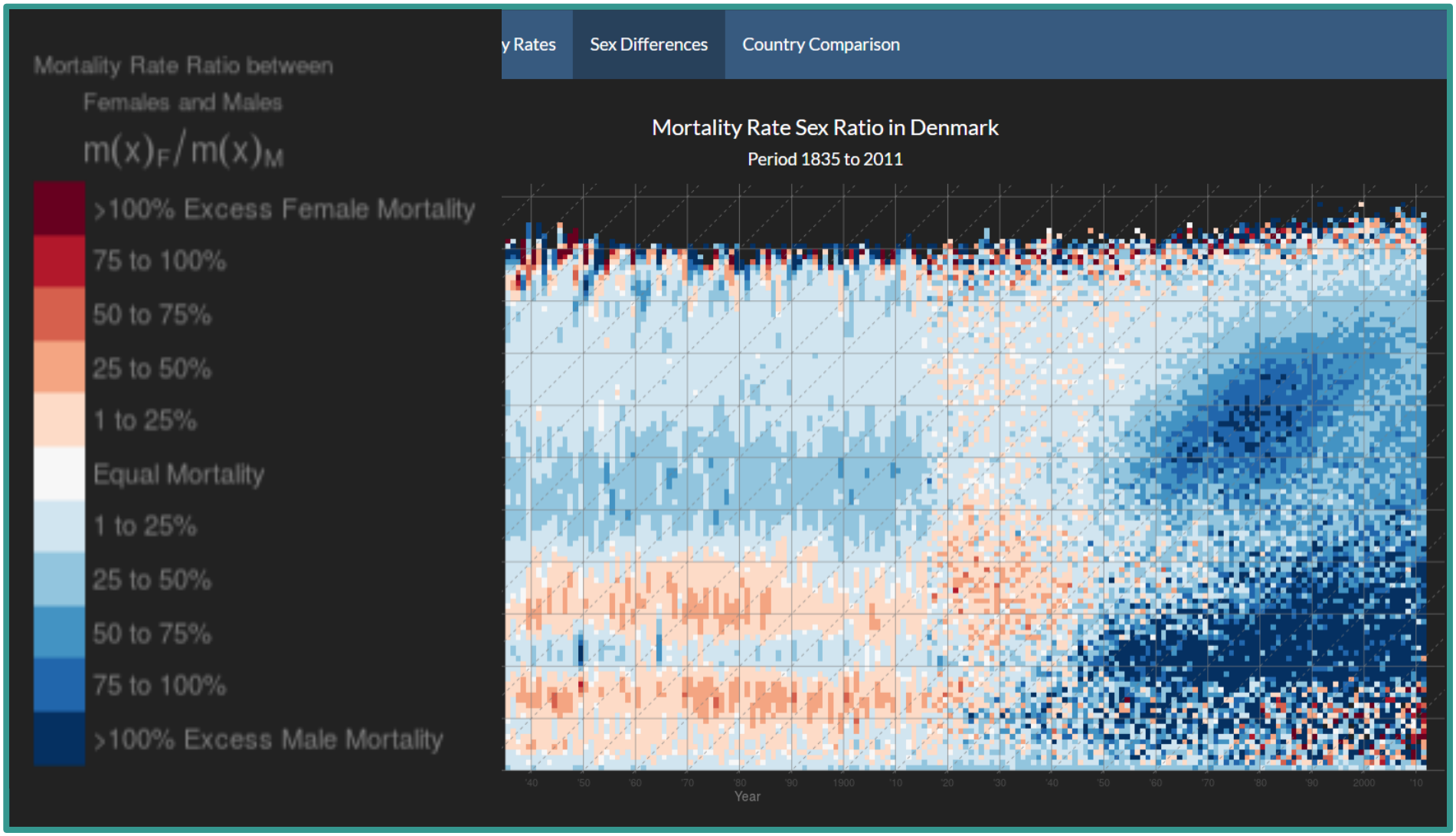
Sex ratio at birth





Sex ratio at birth





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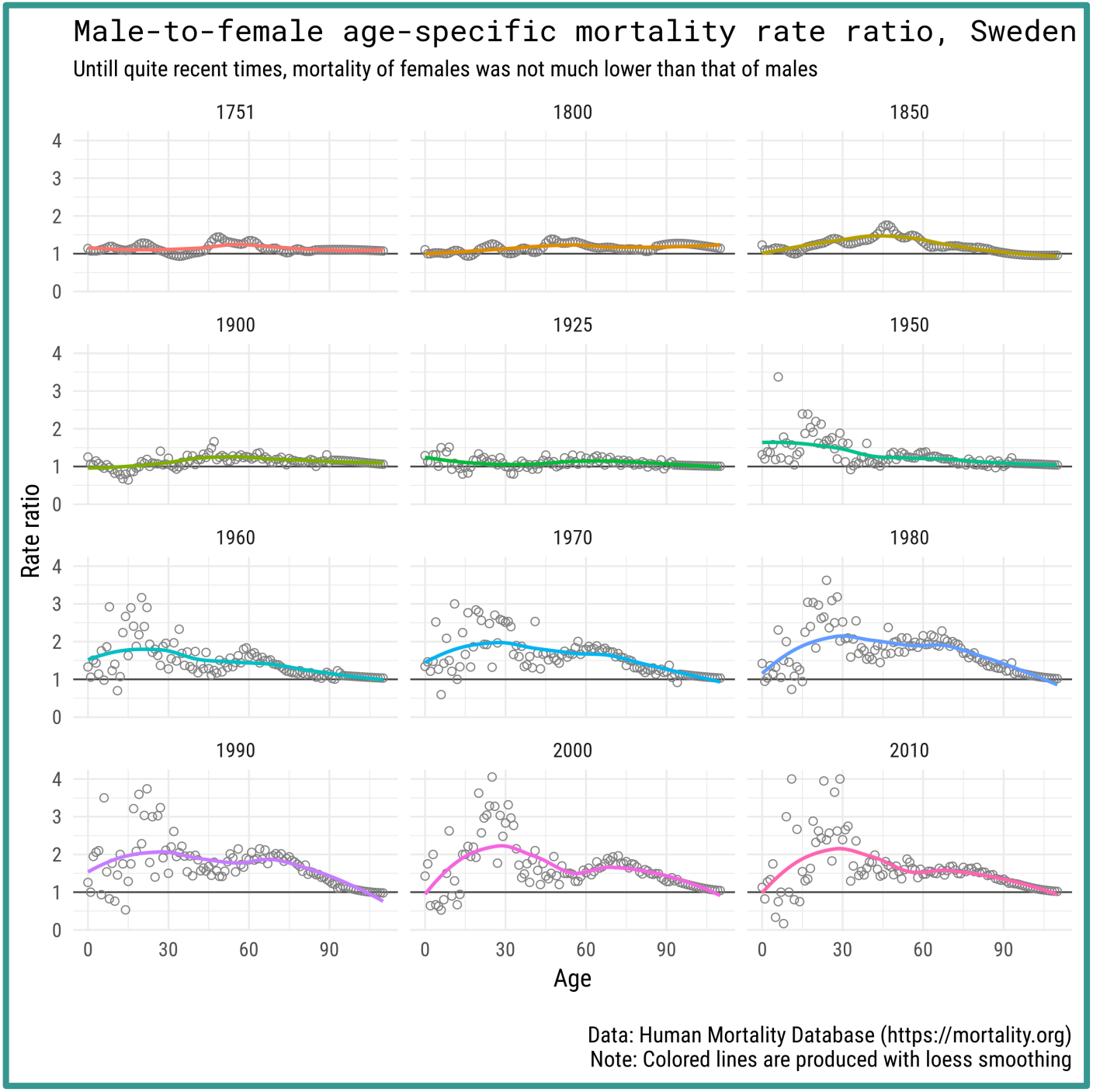


https://ikashnitsky.github.io/2017/gender-gap-in-swedish-mortality/

…even during severe crisis

Table 1. Absolute and relative differences in male and female life expectancy for seven high-mortality populations during (and, when available, before and after) extreme mortality conditions

	Life expectancy						Female–male difference in life expectancy					
	Male			Female			Absolute, y			Relative		
Population	Pre	Crisis	Post	Pre	Crisis	Post	Pre	Crisis	Post	Pre	Crisis	Post
Liberia 1820–1843	—	1.68	22.87*	—	2.23	24.62*	—	0.55	1.25*	—	0.33	0.05*
Trinidad 1813–1816	—	15.18–19.45†	—	—	13.21–20.58†	—	—	−1.27–1.13†	—	—	−0.08–0.06†	—
Ukraine 1933	41.58	7.30	45.12	45.93	10.85	50.49	4.35	3.55	5.37	0.1	0.49	0.12
Sweden 1773	32.31	17.15	37.61	35.19	18.79	39.85	2.88	1.64	2.24	0.09	0.09	0.06
Iceland 1846	35.35	17.86	33.13	40.81	18.82	38.31	5.46	0.96	5.18	0.15	0.05	0.16
Iceland 1882	37.62	16.76	37.82	43.99	18.83	43.74	6.37	2.07	5.92	0.17	0.12	0.16
Ireland 1845–1849	38.3	18.7	—	38.3	22.4	—	0	3.70	—	0	0.20	—

*Life expectancy at age 1 y.
†Values refer to lower and upper bound.

Zarulli, V., Jones, J. A. B., Oksuzyan, A., Lindahl-Jacobsen, R., Christensen, K., & Vaupel, J. W. (2018). Women live longer than men even during severe famines and epidemics. Proceedings of the National Academy of Sciences, 201701535. https://doi.org/10.1073/pnas.1701535115

Life expectancy

The common misconception

People of the past all lived ~30 years

The common misconception

People of the past all lived ~30 years

That's *life expectancy*

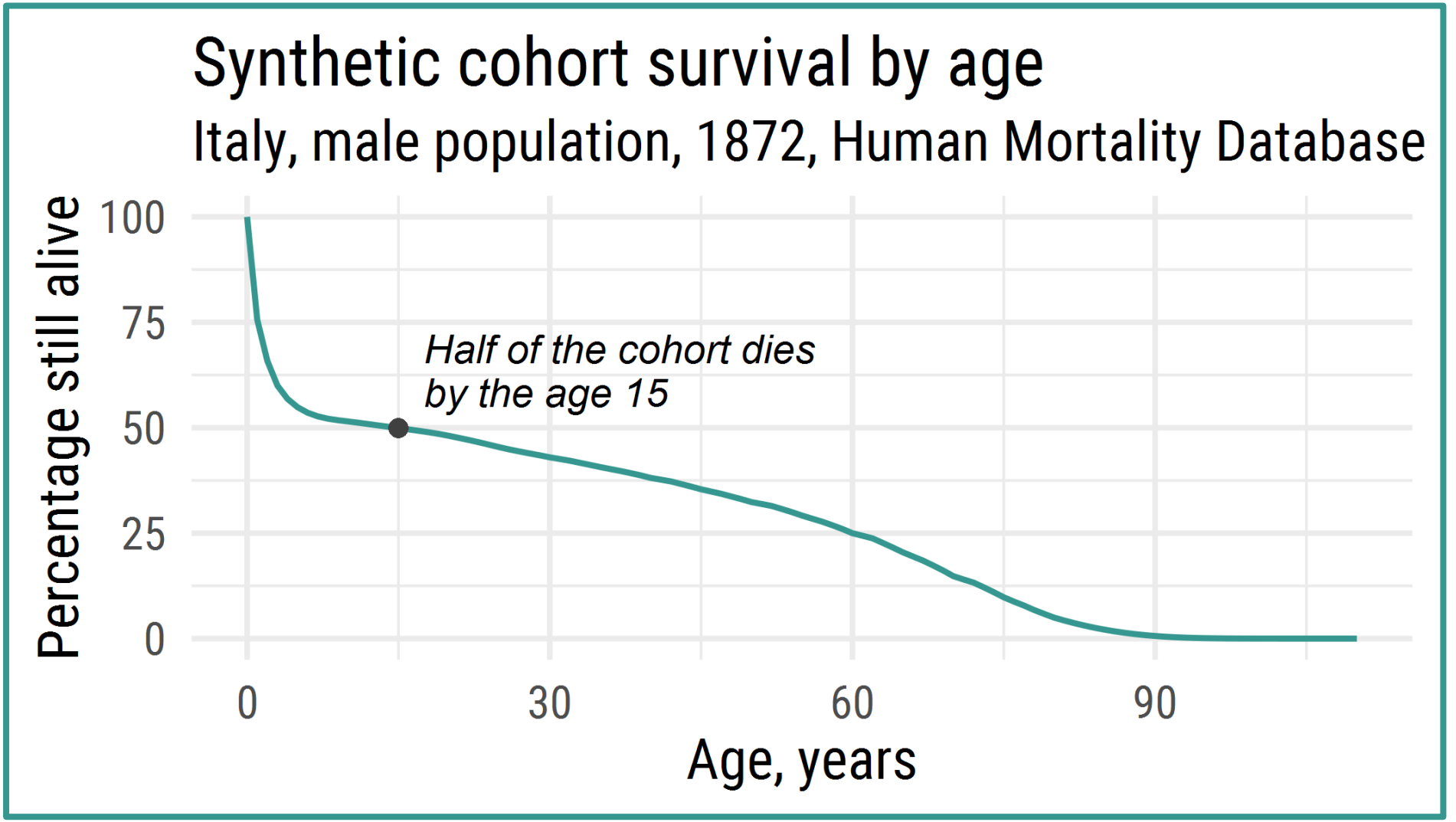
The common misconception

People of the past all lived ~30 years

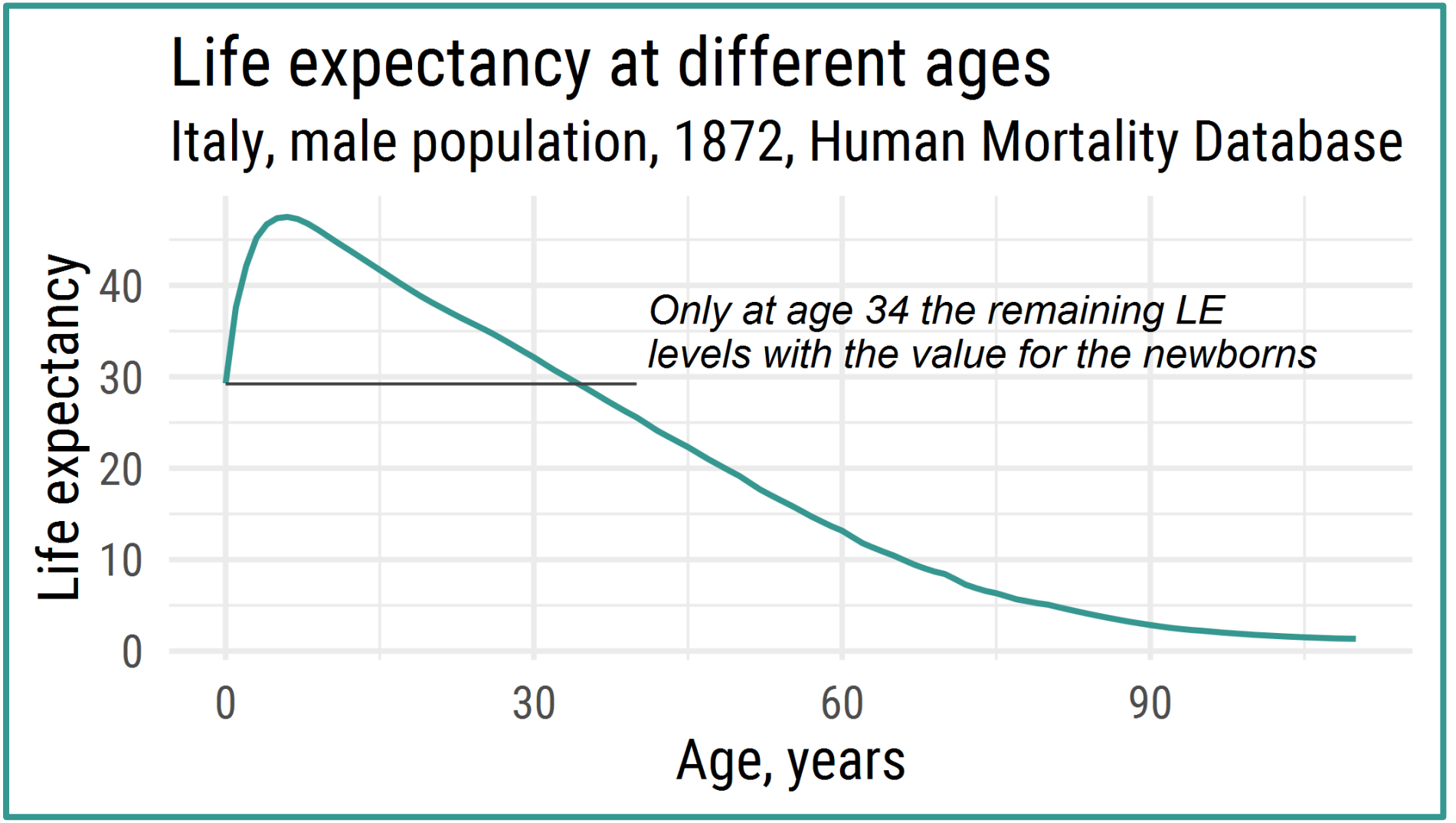
That's *life expectancy*

Life expectancy at age x is the *average* number of years a person aged x can expect to live under *current mortality regime*

High mortality regime



High mortality regime



Life table

The Life Table

Age	Dx	Nx	mx	ax	qx	lx	dx	Lx	Tx	ex
0	3315	17789	0.18635	0.35	0.166217	100000	16621.66	89195.92	4021643	40.22
1	866	15432	0.05612	0.5	0.054588	83378.34	4551.478	81102.6	3932447	47.16
2	582	14137	0.04117	0.5	0.04034	78826.86	3179.845	77236.94	3851344	48.86
3	366	13247	0.02763	0.5	0.027253	75647.02	2061.645	74616.19	3774107	49.89
4	243	12985	0.01871	0.5	0.018537	73585.37	1364.022	72903.36	3699491	50.27
5	202	12973	0.01557	0.5	0.01545	72221.35	1115.8	71663.45	3626588	50.21
6	180	12990	0.01386	0.5	0.013765	71105.55	978.7403	70616.18	3554924	50
7	161	13166	0.01223	0.5	0.012156	70126.81	852.4382	69700.59	3484308	49.69
8	145	13438	0.01079	0.5	0.010732	69274.37	743.4595	68902.64	3414607	49.29
9	132	13633	0.00968	0.5	0.009633	68530.91	660.1839	68200.82	3345705	48.82
10	122	13872	0.00879	0.5	0.008752	67870.73	593.9732	67573.74	3277504	48.29
11	113	13947	0.0081	0.5	0.008067	67276.75	542.7436	67005.38	3209930	47.71
12	104	13742	0.00757	0.5	0.007541	66734.01	503.2716	66482.38	3142925	47.1
13	96	13498	0.00711	0.5	0.007085	66230.74	469.2324	65996.12	3076442	46.45
14	88	13224	0.00665	0.5	0.006628	65761.51	435.8648	65543.57	3010446	45.78
15	82	13060	0.00628	0.5	0.00626	65325.64	408.9609	65121.16	2944903	45.08
16	77	12920	0.00596	0.5	0.005942	64916.68	385.7539	64723.8	2879781	44.36
17	74	12677	0.00584	0.5	0.005823	64530.93	375.7634	64343.05	2815058	43.62
18	74	12334	0.006	0.5	0.005982	64155.16	383.7796	63963.27	2750715	42.88
19	75	11770	0.00637	0.5	0.00635	63771.38	404.934	63568.92	2686751	42.13
20	78	11022	0.00708	0.5	0.007055	63366.45	447.0519	63142.92	2623182	41.4
...	...	...	...	...	...	...	...	...	...	...

Period life table, Danish females, 1835
Source: Human Mortality Database

Abridged Life Table

Age	mx	qx	ax	lx	dx	Lx	Tx	ex
0	0.18635	0.16621	0.35	100000	16621	89196	4020898	40.21
1-4	0.03648	0.13381	1.52	83379	11157	305860	3931702	47.15
5-9	0.01248	0.06031	2.24	72222	4356	349078	3625843	50.20
10-14	0.00765	0.03751	2.35	67866	2546	332582	3276765	48.28
15-19	0.00609	0.02999	2.49	65320	1959	321694	2944183	45.07
20-24	0.00802	0.03934	2.58	63361	2493	310777	2622489	41.39
25-29	0.00915	0.04471	2.48	60869	2722	297499	2311712	37.98
30-34	0.00940	0.04594	2.56	58147	2671	284206	2014213	34.64
35-39	0.01210	0.05875	2.55	55476	3259	269382	1730007	31.18
40-44	0.01228	0.05958	2.49	52217	3111	253271	1460625	27.97
45-49	0.01452	0.07013	2.57	49106	3444	237149	1207354	24.59
50-54	0.01824	0.08730	2.54	45662	3986	218507	970205	21.25
55-59	0.02197	0.10430	2.57	41675	4347	197815	751698	18.04
60-64	0.03333	0.15436	2.61	37328	5762	172887	553883	14.84
65-69	0.04743	0.21209	2.51	31567	6695	141168	380996	12.07
70-74	0.06571	0.28261	2.53	24872	7029	106968	239827	9.64
75-79	0.10151	0.40352	2.46	17843	7200	70925	132859	7.45
80-84	0.14579	0.52476	2.33	10643	5585	38308	61934	5.82
85-89	0.19213	0.62606	2.22	5058	3167	16481	23627	4.67
90-94	0.24694	0.71971	2.10	1891	1361	5512	7146	3.78
95-99	0.31115	0.80075	1.97	530	425	1364	1633	3.08
100-104	0.38339	0.86486	1.83	106	91	238	269	2.55
105-109	0.46117	0.91142	1.68	14	13	28	31	2.15
110+	0.52464	1.00000	1.91	1	1	2	2	1.91

Period life table, Danish females, 1835
Source: Human Mortality Database

Age-specific death rates, ${}_nm_x$

Age	Dx	Nx	mx
0	3315	17789	0.18635
1	866	15432	0.05612
2	582	14137	0.04117
3	366	13247	0.02763
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6	180	12990	0.01386
7	161	13166	0.01223
8	145	13438	0.01079
9	132	13633	0.00968
10	122	13872	0.00879
11	113	13947	0.0081
12	104	13742	0.00757
13	96	13498	0.00711
14	88	13224	0.00665
15	82	13060	0.00628
16	77	12920	0.00596
17	74	12677	0.00584
18	74	12334	0.006
19	75	11770	0.00637
20	78	11022	0.00708
...	...	...	...

Period life table, Danish females, 1835
Source: Human Mortality Database

$$ {}_nm_x \simeq {}_nM_x = \frac{{}_nD_x}{{}_nN_x} $$

where:

n → Age-interval (here 1 year)

${}_nm_x$ → Death rate between ages x and $x+n$

${}_nD_x$ → Deaths between ages x and $x+n$

${}_nN_x$ → Pop. at risk between ages x and $x+n$

e.g.
m_0 = 0.18635 means that there are around 186 deaths per 1000 female children during the first year of life

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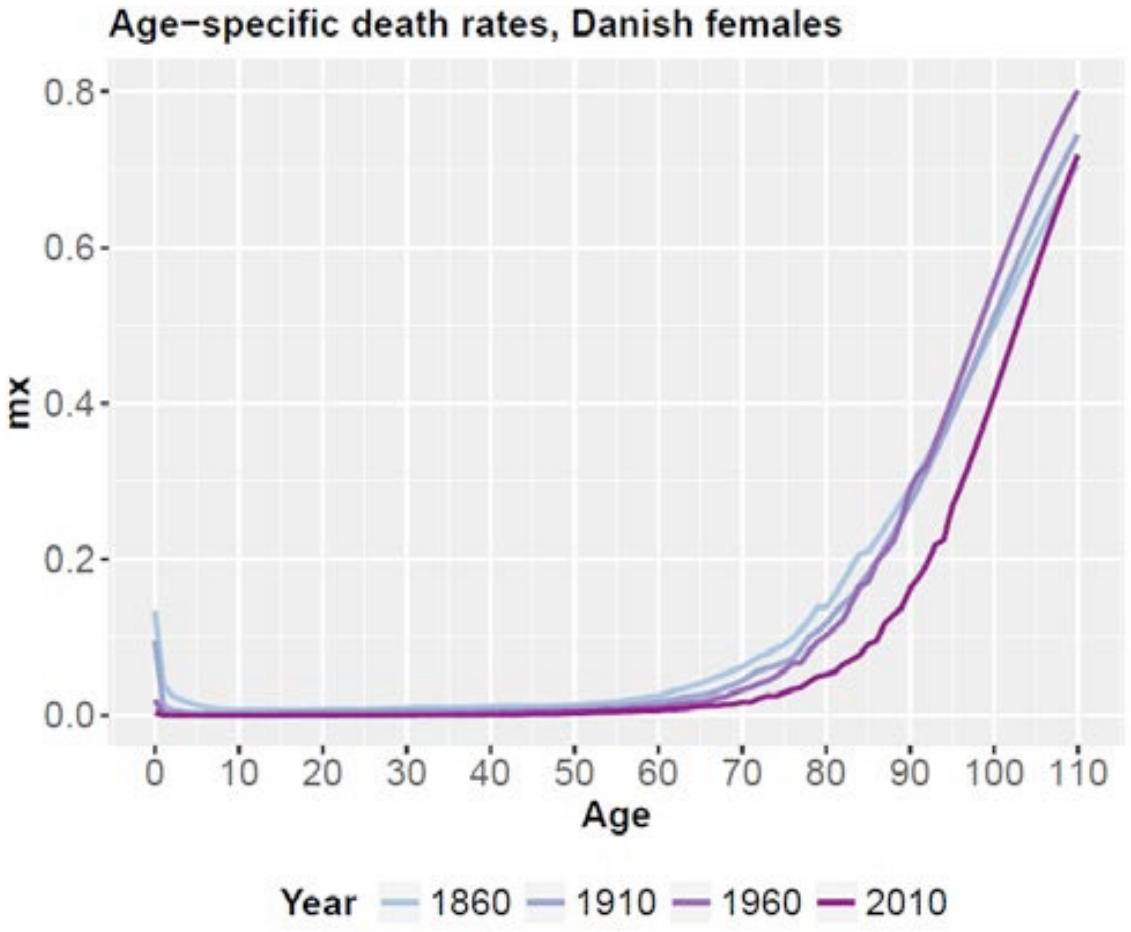
...	...	...	...



Period life table, Danish females, 1835

Source: Human Mortality Database

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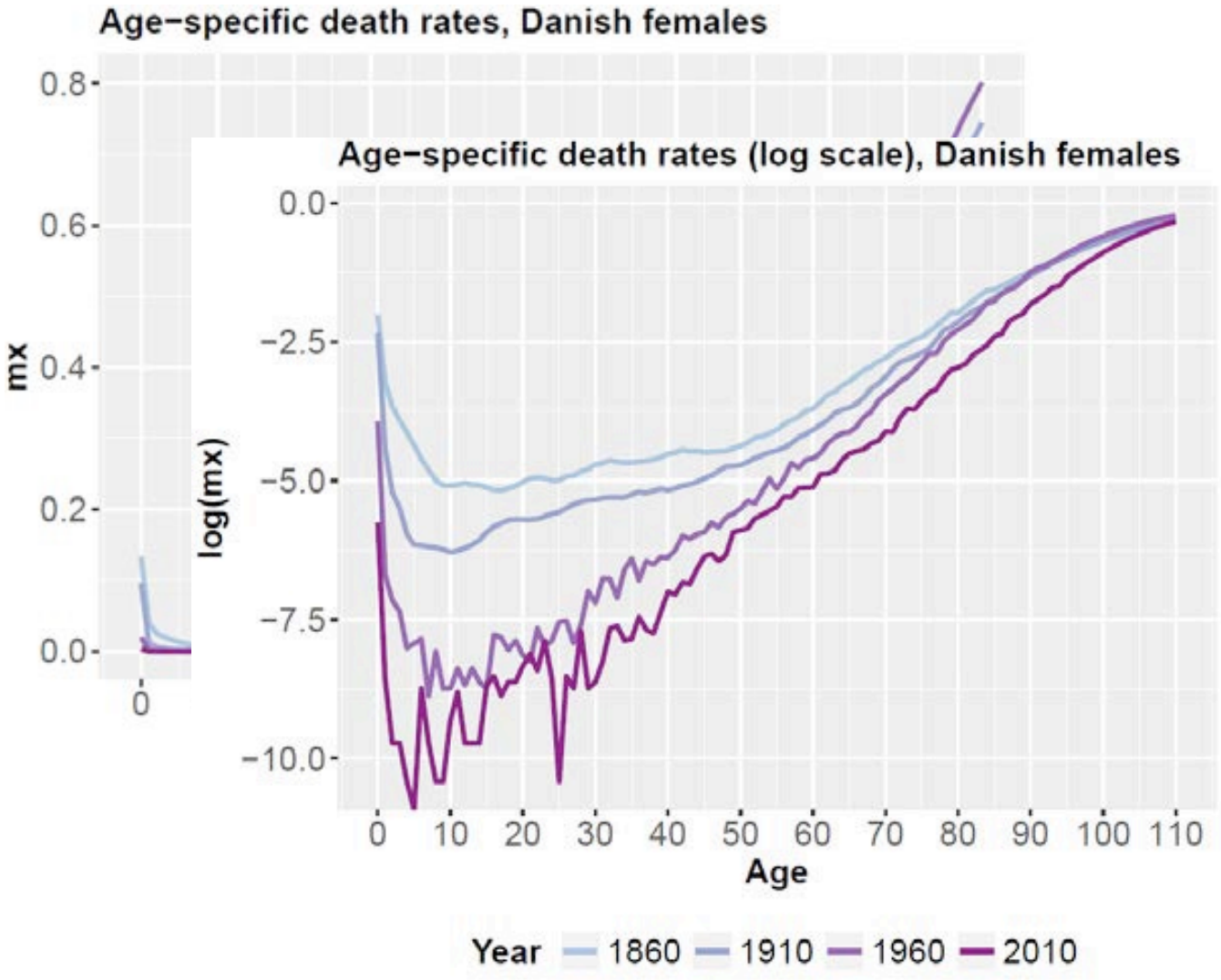

Period life table, Danish females, 1835
Source: Human Mortality Database

Person-years lived by those who die, ${}_n a_x$

Age	ax
0	0.35
1	0.5
2	0.5
3	0.5
4	0.5
5	0.5
6	0.5
7	0.5
8	0.5
9	0.5
10	0.5
11	0.5
12	0.5
13	0.5
14	0.5
15	0.5
16	0.5
17	0.5
18	0.5
19	0.5
20	0.5
...	...

Period life table, Danish females, 1835
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${}_n a_x$ is the average number of person-years lived in the interval x to $x + n$ by those who die in that interval.

Common assumptions: ${}_n a_x = n/2$ i.e. deaths occur, on average, halfway through the age-interval

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20	0.5
...	...

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Common assumptions: ${}_n a_x = n/2$ i.e. deaths occur, on average, halfway through the age-interval

Exception: youngest age groups

Values of ${}_n a_x$ for use below age 5, from Preston et al. 2001, p. 48

	Males	Females
Value of ${}_1 a_0$		
If ${}_1 m_0 >= 0.107$	0.330	0.350
If ${}_1 m_0 < 0.107$	$0.045 + 2.684 * {}_1 m_0$	$0.053 + 2.8 * {}_1 m_0$
Value of ${}_4 a_1$		
If ${}_1 m_0 >= 0.107$	1.352	1.361
If ${}_1 m_0 < 0.107$	$1.651 - 2.816 * {}_1 m_0$	$1.522 - 1.518 * {}_1 m_0$

Period life table, Danish females, 1835
Source: Human Mortality Database

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8	0.5
9	0.5
10	0.5
11	0.5
12	0.5
13	0.5
14	0.5
15	0.5
16	0.5
17	0.5
18	0.5
19	0.5
20	0.5
...	...

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If ${}_1 m_0 < 0.107$	$1.651 - 2.816 * {}_1 m_0$	$1.522 - 1.518 * {}_1 m_0$

A set of ${}_n a_x$ values can also be borrowed from another similar population.

Age-specific death probabilities, ${}_nq_x$

Age	Dx	Nx	mx
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2	582	14137	0.04117
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15	82	13060	0.00628
16	77	12920	0.00596
17	74	12677	0.00584
18	74	12334	0.006
19	75	11770	0.00637
20	78	11022	0.00708
...	...	...	...

Period life table, Danish females, 1835
Source: Human Mortality Database

Convert the set of observed period age-specific death rates, ${}_n\boldsymbol{M}_x$, into a set of age-specific probabilities of dying, ${}_n\boldsymbol{q}_x$

$$ {}_nq_x = \frac{n \times {}_nm_x}{1 + (n - {}_na_x) \times {}_nm_x} $$

For the last, open-ended age category, set ${}_\infty q_x = 1$

Age-specific death probabilities, ${}_nq_x$

Age	Dx	Nx	mx
0	3315	17789	0.18635
1	866	15432	0.05612
2	582	14137	0.04117
3	366	13247	0.02763
4	243	12985	0.01871
5	202	12973	0.01557
6	180	12990	0.01386
7	161	13166	0.01223
8	145	13438	0.01079
9	132	13633	0.00968
10	122	13872	0.00879
11	113	13947	0.0081
12	104	13742	0.00757
13	96	13498	0.00711
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Period life table, Danish females, 1835
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The probability to survive between ages x and $x + n$, ${}_nq_x$, is:

$$ {}_np_x = 1 - {}_nq_x $$

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Period life table, Danish females, 1835
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$$ {}_np_x = 1 - {}_nq_x $$

Note that the conversion from ${}_nm_x$ to ${}_nq_x$ depends only on ${}_na_x$

Survival function, l_x

Age	qx	lx
0	0.166217	100000
1	0.054588	83378.34
2	0.04034	78826.86
3	0.027253	75647.02
4	0.018537	73585.37
5	0.01545	72221.35
6	0.013765	71105.55
7	0.012156	70126.81
8	0.010732	69274.37
9	0.009633	68530.91
10	0.008752	67870.73
11	0.008067	67276.75
12	0.007541	66734.01
13	0.007085	66230.74
14	0.006628	65761.51
15	0.00626	65325.64
16	0.005942	64916.68
17	0.005823	64530.93
18	0.005982	64155.16
19	0.00635	63771.38
20	0.007055	63366.45
...	...	...

The survival function shows the number of survivors at age x

Choose a value of l_0, the *radix*. Commonly 100,000 or 1

Then compute:

$$l_{x+n} = l_x \times {}_np_x$$

Period life table, Danish females, 1835
Source: Human Mortality Database

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The survival function shows the number of survivors at age x

Choose a value of l_0, the *radix*. Commonly 100,000 or 1

Then compute:

$$l_{x+n} = l_x \times {}_np_x$$

e.g.
l_5 = 72221.35 means that there is a 72,2% survival chance from birth until age 5

Period life table, Danish females, 1835
Source: Human Mortality Database

Survival function, l_x

Age	qx	lx
0	0.166217	100000
1	0.054588	83378.34
2	0.04034	78826.86
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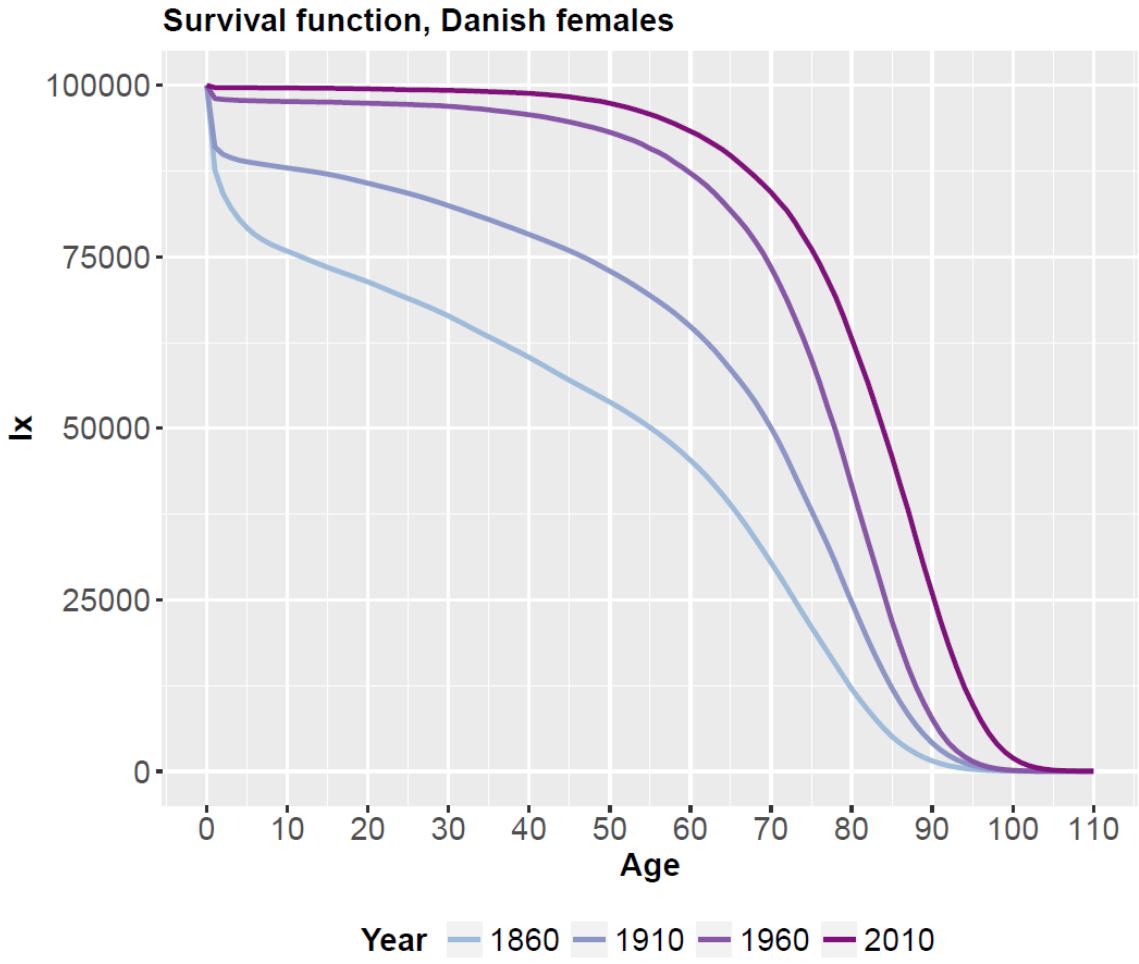

Period life table, Danish females, 1835
Source: Human Mortality Database

Distribution of deaths, ${}_nd_x$

Age	qx	lx	dx
0	0.166217	100000	16621.66
1	0.054588	83378.34	4551.478
2	0.04034	78826.86	3179.845
3	0.027253	75647.02	2061.645
4	0.018537	73585.37	1364.022
5	0.01545	72221.35	1115.8
6	0.013765	71105.55	978.7403
7	0.012156	70126.81	852.4382
8	0.010732	69274.37	743.4595
9	0.009633	68530.91	660.1839
10	0.008752	67870.73	593.9732
11	0.008067	67276.75	542.7436
12	0.007541	66734.01	503.2716
13	0.007085	66230.74	469.2324
14	0.006628	65761.51	435.8648
15	0.00626	65325.64	408.9609
16	0.005942	64916.68	385.7539
17	0.005823	64530.93	375.7634
18	0.005982	64155.16	383.7796
19	0.00635	63771.38	404.934
20	0.007055	63366.45	447.0519
...	...	...	...

Shows the number of life table deaths between ages x and $x + n$

$$ {}_nd_x = l_x - l_{x+n} = l_x * {}_nq_x $$

Period life table, Danish females, 1835
Source: Human Mortality Database

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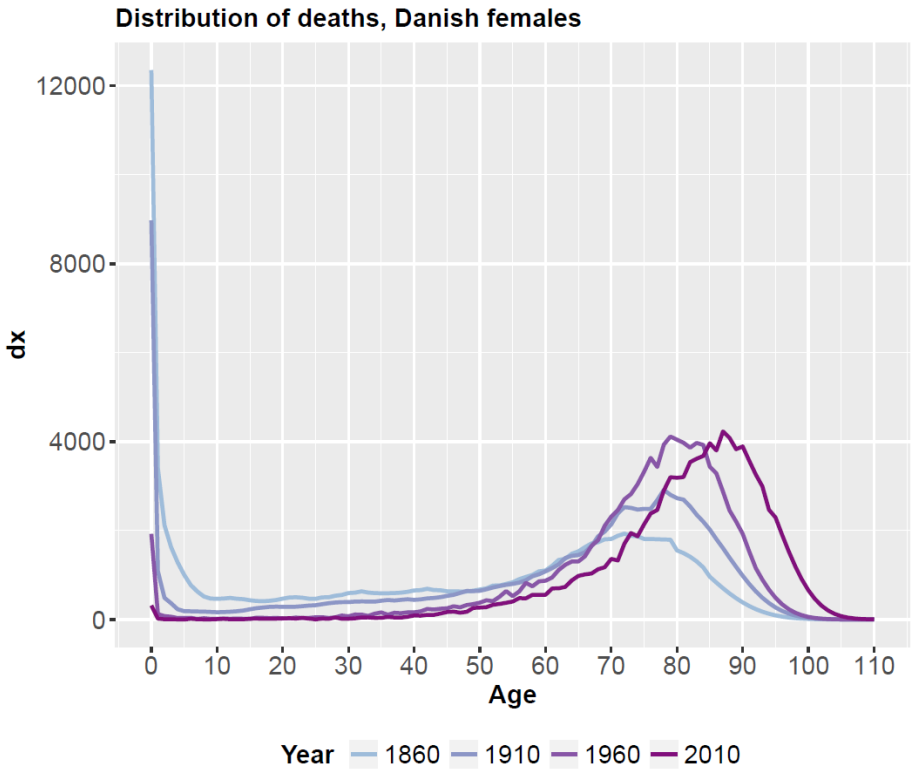
Shows the number of life table deaths between ages x and $x + n$

$$ {}_nd_x = l_x - l_{x+n} = l_x * {}_nq_x $$



Period life table, Danish females, 1835
Source: Human Mortality Database

Person-years lived in an age-interval, ${}_nL_x$

Age	ax	lx	dx	Lx
0	0.35	100000	16621.66	89195.92
1	0.5	83378.34	4551.478	81102.6
2	0.5	78826.86	3179.845	77236.94
3	0.5	75647.02	2061.645	74616.19
4	0.5	73585.37	1364.022	72903.36
5	0.5	72221.35	1115.8	71663.45
6	0.5	71105.55	978.7403	70616.18
7	0.5	70126.81	852.4382	69700.59
8	0.5	69274.37	743.4595	68902.64
9	0.5	68530.91	660.1839	68200.82
10	0.5	67870.73	593.9732	67573.74
11	0.5	67276.75	542.7436	67005.38
12	0.5	66734.01	503.2716	66482.38
13	0.5	66230.74	469.2324	65996.12
14	0.5	65761.51	435.8648	65543.57
15	0.5	65325.64	408.9609	65121.16
16	0.5	64916.68	385.7539	64723.8
17	0.5	64530.93	375.7634	64343.05
18	0.5	64155.16	383.7796	63963.27
19	0.5	63771.38	404.934	63568.92
20	0.5	63366.45	447.0519	63142.92
...	...	...	...	...

Period life table, Danish females, 1835
Source: Human Mortality Database

$$ {}_nL_x = n \times l_{x+n} + {}_na_x \times {}_nd_x $$

$$ \text{(open–ended interval: } {}_\infty L_x = \frac{l_x}{{}_\infty m_x}) $$

e.g.

$$ L_0 = (n * l_1) + ({}_1a_0 * {}_1d_0) = $$
$$ (1 * 83378.34) + (0.35 * 16621.66) $$
$$ = 89195.92 $$

Person-years lived above age x, T_x

Age	Lx	Tx
0	89195.92	4021643
1	81102.6	3932447
2	77236.94	3851344
3	74616.19	3774107
4	72903.36	3699491
5	71663.45	3626588
6	70616.18	3554924
7	69700.59	3484308
8	68902.64	3414607
9	68200.82	3345705
10	67573.74	3277504
11	67005.38	3209930
12	66482.38	3142925
13	65996.12	3076442
14	65543.57	3010446
15	65121.16	2944903
16	64723.8	2879781
17	64343.05	2815058
18	63963.27	2750715
19	63568.92	2686751
20	63142.92	2623182
...	...	...

$$T_x = \sum_{a=x}^{\infty} {}_nL_a$$

Period life table, Danish females, 1835
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Person-years lived above age x, T_x

Age	Lx	Tx
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$$T_x = \sum_{a=x}^{\infty} {}_nL_a$$

T_0 = 4 021 643 means that the LT population has this many person-years to live above age 0

Period life table, Danish females, 1835
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0	89195.92	4021643
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$$T_x = \sum_{a=x}^{\infty} {}_nL_a$$

T_0 = 4 021 643 means that the LT population has this many person-years to live above age 0

Recall that the radix (initial population) is 100 000

Period life table, Danish females, 1835
Source: Human Mortality Database

Life expectancy at age x, e_x

Age	lx	Tx	ex
0	100000	4021643	40.22
1	83378.34	3932447	47.16
2	78826.86	3851344	48.86
3	75647.02	3774107	49.89
4	73585.37	3699491	50.27
5	72221.35	3626588	50.21
6	71105.55	3554924	50
7	70126.81	3484308	49.69
8	69274.37	3414607	49.29
9	68530.91	3345705	48.82
10	67870.73	3277504	48.29
11	67276.75	3209930	47.71
12	66734.01	3142925	47.1
13	66230.74	3076442	46.45
14	65761.51	3010446	45.78
15	65325.64	2944903	45.08
16	64916.68	2879781	44.36
17	64530.93	2815058	43.62
18	64155.16	2750715	42.88
19	63771.38	2686751	42.13
20	63366.45	2623182	41.4
...	...	...	...

Period life table, Danish females, 1835
Source: Human Mortality Database

$$T_x = \sum_{a=x}^{\infty} {}_nL_a$$

T_0 = 4 021 643 means that the LT population has this many person-years to live above age 0

Recall that the radix (initial population) is 100 000

$$e_x^0 = \frac{T_x}{l_x}$$

Average number of additional years that a survivor to age x will live beyond that age. Average age at death.

Life expectancy at age x, e_x

Age	lx	Tx	ex
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Period life table, Danish females, 1835
Source: Human Mortality Database

$$T_x = \sum_{a=x}^{\infty} {}_nL_a$$

T_0 = 4 021 643 means that the LT population has this many person-years to live above age 0

e_0 = 40.2 means that a Danish female born in 1835 would be expected to live 40.22 years if she experienced, at each age, the age-specific death rates prevailing in the year 1835.

Recall that the radix (initial population) is 100 000

$$e_x^0 = \frac{T_x}{l_x}$$

Average number of additional years that a survivor to age x will live beyond that age. Average age at death.

Life expectancy at age x, e_x

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4	73585.37	3699491	50.27
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6	71105.55	3554924	50
7	70126.81	3484308	49.69
8	69274.37	3414607	49.29
9	68530.91	3345705	48.82
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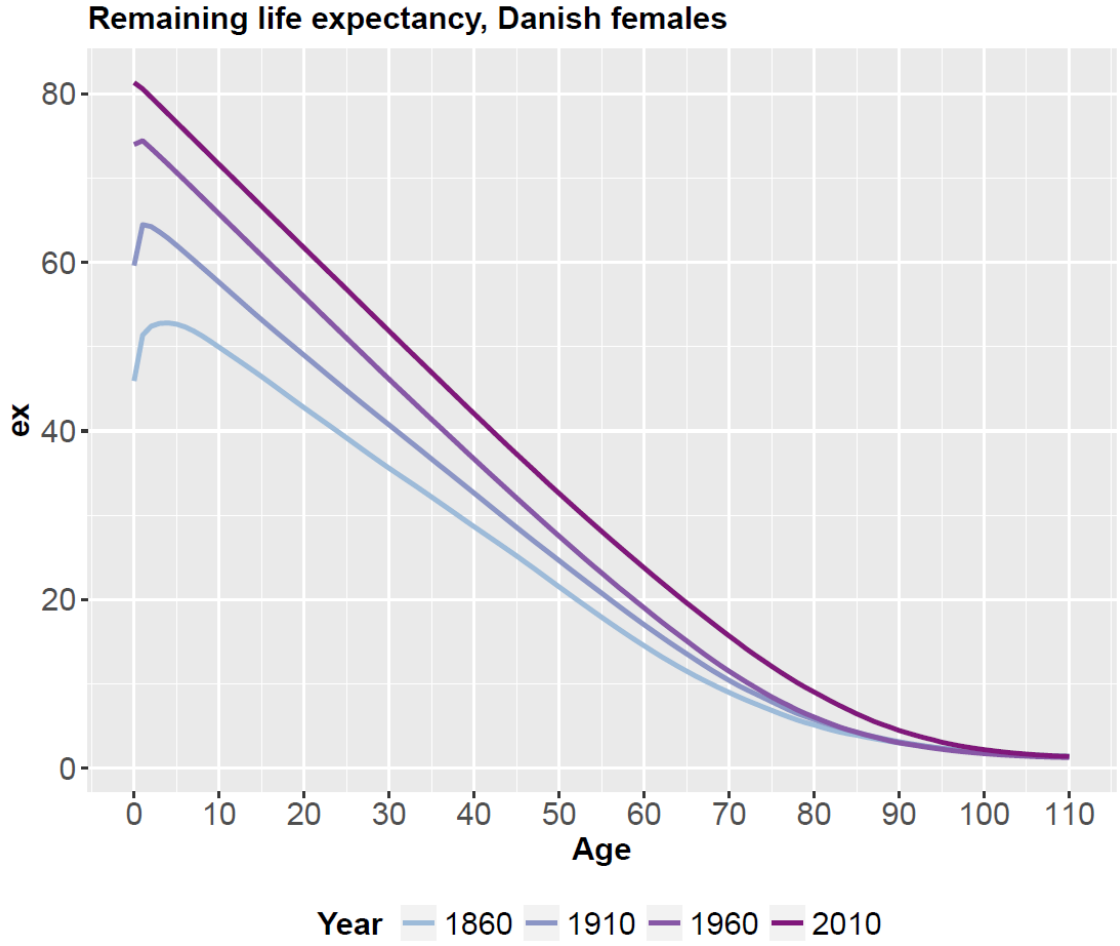

Period life table, Danish females, 1835
Source: Human Mortality Database

thank you

ikashnitsky@sdu.dk

@ikashnitsky