Mobile Robot Jacobians

- Recall – Jacobians relate Joint Velocities to end-effector velocities (or task velocities) for articulated robots.

→ For mobile robots Jacobians basically do the same thing!

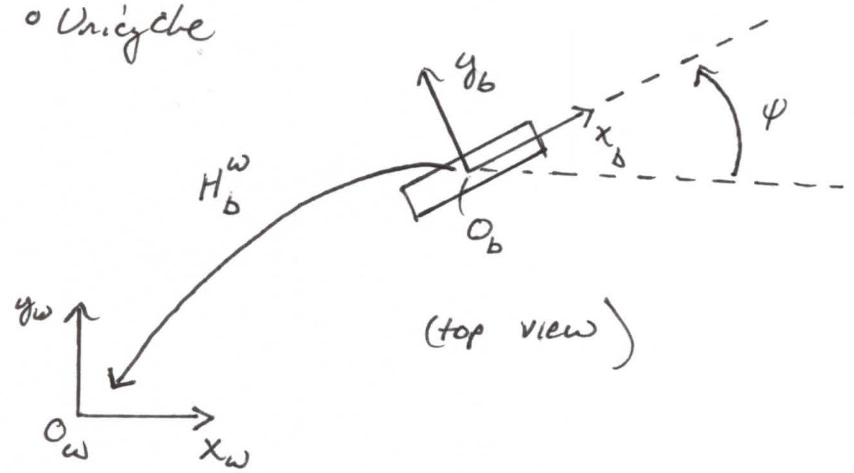
Articulated Robots

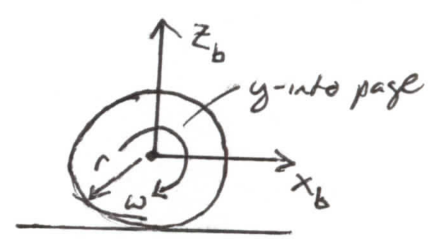
- Joint Velocities
- End-effector Velocities
  ⇝ Relative to the base frame

Mobile (Ground) Robots

- Wheel velocities
- Body velocities
  ⇝ Relative to the world frame

- Unicycle



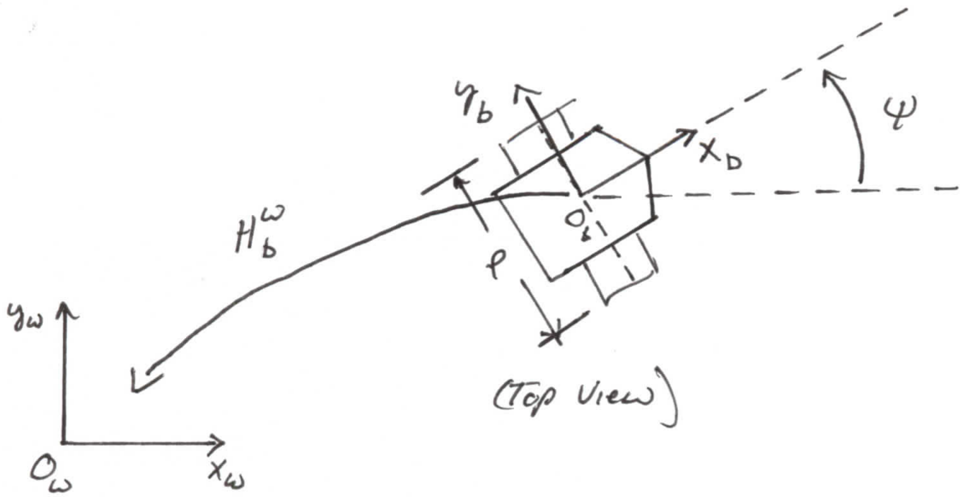


$$H_b^w = \underbrace{\begin{bmatrix} c\psi & -s\psi & 0 & x \\ s\psi & c\psi & 0 & y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{3D} = \underbrace{\begin{bmatrix} c\psi & -s\psi & x \\ s\psi & c\psi & y \\ 0 & 0 & 1 \end{bmatrix}}_{2D}$$

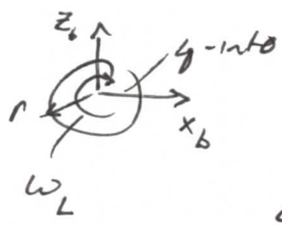
$$V_{body} = \begin{bmatrix} \omega r \\ 0 \end{bmatrix} \longrightarrow \begin{bmatrix} \dot{x}_w \\ \dot{y}_w \\ \dot{\psi}_w \end{bmatrix} = \begin{bmatrix} r\cos\psi & 0 \\ r\sin\psi & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \omega \\ \dot{\psi} \end{bmatrix}$$

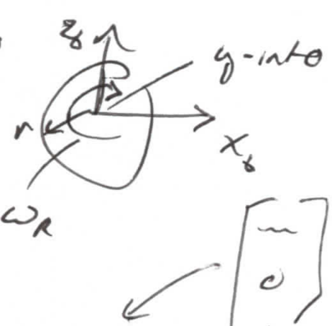
$$\dot{\psi}_{body} = \dot{\psi}$$

- Differential Drive



$$H_b^\omega = \begin{bmatrix} c\psi & -s\psi & x \\ s\psi & c\psi & y \\ 0 & 0 & 1 \end{bmatrix}$$





$$V_b = \frac{(\omega_R + \omega_L)\, r}{2} \longrightarrow$$

$$\dot{\psi}_b = \frac{(\omega_R - \omega_L)\, r}{\ell} \longrightarrow$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} \frac{r}{2} c\psi & \frac{r}{2} c\psi \\ \frac{r}{2} s\psi & \frac{r}{2} s\psi \\ -\frac{r}{\ell} & \frac{r}{\ell} \end{bmatrix} \begin{bmatrix} \omega_L \\ \omega_R \end{bmatrix}$$

What do we notice about both Jacobians?

→ We have more task variables than Joint variables

⇝ The robot's body has more velocities than we have wheel velocities to control!

⇒ So... we <u>cannot</u> controll all of our velocities at once

⇝ Let's try to just control $\dot{x}$ and $\dot{y}$ for the differential drive example

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{r}{2} c\varphi & \frac{r}{2} c\varphi \\ \frac{r}{2} s\varphi & \frac{r}{2} s\varphi \end{bmatrix}}_{\text{Jacobian}} \begin{bmatrix} \omega_L \\ \omega_R \end{bmatrix}$$

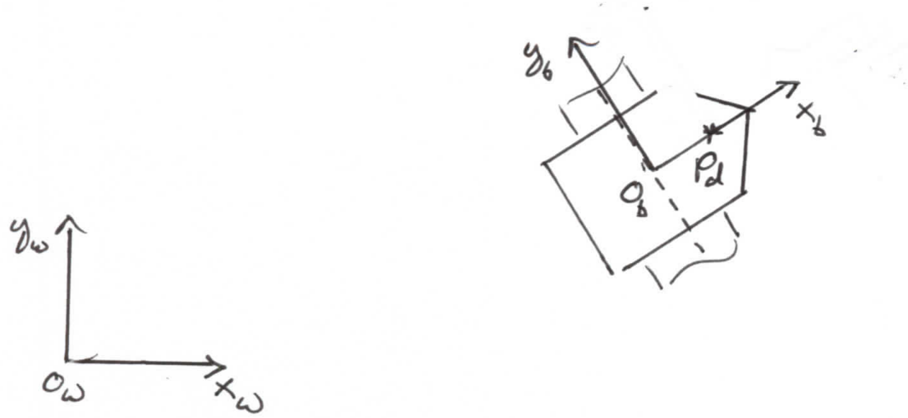
→ What is the rank of our Jacobian?

⇒ 1! No matter what φ we put in...

⇝ This means that no matter what we try to do with this Jacobian,

$$\begin{bmatrix} \omega_L \\ \omega_R \end{bmatrix} = J(\varphi)^{-1} \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} \quad \text{won't work!}$$

Let's add a control point offset from the ~~car~~ body frame.



$$H_b^{\omega} = \begin{bmatrix} c\psi & -s\psi & x \\ s\psi & c\psi & y \\ 0 & 0 & 1 \end{bmatrix}$$

$$p_d^b = \begin{bmatrix} d \\ 0 \end{bmatrix}$$

Recall

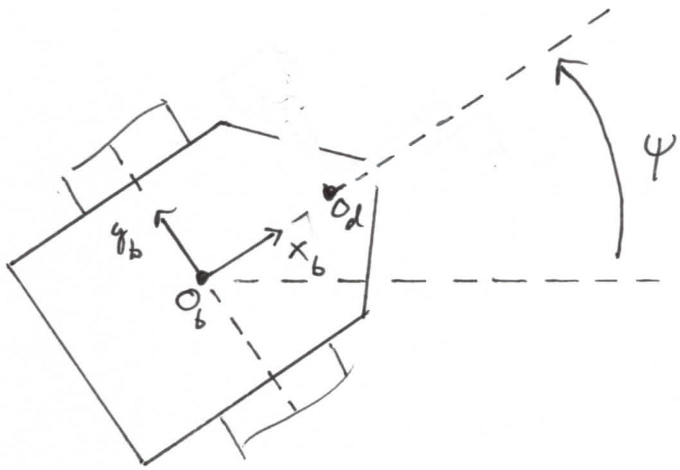
$$\dot{\vec{X}}^b = \begin{bmatrix} \frac{(\omega_L + \omega_R)\, r}{2} \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{r}{2} & \frac{r}{2} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \omega_L \\ \omega_R \end{bmatrix}$$

$$\dot{\vec{X}}^{\omega} = R_b^{\omega} \dot{\vec{X}}^b = \begin{bmatrix} c\psi & -s\psi \\ s\psi & c\psi \end{bmatrix} \begin{bmatrix} \frac{r}{2} & \frac{r}{2} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \omega_L \\ \omega_R \end{bmatrix}$$

$\Rightarrow$ How do we find

$\dot{\vec{X}}^d$?

$\rightsquigarrow$ Let's Look at the picture...



$\rightsquigarrow$ Will anything change about the velocity in the x-direction at Point d?

* NO!

$$\dot{x}_d^b = \begin{bmatrix} \frac{r}{2} & \frac{r}{2} \end{bmatrix} \begin{bmatrix} \omega_L \\ \omega_R \end{bmatrix}$$

$\rightsquigarrow$ What about the y-direction?

* Yes!

$$\dot{y}_d^b = d\,\dot{\psi} = \begin{bmatrix} \frac{-dr}{\ell} & \frac{dr}{\ell} \end{bmatrix} \begin{bmatrix} \omega_L \\ \omega_R \end{bmatrix}$$

Applying the same method as previously

$$\dot{\vec{X}}^{w}_{d} = \begin{bmatrix} c\psi & -s\psi \\ s\psi & c\psi \end{bmatrix} \begin{bmatrix} \frac{r}{2} & \frac{r}{2} \\ \frac{-dr}{\ell} & \frac{dr}{\ell} \end{bmatrix} \begin{bmatrix} \omega_L \\ \omega_R \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} \left(\frac{r}{2}\right)c\psi + \left(\frac{dr}{\ell}\right)s\psi & \frac{r}{2}c\psi - \left(\frac{dr}{\ell}\right)s\psi \\ \left(\frac{r}{2}\right)s\psi - \left(\frac{dr}{\ell}\right)c\psi & \frac{r}{2}s\psi + \left(\frac{dr}{\ell}\right)c\psi \end{bmatrix}}_{\text{New } J(\psi)!} \begin{bmatrix} \omega_L \\ \omega_R \end{bmatrix}$$

→ Can we invert this new Jacobian?

Yes!

⇒ Using this

$$\begin{bmatrix} \omega_L \\ \omega_R \end{bmatrix} = J(\psi)^{-1} \begin{bmatrix} \dot{x}^{w}_{d} \\ \dot{y}^{w}_{d} \end{bmatrix}$$

→ we can use current position, and heading along with a desired position to solve for required wheel velocities!