Robot Motion Planning: the GJK Algorithm

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1 The GJK Algorithm

It is a efficient method to compute the distance between two convex polygons. The procedure is from [1, 2]. C denotes a convex polygon, which represents the Minkowski difference from two polygons A and B.

1.1 Procedure

1. Initialize the simplex set V (randomly or in other ways) from the convex hull C.
2. Compute point $\mathbf{p}$ of minimum norm in V.
3. If $\mathbf{p}$ is the origin, or the origin is inside V, return 0 and exit.
4. Reduce V to the smallest subset Q such that $\mathbf{p}$ is in Q.
5. Let $\mathbf{q} = s_C(-\mathbf{p})$. If $\mathbf{q}$ is no more extreme in the direction of $-\mathbf{p}$ than $\mathbf{p}$ itself, return $\|\mathbf{p}\|$ and exit.
6. Add $\mathbf{q}$ to Q to update V. Go to step 2.

1.2 Pseudocode

function distance = GJK_2D(C)
  $V \leftarrow \{\mathbf{y}_1, \mathbf{y}_2, \mathbf{y}_3\} \in C$.
  $k \leftarrow 1$ (Note: Matlab index starts from 1)
  $\mathbf{p}_k = \text{argmin}_{\mathbf{p}\in V_k} \|\mathbf{p}\|$.
  if $\mathbf{p}_k = \mathbf{0}$ or $\mathbf{0} \in V_k$ **then**:
    return 0 and exit
  end if
  Set $Q_k \subset V_k$ such that $\mathbf{p}_k \in Q_k$.
  $\mathbf{q}_k \leftarrow s_C(-\mathbf{p}_k)$
  if $\mathbf{q}_k \in Q_k$ **then**:
    return $\|\mathbf{p}_k\|$ and exit
  end if
  $V_k = \{\mathbf{q}_k\} \cup Q_k$.
  $k \leftarrow k + 1$ and repeat

1.3 Without explicitly computing the Minkowski difference

Instead of explicitly computing $C = A \ominus B$ which requires a significant computational cost in general, you can only sample the Minkowski difference point set by using a support mapping function of the Minkowski difference. Then use the fact that

$$s_{A\ominus B}(\mathbf{v}) = s_A(\mathbf{v}) - s_B(-\mathbf{v}).$$

For more details, see [2, Sec.9.5].

References

[1] C. Ericson. The Gilbert-Johnson-Keerthi (GJK) algorithm. SIGGRAPH Presentation, Sony Computer Entertainment America, 2004.

[2] C. Ericson. *Real-Time Collision Detection*. Elsevier, San Francisco, CA, USA, 2005.