LowPassFilter

November 28, 2022

[ ]:
```python
# Packages and adjustments to the figures
from scipy import signal
import matplotlib.pyplot as plt
#plt.rcParams.update(plt.rcParamsDefault)
import numpy as np
import math
#plt.rcParams["figure.figsize"] = 10,5
#plt.rcParams["font.size"] = 12
#plt.rcParams.update({"text.usetex": True,"font.family": "sans-serif","font.
 ↪sans-serif": ["Helvetica"]})
```

0.1 1. Generate a test signal

- A simple test signal $y = \{y_i\}$ is generated with a fixed sampling frequency using the function:

$$y(t) = m_0 \sin(2\pi f_0 t) + m_1 \sin(2\pi f_1 t)$$

- The power spectrum is plotted as the magnitude of the discrete fourier transform (DFT): $|\hat{y}|$

[ ]:
```python
# Generate a signal
samplingFreq = 1000; # sampled at 1 kHz = 1000 samples / second
tlims = [0,1]        # in seconds
signalFreq = [2,50]; # Cycles / second
signalMag = [1,0.2]; # magnitude of each sine
t = np.linspace(tlims[0],tlims[1],(tlims[1]-tlims[0])*samplingFreq)
y = signalMag[0]*np.sin(2*math.pi*signalFreq[0]*t) + signalMag[1]*np.sin(2*math.
 ↪pi*signalFreq[1]*t)

# Compute the Fourier transform
yhat = np.fft.fft(y);
fcycles = np.fft.fftfreq(len(t),d=1.0/samplingFreq); # the frequencies in␣
 ↪cycles/s

# Plot the signal
plt.figure()
plt.plot(t,y);
plt.ylabel("$y(t)$");
```

```
plt.xlabel("$t$ (s)");
plt.xlim([min(t),max(t)]);

# Plot the power spectrum
plt.figure()
plt.plot(fcycles,np.absolute(yhat));
plt.xlim([-100,100]);
plt.xlabel("$\omega$ (cycles/s)");
plt.ylabel("$|\hat{y}|$");
```

0.2 2. Low-pass filter transfer function

- A cutoff frequency is selected and the transfer function for the low-pass filter is computed using signal.TransferFunction
- The low-pass filter transfer function is

$$H(s) = \frac{\omega_0}{s + \omega_0}$$

- The Bode plot shows the frequency response of H by plotting the magnitude and phase of the frequency response
- Low frequencies are not attenuated (this is the *pass band*)
- High frequencies are attenutated (this is the *stop band*)

numpy.logspace(start, stop, num=50, endpoint=True, base=10.0, dtype=None, axis=0)

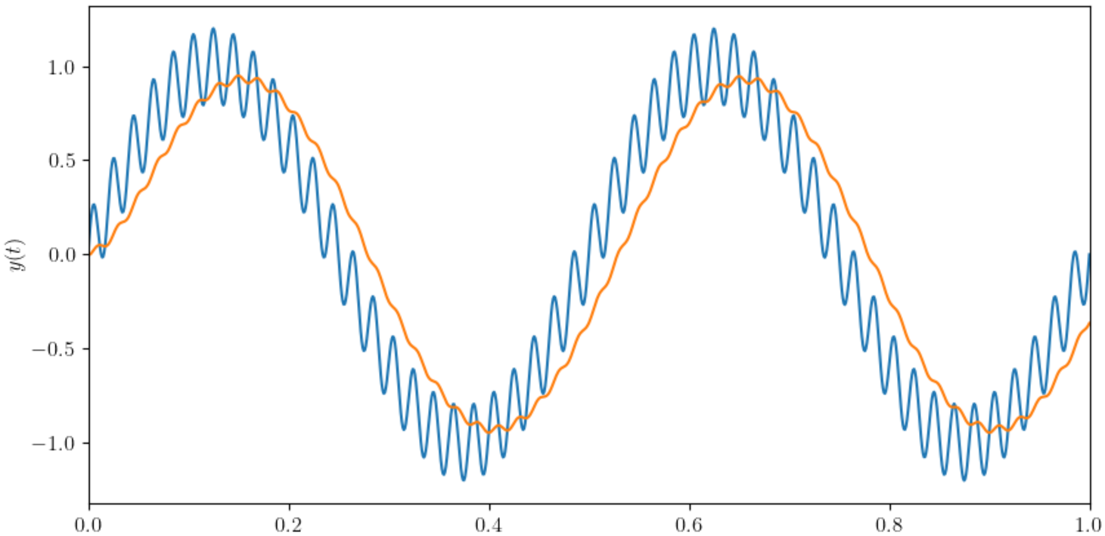
Return numbers spaced evenly on a log scale.

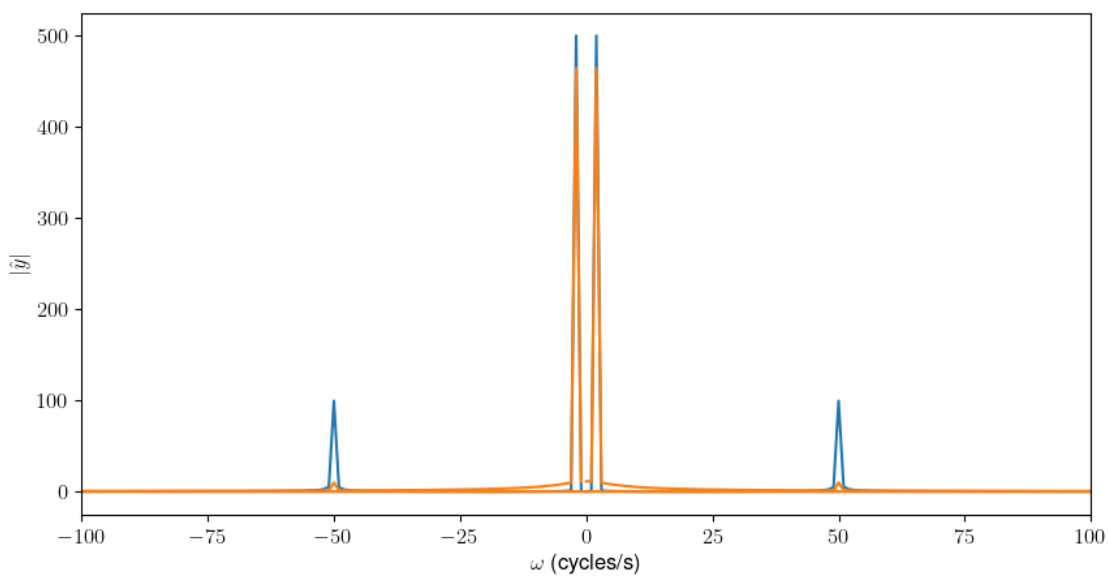
np.log10(min(signalFreq)*2*np.pi/10)

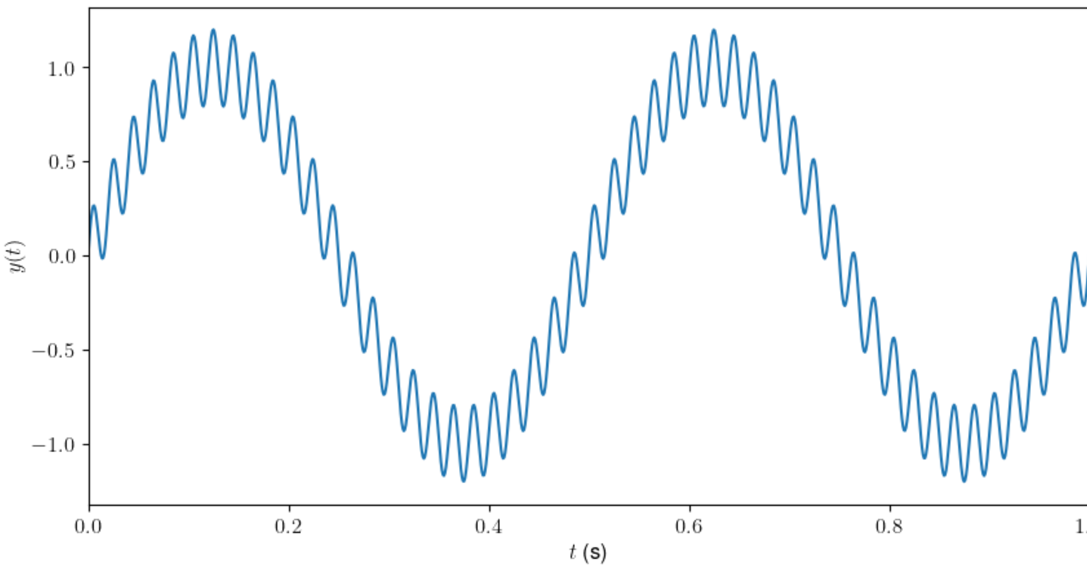
$$log_{10}(min(\frac{F_{signal} * 2 * \pi}{10})$$

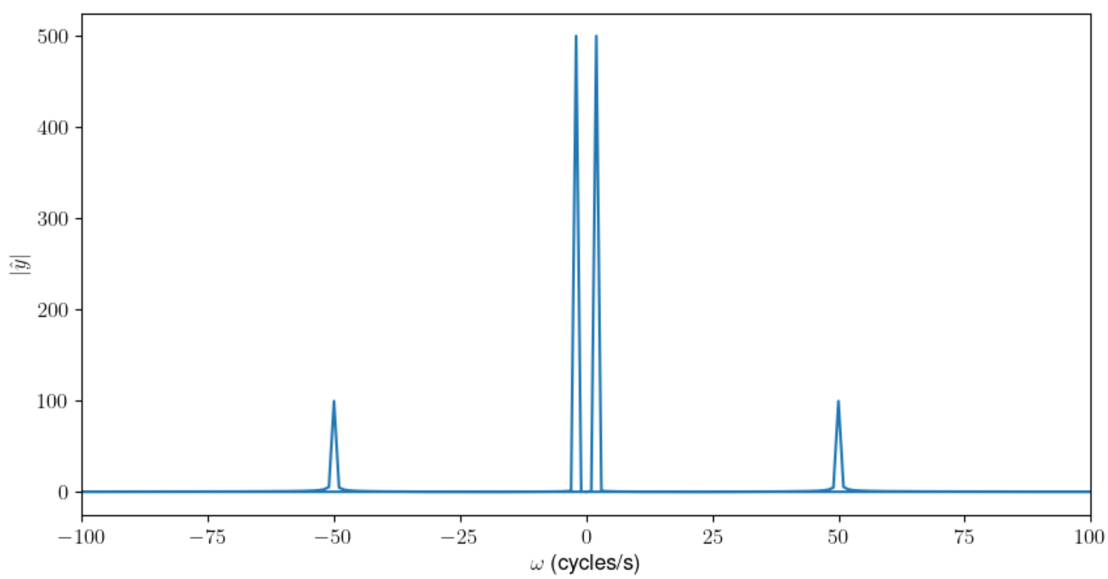
[ ]:
```
# Low-pass filter
w0 = 2*np.pi*5; # pole frequency (rad/s)
num = w0        # transfer function numerator coefficients
den = [1,w0]    # transfer function denominator coefficients
lowPass = signal.TransferFunction(num,den) # Transfer function

# Generate the bode plot
w = np.logspace( np.log10(min(signalFreq)*2*np.pi/10), np.
 ↪log10(max(signalFreq)*2*np.pi*10), 500 )
w, mag, phase = signal.bode(lowPass,w)

# Magnitude plot
plt.figure()
plt.semilogx(w, mag)
for sf in signalFreq:
    plt.semilogx([sf*2*np.pi,sf*2*np.pi],[min(mag),max(mag)],'k:')
plt.ylabel("Magnitude ($dB$)")
```

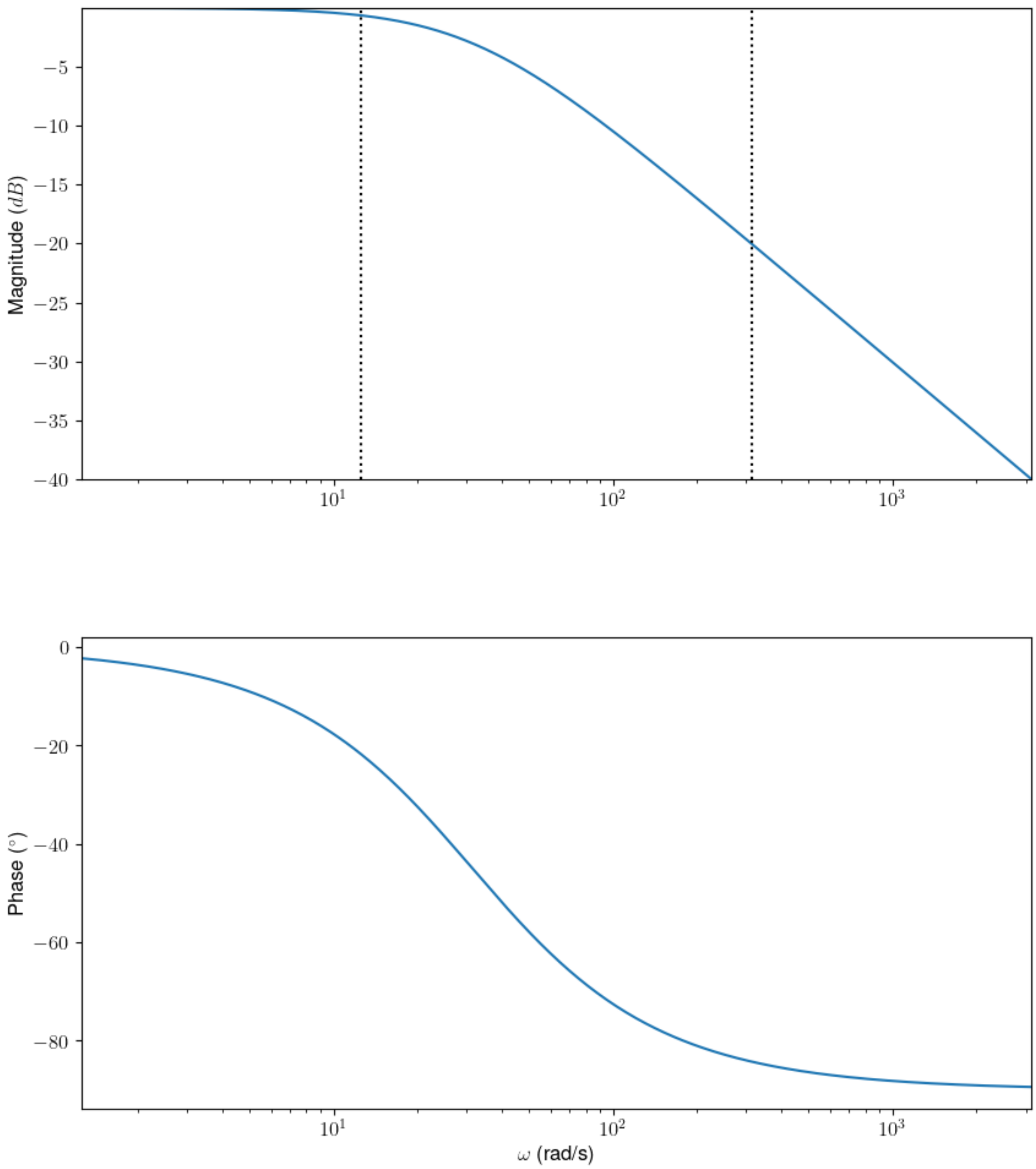
```python
plt.xlim([min(w),max(w)])
plt.ylim([min(mag),max(mag)])

# Phase plot
plt.figure()
plt.semilogx(w, phase)  # Bode phase plot
plt.ylabel("Phase ($^\circ$)")
plt.xlabel("$\omega$ (rad/s)")
plt.xlim([min(w),max(w)])
plt.show()
```






1.0
0.5
0.0
−0.5
−1.0
y(t)
0.0
0.2
0.4
0.6
0.8
1.0
t (s)


500
400
300
200
100
0
|ŷ|
−100
−75
−50
−25
0
25
50
75
100
ω (cycles/s)



0.3 3. Discrete transfer function

To implement the low-pass filter on hardware, you need to compute the discrete transfer function using the signal's sampling frequency. * The time step is $\Delta t = 1/f_s$ * Computing the discrete transfer function using Tustin's method, set $s = \frac{2}{\Delta t}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$, so

$$H(z) = \frac{\omega_0}{\frac{2}{\Delta t}\frac{1-z^{-1}}{1+z^{-1}} + \omega_0} = \frac{\Delta t \omega_0 (z+1)}{(\Delta t \omega_0 + 2)z + \Delta t \omega_0 - 2}$$

* You don't have to compute it by hand. The to_discrete method is used to compute the bilinear transform (Tustin's method)

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```
dt = 1.0/samplingFreq;
discreteLowPass = lowPass.to_discrete(dt,method='gbt',alpha=0.5)
print(discreteLowPass)
```

```
TransferFunctionDiscrete(
array([0.01546504, 0.01546504]),
array([ 1.        , -0.96906992]),
dt: 0.001
)
```

0.4 4. Filter coefficients

We want to find the filter coefficients for the discrete update:

$$y[n] = a_1 y[n-1] + a_2 y[n-2] + ... + b_0 x[n] + b_1 x[n-1] + ...$$

The coefficients can be taken directly from the discrete transfer function of the filter in the form:

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + ...}{1 - a_1 z^{-1} - a_2 z^{-2} + ...}$$

(This is a result of taking the Z-transform which is not shown here)

Compare this to a transfer function with coefficients num = [b_0, b_1, b_2] den = [1, a_1, a_2] is

$$H(z) = \frac{b_0 z^2 + b_1 z + b_2}{z^2 + a_1 z + a_2}$$

which is equivalent to

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

So you can take the coefficients in the same order that they are defined in the numerator and denominator of the transfer function object. The only difference is that the **coefficients in the denominator need a negative sign**.

- To filter the signal, apply the filter using the discrete update
- The filtered signal and filtered signal power spectrum are plotted alongside the unfiltered signal

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```
# The coefficients from the discrete form of the filter transfer function (but␣
↪with a negative sign)
b = discreteLowPass.num;
a = -discreteLowPass.den;
print("Filter coefficients b_i: " + str(b))
print("Filter coefficients a_i: " + str(a[1:]))

# Filter the signal
```

```python
yfilt = np.zeros(len(y));
for i in range(3,len(y)):
    yfilt[i] = a[1]*yfilt[i-1] + b[0]*y[i] + b[1]*y[i-1];

# Plot the signal
plt.figure()
plt.plot(t,y);
plt.plot(t,yfilt);
plt.ylabel("$y(t)$")
plt.xlim([min(t),max(t)]);

# Generate Fourier transform
yfilthat = np.fft.fft(yfilt)
fcycles = np.fft.fftfreq(len(t),d=1.0/samplingFreq)

plt.figure()
plt.plot(fcycles,np.absolute(yhat));
plt.plot(fcycles,np.absolute(yfilthat));
plt.xlim([-100,100]);
plt.xlabel("$\omega$ (cycles/s)");
plt.ylabel("$|\hat{y}|$");
```

```
Filter coefficients b_i: [0.01546504 0.01546504]
Filter coefficients a_i: [0.96906992]
```

[ ]:

```python

```