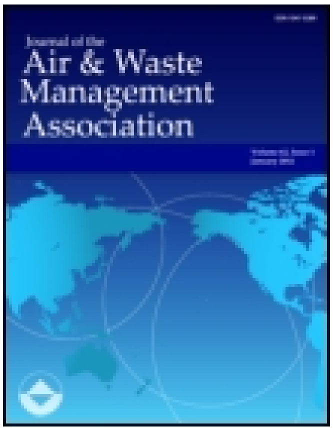

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The Validity of Several Plume Rise Formulas

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The Validity of Several Plume Rise Formulas

The ground level concentration of pollutants downwind of a tall chimney decreases as the effective height of the stack increases. The effective height of the stack is the actual height plus the rise of the plume centerline due to momentum and buoyancy of the effluent. Over twenty formulas to predict plume rise from stack and meteorological parameters have been proposed; none is uniformly accepted. In this paper, 711 plume rise observations were used to test the ability of fifteen of the published and commonly used formulas to predict plume rise. The plume rise data were obtained from single stacks whose heat emission rate varied over four orders of magnitude. None of the formulas tested was found to be significantly better than the others. Research was performed under the auspices of the U.S. Atomic Energy Commission.

Very tall stacks are an increasingly used tool for reducing ground level concentrations downwind from large pollution sources, such as coal and oil fueled electric power plants. The reason is that, considering current techniques for removing sulfur and other noxious materials from the fuel either before or after burning and despite the cost of construction, a very tall stack is the least expensive and most effective technique now available.

The reasons for the reduced ground level concentrations are the greater wind speed at the higher elevations, a longer time for mixing to take place before the plume diffuses to the ground, and the change in the character of turbulence at the higher elevations. Smoke emitted at such altitudes is above the nocturnal inversion and thus is effectively prevented from reaching the ground at night. During the day, the plume is generally above the highly unstable surface layer which frequently contains large eddies (looping) that bring momentary high concentrations to ground level.

Most formulas predict that ground level concentrations from stacks is inversely proportional to the square of the effective stack height (true height plus plume rise). Thus, plume rise is an important consideration in stack design. Plume rise is usually defined as the height above the stack orifice that the plume centerline rises due to the momentum and buoyancy of the stack gases.

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The state of the art of predicting height-of-rise from meteorological and stack data at present is quite poor. The number of formulas for calculating plume rise appears to vary inversely with our understanding of the processes involved: over twenty such formulas have been proposed and new ones appear annually. Some are based on theoretical considerations including dimensional analysis, others on strictly empirical grounds. None is universally accepted or used.

Factors Influencing Plume Rise

Three sets of parameters control the behavior of a smoke plume injected into the atmosphere from a stack. These are stack engineering factors, meteorological conditions, and the nature of the effluent itself. Stack design parameters which influence plume rise include speed and temperature of the effluent, terrain and buildings near the stack (which create mechanical turbulence and whose effect usually varies with wind direction), local heat sources which create convective turbulence, the shape of the stack, and the number of stacks in the area. Stone and Clarke[1] also include stack height as a parameter. Atmospheric turbulence controls both the rate of mixing of the plume with outside air and the motion of the plume before and after mixing has reduced the buoyancy and momentum of the effluent to near zero. Wind speed, terrain roughness, and stability are the primary factors determining the intensity and spectrum of turbulence.

Some of the formulas for predicting plume rise contain two terms, one for momentum of the effluent, the other proportional to the heat content or buoyancy of the plume raised to some power. In many of the formulas, the momentum term is not included as a separate entity, but its effect is included in the coefficients for the heat term. Momentum and heat content are, of course, not independent for a given stack.

If stack velocity, V_s, is small compared with wind speed, U, there will be downwash behind the stack. If V_s is very large, the rate of mixing with ambient air will be increased, resulting in a loss in buoyancy and consequently a reduction in plume rise. Thus, plume rise will reach a maximum for some intermediate value of V_s; Stone and Clarke[1] report this to be about 25 m/sec.

The major area of disagreement in the literature is the power to which the heat term should be raised; values from 0.25 to 1.00 have been published. Investigators also are not in agreement as to whether the parameter to be used is the temperature difference between the plume and ambient air (buoyancy) or the heat content of the effluent.

The meteorological parameters which influence plume rise are the horizontal wind speed and stability, as denoted by the vertical temperature or potential temperature gradient. Stability does not enter most equations as a variable, but different formulas may be indicated for various stability classes. Wind speed, raised to some power, appears in all of the formulas; the power varies from $U^{-1/3}$ to U^{-3}, but is usually U^{-1}. Also of importance is wind direction if the terrain surrounding the stack is not uniform. Other factors influencing plume rise through their effect on turbulence intensity are the intensity of solar radiation and the distribution and type of cloudiness. Even over uniform, smooth terrain, cumulus clouds cause irregular ground heating which creates large thermal eddies with appreciable vertical components. Under such conditions, plumes from tall stacks may be brought to the ground.

Table I. Summary of data sources.

	Stable	Neutral	Unstable	Total
Argonne I	20	10	6	36
Argonne II	26	6	102	134
Harwell	8	27	11	46
Duisburg	51	328		379
Gernsheim	10	33		43
Bosanquet		7		7
Tennessee Valley Authority				
Widow's Creek	1	9		10
Gallatin	2	24		26
Paradise	4	26		30
Totals	122	470	119	711

Data Used in Study

A comprehensive set of data were used to carry out this comparison study relating plume rise with stack and meteorological variables. A total of 711 plume rise observations, obtained from ten stacks with plume rises and heat emission rates ranging over three to four orders of magnitude respectively, were used. Only data from single stacks were used.

Table I presents a summary of the data sources used in this report. Two sets of plume rise observations were obtained from the small Argonne Meteorology Stack (Moses and Strom[2]; Carson and Moses[3]). Data from three industrial stacks in England (Bosanquet, *et al.*[4]), one stack at Harwell, England (Stewart, *et al.*[5]), two electrical generating stacks in Germany (Rauch[6]), and three power stacks of the Tennessee Valley Authority (unpublished data) were also used.

There is an appreciable nonuniformity among the data from these stacks. For example, averaging times of the plume rise measurements differed from 1.5 to 2.5 minutes for the German data, 4 minutes for Argonne, and up to 120 minutes for the Tennessee Valley Authority. Plume rise definitions and techniques of measurement varied; wind measuring levels and procedures were not always comparable. For this study, all winds were corrected to represent the wind speed at the top of the stack. Table II shows the ranges of meteorological and stack parameters which prevailed during data acquisition. There is also a lack of agreement on techniques for measuring plume rise and associated meteorological variables. Most plume observations are made optically (photographic or by eye), a few with samplers. Optical considerations, such as the contrast of the plume against the sky and horizon, length of plume observed or photographed, visibility, cloudiness, etc., place serious limitations on observing procedures. Very few plume observations at night have been made. Since plumes under most weather conditions meander, most data refer to some type of time-averaged position of the plume. There is little uniformity between the various sets of data on how such average positions were determined. The lack of standardized observing techniques, of course, reduces the comparability of the various sets of plume rise data. Optical and other considerations usually do not permit direct measurement of the final plume rise.

To facilitate comparisons, a uniform set of symbols, using meters, grams, seconds, kilocalories, and degrees K, was adapted:

Δh_0 = observed plume rise, m
Δh_c = calculated plume rise, m
V_s = stack exit velocity, m/s
d = stack diameter, m
U = mean wind speed at top of stack, m/s
Q_h = heat emission rate, kilocalories/sec
H_s = height of stack, m
T_a = air temperature, °K
θ = potential temperature, °K
T_s = stack gas temperature, °K
L = horizontal distance from point of emission
$C_1, C_2, C_3, C_4, K, y, z$ = regression coefficients
F = buoyancy flux parameter (Csanady[7]; Briggs[8])
S = stability parameter (Csanady; Briggs)

Method of Comparison

Fifteen plume rise formulas, described in the next section, were used in this study. The appropriate stack and meteorological data were inserted into each of the formulas, and the plume rise computed for each of the 711 observations. The data were stratified into three stability classes: stable if $\partial\theta/\partial z \geq 0.85$°K/100 m; unstable if $\partial\theta/\partial z < -0.22$°K/100 m; neutral if between. Certain of the formulas tested are indicated as applicable for all stability classes; these were tested using all observations. Others were tested using only those data in the appropriate stability class.

The standard error of estimate for each formula,

$$\text{S.E.} = \left[\frac{\Sigma(\Delta h_0 - \Delta h_c)^2}{(N-1)}\right]^{1/2}, \quad (1)$$

was then computed. This parameter was used as the index of goodness of fit of the formula and is called S.E.#1 in Tables III and IV.

The ability of a formula to fit the observations can be improved if the heights calculated by a given equation

Table II. Ranges of stack and meteorological parameters.

Station	Wind Speed m/sec	Stack Effluent Vel. m/sec	Heat Emission Rate kilocal/sec	Plume Rise m	Stack Height m	Stack Dia. m
Argonne I	7.3 - 1.0	13.2 - 4.7	12.7 - 2.0	16.9 - 1.4	34	0.44
Argonne II	10.3 - 3.1	14.0 - 4.56	34.4 - 3.3	34.3 - 0.2	34	0.44
Harwell	11.7 - 3.0	9.90	1215	116.0 - 18.0	61	3.46
Duisburg	10.5 - 1.6	12.1 - 3.5	2475 - 979	112.0 - 9.0	125	3.50
Gernsheim	9.2 - 1.5	5.3 - 2.2	1003 - 384	78.0 - 21.0	75	2.30
Bosanquet	10.0 - 3.1	10.8 - 8.5	1530 - 72	61.0 - 14.0	?	2.5,1.25
Tennessee Valley Authority						
Widow's Creek	6.6 - 1.5	24.5 - 22.9	$1.79 - 1.57 \times 10^4$	356 - 73	153	6.34
Gallatin	8.2 - 1.6	16.4 - 14.8	$1.77 - 1.50 \times 10^4$	476 - 81	153	7.63
Paradise	11.3 - 2.3	19.2 - 15.3	$2.46 - 1.70 \times 10^4$	457 - 64	183	7.93

are multiplied by a "correction factor" A. The standard error of estimate of the altered equation,

$$\text{S.E.} = \left[\frac{\Sigma(\Delta h_0 - A\,\Delta h_c)^2}{(N-1)}\right]^{1/2}, \quad (2)$$

is a minimum if A is defined as

$$A = \left[\frac{\Sigma(\Delta h_0)(\Delta h_c)}{\Sigma(\Delta h_c)^2}\right]. \quad (3)$$

The value of A for each formula and the reduced standard error (S.E.#2) are also listed in Tables III and IV. Note that the "A factor" is not identical to Stümke's K (Stümke[9]).

It would be tempting to say that the formula giving the smallest standard error of estimate is the best. Considering the quality of the basic data, it would be premature to make such a statement. More and better data, plus more refined analyses, are required before even a statement that plume rise depends on a specific power of Q_h or $(T_s - T_a)/T_a$ can be made with certainty.

Other studies comparing observed and calculated plume rises have been made by (1) Moses and Strom,[2] (2) Rauch,[6] (3) Stümke,[9,10] (4) Moses, Carson, and Strom,[11] (5) the CONCAWE Working Group (Brummage, *et al.*[12]), and (6) Carpenter, *et al.*[13]

Table III. Summary of plume rise formula comparison all data.

Formula	S.E.#1	A Factor	S.E.#2	$\overline{\Delta h_c}$
Holland (4)	39.83	0.81	33.00	35.24
Stümke (5)	53.64	0.61	29.53	68.24
CONCAWE (7)	40.66	0.71	26.89	63.61
CONCAWE Simplified (8)	37.86	0.73	29.91	65.24
Lucas-Moore-Spurr (9)	133.25	0.35	36.14	151.96
Rauch (10)	36.14	1.00	36.14	53.13
Stone-Clarke (11)	121.73	0.37	34.70	140.11
Carson-Moses 1967 (13)	35.83	0.79	27.49	50.54
Moses-Carson 1968 (17)	29.66	1.00	29.66	43.08
Briggs Transitional (21)	80.00	0.49	25.73	76.92
Csanady (24)	3742.17	0.012	57.36	662.81

S.E.#1 = standard error of formula, meters (Equation 1).
S.E.#2 = standard error of formula with "A factor," meters (Equation 2).
$\overline{\Delta h_0}$ = 48.46 m = observed mean plume rise
$\overline{\Delta h_c}$ = calculated average plume rise, m
Sample size = 711.
Standard deviation of sample = 60.95 m.

The Plume Rise Equations

Listed below are the plume rise equations used in this comparison study. A consistent notation system is used.

The Holland Formula

Holland[14] presented the following empirical equation for predicting plume rise:

$$\Delta h_c = \left[\frac{1.5V_s d + 0.04Q_h}{U}\right] \quad (4)$$

This expression is based on wind tunnel tests and data from three stacks, two operated by Oak Ridge National Laboratory and one by the Tennessee Valley Authority. This formula contains both momentum and heat flux terms. Since the height of plume rise also depends on stability, Holland suggested that 10 to 20% of the rise given by the equation be added for unstable conditions and an equal amount subtracted for inversions. In this paper, 20% corrections have been applied.

This and many other equations have been criticized on the grounds that they are empirical. It should be remembered that empirical equations cannot be used for conditions outside those used in their formulation. Stümke[9] concluded that this equation, if multiplied by 2.92, gave the best fit of all formulas tested.

The Stümke Formula

Stümke[10] later introduced an empirical modification of the Holland formula:

$$\Delta h_c = \frac{1}{U} \times \left[1.5V_s d + 65d^{3/2}\left(\frac{T_a - T_s}{T_s}\right)^{1/4}\right]. \quad (5)$$

The momentum term was not changed, but the thermal term depends on the temperature excess of the effluent to the one-fourth power.

The CONCAWE Formula

The CONCAWE Working Group on Stack Height and Atmospheric Dispersion made an analysis (using 438 plume rise observations) to fit an equation of the form

$$\Delta h_c = \left[K\frac{Q_h^{x}}{U^{y}}\right], \quad (6)$$

where K, x, and y were determined empirically by regression techniques (Brummage, *et al.*[12]). The resulting equation is:

$$\Delta h_c = \left[2.58\frac{Q_h^{0.58}}{(U)^{0.70}}\right]. \quad (7)$$

The CONCAWE Simplified

The above regression equation is rather unwieldy. For this reason, an equation of the form

$$\Delta h_c = \left[K\frac{Q_h^{1/2}}{U^{3/4}}\right] \quad (8)$$

was postulated. The value of K, derived from regression techniques, was 5.53 (Brummage, *et al.*[12]).

The Lucas-Moore-Spurr Formula

These authors[15] showed that plume rise measurements from two United Kingdom power plants could be reconciled with the theoretical results of Priestley.[16] Using the value K for neutral conditions, this formula becomes

$$\Delta h_c = \left[135\frac{Q_h^{1/4}}{U}\right]. \quad (9)$$

The Rauch Formula

Rauch[6] proposed a plume rise equation of the same form as the Priestley and Lucas-Moore-Spurr expressions, with the value of K computed from observations made at two stacks in Germany. His equation is

$$\Delta h_c = \left[47.2\frac{Q_h^{1/4}}{U}\right]. \quad (10)$$

Rauch's value of K is much smaller than that proposed by the British authors.

The Stone-Clarke Formula

Plume rise studies conducted by the Central Electricity Generating Board in England show that plume rise depends in part on the height of the stack itself. The higher the stack, the greater the rise due to the decrease in turbulence intensity and slower mixing, especially above 100 m (Stone and Clarke[1]). They proposed the following, a modification of the Lucas-Moore-Spurr formula:

$$\Delta h_c = \left[(104.2 + 0.171H_s)\frac{Q_h^{1/4}}{U}\right]. \quad (11)$$

Stone and Clarke emphasized that this is a tentative formula, and that the coefficient for H_s may be underestimated. They also state that exit speed and chimney diameter should enter the expression.

The Carson-Moses 1967 All-data Formulas

Carson and Moses[17] used multiple regression techniques to determine the coefficients in a plume rise equation containing both momentum and heat flux terms:

Table IV. Summary of plume rise formula comparison data stratified by stability classes.

Formula - Stability	S.E.#1	A Factor	S.E.#2	$\overline{\Delta h_c}$	$\overline{\Delta h_0}$
Holland					
Unstable (4) + 20%	20.22	4.17	9.67	3.3	15.2
Neutral (4)	38.33	0.89	37.06	45.0	60.1
Stable (4) − 20%	41.97	0.64	31.42	23.7	35.9
Carson-Moses 1967 for specific stability classes					
Unstable (14)	11.47	0.77	9.19	19.5	
Neutral (15)	33.24	0.88	30.95	62.0	
Stable (16)	42.97	0.60	23.78	41.5	
Moses-Carson 1968 for specific stability classes					
Unstable (18)	9.07	1.00	9.07	19.5	
Neutral (19)	30.25	1.00	30.25	57.8	
Stable (20)	23.35	1.00	23.35	26.4	
Briggs Transitional					
Unstable (21)	10.34	1.09	10.18	10.1	
Neutral (21)	86.03	0.51	28.30	97.4	
Stable (21)	94.12	0.37	26.24	63.3	
Briggs stable windy					
Stable (22)	31.38	0.72	29.93	56.9	
Briggs neutral					
Neutral (23)	5043.22	0.011	67.40	1127.7	

$$\Delta h_c = \left[C_1 d \left(\frac{V_s}{U}\right)^{C_2} + C_3 \frac{(Q_h{}^{C_4})}{U} \right], \tag{12}$$

where C_1, C_2, C_3, and C_4 are constants determined by least-square regression techniques. After experimentation with the constants, it was decided to let $C_2 = 1$ and $C_4 = 0.5$ for all data. C_1 and C_3 were then determined by regression analysis.

A total of 752 plume rise observations, all that could be found in the literature, were used. The resulting equation, using data from all stability classes, is

$$\Delta h_c = \left[4.12 \frac{V_s d}{U} + 2.92 \frac{Q_h{}^{1/2}}{U} \right]. \tag{13}$$

The Carson-Moses 1967 Formulas for Stability Classes

If the data are separated into stability classes before the regression coefficients are computed, the resulting equations are

Unstable

$$\Delta h_c = \left[-1.28 \frac{V_s d}{U} + 15.65 \frac{Q_h{}^{1/2}}{U} \right], \tag{14}$$

Neutral

$$\Delta h_c = \left[3.72 \frac{V_s d}{U} + 3.15 \frac{Q_h{}^{1/2}}{U} \right], \tag{15}$$

Stable

$$\Delta h_c = \left[1.22 \frac{V_s d}{U} + 4.90 \frac{Q_h{}^{1/2}}{U} \right]. \tag{16}$$

The Moses-Carson 1968 All-data Formula

In our 1967 study, the Tennessee Valley Authority plants at Colbert, Gallatin, and Paradise furnished the bulk of the data with large heat emission rates, a total of 107 observations. More recently, these data have been considerably reworked by the Tennessee Valley Authority investigators. In particular, the averaging periods have been changed (generally made longer) and the values of Q_h have been reduced by a factor of two or more. Additional observations were included.

It was therefore decided to recompute the regression equations using the new, improved data provided by the Tennessee Valley Authority. In addition, it was decided to use only those observations made with a single stack in operation, as the effect on plume rise of two or more nearby stacks is not known. This change in input data reduced our Tennessee Valley Authority data to 66 observations, and the entire set to 711.

As expected, the lowering of the Q_h values for these data had a marked effect on the regression coefficients. The resulting equation for data from all stability classes is (Moses and Carson[18]):

$$\Delta h_c = \left[-0.029 \frac{V_s d}{U} + 5.35 \frac{Q_h{}^{1/2}}{U} \right]. \tag{17}$$

The negative sign for the momentum term is at first disturbing. This result was arrived at by regression computation and implies that a plume with no heat content ($Q_h = 0$) would fall. This is not a proper application of the equation, since no empirical equation should be used outside its range of formulation.

The Moses-Carson 1968 Formulas for Stability Classes

This revised set of data were then subdivided into the three stability classifications and regression equations calculated for each class. The results are

Unstable

$$\Delta h_c = \left[3.47 \frac{V_s d}{U} + 10.53 \frac{(Q_h)^{1/2}}{U} \right], \tag{18}$$

Neutral

$$\Delta h_c = \left[0.35 \frac{V_s d}{U} + 5.41 \frac{(Q_h)^{1/2}}{U} \right], \tag{19}$$

Stable

$$\Delta h_c = \left[-1.04 \frac{V_s d}{U} + 4.58 \frac{(Q_h)^{1/2}}{U} \right]. \tag{20}$$

The Briggs Transitional Case

Briggs[8] by dimensional analysis techniques suggested the following formula for the rise of a bent-over plume:

$$\Delta h_c = \left[\frac{F^{1/3} L^{2/3}}{U} \right] \tag{21}$$

where F is the buoyancy flux ($=0.038 Q_h$) and L is the distance downwind of the stack at which Δh_0 is measured. Note that plume rise is proportional to $Q_h{}^{1/3}$ and U^{-1}.

Briggs Windy Stable Formula

Again by dimensional analysis, Briggs[8] suggested that plume rise in a stable atmosphere is given by:

$$\Delta h_c = \left[2.6 \left(\frac{F}{US}\right)^{1/3} \right], \tag{22}$$

where S is a stability parameter $= [(g/T_a)x(\partial\theta/\partial z)]$.

Note that plume rise is again proportional to $Q_h{}^{1/3}$, but now to $U^{-1/3}$.

Briggs Neutral Windy Formula

In a neutral atmosphere, mixing of stack effluent with ambient air will never reduce buoyancy to zero. Therefore, the plume will continue to rise. Briggs[8] suggested that the final plume rise would be greater than $(400F/U^3)$.

In this paper, using his suggested relationship, the formula tested was

$$\Delta h_c = \left[400 \frac{F}{U^3} \right]. \tag{23}$$

In a neutral atmosphere, this equation states that plume rise is proportional to Q_h and U^{-3}.

Csanady Formula

Csanady[7], also using dimensional analysis and data from stacks in Canada, suggested

$$\Delta h_c = \left[250 \frac{F}{U^3} \right]. \tag{24}$$

This formula, except for the nondimensional constant, is similar to Briggs' neutral windy case.

Results of Comparison

Tables III and IV summarize the results of these computations. Table III lists the standard errors of estimate of the various plume rise equations for those equations indicated as applying without regard to stability. Table IV lists the standard error of estimate when the data are stratified into stability classes. Also listed in both tables are the A-factors, the standard error of estimate of the "corrected" equation, and the average calculated plume rise. The average observed plume rise for all data was 48.46 m; the observed standard deviation was 60.95 m. It should be pointed out that the A-factor is not a multiplying constant whose properties are to make the average calculated plume rise, $\overline{\Delta h_0}$, equal the average observed, $\overline{\Delta h_c}$.

Statistically, the Moses and Carson 1967 and 1968 formulas should not be included in this comparison. The 1968 equations were derived by regression techniques using the same set of observations as in this comparison. 645 of the 752 observations used to compute 1967 formulas were included in this test.

Table III shows that the Holland, CONCAWE simplified and Rauch formulas all had standard errors below 40 m. A comparison of the Rauch formula results with those of the Lucas-Moore-Spurr and Stone-Clarke relations indicate the coefficients used in England are too large by a factor of three. An examination of the detailed calculations show that formulas with plume rise proportional to U^{-3} (Briggs neutral windy formula and the Csanady equation) yield computed heights that are much too high under light wind conditions. It is somewhat unexpected to find the CONCAWE simplified formula is slightly better than the CONCAWE standard relation.

Of interest is that none of the A-factors is greater than one, and that all of the formulas except the Moses-Carson 1968 and Holland overestimate the average plume rise. The Briggs transitional formula with the A-factor has the smallest standard error of any equation tested; however, it considerably overestimates plume rise when used as published.

When the data are separated according to stability, and tested by means of formulas indicated as valid for that class, the Moses-Carson 1968 equations are superior, with the Briggs transitional formulas with the A-factor next.

By far the best set of plume rise data available are those provided by the Tennessee Valley Authority. Our calculations, using data from this source only, show that the Moses-Carson 1968 all-data formula yielded the lowest standard error of any equation tested, and somewhat better than the Carson-Moses 1967 relation.

Conclusions

No plume rise equation can be expected to accurately predict short-term plume rise. In this comparison, plume rises observed for periods of four minutes and less comprised the bulk of data. The large variation of plume rise from one averaging period to the next reveals the presence of eddies with much longer periods. It is planned to continue this study after a detailed re-examination of the basic plume rise observations.

Two of the more important questions in plume rise calculations were not resolved in this study: the power to which the heat flux term should be raised and whether or not a separate term for momentum is needed. On the basis of these calculations, formulas with Q_h raised to the 0.5 power were slightly superior. What is clearly shown is that the quality of the presently available plume rise data is simply inadequate; more and better data are needed.

A study of Tables III and IV, plus data not included in this report, indicates that none of the equations is markedly superior and can be recommended for universal use.

From the standpoint of goodness of fit and ease of computation, one is included to suggest

$$\Delta h_c = A\left[-0.029\,\frac{V_s d}{U} + 5.35\,\frac{(Q_h)^{1/2}}{U}\right] \qquad (25)$$

as the preferred plume rise equation. Corrections for stability are made by using the A factors of 2.65, 1.08, and 0.68 for unstable, neutral, and stable conditions respectively (Moses and Carson[18]).

It should be emphasized that no equation is designed for day-to-day operation with a single stack. It is to be used for general design considerations. As more data are accumulated, it is quite possible that the regression coefficients will change. Since the equation recommended is an empirical one, it is important to caution that it only be used over the range of variables upon which it is derived. It is also essential to realize that these results apply to relatively smooth terrain without the undue influence of buildings. The equation, however, does represent a least-squares fit of the available data that should serve as a useful guide in stack design.

It is believed that many plume rise observations are available that have not been included in this analysis. The authors would greatly appreciate receiving listings of these plume rise data for inclusion in future investigations.

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