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MODELLING SLOPE, WIND AND MOISTURE CONTENT EFFECTS ON FIRE SPREAD

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Abstract. *A numerical method is developed for fire spread simulation modelling. The two dimensional model presented takes into account moisture content, radiation, wind and slope effects, which are by far the most important mechanisms in fire spread. We consider the combustion of a porous solid, where the energy conservation equation is applied. The influence of the moisture content and eventually heat absorption by pyrolysis, can be represented as two free boundaries, and is treated in this work using a multivalued operator representing the enthalpy. The maximal monotone property of this operator allows the implementation of a numerical algorithm with good convergence properties.*

1 INTRODUCTION

Many existing physical models for fire spread in porous fuel bed use the principle of energy conservation applied to the preheated fuel. Generally, radiation is considered as the dominant mechanism of the fuel preheating. On the other hand slope and wind effects as well as the initial vegetation moisture have to be taken into account in order to obtain reliable rates of fire spread. Physical models from fundamental conservation equations and complex physics have been developed [1]. These valuable approaches are computationally expensive and too slow to be used in real time mode, even with fast and parallel processing. Besides, several works have appeared recently where one or two dimensional physical models are considered in order to simulate fire spread in small computers, with moderate simulation times, see for example [2], [3], [4]. This paper is a contribution to generally applicable models of fire spread through fuel beds, by means of simple models, but taking into account non local radiation, moisture content and wind and slope effects. Particularly the influence of the moisture content and eventually heat absorption by pyrolysis, can be represented as two free boundaries, and are treated in this paper using a multivalued operator representing the enthalpy. The maximal monotone property of this operator allows the implementation of a numerical algorithm with well-known convergence properties.

2 GOVERNING EQUATIONS

The non dimensional equations governing the fire spread in a region Ω with boundary Γ are:

$$\partial_t e + w.\nabla e - \kappa\Delta u + \alpha u = R(u, y), \tag{1}$$

$$e \in G(u), \tag{2}$$

$$\partial_t y = -g(u)y. \tag{3}$$

The boundary and initial conditions are given, by

$$u(x, t) = 0 \quad x \in \Gamma,\ t > 0, \tag{4}$$

$$u(x, 0) = u_0(x) \quad x \in \Omega, \tag{5}$$

$$y(x, 0) = y_0(x) \quad x \in \Omega, \tag{6}$$

The unknowns e, u and y are the non-dimensional enthalpy, the non-dimensional temperature and the mass fraction of solid fuel, respectively. w is a re-scaled wind velocity and g is defined by $g(u) = s^+(u, y)\beta$ where the function s^+ is given by

$$s^+(u, y) = \begin{cases} 1 & \text{if} \quad u \geq u_p \text{ and } y > y_e \\ 0 & \text{in another case} \end{cases}$$

where y_e is the mass fraction lower bound of extinction.

The non-dimensional enthalpy e is an element of a multivalued operator G, given by:

$$G(u) = \begin{cases} u & if \quad u < u_v \\ [u_v \ , \ u_v + \lambda_v] & if \quad u = u_v \\ u + \lambda_v & if \quad u_v < u < u_p \\ [u_p + \lambda_v \ , \ u_p + \lambda_v + \lambda_p] & if \quad u = u_p \\ u + \lambda_v + \lambda_p & if \quad u > u_p \end{cases}$$

Where u_v and u_p, are the non-dimensional water evaporation temperature and the non-dimensional solid fuel pyrolysis temperature, respectively. The quantities λ_v and λ_p are the non-dimensional evaporation heat and pyrolysis heat, respectively. The convective term $w.\nabla e$ takes into account the energy convected by the gas pyrolisated through the elementary control volume.

The right hand side of equation (1) describes the thermal radiation from the flame above the layer. A derivation of this term is given in the appendix. This term is a convolution operator given by,

$$R(x) = \delta \int_{\Omega_f(u,y)} f(x - \tilde{x}) \, d\tilde{x},$$

which takes into account non local radiation from the flame, where $\Omega_f(u,y) = \{x \in \Omega; \ u(x) > u_p \text{ and } y(x) > y_e\}$ is the fire domain on the surface.

The term $\kappa\Delta u$ describes thermal conduction and αu represents the energy lost by convection in the vertical direction.

Equation (3) represents the fuel mass variation by pyrolysis.

It should be noticed that in the burnt zone the multivalued operator does not exactly represent the physical phenomena as the water vapor is no more in the porous medium. This drawback can be circumvented setting $\lambda_v = 0$ and $\lambda_p = 0$ in the burnt area.

This model is a variant of the models in [5](chapter one), Model I in [4] or the model in [6] where we have introduce the influence of the moisture content and the heat absorption by pyrolysis, by using the enthalpy multivalued operator. The non local radiation term used in this paper is derived in the appendix.

3 NUMERICAL METHOD

3.1 Time integration

Let $\Delta t = t^{n+1} - t^n$ a time step and let y^n, e^n and u^n denote approximations at time step t^n to the exact solution y, e and u, respectively.

We consider a semi-implicit scheme by discretizing the total derivative, see [8],

$$\partial_t e + w.\nabla e \approx \frac{1}{\Delta t}(e^{n+1} - \bar{e}^n),$$

where $\bar{e}^n = e^n \circ X^n$, and $X^n(x) = X(x, t^{n+1}, t^n) \approx x - w\Delta t$ is the position at time t^n of the particle which is at position x at time t^{n+1}. At each time step we solve,

$$\frac{y^{n+1}-y^n}{\Delta t} = -y^{n+1}g(u^{n+1}), \tag{7}$$

$$\frac{e^{n+1}-\bar{e}^n}{\Delta t} - \kappa\Delta u^{n+1} + \alpha u^{n+1} = R^n, \tag{8}$$

$$e^{n+1} \in G(u^{n+1}). \tag{9}$$

The basic idea is to treat implicitly the positive terms. Instead the non local radiation term, which is costly computed, is evaluated explicitly at time t^n.

3.2 Iterative solution at each time step

Problem (7),(8),(9) is non linear due to the multivalued operator G. We first consider an exact perturbation of this problem.
Let $\omega > 0$ be a given parameter and set,

$$G^\omega = G - \omega I,$$

where I is the identity, then (9) can be written,

$$z^{n+1} = e^{n+1} - \omega u^{n+1} \in G^\omega(u^{n+1}). \tag{10}$$

For λ and ω verifying $\lambda\omega < 1$, the resolvent,

$$J_\lambda^\omega = (I+\lambda G^\omega)^{-1} = ((1-\lambda\omega)I + \lambda G)^{-1}$$

is a well defined univalued operator and the Yosida approximation of G^ω is given by

$$G_\lambda^\omega = \frac{I - J_\lambda^\omega}{\lambda}.$$

It is easy to check that inclusion (10) is equivalent to equation

$$z^{n+1} = G_\lambda^\omega(u^{n+1} + \lambda z^{n+1}).$$

This suggest the following algorithm for solving (7),(8),(9):
For u^n, y^n and z^n given,

1. Set $u^{n+1,0} = u^n$, $z^{n+1,0} = z^n$.
2. Compute
$$y^{n+1,i+1} = \frac{y^n}{1+\Delta t g(u^{n+1,i})}.$$
3. Compute $u^{n+1,i+1}$ solving
$$(\alpha\Delta t + \omega)u^{n+1,i+1} - \Delta t\kappa\Delta u^{n+1,i+1} = \bar{e}^n - z^{n+1,i} + \Delta t R^n.$$

4. Compute $z^{n+1,i+1} = G_\lambda^\omega(u^{n+1,i+1} + \lambda z^{n+1,i})$.
5. If $\|z^{n+1,i+1} - z^{n+1,i}\| > Tol$, update $i \leftarrow i+1$ and go to step 2.
   else end of the loop.

For $\lambda\omega \leq 1/2$ the Yosida approximation G_λ^ω is a Lipschitz operator with constant $1/\lambda$ and the convergence of the algorithm can be proved [9].

3.3 Practical computation of $z^{n+1,i+1}$

In the following we take $\lambda\omega = 1/2$. Set $\bar{u} = u^{n+1,i+1} + \lambda z^{n+1,i}$, then

$$G_\lambda^\omega(\bar{u}) = \frac{1}{\lambda}\bar{u} - \frac{1}{\lambda}J_\lambda^\omega(\bar{u}).$$

It remains to explain how to calculate

$$\bar{z} = J_\lambda^\omega(\bar{u}),$$

which is equivalent to solve (for $\lambda\omega = \frac{1}{2}$)

$$(\omega I + G)\bar{z} \ni 2\omega\bar{u}.$$

Then $\bar{z}$ is given by (see figure 1):

$$\bar{z} = \begin{cases} \frac{2\omega\bar{u}}{1+\omega} & if \quad 2\omega\bar{u} < (1+\omega)u_v \\ u_v & if \quad (1+\omega)u_v < 2\omega\bar{u} < (1+\omega)u_v + \lambda_v \\ \frac{2\omega\bar{u}-\lambda}{1+\omega} & if \quad (1+\omega)u_v + \lambda_v < 2\omega\bar{u} < (1+\omega)u_p + \lambda_v \\ u_p & if \quad (1+\omega)u_p + \lambda_v < 2\omega\bar{u} < (1+\omega)u_p + \lambda_v + \lambda_p \\ \frac{2\omega\bar{u}-\lambda_v-\lambda_p}{1+\omega} & if \quad (1+\omega)u_p + \lambda_v + \lambda_p < 2\omega\bar{u} \end{cases}$$

4 NUMERICAL RESULTS

First we consider the influence of the moisture content defined as the ratio of the weight of water absorbed to the weight of dry wood in a case without wind and slope. The numerical calculation corresponds to a square fuel bed of $3 \times 3\ m^2$ composed with *Pinus Pinaster* with a fuel load of $1\ kg/m^2$. We have studied the propagation of a fire front for different moisture contents, neglecting first the pyrolysis heat. We used the following set of parameters:

$$\kappa = 1.3 \times 10^{-4},\ \alpha = 0.02$$

The parameter δ in the radiation term which takes into account flame and fuel properties, particularly the temperature of the flame T_f and the absorption coefficient a (see Appendix), is adjusted for a given fuel in accordance with the rate of spread. The fire is

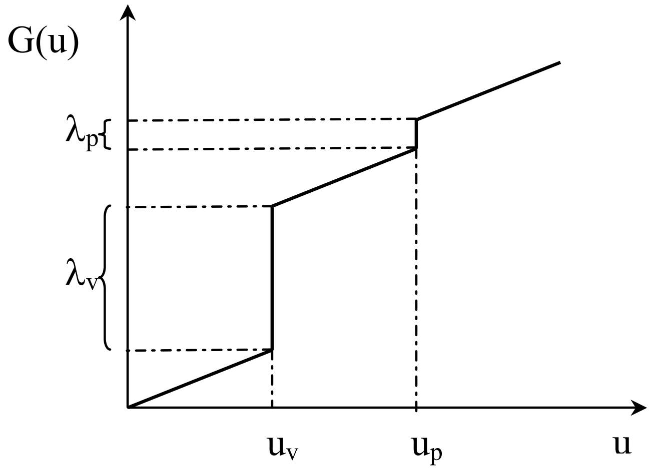

Figure 1: G operator

ignited at the center of the square. We obtain a circular fire front as it is foreseen. Figure 2 shows the temperature profile on the direction $x_2 = x_1$ at time $150s$ for different moisture contents. We have considered three values for $\lambda_v = 0.6,\ 0.9,\ 1.2$ which correspond to a moisture content of $0.1,\ 0.15$, and 0.2 (kg of water/kg of dry fuel), respectively. The corresponding rates of spread are $1.07 cm/s$, $0.76 cm/s$, $0.49 cm/s$, respectively . The effect of the moisture content can be clearly appreciated in the plate before the fire front. For a given fuel load there is an upper value of fuel moisture above which the fire will not propagate, in this example this critical value is $\lambda_v = 1.32$ (0.22 kg of water/kg of dry fuel). The heat absorbed by pyrolysis is usually much lower than the heat absorbed by water evaporation and sometimes is neglected or emulated by an equivalent heat mechanism modifying the specific heat. To see the effect of the pyrolysis heat we show in figure 3 for $\lambda_v = 0.3$ (0.05 kg of water/kg of dry fuel) and for $\lambda_p = 0$ and 0.09, the position of the fire front at time $150s$. This affects the critical value of the fuel moisture.

In the second example the influence of the slope and the wind are considered. The slope modify the non local radiation term in a similar manner as the wind does, so its effect in the propagation of the fire front is similar. The wind has two effects, on one hand through the convective term, and on the other hand determining the tilt angle of the flame, increasing or decreasing non local radiation. We take a terrain surface given by $z = |x| \tan(\pi/6) + y \tan(\pi/6)$ representing an ideal canyon. The dimension of the table is $2.8 \times 2.9\ m^2$. A wind velocity of $1\ m/s$, above the table is considered in the x_2 direction. The parameters δ and β are adjusted so that the rate of spread as a function of the slope

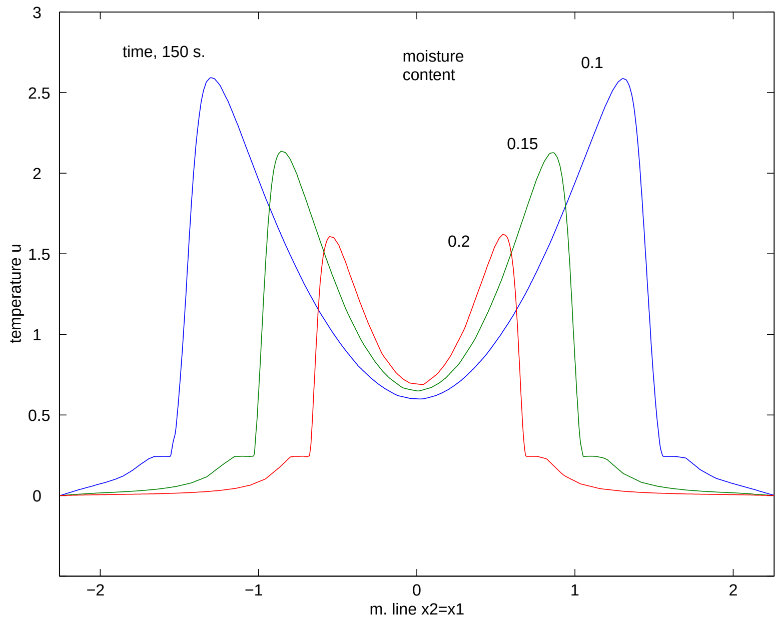

Figure 2: Temperature at time 150s for different moisture contents

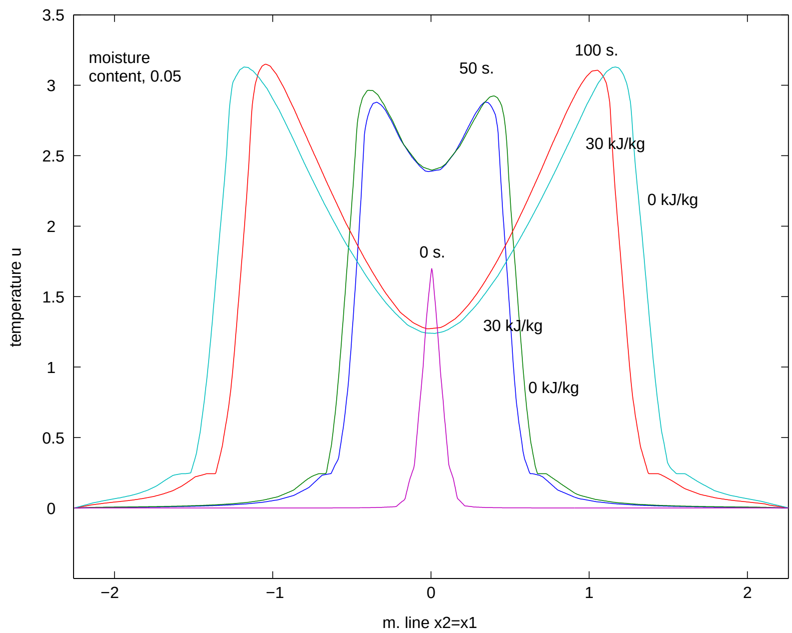

Figure 3: Temperature at times 50s. and 100s. for different pyrolysis heat

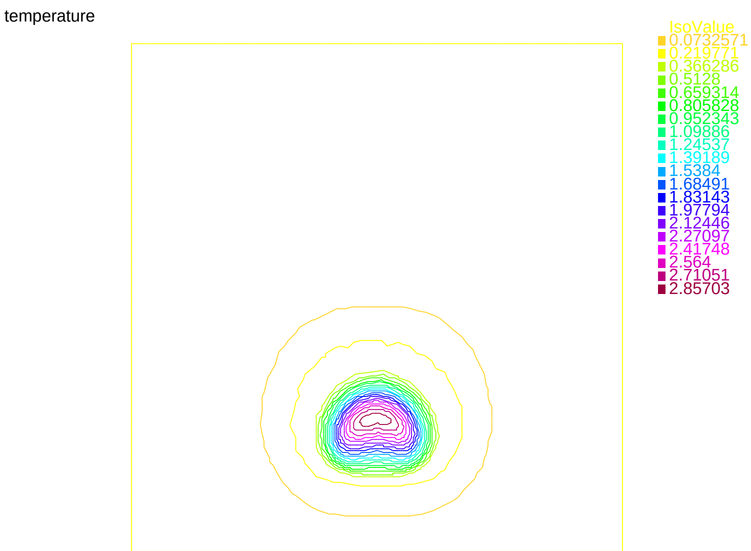

Figure 4: Front position and isotherms at time 50s.

angle and of the wind velocity fits in the values given in [7]. Figures 4 and 5 show the temperature contours at times 50 s and 100 s. These numerical results reasonably agree with the images of the experiment described in [7].

All the computations have been done using P1-Lagrange finite elements approximation and anisotropic adaptivity, implemented with FreeFem++, a finite element package of Pironneau and Hetcht[11].

5 CONCLUSIONS

A simplified fire spread model is presented, taking into account the dominant thermal transfer mechanism in this kind of combustion.
The influence of the moisture content and the heat absorption by pyrolysis is modelled as a double free boundary problem using a multivalued operator representing the enthalpy. The maximal monotone property of this operator allows the implementation of a numerical algorithm with well-known convergence properties.
On the other hand the heat source is a convolution operator representing the non local radiation from the flame above the layer whith temperature assumed to be known. From a numerical point of view this term is costly. Anyway in most cases the burning zone is a small part of the total domain, so that, the calculation of this term can be reduced to this part.
The numerical examples show the effect of the vegetation moisture in decreasing the velocity of the fire spread, as well as, the effect of the wind and slope on the fire propagation.

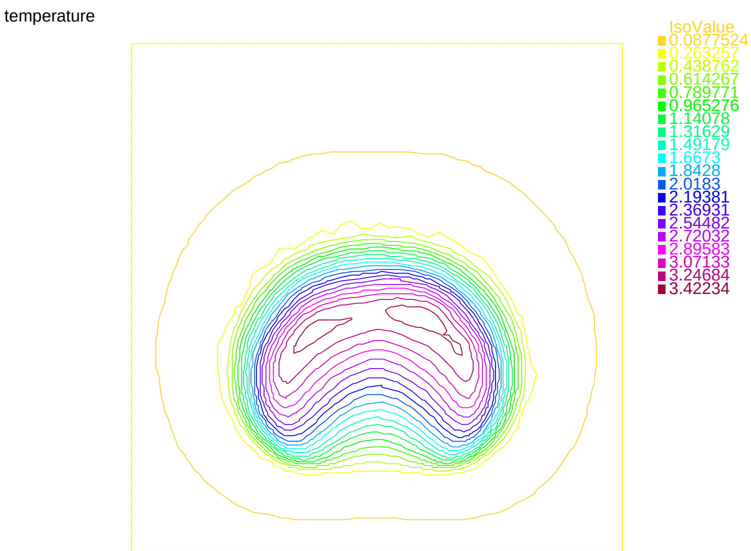

Figure 5: Front position and isotherms at time 100s.

6 APPENDIX

In this appendix we estimate the non local radiation term R in equation (1). In a non scattering medium, for a gray gas, the incident radiation intensity at a given point of the terrain surface for a fixed direction, integrated over all wavelengths, is given by (see[10] and Figure 6):

$$i(\mu) = i(0)e^{-\mu} + \int_0^{\mu} i_b(\mu^*)e^{-(\mu-\mu^*)}d\mu^*, \tag{11}$$

where $\mu(s) = \int_0^s a(s^*)ds^*$ is the optical thickness for a path of length s, being a the absorption coefficient, and i_b the intensity radiation from a black body.
Assuming that the gas out of the flame is a transparent medium, $i(0) = 0$ and taking into account that inside the flame $i_b(\mu) = \frac{\sigma T_f^4(\mu)}{\pi}$, being σ the Stefan-Boltzmann constant and T_f the flame temperature, and assuming a flame with small width, we obtain

$$i(\mu(s)) = \frac{\sigma T_f^4}{\pi}(1 - e^{-a(s_2-s_1)}) \approx \frac{a\sigma T_f^4}{\pi}(s_2 - s_1),$$

where $s_2 - s_1$ is the travel length inside the flame.
The energy flux q at a point $\mathbf{P}(x, z)$ of the terrain surface is obtained integrating for all directions

$$q = \int_{\theta=0}^{\theta=2\pi} \int_{\beta=\beta_H}^{\beta=\pi/2} i(\beta, \theta)\cos(\beta)\sin(\beta)d\beta d\theta.$$

For a flame with triangular section of vertex $\mathbf{V}$ and base centered in $\mathbf{O}$ (see Figure 6) setting α_f the angle between the flame and the horizontal plane, γ the angle between the

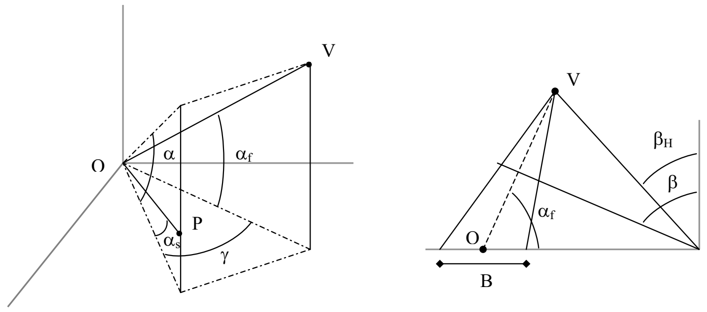

Figure 6: Flame position

horizontal projection of the flame with the position vector of the point x with respect to the center of the flame base, α_s the slope of the terrain surface, and B the width of the flame base, we get,

$$q(x) = \frac{a\sigma T_f^4}{\pi} \int_0^{2\pi} \int_{\beta_H}^{\pi/2} \frac{B \sin\beta(\tan\beta - \tan\beta_H)}{(\frac{\cos\gamma + \tan\alpha_f \tan\alpha_s}{\tan\alpha_f - \cos\gamma\tan\alpha_s} + \tan\beta)^2 - (\frac{B}{2H})^2} d\beta d\theta. \tag{12}$$

Now using a Simpson rule to integrate with respect to β and observing that $d\theta = \frac{d\tilde{A}}{B\|x-\tilde{x}\|}$ we obtain,

$$q(x) = \frac{a\sigma T_f^4}{\pi} \int_{\Omega_f} \frac{g(\alpha, \gamma, \alpha_s, \beta_H)}{\|x - \tilde{x}\|} d\tilde{A}, \tag{13}$$

where,

$$g(\alpha_f, \gamma, \alpha_s, \beta_H) = \frac{2}{3}(\frac{\pi}{2} - \beta_H) \frac{\sin(\frac{\beta_H + \frac{\pi}{2}}{2})(\tan(\frac{\beta_H + \frac{\pi}{2}}{2}) - \tan\beta_H)}{(\frac{\cos\gamma + \tan\alpha_f \tan\alpha_s}{\tan\alpha_f - \cos\gamma\tan\alpha_s} + \tan(\frac{\beta_H + \frac{\pi}{2}}{2}))^2},$$

and $\beta_H = \arg\tan(\frac{\|x-\tilde{x}\|}{H} - \frac{\cos\gamma + \tan\alpha_f \tan\alpha_s}{\tan\alpha_f - \cos\gamma\tan\alpha_s})$.
Finally the expression for R in (1) is obtained by adimensionalization.

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