

A MATHEMATICAL MODEL OF SPOT FIRES AND THEIR MANAGEMENT  
IMPLICATIONS

HUMBOLDT STATE UNIVERSITY

By

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## ABSTRACT

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Holly A. Perryman

In spite of considerable effort to predict wildfire behavior, the effects of firebrand lift-off and the resulting spot fires on fire propagation are still poorly understood. Horizontal discontinuities termed “fuelbreaks” can impede surface fire spread, however long-distance spot fires allow wildfires to ignite and propagate beyond (i.e., jump) fuelbreaks. Current fire behavior programs aid fire managers in predicting potential wild fire behavior, but these programs lack appropriate methods to determine firebrand and spot fire behavior. In this thesis, we developed a cellular automata model integrating key mathematical models and a recent model that determines firebrand landing patterns. Using our model we varied values of wind speed 6 meters above treetop, surface fuel loading, surface fuel moisture content, and canopy base height to examine two scenarios: the probability of a spot fire igniting beyond a fuelbreak of various widths, and how spot fires affect the surface fire’s rate of spread. This was tested in the context of a pine forest. Canopy base height had the greatest influence for both scenarios, followed by wind speed 6 meters above treetop, surface fuel moisture content and surface fuel loading. Our results suggest that the average rate of spread with spot fires is constant, and fire managers would benefit from a mathematical model that determines this rate. Since spot fires impact fire propagation updating fire behavior programs utilized by fire managers, or creating a new fire behavior program, is essential to improve wild fire prediction.

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## INTRODUCTION

Because of the devastating effects that uncontrolled wild fires can have on forest ecosystems, fire managers need reliable techniques to predict how wild fires propagate. Most fire propagation models to focus on two different types of wild fire spread: surface fires and canopy fires. Surface fires are fires that propagate along the forest floor or in herbaceous fuels while canopy fires occur when surface fires spread into tree crowns. Another equally important aspect of wild fire spread that lacks substantial research is the phenomenon where fires are ignited miles ahead of the flaming front by burning pieces of forest fuels propelled from the main fire (Pyne et al., 1996), an event known as spotting. Advancing our understanding of fire propagation would aid in minimizing risk to life and property, projecting the growth of ongoing fires, planning prescribed fires and examining the trade-offs in fuel treatment options designed to impede fire spread (Butler et al., 2004; Manzello et al., 2006b).

During wildland fires fuels such as twigs, bark, pine needles and pine cones are ignited; occasionally these fuels are lofted outside of the main fire and carried away by the wind, at which point they are termed *firebrands* (Clements, 1977). Firebrands not consumed while aloft eventually land on unburned surface fuels beyond the front, potentially igniting the neighboring vegetation and creating a *spot fire* (Figure 1). In forests, the un-ignited canopy stratum reduces spot fire ignition by prohibiting the generation of strong updrafts which loft firebrands out of the fire as well as by acting as a physical barrier or filter that prevents the wind from carrying lofted firebrands beyond the fireline (Albini, 1983a). However, intense surface fires may ignite the crown of a single tree or small group of trees, an event termed *torching*. Torching not only produces more firebrands than surface fires, it also creates a strong transitory convective flow that can loft firebrands to great heights (Albini, 1983a;





Figure 1: The above image displays spot fires igniting during the 1998 Alpine National Park Fire in Victoria, Australia (<http://www.dse.vic.gov.au>).

Scott, 2003) which allows lofted firebrands to land far ahead (ca., 4 km) of the fire front (Albini, 1983a; Pyne et al., 1996). Spot fires that ignite kilometers ahead of the fire front are called long-distance spot fires.

Spot fires have had a substantial impact on human life, property loss and local economies within the urban-wildland interface (Manzello et al., 2006b). A wild fire may be burning miles away from urban communities, but conditions can allow lofted firebrands to land within the community to potentially ignite vegetation or structures within this zone. Manzello et al. (2006b) studied how firebrands could ignite spot fires on rooftops and gutters. They did this by using an apparatus which allowed for the ignition and deposition of single and multiple firebrands onto a target fuel-bed. Varying the moisture content for the tested fuel-beds, Manzello et al. determined that flux of firebrands, size of firebrands, and the degree of air flow are important parameters to determine the ignition propensity of a fuel-bed due to a firebrand. Manzello et al. performed a similar analysis considering

shredded hardwood mulch, pine straw mulch and cut grass. They found that the size of the firebrands, degree of air flow and the fuel-bed moisture content are important parameters to determine ignition due to a firebrand. Both Manzello et al. (2006b) and Manzello et al. (2006a) desired to use their results with other studies to validate firebrand ignition mathematical models. A better understanding of spot fire behavior and improved mathematical models would aid in the protection of wildland-urban interface communities.

Spot fires that ignite near the main front may have little effect on surface fire propagation because the front can overcome the spot fire before it can contribute to the spread of the wild fire (Pyne et al., 1996). However, long-distance spot fires create substantial difficulties for fire suppression (Albini, 1979). A fire front can be detained using fuelbreaks, a fire suppression method where fuel is removed from strips of land (Agee et al., 2000). Agee and Skinner (2005) summarize a set of principles to address when creating fuelbreaks and shaded fuelbreaks. Fuelbreaks reduce the amount of surface fuels in order to lower the fireline intensity and hinder surface fire propagation. Common strategies to establish fuelbreaks consist of removing fuel by prescribed burning or adjusting fuel arrangement to produce a less flammable fuel-bed (Agee et al., 2000). Shaded fuelbreaks are variants that retain some high canopy shade, sufficient to prevent torching, but enough shade to inhibit surface fuel-bed recovery. If conditions permit, however, the main fire may loft ignited firebrands, thereby allowing the ignition of spot fires well beyond fuelbreaks. If fire fighters fail to extinguish spot fires igniting beyond the fuelbreaks, the wild fire will continue to spread ahead of the fuelbreak. Land managers can reduce firebrand dispersal by reducing the probability of torching by creating shaded fuelbreaks, however there are no set guidelines to determine fuelbreak placement or size (Agee et al., 2000). To operationally accomplish the establishment of an effective fuelbreak, land managers need reliable prediction tools. Finney (2001) explored using FARSITE to see how fires propa-

gate through different designed fuelbreaks, however more research and development was necessary to understand the role the treatment area's size, shape, and placement plays in modifying fire growth and behavior. In this document, we use the term fuelbreak to identify both fuelbreaks as well as shaded fuelbreaks.

Mathematical models have been developed to aid in predicting and understanding wild fires. Commonly utilized programs predicting fire behavior are BehavePlus, FARSITE, FlamMap, and FSPro (Andrews, 2009). BehavePlus, an updated version of BEHAVE (Andrews and Chase, 1989), is a PC-based fire modeling system that generates information such as fire spread rate, fireline intensity, and maximum potential spotting distance (Andrews et al., 2008). FARSITE (Finney, 2004) is a fire growth simulator designed to predict fire spread over a GIS layer. FlamMap (Finney, 2006) is a fire mapping and analysis program used to examine potential fire behavior across a landscape. FSPro displays fire spread probability across a GIS layer based on thousands of FARSITE simulations (Andrews, 2009). Hereafter, we focus on BehavePlus and FARSITE since they are more commonly utilized by fire managers (Andrews, 2009). Fortunately, BehavePlus and FARSITE are based on many of the same mathematical models (Andrews and Bevins, 2003); (Table 1).

Table 1: Mathematical Models in both BehavePlus and FARSITE

Author	Purpose
Rothermel (1972)	Surface Fire Spread
Van Wagner (1977)	Tree Torch Condition
Byram (1959)	Fire Intensity
Albini (1979)	Maximum Spot Distance
Albini and Baughman (1979)	Wind Adjustment Factor
Schroeder (1969) (BehavePlus Only)	Probability of Spot Ignition

Although BehavePlus and FARSITE are useful for many land management applications, neither program takes into account important aspects of spot fire ignition and prop-

agation and their effects on surface fire spread. BehavePlus calculates the maximum distances spot fires may ignite using work by Albini (1979), Chase (1981), Albini (1981), Albini (1983b), Albini (1983a) and Chase (1984). BehavePlus also calculates the probability of spot fire ignition by lofted firebrands based on work by Schroeder (1969). FARSITE simulates spot fire ignition and propagation using the models from Albini (1979) and Albini and Baughman (1979) and determines the probability of spot fire ignition by a user-defined percentage. Recent development on the ground-level landing patterns that lofted firebrands follow has been done by Sardoy et al. (2007) and Sardoy et al. (2008); currently neither of these models are considered in BehavePlus or FARSITE. If these models were incorporated, fire managers might be more prepared for potential spot fires, which may aid in suppression and prevention methods (i.e., fuelbreak placement and width).

The surface fire model used in the United States is Rothermel's fire spread equation (1972) (Johnson and Miyanishi, 2001). This mathematical model calculates the mean rate of spread for a propagating surface fire. Rothermel (1972) extended a model by Frandsen (1971) that applied the conservation of energy principle to a unit volume of fuel ahead of the propagating fire in a homogeneous fuel-bed. Frandsen's formula contains heat flux terms and as the mechanisms of heat transfer were unknown, it could not be solved analytically. Each term needed to be examined and evaluated experimentally or analytically in order to solve the equation. Rothermel (1972) found an approximate solution for the Frandsen (1971) model:

$$ROS = \frac{I_R \xi (1 + \phi_w + \phi_s)}{\rho_b \varepsilon Q_{ig}} \quad (1)$$

where  $ROS$  is the spread rate of the surface fire ( $\text{m s}^{-1}$ ),  $I_R$  is the reaction intensity which is the heat release rate per unit area of the front ( $\text{kJ m}^{-2} \text{sec}^{-1}$ ),  $\xi$  is the propagating flux ratio which relates the propagating flux to the reaction intensity, (dimensionless),  $\phi_w$  is

wind coefficient (dimensionless),  $\phi_s$  is the slope coefficient (dimensionless),  $\rho_b$  is oven-dry bulk density of the fuel ( $\text{kg m}^{-3}$ ),  $\varepsilon$  is the effective heating number which is the ratio of the effective bulk density to the actual bulk density (dimensionless) and  $Q_{ig}$  is the heat of ignition of the surface fuels which is the energy per unit mass required for ignition ( $\text{kJ kg}^{-1}$ ). The formulas for the individual terms in equation (1) can be found in Appendix 1. Rothermel (1972) further modified the equation for the spread model to compensate for the fact that fuels are composed of heterogeneous mixtures of fuel types and particle sizes, but as we assume the fuel-bed is homogenous we do not consider these formulas.

Rothermel's model is ideal for fire managers because it does not require values of fuel characteristics during combustion and it is applicable for a wide range of fuels. However, utilizing the model would have required extensive data collection to find values for the input parameters. For this reason, Rothermel (1972) introduced fuel "models". Fuel models identify the input parameters to compute equation (1) which remain reasonably constant between different terrains with the same fuel type (Table 2). To use Rothermel's model to determine a potential wild fire rate of spread, land managers simply select a fuel model that best represents the area of concern to define all but three input parameters. The user-defined input parameters are wind velocity at mid-flame height ( $\text{miles h}^{-1}$ ) the fuel moisture content ( $\text{kg moisture / kg oven-dry wood}$ ) and the slope steepness (%). Rothermel (1972) introduced eleven fuel models which were further refined by Albini (1976), who also created two additional models. Anderson (1982) described aids in determining the fuel model appropriate for a specific situation to ensure the most accurate results from Rothermel's model. Scott and Burgan (2005) expanded the quantity of fuel models to 40.

Rothermel (1972) acknowledged that his model lacked a description of spot fire ignition and propagation due to firebrand dispersal. He highlighted findings from Berlad (1970) to argue that lofted firebrands do not necessarily advance wild fires. However, other studies

Table 2: Parameters given by Fuel Model

Variable	Definition
$w_o$	ovendry fuel loading ( $\text{kg m}^{-2}$ )
$\delta$	fuel depth (m)
$\sigma$	fuel particle surface-area-to-volume ratio ( $\text{m}^{-1}$ )
$h$	fuel particle heat content ( $\text{kJ kg}^{-1}$ )
$\rho_p$	ovendry particle density ( $\text{kg m}^{-3}$ )
$S_T$	fuel particle total mineral content (kg minerals / kg ovendry wood)
$S_e$	fuel particle effective mineral content (kg silica-free minerals / kg ovendry wood)
$m_x$	moisture content of extinction (kg moisture / kg ovendry wood)

argue that spot fires contribute to the propagation of wild fires (Blackmarr, 1969; Albini, 1979; Scott and Reinhardt, 2001; Porterie et al., 2007; Sardoy et al., 2007).

Tree torches influence the number of firebrands dispersed (Albini, 1979; Viegas, 1998; Finney, 2004; Sardoy et al., 2007), so when considering the mechanisms of spot fires torching must be examined as well. Van Wagner (1977) studied the interaction between surface fuels and canopy fuels during wild fires and developed criteria for crown ignition to aid the study of canopy fires. He proposed that the ignition of the canopy depended on the bottom of the canopy, also called the canopy base, reaching a critical temperature. The canopy base reaching a critical temperature depends on the moisture content of the canopy,  $m_c$ , (kg moisture / kg ovendry wood) and the height from the forest floor to the canopy base  $z$  (m); (Figure 2). Van Wagner used the relationship proposed by Thomas (1963) to determine the critical surface fire intensity needed to ignite the canopy  $I_o$ :

$$I_o = (czQ_{ig}^c)^{3/2} \quad (2)$$

where  $c$  is an empirical constant (dimensionless) and  $Q_{ig}^c$  is the heat of ignition of the canopy ( $\text{kJ kg}^{-1}$ ). Van Wagner calculated  $Q_{ig}^c$  with the formula

$$Q_{ig}^c = 460 + 2590m_c. \quad (3)$$

To obtain equation (2) Van Wagner assumed that Thomas's relationship gives the temperature above ambient for crown ignition only at an arbitrary value of  $Q_{ig}^c$  which is denoted by  $Q_{ig}^{co}$ , and that the actual required temperature rise at the crown base varies with the ratio  $Q_{ig}^c/Q_{ig}^{co}$ . Van Wagner estimated the constant  $c$  to be 0.010 by considering data from three experimental crown fires.

Van Wagner may have considered his own 1977 research to be simple and incomplete since it lacks descriptions on initial surface intensity, canopy fire spread rate and rate of forward heat transfer to unburned canopy fuels, however his work stimulated further research on canopy fires (e.g., Turner et al., 1994; Scott and Reinhardt, 2001; Butler et al., 2004; Cruz et al., 2005; Dickinson et al., 2009), it is still incorporated into fire behavior programs (Finney, 2004; Andrews et al., 2008), fire managers still use it to determine fuelbreak development (Keyes and O'Hara, 2002), and his research provides a guide to calculate the probability of tree torching (Albini, 1979).

To use equation (2), surface fireline intensity must be determined. Byram (1959) described fire intensity to be the rate of heat release per unit time per unit length of the fire front ( $\text{kW m}^{-1}$ ). Byram claimed this is numerically equal to the product of the available fuel, the heat yield, and the forward rate of spread:

$$I = Hw_oROS \quad (4)$$

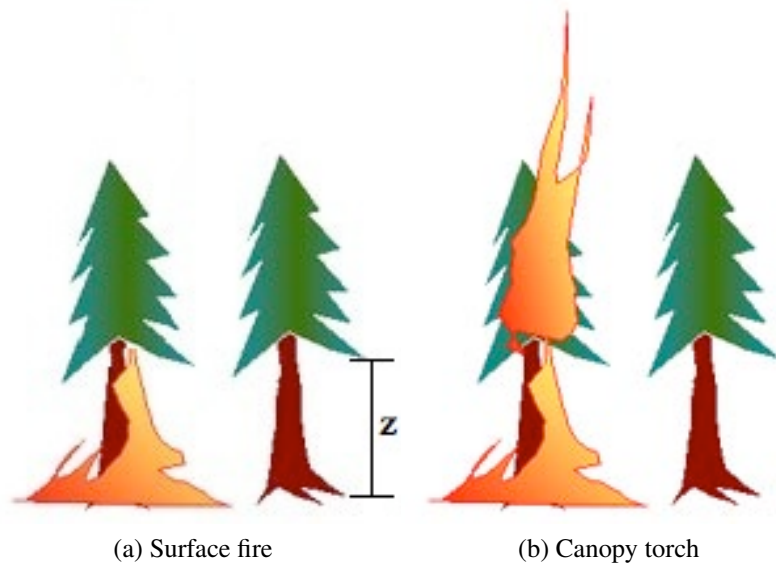


Figure 2: A surface fire interacting with the canopy layer of a forest (a) which can lead to a torch if conditions are right (b).

where  $I$  is the surface fire intensity ( $\text{kW m}^{-1}$ ),  $H$  is the heat of combustion ( $\text{kJ kg}^{-1}$ ),  $w$  is the mass of the fuel-bed per unit area ( $\text{kg m}^{-2}$ ) and  $ROS$  is the surface fire rate of spread ( $\text{m s}^{-1}$ ). The user must define  $H$  and  $w$  while  $ROS$  is calculated by equation (1). We employ the models by Van Wagner (1977) and Byram (1959) to determine if torching occurs during a wild fire.

Assuming that firebrands are lofted by torching trees, Albini (1979) developed a mathematical model that calculates the maximum distance at which a spot fire could be ignited. This work was extended when Albini (1981) adapted his model to include firebrands produced by burning piles. He also added formulas to estimate maximum spot distance when the terrain downwind is not forest (i.e., grasslands and shrublands). Later, Albini (1983b) examined the transport of firebrands by line fire thermals, which are columns of rising air created by line fires. Finally Albini developed the model to calculate the maximum spotting distance for a wind-driven surface fire (Albini, 1983a). Albini's work represents the first



attempts to be theoretically complete in the field of modeling spot fires (Pastor et al., 2003). The authors partially confirmed their work through empirical data.

The spot fire process Albini (1979) modeled is initiated by a torching tree, or a small group of torching trees. For a brief period of time a steady flame is created within the canopy and a buoyant plume forms above the flame and canopy. The fluid flow field of the plume lofts the firebrands *vertically*. When the steady flame is extinguished, Albini assumed the flow field begins to break down. Once the flow field collapses, firebrands begin to descend during which they are carried away from the fire front by the wind above the top of the canopy ( $U_{z_o}$ ). Considering that firebrands burn while falling, Albini determines the maximum spotting distance to be where a firebrand is completely consumed the instance it reaches the ground (Figure 3). Albini modeled this complicated process by devising sub-models for 1) the structure of the steady flame within the canopy, 2) the structure of the buoyant plume above the canopy, 3) the burning rate of firebrands, 4) the lofting of firebrands into the plume, 5) the surface wind carrying lofted firebrands, and 6) the trajectories of burning firebrands. Each sub-model is based on physical principles, includes assumptions and approximations for simplicity, and are supported by empirical relationships.

Albini's work culminated in the following formula for calculating the maximum spot distance for flat terrain:

$$x^* = 0.0136 U_{z_o} (z_o/g)^{0.5} (0.362 + 0.5 (z_v/z_o)^{0.5} \ln(z_v/z_o)) \quad (5)$$

where  $x^*$  is the maximum distance away from the fire front a spot may occur (m),  $U_{z_o}$  is the wind speed at the treetop ( $\text{m s}^{-1}$ ),  $z_o$  is the tree height (m),  $g$  is the acceleration of gravity ( $\text{m s}^{-2}$ ) and  $z_v$  is the maximum *vertical* distance a firebrand travels in order to land on the ground the moment of complete consumption (m). The parameters  $U_{z_o}$  and  $z_o$  can be

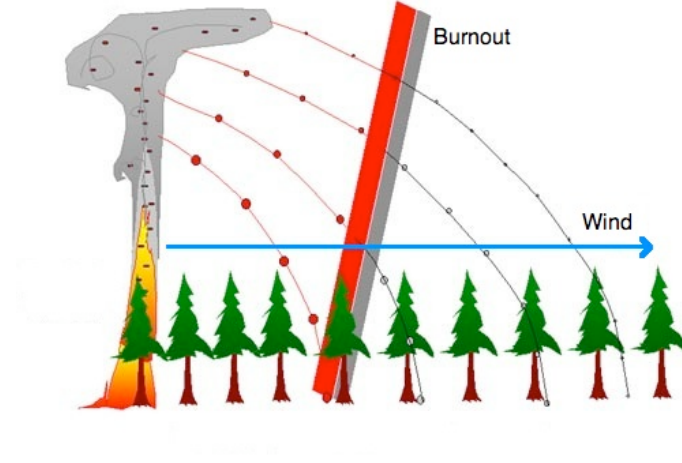


Figure 3: Firebrands are lofted from a torching tree, or small group of trees. Once the plume breaks down, firebrands are carried by the wind until they are consumed or land. Figure provided by Andrews et al. (2008).

measured while  $z_v$  is calculated. To do this, Albini proposed that a firebrand's total travel time from the initial height  $z_o$  to  $z_v$  ( $T_T$ ) is equal to the sum of three components: the time elapsed for the firebrand to travel from the initial height  $z_o$  to the flame tip ( $T_f$ ), the time elapsed for the firebrand to travel from the flame tip to the base of the plume ( $T_t$ ) and the time elapsed for the firebrand to travel from the base of the plume to  $z_v$  ( $T_p$ ). This leads to

$$T_T = \left( \frac{w_F}{2z_F} \right) (T_f + T_t + T_p), \quad (6)$$

where formulas for  $T_f$ ,  $T_t$ , and  $T_p$  normalized by the characteristic time  $\left( \frac{2z_F}{w_F} \right)$  are as follows:

$$\begin{aligned}
\frac{w_F T_f}{2z_F} &= 1 - \left(\frac{z_o}{z_F}\right)^{0.5} + \left(\frac{v_o}{w_F}\right) \ln \left( \frac{1 - \frac{v_o}{w_F}}{\left(\frac{z_o}{z_F}\right)^{0.5} - \frac{v_o}{w_F}} \right) \\
\frac{w_F T_t}{2z_F} &= 0.2 + B \left(\frac{D}{z_F}\right)^{0.5} \left( 1 + B \left(\frac{D}{z_F}\right)^{0.5} \ln \left( 1 + \frac{1}{1 - B \left(\frac{D}{z_F}\right)^{0.5}} \right) \right) \\
\frac{w_F T_p}{2z_F} &= \frac{a}{\left(0.8 \frac{v_o}{w_F}\right)^3} \left( \ln \left( \frac{1 - 0.8 \frac{v_o}{w_F}}{1 - 0.8r \frac{v_o}{w_F}} \right) - 0.8 \frac{v_o}{w_F} (r - 1) - 0.5 \left(0.8 \frac{v_o}{w_F}\right)^2 (r^2 - 1) \right).
\end{aligned} \tag{7}$$

Moreover, Albini proposed that  $T_T$  is analogous to the total time required for the flow structure to rise to the height  $z_v$ , which he derived to be the following:

$$T_T = t_o + 1.2 + \frac{a}{3} \left( \left( \frac{b + \frac{z_v}{z_F}}{a} \right)^{3/2} - 1 \right) \tag{8}$$

The only unknown parameter within equation (7) and equation (8) is  $z_v$ , and therefore equation (6) can be solved for  $z_v$ , which is then substituted into equation (5) to solve for the maximum spotting distance (the parameters used to calculate  $T_T$ ,  $T_f$ ,  $T_t$ , and  $T_p$  are described in Appendix 2).

Although Albini's 1979 model is useful to fire managers, Pastor et al. (2003) state that some of the simplifications and assumptions made to create the model detract from its reliability in analyzing real fires. In addition, Albini's model omits 1) long distance firebrands that can occur during intense wild fires, 2) the likelihood of a tree torching, 3) the availability of optimum firebrand material, 4) the probability of spot ignition and 5) the number of spot fires that can occur during a fire.

Between Rothermel's 1972 model and Albini's 1979 model there are two different wind speed measurements; Rothermel's model considers wind speed at mid-flame height

$U$  while Albini's model considers wind speed at treetop height  $U_{z_o}$ . However, it is common practice among fire managers to measure the wind 6 meters above the treetop  $U_{z_o+6}$  (Fischer and Hardy, 1972). Implementing both Rothermel's and Albini's models was made easier by a model developed by Albini and Baughman (1979) of a wind adjustment factor that relates wind speed within and below a uniform forest canopy to the wind speed 6 meters above the treetop. The authors limit their research to studying a steady, undisturbed windfield's influence on a surface fire. Using the logarithmic wind profile developed by Sutton (1953), the authors relate  $U_{z_o+6}$  ( $\text{m sec}^{-1}$ ) to  $U_{z_o}$  ( $\text{m sec}^{-1}$ ):

$$U_{z_o} = \frac{U_{z_o+6}}{\ln \left( \frac{20 + 1.18z_o}{0.43z_o} \right)} \quad (9)$$

where  $z_o$  is the treetop height (m), as well as  $U_{z_o+6}$  to  $U$  ( $\text{m sec}^{-1}$ ):

$$U = \frac{0.555U_{z_o+6}}{\sqrt{3.28fz_o} \ln \left( \frac{20 + 1.18z_o}{0.43z_o} \right)} \quad (10)$$

where  $f$  is the crown filling fraction:

$$f = \frac{\pi}{12} \frac{C}{100} \quad (11)$$

where  $C$  is the canopy cover (%). Upon comparing results obtained by equations (9) and (10) to reported experimental measurements, Albini and Baughman (1979) concluded that their model produces results that compare well with actual measurements. It is important to note that this model was derived considering a flat, homogeneous terrain, and the authors fail to account for the substantial influence fire has on local wind speeds, wind direction, or the profile of wind speed with height. In addition, canopy base height is not considered

in equation (10) and high canopy base heights increase the wind speed beneath the canopy (Varner and Keyes, 2009).

BehavePlus uses a model created by Schroeder (1969) to calculate the probability of spot ignition from a firebrand (Andrews et al., 2008). Schroeder's formula is a function of the heat of ignition  $Q_{ig}$  ( $\text{kJ kg}^{-1}$ ) which he defined to be the sum of five components: 1) the heat required to raise the temperature of the dry fuel from its initial temperature  $T_o$  ( $^{\circ}\text{C}$ ) to its ignition temperature (ignition temperature was assumed to be  $320^{\circ}\text{C}$ ), 2) the heat required to raise the moisture contained in the fuel from its initial temperature to the boiling point, 3) the heat of desorption, 4) the heat required to vaporize the moisture and 5) the heat required to raise the temperature of water vapor contained in the fuel voids from the boiling point to ignition temperature (Schroeder eventually omits this quantity as its effect is negligible). This gives the following formula for  $Q_{ig}$ :

$$Q_{ig} = 4.187 (144.51 - 0.266T_o - 0.00058T_o^2 - T_o m_f + 18.54 (1 - e^{-15.1m_f}) + 640m_f) \quad (12)$$

where  $m_f$  is the moisture content for the fuel ( $\text{kg moisture} / \text{kg oven-dry wood}$ ).

Schroeder postulated that if a firebrand lands on ignitable fuel then the probability of ignition  $P(I)$  is the product of the probability that the firebrand will be a specific size and the probability that a firebrand of that size will cause an ignition. He used the findings of Blackmarr (1969) to determine the probability that a specific size firebrand will cause an ignition. From this, Schroeder was able to derive  $P(I)$  as a function of the heat from the firebrand at the critical moisture content (moisture content where  $P(I) = 0$ ) minus  $Q_{ig}$ . He assumed that the size distribution of firebrands was a log normal distribution and using

findings from Keetch (1960) numerically derived a formula for  $P(I)$  as a function of  $Q_{ig}$ . Bradshaw et al. (1983) gave its general form as

$$P(I) = \frac{k_3 (0.0239 (Q_{igmx} - Q_{ig}))^{k_4}}{50} \quad (13)$$

where  $Q_{igmx}$  is the heat of pre-ignition for a fuel particle at the moisture content of extinction ( $\text{kJ kg}^{-1}$ ), and  $k_3$  and  $k_4$  are empirical constants. Bradshaw et al. (1983) identified  $Q_{igmx}$ ,  $k_3$  and  $k_4$  as functions of the moisture content of extinction  $m_x$  ( $\text{kg moisture} / \text{kg oven-dry wood}$ ).

In the previous we discussed some key mathematical models used in current fire behavior programs BehavePlus and FARSITE. These programs, however, lack current research concerning firebrand and spot fire behavior; preventing them from addressing additional topics (e.g., the probability of a spot fire igniting beyond a fuelbreak, or how spots fires influence the average rate of spread of the surface fire). Pastor et al. (2003), Sullivan (2007a) and Sullivan (2007b) review mathematical models developed concerning wild fire propagation. Most models may focus on surface fires and canopy fires, but there are other models which consider firebrand and spot fire behavior. In the following we will discuss some of these models.

A fire behavior program developed by Lynn (1997) appears to show some promise in depicting spot behavior (Sullivan, 2007a). FIRETEC is a multi-stage transport/wildland fire model based on the principles of conservation of mass, momentum, and energy. FIRETEC has the ability to track the probability fraction of mass in the plume above a critical temperature, thus providing a method of determining the occurrence of spotting downwind from a fire (Sullivan, 2007a). With more development, FIRETEC could become a reliable fire behavior prediction program for land managers.

Pastor et al. (2003) classified mathematical models concerning spot behavior as theoretical and empirical. They acknowledged Ellis (2000) and Woycheese et al. (1999) for creating the most significant theoretical models to date. Ellis (2000) developed a model of firebrand behavior to predict spotting distance from various sources of burning bark under various weather conditions while Woycheese et al. (1999) developed a formula for the maximum spotting distance based on the rate of heat release, ambient wind velocity and wood type. The model's strictly theoretical approach provides acceptable reliability, but has not yet been incorporated into a fire behavior program (Pastor et al., 2003). Woycheese et al. (1999) considered disk-shaped firebrands because they found their terminal velocity was substantially smaller than that for cylinders and spheres of the same diameter, so Woycheese (2001) continued his research on spot behavior by conducting 500 experiments exploring the effects of wood type, sample size, related velocity and grain orientation on the combustion of disk-shaped firebrands. Anthenien et al. (2006) also found that disk-shape firebrands represent the highest potential hazard for long-range spotting. Woycheese (2001) concluded that more study will be required to develop an accurate prediction of firebrand landing patterns and distances.

Sardoy et al. (2008) determined spatial landing distributions, also referred to as ground-level distribution, of lofted disk-shaped firebrands using numerical solutions of a fluid dynamical model proposed by Sardoy et al. (2007). Sardoy et al. (2007) created a numerical model for the transport of burning firebrands lofted by a three-dimensional steady-state buoyant plume from a crown fire. The steady-state gas flow and thermal fluids of the plume are pre-computed using a reduced version of a physics-based two-phase model by Porterie et al. (2000) and Porterie et al. (2005). Sardoy et al. (2007) developed a model concerning the pyrolysis (the chemical decomposition of a condensed substance by heating) and combustion of disk-shaped firebrands. This model was validated using data from

experiments conducted on disk-shaped firebrands. The authors emphasized the combustion model considering the diameter of burning firebrands at landing determine the potential for ignition of neighboring vegetation. Sardoy et al. (2007) observed lofted firebrands landing anywhere between 0 to 900 m in their computer simulations. After studying the effects of varying the wind speed and fire intensity to represent moderate to high-intensity wild fires as well as firebrand diameter, thickness, and density, they found that the distance reached by the lofted firebrand varies almost linearly with wind speed and depends weakly on fire intensity. In addition, spotting distance was independent of the initial particle diameter and followed a decreasing power-law function of the product of initial firebrand density and firebrand thickness. The relationships found by Sardoy et al. (2007) were independent of the initial placement of the firebrand within the canopy.

Sardoy et al. (2008) continued their work by focusing on determining a ground-level distribution of lofted disk-shape firebrands and their characteristics upon landing. Their model was composed of the following: an extension of the model from Mercer and Weber (1994) to describe the behavior of a buoyant plume from a line fire, an extended model from Sardoy et al. (2007) which calculates the path of the firebrand undergoing lift and rotation, and a thermal degradation and combustion model of firebrands that determines landing characteristics (Sardoy et al., 2007). Sardoy et al. (2008) ran their model with various fire intensities and wind speeds to represent moderate to high-intensity wild fires. Each scenario lofted 10,000 firebrands with different dimensions and initial location and properties were randomly generated. Their simulation results show that the firebrands' char content determines whether lofted firebrands have a single-mode or bimodal landing behavior independently of changes in fire intensity and wind speed. Bimodal behavior occurs when pyrolysis and char oxidation are present in the firebrand properties. Firebrands within the two modes were referred to as "short-distance" and "long-distance". Short-



distance firebrands traveled up to and around 1000 m, and approximately 72.4% of all landed firebrands landed in this mode. More than 99% of short-distance firebrands landed while still glowing. Long-distance firebrands landed at distances between 4500 and 6100 m. Approximately 27.6% of all landed firebrands landed in the long-distance mode and the majority of long-distance firebrands landed in a char oxidation state (i.e., not glowing). The product of the firebrand's initial density and initial thickness determines which mode receives the lofted firebrand. Sardoy et al. concluded that short-distance firebrands present the greater fire danger since more of the firebrands land glowing and they have a greater unburned mass remaining.

Focusing on the short-distance mode produced in the bimodal behavior, the authors discretized the x-axis into intervals of equal size and chose an interval size large enough to avoid empty segments and small enough to obtain sufficient detail. The number of particles in each interval was then divided by the total number of short-distance firebrands. The short-distance firebrand distribution was approximated by a lognormal function of the landing distance,

$$p(d) = \frac{1}{\sqrt{2\pi}\sigma_{fb}d} \exp\left(\frac{-(\ln(d) - \mu_{fb})^2}{2\sigma_{fb}^2}\right) \quad (14)$$

where  $p(d)$  is the probability firebrands land at distance  $d$  (m) away from the fireline,  $\sigma_{fb}$  is the standard deviation,  $\mu_{fb}$  is the mean. In determining  $\sigma_{fb}$  and  $\mu_{fb}$  Sardoy et al. identify two plume regimes: buoyancy-driven and wind-driven. These illustrate how fire and wind conditions affect the short-distance firebrands in the bimodal distribution. To define the two regimes the authors introduced the Froude number  $Fr$ :

$$Fr = \frac{U_H}{\sqrt{g \left( \frac{I_F}{\rho_\infty c_{eg} T_\infty g^{1/2}} \right)^{2/3}}} \quad (15)$$

where  $U_H$  is the wind speed at a defined height  $H$  ( $\text{m s}^{-1}$ ),  $g$  is the acceleration due to gravity ( $\text{m s}^{-1}$ ),  $I_F$  is the fire intensity incorporating the intensity produced by torching trees ( $\text{kW m}^{-1}$ ),  $\rho_\infty$  is ambient firebrand density ( $\text{kg m}^{-3}$ ),  $c_{cg}$  is the specific heat of gas ( $\text{kJ kg}^{-1} \text{K}^{-1}$ ) and  $T_\infty$  is ambient temperature (K). Buoyancy-driven regimes have a Froude number less than or equal to one while wind-driven regimes have a Froude number greater than one. Performing a least-squares fitting of the model results showed  $\sigma_{fb}$  and  $\mu_{fb}$  follow relationships with powers of fire intensity  $I$  and wind speed  $U_H$  (the authors results were gathered assuming the wind was measured at tree top height). For a buoyancy-driven plume

$$\begin{aligned}\sigma_{fb} &= 2.5743(I_F^{-0.21}U_H^{0.44}) + 0.19 \\ \mu_{fb} &= 0.0549(I_F^{0.54}U_H^{-0.55}) + 1.14\end{aligned}\tag{16}$$

whereas a wind-driven plume satisfies

$$\begin{aligned}\sigma_{fb} &= 5.3901(I_F^{-0.01}U_H^{-0.02}) - 3.48 \\ \mu_{fb} &= 0.2005(I_F^{0.26}U_H^{0.11}) - 0.02.\end{aligned}\tag{17}$$

There are a variety of methods used to model wild fire propagation: reaction-diffusion, diffusion-limited aggregation, percolation and fractals, etc., but the most prevalent is the cellular automata (Sullivan, 2007b). The earliest cellular automata model that included spotting was EMBYR, developed by Hargrove et al. (2000). EMBYR is a probabilistic model depicting fire spreading across a heterogeneous landscape to simulate the ecological effects of large fires in Yellowstone National Park, USA. Using a lattice where each cell is 50m by 50m, a stochastic model predicted the fire spread by assuming a burning cell would ignite any of its eight neighboring cells based on a determined ignition probability. The authors based spotting on the maximum spot distance determine by BEHAVE, considering

three different wind classes, a  $3^\circ$  random angle from the prevailing wind direction and the moisture content of the fuel in potential spotting cells (Sullivan, 2007b). Hargrove et al. (2000) found that the inclusion of spot fires in simulations increased both rate of fire spread and the total area burned. Validation of their model has yet to be performed due to difficulties parameterizing the model and the poor resolution of historical data (Sullivan, 2007b).

More current cellular automata models with spot fire ignition and propagation have been developed by Porterie et al. (2007) and Alexandridis et al. (2008). Porterie et al. (2007) uses a weighting procedure on the cells based on the characteristic times and thermal degradation and combustion of the flammable site. After validating the model against non-spotting experiments in homogeneous fuel, the authors simulated fire spread in heterogeneous fuel and incorporated spot fires. They selected cells in which spot fires might ignite based on an exponential distribution. The authors considered observations from experiments and fine-scale physically based deterministic models that showed that the density of firebrands on the ground decreased exponentially with the distance from the fire front. Then, Porterie et al. (2007) determined the probability of spot fire ignition based on the moisture content of the fuel receiving the firebrand. They found that in homogeneous fuels the wild fire rate of spread increased under certain conditions, however in heterogeneous fuels the rate of spread decreased as the degree of heterogeneity increased. This effect of heterogeneity indicates that critical channels exist in the landscape which, if maintained by fire fighters, can hinder wild fire propagation (Porterie et al., 2007).

Alexandridis et al. (2008) present an enhanced methodology for predicting the spread of a wild fire based on a cellular automata framework. They consider wild fires in a mountainous terrain and take into account vegetation type and density, wind speed and direction. They simulated the 1990 fire of Spetses Island in an attempt to validate the model. Upon

comparing the simulation results to the actual data from the fire, Alexandridis et al. concluded that the model adequately predicts a fire's evolution in time and space. They claimed that the model has the potential to be developed into an effective fire risk-management tool for land managers, although it appears this has not yet been accomplished.

These models are major contributions in the knowledge of spot behavior, but there are no known research that attempts to incorporate them into current fire behavior systems. In the remainder of this work we propose a mathematical model of a propagating wild fire incorporating spot fire ignition and propagation based on models used in current fire behavior programs (Table 1) and a new model that determines landing distributions of lofted firebrands (Sardoy et al., 2008). Basing our model on the mathematical model that govern utilized fire behavior programs would encourage the possibility to incorporate newer models into these fire behavior systems. Using our model we vary values for canopy base height, surface fuel loading, surface fuel moisture content and wind speed 6 meters above the treetop to investigate how spot fires aid the propagation of wild fires. This investigation is preformed by examining two scenarios: the probability of a spot fire igniting beyond a fuelbreak and how spot fire effect the average rate of spread. In the following we describe the methods taken to construct our model. A description of variables that represent key parameters and constants is presented in Table 3.

Table 3: Variable Definitions

Variable	Definition
$ROS$	surface fire rate of spread ( $\text{m s}^{-1}$ )
$I$	surface fire intensity ( $\text{kW m}^{-1}$ )
$I_o$	critical surface fire intensity to torch ( $\text{kW m}^{-1}$ )
$x^*$	maximum distance away from fire line spots ignite (m)
$T_T$	total time for which the flow structure exists to height $z_v$ (s)
$T_f$	time for particle to travel from $z_o$ to flame tip (s)
$T_t$	time for particle to travel from flame tip to plume base (s)
$T_p$	time for particle to travel from plume base to $z_v$ (s)
$z_v$	maximum <i>vertical</i> height achieved by a lofted firebrand (m)
$d$	distance away from fire line a firebrand may land (m)
$P(ROS)$	surface fire rate of spread (cells $\Delta t^{-1}$ )
$P(I)$	probability of spot fire ignition (Schroeder, 1969)
$P(I)_d$	probability of spot fire ignition adjusted $d$
$P(d)$	probability firebrand lands at $d$ (Sardoy et al., 2008)
$P(torch)$	probability the tree will torch (Van Wagner, 1977)
$x_{fb}$	distance firebrands travel downwind of fire (m)
$rand_x$	random value used to determine $x_{fb}$
$y_{fb}$	distance firebrands travel perpendicular to wind (m)
$rand_y$	random value used to determine $y_{fb}$
$t_i$	time steps before spot fire ignition
$t_v$	time steps required for firebrand to reach $z_v$
$t_s$	time steps required for firebrand to go from $z_v$ to the ground
$t_I$	time steps required for firebrand to match surface fire intensity
User defined:	
$U_{z_o+6}$	windspeed 6 meters above tree top ( $\text{miles h}^{-1}$ )
$m_f$	moisture content of fuel ( $\text{kg moisture} / \text{kg oven-dry wood}$ )
$m_c$	moisture content of canopy (%)
$z$	crown height (m)
$z_o$	tree height (m)
$t_d$	tree diameter (in)
$C$	canopy cover (%)
$t_{sim}$	number of trees in a torch
Control constants:	
$\sigma_y$	constant used to determine $y_{fb}$
$\sigma_t$	constant for $P(no\ torch)$
$\lambda_s$	constant for $P(I)_d$
$fb$	number of ignited firebrands that land on the ground

## METHODS

The process we model starts with a propagating surface fire (Figure 4). If certain conditions are met, a tree torch occurs. Once a tree torches, firebrands are dispersed throughout the terrain. Upon landing, some of these firebrands may ignite a spot fire. For simplicity, we assume spot fires have the same intensity as the main fire upon ignition. Thus, after an appropriate amount of time passes the spot fire ignites and spreads like the main fire; potentially torching additional trees.

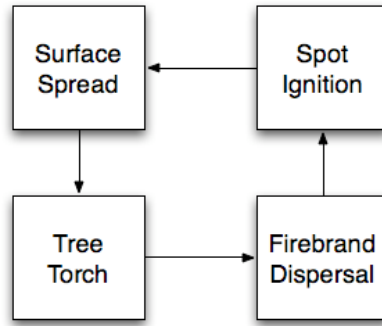


Figure 4: The model begins with a spreading surface fire. As the fire spreads through a forest, each tree has an associated probability of torching. When a tree torches, firebrands are lofted into the air and dispersed along the ground. The area receiving a firebrand has an associated probability of ignition due to the firebrand. If the conditions are met, a spot fire ignites. After an appropriate amount of time the spot fire behaves like a surface fire.

Using cellular automata to model these processes allows flexibility in simulating a wild fire in some predefined terrain. For simplicity, we assume the terrain has homogeneous surface and canopy fuels but the model can easily be adapted to heterogeneous environments. The terrain is represented by a grid with  $n$  rows and  $m$  columns. Each cell in the grid has a length and width of  $\Delta x$  meters. At any time each cell can be in one of the following states: unburned, burned, or torched (Figure 5). For each time step  $\Delta t$  an unburned cell

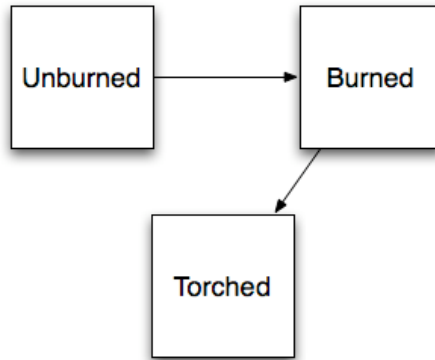


Figure 5: Cells in the grid begin as unburned and transition to burned upon being ignited by the surface fire or a spot fire. Burned cells have the ability to become torched if conditions are met to do so. Torched cells will disperse firebrands which have the ability to trigger an unburned cell to burning via spot fire ignition.

can become burned if one of the four nearest neighbors is burned (Figure 6). Cells that are currently burning or have burned and extinguished are considered are in the “burned” state. If a cell that satisfies conditions to torch ignites it will instantaneously transition from burned to torched. Only cells that become torched generate firebrands that have the potential to ignite spot fires. In the following subsections we discuss the methods behind the major processes modeled (Figure 4).

### Surface Spread

To determine the likelihood of an unburned cell adjacent to a burned cell igniting, we use the model developed by Rothermel (1972), equation (1). Equation (1) produces the wild fire’s rate of spread  $ROS$  ( $\text{m s}^{-1}$ ). The input parameters required by the user are the wind-speed at mid-flame height  $U$  (mph), moisture content of the fuel-bed  $m_f$  (kg moisture / kg ovendry wood) and slope steepness  $\phi$  (%). For simplicity, we assume the terrain is flat.

	$(i + 1, j)$	
$(i, j - 1)$	$(i, j)$	$(i, j + 1)$
	$(i - 1, j)$	

Figure 6: Cell  $(i, j)$  refers to an unburned cell in row  $i$ , column  $j$ ; the other labeled cells refer to the four nearest neighbors. An unburned cell can become burned if one of the four nearest neighbors are burned.

Since we are implementing the wind adjustment factor developed by Albini and Baughman (1979), our model calculates wind-speed at mid-flame height with equation (10). Thus, the wind speed 6 meters above the treetop  $U_{z_o+6}$  is an input parameter. The remaining input parameters are defined by a fuel model (Scott and Burgan, 2005). For a description of all the input parameters and the equations to solve equation (1), see Appendix 1. Naturally occurring wild fires do not travel at a constant speed across the ground. Changes in the fuel-bed alter the fire's progress, so equation (1) is considered to compute an average rate of spread. Our model propagates fire based on a computed probability,

$$P(ROS) = ROS \left( \frac{\Delta t}{\Delta x} \right) f_{dim} \quad (18)$$

where  $P(ROS)$  is the probability of propagation (cells  $\text{step}^{-1}$ ),  $ROS$  is the surface fire's rate of spread as computed by equation (1) ( $\text{m s}^{-1}$ ),  $\Delta t$  is the time step (seconds  $\text{step}^{-1}$ ),  $\Delta x$  is the cell width ( $\text{m cell}^{-1}$ ) and  $f_{dim}$  is a dimensionality factor (dimensionless). Since



our model considers a two-dimensional space, the way we implement the probability of propagation increases the modeled surface fire's spread rate. The dimensionality factor is incorporated to equalize the average rate of spread with no spotting to that which is calculated by equation (1). We initialize  $\Delta t$  so that  $P(ROS)$  is between zero and one and  $P(ROS)$  is a probability. If an unburned cell is a neighbor to at least one burned cell and  $P(ROS)$  is greater than a randomly generated uniform number (i.e., between 0 and 1) then the unburned cell ignites.

### Tree Torch

We assume firebrands are lofted by torching trees, so our model determines cells that will torch by utilizing the 1977 condition developed by Van Wagner (Equation 2) and Byram's 1959 model to determine fire front intensity (Equation 4). Recall equation (2) calculates the critical surface fire intensity that may cause the tree canopy to ignite  $I_o$  and equation (4) calculates the surface fire intensity  $I$ . If  $I$  is larger than  $I_o$  a torch may occur, however nature is not so predictable. For this reason, we assume the ignition of the tree canopy is probabilistic, so we use equation (2) to develop a probability logistic function (Stauffer, 2008) of fire intensity to determine the probability that the canopy ignites,  $P(torch)$ :

$$P(torch) = \frac{\exp(-\sigma_t(I_o - I))}{1 + \exp(-\sigma_t(I_o - I))}. \quad (19)$$

With  $P(torch)$  as  $I$  approaches and surpasses  $I_o$  the probability of torching increases but there is still a probability the tree canopy will not torch (Figure 7). There is no known information to aid in determining  $\sigma_t$ , but it likely depends on crown conditions (i.e., crown moisture content, crown density, etc.). Our model simulates the probability that cell will not torch. In a cell, if this random number is less than the value produced by equation (19)

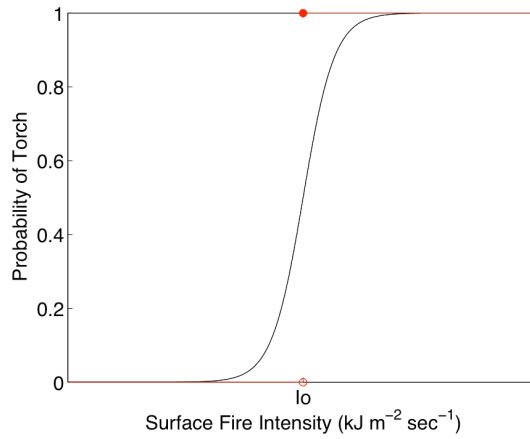


Figure 7: In the above, the red lines display the 1977 torching condition developed by Van Wagner (Equation 2) while the black curve reflects the cell torch probability function (Equation 19). The x-axis represents values for the surface fire intensity  $I$  ( $\text{kJ m}^{-2} \text{sec}^{-1}$ ) and the y-axis is the probability of torching  $P(\text{torch})$ . Concerning the black curve, the point of inflection is the critical surface fire intensity needed to ignite the canopy  $I_o$  ( $\text{kJ m}^{-2} \text{sec}^{-1}$ ) as calculated by equation (2). The scale of the x-axis depends on  $\sigma_t$ : a larger  $\sigma_t$  will compress the scale while a smaller  $\sigma_t$  will expand it.

than the cell will torch when it becomes burned. For the simulations, we assume torching is instantaneous.

### Firebrand Dispersal

Upon a cell torching, the location of the cell in which each firebrand lands must be determined. This is done using the models developed by Sardoy et al. (2007, 2008). To calculate the mean and standard deviation ( $\mu_{fb}$  and  $\sigma_{fb}$ , respectfully) for the lognormal function which determines where firebrands land, equation (14), our model first determines if the plume is buoyancy-driven or wind-driven by computing the Froude number, equation (15). Like Sardoy et al. (2008), we use  $g = 9.8$  ( $\text{m s}^{-1}$ ),  $c_{pg} = 1121$  ( $\text{kJ kg}^{-1} \text{K}^{-1}$ ),  $T_\infty = 300$  (K). For simplicity, we let  $\rho_\infty = 1.1$  ( $\text{kg m}^{-3}$ ) and  $I_F = I + 0.015$  ( $\text{kW m}^{-1}$ ). Recall,  $I$

represents the surface fire intensity as computed by equation (4). If the Froude number is greater than one the plume is buoyancy-driven and equation (16) is used to calculate  $\sigma_{fb}$  and  $\mu_{fb}$ , however if the Froude number is less than or equal to one the plume is wind-driven and equation (17) will be used to calculate  $\sigma_{fb}$  and  $\mu_{fb}$ . There was no known mathematical model that determines the number of firebrands from a tree torch that land on the ground ignited, so we assume each torch produces 50 firebrands that can potentially ignite a spot fire.

Rewriting the lognormal distribution given by equation (14) provides us with the distances the firebrands land downwind from the torch,  $d$ ,

$$d = \exp(\mu_{fb} + \sigma_{fb}rand_{fb}) \quad (20)$$

where  $\mu_{fb}$  and  $\sigma_{fb}$  are the mean and standard deviation as determined by equation (16) or equation (17) and  $rand_{fb}$  is a normally distributed random number with mean zero and standard deviation one. Dividing  $d$  by  $\Delta x$  converts the units to cells  $x_{fb}$ . We randomly distribute the distances firebrands travel perpendicular to the wind  $d_{per}$  (m):

$$d_{per} = \sigma_v rand_v \quad (21)$$

where  $\sigma_v$  is the standard deviation and  $rand_v$  is a normally distributed random number with mean zero and standard deviation one. We convert  $d_{per}$  from meters to cells  $y_{fb}$ . There are no known mathematical models to aid us determine  $\sigma_v$ , so we estimate it. Using equations (20) and (21) firebrands land in cells in various columns and rows downwind of the torched

cell. The model tracks how many firebrands from a single torch land in any given cell as well as the distance from the torch  $d$  (cells). A counter tracks the number of firebrands that land outside of the grid.

### Spot Ignition

The probability of spot ignition,  $P(I)$ , is calculated using the model by Schroeder (1969), equation (13), however instead of using Schroeder's formula to calculate  $Q_{ig}$ , equation (12), we use the formula used by Rothermel (1972),

$$Q_{ig} = 250 + 1116m_f \quad (22)$$

where  $m_f$  is the moisture content of the fuel (kg moisture / kg oven-dry wood). We do this because equation (22) is more recent and eliminates the need for the user-defined variable  $T_o$ . It is reasonable to assume the probability of ignition would decrease the farther away the firebrand is from the torch, in addition, it is reasonable to assume that the probability of ignition would increase the more glowing firebrands land in the cell. We re-calculated the probability of spot fire ignition based on an exponentially decreasing factor determined by the firebrands landing distance away from the torch as well as by the number of firebrands that land in the cell,  $P(I)_d^{fb}$ :

$$P(I)_d^{fb} = 1 - (1 - P(I)_d)^{fb} \quad (23)$$

where  $fb$  is the number of firebrands that land in the cell and  $P(I)_d$  is the probability of ignition based on the distance between the torch and where the firebrand lands:

$$P(I)_d = P(I) \exp(-\lambda_s d), \quad (24)$$

where  $P(I)$  is the probability of spot fire ignition calculated by equation (13),  $\lambda_s$  is a positive number representing the decay constant and  $d$  is the firebrand's landing distance away from the torch (cells). If an unburned cell receives  $fb$  firebrands and  $P(I)_d^{fb}$  is greater than a randomly generated uniform (0,1) number then a spot fire ignites in that cell.

Before spot fires ignite, an appropriate amount of time needs to pass. Considering the assumption that firebrands do not descend to the ground until the gas flow breaks down (Albini, 1979), we assume that the number of time steps that pass before spot fire ignition,  $t_i$ , is the sum of three time intervals: the number of time steps required for the firebrand to reach the maximum vertical height to achieve  $d$ ,  $t_v$ , the number of time steps that pass during the firebrands decent from the maximum vertical height to the ground,  $t_g$ , and the number of time steps that pass before the spot fire reaches the steady state, which means the spot fire rate of spread is equivalent to the main fire,  $t_I$ . The parameter  $t_I$  is necessary because we are assuming that a spot fire has the same intensity as the main fire and it can torch cells and potentially start more spot fires. We use Albini's 1979 model in estimating  $t_v$ . Recall,  $T_f$  is the time it takes the particle to travel from the initial height to the flame tip,  $T_t$  is the time it takes the firebrand to travel from the flame tip to the base of the plume, and  $T_p$  the time it takes the firebrand to travel from the base of the plume to the ground. Re-writing the equations (7) for  $T_f$ ,  $T_t$ , and  $T_p$  allows us to solve for  $t_v$  by summing  $T_f$ ,  $T_t$ ,

and  $T_p$ . Due to a lack of information, we make the assumption that  $t_g$  is equivalent to  $t_v$ , and it will take 20 minutes for a spot fire to reach the steady-state. In our model, a spot fire ignites once  $t_I$  time steps pass and behaves exactly as the main fire.

In the previous we discussed the major processes being modeled and the mathematics modeling each process. A schematic of our model's logical process is shown in Figure (8). In the following we discuss the implementation of our model.

### Implementation

We parameterized the model to simulate a wild fire propagating through a ponderosa pine forest (*Pinus ponderosa* Laws.), a widespread ecosystem in fire-prone areas of western North America. Considering canopy diameter of mature ponderosa pine trees we let  $\Delta x = 7$  (m) so we can assume each cell has one tree. This implies that the number of tree crowns in the torch is one; a parameter value necessary for implementing Albini's 1979 model. In order to use equation (18) to determine the probability of fire propagation we need to assume a small enough value for  $\Delta t$ , so we let  $\Delta t = 30$  (sec). The fire is initiated by setting the first column in the grid modeling the fire's propagation to burned. This column is referred to as the source of ignition. We assume that the wind is blowing from left to right, so the fire propagates toward the right-most column from the source of ignition (Figure 9).

We address two basic issues with our model: the probability spot fires will jump fuelbreaks of various widths and how spot fires influence the fire's average rate of spread. For both issues addressed we vary values for surface fuel moisture content  $m_f$ , canopy base height  $z$ , oven-dry surface fuel loading  $w_o$  and wind speed 6 meters above the treetop  $U_{z_o+6}$ . We vary these parameters because they encourage torching or spotting in some aspect. In addition, low canopy heights, low fuel loading, low fuel moisture content and high

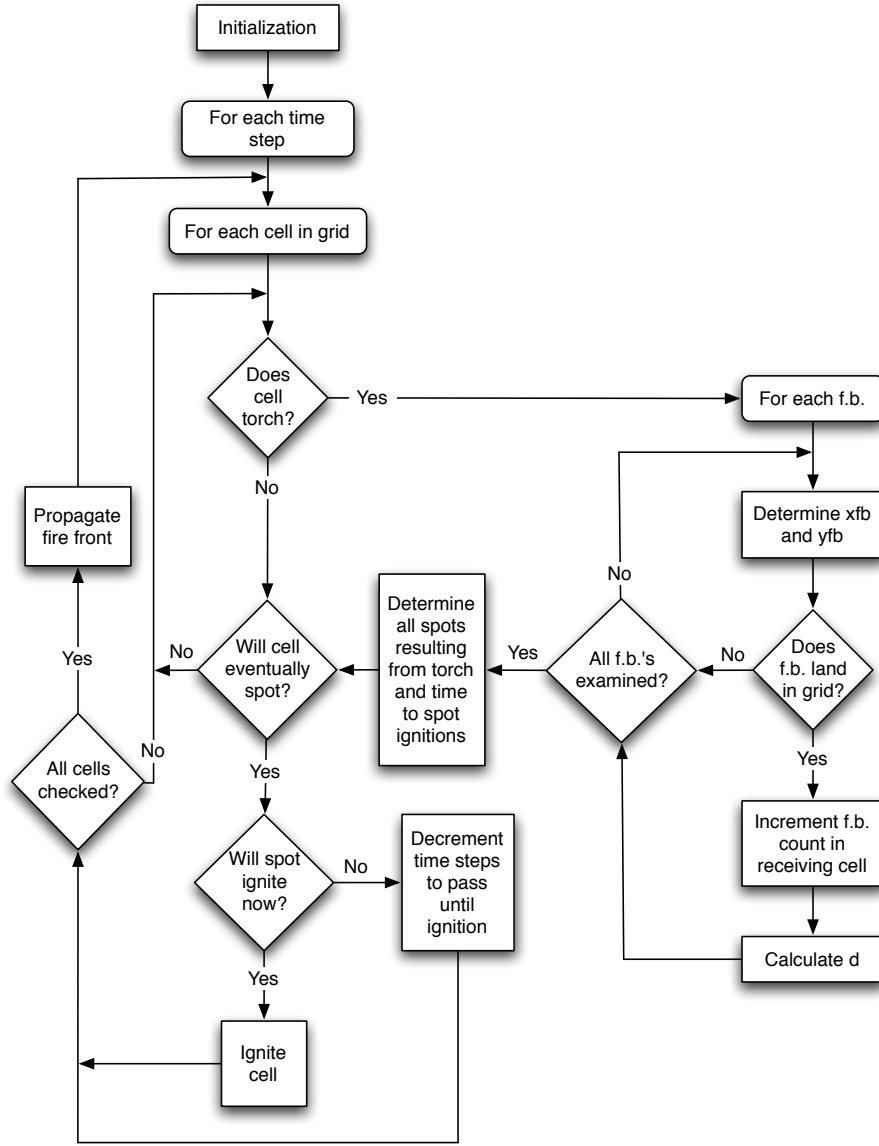


Figure 8: The above displays the model's logical process. The model begins in the Initialization stage and will continue to run until the time step exceeds a set maximum time. Parameters  $x_{fb}$  and  $y_{fb}$  are the distances (cells) firebrands travel from the torch parallel and perpendicular to the wind as described by equations (20) and (21), respectively. The parameter  $d$  is the distance between where the firebrand lands and the cell that torched (m) while 'f.b.' means firebrand.

wind speeds encourage extreme fire behavior. For all varied parameters we estimate low, medium and a high values (Figure 4). While a varied parameter value is being changed, the remaining are set to the medium values. To define values for parameters not varied we use fuel model TL8 as described by Scott and Burgan (2005), research by Susott (1982) and estimation (Table 5). Parameters  $\sigma_t$  (equation 19),  $\lambda_s$  (equation 24), and the number of firebrands which land glowing were estimated while analyzing preliminary results. All results were averaged from 2500 simulations.

Table 4: We vary four key parameter values by identifying a low, medium and high value for each varied parameter. The following identifies the four parameters varied as well as the varied values for each parameter.

Parameter	Values:		
	Low	Medium	High
canopy base height $z$ (m)	10	15	20
surface fuel moisture content $m_f$ (kg moisture / kg oven-dry wood)	0.063	0.094	0.124
oven-dry surface fuel loading $w_o$ (kg m <sup>-2</sup> )	1.1213	2.2426	3.3639
wind speed 6 meters above tree top height $U_{z_o+6}$ (m s <sup>-1</sup> )	4.4704	8.9408	13.4112

To examine the ability spot fires have igniting beyond fuelbreaks, also called *jumping* fuelbreaks, we incorporated a fuelbreak into the grid (Figure 10). Once a propagating fire reaches the fuelbreak the fire ceases propagation (10a), however a spot fire can ignite beyond the fuelbreak (jump) allowing the fire to continue propagating (10b). For this scenario the grid modeling the propagating wild fire has a length equal to the fuelbreak width plus twice the maximum spot fire distance as computed by equation 5 (m) and a width of 301 (m). The fuelbreak's width is initially 7 (m) varied from 25 to 500 (m) with increments of 25 (m). The MATLAB code for gathering these results can be seen in Appendix 3.



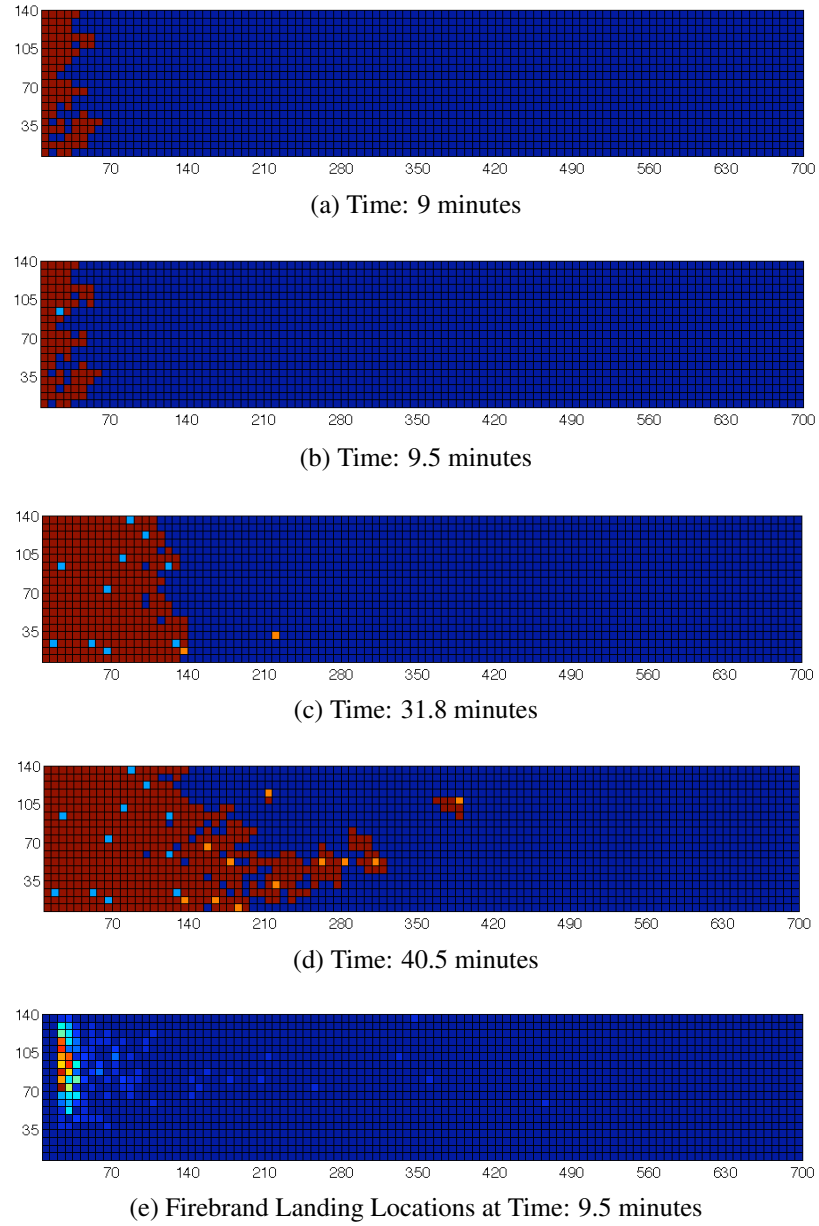
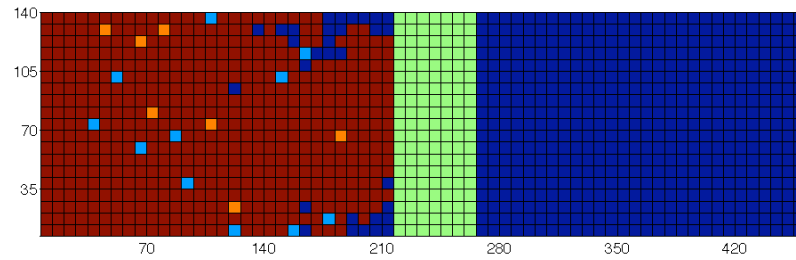
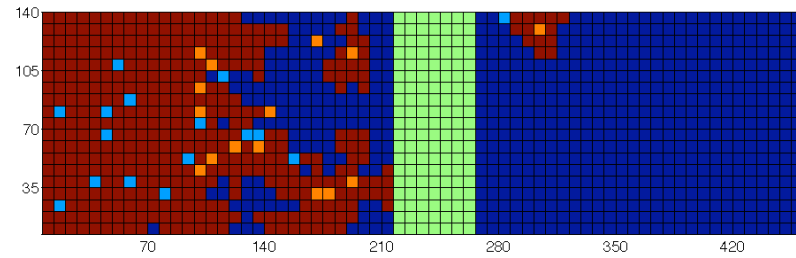


Figure 9: Panels (a) - (d) are a series of snapshots of the grid modeling the fire propagation over time and panel (e) displays the landing locations of firebrands distributed from the torch occurring in panel (b). For these images, the grid length is 700 (m) and the length is 140 (m). In panels (a) - (d), brown cells are burned, dark blue cells are unburned, light blue cells are torched and orange cells represent the ignition of a spot fire. In panel (e) the color of the cell corresponds to the number of firebrands that land there. The colors range from dark blue to brown; dark blue representing no firebrands landed in the cell while brown represents the most firebrands landed in the cell.



(a) Time: 112.98 minutes



(b) Time: 37.5 minutes

Figure 10: In the above brown cells are burned, dark blue cells are unburned, light blue cells are torched, orange cells represent spot fire ignition and the green cells represent the fuelbreak. Panel (a) shows a simulation when the fuelbreak succeeded in prohibiting the surface fire propagation. The propagating fire front met the fuelbreak and a spot fire ignition did not occur beyond the fuelbreak. Panel (b) shows a simulation where a spot fire ignited beyond a fuelbreak, allowing the fire to propagate beyond the fuelbreak. The length and width of the grid is in meters. For these images the grid length is 455 (m), including the 56 (m) fuelbreak, and the width is 140 (m).

Table 5: The following is a list of parameters that are not varied throughout the study, the values, and how we determined a value. TL8 indicates which fuel model from by Scott and Burgan (2005) was used to retrieve said parameter values, Susott (1982) provided a value for the heat of combustion  $H$  and remaining parameters were estimated.

Parameter	Value	Source
fuel depth $\delta$ (m)	0.0915	TL8
fuel particle surface-area-to-volume ratio $\sigma$ ( $\text{m}^{-1}$ )	5807.016	TL8
fuel particle heat content $h$ ( $\text{kJ kg}^{-1}$ )	18608.003	TL8
ovendry particle density $\rho_p$ ( $\text{kg m}^{-3}$ )	512.591	TL8
fuel particle total mineral content $S_T$ (kg minerals / kg ovendry wood)	0.0555	TL8
fuel particle effective mineral content $S_e$ (kg silica-free minerals / kg ovendry wood)	0.010	TL8
moisture content of extinction $m_x$ (kg moisture / kg ovendry wood)	0.35	TL8
heat of combustion $H$ ( $\text{kJ kg}^{-1}$ )	22000	(Susott, 1982)
moisture content of canopy $m_c$ (kg moisture / kg ovendry wood)	0.80	estimated
tree height $z_o$ (m)	30	estimated
canopy cover $C$ (%)	40	estimated
tree diameter $t_d$ (cm)	50.8	estimated
$P(ROS)$ dimensionality factor $f_{dim}$	0.5185	estimated
$d_{per}$ constant $\sigma_v$	$\frac{\Delta x}{2}$	estimated
$P(I)_d$ constant $\lambda_s$	0.005	estimated
$P(torch)$ constant $\sigma_t$	0.001	estimated
number of ignited firebrands landing on ground	50	estimated

To examine how spot fires affect the average rate of spread of a wild fire we compared the average rate of spread without spot fires  $ROS$ , the value given by equation (1), to the average rate of spread with spot fires  $ROS_{spots}$  ( $\text{m s}^{-1}$ ). We made the grid modeling the propagating wild fire have a length of six times the maximum spot fire distance  $x^*$  (m) and a width of 301 (m). The fire was allowed to propagate for 10 hours. To determine the distance the front is from the source of ignition we located the first unburned cell for each row in the grid modeling the fire's propagation (Figure 11). We calculate the average rate

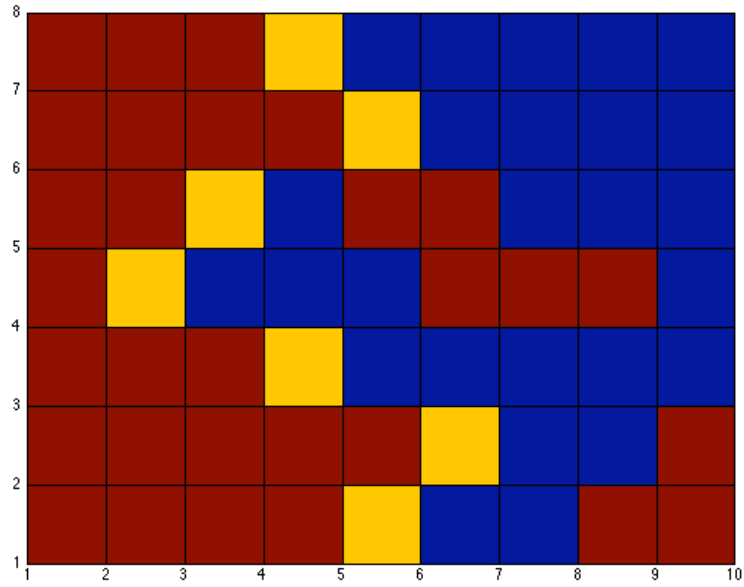


Figure 11: The source of ignition is the left-most column in the grid. Brown cells are burned while blue cells are unburned. Light orange cells represent that row's unburned cell which is farthest to the left. For each time step, the distances from the source of ignition and the individual light orange cells are determined and averaged to calculate the average front distance from source.

of spread with spot fires by averaging each row's front distance from source every for time step  $\Delta t$ . The MATLAB code for gathering these results can be seen in Appendix 4.

## RESULTS

The following describes the results obtained implementing our model to address the probability of a spot fire igniting beyond a fuelbreak (also referred as jumping a fuelbreak) of various widths and how spot fires effect the average rate of spread of the surface fire.

The probability of a spot fire jumping a fuelbreak  $P(jump)$  was affected by changes in fuelbreak width (m), surface fuel moisture content  $m_f$ , canopy base height  $z$ , overdry surface fuel loading  $w_o$  and wind speed 6 meters above the treetop  $U_{z_o+6}$  (Figure 12). Changes in canopy base height had the greatest influence on the probability of a spot fire jumping a fuelbreak while wind speed and surface fuel moisture content had a moderate impact. Fuel loading had the least influence.

To examine how spot fires influence the surface fire's propagation we compared the average wild fire front distance from the source when no spotting occurs to the average wild fire front distance from the source when spotting occurs. Considering individual simulations (Figure 13), when no spotting occurs there is a steady increase in the front distance with random moments of slight increases (ca., 100 to 800 meters; Figure 13a). However allowing spot fires to ignite and propagate produced both: simulations with a steady increase from the source (little to no spot fires igniting), and simulations with dramatic increases in the average front distance (Figure 13b). These dramatic peaks indicate instances when the main fire front merged with a spot fire. To obtain an average wild fire front distance with and without spotting we averaged 2500 individual simulations (Figures 14a and 14b), and the average rate of fire spread is shown in Figure 14c.

Without spotting all parameter variations produced results where the average rate of spread of the modeled fire equalized to approximately the rate of spread computed by equation (1). With spot fires, all parameter variations approached a constant rate of spread

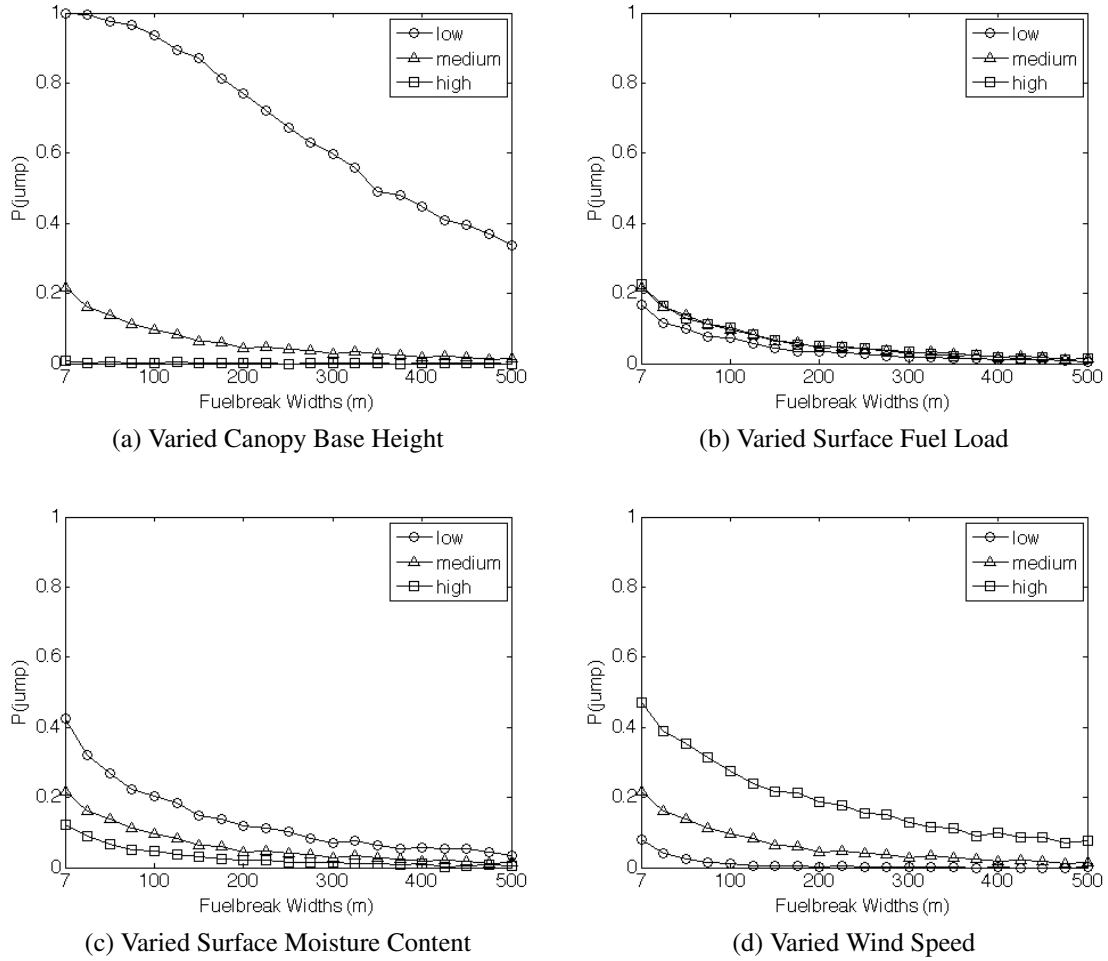
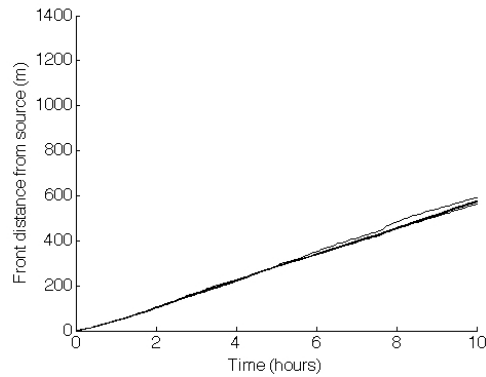
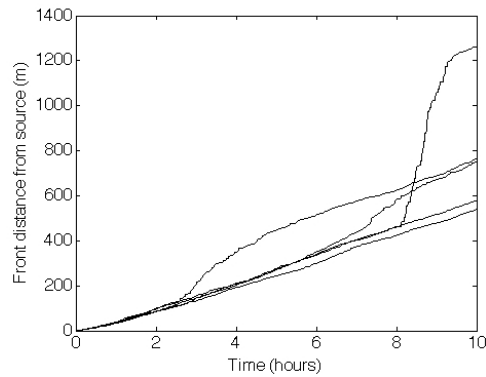


Figure 12: The above display the probability of a spot fire igniting beyond a fuelbreak  $P(jump)$  versus the fuelbreak width (m). Except for the stated parameter, other varied parameters are set to the medium values (Table 4). Lines with circles, triangles and squares indicate the stated parameter is set to the low, medium and high value, respectively.



(a) No Spotting



(b) Spotting

Figure 13: In the above, panel (a) displays the average front distance from the source for five individual simulations when no spotting occurs while panel (b) displays the average front distance from the source for five individual with simulations with spotting occurring. In panel (a) simulations produce a front which moves fairly constant with moments of random, minor peaks. In panel (b) there are simulations which appear analogous to panel (a), but there are also simulations which produce dramatic spikes in the average front distance. This displays instances when the main fire front merges with a spot fire front.

that was greater than that given by equation (1). Table 6 shows the average rate of spread without spotting as computed by equation (1), the average rate of spread with spotting, and the time to reach the average rate of spread with spotting for each varied parameter value.

All variations increase the average rate of spread when spotting occurs (Table 6). Although a low surface fuel load and a high wind speed produce higher rates of spread without

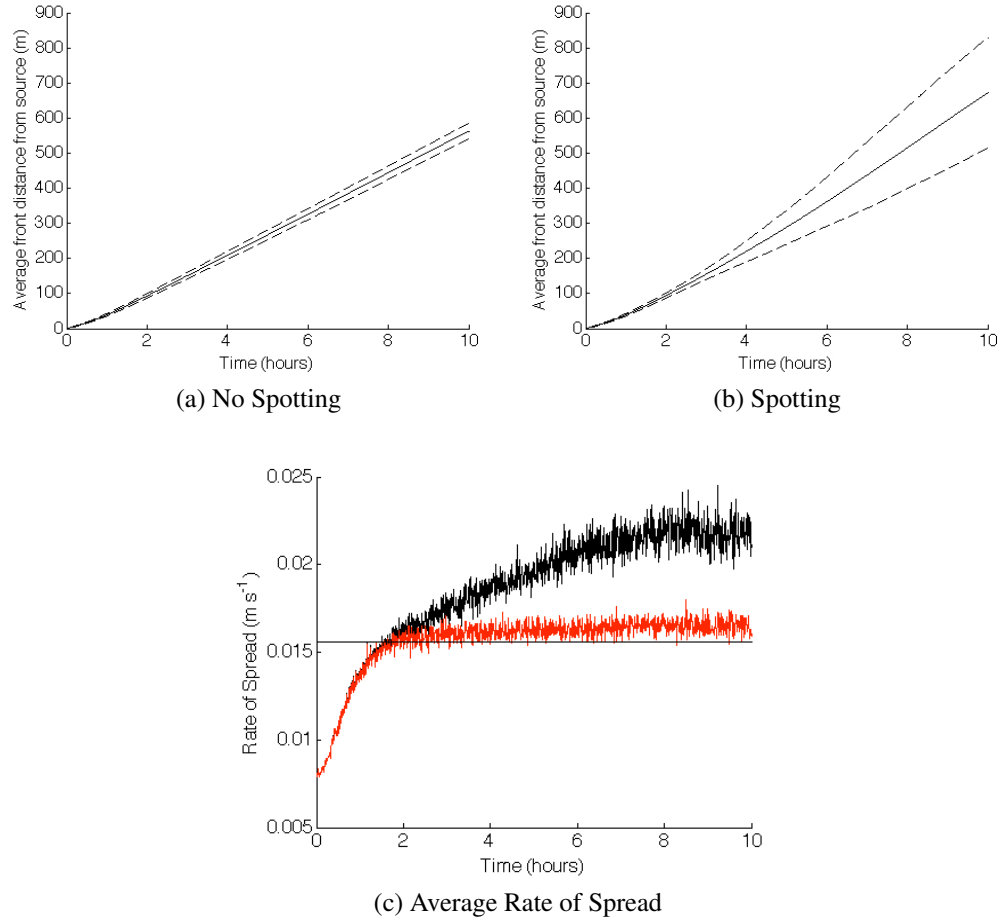


Figure 14: In the above, panel (a) displays the average distance from the source with no spot fires, panel (b) displays the average distance from the source with spot fires, and panel (c) displays the average rate of spread for panels (a) and (b). All varied parameters are set to the medium value (Table 4). In panels (a) and (b) the solid line represents the average distance from source while the dashed lines are the standard deviation. In panel (c) the straight line is the rate of spread as computed by Rothermel's 1977 model (Equation 1), the red curve represents the average rate of spread for the result in panel (a), and the black curve is the average rate of spread for the result in panel (b). Panel (c) displays the average rate of spread with spotting approaches a constant as well as the time it takes for the average rate of spread to equalize from the value computed by equation (1) to the constant average rate of spread with spotting. In addition, panel (c) displays the time it takes for the fire to equalize to average rate of spread as computed by equation (1). The information displayed in the above panels were gathered from 2500 simulations.



Table 6: To examine how spotting effects the average rate of surface fire spread we executed our model with the below variations on the varied parameter values (Table 4). For each variation we collected the rate of spread without spotting as calculated by equation (1), the constant rate of spread with spot fire ignition and propagation, and the time to equalize from the average rate of spread without spotting to the average rate of spread with spotting.

		Average Rate of Spread No Spotting ( $\text{m s}^{-1}$ )	Average Rate of Spread With Spotting ( $\text{m s}^{-1}$ )	Time to equalize (hours)
All Parameters	Medium	0.016	0.022	6
Canopy Base Height	High	0.016	0.017	4
	Low	0.016	0.128	7
Fuel Load	High	0.010	0.014	7
	Low	0.023	0.029	4
Moisture Content	High	0.013	0.016	6
	Low	0.019	0.038	7
Wind Speed	High	0.024	0.053	7
	Low	0.009	0.010	4

spotting, a low canopy base height (10 m) has the highest increase in the average rate of spread and produces the highest average rate of spread with spotting. A high wind speed and a low surface fuel moisture content have a moderate increase in the average rate of spread. In contrast, a low wind speed has the least influence on the average rate of spread with or without spotting. Times to equalize from the average rate of spread without spotting to the average rate of spread with spotting ranged between 4 - 8 hours. Variations that have little to moderate influence on the average rate of spread have higher equalization times whereas variations that have moderate to high influence on the average rate of spread tend to have smaller equalization times.

## DISCUSSION

Our model makes plausible predictions. Specifically, low canopy base heights, high surface fuel loads, low surface fuel moisture contents, and high wind speeds 6 meters above the treetop have the largest impacts on both the probability of a spot fire igniting beyond a fuelbreak and the average rate of spread with spotting (Figure 12, Table 6), and that larger fuelbreak widths decrease the probability of spot fires jumping a fuelbreak.

Out of all parameter variations, a low canopy base height had the greatest influence on the probability of a spot fire igniting beyond a fuelbreak as well as the average rate of spread with spotting, followed by wind speed 6 meters above the treetop, surface fuel moisture content, and surface fuel load. It is surprising how large of an effect canopy base height has on both scenarios in comparison to the other variations in our model. In addition, the difference in the probability of jumping for a medium canopy base height and a low canopy base height indicates the probability of jumping is sensitive concerning variations of canopy base height. From these results, canopy base height above all other parameter variations considered in this study is of the most importance for fire managers concerning spot prevention. This is inspiring considering canopy base height is an important component of fuels management (Agee et al., 2000; Agee and Skinner, 2005; Keyes and O'Hara, 2002). Managing canopy base height will aid in designing effective fuelbreaks as well as hinder the potential increase in the average rate of surface fire spread.

For the development of effective fuelbreaks, canopy base height and fuelbreak width are equally important factors. Our model could aid in determining appropriate fuelbreak widths given forest conditions; however the unknown constants used to determine the probability of a tree torch  $\sigma_t$  (Equation 19), the distances firebrands travel perpendicular to the wind  $\sigma_y$  (Equation 21), and the probability of a spot fire to ignite  $\lambda_s$  (Equation 23) limit

the quantitative accuracy of our results. Investigation concerning these constants is limited likely because of the considerable empirical data collection and experimental burning required to determine said constants, but our results highlight the importance in investigating these constants for future models as well as achieving quantitative accuracy. However, despite quantitative inaccuracy, preliminary modeling show that qualitative aspects of our results remain consistent. More specifically, the order of importance of the various factors (i.e., canopy base height, wind speed 6 meters above treetop, surface fuel moisture content, and surface fuel loading) remains unchanged.

According to our results, low surface fuel loads influence the average rate of spread without spotting, but higher surface fuel loads have a greater impact on the average rate of spread with spotting (Table 6). These results seem contradictory. This is occurring because surface bulk density  $p_b$  is based on surface fuel load and surface fuel depth (Appendix 1). By varying surface fuel load and neglecting to vary surface fuel depth in our model we produce lower bulk density with lower fuel load. Thus, low surface fuel loads are corresponding to unrealistically “airy” fuel-beds. In addition, our results concerning the probability of a spot fire igniting beyond a fuelbreak indicate that low surface fuel loads impede spot fire ignition as well as the potential existence of a critical value such that if surface fuel load is greater than or equal to this critical value there is no change in probability of a spot fire igniting beyond a fuelbreak (Figure 12b). This is likely due to the varying surface bulk density. To remedy this, appropriate values of surface fuel depth should vary with fuel load.

Because after some period of time the average rate of spread with spotting approached a constant rate of spread, it should be possible to construct a simple mathematical model to determine the average rate of spread with spotting. The scattered colony model for the spread of invasive species proposed by Shigesada and Kawasaki (1997) may be an appro-

priate method to model surface fire with spot fire ignition and propagation. The main fire would extend into the surrounding area by random diffusion, while at the same time produce individual fires far away from the initial fire that extend into the surrounding area (Shigesada and Kawasaki, 1997). The model would likely incorporate the average rate of spread as computed by Rothermel's 1972 model (Equation 1) as well as other important parameters. For the latter, our results indicate that canopy base height is one of the most important parameters to be included in this formula, followed by wind speed 6 meters above the treetop, surface fuel moisture content and then surface fuel loading. More research is necessary to determine if other parameters impact the average rate of spread with spotting, as well as to determine if all four parameters which we studied are necessary when developing a predictive model for the average rate of spread with spotting. In addition, considering all variations equalized to the average rate of spread with spotting near the end of the ten hours allocated to propagate, future simulations should allow the fire to propagate for longer periods of time to retain better estimates of the time to equalize to the average rate of spread with spotting, as well as, to reflect the long durations of wild fires (i.e., weeks to months).

Our modeled fire's average rate of spread without spotting indicates the need for altering our model's method of propagating a wild fire (Figure 14c). Initially, the model's average rate of spread without spotting begins below the value produced by Rothermel's model and approaches it over time. This could be occurring because our model initializes a fire by igniting an entire column (i.e., line ignition) but fires typically propagate in an elliptical pattern (Rothermel, 1991; Anderson, 1983). This may be corrected by igniting the fire within a single cell (i.e., spot ignition) instead of an entire column. In addition, even though a dimensionality factor was incorporated to the probability of propagation to force the average rate of spread without spotting produced by the modeled fire to be ap-

proximately equal to the value produced by Rothermel's 1972 model, the modeled fire still equalized to an average rate of spread without spotting slightly higher (ca., between  $0.001$  and  $0.01 \text{ m s}^{-1}$ ) than that produced by Rothermel's model. Even if the dimensionality factor worked as desired, future work with this model should include developing a different formula for the probability of propagation that would be appropriate for a two-dimensional model.

This model was created as a preliminary model. There is substantial work that could be done to improve it. For instance, firebrands can be lofted without torching trees. Albin (1981, 1983a, 1983b) adapts his model to consider this, however Sardoy et al. (2008) only considers firebrands dispersed from torching trees. More research concerning firebrand dispersal is necessary in order to consider firebrands lofting during non-torching events. In addition, our model does not incorporate a process that distinguishes burning cells from cells where the fire has burned out. Doing this would increase the accuracy of the model. Also, the model assumes the time it takes firebrands to land on the ground is twice the time it takes firebrands to reach the maximum vertical distance, a crude approximation of this important phenomenon. Future work to determine the time elapsed for firebrands to land from the maximum vertical distance would be important for future spot fire modeling, as well as the time required for spot fires to begin spreading with the same intensity as the main fire.

Improving fire behavior decision-support programs is of great importance since they are considered one of the most important tools implemented by fire managers (Johnson and Miyanishi, 2001). Our model is a compilation of key mathematical models governing the most utilized software (i.e., BehavePlus and FARSITE; Table 6) and a current firebrand dispersal model (Sardoy et al., 2007, 2008). Updating commonly implemented fire behavior prediction programs with new mathematical models is possible, but considering the com-

plexity of these programs it may be easier and more beneficial to create a new fire behavior program all together (Andrews, 2009). As our work has demonstrated, having fire prediction software include firebrand and spot fire behavior would not only improve prediction accuracy, it would also aid fire managers in answering a variety of unanswered questions which are of great concern, including the silvicultural practices necessary to design an effective fuelbreak or how spot fires may influence the average rate of spread for potential wild fires.

## BIBLIOGRAPHY

- Agee, J., B. Bahro, M. A. Finney, P. N. Omi, D. B. Sapsis, C. N. Skinner, J. W. van Wagtendonk, and C. P. Weatherspoon. 2000. The use of shaded fuel breaks in landscape fire management. *Forest Ecology and Management*, **127**:55–66.
- Agee, J. K. and C. N. Skinner. 2005. Basic principles of forest fuel reduction treatments. *Forest Ecology and Management*, **211**:83–96.
- Albini, F. A. 1976. Estimating wildfire behavior and effects. General Technical Report INT-30, USDA Forest Service, Intermountain Forest and Range Experiment Station, Odgen, UT.
- Albini, F. A. 1979. Spot fire distance from burning trees - a predictive model. General Technical Report INT-56, USDA Forest Service, Intermountain Forest and Range Experiment Station, Odgen, UT.
- Albini, F. A. 1981. A model for the wind-blown flame from a line fire. *Combustion and Flame*, **43**:155–174.
- Albini, F. A. 1983a. Potential spotting distance from wind-driven surface fires. Research Paper INT-309, USDA Forest Service, Intermountain Forest and Range Experiment Station, Odgen, UT.
- Albini, F. A. 1983b. Transport of firebrands by line thermals. *Combustion Science Technology*, **32**:277–288.
- Albini, F. A. and R. G. Baughman. 1979. Estimating windspeeds for predicting wildland fire behavior. Research Paper INT-221, USDA Forest Service, Intermountain Forest and Range Experiment Station, Odgen, UT.
- Alexandridis, A., D. Vakalis, C. I. Siettos, and G. V. Bafas. 2008. A cellular automata model for forest fire spread prediction: The case of the wildfire that swept through Spetses Island in 1990. *Applied Mathematics and Computation*, **204**:191–201.
- Anderson, H. E. 1982. Aids to determining fuel models for estimating fire behavior. General Technical Report INT-122, USDA Forest Service, Intermountain Forest and Range Experiment Station, Odgen, UT.
- Anderson, H. E. 1983. Predicting wind-driven wild land fire size and shape. General Technical Report INT-305, USDA Forest Service, Intermountain Forest and Range Experiment Station, Odgen, UT.
- Andrews, P. L. 2009. Behaveplus workshop. Fourth International Fire Congress.

- Andrews, P. L. and C. D. Bevins. 2003. BehavePlus fire modeling system version 2: Overview. In *2nd International Wildland Fire Ecology and Fire Management Congress*.
- Andrews, P. L., C. D. Bevins, and R. C. Seli. 2008. BehavePlus fire modeling system version 4.0 user's guide. General Technical Report 106WWW Revised, USDA Forest Service, Rocky Mountain Research Station.
- Andrews, P. L. and C. H. Chase. 1989. Behave: fire behavior prediction and fuel modeling system - burn subsystem, part 2. Technical Report INT-260, USDA Forest Service, Ogden, UT.
- Anthenien, R. A., S. D. Tse, and A. C. Fernandez-Pello. 2006. On the trajectories of embers initially elevated or lofted by small scale ground fire plumes in high winds. *Fire Safety Journal*, **41**(349-363).
- Berlad, A. 1970. Fire spread in solid fuel arrays. *Combustion and Flame*, **14**:123-236.
- Blackmarr, W. H. 1969. Instability of pine litter influenced by moisture content. Manuscript, USDA Forest Service, Southeastern Forest Experiment Station, Asheville, NC.
- Bradshaw, L. S., J. E. Deeming, R. E. Burgan, and J. D. Cohen. 1983. The 1978 national fire-danger rating system: Technical documentation. General Technical Report INT-169, USDA Forest Service, Intermountain Forest and Range Experiment Station, Ogden, UT.
- Butler, B., M. Finney, P. Andrews, and F. Albini. 2004. A radiation-driven model for crown fire spread. *Canadian Journal of Forest Research*, **34**:1588-1599.
- Byram, G. M. 1959. Combustion of Forest Fuels. In K. P. Davis, ed., *Forest Fire: Control and Use*. McGraw-Hill.
- Chase, C. H. 1981. Spot fire distance equations for pocket calculators. Research Paper INT-310, USDA Forest Service, Intermountain Forest and Range Experiment Station, Ogden, UT.
- Chase, C. H. 1984. Spotting distance from wind-driven surface fires-extensions of equations for pocket calculators. Note INT-346, USDA Forest Service, Intermountain Forest and Range Experiment Station, Ogden, UT.
- Clements, H. B. 1977. Liftoff of forest firebrands. Research Paper SE-159, USDA Forest Service, Southeastern Forest Experiment Station, Asheville, NC.



- Cruz, M. G., M. E. Alexander, and R. H. Wakimoto. 2005. Development and testing of model for predicting crown fire rate of spread in conifer forest stands. *Canadian Journal of Forest Research*, **35**:1626–1639.
- Dickinson, J. D., A. P. Robinson, P. E. Gessler, R. J. Harrod, and A. M. S. Smith. 2009. Flatland in flames: a two-dimensional crown fire propagation model. *International Journal of Wildland Fire*, **18**(5):527–535.
- Ellis, P. 2000. *The Aerodynamics and Combustion Characteristics of Eucalypt Bark-A Firebrand Study*. Ph.D. thesis, Australian National University.
- Finney, M. A. 2001. Design of regular landscape fuel treatment patterns for modifying fire growth and behavior. *Forest Science*, **47**(2):219–228.
- Finney, M. A. 2004. FARSITE: Fire area simulator-model development and evaluation. Research Paper RMRS-RP-4 Revised, USDA Forest Service, Rocky Mountain Research Station.
- Finney, M. A. 2006. An overview of FlamMap fire modeling capabilities. No. RMRS-P-41.
- Fischer, W. C. and C. E. Hardy. 1972. *Fire-weather observers' handbook*. USDA Forest Service.
- Frandsen, W. H. 1971. Fire spread through porous fuels from the conservation of energy. *Combustion and Flame*, **16**:9–16.
- Hargrove, W. W., R. H. Gardner, M. G. Turner, W. H. Romme, and D. G. Despain. 2000. Simulating fire patterns in heterogeneous landscapes. *Ecological Modeling*, **135**:243–263.
- Johnson, E. A. and K. Miyanishi, eds. 2001. *Forest Fires: Behavior and Ecological Effects*. Academic Press.
- Keetch, J. J. 1960. United States Forest Service circular letter 4400 dated april 19, 1960.
- Keyes, C. R. and K. L. O'Hara. 2002. Quantifying stand targets for silvicultural prevention of crown fires. *Western Journal of Applied Forestry*, **17**(2):101–109.
- Lynn, R. R. 1997. *A Transport Model for Prediction of Wildfire Behaviour*. Ph.D. thesis, New Mexico State University.
- Manzello, S. L., T. G. Cleary, J. R. Shields, and J. C. Yang. 2006a. Ignition of mulch and grasses by firebrands in wildland-urban interfaces fires. *International Journal of Wildland Fire*, **15**:427–431.

- Manzello, S. L., T. G. Cleary, J. R. Shields, and J. C. Yang. 2006b. On the ignition of fuel beds by firebrands. *Fire Mater*, **30**:77–87.
- Mercer, G. and R. Weber. 1994. Plumes above line fires in a cross-wind. *International Journal of Wildland Fire*, **4**:201–207.
- Pastor, E., L. Zarate, E. Planas, and J. Arnaldos. 2003. Mathematical models and calculation systems for the study of wildland fire behaviour. *Progress in Energy and Combustion Science*, **29**:139–153.
- Porterie, B., J. L. Consalvi, A. Kaiss, and J. C. Loraud. 2005. Predicting wildland fire behavior and emissions using a fine-scale physical model. *Numerical Heat Transfer, Part A: Applications*, **47**(6):571–591.
- Porterie, B., D. Morvan, J. Loraud, and M. Larini. 2000. Firespread through fuel beds: Modeling of wind-aided fires and induced hydrodynamics. *Physics of Fluids*, **12**:1762–1782.
- Porterie, B., N. Zekri, J.-P. Clerc, and J.-C. Loraud. 2007. Modeling forest fire spread and spotting process with small world networks. *Combustion and Flame*, **149**:63–78.
- Pyne, S. J., P. L. Andrews, and R. D. Laven. 1996. *Introduction to Wildland Fire*. John Wiley and Sons, INC.
- Rothermel, R. C. 1972. A mathematical model for predicting fire spread in wildland fuels. Research Paper INT-115, USDA Forest Service, Intermountain Forest and Range Experiment Station, Odgen, UT.
- Rothermel, R. C. 1991. Predicting behavior and size of crown fires in the northern rocky mountains. Research Paper INT-438, USDA Forest Service, Intermountain Forest Range Experiment Station.
- Sardoy, N., J.-L. Consalvi, A. Kaiss, A. C. Fernandez-Pello, and B. Porterie. 2008. Numerical study of ground-level distribution of firebrands generated by line fires. *Combustion and Flame*, **154**:478–488.
- Sardoy, N., J.-L. Consalvi, B. Porterie, and A. C. Fernandez-Pello. 2007. Modeling transport and combustion of firebrands from burning trees. *Combustion and Flame*, **150**:151–169.
- Schroeder, M. J. 1969. Ignition probability. Office Report 2106-1, Rocky Mountain Research Station.

- Scott, J. H. 2003. Canopy fuel treatment standards for the wildland-urban interface. In *Fire, Fuel Treatments and Ecological Restoration*, USDA Forest Service Proceedings RMRS-P-29, pp. 29–38. Fort Collins, Colorado.
- Scott, J. H. and R. E. Burgan. 2005. Standard fire behavior fuel models: A comprehensive set of use with rothermel's surface fire spread model. General Technical Report RMRS-GTR-153, USDA Forest Service, Rocky Mountain Research Station.
- Scott, J. H. and E. D. Reinhardt. 2001. Assessing crown fire potential by linking models of surface and crown fire behavior. Research Paper RMRS-RP-29, USDA Forest Service, Rocky Mountain Research Station.
- Shigesada, N. and K. Kawasaki. 1997. *Biological Invasions Theory and Practice*. Oxford University Press.
- Stauffer, H. B. 2008. *Contemporary Bayesian and Frequentist Statistical Research Methods for Natural Resource Scientists*. Wiley - Interscience.
- Sullivan, A. L. 2007a. A review of wildland fire spread modelling, 1990-present 1: Physical and quasi-physical models. *arXiv:0706.3074 [physics.geo-ph]*, p. 46.
- Sullivan, A. L. 2007b. A review of wildland fire spread modelling, 1990-present 3: Mathematical analogues and simulation models. *arXiv:0706.4130v1 [physics.geo-ph]*.
- Susott, R. A. 1982. Characterization of the thermal properties of forest fuels by combustible gas analysis. *Forest Science*, **28**(2):404–420.
- Sutton, O. G. 1953. *Micrometeorology*. McGraw-Hill, New York.
- Thomas, P. H. 1963. Size of flames from natural fires. In *Ninth International Symposium on Combustion*, pp. 844–859. Academic Press.
- Turner, M. F., W. W. Hargrove, R. H. Gardner, and W. H. Romme. 1994. Effects of fire on landscape heterogeneity in Yellowstone National Park, Wyoming. *Journal of Vegetation Science*, **5**(5):731–742.
- Van Wagner, C. 1977. Conditions for the start and spread of crown fire. *Canadian Journal of Forest Research*, **7**(1):23–24.
- Varner, J. M. and C. R. Keyes. 2009. Fuels treatments and fire models: Errors and corrections. *Fire Management Today*, **69**(3).
- Viegas, D. X. 1998. Forest fire propagation. *Philosophical Transactions of The Royal Society*, **356**(1748):2907–2928.

- Woycheese, J. P. 2001. Wooden disk combustion for spot fire spread. In *International Interflam Conference 9th Proceedings*, vol. 1, pp. 101–112.
- Woycheese, J. P., P. J. Pagni, and D. Liepmann. 1999. Brand propagation from large-scale fires. *Journal of Fire Protection Engineering*, **10**:32–44.

## APPENDIX

### Appendix 1: Input Parameters and Equations for Rothermel (1972) Model

#### Input Parameters Defined by User

$U$ , wind velocity at mid-flame height (mph)

$m_f$ , fuel particle moisture content (kg moisture / kg oven-dry wood)

$\phi$ , slope steepness (%)

#### Rothermel's Surface Fire Rate of Spread Model:

$$ROS = \frac{I_R \xi (1 + \phi_w + \phi_s)}{\rho_b \varepsilon Q_{ig}}$$

where

$$\begin{aligned}
I_R &= \Gamma' w_n h \eta_M \eta_s \\
\Gamma' &= \Gamma'_{max} (\beta / \beta_{op})^A e^{(A(1-\beta/\beta_{op}))} \\
\Gamma'_{max} &= \sigma^{1.5} (495 + 0.0594 \sigma^{1.5})^{-1} \\
\beta_{op} &= 3.348 \sigma^{-0.8189} \\
A &= 1 / (4.774 \sigma^{0.1} - 7.27) \\
\eta_M &= 1 - 2.59 (m_f / m_x) + 5.11 (m_f / m_x)^2 - 3.52 (m_f / m_x)^3 \\
\eta_s &= 0.174 S_e^{-0.19} \\
\xi &= (192 + 0.2595 \sigma)^{-1} e^{(0.792 + 0.681 \sigma^{0.5})(\beta + 0.1)} \\
\phi_w &= C U^B (\beta / \beta_{op})^{-E} \\
C &= 7.47 e^{(-0.133 \sigma^{0.55})} \\
B &= 0.02526 \sigma^{0.54} \\
E &= 0.715 e^{(-3.59 \times 10^{-4} \sigma)} \\
w_n &= w_o / (1 + S_T) \\
\rho_b &= w_o / \delta \\
\varepsilon &= e^{(-138 / \sigma)} \\
Q_{ig} &= 250 + 1116 m_f \\
\beta &= \rho_b / \rho_p
\end{aligned}$$

To see a full description of parameters and formulas, see Rothermel's 1972 article.

## Appendix 2: Formulas for Albini (1979) Model

In order to determine the maximum spot fire distance, Albini proposes the formulas given in equations (6), (7) and (8) where  $t_o$  is the time period of steady burning of the tree(s) canopy normalized by the characteristic time  $\left(\frac{2z_F}{w_F}\right)$ ,  $z_F$  is the horizontal height at the flame time (ft),  $z_o$  is the initial height of the firebrands (ft) which is equivalent to tree height,  $v_o$  is the initial vertical velocity of the firebrand ( $\text{ft sec}^{-1}$ ),  $w_F$  is the gas velocity at flame tip ( $\text{ft sec}^{-1}$ ),  $D$  is the diameter of the cylindrical firebrand,  $r$  is a parameter substitution for simplicity, and  $a$ ,  $b$  and  $B$  are empirical constants equal to 5.963 and 4.563 and 40 respectively. Albini include that

$$\begin{aligned}\frac{v_o}{w_F} &= B(D/z_F)^{0.5} \\ w_F &= 4.2z_F^{0.5} \\ D &= \frac{z_v + z_o}{0.39 \times 10^5} \\ r &= ((b + z_v/z_F)/a)^{0.5} \\ z_F &= 10.561DBH^{-0.1998} \\ t_o &= 8.1479DBH^{-0.3194}\end{aligned}$$

where  $DBH$  is the average tree diameter at breast height (inches). We retrieved formulas for  $z_F$  and  $t_o$  by performing a regression on the data provided by Albini (1979). Substituting the previous formulas and equations (7) and (8) into equation (6) solves for the maximum vertical height a firebrand can travel in order for it to land on the ground just as it is consumed,  $z_v$ , (ft). This value is used to solve equation (5) for the maximum spotting distance.

### Appendix 3: Program Code to Determine Probability of a Spot Fire Jumping a Fuelbreak

This program calls the `solve_for_traveltimes_function` (Appendix 5).

```
%*****Spot Fire Model*****
%Creators: Holly Perryman, Dr. Chris Dugaw,
%Dr. Morgan Varner, and Dr. Diane Johnson
%*****
deltat=30;%time step
deltax=7;%cell width [Each cell is deltax by deltax]
zometers = 30; %(m)
%Initial height of particle (equivalent to tree
%height) in feet
zo=zometers/0.3048; %(feet)
%Tree height converted to meters
UHplus=20*0.44704;
%Wind speed above tree top (meters per sec)-The first
%factor is MPH.
%VARIED: 10, 20, 30 mph
UH=UHplus/(log((20+1.18*zo)/(0.43*zo)));
%Finney page 15
%Tree top wind speed (meters per sec)
UHmph = UH*2.23693629;
%Converting tree top wind speed to mph b/c
%solve_for_traveltimes_function needs it as such
```



```

Td=20;

%Tree diameter (in)-this is used to find zF & tnot

Tc=1;

%Tree simulations-number of trees in the torch

[traveltimes, maxspotdist, maxspotcell] =
solve_for_traveltimes_function
    (deltax,deltat,zo,UHmph,Td,Tc);

%Returns traveltimes matrix, max spot dist (m), max spot
%dist (cells) Our model assumes that the trees recieving
%the firebrands are the same height as the trees sending
firebrandsbreak_widths=0:25:500;

%width 0 will make a breakwidth of 1 cell
meds=zeros(size(break_widths));

%Matrix to record results
for breakindex=1:length(break_widths)
%for gathering probability for each index
breakwidth = break_widths(breakindex);
%fuelbreak width (m)
breakcolumns = round(breakwidth/deltax);
%fuelbreak width (columns)
L=2*maxspotcell+breakcolumns;%Length in columns
W=301;%Width in meters
n=W/deltax;%Number of rows in the burning area
m=L;%Number of columns in the burning area

%*****Identify Parameters*****

```

```

%%*****PARAMETERS VARIED:
moisturecontent=0.094;
%Fuel moisture content
%Units: (kg moisttrue / kg ovendry wood)
%VARIED : 0.063, 0.094, 0.124
mf=(moisturecontent)*ones(n,m);
%Generates matrix of moisture content values
crownheight=15; %(m)
%Canopy base height
%VARIED : 10, 15, 20 (Morgan 9/9/09)
z=(crownheight)*ones(n,m);
%Generating matrix for canopy base height values
fuelloading = 10*3.28*3.28*2000*.454*(1/43560);
%Ovendry fuel loading [Roth 72 pg 10]
%Units: converted to (kg m-2)
%Value from Scott and Burgan 2005 TL8 [pg. 18]
%VARIED: 5, 10, 15 (Morgan on 2/20/09)
wo=(fuelloading)*ones(n,m);
%Generating data matrix for fuel loading values
jumpcount = 0;
%Defines the counter tracking the total jumps
sims = 2500;
%Defining the number of simulations to run
for s=1:sims %Loop beginning simulations
%%***** UNVARIED PARAMETERS

```

```

betatorch=0.001;

%This is the beta for the logistic that is the
%probability of a torch
sigmay=deltax/2;

%sigma for the normal distribution of the firebrands
%landing perpendicular to wind
spotlambda = 0.005;

%The lambda for altering the probability of spot ignition
%based on the distance the firebrand lands away from
%the torch
numbf=50;

%Number of firebrands which land on ground ignited
CanopyCover = 40;

%Canopy cover (%)
cffactor = (CanopyCover/100)*(pi()/12);

%Crown Filling Factor pg 18 Finney 04;
Umps = (0.555*UHplus)/(sqrt(cffactor*3.28*zometers)*
    log((20+1.18*zometers)/(0.43*zometers)));

%Wind speed at midflame height (meters per second)
%Roth 77 page 27;
Umph=Umps*2.23693629;

%Convert Umps to miles per hour
depth = 0.3*(1/3.2802)*ones(n,m);

%Value from Scott and Burgan 2005 pg 18
%Units: converted to m

```

```

pb = wo./depth;

%Bulk density-Units: converted to (kg m-3)

%density of the fuel bed [pg 157 Johnson & Miyanishi 01]
fuelmineralcontent=0.0555;

%Fuel mineral content [Roth 72 pg 10]

%Units: (kg minerals / kg oven-dry wood)

%Got value from Roth 72 page 36

%The higher the value, the more it should slow the spread
St=(fuelmineralcontent)*ones(n,m);

%Generating data matrix for fuel mineral content values

%Wo and St are needed to calculate wn [Eqn 24 Roth 72]
heatcontent=8000*(2.20462262)*(1.055056);

%heat content of fuel-Units: converted to (kJ kg-1)

%Roth 72 page 6

h=(heatcontent)*ones(n,m);

%Generating matrix for heat content values

mc=80;

%canopy moisture content.
moistureextinction=0.35;

%moisture content of extinction - If the moisture content
%was this, fire would not propagate through the fuel

%Units: (kg moisture / kg oven-dry wood)

%Value from Scott and Burgan 2005
mx=(moistureextinction)*ones(n,m);

%Generating data matrix for moisture extinction values

```

```

%mx and mf are needed to calculate nm [Eqn 29 Roth 72]
mineralcontent=0.010;
%Effective mineral content silica free
%Units: (kg silica-free minerals / kg oven-dry wood)
%Got value from Roth 72 page 36
Se=(mineralcontent)*ones(n,m);
%Generating data matrix for mineral content values
%Se needed for calculating ns [Eqn 30 Roth 72]
particledensity = 32*(0.45359237)*(1/0.3048)^3;
%Fuel particle density (weighted average)
%Units: converted to (kg m-3)
%Value from Roth 72 (However he calls it pb!)
%Scott and Burgan 2005 clarified the error
beta = pb/particledensity;
%Packing ratio of fuel array [pg 13 Roth 72]
%Units: (dimensionless)
SAVR= 1770*(3.2808);
%Defining the surface-area-to-volume-ratio
%Units: converted to (m-1)
%Value from Scott and Burgan 2005
sigma=(SAVR)*ones(n,m);
%Defining fuel particle surface-area-to-volume-ratio
%***** PARAMETERS FOR THE TORCH MODEL
c=0.010;
%constant for Eqn 4 Van Wagner 77 for determining Io

```

```

heatofcombustion=22*(1000);
%heat of combustion of fuel
%Units: (kJ kg-1)
%Van Wagner 77 page 3
%Got value from Susott 1982 Table 3 (MJ)
H=(heatofcombustion)*ones(n,m);
%Generating matrix for heat of combustion values
%%***** PARAMETERS FOR FROUDE NUMBER (SARDOY 08)
cpg = 1121;%Specific heat of gas (Sardoy 08)
pambient = 1.1;%Ambient density (Sardoy 08)
tambient = 300;%Ambient temperature (Sardoy 08)
gravity = 9.8;%Gravity meters per second
%*****
%-----Surface Fire Model-----
% The surface fire model originates from Roth. 1972.
%However, some of the calculations have been adjusted
%for unit conversions. These adjusted equations were
%given by 'Wildland Fire Fundamentals' starting on pg 37.
%NOTE: Roth. 72 has a nice summary of equations on pg 26
%*****CALCULATING Ir [Eqn 27 Roth 72]*****
%Calculate moisture damping coefficient (nm)
%(dimensionless) [Eqn 29 Roth 72]
nm=ones(n,m);
%Defining moisture damping coefficient matrix
nm=1-(2.59)*(mf./mx)+5.11*(mf./mx).^ (2)-3.52*(mf./mx).^ (3);

```

```

%Calculating moisture damping coefficient
%INPUT: mf, unitless ; mx, unitless
%Calculate mineral damping coefficient (ns)
%(dimensionless) [Eqn 30 Roth 72]
ns=ones(n,m);
%Defining mineral damping coefficient
ns=0.174*(Se).^(-0.19);
%Calculating mineral damping coefficient
%Input: Se, unitless
%Calculate net initial fuel loading (wn)
%Units: (Kg m(-2)) [Eqn 24 Roth 72]
%Since we converted wo, wn will be converted as well.
wn=ones(n,m);
%Defining net initial fuel loading
wn=(wo).*(1+St);
%Calculating net initial fuel loading (kg m(-2))
%INPUT: wo, (kg m(-2)); St, unitless
%In Roth. 72, he says the equation is wn=(wo)./(1+St),
%however 'Wildland Fire Fundamentals' states the equation is
%as above.
%Note: sigma in Roth. 72 has units (ft(-1)) or (cm(-1))
%from 'Wildland Fire Fundamentals'. Thus, we must convert
%the coefficients on sigma to account that sigma is (m(-1))
%This is also done when we A, betaop, C, B, E, xi, and e.
%Calculating maximum reaction velocity (gammamax)

```

```

%[Eqn 36 Roth 72]

%Generating matrix of converted sigma values
gammamax=ones(n,m);

%Defining maximum reaction velocity
gammamax=(0.0591+2.926*100^(1.5)*sigma.^(-1.5)).^(-1).*(
    (1/60));

%Calculating maximum reaction velocity
%Units: Converted to (sec^{-1})
%Eqn: From 'Wildland Fire Fundamentals' with
%[sigma]=(cm^{-1})
%Calculating arbitrary constant (A) (dimensionless)
%[Eqn 39 Roth 72]
A=ones(n,m);

%Defining arbitrary constant matrix
A=8.9033*100^(0.7913)*sigma.^(-0.7913);

%Calculating arbitrary constant
%Eqn: From 'Wildland Fire Fundamentals' with
%[sigma]=(cm^{-1})
%Calculating optimum packing ratio of fuel array (betaop)
%(dimensionless) [Eqn 37 Roth 72]
betaop=ones(n,m);

%Defining optimum packing ratio matrix
betaop=3.348*((3.2808^(0.8189))*sigma.^(-0.8189));

%Calculating optimum packing ratio
%INPUT: sigma, 1/ft

```



```

%Calculating optimum reaction velocity (gamma)
%Units: (sec-1) [Eqn 38 Roth 72]
%Since we converted gammamax, gamma converted as well.
gamma=ones(n,m);
%Defining optimum reaction velocity
gamma=gammamax.*(beta./betaop).^ (A).*
    exp(A.*(1-(beta./betaop)));
%Calculating optimum reaction velocity
%INPUT: gammamax, sec-1 ; beta, unitless ; betao,
%unitless; A, unitless
%Calculating reaction intenstity (Ir)
%Units: (kJ m-2 sec-1) [Eqn 27 Roth 72]
Ir=ones(n,m);
%Defining reaction intenstity
Ir=wn.*h.*gamma.*nm.*ns;
%Calculating reaction intenstity
%*****
%*****CALCULATE HEAT OF IGNITION*****
%Calculate heat of ignition (Qig)
%Units:(kJ kg-1) [Eqn. from 'Wildland Fire
%Fundamentals' Figure 1.24]
QigRoth = 250+1116*mf;
%Equation for Qig from Roth 72
QigRoth = QigRoth*(2.25)*(1.055056);
%Converting units from BTU / lb to (kJ kg-1)

```

```

%Calculating heat of ignition
%Units: (kJ kg{-1})

QigWagner = 460+25.9*mc;

%Calculating heat of Ignition from Van Wagner 76
%Units: (kJ kg{-1})

%*****
%*****CALCULATE WIND FACTOR*****

%Although the concept is from Roth. 72, these equations are
%converted to take sigma with units (cm{-1}) ('Wildland
%Fire Fundamentals')

U = Umph*1609.344*(1/60)*ones(n,m);

%Windspeed at midflame height-units (m/min)

C = 7.47*exp(-0.87113*100{-0.55}*sigma.{.55});

%Roth 72 eqn 82

B = 0.15988*100{-0.54}*sigma.{.54};

%Roth 72 eqn 83

E = 0.715*exp(-0.01094*(100{-1})*sigma);

%Roth 72 eqn 84

wind = C.*(3.281*U).{B}.*(beta./betaop).{-E};

%Calculating wind factor

%Roth 72 eqn 47

%*****
%*****CALCULATE RATE OF SPREAD*****

xi = (192+(0.2595)*3.2808{-1}*sigma).{-1}.*

```

```

exp((0.792+0.681*(3.2808^(-.5))*sigma.^(.5)).*(beta+0.1));
%Calculating Propagating Flux Ratio
%Roth 72 Eqn. 76 page 32
e = exp(-4.528*(100)*sigma.^(-1));
%Calculating Effective Heating Number
%Roth 72 eqn 14
ROSwind = Ir.*xi.*(1+wind)./(pb.*e.*QigRoth);
%Calculating ROS-Units: m/s
%Roth 72 equation 75 (no slope)
ROSchain = ROSwind*60*60*3.28*(1/66);
%Converting ROS to chains per hour
%*****END OF SURFACE FIRE MODEL*****

%%-----DETERMINE PROBABILITY OF A TORCH-----
%Calculate Headfire intensity (I) (kW m{-1})
%[Eqn. 3.3 Byram 59]
I=ones(n,m);
%Defining Intensity matrix
I=H.*wo.*ROSwind;
%Setting Intensity matrix to Calculated intensity values
Iunit = I(1)*(1/3.28)*(1/1.055056);
%Convert I to btu per sec per feet
%Calculate the critical value for the fire intensity (Io)
%Using Eqn. 4 Van Wagner 77
Io=ones(n,m);

```

```

%Defining matrix of Critical fire intensity
Io=(c*z.*QigWagner).^(3/2);
%Calculating Critical fire intensity values for matrix
%Using Eqn. 4 Van Wagner 77
%Units on Io: kW m-1
torchfunction=exp(-betatorch*(Io-I))./(
    (1+exp(-betatorch*(Io-I))));
%lognormal distribution
%function of Intensity, I
randomtorch=rand(n,m);
%Matrix of random numbers between 0 and 1
treetorch = randomtorch<torchfunction;
%If 1 it will torch, if 0 it will not torch
%%-----END OF DETERMINING PROBABILITY OF A TORCH-----

%%*****CALCULATE MATRIX OF TRAVEL TIMES*****
%This was done in the initial set up of the matrix
%In traveltimes:
%row 1 is the Number of cells away from torch
%row 2 is the vertical travel time to reach value in row 1
%(time steps)
%%*****END OF CALCULATING MATRIX OF TRAVEL TIMES*****

%-----CALCULATE PROBABILITY OF IGNITION-----
%Used Schroeder 69 & Bradshaw 83(based on Schroeder 69)

```

```

Qigmf = QigRoth*(1/4.1868);
Qigmx = (250+1116*mx)*(2.25)*(1.055056)*(1/4.1868);
%Using the eqn from Roth 72 to calculate Qig for mf and mx.
%Units converted to cal per gram
probignx = (Qigmx - Qigmf)/10;
%Eqn 79 Bradshaw 1983
k3=.0000124;
k4=4.58;
%Empirical constants (Bradshaw 1983 page 27)
%Functions of the dead moisture of extinction
probign = ((k3*probignx.^k4)/50);
%Calculates the probability of ignition (decimal)
%-----END CALCULATE PROBABILITY OF IGNITION-----

%%*****FROUDE NUMBER*****
%The Froude number was used by Sardoy 08 to determine the
%type of plume the fire is producing. The plume type
%determined the way xsigma and xmu are determined for
%the probability distribution determining where firebrands
%landed parallel with the wind.
Imw=I*0.001;
%Convert Intensity to MW per m for finding xmu
Icanopy = I(1) + 15;
%The Froude number depends on the intensity produced by the
%entire fire. When torching occurs, the fire intensity

```

```

%increases. Sardoy developed a model for this, however for
%our simplicity we just increases the fire intensity by 15.
%This is crude simplification and future work on this model
%would include including Sardoy et. al.s model adapting
%the fire intensity with the torch.
LC = (10^3*Icanopy/(pambient*cpg*gravity^.5))^.66;
%For determining the Froude Number (FR)
FR = UH/sqrt(gravity*LC);
%The Froude Number
if (FR > 1)
%If the Froude number is greater than one use the following
%to calculate mean and standard deviation
xmu=Imw(1)^0.26*UH^0.11;
%Sardoy 08; this calculates parameter is needed to
%calculate the appropriate sigma for the FB distribution
mufb=1.32*xmu-0.02;
%mu from Sardoy 08 page 488
xsigma=Imw(1)^-0.01*UH^-0.02;
%Sardoy 08; this calculates parameter is needed to
%calculate the appropriate sigma for the FB distribution
sigmafb=4.95*xsigma-3.48;
%sigma from Sardoy 08 page 488
else %(Fr < 1)
%If the Froude number is greater than one, than use the
%following calculations determine xsigma

```

```

xmu=Imw(1)^0.54*UH^-0.55;

%Sardoy 08; this parameter is needed to calculate
%the appropriate sigma for the FB distribution
mufb=1.47*xmu+1.14;

%mu from Sardoy 08 page 488
xsigma=Imw(1)^0.21*UH^0.44;

%Sardoy 08; this parameter is needed to calculate the
%appropriate sigma for the FB distribution
sigmafb=0.86*xsigma+0.19;

%sigma from Sardoy 08 page 488
end;

%%*****END CALCULATE FROUDE NUMBER*****

%-----PARAMETERS FOR MAIN LOOP-----

fbcount=zeros(n,m);

%Matrix counting how many fire brands are in each cell
fbtotalcount=zeros(n,m);

%Counting the total number of fbcount
distfromtorch=zeros(n,m);

%Defines matrix monitoring fb dist from torch cell
ignitesteps=inf*ones(n,m);

%Creating matrix that monitors the time steps that must
%pass before a spot fire from any torch ignites. It will
%ignite when the the cell value is less than or equal to
%zero, that is why it is a matrix of inf
fboutcount=0;

```

```

%This counter tracks the number of firebrands that land
%outside of the grid containing the wildfire
burnstate=zeros(n+2,m+2);
%Creating matrix for determining burn state of the grid.
%Burnstate monitors a fire with spots. There
%is buffer of zeros around it. The buffer helps
%determine if the fireline spreads
burnstate(2:n+1,maxspotcell+2:maxspotcell+2+breakcolumns)=2;
%Defining fuelbreak
burnstate(2:n+1,2)=4;
%Creates a line of 4's in the far left of the data grids
%initializing the fire
%0 means unburned
%2 means torch
%1 means fuelbreak
%3 means spot
%4 means burning
probrand = rand(n,m);
%A matrix of random numbers between 0 and 1. Used to
%determine if cell ignites if firebrands land there
%%*****END OF PARAMTERS FOR LOOP
%*****MAIN LOOP*****
for i=1:n %Changing the rows
for j=1:m %Changing the columns
if ((burnstate(i+1,j+1)>=3) && (treetorch(i,j)==1))

```



```

treetorch(i,j)=0;

%Clears torch from treetorch so it won't send out more
%firebrands the next time around
fbcount=zeros(n,m);

%Clears fbcount for the new torch
burnstate(i+1,j+1)=1;

distfromtorch=inf*ones(n,m);

%Clears distfromtorch for new torch
randfb=randn(numfb,1);

%matrix of normally random numbers
distxfb=exp(mu_fb+sigma_fb*randfb);

%Converts randfb to a lognormal distribution with mu
%and sigma from Sardoy 08. This will give the distances
%that the firebrands are going to travel based on the
%findings of Sardoy 08.
cellxfb=round(distxfb./deltax);

%Converts the distances the firebrands will travel
%horizontally to cells in the matrix
distyfb=sigma_y*randn(numfb,1);
cellyfb=round(distyfb);

if ((i+cellyfb(k)<=0) || (j+cellxfb(k)<=0) ||
    (i+cellyfb(k)>n) || (j+cellxfb(k)>m))

fboutcount = fboutcount + 1;

end; %Recording FB's landing outside burnstate
if ((i+cellyfb(k)>0) && (j+cellxfb(k)>0) &&

```

```

        (i+cellyfb(k)<=n) && (j+cellxfb(k)<=m))
fbcount(i+cellyfb(k), j+cellxfb(k))=
    fbcount(i+cellyfb(k), j+cellxfb(k)) + 1;
if (fbcount(i+cellyfb(k), j+cellxfb(k))==1)
distfromtorch(i+cellyfb(k), j+cellxfb(k)) =
    round(sqrt(cellxfb(k)^2+cellyfb(k)^2));
end %Recording FB distances from torch (cells)
end; %Recording FB's landing within burnstate
end; %End creating fbcount matrix & distfromtorch matrix
fbtotalcount = fbtotalcount + fbcount;
probignmulti=exp(-spotlambda*distfromtorch);
probspotign=probign.*probignmulti;
fbign = probrand<probspotign;
end; %End statement checking for a torch in burnstate
if ((burnstate(i+1,j+1)==0) &&
    (fbtotalcount(i,j)>=1) && (fbign(i,j)==1))
%If all true a spot fire will ignite at that cell
burnstate(i+1,j+1)=3;
end;
end; %End loop for j counter
end; %End loop for i counter
%The i & j loops to check individual cells
jump = sum(sum(burnstate(
    2:n+1,maxspotcell+breakcolumns+3:m+1)))/3;
%Displays how many spots occur beyond the break

```

```

if (jump>0)
jumpcount = jumpcount + 1;
end;
end; %end loop for simulations
meds(breakindex)=jumpcount/sims;
end; %for loop through breakwidths
%Display ***
figure(2)
h=plot(break_widths,meds,'k-o');
set(gca,'FontSize',16)
xlabel('Fuelbreak Widths (m)')
ylabel('P(jump)')
set(gcf,'Color',[1,1,1])
set(h,'MarkerSize',8)
%End Display ***
save prob_jump_ch_meds.mat; %Save data
%*****

```

#### Appendix 4: Program Code to Determine Average Rate of Spread with Spotting

This program calls `solve_for_traveltimes_function` (Appendix 5) and `create_ignitetimes_function_new` (Appendix 6).

```
%*****Spot Fire Model*****
%Creators: Holly Perryman, Dr. Chris Dugaw,
%Dr. Morgan Varner, and Dr. Diane Johnson
%*****
fronts=[];
frontsno=[];
%Initializes matrices that record front distance for
%all simulations
for frontloop=1:1:2500;
%Initializes 2500 simulations
front=[];
frontnospot=[];
%initial matrices that record front distance for
%individual simulations
tmax=10*60*60;%total time in secs the for loop will run
deltat=30;%time step
deltax=7;%cell width [Each cell is deltax by deltax]
zometers = 30; %(m)
%Initial height of particle (equivalent to tree
%height) in feet
```

```

zo=zometers/0.3048; %(feet)

%Tree height converted to meters
UHplus=20*0.44704;

%Wind speed above tree top (meters per sec)-The first
%factor is MPH.

%VARIED: 10, 20, 30 mph
UH=UHplus/(log((20+1.18*zo)/(0.43*zo)));

%Finney page 15

%Tree top wind speed (meters per sec)
UHmph = UH*2.23693629;

%Converting tree top wind speed to mph b/c
%solve_for_traveltimes_function needs it as such
Td=20;

%Tree diameter (in)-this is used to find zF & tnot
Tc=1;

%Tree simulations-number of trees in the torch
[traveltimes, maxspotdist, maxspotcell] =
solve_for_traveltimes_function
    (deltax,deltat,zo,UHmph,Td,Tc);

%Returns traveltimes matrix, max spot dist (m), max spot
%dist (cells) Our model assumes that the trees recieving
%the firebrands are the same height as the trees sending
%the firebrands

L=6*maxspotcell;%Length in columns
W=301;%Width in meters

```

```

n=W/deltax;%Number of rows in the burning area
m=L;%Number of columns in the burning area

%*****Identify Parameters*****

%%*****PARAMETERS VARIED:
moisturecontent=0.094;
%Fuel moisture content
%Units: (kg moisture / kg oven-dry wood)
%VARIED : 0.063, 0.094, 0.124
mf=(moisturecontent)*ones(n,m);
%Generates matrix of moisture content values
crownheight=15; %(m)
%Canopy base height
%VARIED : 10, 15, 20 (Morgan 9/9/09)
z=(crownheight)*ones(n,m);
%Generating matrix for canopy base height values
fuelloading = 10*3.28*3.28*2000*.454*(1/43560);
%Oven-dry fuel loading [Roth 72 pg 10]
%Units: converted to (kg m-2)
%Value from Scott and Burgan 2005 TL8 [pg. 18]
%VARIED: 5, 10, 15 (Morgan on 2/20/09)
wo=(fuelloading)*ones(n,m);
%Generating data matrix for fuel loading values

%%***** UNVARIED PARAMETERS
betatorch=0.001;
%This is the beta for the logistic that is the

```

```

%probability of a torch
sigmay=deltax/2;
%sigma for the normal distribution of the firebrands
%landing perpendicular to wind
spotlambda = 0.005;
%The lambda for altering the probability of spot ignition
%based on the distance the firebrand lands away from
%the torch
numbf=50;
%Number of firebrands which land on ground ignited
CanopyCover = 40;
%Canopy cover (%)
cffactor = (CanopyCover/100)*(pi()/12);
%Crown Filling Factor pg 18 Finney 04;
Umps = (0.555*UHplus)/(sqrt(cffactor*3.28*zometers)*
    log((20+1.18*zometers)/(0.43*zometers)));
%Wind speed at midflame height (meters per second)
%Roth 77 page 27;
Umph=Umps*2.23693629;
%Convert Umps to miles per hour
depth = 0.3*(1/3.2802)*ones(n,m);
%Value from Scott and Burgan 2005 pg 18
%Units: converted to m
pb = wo./depth;
%Bulk density-Units: converted to (kg m-3)

```

```

%density of the fuel bed [pg 157 Johnson & Miyanishi 01]
fuelmineralcontent=0.0555;
%Fuel mineral content [Roth 72 pg 10]
%Units: (kg minerals / kg oven-dry wood)
%Got value from Roth 72 page 36
%The higher the value, the more it should slow the spread
St=(fuelmineralcontent)*ones(n,m);
%Generating data matrix for fuel mineral content values
%Wo and St are needed to calculate wn [Eqn 24 Roth 72]
heatcontent=8000*(2.20462262)*(1.055056);
%heat content of fuel-Units: converted to (kJ kg-1)
%Roth 72 page 6
h=(heatcontent)*ones(n,m);
%Generating matrix for heat content values
mc=80;
%canopy moisture content.
moistureextinction=0.35;
%moisture content of extinction - If the moisture content
%was this, fire would not propagate through the fuel
%Units: (kg moisture / kg oven-dry wood)
%Value from Scott and Burgan 2005
mx=(moistureextinction)*ones(n,m);
%Generating data matrix for moisture extinction values
%mx and mf are needed to calculate nm [Eqn 29 Roth 72]
mineralcontent=0.010;

```



```

%Effective mineral content silica free
%Units: (kg silica-free minerals / kg oven-dry wood)
%Got value from Roth 72 page 36
Se=(mineralcontent)*ones(n,m);
%Generating data matrix for mineral content values
%Se needed for calculating ns [Eqn 30 Roth 72]
particledensity = 32*(0.45359237)*(1/0.3048)^3;
%Fuel particle density (weighted average)
%Units: converted to (kg m-3)
%Value from Roth 72 (However he calls it pb!)
%Scott and Burgan 2005 clarified the error
beta = pb/particledensity;
%Packing ratio of fuel array [pg 13 Roth 72]
%Units: (dimensionless)
SAVR= 1770*(3.2808);
%Defining the surface-area-to-volume-ratio
%Units: converted to (m-1)
%Value from Scott and Burgan 2005
sigma=(SAVR)*ones(n,m);
%Defining fuel particle surface-area-to-volume-ratio
%***** PARAMETERS FOR THE TORCH MODEL
c=0.010;
%constant for Eqn 4 Van Wagner 77 for determining Io
heatofcombustion=22*(1000);
%heat of combustion of fuel

```

```

%Units: (kJ kg{-1})
%Van Wagner 77 page 3
%Got value from Susott 1982 Table 3 (MJ)
H=(heatofcombustion)*ones(n,m);
%Generating matrix for heat of combustion values
%%***** PARAMETERS FOR FROUDE NUMBER (SARDOY 08)
cpg = 1121;%Specific heat of gas (Sardoy 08)
pambient = 1.1;%Ambient density (Sardoy 08)
tambient = 300;%Ambient temperature (Sardoy 08)
gravity = 9.8;%Gravity meters per second
%*****
%-----Surface Fire Model-----
% The surface fire model originates from Roth. 1972.
%However, some of the calculations have been adjusted
%for unit conversions. These adjusted equations were
%given by 'Wildland Fire Fundamentals' starting on pg 37.
%NOTE: Roth. 72 has a nice summary of equations on pg 26
%*****CALCULATING Ir [Eqn 27 Roth 72]*****
%Calculate moisture damping coefficient (nm)
%(dimensionless) [Eqn 29 Roth 72]
nm=ones(n,m);
%Defining moisture damping coefficient matrix
nm=1-(2.59)*(mf./mx)+5.11*(mf./mx).^ (2)-3.52*(mf./mx).^ (3);
%Calculating moisture damping coefficient
%INPUT: mf, unitless ; mx, unitless

```

```

%Calculate mineral damping coefficient (ns)
%(dimensionless) [Eqn 30 Roth 72]
ns=ones(n,m);
%Defining mineral damping coefficient
ns=0.174*(Se).^(-0.19);
%Calculating mineral damping coefficient
%Input: Se, unitless
%Calculate net initial fuel loading (wn)
%Units: (Kg m(-2)) [Eqn 24 Roth 72]
%Since we converted wo, wn will be converted as well.
wn=ones(n,m);
%Defining net initial fuel loading
wn=(wo).*(1+St);
%Calculating net initial fuel loading (kg m(-2))
%INPUT: wo, (kg m{-2}); St, unitless
%In Roth. 72, he says the equation is wn=(wo)./(1+St),
%however 'Wildland Fire Fundamentals' states the equation is
%as above.
%Note: sigma in Roth. 72 has units (ft(-1)) or (cm(-1))
%from 'Wildland Fire Fundamentals'. Thus, we must convert
%the coefficients on sigma to account that sigma is (m(-1))
%This is also done when we A, betaop, C, B, E, xi, and e.
%Calculating maximum reaction velocity (gammamax)
%[Eqn 36 Roth 72]
%Generating matrix of converted sigma values

```

```

gammamax=ones(n,m);

%Defining maximum reaction velocity
gammamax=(0.0591+2.926*100^(1.5)*sigma.^(-1.5)).^(-1).*(
    (1/60));

%Calculating maximum reaction velocity
%Units: Converted to (sec^{-1})
%Eqn: From 'Wildland Fire Fundamentals' with
%[sigma]=(cm^{-1})
%Calculating arbitrary constant (A) (dimensionless)
%[Eqn 39 Roth 72]
A=ones(n,m);

%Defining arbitrary constant matrix
A=8.9033*100^(0.7913)*sigma.^(-0.7913);

%Calculating arbitrary constant
%Eqn: From 'Wildland Fire Fundamentals' with
%[sigma]=(cm^{-1})
%Calculating optimum packing ratio of fuel array (betaop)
%(dimensionless) [Eqn 37 Roth 72]
betaop=ones(n,m);

%Defining optimum packing ratio matrix
betaop=3.348*((3.2808^(0.8189))*sigma.^(-0.8189));

%Calculating optimum packing ratio
%INPUT: sigma, 1/ft
%Calculating optimum reaction velocity (gamma)
%Units: (sec^{-1}) [Eqn 38 Roth 72]

```

```

%Since we converted gammamax, gamma converted as well.
gamma=ones(n,m);
%Defining optimum reaction velocity
gamma=gammamax.*(beta./betaop).^ (A).*
    exp(A.*(1-(beta./betaop)));
%Calculating optimum reaction velocity
%INPUT: gammamax, sec-1 ; beta, unitless ; betao,
%unitless; A, unitless
%Calculating reaction intensity (Ir)
%Units: (kJ m-2 sec-1) [Eqn 27 Roth 72]
Ir=ones(n,m);
%Defining reaction intensity
Ir=wn.*h.*gamma.*nm.*ns;
%Calculating reaction intensity
%*****
%*****CALCULATE HEAT OF IGNITION*****
%Calculate heat of ignition (Qig)
%Units:(kJ kg-1) [Eqn. from 'Wildland Fire
%Fundamentals' Figure 1.24]
QigRoth = 250+1116*mf;
%Equation for Qig from Roth 72
QigRoth = QigRoth*(2.25)*(1.055056);
%Converting units from BTU / lb to (kJ kg-1)
%Calculating heat of ignition
%Units: (kJ kg-1)

```

```

QigWagner = 460+25.9*mc;

%Calculating heat of Ignition from Van Wagner 76
%Units: (kJ kg{-1})

%*****
%*****CALCULATE WIND FACTOR*****

%Although the concept is from Roth. 72, these equations are
%converted to take sigma with units (cm{-1}) ('Wildland
%Fire Fundamentals')

U = Umph*1609.344*(1/60)*ones(n,m);
%Windspeed at midflame height-units (m/min)

C = 7.47*exp(-0.87113*100{-0.55}*sigma.{.55});

%Roth 72 eqn 82

B = 0.15988*100{-0.54}*sigma.{.54};

%Roth 72 eqn 83

E = 0.715*exp(-0.01094*(100{-1})*sigma);

%Roth 72 eqn 84

wind = C.*(3.281*U).{B}.*(beta./betaop).{-E};

%Calculating wind factor

%Roth 72 eqn 47

%*****
%*****CALCULATE RATE OF SPREAD*****

xi = (192+(0.2595)*3.2808{-1}*sigma).{-1}.*
exp((0.792+0.681*(3.2808{-0.5})*sigma.{.5})).*(beta+0.1));

%Calculating Propagating Flux Ratio

```

```

%Roth 72 Eqn. 76 page 32
e = exp(-4.528*(100)*sigma.^(-1));
%Calculating Effective Heating Number
%Roth 72 eqn 14
ROSwind = Ir.*xi.*(1+wind)./(pb.*e.*QigRoth);
%Calculating ROS-Units: m/s
%Roth 72 equation 75 (no slope)
ROSchain = ROSwind*60*60*3.28*(1/66);
%Converting ROS to chains per hour
%*****END OF SURFACE FIRE MODEL*****

%%-----DETERMINE PROBABILITY OF A TORCH-----
%Calculate Headfire intensity (I) (kW m-1)
%[Eqn. 3.3 Byram 59]
I=ones(n,m);
%Defining Intensity matrix
I=H.*wo.*ROSwind;
%Setting Intensity matrix to Calculated intensity values
Iunit = I(1)*(1/3.28)*(1/1.055056);
%Convert I to btu per sec per feet
%Calculate the critical value for the fire intensity (Io)
%Using Eqn. 4 Van Wagner 77
Io=ones(n,m);
%Defining matrix of Critical fire intensity
Io=(c*z.*QigWagner).^(3/2);

```

```

%Calculating Critical fire intensity values for matix
%Using Eqn. 4 Van Wagner 77
%Units on Io: kW m{-1}
torchfunction=exp(-betatorch*(Io-I))./(
    (1+exp(-betatorch*(Io-I))));
%lognormal distribution
%function of Intensity, I
randomtorch=rand(n,m);
%Marix of random numbers between 0 and 1
treetorch = randomtorch<torchfunction;
%If 1 it will torch, if 0 it will not torch
%%-----END OF DETERMINING PROBABILITY OF A TORCH-----

%%*****CALCULATE MARTIRX OF TRAVEL TIMES*****
%This was done in the initial set up of the matrix
%In traveltimes:
%row 1 is the Number of cells away from torch
%row 2 is the vertical travel time to reach value in row 1
%(time steps)
%%*****END OF CALCULATING MARTIRX OF TRAVEL TIMES*****

%-----CALCULATE PROBABLITY OF IGNITION-----
%Used Schroeder 69 & Bradshaw 83(based on Schroeder 69)
Qigmf = QigRoth*(1/4.1868);
Qigmx = (250+1116*mx)*(2.25)*(1.055056)*(1/4.1868);

```



%Using the eqn from Roth 72 to calculate Qig for mf and mx.

%Units converted to cal per gram

probignx = (Qigmx - Qigmf)/10;

%Eqn 79 Bradshaw 1983

k3=.0000124;

k4=4.58;

%Empirical constants (Bradshaw 1983 page 27)

%Functions of the dead moisture of extinction

probign = ((k3\*probignx.^k4)/50);

%Calculates the probability of ignition (decimal)

%-----END CALCULATE PROBABILITY OF IGNITION-----

%%\*\*\*\*\*FROUDE NUMBER\*\*\*\*\*

%The Froude number was used by Sardoy 08 to determine the

%type of plume the fire is producing. The plume type

%determined the way xsigma and xmu are determined for

%the probability distribution determining where firebrands

%landed parallel with the wind.

Imw=I\*0.001;

%Convert Intensity to MW per m for finding xmu

Icanopy = I(1) + 15;

%The Froude number depends on the intensity produced by the

%entire fire. When torching occurs, the fire intensity

%increases. Sardoy developed a model for this, however for

%our simplicity we just increases the fire intensity by 15.

```

%This is crude simplification and future work on this model
%would include including Sardoy et. al.s model adapting
%the fire intensity with the torch.
LC = (10^3*Icanopy/(pambient*cpg*gravity^.5))^.66;
%For determining the Froude Number (FR)
FR = UH/sqrt(gravity*LC);
%The Froude Number
if (FR > 1)
%If the Froude number is greater than one use the following
%to calculate mean and standard deviation
xmu=Imw(1)^0.26*UH^0.11;
%Sardoy 08; this calculates parameter is needed to
%calculate the appropriate sigma for the FB distribution
mufb=1.32*xmu-0.02;
%mu from Sardoy 08 page 488
xsigma=Imw(1)^-0.01*UH^-0.02;
%Sardoy 08; this calculates parameter is needed to
%calculate the appropriate sigma for the FB distribution
sigmafb=4.95*xsigma-3.48;
%sigma from Sardoy 08 page 488
else %(Fr < 1)
%If the Froude number is greater than one, than use the
%following calculations determine xsigma
xmu=Imw(1)^0.54*UH^-0.55;
%Sardoy 08; this parameter is needed to calculate

```

```

%the appropriate sigma for the FB distribution
mufb=1.47*xmu+1.14;

%mu from Sardoy 08 page 488
xsigma=Imw(1)^0.21*UH^0.44;

%Sardoy 08; this parameter is needed to calculate the
%appropriate sigma for the FB distribution
sigmafb=0.86*xsigma+0.19;

%sigma from Sardoy 08 page 488
end;

%%*****END CALCULATE FROUDE NUMBER*****

%-----PARAMETERS FOR MAIN LOOP-----

fbcount=zeros(n,m);

%Matrix counting how many fire brands are in each cell
ignitesteps=inf*ones(n,m);

%Creating matrix that monitors the time steps that must
%pass before a spot fire from any torch ignites. It will
%ignite when the the cell value is less than or equal to
%zero, that is why it is a matrix of inf
fboutcount=0;

%This counter tracks the number of firebrands that land
%outside of the grid containing the wildfire
spotcount=0;

%Counter tracking firebrands landing inside grid
torchcount = 0;

```

```

%Counter tracking number of torches which occur
timeindex=0;

%Counter index to create vectors for average front distance
burnstate=zeros(n+2,m+2);
burnstatenospot=zeros(n+2,m+2);

%Creating matrixs for determining burn state of the grid.
%Burnstate monitors a fire with spots and burnstatenospot
%does not have spotting. we use burnstatenospot to aid
%in determing the adjustment factor for p(ROS). There
%is buffer of zeros around both. The buffer helps
%determine if the fireline spreads
burnstate(2:n+1,2)=4;
burnstatenospot(2:n+1,2)=4;

%Creates a line of 4's in the far left of the data grids
%initializing the fire
%0 means unburned
%2 means torch
%1 means fuelbreak
%3 means spot
%4 means burning
probrand = rand(n,m);

%A matrix of random numbers between 0 and 1. Used to
%determine if cell ignites if firebrands land there

%*****MAIN LOOP*****

```

```

for t=0:deltat:tmax;
%For loop for time steps
timeindex=timeindex+1;
%For recording front data
statesum=(burnstate(2:n+1,1:m)+burnstate(1:n,2:m+1)+
    burnstate(2:n+1,3:m+2)+burnstate(3:n+2,2:m+1));
statesumno=(burnstatenospot(2:n+1,1:m)+
    burnstatenospot(1:n,2:m+1)+burnstatenospot(2:n+1,3:m+2)+
    burnstatenospot(3:n+2,2:m+1));
%Sums the cell to the left, right, above & below fore
%each cell
randmatrix=rand(n,m);
%Generates a matrix of random numbers (size n by m)
pmatrix=ROSwind*deltat/deltax*0.5185;
%Probability that fire will spread into the next cell
%The adjustment factor is 0.5185. This factor equalizes the
%spread rate. We determined the factor by running 2500
%simulations with varied parameters set to medium values.
for i=1:n %Changing the rows
for j=1:m %Changing the columns
if ((burnstate(i+1,j+1)==4) && (treetorch(i,j)==1))
%The tree torches and launches firebrands
torchcount=torchcount+1;
fbcount=zeros(n,m);

```

```

%Clears fbcount for the new torch
distfromtorch=inf*ones(n,m);
%Clears distfromtorch for new torch
treetorch(i,j)=0;
%Clears torch from treetorch so it won't send out more
%firebrands the next time around
burnstate(i+1,j+1)=2;
%Changes the cell from burned to torched in burnstate
randfb=randn(numfb,1);
%matrix of normally random numbers
distxfb=exp(mufb+sigmafb*randfb);
%Converts randfb to a lognormal distribution with mu
%and sigma from Sardoy 08. This will give the distances
%that the firebrands are going to travel based on the
%findings of Sardoy 08.
cellxfb=round(distxfb./deltax);
%Converts the distances the firebrands will travel
%horizontally to cells in the matrix
distyfb=sigmay*randn(numfb,1);
%This is our normal distribution for where the
%firebrands will land perpendicular to the wind
cellyfb=round(distyfb);%Rounds distyfb
for k=1:numfb
if ((i+cellyfb(k)<=0) || (j+cellxfb(k)<=0) ||
    (i+cellyfb(k)>n) || (j+cellxfb(k)>m))

```

```

%If a firebrand lands outside of the grid, count it
fboutcount = fboutcount + 1;
end;
if ((i+cellyfb(k)>0) && (j+cellxfb(k)>0) &&
    (i+cellyfb(k)<=n) && (j+cellxfb(k)<=m))
%If the firebrand lands withing the grid, record it
%in fbcount
fbcount(i+cellyfb(k), j+cellxfb(k))=fbcount(i+cellyfb(k),
    j+cellxfb(k)) + 1;
if (fbcount(i+cellyfb(k), j+cellxfb(k))==1)
%If a cell has 1 firebrand, find the distance that cell is
%from the torch cell. I can get away with looking at a
%cell with just one because I am recording firebrands
%landing in fbcount one at a time.
distfromtorch(i+cellyfb(k), j+cellxfb(k)) =
    round(sqrt(cellxfb(k)^2+cellyfb(k)^2));
end %Recording FB distances from torch
end; %Recording FB's landing within burnstate
end; %End creating fbcount matrix & distfromtorch matrix
probignmulti=exp(-spotlambda*distfromtorch);
%Determines individual multipliers for re-calclating the
%probability of spot ignition based on a exponential decay
%determined by the distance the firebrand landed away from
%the torch
probspotign=1-(1-probign.*probignmulti).^fbcount;

```

```

%Re-calculates the probability of spot ignition based on
%the multiplier and if there are more firebrands in the
%cell than there is a great chance of ignition
fbign = probrand<probspotign;
%Determines if a cell ignites for this torch
ignitetimes = create_ignitetimes_function_new
    (distfromtorch,traveltimes,fbign,n,m,maxspotcell);
%This is the time steps that will need to pass before THIS
%torch will ignite spot fires. To keep track of the time
%steps for ALL torches we use ignitesteps.
ignitesteps(ignitetimes<ignitesteps)=
    ignitetimes(ignitetimes<ignitesteps);
%This replaces times in ignitesteps (time steps that need
% to pass before ALL spots ignite) with times in
% ignitetimes (time steps that need to pass before spots
% from the recent torch to ignite) if and only if the
% ignite times from the recent torch will ignite BEFORE the
% spots that are already waiting to ignite
end; %End statement checking for a torch in burnstate
if ((burnstate(i+1,j+1)==0) && (ignitesteps(i,j)~=inf))
%If all true a spot fire will ignite at that cell
if(ignitesteps(i,j)==0)
%If true the spot ignites during this time step
burnstate(i+1,j+1)=3; %ignite spot
spotcount = spotcount+1; %increment counter

```



```

end;

%Else, decrement ingitesteps and exit out of loop
ignitesteps(i,j)=ignitesteps(i,j)-1;
end;
end; %End loop for j counter
end; %End loop for i counter

%The i and j counters check individual cells in grid
%Propagate Fire
burnstate(2:n+1,2:m+1)=burnstate(2:n+1,2:m+1)+
    (4*(statesum>=2).*(randmatrix<pmatrix)).*
    (burnstate(2:n+1,2:m+1)==0));
burnstatenospot(2:n+1,2:m+1)=burnstatenospot(2:n+1,2:m+1)+
    (4*(statesumno>=2).*(randmatrix<pmatrix)).*
    (burnstatenospot(2:n+1,2:m+1)==0));

first=[];
firstno=[];

%Matrices that monitor the front distance
for frontindex=2:n+1;
first(frontindex-1)=
    find(burnstate(frontindex,2:end)==0,1,'first')-2;
firstno(frontindex-1)=
    find(burnstatenospot(frontindex,2:end)==0,1,'first')-2;
%Locate the first unburned cell in each row
end;

front(timeindex)=mean(first)*deltax;

```

```

frontnospot(timeindex)=mean(firstno)*deltax;
%Average front distance for each timeindex for timestep
end;%for loop
fronts=[fronts;front];
frontsno=[frontsno;frontnospot];
%Row indicates the simulation and columns are
%the front distances at each timeindex for that simulation
end;%firefront loop
meanfront=mean(fronts,1);
meanfrontnospot=mean(frontsno,1);
%averages the front dist for each timeindex over all
%simulations
%%%PLOTING RESULTS
stdadd=mean(fronts,1)+std(fronts,0,1);
stdsub=mean(fronts,1)-std(fronts,0,1);
stdnospotadd=mean(frontsno,1)+std(frontsno,0,1);
stdnospotsub=mean(frontsno,1)-std(frontsno,0,1);
figure(1);
clf('reset');
hold on;
plot((0:deltat:tmax)/60/60,meanfront,'k');
plot((0:deltat:tmax)/60/60,stdadd,'k--');
plot((0:deltat:tmax)/60/60,stdsub,'k--');
set(gca,'FontSize',16)
xlabel('Time (hours)')

```

```

ylabel('Average front distance from source (m)')
set(gcf,'Color',[1,1,1])
figure(2)
clf('reset');
hold on;
plot((0:deltat:tmax)/60/60,meanfrontnospot,'k');
plot((0:deltat:tmax)/60/60,stdnospotadd,'k--');
plot((0:deltat:tmax)/60/60,stdnospotsub,'k--');
set(gca,'FontSize',16)
xlabel('Time (hours)')
ylabel('Average front distance from source (m)')
set(gcf,'Color',[1,1,1])
%*****Create Average rate of spread plot*****
deriv=zeros(1,length(meanfront));
derivnospot=zeros(1,length(meanfrontnospot));
deriv(1)=(meanfront(1))/deltat;
derivnospot(1)=(meanfront(1))/deltat;
for i=1:length(meanfront)-1
deriv(i+1)=(meanfront(i+1)-meanfront(i))/deltat;
derivnospot(i+1)=(meanfrontnospot(i+1)-
    meanfrontnospot(i))/deltat;
end;
figure(3)
clf('reset');
hold on;

```

```

plot((0:deltat:tmax)/60/60,deriv,'k');
plot((0:deltat:tmax)/60/60,derivnospot,'r');
plot((0:deltat:tmax)/60/60,ROSwind(1),'k');
set(gca,'FontSize',16)
xlabel('Time (hours)')
ylabel('Rate of Spread (m s^{-1})')
set(gcf,'Color',[1,1,1])
ROSspots=mean(deriv(9*60*60/deltat+1:length(deriv)))
%print out Average rate of spread with spotting
%Saving Results
save Dist_From_Source_Meds.mat;
%*****

```

### Appendix 5: Program Code: solve\_for\_traveltimes\_function

```

function [traveltimes, spotdistmeter, distcell]=
    solve_for_traveltimes_function(deltax,deltat,zo,UH,Td,Tc)
%solve_for_traveltimes_function(deltax,zo,UH,Td)
%zo=firebrand initial height (tree height); UH=Wind
%speed; Td=tree diameter
a=5.963;%Albini 1979 D33 pg 52
b=4.563;%Albini 1979 D34 pg 52
B=40;%Albini 1979 D19 pg 49
g=32.174;
%acceleration of gravity ft/s^2[Albini 79 A54 page 34]
zF=26.184*log(Td)-18.528;
%Flame height in feet (Formula from Excel regression)
zFM = 1.0353*Tc^0.3882;%Flame height multiplier
zF = zF*zFM;%Adjusting flame height
tnot=10.301*Td^(-0.1909);
%Flame duration (Formula from Excel regression)
tnotM = 1.0061*Tc^(-0.1974);%Flame duration Multiplier
tnot = tnot*tnotM;%Adjusting flame duration
x=zo/zF;%Self made variable definition
wF=4.2*zF^(1/2);
%Note: If zF is in meters use: 2.3*zF^(1/2)
%Note: If zF is in feet use: 4.2*zF^(1/2)
%Albini 79 A58 page 35

```

```

UH=UH*0.0002777777778;

%Convert UH from mph to mps.
solvedz=fzero(@f,300)+zo/2;

%Solve for z (solvedz), the maximum vertical distance a
%particle will travel and will just burn out once it hits
%the ground (feet).0.3048*
spotdistmeter=1609.344*(21.9*UH*(zo/g)^.5*
    (0.362+(solvedz/zo)^.5*.5*log(solvedz/zo)));
%Returns the max spot distance converted to meters
%(F22 returns miles)
distcell = round((1/deltax)*spotdistmeter);
%Converting max spot distance to cells
traveltimes = zeros(2,distcell);
%row 1: Number of cells away from torch
%row 2: vertical travel time to reach value in row 1
%(time steps)
for k=1:distcell %Incrementing horizontal distance (cells)
for j=1:round(solvedz) %Incrementing vertical distance (z)
dcell = round(1609.344*(21.9*UH*(zo/g)^.5*
    (0.362+(j/zo)^.5*.5*log(j/zo)))*(1/deltax));
%calculating the cells away from torch given j value
if ((k==dcell-1))
%If true; we reached the next cell so we need to record
%current travel time before we continue
traveltimes(1,k)=k;

```

```

traveltimes(2,k)=round((2*solve_for_tt(j-1))/deltat)+1;
%We multiply by two because we are assuming the time to get
%the the vertical height (z) is equal to the time for the
%fb to go from z to hit the ground. We add one
%because we are assuming it takes one time step for the
%tree to start lofting fb's
end; %End creating traveltimes
if ((traveltimes(1,distcell)==0) && (k==distcell))
%If true; k is on final round and we have yet to record the
%travel time for distcell. This makes sure that time gets
%recorded in traveltimes.
traveltimes(1,k)=k;
traveltimes(2,k)=round(2*solve_for_tt(solvedz))+1;
end; %End check for recording travel time for distcell
end; %End for loop incrementing vertical distances (z)
end; %End for loop incrementing horiz. distances (cells)
%traveltimes(2,:)=traveltimes(2,:);
%-----%
function out=f(z)%Calculating z
D=(z+zo)/(0.39*10^5);%Equation D45
y=B*(D/zF)^(1/2);%Equation D18
m=D/zF;%Personal parameter for simplicity
r=((b+z/zF)/a)^(1/2);%Equation D37
tt=tnot+1.2+a/3*(((b+z/zF)/a)^1.5-1); %Albini eqn D43
tF=1-x^.5+y*log((1-y)/(x^.5-y)); %Albini eqn D17

```

```

tT=0.2+B*m^.5*(1+B*m^.5*log(1+1/(1-B*m^.5)));
%Albini eqn D30
tP=a/(.8*y)^3*(log((1-.8*y)/(1-.8*r*y))-.8*y*(r-1)-
    .5*(.8*y)^2*(r^2-1));
%Albini eqn D38
out=tt-(tF+tT+tP);
%2*zF/wF is the characteristic time scale. Albini's model
%uses dimensionless time, so this factor is left out of
%the sum to keep time dimensionless
end
%-----%
function verticaltraveltime=solve_for_tt(z)
%Calculates vertical travel time for different z values
D=(z+zo)/(0.39*10^5);
y=B*(D/zF)^(1/2);
m=D/zF;
r=((b+z/zF)/a)^(1/2);
tF=(2*zF/wF)*(1-x^.5+y*log((1-y)/(x^.5-y)));
%Albini eqn D17: Time it takes for the fb to get from
%initial spot to the flame tip
tT=(2*zF/wF)*(0.2+B*m^.5*(1+B*m^.5*log(1+1/(1-B*m^.5))));
%Albini eqn D30: Time it will take the partical to get
%from the flame tip to the plume
tP=(2*zF/wF)*(a/(.8*y)^3*(log((1-.8*y)/(1-.8*r*y))-
    .8*y*(r-1)-.5*(.8*y)^2*(r^2-1)));

```



```

%Albini eqn D38: Time it will take for partical to get from
%plume base to max vertical height
if (z < zF)
verticaltraveltime = tF;
elseif (z > zF) && (z < 1.4*zF)
verticaltraveltime = tF+tT;
elseif (z > 1.4*zF)
verticaltraveltime=tF+tT+tP;
end;
end
%-----%
end

```

### Appendix 6: Program Code: create\_ignitetimes\_function\_new

```

function [ignitetimes]=create_ignitetimes_function_new
    (distfromtorch,traveltimes,fbign,n,m,maxspot)
%create_ignitetimes_function
%(deltat,distfromtorch,traveltimes,fbign,n,m)
ignitetimes = inf*ones(n,m);
reduceddft=distfromtorch.*((distfromtorch>0).*(fbign>0));
%Eliminates cells w/ firebrands that will not ignite spots
for i=1:n
    for j=1:m
        if reduceddft(i,j)>0
            if (reduceddft(i,j)>maxspot)
                %If the number of cells the fb lands surpasses the max
                %spot distance than just use the traveltime value for
                %the max spot distance
                ignitetimes(i,j) = traveltimes(2,maxspot)+40;
                %We add seven here because we are assuming it takes
                %20 min for a spot to reach steady state
                %and ignite
            else
                %else, find the appropriate traveltime value
                ignitetimes(i,j) = traveltimes(2,reduceddft(i,j))+40;
                %We add forty here because we are assuming it takes
                %20 min for a spot to reach steady state
            end
        end
    end
end

```

```
%and ignite  
end; %End if statement checking if the fb lands beyond the  
%max spot dist  
end; %End making ignitetimes  
end; %End of j incrimenator  
end; %End of i incrimenator  
end
```