Mathematical Modeling for Network Selection in Heterogeneous Wireless Networks – A Tutorial

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***Abstract*—In heterogeneous wireless networks, an important task for mobile terminals is to select the best network for various communications at any time anywhere, usually called network selection. In recent years, this topic has been widely studied by using various mathematical theories. The employed theory decides the objective of optimization, complexity and performance, so it is a must to understand the potential mathematical theories and choose the appropriate one for obtaining the best result. Therefore, this paper systematically studies the most important mathematical theories used for modeling the network selection problem in the literature. With a carefully designed unified scenario, we compare the schemes of various mathematical theories and discuss the ways to benefit from combining multiple of them together. Furthermore, an integrated scheme using multiple attribute decision making as the core of the selection procedure is proposed.**

***Index Terms*—Network selection, heterogeneous wireless networks (HWNs), utility theory, multiple attribute decision making (MADM), fuzzy logic, game theory, combinatorial optimization, Markov chain.**

I. INTRODUCTION

THE RECENT development of wireless technologies has totally revolutionized the world of communications. Multiple technologies are evolving simultaneously towards providing users with high-quality services of broadband access and seamless mobility. On one hand, wireless wide area networks (WWANs) evolve from GSM to UMTS and beyond 3G, providing wide coverage and good mobility capabilities. On the other hand, a series of standards of wireless local area networks (WLANs), including IEEE 802.11a, IEEE 802.11b, IEEE 802.11g, IEEE 802.11n, etc., have been established for local-area high-speed economic wireless access. To complement them, wireless personal area networks (WPANs), e.g., Bluetooth and Zigbee, and wireless metropolitan area networks (WMANs), e.g., WiMAX, are developed for short-range and metropolitan coverages, respectively. All the above networks have been deployed with coverage overlapping one another, hence forming a hybrid network for wireless access, which is usually called heterogeneous wireless networks (HWNs).

To access the Internet through HWNs, current terminals, e.g., laptops and cellphones, are usually installed with multiple wireless access network interfaces. One type of terminals widely used nowadays is those with multiple interfaces but no functionality to support IP mobility or multihoming, called multi-mode mobile terminals. The other is with IP mobility and multihoming functionalities, called multi-homed mobile terminals. Mobility means that a terminal can switch between networks without breaking on-going communications. Multihoming means that a terminal has multiple IP connections to one or multiple networks simultaneously. Multi-homed terminals use multiple interfaces to share load for the same session and support session continuity with low (or no) packet loss during mobility or link break. By contrast, multi-mode terminals can only select and use one interface for certain session at a time.

Both multi-mode and multi-homed terminals require always to rank the access networks and select the best at any time anywhere, which is well known as always best connected (ABC). ABC brings plenty of advantages to users. With ABC functionality, terminals select appropriate access networks to fit for various QoS requirements of applications; terminals avoid selecting a network with high traffic load for avoiding congestion; terminals predict networks' availability so that they do not connect to networks which disappear soon; and terminals minimize signalling costs by using network selection and handover decision strategies specifically for this purpose. Moreover, ABC benefits operators. Since ABC has the feature of assisting the assignment of traffic load to multiple networks, operators maximize the utilization rate of the resources of the networks they operated, hence maximizing revenue. According to network selection strategies, operators analyze and decide the number of WiFi access points they should deploy to attract users to WLANs. Finally, ABC is suitable to synthetically consider users' and operators' benefits, so that a win-win partnership can be achieved.

ABC contains many necessary components [1], such as network discovery, network selection, handover execution, authentication, authorization and accounting (AAA), mobility management, profile handling, content adaptation, etc., in which network selection is a key component and will be extensively discussed in this paper. In recent years, a large number of research works have discussed the selection of the best network. Among them, different mathematical theories have been used for modeling this problem. Although two survey papers on this topic [2], [3] have been published, they were not focused on the mathematical theories used to model this problem. Based on our study, the mathematical model used for representing the problem is the first thing and the most important thing we should consider when designing a

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TABLE I
NETWORKS AND SELECTED ATTRIBUTES IN THE UNIFIED SCENARIO

	Bandwidth	Price	Cell radius	Security	Power consumption	Traffic
WWAN	2	50	2000	3	1/100	X
WMAN	10	20	2000	3	1/100	X
WLAN	54	5	75	1	1/50	X
WPAN	1	1	10	2	1/1000	X

TABLE II
SELECTED PROPERTIES OF THE 16 USERS IN THE UNIFIED SCENARIO

User No.		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Application	Conversational	●	●	●	●												
	Streaming					●	●	●	●								
	Interactive									●	●	●	●				
	Background													●	●	●	●
User	Money first	●	●			●	●			●	●			●	●		
	Quality first			●	●			●	●			●	●			●	●
Terminal	Battery first	●		●		●		●		●		●		●		●	
	Mobility first		●		●		●		●		●		●		●		●

network selection strategy. It decides the aim of optimization, the utilization of different parameters, and the performance of the selection strategy. Therefore, to fill out this blank, we conduct a serious survey and provide a systematic tutorial on mathematical theories for modeling the network selection problem.

Throughout this paper, we use a unified scenario to help explain schemes using different mathematical theories. On the network side, we consider 4 types of available networks (i.e., WWAN, WMAN, WLAN and WPAN) and 6 attributes (i.e., bandwidth, price, cell radius, security, power consumption and traffic), as given in Table I. These attributes are carefully selected, so that there is upward attribute e.g., bandwidth, downward attribute e.g., price, dynamic attribute e.g., traffic, terminal-related attribute e.g., power consumption, application-related attribute e.g., security and mobility-related attribute e.g., cell radius. Note that one attribute could have multiple of these features. Moreover, we design WMAN as a dominant alternative of WWAN, so that we could clearly see the load balancing feature of the schemes with different mathematical theories. On the user side, we consider 4 types of applications with different QoS requirements including conversational, streaming, interactive and background [4]. For each application type, we consider 4 users with different user preferences (i.e., money first and quality first) and different terminal properties (i.e., battery first and mobility first). Totally, there are 16 users with different user-side features, as summarized in Table II.

VHO represents handover between different types of access technologies, which is needed not only for connectivity reason but also for other ones, such as user preference and network load balancing. In the literature, VHO decision is sometimes confused with the term network selection, so in this paper, we strictly distinguish the two terms: network selection is to rank networks and find the best one, while VHO decision is to decide whether it is worth the handover to the best network or a network better than the current one. VHO decision is not to simply check whether the difference between the two networks is larger than the VHO cost. In fact, this decision takes into account the predicted information of many parameters as long as they are predictable, including the expected time point that a better network will be available, the average duration that a better network can last, the probability density function of a better network's dwelling time, the utilities of networks, etc. However, since the subject of this tutorial is network selection, we are not going to discuss too much on VHO decision.

The rest of this paper is organized as follows. From Sections II to VII, we systematically discuss the existing studies on network selection using utility theory (cost function), multiple attribute decision making, fuzzy logic, game theory, combinatorial optimization, Markov chain, respectively. In Section VIII, we compare schemes using different mathematical theories, discuss the ways to combine multiple of these theories together, and propose an integrated scheme in the end. Section IX concludes the paper. Finally, Section X and Section XI provides the notations and the glossary.

II. UTILITY THEORY (COST FUNCTION)

For making a decision, utility refers to the satisfaction that a goods or service provides to the decision maker [5]. An associated term is utility function which relates to the utility derived by a consumer from a goods or service. Different consumers with different user preferences will have different utility values for the same product. Thus, the individual preferences should be taken into account in the utility evaluation.

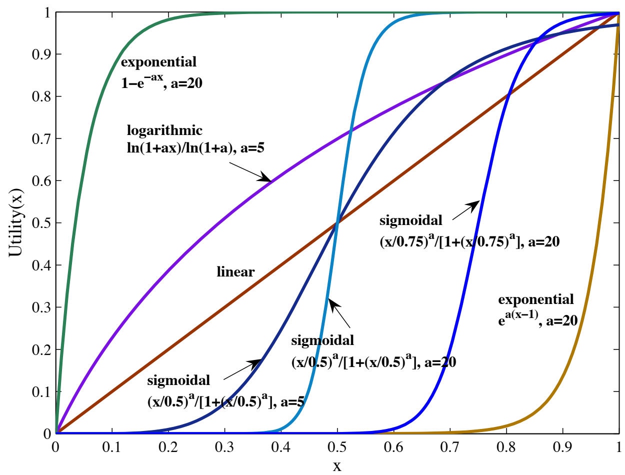

Fig. 1. Typical utility functions.

A. *Utility functions in network selection*

Utilities can be classified into monotonic utilities and non-monotonic ones. The utility is said to be monotonic if the measure of satisfaction associated with the attribute shows a monotonic increase and decrease with an increase in attribute value. Otherwise, it is said to be non-monotonic. Normally, monotonic utilities are used, except if the attribute is considered as the nominal-the-best. For a nominal-the-best attribute, instead of considering the best (either the largest or the smallest) as the most desired network, the one that is closest to the service requirement is preferred [6]. When evaluating the utility of an attribute, we should distinguish between the upward and downward attributes. The attributes of which the higher preference relation is in favor of the higher value are called upward attributes. Conversely, the downward attributes encompass various costs. Given an attribute, its utility can be calculated based on certain utility function. And, the utility function of one attribute could be different from that of others. Some examples of common utility functions are shown in Fig. 1. It is important to select the suitable utility function for each attribute. Sigmoidal utility function is considered to be suitable for the network selection problem [7], but the parameters in the sigmoidal function might be different to fit for different attributes' features.

During the network selection procedure, we consider multiple attributes together, so the utilities of multiple attributes are combined as a total utility. It has been pointed out that a valid form to combine these attributes together satisfies the following requirements [7]:

$$\begin{cases} \frac{\partial U}{\partial u_j} \geq 0 \\ \lim\limits_{u_j \to 0} U = 0, \forall j = 1, ..., M \\ \lim\limits_{u_1,...,u_M \to 1} U = 1 \end{cases} \tag{1}$$

where U is the total utility of all the attributes and u_j is the utility of attribute j. M denotes the number of attributes throughout this paper.

Cost function is a measurement of the cost caused by using certain network. Usually, the cost of a network can be considered as the inverse of its utility, but the form of this inversion is related with the way to combine multiple attributes. For example, if these attributes are summed up, the total cost is calculated as the cost minus the utility. A general form of cost function for the network selection problem was given in [8], which integrates a large number of attributes, their weights, and furthermore, network elimination factors given by

$$\mathcal{F}_i = \sum_k (\prod_j \epsilon_{ij}^k) \sum_j [f_j^k(w_j^k)\mathcal{N}(u_{ij}^k)], \tag{2}$$

where $\mathcal{N}(u_{ij}^k)$ represents the normalized utility of application k in network i in terms of attribute j. $f_j^k(w_j^k)$ is the weighting function of attribute j for application k. ϵ_{ij}^k is the network elimination factor, either 1 or infinite, to reflect whether current network conditions are suitable for requested applications. For example, if a network cannot guarantee the delay requirement of certain real-time application, its corresponding elimination factor will be set to infinite. Thus, the corresponding cost becomes infinite, which eliminates this network.

One study that is worth mentioning is the usage of the consumer-surplus concept of microeconomics in [9]. Users always search for cost effective solutions to meet their expectations. If the price is less than the value the user is willing to pay, he saves money. Consumer-surplus represents the difference between the monetary value of the data to the user and its actual price, so the network with the best predicted consumer-surplus, which is also predicted to meet the service completion deadline, will be selected.

B. *Attributes in network selection*

A lot of studies model the network selection issue with cost or utility functions, but they may consider different attributes and measure them in different manners. A summary of attributes and their usage in different papers is provided in Table III. For types of attributes, we first classify them into upward and downward attributes, then static, dynamic and semi-dynamic attribute. Semi-dynamic attributes are those that are not totally static but not quite dynamic either. For example, bandwidth is sometimes used statically as the total bandwidth of each network, but sometime used dynamically as the average bandwidth a user obtains. Bit error rate (BER), jitter and service completion time are changeable along with the environment and the network condition, but it is difficult to dynamically evaluate their instantaneous values for network selection, so they are classified as semi-dynamic attributes. Moreover, we also consider some other features of attributes, such as mobility-related, QoS-related, terminal-related and inter-network. For lists of references, considering that every study on network selection will use one or multiple attributes as decision criteria and some key attributes are even used by most studies on this issue, so it is tedious to provide complete lists for all the attributes. Instead, Table III just aims to list some most typical examples of each attribute. For utility functions used in the literature, most studies that do not specifically discuss utility functions could be considered as using linear utility functions. While in some recent studies, polynomial, logarithmic, exponential and sigmoidal utility functions are

TABLE III
KEY ATTRIBUTES AND THEIR UTILITY FUNCTIONS

Attribute	Types	References	Utility functions
Bandwidth	upward/semi-dynamic/QoS-related	[7], [8], [10], [20], [21], [23], [24], [28], [30], [37], [38], [46], [51]	linear, logarithmic, sigmoidal
Cell radius (diameter)	upward/static/mobility-related	[38]	linear
Security	upward/static/QoS-related	[10], [21], [23], [24], [51]	linear, sigmoidal
Battery	upward/dynamic/terminal-related	[21], [22], [28]	linear
SNR/SIR	upward/dynamic/QoS-related	[21], [22]	linear
RSS	upward/dynamic/QoS-related	[11]–[13], [21], [28], [51]	linear
Price	downward/static	[7]–[10], [13], [21], [23], [24], [28], [34], [38]	linear, logarithmic
VHO signaling cost	downward/static/mobility-related/inter-network	[12], [24], [54]	linear
VHO latency	downward/static/mobility-related/inter-network	[12], [27]	linear
HHO signaling cost	downward/static/mobility-related	[12], [54]	linear
HHO latency	downward/static/mobility-related	[12], [38]	linear
Handover failure probability	downward/static/mobility-related	[27]	linear
Interruption probability	downward/static/mobility-related	[27]	linear
Size of unsent messages	downward/static/mobility-related	[27]	linear
Traffic	downward/dynamic	[7], [11], [24], [34], [37]	linear, sigmoidal
Power consumption	downward/static/terminal-related	[24], [38], [51]	linear
BER	downward/semi-dynamic/QoS-related	[21], [23], [24]	linear, sigmoidal
Delay	downward/semi-dynamic/QoS-related	[20], [21], [23], [51]	linear, sigmoidal
Packet loss	downward/semi-dynamic/QoS-related	[20], [23]	linear, sigmoidal
Jitter	downward/semi-dynamic/QoS-related	[20], [21], [23], [24], [51]	linear, sigmoidal
Response time	downward/semi-dynamic/QoS-related	[23]	linear
Service completion time	downward/semi-dynamic/QoS-related	[9]	linear, polynomial, exponential

utilized for some attributes, which are summarized in this table.

In the above presentations, we discussed utility functions for various attributes. To avoid a potential misunderstanding, we would like to point out that utility function for a certain attribute could be totally different in different scenarios. For example, the utility of bandwidth should jump to a fixed value after certain thresholds for voice and video applications, but kind of linearly increase for data application [13]. If sigmoidal functions are used, the parameter a, as shown in Fig. 1, should be large for voice and video applications while small for data applications. For voice and video applications, the mid values, corresponding to the thresholds, should be also different.

Moreover, it is important to state clear that other studies on the network selection issue could also evaluate networks based on utility/cost functions which combine multiple attributes. However, those studies focus on other mathematical models, which will be presented in later sections.

C. Case study

We consider the unified scenario presented in Section I with Tables I and II. Since it would be unfair by assuming different networks with different traffic conditions, we assume that they have the same traffic condition, which means that the attribute ‘traffic’ is not considered in this case study. Based on the above studies, sigmoidal utility functions with different configurations of mid value and parameter a, as shown in Fig. 1, are used for different attributes under the cases of different user-side properties. For example, user 5 requires streaming application while user 1 requires conversational application, so the mid value in the sigmoidal utility function of ‘bandwidth’ is much larger for user 5 than for user 1; user 7 prefers better service, so a in the sigmoidal utility function of ‘price’ can be small but that of ‘bandwidth’ should be large. In other words, sigmoidal utility functions could be different for different users and different attributes, so there are 5×16 sigmoidal utility functions. For the sake of conciseness, we are not going to list them.

TABLE IV
OBJECTIVE AND SUBJECTIVE WEIGHTING METHODS

	Category	Calculation
Entropy	Objective weighting	$w_j = 1 - \frac{1}{\ln N}\sum_{i=1}^{N}\left[x_{ij}\ln(x_{ij})\right]$
Variance	Objective weighting	$w_j = \sqrt{\sum_{i=1}^{N}(x_{ij}-\bar{x}_j)^2/N\bar{x}_j}, \bar{x}_j = \frac{1}{N}\sum_{i=1}^{N}x_{ij}$
Eigenvector	Subjective weighting	$(\mathbf{B}-\lambda\mathbf{I})\cdot\mathbf{w} = 0$
Weighted least square	Subjective weighting	$\min \mathcal{Z} = \sum_{i=1}^{M}\sum_{j=1}^{M}(b_{ij}w_j - w_i)^2, s.t. \sum_{i=1}^{M}w_i = 1$
TRUST	Subjective weighting	$\mathbf{w} = \mathbf{e}\times(\mathbf{d}\times\mathbf{I})\times\mathbf{R}$

In order to prominently reflect the effect of the sigmoidal utility functions, we simply sum the utilities of these attributes with equal weights. Moreover, we use the Enhanced Max-Min method in Table V to normalize the values of attributes for all the case studies throughout this paper. We want to mention that, with Enhanced Max-Min method, the utilities of the best and the worst networks on any attribute will be stretched close to 1 and 0, respective. Then, if the utilities are going to be summed up with equal weights as we said above, multiple trivial attributes could conceal the importance of the key attribute and dominant the final decision. To avoid this pitfall, we compress all the utilities from $[0,1]$ to $[0.1,0.9]$ and set the mid value of sigmoidal function to 0.01 (or 0.99) when the attribute is trivial (or dramatically important). Network selection results of the 16 users are given in Table VII, together with the results from schemes using other mathematical theories for comparison.

III. MULTIPLE ATTRIBUTE DECISION MAKING

Multiple attribute decision making (MADM) refers to making preference decision over the available alternatives that are characterized by multiple (usually conflicting) attributes. MADM is a branch of multiple criteria decision making (MCDM) which also includes multiple objective decision making (MODM). MODM problems involve designing the best alternative given a set of conflicting objectives, which creates a product in the design process. For example, automobile manufacturers want to design a car that maximizes riding comfort and fuel economy and minimizes production cost. Apparently, network selection does not create any physical product but only makes a decision, so MADM is more suitable for this problem.

A. MADM basics

MADM problems have several common characteristics [14]:

Alternatives: a finite number of alternatives are screened, prioritized, selected and/or ranked for making the final decision. The term 'alternative' is synonymous with 'option,' 'policy,' 'action,' 'candidate,' etc.

Multiple attributes: the decision maker does consider multiple attributes of these alternatives. The term 'attribute' can be referred to as 'goal,' 'criterion,' 'property,' 'characteristic,' etc.

Decision matrix: a MADM problem can be concisely expressed in a matrix format, where columns indicate attributes and rows indicate alternatives. Thus, a typical element x_{ij} of the matrix indicates the value of the ith alternative with respect to the jth attribute.

Attribute weights: different decision makers might focus on different aspects when ranking the alternatives, so weights must be calculated to represent multiple attributes' relative importance. Table IV gives some common weighting methods including objective and subjective methods. The objective weights are calculated directly based on the relative difference between attributes, given by w_j for attribute j. Then, the objective weights are obtained as the normalized values of w_j. By contrary, subjective weights $\mathbf{w}$ are usually calculated based on the decision maker's pair-wise comparison between all the attributes, given by b_{ij} as the comparison value between the ith and jth attributes and $\mathbf{B}$ as the matrix containing all the comparison values. Moreover, for the eigenvector method in the table, λ is the eigenvalue and $\mathbf{I}$ is an identity matrix. N denotes the number of networks throughout this paper.

However, these traditional methods to calculate subjective weights do not work well for the network selection problem since its pair-wise comparison process is slow and not automatic. Therefore, we proposed a TRigger-based aUtomatic Subjective weighTing (TRUST) method [15] to calculate subjective weights, as shown in the weighting module of Fig. 6. Since some events can trigger the network selection procedure, there should be some relationship between these events and selection results. Our method uses a mapping pot to store this relationship in order to calculate the subjective weights. Two parameters are stored in the mapping pot and used for the calculation of subjective weights. One is a E-by-M matrix $\mathbf{R}$ representing the relationship between events and network attributes, where E is the number of events and r_{ij} in the matrix represents the strength of the effect from the ith event to the jth attribute, e.g., the event 'speed up' increases the weight of mobility-related attributes. The other is a 1-by-E vector $\mathbf{e}$ representing the weights of events, which can be calculated in advance or obtained from the operator during the initiation of the mobile terminal. Finally, the subjective weights of network attributes can be calculated as shown in Table IV, where $\mathbf{d}$ is a 1-by-E binary vector denoting true or false of the trigger events.

Normalization: different attributes have different measurement units, so normalization is treated as a necessary step of network selection. There are several methods of normalization, compared in Table V. For a given attribute j, x_{ij} represents

TABLE V
NORMALIZATION METHODS FOR ATTRIBUTES IN NETWORK SELECTION

	Normalization Function
Max-Min	$v_{ij} = (x_{ij} - \min_i(x_{ij}))/(\max_i(x_{ij}) - \min_i(x_{ij}))$
Square root	$v_{ij} = x_{ij}/\sqrt{\sum_{i=1}^{N} x_{ij}^2}$
Sum	$v_{ij} = x_{ij}/\sum_{i=1}^{N} x_{ij}$
Enhanced Max-Min	$v_{ij} = \begin{cases} 1 - \lvert x_{ij} - \max_i(x_{ij})\rvert/(\max_i(x_{ij}) - \min_i(x_{ij})) & \text{for upward attributes} \\ 1 - \lvert x_{ij} - \min_i(x_{ij})\rvert/(\max_i(x_{ij}) - \min_i(x_{ij})) & \text{for downward attributes} \\ 1 - \lvert x_{ij} - \Lambda_j\rvert/\max_i\{\max_i(x_{ij}) - \Lambda_j, \Lambda_j - \min_i(x_{ij})\} & \text{for nominal-the-best attributes} \end{cases}$

the value of the ith network in terms of this attribute, and v_{ij} represents its normalized value. The enhanced Max-Min method consider three groups of network-side attributes, i.e., upward, downward and nominal-the-best, where Λ_j represents the nominal value of attribute j. There are two differences between Max-Min and enhanced Max-Min methods: first, the latter considers the nominal-the-best group; second, the latter adjusts downward attributes into upward attributes. For the sake of the second difference, the outputs of the enhanced Max-Min method are all considered as upward attributes, while for the other three methods, we have to distinguish between upward and downward attributes while combining them together. For examples of the usages of these normalization methods, refer to [16]–[19].

B. MADM algorithms in network selection

MADM algorithms can be divided into compensatory and non-compensatory ones [20]. Non-compensatory algorithms, e.g., dominance, conjunctive, disjunctive or sequential elimination, are used to find acceptable alternatives which satisfy the minimum cutoff. By contrary, compensatory algorithms combine multiple attributes to find the best alternative. Most MADM algorithms that have been studied for the network selection problem are compensatory algorithms, including simple additive weighting (SAW), multiplicative exponential weighting (MEW), gray relational analysis (GRA), Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS), ELimination Et Choix Traduisant la REalité (ELECTRE), etc.

SAW is widely used by most studies of the network selection problem using cost or utility functions, generally given by

$$\mathcal{C}_{SAW} = \sum_{j=1}^{M} w_j v_{ij}, \tag{3}$$

where w_j represents the weight of the jth attribute, and v_{ij} represents the adjusted value of the jth attribute of the ith network.

MEW is to calculate the coefficient by multiplicative operation [7], [21], given by

$$\mathcal{C}_{MEW} = \prod_{j=1}^{M} v_{ij}^{w_j}. \tag{4}$$

(4) can be further modified as $\mathcal{C}^*_{MEW} = \ln(\mathcal{C}_{MEW}) = \sum_{j=1}^{M} w_j \ln(v_{ij})$. Considering the characteristic of the natural logarithm, the attribute whose cost is close to 0 has larger impact on the total cost than others. For example, Bluetooth is more often selected by MEW than by other algorithms due to its low monetary and power costs.

Another two MADM algorithms used for network selection are TOPSIS [17], [22] and GRA [6], [23], which both consider the distance from the evaluated network to one or multiple reference networks. Coefficient of TOPSIS can be calculated as

$$\mathcal{C}_{TOPSIS} = \frac{\mathcal{D}^\alpha}{\mathcal{D}^\beta + \mathcal{D}^\alpha}, \tag{5}$$

where $\mathcal{D}^\alpha = \sqrt{\sum_{j=1}^{M} w_j^2 (v_{ij} - \mathcal{V}_j^\alpha)^2}$ and $\mathcal{D}^\beta = \sqrt{\sum_{j=1}^{M} w_j^2 (v_{ij} - \mathcal{V}_j^\beta)^2}$ represent the Euclidean distances from the current network to the worst and best reference networks, respectively. $\mathcal{V}_j^\alpha$ and $\mathcal{V}_j^\beta$ represent the values of the jth attribute of the worst and best reference networks, respectively.

Different from TOPSIS, GRA uses only the best reference network to calculate the coefficient, given by

$$\mathcal{C}_{GRA} = \frac{1}{\sum_{j=1}^{M} w_j |v_{ij} - \mathcal{V}_j^\beta| + 1}. \tag{6}$$

ELECTRE, another well-known MADM algorithm but different from the above four algorithms, does not calculate certain coefficient for network ranking. It contains the following steps [16]:

1) identifying attributes of different networks as a decision matrix;
2) defining an ideal network;
3) calculating the absolute difference between each network and the ideal network;
4) normalizing the absolute difference;
5) multiplying weights of attributes;
6) calculating concordance and discordance matrices; and
7) making decision based on concordance and discordance matrices.

Among them, the key step is 6), in which concordance and discordance matrices are calculated based on concordance and discordance sets, denoted by $\mathscr{C}$ and $\mathscr{D}$, respectively. $\mathscr{C}_{kl}$ contains the attributes on which network k is better than network l, and $\mathscr{D}_{kl}$ is inverse.

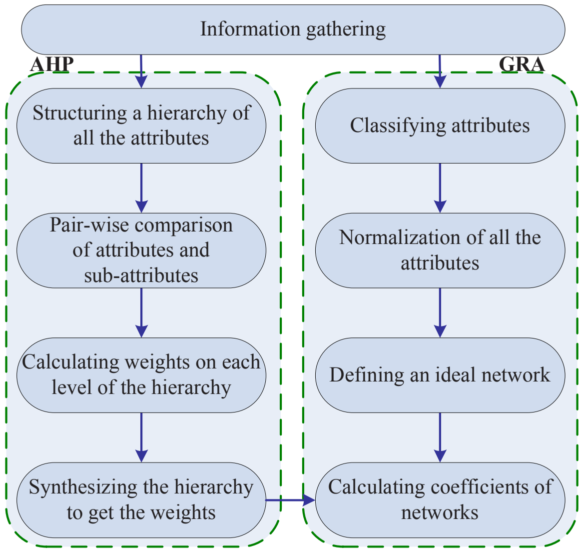

Fig. 2. An example of combining MADM with AHP-based subjective weighting.

Then, the elements in concordance and discordance matrices are calculated as follows:

$$\begin{cases} c_{kl} = \sum_{j \in \mathscr{C}_{kl}} w_j \\ d_{kl} = \frac{\sum_{j \in \mathscr{D}_{kl}} |v_{kj} - v_{lj}|}{\sum_{j=1}^{M} |v_{kj} - v_{lj}|}. \end{cases} \quad (7)$$

Among all the MADM algorithms, [7] pointed out that MEW is the only one that satisfies all the requirements indicated by (1), while [6] argued that GRA is more suitable than others in the scenarios when some attributes have non-monotonic utilities. [21] showed that SAW, MEW and TOPSIS have similar performance to all traffic classes, while GRA provides a slightly higher bandwidth and lower delay for interactive and background traffic. [24] showed that MEW gives larger probability to select WPAN than other algorithms due to its multiplication operation. Moreover, it is easy to combine compensatory MADM algorithms with the eigenvector subjective weighting method based on analytical hierarchy process (AHP), such as the scheme shown in Fig. 2 [23]. AHP is a procedure to divide a complex problem into a number of deciding factors and integrate the relative dominances of the factors with the solution alternatives to find the optimal one. For weighting the attributes in a network selection scheme, AHP structures attributes into a hierarchy. For example, [23] structures all the QoS-related attributes into five groups (i.e., throughput, timeliness, reliability, security and cost) and each group has one or multiple attributes (e.g., delay, response time and jitter are in the group of timeliness). Therefore, QoS is on the first level, the five groups are on the second level, while attributes in each group are on the third level. Then, on each level in the hierarchy, weights are calculated based on certain weighting method, e.g., those in Table IV. Finally, weights of different levels are synthesized to achieve the overall weight of each attribute.

Note that MADM is not the only mathematical theory that combines multiple attributes together. Theories in the other sections also prefer to combine multiple attributes for decision, using usually SAW. Moreover, weighting and normalization are common operations for schemes using all kinds of mathematical theories, not only for MADM. We present them in this section since they are mainly studied in the scope of MADM-based network selection.

C. Case study

We consider the unified scenario presented in Section I with Tables I and II. Similar to the case study in Section II, attribute 'traffic' is not considered in this case study. Based on the above studies, we choose the widely used MADM algorithm, SAW, for this case study. Enhanced Max-Min method is used for normalization. Eigenvector method is used for calculating the subjective weights. For each user, a pair-wise comparison matrix is obtained by the decision maker based on user-side properties. For example, the pair-wise comparison matrix of user 1 could be

$$\mathbf{B} = \begin{pmatrix} 1 & 1/7 & 1 & 1 & 1/7 \\ 7 & 1 & 7 & 7 & 1 \\ 1 & 1/7 & 1 & 1 & 1/7 \\ 1 & 1/7 & 1 & 1 & 1/7 \\ 7 & 1 & 7 & 7 & 1 \end{pmatrix}.$$

Weights are calculated as the eigenvector of the above pair-wise comparison matrix corresponding to the largest eigenvalue, given by $\{0.0588, 0.4118, 0.0588, 0.0588, 0.4118\}$. Sometimes, the eigenvector could be negative, so we should always normalize the obtained eigenvector to avoid treating the worst network as the best.

We can see from this matrix that two attributes are key factors for the decision, i.e., price (as the user preference is 'money first') and power consumption (as the terminal property is 'battery first'). For the other three attributes, it is really difficult for us to say which one is the most important one, so we give them equal weights. For the sake of conciseness, we are not going to list the pair-wise comparison matrices for all the users, but we would like to remind that pair-wise comparison matrices are different from user to user and from scenario to scenario.

Network selection results of the 16 users are given in Table VII, together with the results from schemes using other mathematical theories for comparison. Notice that the selection results by using other MADM algorithms are quite close to SAW. For example, with TOPSIS, the only difference in the results is that user 6 selects WLAN instead of WMAN.

IV. Fuzzy Logic

Humans usually think in terms of linguistic descriptions, so giving these descriptions some mathematical form helps exploit human knowledge. Fuzzy logic utilizes human knowledge by giving the fuzzy or linguistic descriptions a definite structure.

A. Fuzzy logic basics

To understand well this section, it is necessary to know the following concepts [25]:

Fuzzy set: a fuzzy set is a class of objects with a continuum of grades of membership, which is characterized by

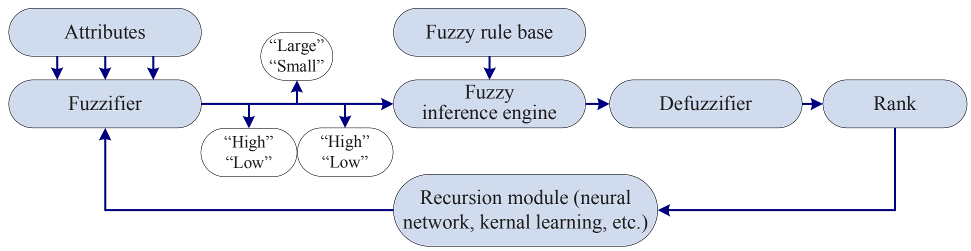

Fig. 3. A combined framework of fuzzy logic based network selection.

a membership function assigning to each object a grade of membership ranging between zero and one [26]. Fuzzy set is considered as an extension of the classical notion of set. In the classical set theory, the membership of elements in a set is assessed in binary terms, which means either belongs or does not belong to the set. By contrast, the fuzzy set theory permits the gradual assessment of membership using a membership function valued within $[0, 1]$. The classical set is usually called *crisp set* in the fuzzy logic theory.

Fuzzifier: the module to map a crisp point into a fuzzy set.

Fuzzy rule base: the module consisting of a collection of fuzzy IF-THEN rules. A typical form of a rule is

$$\text{IF } \mathcal{X}_1 \text{ is } \mathscr{F}_1^l \text{ and ... and } \mathcal{X}_M \text{ is } \mathscr{F}_M^l, \text{ THEN } \mathcal{Y} \text{ is } \mathscr{G}^l, \quad (8)$$

where l denotes the index of the rule in the fuzzy rule base, $\mathcal{X}_j$ represents the jth input variable, $\mathcal{Y}$ represents the output variable, and $\mathscr{F}_j^l$ and $\mathscr{G}^l$ are corresponding fuzzy sets for $\mathcal{X}_j$ and $\mathcal{Y}$, respectively.

Fuzzy inference engine: the module which uses fuzzy logic principles to combine the fuzzy IF-THEN rules in the fuzzy rule base.

Defuzzifier: the module to map a fuzzy set into a crisp point (the opposite of fuzzifier).

Membership function: representing the degree of truth in fuzzy logic theory.

B. Fuzzy logic in network selection

There are different ways to use the fuzzy logic theory in a network selection scheme: some studies use it as the core of the selection scheme, some combine fuzzy logic with MADM algorithms, while some use the fuzzy logic with recursion (neural network, kernel learning, etc.).

A very basic framework without combining with any other theory is used by [27] for fuzzy logic based network selection, as shown in Fig. 3, eliminating the recursion part. In their scheme, three input fuzzy variables are considered (i.e., the probability of a short interruption, the failure probability of handover to radio, and the size of unsent messages), while we could surely consider more attributes as input fuzzy variables for network selection. At the beginning of the procedure, the fuzzy variables are fuzzified and converted into fuzzy sets by a singleton fuzzifier. Then, based on the fuzzy rule base, the fuzzy inference engine maps the input fuzzy sets into output fuzzy sets by the algebraic product operation. Finally, the output fuzzy sets are defuzzified into a crisp decision point.

Many studies proposed schemes by combining fuzzy logic with MADM algorithms [2], [22], [28], coined *fuzzy MADM*. The idea is to use MADM for the fuzzy interference engine and defuzzifier parts. Fuzzy MADM is particularly interesting for the case when some attributes cannot be precisely obtained or when some attributes are better to be set with fuzziness due to the complex HWNs environment in an MADM scheme. According to the data type of the alternative's performance, fuzzy MADM can be categorized into three groups: data being all fuzzy, all crisp, and either fuzzy or crisp [22].

Since some dynamic factors change frequently, the recursion is used to combine the latest information with previous ranking result to obtain the latest rank. In the literature, there are several proposals combining fuzzy logic with a recursion procedure. The recursion procedure can be a simple recursion without any further operation or certain learning procedure, such as neural network or kernel learning, as shown in Fig. 3. [29] proposed a fuzzy logic based scheme using simple recursion, which considers the requirements of both operator and user. The rank produced by the fuzzy module is fed back to this module, so that it could produce a new rank when some factors change. [30] combined the fuzzy logic with neural network for network selection. Elman neural network is used to predict the number of users using certain network after the selection and feeds it back to the fuzzifier. And, [31] proposed a scheme to combine the fuzzy logic with kernel learning for similar purpose.

C. Case study

We consider the unified scenario presented in Section I with Tables I and II. For the same reason as the case studies in previous sections, attribute 'traffic' is not considered in this case study. We consider two fuzzy sets for each attribute, e.g. bandwidth has 'large' and 'small' fuzzy sets. Thus, with five attributes, there are maximum 2^5 fuzzy rules in the fuzzy rule base. For example, a basic fuzzy rule could be '*IF bandwidth is large & price is low & cell radius is large & security is high & power consumption is low, THEN utility is high*'. Membership function of each attribute is carefully designed based on the property of the attribute, as shown in Fig. 4. For example, bandwidth is an attribute which has some kind of threshold to guarantee QoS, so the slope of its membership function is large.

In order to combine the user-side properties into the scheme and to simplify the fuzzy rule base, each user maintains his/her own bunch of fuzzy rules and each fuzzy rule contains only some of the five inputs. For example, user 1 uses conversational applications with money first and battery first properties, so one of his fuzzy rules could be '*IF price is low & power consumption is low, THEN utility is high*'. For the

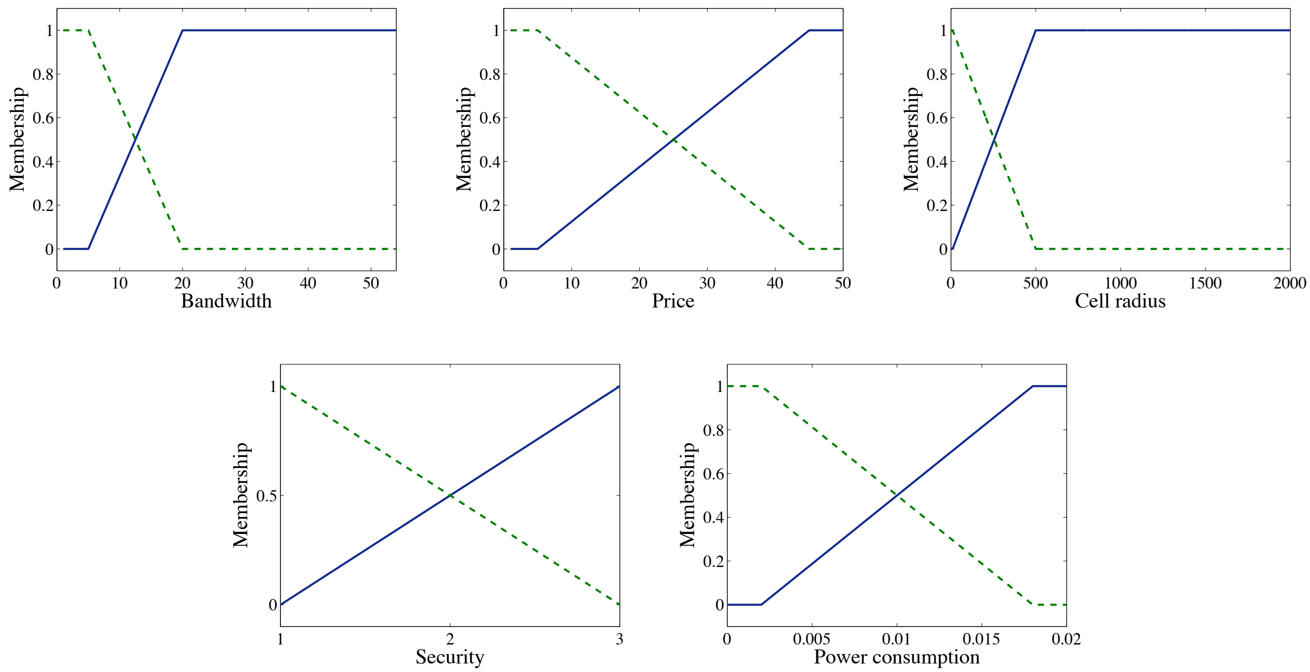

Fig. 4. Membership functions of different attributes in the unified scenario.

sake of conciseness, we are not going to list all the fuzzy rules. For each network, the fuzzy inference engine combines all the fuzzy rules in the user's fuzzy rule base and the defuzzifier transfers the fuzzy output into a crisp value to represent the utility of the network. In the end, the network with the highest utility is selected.

Network selection results of the 16 users are given in Table VII, together with the results from schemes using other mathematical theories for comparison. Since fuzzy logic module ignores trivial difference, there is a non-negligible probability that several networks might have the same priority. Therefore, in Table VII, we mark all the best networks when we could not distinguish them.

V. Game Theory

Game theory is related to the actions of decision makers who are conscious that their actions affect each other. The essential elements of a game include [32]:

Player: the individual who makes the decision. The goal of each player is to maximize his/her own payoff by a choice of strategy.

Strategy set: the set containing all the strategies a player can choose. In each round, the player chooses one strategy from the set.

Payoff: the utility that a player can receive by taking certain strategy when all the other players' strategies are decided.

Equilibrium: the combination of strategies containing the best strategy for every player. Nash equilibrium (NE) is the solution of a game, in which no player can achieve more payoffs by unilaterally changing his own strategy.

The techniques of game theory are widely adapted in resource management mechanisms in HWNs. We categorize game theoretical network selection scheme into three groups: game between users, game between networks and game between users and networks.

A. Game between users

The game between users considers the problem in which users selfishly select their believed best network, hence causing network congestion and performance degradation. [33] modeled the network selection problem into a non-cooperative game belonging to the class of congestion games between selfish users. In this game, the users are the players who take their actions on selecting one network among the available ones. Analytical upper bounds for the price-of-anarchy and price-of-stability are derived, which are considerably tighter than well known bounds for generic congestion games. The cost of each user depends on the congestion of the selected network, given by $c_k(i, \sum_{l\in\mathscr{K}} \eta_{li})$, where i indicates that user k selects network i. η_{ki} is a binary variable representing whether user k selects network i, so $\sum_{l\in\mathscr{K}} \eta_{li}$ indicates the total number of users selecting network i. This game becomes a problem in which all the users try to choose the network with minimum cost, whose NE can be indicated as

$$\zeta_{ki'}\eta_{ki}c_k(i, \sum_{l\in\mathscr{K}} \eta_{li}) \leq c_k(i', \sum_{l\in\mathscr{K}} \eta_{li'}), \forall i, i' \in \mathscr{N}, \forall k \in \mathscr{K}, \tag{9}$$

where $\mathscr{K}$ and $\mathscr{N}$ represents the sets of users and networks, respectively. $\zeta_{ki'}$ is a binary variable representing whether user k is within the coverage of network i'.

Another game model used for network selection is the evolutionary game, which extends the formulation of a non-cooperative game by including the concept of population, i.e. a group of players. In an evolutionary game, there could be a single or multiple populations, and the players from one population may choose strategies against players from another population. In a word, an evolutionary game defines a foundation to obtain equilibrium for the game of populations.

Beside the concept of population, there are two other important concepts in an evolutionary game: replicator and replicator dynamics. A replicator is a player from a population who is able to replicate itself through the process of mutation and selection. This replication process can be modeled by a set of ordinary differential equations, called replicator dynamics, given by

$$\dot{p}_i(t) = p_i(t)[\pi_i(t) - \bar{\pi}(t)], \tag{10}$$

where $p_i(t) = \mathcal{K}_i/K$ denotes the proportion of players choosing strategy i, with $\mathcal{K}_i$ is the number of players choosing strategy i and K is the total number of players in the game. $\pi_i(t)$ is the payoff of the players choosing strategy i and $\bar{\pi}(t)$ is the average payoff of the entire population. Based on the above replicator dynamics, the evolutionary equilibrium is defined as the set of fixed points of the replicator dynamics that are stable. In other words, none of the players wants to change its strategy since its payoff is equal to the average payoff of the population.

[34] studied the evolutionary game for network selection. In this game, users are players, users in a service area forms a population, the selection of one network is considered as the strategy and utility of a user is its payoff. For service area a, the evolutionary equilibrium is obtained by solving the set of equations indicated by $\{\dot{p}_i^{(a)} = 0 | i = 1, ..., N\}$, where N is the total number of candidate networks in service area a and $p_i^{(a)}$ denotes the proportion of users choosing network i in service area a. The evolutionary equilibrium is stable if all the eigenvalues of the Jacobian matrix corresponding to the replicator dynamics have a negative real part. [34] also studied a non-cooperative game between users in different service areas. In this game, users in the same area collaborate with each other to compete for bandwidth with other groups of users in other areas. A strategy is the proportion of users choosing network i, denoted by $p_i^{(a)}$. The payoff of a player is the total utility from all users in the group choosing all different networks, denoted by $\pi(\mathbf{p}^{(a)}, \mathbf{p}^{(-a)})$, where $\mathbf{p}^{(a)}$ denotes the vector of proportion of users choosing different networks in service area a, and $\mathbf{p}^{(-a)}$ denotes a vector of the proportion of users in all service areas except a. This game is similar to the congestion game presented above, except it is a game between groups of users in different service areas, instead of single users.

Another idea is to model network selection as a Bayesian game with incomplete information since it is usually difficult to inform all the players about the required information from other users. In a Bayesian game, the incomplete information is considered as private information of players before the game begins, called the *type* of the player. [35] modeled network selection into a Bayesian game by defining the type of player k as its minimum bandwidth requirement $B_k \in \mathbb{B}_k$, where $\mathbb{B}_k$ is the type space of player k. B_k is a variable obeying certain probability distribution function. Then, the expected payoff $\bar{\pi}_k$ is defined as bandwidth utility minus connection fee, where bandwidth utility is the benefit the user gets from selecting certain network, which could be zero if the allocated bandwidth is smaller than B_k. In a Bayesian game, for every type of player k, the best response can be obtained by

$$\mathcal{B}_k(\mathbf{q}_{-k}, B_k) = \arg\max_{q_k \in \mathscr{Q}} \bar{\pi}_k(q_k, \mathbf{q}_{-k}, B_k), \tag{11}$$

where $\mathscr{Q}$ is the set of Bayesian strategies.

A NE is indicated by strategy $\{q_k^*, \mathbf{q}_{-k}^*\}$, if and only if $\forall q_k \in \mathscr{Q}$, $\forall k \in \mathscr{K}$, $\bar{\pi}_k(q_k^*, \mathbf{q}_{-k}^*) > \bar{\pi}_k(q_k, \mathbf{q}_{-k}^*)$. Moreover, a combination of Bayesian game and evolutionary game is also tried for the network selection issue by [35].

In the above studies of game between users, they assume that multiple users are waiting for service at the time of decision. However, we all know that users usually come for service one by one. [36] studied a WLAN access point selection case where selection requirement of multiple terminals are not coming concurrently and all the terminals in the WLAN coverage area are informed immediately with the network selection information of each terminal. It was proved that the outcome of a one-by-one optimization process of these terminals corresponds to the NE of a one-shot game with multiple terminals' concurrent selection.

One special scenario where multiple users might do network selection at the same time is called group handover in [37]. This happens when multiple users move together, e.g., in a bus, or when certain network has some sudden problem. Three options were proposed:

1) if each mobile terminal knows the traffic loads of the other terminals, a NE based algorithm can be used. In this algorithm, the selection of each terminal is the corresponding strategy of the computed NE;

2) another algorithm is to separate terminals' handovers by using random delays, similar to the algorithm avoiding handover synchronization in [38]. In this algorithm, each terminal that has decided its selection should announce that to the others or to an independent function entity, so that others know its selection; and

3) sometimes, a terminal decides to select a target network and announces its selection to others, but it may not be able to really handover to it due to failure or rejection by that network. In this case, other terminals get incorrect information about the handover of this terminal. Therefore, the third algorithm is to announce its selection after the terminal has already finished its handover to the target network.

B. Game between networks

In an HWNs environment, different networks might be managed by different service providers, so their competition to attract and get more users become an important issue. Game between networks does not provide us a network selection scheme for users, but it indirectly guides users to think about their corresponding schemes for network selection under this network competition environment.

One model is to consider pricing strategies as the strategies of networks. For non-cooperative case, the problem is modeled as a Bertrand game [13], which describes interactions among sellers that consider their prices and buyers that choose their product at that price. Assuming that each user chooses the network with the maximum performance-cost ratio (PCR), each network chooses the pricing strategy that maximizes its own payoff (related to the price of service and the number of users choosing this network), fixing the other networks' pricing strategies, which indicates the NE. However, severe competition may result in low price and shrink total payoff in turn, which is not acceptable for network operators. Therefore, cooperation between several or all network operators may be established to provide the same QoS to users with the coalition price.

Another model is to consider the strategy of a network as the selection of a user for service, in which the users are totally passive and have no right to decide which network he wants to use. As an example, [39] described such a multi-round game model as follows:

1) a bunch of users send service requests to multiple networks;

2) a centralized entity gathers requests and put the users into a waiting list. Networks calculate payoffs based on gathered information;

3) in each round, each network selects one user for service and this user is removed from the waiting list;

4) multiple rounds are performed until all the users are served.

In this game, the best strategy for each network is to select the user with the maximum payoff from the waiting list of users that have not been served.

C. Game between users and networks

The set of users and the set of networks are considered as two players. The users' strategies are to select their favorable networks to maximize their payoffs, such as quality of services and price. Meanwhile, the networks' strategies are to select their favorable users to maximize their payoffs, such as the revenue [40]. If NE exists, the users and the networks correspondingly select each other. Otherwise, a sub-optimal solution will be used.

At the end of this section, we would like to mention that, for studies using game theory, it is important to not only indicate NE but also study how to reach the NE. Studies on network selection have utilised different approaches for this purpose, such as a centralized approach called population evolution in [34], and some decentralized approaches in which users could independently adapt themselves to reach the equilibria, e.g., Q-learning in [34] and no-regret learning in [41]. Moreover, in the literature of game theory, there are numerous algorithms for NE searching, e.g., Lemke-Howson algorithm [42] searching for one NE and Dickhaut-Kaplan algorithm [43] searching for the support of all NE. However, explanation of these algorithms is out of the scope of this tutorial.

D. Case study

We consider the unified scenario presented in Section I with Tables I and II. First and foremost, we emphasize that the feature and result of a game is largely related to the definition of the utility in the game. If the utility is defined highly correlated to the average bandwidth obtained by selecting certain network, the equilibrium of this game has the trend to uniformly distribute users into different networks. However, when networks are all with enough resource at certain moment, this kind of equilibrium is apparently not a good solution. Therefore, we define the utility of the game as follows in this case study: when the selected network could support all its users, the utility of each user is calculated as the total utility of the five normalized attributes by SAW algorithm, similar to the case study in Section III; otherwise, we assume that congestion in this network occurs, so the utility of each user in this network is zero.

With the above utility function, the equilibrium provides the same result as SAW algorithm when networks have enough resource. In order to show the difference between this game model and MADM, we consider the situation when networks' capacities are quite limited and we could not let all the users select their favorite networks as in MADM-based schemes. We set each network a limited capacity for these 16 users. In other words, you could imagine that these networks' capacities have already been largely occupied by other users at the moment of the coming of these 16 users. For fairness, we assume that the 4 networks have the same limited capacity, given by 12, so that we could avoid the case where the previous traffic of networks dominates these users' selection. Moreover, in order to let all the users being served by the end of the selection procedure, we set the capacity cost of each user, based on their applications, as $\{1, 1, 1, 1, 5, 5, 5, 5, 1, 1, 1, 1, 2, 2, 2, 2\}$. We intentionally set the whole capacity of the 4 networks (i.e. 48) larger than the total required capacity of the 16 users (i.e. 36), so as to see the possibility of some networks having more users than others.

Based on Nash's theorem in [44], this game has at least one NE. We could definitely use certain algorithm mentioned above to find the NE, but the usage of these algorithms could not show us the difference between using game theory of this network selection issue or other mathematical theories. In order to show an intuitionistic comparison between game theoretical network selection scheme and other schemes, e.g., MADM-based schemes, we use the following method to simply find a pure strategy NE: First, we put all the users into their favorite networks based on the calculated utilities using SAW. Second, we check if there are some networks getting congested. If so, we choose the user with minimum capacity cost from this network and put it into the network with maximum utility among all the networks with enough capacity. We continue this procedure until no network is under congestion. Third, in the obtained allocation state, we search and switch for each user if there is a better network until no user could increase its utility by unilaterally changing to another network. Finally, we reach a pure strategy NE.

We can see that the objective of the first and second steps in the above method is just to get to an initial state for the third step. We use SAW in the first step instead of a random initial state, so that we could compare the results with MADM-based schemes. We find that the allocation in the first step is quite similar to that of MADM-based schemes without traffic consideration in Section III.

Network selection results of the 16 users using the above game theoretical scheme are given in Table VII, together with the results from schemes using other mathematical theories for comparison. With the above configuration of networks and users, these results are actually obtained by the first and the second steps. When we check for the possibility of any user could unilaterally increase its own utility by changing to another network, we find that the allocation state obtained by the first two steps is coincidently already a pure strategy NE.

VI. Combinatorial Optimization

Combinatorial optimization searches for an optimum object in a finite collection of objects. The number of objects grows exponentially in the size of the collection, so scanning all objects one by one and selecting the best one is not an option [45]. Based on the time complexity, combinatorial optimization problems can be classified into several groups, e.g., NP-hard problems which are considered at least as hard as NP problems. NP is short for *non deterministic polynomial time*.

A. Combinatorial optimization in network selection

Two NP hard models, i.e., knapsack and bin packing, have been considered for the network selection problem.

Knapsack problems are a family of optimization problems that require a subset of some given items to be chosen so that the corresponding profit sum is maximized without exceeding the capacity of the knapsack(s).

A generalized knapsack model fitting for the network selection problem is a combination of the 0-1 knapsack model and the multiple choice multiple dimension knapsack (MMKP) model [46], given by

$$\max \mathcal{U} = \sum_{k=1}^{N} \sum_{i=1}^{M} \psi_{ki} z_{ki}, s.t. \sum_{k=1}^{N} c_{ki} z_{ki} \leq C_i, \tag{12}$$

where $\mathcal{U}$ is the total profit, ψ_{ki} is the profit of item k placed in knapsack i, c_{ki} is the capacity cost of item k placed in knapsack i, z_{ki} is a binary variable representing the placement (or not) of item k in knapsack i, and C_i is the capacity of knapsack i.

Mappings between network selection and the knapsack problem are given as follows:

1) Applications map to the items,
2) Networks map to the knapsacks,
3) Resource constraint of a network maps to the capacity of a knapsack,
4) Cost of an application in a network maps to the cost of an item in a knapsack,
5) User utility maps to the profits, and
6) Utility of an application in a network maps to the profit of an item in a knapsack.

It is worth mentioning that the knapsack model fits for the case when networks' capacities are quite limited and load balancing is strongly demanded. When the capacity of networks is large enough for a coming application, the above model becomes a SAW algorithm presented in Section III.

Another NP hard model used to solve the network selection problem is bin packing. The classical bin packing problem is a well studied optimization problem: given K objects with sizes $c_1, ..., c_K$ belonging to $(0, 1]$, find a packing in unit-sized bins that minimizes the number of bins used. In the off-line version of this problem, it is possible to consider all the objects and choose the order of assignment. In the online version however, each object must be assigned in turn without knowledge of the next objects. That is, given $K-1$ already packed objects with sizes $c_1, ..., c_{K-1}$ belonging to $(0, 1]$, the new object K with size c_K belonging to $(0, 1]$ must be packed in such a manner that the number of used bins is minimized.

Network selection can be formulated as a bounded-space variable-size online bin packing problem, in which the number of available bins at any time is a restricted to a predefined number (i.e., bounded-space) and the capacities of bins can be different (i.e., variable-size). The objective is to find the best way of allocating applications into the networks in order to minimize the number of rejected applications, i.e., the blocking probability, hence maximizing the whole system's capacity. Moreover, one obvious difference from the classical bin packing problem is that the bandwidth required by one application is determined by the selected network, so we use c_{ki} to denote the size of application k in network i. In [47], the authors mapped the problem of network selection into the bin packing problem in this way and compared five algorithms, including FirstFit, BestFit, WorstFit, LessVoice and Random. The selection rules of these algorithms are summarized as follows:

FirstFit: the first randomly selected network that has enough space for the application.

BestFit: the network with minimum free space left after serving the application.

WorstFit: the network with maximum free space left after serving the application.

LessVoice: the network with minimum $c_{ki}/c_{voice,i}$.

Random: a totally random network, rejecting to serve the application when no enough space for it.

Based on the above studies, [35] proposed a greedy heuristic algorithm to match between the users and the networks. For the case of K users allocating to N networks, the algorithm starts with an $K \times N$ utility-to-resource ratio list where a utility-to-resource ratio is between the utility of a user and the resource that a network could allocate to this user. In each round of the algorithm, the user-network pair with the largest utility-to-resource ratio is picked and all the ratios for this user are removed from the list. The time complexity of this algorithm is bounded by $O(K^2 \times N)$. This greedy heuristic algorithm was compared with three bin-packing algorithms (including FirstFit, BestFit and WorstFit) and was shown that it out-performs them on both total utility and blocking probability.

B. Case study

We consider the usage of the MMKP knapsack model in the unified scenario presented in Section I with Tables I and II. Similar to the case study in Section V, this model also fits for the situation when networks' capacities are quite limited. Otherwise, it becomes a SAW algorithm of MADM, as explained in the case study of Section III. Therefore, in order to show the difference between schemes with this mathematical model and others, the capacity of networks and the capacity cost of users are set in the same way as explained in the case study of Section V.

The profit of each user is obtained as the combination of the normalized values of the five attributes based on their weights obtained by the eigenvector method, similar to our configuration in the case study of Section III. Finally, we use simulated annealing (SA) algorithm [48] to find a sub-optimal solution for this problem.

We state the algorithm from an initial state with a total profit of 6.16, given by $\{W, M, L, P, W, M, L, P, W, M, L, P, W, M, L, P\}$, in which network serves one user of each application. With $1,000,000$ rounds, we finally find a sub-optimal solution with a total profit of 11.67 and the allocation in Table VII. Based on the selection results of MADM in Table VII, we predicted that users should first occupy the capacities of WMAN and WPAN as much as possible, then choose WWAN or WLAN. This is proved true by the results, in which the four networks' capacities are occupied as $\{2, 12, 10, 12\}$.

VII. Markov Chain

Markov chain is a common tool for decision making. In this section, we present three types of Markovian approaches for network selection: Markov decision process (MDP) based scheme, permutation-based scheme and rank aggregation based scheme.

A. MDP-based scheme

In many situations in the optimization of dynamic systems, a single utility for the optimizer might not suffice to describe the real objectives involved in the sequential decision making. A natural approach is to optimize each objective with constraints on others. MDP can be used to handle this kind of multi-objective dynamic decision making problem [49]. In the literature, several network selection schemes based on MDP theory have been proposed.

An MDP is defined through the following objects [50]: a state space $\mathbb{S}$, sets $\mathscr{A}(s)$ of available actions at states $s \in \mathbb{S}$, transition probabilities $\rho(Y|s, a)$ and reward functions $r(s, a)$ denoting the one-step reward using action a in state s.

The above objects indicate a stochastic system with a state space $\mathbb{S}$. When the system is at state $s \in \mathbb{S}$, a decision maker selects an action a from the set of actions $\mathscr{A}(s)$. After an action a is selected, the system moves to the next states according to the probability distribution $\rho(Y|s, a)$ and the decision-maker collects a one-step reward $r(s, a)$. The selection of an action a may depend on the current state of the system, the current time, and the available information about the history of the system. At each step, the decision maker may select a particular action or, in a more general way, a probability distribution on the set of available actions $\mathscr{A}(s)$, which are called nonrandomized and randomized decisions, respectively. An MDP is called *discrete* if the state and action sets are discrete, which is the case for network selection. For discrete MDP, we denote the transition probabilities by $\rho(y|s, a)$.

[51], [52] provides an idea for modeling the network selection problem into an MDP. They put many decision epochs during the lifetime of a session with either equal or variable time intervals, represented by $\mathbf{t} = \{1, ..., T\}$, where T denotes the time that the session terminates. At decision epoch $t \in \mathbf{t}$, s_t and a_t are used to represent the current state and the chosen action, respectively. The state transition probability is denoted by $\rho(y|s_t, a_t)$. The reward is defined by $r(s_t, a_t) = f(s_t, a_t) - g(s_t, a_t)$, where $f(s_t, a_t)$ represents the benefit from using another network rather than the current one and $g(s_t, a_t)$ represents the signalling cost (may also consider packet loss) for handing-over to that network. For the whole session period, a policy $\theta = (\delta_1, ..., \delta_T) \in \Theta$, $\Theta = \mathscr{A}(s_1) \times ... \times \mathscr{A}(s_T)$, is defined as a sequence of action rules at all the decision epochs, where $\delta_t, t \in \{1, ..., T\}$ represents the action rule at decision time t. Given an initial state s_1, the objective of this MDP is to determine an optimal policy θ to maximize the expected total reward, denoted by $v(s_1) = \max_{\theta \in \Theta} v^{\theta}(s_1)$. $v^{\theta}(s_1)$ is calculated as the mean value of the total reward of all epochs with respect to the policy θ and the initial state s_1. To satisfy the Bellman optimality equation, the above equation could be further written as

$$v(s_1) = \max_{a \in \mathscr{A}(s_1)} \left\{ r(s_1, a) + \gamma \sum_{y \in \mathbb{S}} \left[\rho(y|s_1, a) v(y) \right] \right\} \quad (13)$$

where γ is the discount factor mapping the future reward to the current state. The future reward is less reliable and predictable, so it is less important than the current reward, denoted by $\gamma \leq 1$.

One key feature of MDP model is that it considers a bunch of consecutive decision epochs and makes a combined decision at the beginning, but this also requires an ambitious assumption that we need to predict, at the beginning of a session, the state information for all the future decision epochs during this session. Another feature is that MDP model solves network selection and VHO decision at one time by considering both benefit $f(s_t, a_t)$ and handover cost $g(s_t, a_t)$. If we only consider $f(s_t, a_t)$, this model tells us the best network at all the decision epochs.

Moreover, [53] used MDP for user/operator negotiation after network ranking. State is defined by the number of ongoing calls and the events, e.g., new call arrival, handover call arrival and call departure. Action is defined as admitting a call, rejecting a call and no action for call departure case. Reward is defined as the benefit for the operator from the acceptance of a call, which is related to service class. Based on these definitions, an operator could find the best strategy for a sequence of calls, which satisfies the Bellman optimality equation. Due to the fact that [53] is mainly about user/operator negotiation, not network selection, we are not going to discuss more on it.

B. Permutation-based scheme

To select the best network, an important task is to distinguish between networks. Since we consider network selection for mobile terminals, one important type of attributes to distinguish between networks is the mobility-related attributes, such as cell radius, coverage percentage, VHO properties, etc. Traditional attributes, e.g., price, bandwidth, etc., usually lead to the discovery of the best network, but mobility-related factors show us the priorities of networks. For example, noticing that certain nomadic terminal's VHO cost between

3G and WLAN is acceptably small, a strategy called *WLAN first* for this terminal should be used. This strategy does not mean the terminal always connects to WLAN, but WLAN has a higher priority than 3G.

In this tutorial, we use the concept *permutation* to represent the priorities of all the networks, without considering their availability. At anytime and anywhere, the first available network in the permutation should be selected. When there are N networks, we have N factorial permutations, so the network selection issue becomes the selection of the best permutation for usage, while the definition of the 'best' permutation is largely related to the VHO cost between networks. In our previous work [54], the total cost of a permutation was modeled as follows:

With N networks and M attributes, we use v_{ij} to denote the value of the jth attribute of the ith network, σ_i to denote the probability that network i is available, w_H to denote the weight of average handover cost and w_i to denote the weight of the ith attribute except the average handover cost, respectively. The total cost of each permutation can be written as

$$\mathcal{C}_{PERM} = (h_H + h_V^+ + h_V^-) \cdot w_H + \sum_{i=1}^{N} \left[\mathcal{R}_i \sigma_i \prod_{j=0}^{i-1} (1-\sigma_j)\right] \cdot (1 - w_H), \tag{14}$$

where $\mathcal{R}_i = \sum_{j=1}^{M} v_{ij} w_j$ is the combination of all the other attributes except VHO cost for network i, h_H is the average HHO cost, h_V^+ and h_V^- represent the average VHO cost of moving into a network better than the current one and the average VHO cost of moving out of the first available network, respectively.

Markov chain is used to help calculate h_V^+ and h_V^-. A state $\mathcal{S}(\cdot)$ in the Markov chain is defined as the state of a terminal staying in an area covered by a certain bunch of networks. For example, $\mathcal{S}(\{n_1 > n_2 > n_3\})$ represents that the terminal is covered by network n_1, n_2 and n_3, while $\mathcal{S}(\{n_2 > n_3\})$ represents that the terminal is covered by network n_2 and network n_3. Symbol '$>$' represents the left-side network is better than the right-side one. Therefore, when the terminal is moving from $\mathcal{S}(\{n_1 > n_2 > n_3\})$ to $\mathcal{S}(\{n_2 > n_3\})$, this movement leads to a VHO, contributing to h_V^-.

Since the number of permutations is the factorial of the number of networks, a permutation-based scheme could take too much time on the calculation of all the permutations' total costs, which causes a problem of slow decision. One idea to simplify the scheme is to divide all the networks into a few groups. As an example, [55] used sigmoidal utility functions for attribute adjustment, hence dividing all the networks into two groups. One group is small-scale networks, while the other group is large-scale networks. Using the above model, a threshold could be obtained for this two-group case, given by

$$\mathcal{T}(w_H) = \frac{\mathcal{R}_{L-S}}{\mathcal{R}_{L-S} + h_{\{S>L\}-\{L>S\}}/\rho_S}, \tag{15}$$

where the subscripts L and S represents large-scale and small-scale networks, respectively. Hence, $\mathcal{R}_{L-S}$ is the difference between $\mathcal{R}_L$ and $\mathcal{R}_S$, and $h_{\{S>L\}-\{L>S\}}$ represents the difference between average handover costs of the two permutations $\{S > L\}$ and $\{L > S\}$, respectively. Seen from the above threshold, the decision is dependent on w_H. If a scheme uses a weight smaller than $\mathcal{T}(w_H)$, $\{S > L\}$ is the best permutation. Otherwise, $\{L > S\}$ is the best.

Beside the consideration of mobility-related factors, another key advantage of permutation-based scheme is that it decreases the scheme trigger rate. When the best permutation is obtained, we do not have to trigger the scheme by terminal movement, but all the other schemes have to trigger network selection when the terminal moves to a new place where network coverage is different (i.e. from state to state in the Markov chain of permutation-based scheme).

C. Rank aggregation based scheme

Network selection can be formulated into a rank aggregation problem, in which a better rank can be derived by combining multiple ranks of different decision factors. [56] proposed a weighted Markov chain (WMC) scheme, falling into this branch, which finds the best network with the following algorithm:

1) Based on each attribute j, a rank of all the networks is obtained, given by $\tau_j = \{n_1^j \geq ... \geq n_N^j\}$, where n_i^j represents the ith network in the rank by this attribute and N represents the number of candidate networks. $\tau_j(i)$ denotes the rank of network i in τ_j. w_j denotes the weight of attribute j.

2) An $N \times N$ weighted Markov chain transition matrix $\mathbf{Y}$ is initialized and updated with certain method below.

3) The stationary distribution vector $\mathbf{f} = \{f_1, ..., f_N\}$, where sd_i is the preference index of network i, calculated by $\mathbf{f} = \mathbf{f} \times \mathbf{Y}$.

4) The best network n_θ is the one satisfying $\theta = \arg\max_i f_i$.

The key step of this algorithm is step 2 to update the $\mathbf{Y}$ matrix. [56] proposed two methods for this task:

Method I: for each attribute j and for each entry y_{kl} in matrix $\mathbf{Y}$, $y_{kl} = y_{kl} + \frac{w_j}{\tau_j(n_k^j)}$ if $\tau_j(n_k^j) \geq \tau_j(n_l^j)$.

Method II: for each attribute j and for each entry y_{kl} in matrix $\mathbf{Y}$, $y_{kl} = y_{kl} + \frac{w_j(N-\tau_j(n_k^j)+1)}{N}$ if $\tau_j(n_k^j) = \tau_j(n_l^j)$, or $y_{kl} = y_{kl} + \frac{w_j}{N}$ if $\tau_j(n_k^j) \geq \tau_j(n_l^j)$.

Another Markovian approach related to network selection was proposed in [57]. State is defined based on the number of users of different services (e.g., voice and data) in different candidate networks. Transitions between states within the Markov chain will occur due to the arrival and departure of voice call or data session. Giving the arrival distributions of voice calls and data sessions, the transition rates between states in the Markov chain will be decided by the network selection policy. The original authors showed that this model could be used to evaluate the performance of many types of network selection schemes, e.g., random selection and load balancing based selection. However, based on our understanding, this approach is more related to call admission control and it is difficult to be used as a scheme to dynamically select the best network in various scenarios. Therefore, we are not going to discuss more on this model.

D. Case study

MDP is an important mathematical model for decision making. An important feature of studies in [51], [52] is that

MDP enlarges the importance of handover cost, so some state information, e.g. the current used network, becomes very important for the decision. By ignoring VHO cost, these consequent decisions become totally independent, and this model provides actually an MADM-based network selection. By considering VHO cost, this model provides actually a VHO decision scheme not a network selection scheme. However, since MDP-based scheme becomes an MADM-based scheme by removing the VHO decision part, we are not going to do any comparison between MDP-based scheme and other schemes. For similar reason, we are going to compare the permutation-based scheme with other schemes. Instead, we select the WMC-based scheme with **MC** update method I for this case study.

We still use the unified scenario presented in Section I with Tables I and II. Weights are calculated by eigenvector method, as explained in Section III. As we assumed in Table I, some features of different networks are totally the same. If we give them different positions in the rank, it is unfair. For example, we assume cell radius of WWAN and WMAN are both 2000, if we give WWAN the first place in the rank and WMAN the second place in the rank, WWAN dominates WMAN based on the rank of cell radius for most 'mobility first' users, which is wrong. Therefore, in our study, we specifically check if some networks have quite similar values for certain attribute. If so, we give them the same position in the rank. For each user, the stationary distribution vector is obtained and the best network is selected as shown in Table VII.

VIII. Integrated Scheme

A. Comparison of using different mathematical theories for network selection

A general comparison of using the above mathematical theories for the network selection issue is provided in Table VI. We compared eight aspects as follows:

Objective: different mathematical theories have different functionalities, which lead to different objectives for their usage in network selection. To sum up, utility theory evaluates the utility of the value of each attribute. For example, a little change of the value of an attribute, that passes some QoS threshold, leads to greatly change of its utility. MADM provides a comprehensive theory for the combination of multiple attributes for a decision, although most studies using other theories also consider SAW by default. Fuzzy logic theory is especially helpful to adjust the values of dynamic attributes since the information of these attributes could be imprecisely collected. Game theory tells us the equilibrium between networks, between users, or between networks and users, which helps us to balance benefits among multiple entities. Combinatorial optimization provides us a sub-optimal allocation of users to networks, which could be quite close to the optimal solution. For the three types of Markovian approaches, the functionalities and objectives are totally different. MDP-based scheme is to optimize a series of consecutive decisions with prediction, permutation-based scheme provides the priorities of networks instead of the best network, while rank aggregation based scheme is to aggregate the ranks of networks obtained by different attributes.

Decision speed: schemes using utility theory, MADM or fuzzy logic are all fast to make a decision. Schemes using combinatorial optimization are really slow. For example, in our case study of the knapsack problem using simulated annealing, it takes dozens of seconds to complete a search of 1,000,000 rounds, which is definitely too late for making the decision on the best network. For game theory, the learning process takes some time. For Markovian approaches, the combination of consecutive decisions in MDP-based scheme, the calculation of the total costs of all the permutations in permutation-based scheme and the update of the MC matrix in rank aggregation based scheme all take some time. Therefore, schemes using game theory and Markov chain are not as fast as the first three theories, but definitely faster than combinatorial optimization.

Implementation complexity: schemes using utility theory, MADM or fuzzy logic are all simple to be implemented. Schemes using combinatorial optimization are complex. The complexity of Markovian approaches is between them due to the fact that the algorithms and calculations in Markovian approaches are more complex than the first three theories and definitely less complex than combinatorial optimization. For the implementation of a game-theoretic scheme, a distributed algorithm by each player is usually used to get to the equilibrium, which is largely more complex than Markovian approaches.

Precision: schemes using MADM, game theory or combinatorial optimization are precise. For Markovian approaches, MDP-based scheme and permutation-based scheme are precise, but rank aggregation based scheme is really imprecise due to the fact that rank only provides networks' priorities not the exact difference between their quantitative values. The precision of schemes using utility theory and fuzzy logic is difficult to judge. Utility functions in utility theory and membership functions in fuzzy logic both have the functionality to adjust attributes, i.e. enlarge or diminish the difference between networks on certain attribute, but this adjustment could loss precision. For example, in our case study of fuzzy logic, we utilized some simple membership functions and some simple fuzzy rules, so some networks are found with the same total utility. Therefore, the precision of schemes using utility theory and fuzzy logic is lower than MADM, game theory and combinatorial optimization.

Decentralized: without considering the problem of information gathering, all the theories could be used for decentralized network selection schemes except combinatorial optimization. Combinatorial optimization provides centralized algorithms to optimize the allocation of applications to networks.

User-centric: schemes using game theory or combinatorial optimization consider too much on the traffic load of networks, which benefits operators a lot but degrades the user's benefit. Schemes using the other theories do not have this feature, which are user-centric.

Mobility-oriented: as we explained in Section VII, it is difficult to take mobility-related attributes, especially VHO-related attributes, into account for the decision of the best network. In the literature, only schemes using fuzzy logic and some Markovian approaches considered VHO-related attributes for the decision. Among these schemes, only permutation-based scheme used VHO-related attributes for network selection,

TABLE VI
COMPARISON OF USING DIFFERENT MATHEMATICAL THEORIES FOR NETWORK SELECTION

	Utility theory	MADM	Fuzzy logic	Game theory	Combinatorial optimization	Markov chain
Objective	Utility evaluation	Combination of multiple attributes	Imprecision handling	Equilibrium between multiple entities	Allocation of applications to networks	Consecutive decisions / rank aggregation / priority evaluation
Decision speed	Fast	Fast	Fast	Middle	Slow	Middle
Implementation complexity	Simple	Simple	Simple	Complex	Complex	Middle
Precision	Middle	High	Middle	High	High	High (but Low for WMC)
Decentralized	Yes	Yes	Yes	Yes	No	Yes
User-centric	Yes	Yes	Yes	No	No	Yes
Mobility-oriented	No	No	Yes	No	No	Yes
Traffic-oriented	No	No	No	Yes	Yes	No

while the others used them for VHO decision. In Table VI, we judge all these schemes as mobility-oriented due to the fact that network selection and VHO decision might be processed together.

Traffic-oriented: similar to our explanation on whether the schemes are user-centric, schemes using game theory or combinatorial optimization take traffic load as a quite important factor for the decision, which even degrades other attributes. Therefore, the two are considered as traffic-oriented, while the others are not.

B. Integration of multiple mathematical theories

We studied the usage of various mathematical theories in this tutorial for the network selection issue. As you can see from the above studies, they have different features and different functionalities. To get all their benefits, we could think about combining them in the way shown in Fig. 5 to achieve an integrated solution:

Utility theory: network attributes, including traffic load, are adjusted by utility functions, but traffic load, which is highly related to combinatorial optimization and game in later operations, may be adjusted in a different way from others.

Fuzzy logic: when there are many access networks, we classify all the networks into several groups to decrease the time cost on the comparison of all the permutations. This operation is based on some key factors, such as cell radius, bandwidth and price, using fuzzy logic.

MADM: after the adjustment of the network attributes, MADM algorithm is used to combine these attributes based on their weights.

Combinatorial optimization: before MADM, we might check whether many networks' available capacity become limited. If so, instead of MADM, we could use certain algorithm of combinatorial optimization for the allocation of new services. Note that this theory is used in a centric manner on the network-side, not by terminals.

Markov chain: MDP might be used in the tradeoff of VHO decision after the networks are ranked

Game theory: after the tradeoff of VHO decision, many simultaneous (or technically considered as simultaneous) handing-over terminals might select the same best network, which causes congestion. We might use game theory for an opportunistic decision, so that these terminals could be distributed into different networks.

C. Case study of an integrated scheme

For this case study, we take our previous design which was very briefly presented in [24] as an example of the integrated scheme. As shown in Fig. 6, the solution contains four steps:

1) monitor the triggers and gather the required information.

2) preparations before combining all the attributes, including weighting procedure and attribute adjustment procedure.

3) combine multiple attributes as a single rank. The left part of this step, which could be any traditional network selection scheme, gives the best network. The right part of this step gives the best permutation, as explained in Section VII.B. Since it takes more time to get the best permutation than to get the best network, we use the best network until the best permutation is obtained.

4) make VHO decision. If the best network or the first available network of the best permutation is better than the current network, this step makes a simple decision on whether the benefit is worth the one-shot VHO cost.

We believe that the network selection procedure implemented in the future terminals should be simple and fast, and the main goal of network selection is to always select the best network for serving the given application, not to pay too much attention to load balancing. A network selection scheme paying too much attention to this attribute degrades other attributes' importance. Taking two networks both with low but totally different traffic loads as an example, the normalization process will ignore the two networks' low traffic loads but retain only the relative large difference, which leads to immoderate traffic load balancing between the two networks and compromises the importance of other attributes. Therefore, traffic loads are usually not required to be strictly balanced

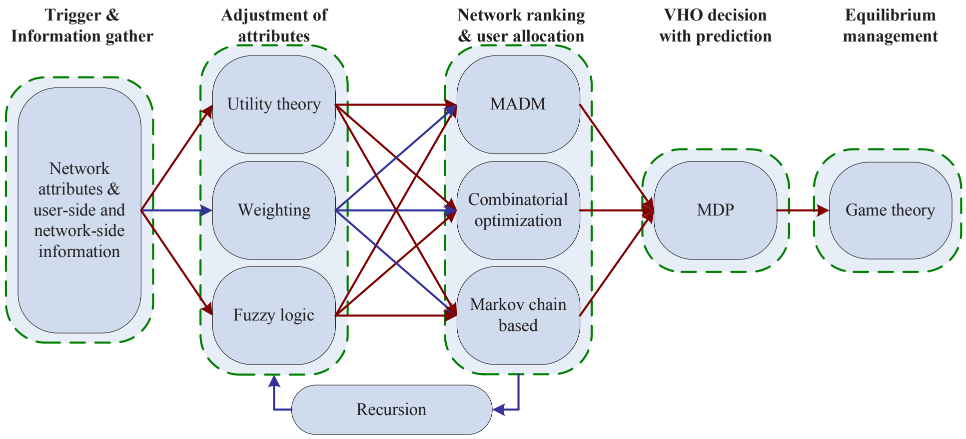

Fig. 5. Relationship between various mathematical theories for network selection.

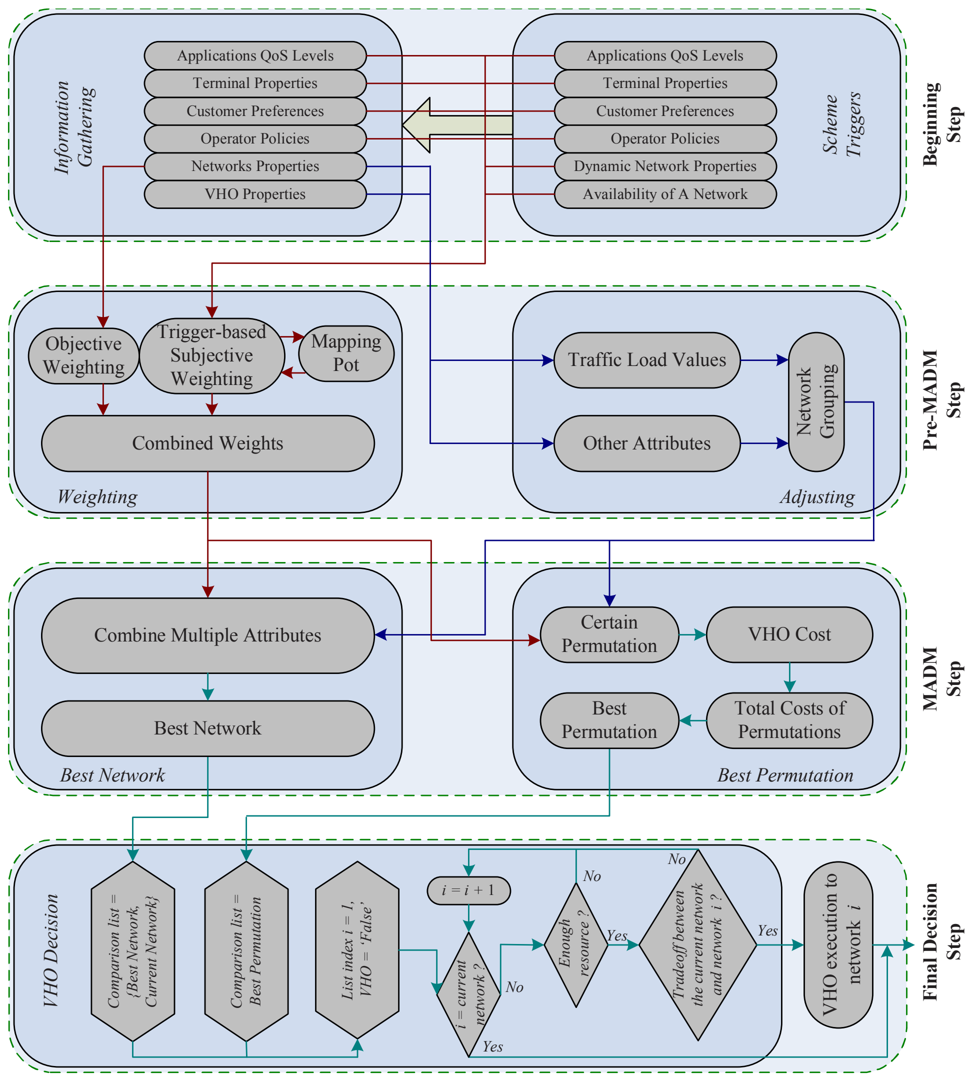

Fig. 6. An example of integrated scheme for case study.

among different networks due to the fact that traffic load is only one of a number of attributes, usually even not a decisive attribute, except when at least one network does not have much resource.

Based on the above analysis, we do not use game theory or combinatorial optimization for a specific load balancing. Instead, we adjust traffic load in a different manner from other attributes in our integrated scheme. First, since the real values are more important than the relative difference between the traffic loads of the two networks, we do not normalize them. Second, we use a special sigmoidal utility function with mid value equals 1 and $a \geq 2$ to calculate the utility. Based on our experience, a large value for a can be used to overcome immoderate traffic load balancing.

For this case study, the unified scenario presented in Section I with Tables I and II is used. In order to provide a fair comparison with other schemes, we use the same attributes, the same sigmoidal utility functions and the same MADM algorithm i.e., SAW. Moreover, we have the following specific configuration to guarantee fairness in this comparison:

• fuzzy logic theory for network grouping is not considered in this study. Otherwise, some networks are going to be totally the same after the adjustment of utility functions and fuzzy membership functions, which hides the load balancing feature of this integrated scheme.

• other schemes do not consider VHO decision, so we are not going to use VHO decision step in this case study, either. Otherwise, the comparison is unfair for other schemes. Therefore, the networks indicated in Table VII are the best networks for those users. Whether those users will handover to their best networks still depends on VHO decision.

The difference between the integrated scheme in this study and other scheme presented in previous sections is as follows:

• our solution combines utility theory and MADM.

• the sigmoidal utility function for 'traffic load' is specifically designed as explained above.

• weights are calculated based on our trigger-based method.

• Markov chain is used for best permutation selection.

• the best network is the first network (because all the four networks are available) in the best permutation obtained by the right side of step 3, not from a best network selection scheme.

• traditional network selection scheme is integrated for a fast decision before the best permutation is found.

• the difficulty of implementation comes from the calculation of total costs of all permutations.

• the precision is high as long as we do not use the 'network grouping' functionality.

• the solution is decentralized, user-centric, mobility-oriented and traffic-oriented.

Note that Fig. 6 is just one example of integrating multiple theories, and the features and network selection results could be totally different if you combine multiple theories in a different way.

In this study, we also consider the case where each network has a limited capacity for these 16 users, as explained in the case study of game theory. We rank the 4 networks based on the integrated scheme as shown in Fig. 6, but at the beginning of the last step, we check if the network has enough resource before VHO decision. We will show that our integrated scheme could achieve a similar load balancing functionality without using game theory or combinatorial optimization.

Network selection results of the 16 users are given in Table VII, together with the results from schemes using other mathematical theories for comparison.

D. Observations on the selection results of different schemes

For the selection results of different schemes in the case studies in this section and previous sections, summarized in Table VII, we have the following important observations:

• different types of users have some general preferences. For example, WLAN is selected by a lot of streaming users but not selected by interactive users at all; interactive users prefer WMAN and WWAN for security reason; conversational users also prefer WMAN and WWAN but for continuity reason; money-first users prefer WPAN and WLAN; mobility-first users prefer WMAN and WWAN; and battery-first users prefer WPAN.

• since we design WWAN as a dominated network by WMAN, users basically prefer WMAN to WWAN. For example, with the first two schemes, no user selects WWAN at all. With schemes using fuzzy logic, WMAN is better than WWAN for most users, but equally good as WWAN for some users for the sake of imprecision of fuzzy logic. With schemes using the other four theories, traffic is considered, so WWAN might be selected when WMAN is full.

• if we consider battery low as an important event, WPAN is obviously preferred. A few exceptions with utility theory and fuzzy logic are due to the imprecision reason, while a few exceptions with the last four theories are due to the reason of traffic load balancing.

• SAW with AHP, fuzzy logic, game between users, knapsack with SA and WMC all define total utility in the same way, i.e., summing up multiple attributes based on linear utility function. Among these five schemes, SAW with AHP provides higher utility than fuzzy logic and WMC since it is precise, while knapsack with SA provides higher utility than game between users since it takes much more time to search for the network with the maximum utility. However, it is unfair to compare the total utilities of all the schemes together since they are actually suitable for different situations: SAW with AHP, fuzzy logic and WMC are suitable for the case when traffic is not a key factor, while game between users and knapsack with SA are suitable for the case when resource of some networks becomes tight. For the scheme sigmoidal utility and the integrated scheme, it is unfair to compare with other schemes on the total utility since they use actually a totally different way to evaluate the total utility. Sigmoidal utility scheme uses sigmoidal functions to adjust the utilities of attributes, so it assumes that the best network should be with the maximum adjusted utility, not the maximum unadjusted utility. The integrated scheme combines traffic into the total utility, so the definition of the total utility is different from others. If we use this definition to evaluate the utility of different schemes, the integrated scheme is surely with the maximum utility, but we feel it unfair for other schemes in this kind of comparison. That is also why we provide general comparison of different schemes' total utilities, instead of demonstrating them in figures.

• with the integrated scheme, traffic loads of different networks are $\{2, 11, 12, 11\}$. Considering that WWAN is dominated by WMAN, it is quite correct to not select WWAN until there is not enough space in WMAN. Traffic loads of different networks using game between users and knapsack with SA are $\{6, 12, 6, 12\}$ and $\{2, 12, 10, 12\}$, respectively. Therefore, considering traffic load balancing, we can see that our integrated scheme is equally good as knapsack with SA, while we do not have to use a slow optimization algorithm, such as SA, in our integrated scheme.

TABLE VII
SELECTION RESULTS OF DIFFERENT SCHEMES IN THE CASE STUDIES

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Sigmoidal utility	M	M	M	M	L	M	L	M	M	M	M	M	P	M	P	M
SAW with AHP	P	M	P	M	P	M	P	M	P	M	P	M	P	M	P	M
Fuzzy logic	L	M	L	WM	LP	L	M	M	P	WM	WMP	WM	P	L	P	WM
Game between users	L	L	W	W	P	M	P	M	W	W	W	W	L	L	P	M
Knapsack with SA	P	M	P	M	L	L	P	M	P	M	M	M	P	M	P	W
WMC	P	M	P	M	P	M	M	M	P	M	M	M	P	M	P	M
Integrated scheme	P	M	P	M	L	L	P	M	M	M	M	M	L	P	P	W

Note: W = WWAN, M = WMAN, L = WLAN and P = WPAN

IX. CONCLUSION

Network selection has been widely studied by using various mathematical theories in the literature. The employed theory is extremely important because it decides the objective of optimization, complexity and performance, but there lacks a tutorial on the mathematical models used for the network selection problem. Therefore, this paper filled the blank by conducting a serious survey and providing a systematic tutorial on the main mathematical theories used for this problem, including utility theory (cost function), MADM, fuzzy logic, game theory, combinatorial optimization, Markov chain. A unified scenario was used to explain and compare selected network selection schemes using these theories. In the end, the integration of multiple of these theories was discussed, and an integrated scheme combining the advantages of several mathematical theories was proposed and compared with selected schemes.

X. NOTATIONS

$\mathcal{C}$: total coefficient of combining multiple attributes
K: number of users or applications
M: number of attributes
N: number of networks
c_{ki}: capacity cost of application k in network i
n_i: network i
v_{ij}: normalized value of attribute j in network i
w_j^k: weights of attribute j for application k
x_{ij}: value of attribute j in network i

Utility Theory (Cost Function):
$\mathcal{N}(\cdot)$: normalization of certain utility
$\mathcal{F}_i$: total cost of network i
U: total utility of all the attributes
$f_j^k(\cdot)$: weighting function of attribute j for application k
u_{ij}^k: utility of application k in network i in terms of attribute j
ϵ_{ij}^k: network elimination factor for application k, network i and attribute j

MADM:
$\mathscr{C}_{kl}$: concordance set including the attributes on which network k is better than network l
$\mathscr{D}_{kl}$: discordance set including the attributes on which network k is worse than network l
$\mathcal{D}^{\alpha}$: Euclidean distance from certain network to the worst reference network
$\mathcal{D}^{\beta}$: Euclidean distance from certain network to the best reference network
$\mathcal{V}_j^{\alpha}$: value of the jth attribute of the worst reference network
$\mathcal{V}_j^{\beta}$: value of the jth attribute of the best reference network
$\mathbf{B}$: pair-wise comparison matrix between all the attributes
$\mathbf{R}$: relationship matrix between events and attributes
$\mathbf{d}$: binary vector denoting true or false of events
$\mathbf{e}$: weights of all the events
$\mathbf{I}$: identity matrix
$\mathbf{w}$: weights of all the attributes
E: number of events
b_{ij}: comparison value between the ith and the jth attributes in $\mathbf{B}$
r_{ij}: strength of the effect from the ith event to the jth attribute
$\bar{x}_j$: mean value of all the networks in terms of attribute j
λ: eigenvalue of $\mathbf{B}$
Λ_j: nominal value of attribute j

Fuzzy Logic:
$\mathscr{F}_j^l$: fuzzy set for the jth input in fuzzy rule l
$\mathscr{G}^l$: fuzzy set for the output in fuzzy rule l
$\mathcal{X}_j$: the jth input of a fuzzy logic system
$\mathcal{Y}$: output of a fuzzy logic system

Game Theory:
$\mathbb{B}_k$: type space of player k in Bayesian game
$\mathscr{K}$: set of users
$\mathscr{N}$: set of networks
$\mathscr{Q}$: set of Bayesian strategies
$\mathcal{B}_k(\mathbf{q}_{-k}, B_k)$: best response of player k in Bayesian game
$\mathcal{K}_i$: number of users choosing network i
$\mathbf{p}^{(a)}$: vector of proportion of users choosing different networks in service area a
$\mathbf{q}_k$: Bayesian strategies of all the players except k
B_k: minimum bandwidth requirement as the type of player k in Bayesian game
$c_k(\cdot)$: cost of user k in the congestion game

$p_i(t)$: proportion of users choosing network i
$p_i^{(a)}$: proportion of users choosing network i in service area a
q_k: Bayesian strategy of player k
$\zeta_{ki'}$: binary variable representing whether user k is within the coverage of network i'
η_{ki}: binary variable representing whether user k selects network i
$\pi_i(t)$: payoff of the users choosing network i in the evolutionary game
$\bar{\pi}(t)$: average payoff of the entire population
$\bar{\pi}_k$: expected payoff of player k as bandwidth utility minus connection fee

Combinatorial Optimization:
$\mathcal{U}$: total profit
C_i: capacity of network i
z_{ki}: binary variable representing whether application k selects network i
ψ_{ki}: profit of application k selecting network i

Markov Chain:
$\mathbb{S}$: state space
$\mathscr{A}(s)$: set of available actions at state s
$\mathcal{R}_i$: combination of all the other attributes except VHO cost
$\mathcal{S}(\cdot)$: state denoted by the area covered by a certain bunch of networks
$\mathcal{T}$: threshold between the selection of different permutations
$\mathbf{Y}$: weighted Markov chain transition matrix
$\mathbf{f}$: stationary distribution vector $\mathbf{t}$: decision epochs
T: number of epochs during a session lifetime in an MDP
a_t: action at epoch t
$f(s_t, a_t)$: benefit of using action a_t from state s_t
$g(s_t, a_t)$: cost of using action a_t from state s_t
h_H: average horizontal handover cost
h_V^+: average cost of vertical handover to a better network
h_V^-: average cost of vertical handover from the current best network
$r(s_t, a_t)$: one-step reward using action a_t from state s_t
s_t: state at epoch t in an MDP
y_{kl}: element in $\mathbf{Y}$, representing the difference between network k and l
δ_t: epoch t during a session lifetime γ: discount factor mapping the future reward to the current state
σ_i: probability that network i is available
$\rho(y|s, a)$: transition probability from state s with action a in discrete MDP
$\rho(Y|s, a)$: transition probability from state s with action a in continuous MDP
τ_j: rank of networks based on attribute j
θ: policy indicating the network selection for each epoch during a session lifetime in an MDP
Θ: policy space

XI. Glossary

AAA: Authentication, authorization and Accounting
ABC: Always Best Connected
AHP: Analytical Hierarchy Process
BER: Bit Error Rate
ELECTRE: ELimination Et Choix Traduisant la REalité
GRA: Gray Relational Analysis
GSM: Global System for Mobile communications
HHO: Horizontal HandOver
HWNs: Heterogeneous Wireless Networks
MADM: Multiple Attribute Decision Making
MCDM: Multiple Criteria Decision Making
MDP: Markov Decision Process
MEW: Multiplicative Exponential Weighting
MMKP: Multiple Choice Multiple Dimension
MODM: Multiple Objective Decision Making
NE: Nash Equilibrium
NP: Non deterministic Polynomial
PCR: Performance-Cost Ratio
QoS: Quality of Service
RSS: Received Signal Strength
SA: Simulated Annealing
SAW: Simple Additive Weighting
SIR: Signal-to-Interference Ratio
SNR: Signal-to-Noise Ratio
TOPSIS: Technique for Order Preference by Similarity to an Ideal Solution
TRUST: TRigger-based aUtomatic Subjective weighTing
UMTS: Universal Mobile Telecommunications System
VHO: Vertical HandOver
WiMAX: Worldwide interoperability for Microwave Access
WLAN: Wireless Local Area Network
WMAN: Wireless Metropolitan Area Network
WMC: Weighted Markov Chain
WPAN: Wireless Personal Area Network
WWAN: Wireless Wide Area Network

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