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PHYC20090 Electronics and Devices

Experiment No.7 Sinusodial Response of the LCR Resonant Circuit

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Contents

Abstract

The aim of this experiment was to observe how electrical alternating current (AC) ciruits exhibit wave properties in the form of sinusodial waves and responses. The circuit was simple, with an inductor (L), resistor (R), and capacitor (C). By varying different input elements of the circuit whilst keeping others fixed, many properties were found. The resonant frequency where current flow and voltage were amplified was found to be at approximately $f_0 = 16\,040 \pm 0.5$Hz, from which the inductance (L) was calculated to be $9.845 \times 10^{-4} \pm 7.86 \times 10^{-7}$H. The quality (Q) factor for this circuit was calculated and observed to be 0.339 ± 0.141 which indicates a less "sharp" peak.

1 Theory

1.1 LCR Circuits

An LCR circuit is made up of inductors (L), capacitors (C), and resistors (R), usually connected in series. Since all the components of the circuit are connected in series, equal amount of the current will flow through each element. [1]

A circuit containing these components, L, C, and R, can act as themselves individually at certain frequencies [2]. The LCR circuit can also magnify the voltages across the L, C, and R such that it is larger than the circuit's input voltage (ie. AC, Alternating Current) [2].

1.1.1 Inductance, Capacitance, Resistance

Inductance, capacitance, and resistance make up the basic parameters that can affect circuits up to some degree [3].

Inductance is a property of a conductor [4] and it's measured by its ability to store energy due to the magnetic field produced by the flow of current [3] and the voltage that is induced by the current's rate of change [4]. With AC (Alternating Current), the magnetic field produced fluctuates with the time-varying properties of AC power sources [3, 4].

The voltage is proportional to the rate of change of the current and this factor of proportionality is known as the inductance [4]. Coils of wire are most commonly used as the inductors in circuits as they amplify [3] the efficiency at which the magnetic field induces the voltage and current in the circuit. By coiling wire (solenoid) the magnetic field is concentrated and magnified at its centre, shown in Figure 1.

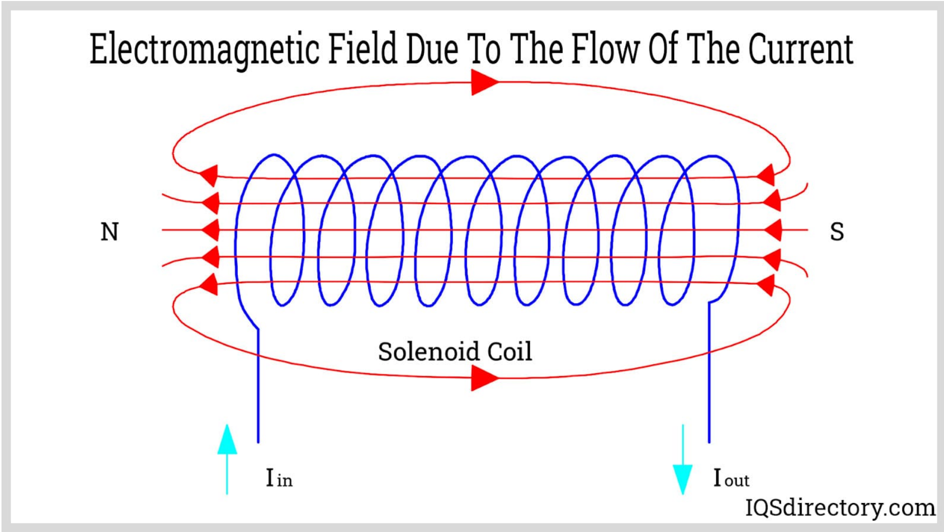

Figure 1: Electromagnetic Field Due to the Flow of the Current in a Solenoid [5]

Capacitance is a circuit's, or circuit component's, ability to collect and store electric charge [6]. Capacitors are made up of two electrically conductive plates separated by some distance. These two plates, when voltage is exchanged between them, become equally charged such that one plate is negatively charged (-Q) and the other is positively charged (+Q) [7, 8]. Overall, the charge of the capacitor will be neutral as the equal charge from both plates (-Q, +Q) cancels out [8] as in figure 2.

In a circuit with an AC supply, the capacitor is alternatively charged and discahrged every half cycle, therefore the amount of total stored charge in that capacitor depends on the frequency of the AC supply as it dictates how long it will charge for [7].

Capacitors are usually assembled with a dielectric material inbetween the two conductive plates [7, 6, 8]. Dielectric materials are poor conductors of electric fields, therefore labelled as insulators [9]. The capacitance of a capacitor increases with a dielectric material as the electric field is decreased and in turn so is the voltage across the two plates [10]. The capacitor ends up storing the same charge as if it were without a dielectric material but at a lower voltage, which is effective in reducing the possibilites of a circuit short [10].

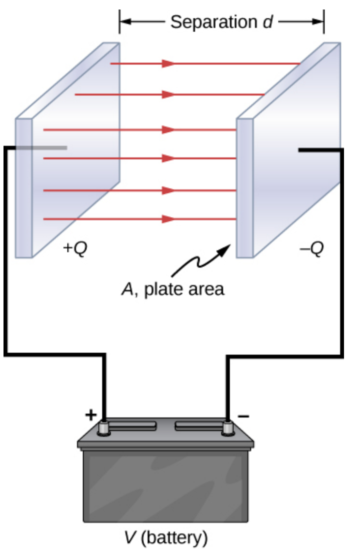

Figure 2: Parallel Plate Capacitor Diagram [8]

Resistance is a force that opposes the flow of current in a circuit [11, 12, 13]. It can be described as the electrical charge's difficulty in moving through a material. [12] Conductors and insulators, mentioned before, are types of materials classified by their resistance. Conductors are materials with little resistance that the electrons can travel through easily, like copper and gold (most metals). Insulators, on the other hand, are materials that make it difficult for the electrical charge to pass through, like wood and rubber [11]. These properties can be seen in electrical wires, with the current-carrying copper wire is encased in an insulating rubber tube for safety.

The resistance of a circuit component usually increases with temperature as the atoms that make up that material get excited, moving around in ways that make it difficult for the electrons to travel through [13, 14].

Resistors are circuit components specifically made to counteract the flow of current in a circuit [15, 14, 12] (Figure). Resistors can be used in a circuit to control the amount of voltage and current flowing in a circuit, which is useful to make sure the circuit doesn't blow and also to correctly distribute the current/voltage throughout the circuit [15, 12] The surplus of electrical energy flowing through a resistor is converted into heat energy which then dissipates [12].

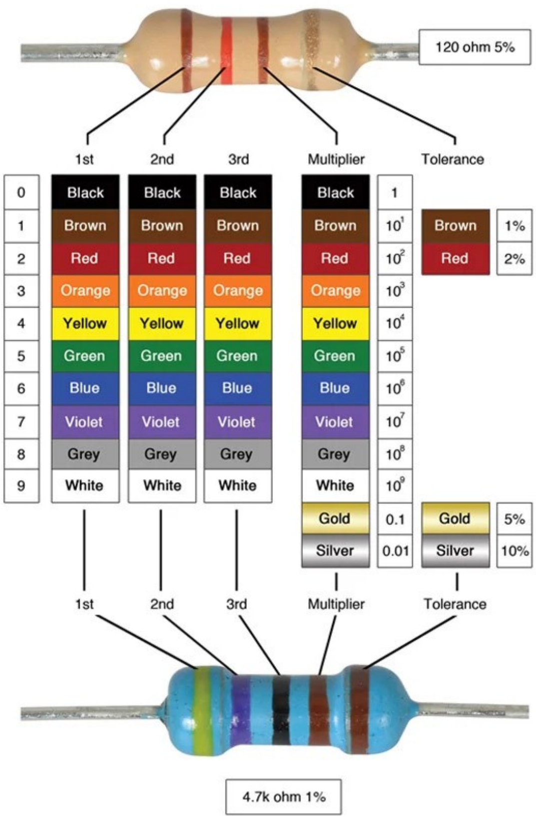

Figure 3: Resistor and How to Read One [16]

1.1.2 Impedance

The impedance of a circuit represents the overall resistance that it offers to AC [17]. The difference between resistance and impedance is that impedance only truly affects AC circuits while resistance affects both AC and DC (Direct Current) [18, 19]. In AC circuits the current is not constant but instead alternating, so the usual ratio of $\frac{V}{I}$ will also not be constant [18].

Impedance is the vector sum of the resistance (R), when the current is in phase and peaks at the same time, and reactance (X), which peaks one quarter of the cycle, within the circuit [17, 18]. The reactance X is composed of the positive reactance of the inductor (X_L) and the negative reactance of the capacitor (X_C) [18].

Impedance will be mathematically expanded on in §1.3, The Mathematics.

1.2 Wave Properties

Charged particles can create electromagnetic fields with their movement which transports electromagnetic (EM) energy, like light and radiation. They generate both an electric field and magnetic field, as the name implies [20]. Electromagnetic waves do need a material in order to be able to move, unlike mechanical waves [20]. As EM waves propagate, the frequency at which they're moving at has its own associated wavelength [21], as shown in figure 4. This is also how we can see colours, each frequency-associated wavelength is processed differently by our eyes, which is actually a very narrow spectrum [22].

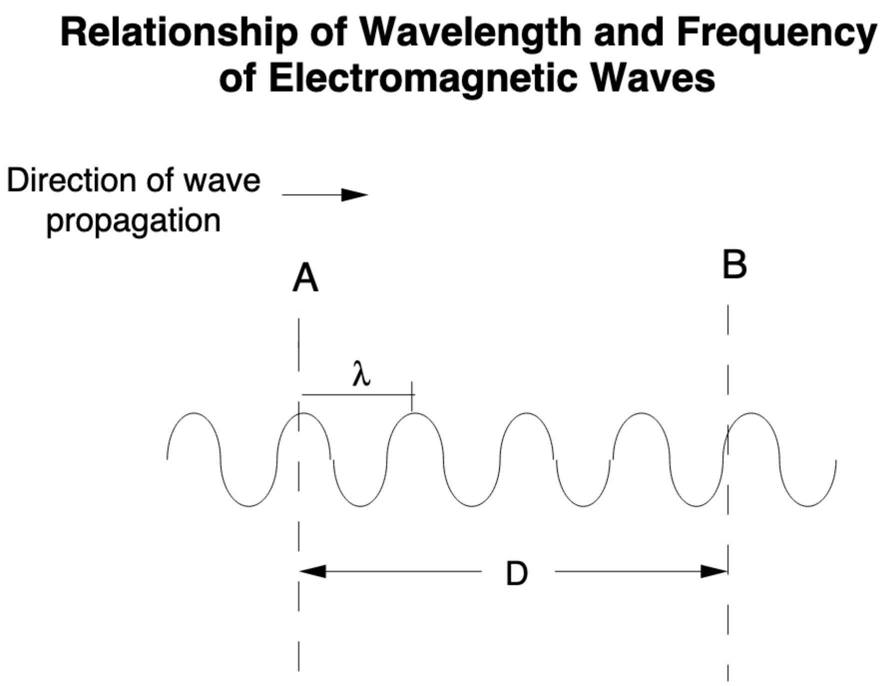

Figure 4: Relationship of Wavelength and Frequency of Electromagnetic Waves [21]

1.2.1 The Oscilloscope

Oscilloscopes are device that display voltage as waves, typically sine waves, in order to visualise the variation of voltage over time [23, 24]. The oscilloscope is an important device that will be used to find the resonant frequency of the LCR circuit being observed in the experiment.

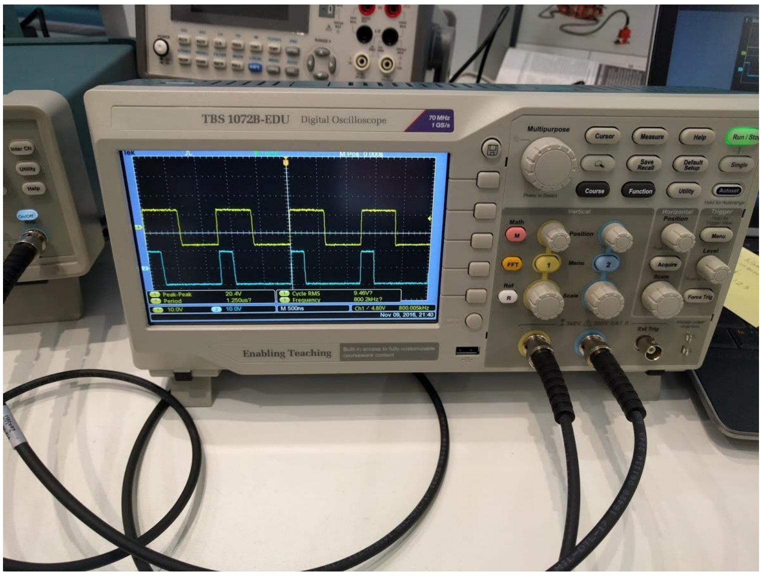

Figure 5: Oscilloscope With 2 Channels [25]

1.2.2 Electrical Resonance

Resonance in electrical circuits takes place at a specific, constant frequency at which the reactance and impedance cancel themselves out [26]. The net reactance, in turn, is zero, so the current flow and voltage amplitude are increased [26].

To get electrical resonance in a circuit the resonant frequency (f_0), at which the circuit is in resonance, must be found [26, 27]. Resonance, as a property of a wave, excites the particles of the same frequency. This is true even in electrical circuits. When there net reactance is zero, the impedance in the circuit becomes entirely resistive so there is no counteraction to the flow of the current. This constant resonance and increase in flow of current can lead to the circuit components overheating and breaking [26].

Electical resonance will be mathematically expanded on in §1.3, The Mathematics.

1.3 The Mathematics

The reactance of the inductor X_L is given by ωL where $\omega = 2\pi f$, the angular frequency [28, 18]. The reactance of the capacitor X_C is given by $-\frac{1}{\omega C}$ with ω once again as the angular frequency [28, 18]. Therefore, by understanding the total impedance formula is $\sqrt{R^2 + X^2}$ [18], we can then substitute X in for our two reactance values to get the impedance, Z:

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \tag{1}$$

The current amplitude is given by $I = \frac{V}{Z}$, Ohm's Law, but for AC circuits extra considerations must be taken. The AC equivalent to Ohm's Law occurs when $X_L = X_C$, where Z is then minimsed and thus the circuit will be in resonance (as discussed in §1.2) [28].

The time domain and frequency domain are two representations of the LCR circuit, as show below in figure 6.

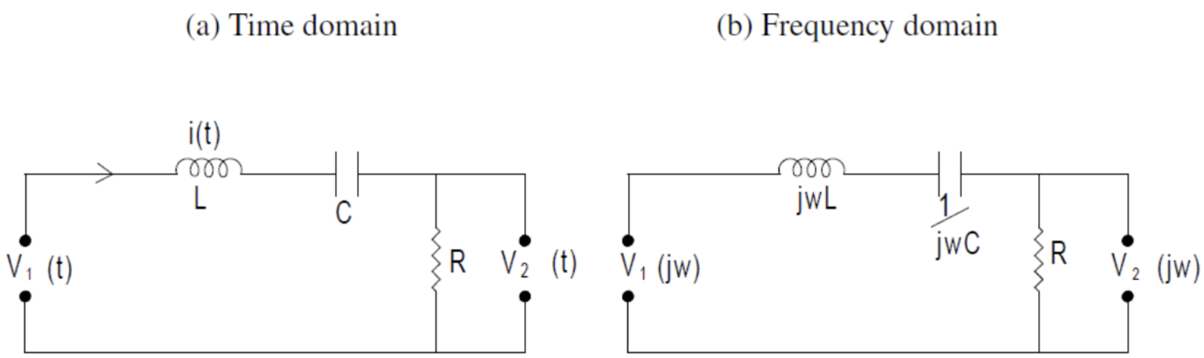

Figure 6: Time and Frequency Domains of the Series LCR Circuit [28]

With the appropriate equations below the mathematical representation becomes clearer [28].

Equation 2 represents Kirchoff's Voltage Law for series LCR circuits in the time domain, and equation 3 is the second-order differential equation for the voltage that represents the time-varying response of the circuit (time domain). Equation 4 is the phasor representation of the circuit in the frequency domain that takes into account the impedances from the capacitor and inductor, as discussed in §1.1.2, the reactances. Equation 5 expresses the output voltage across the resistor in the frequency domain, and how the input voltage is consequently transferred through the circuit.

$$v_1 = L\frac{di}{dt} + \frac{1}{C}\int idt + R_1 \tag{2}$$

$$\frac{d^2v_2}{dt^2} + \frac{Rdv_2}{Ldt} + \frac{1}{LC}v_2 = \frac{dv_1}{dt}\cdot\frac{R}{L} \tag{3}$$

$$V_1 = j\omega LI + \frac{I}{j\omega C} + RI \tag{4}$$

$$V_2 = IR = \frac{R}{j\omega L + \frac{1}{j\omega C} + R}V_1 \tag{5}$$

Equations 2 and 3 representations of time domain. Equations 4 and 5 representations of frequency domain. [28]

The use of complex (j) phasors in the frequency domain allows for the simplification of this above analysis by entirely removing the need for the integro-differential equations that are in the time domain. $H(j\omega)$ is defined at the sinusoidal response function, the ratio of the output to the input phasor in the frequency, equation 6:

$$H(j\omega) = \frac{V_2}{V_1} = \frac{j\omega CR}{1 - \omega^2 LC + j\omega CR} \tag{6}$$

Theoretical analysis shows that for the input signals $v_1(t) = Acos(\omega t)$ and $v_2(t) = Bcos(\omega t + \phi)$ then the gain (7) and phase shift (8) can be defined as, respectively:

$$Gain \equiv \frac{B}{A} = |H|\,\omega CR\ \left[(1-\omega^2 LC)^2 + (\omega CR)^2\right]^{-\frac{1}{2}} \tag{7}$$

$$Phase \equiv \phi = \angle H = 90^\circ - \tan^{-1}\left[\frac{\omega CR}{(1-\omega^2 LC)}\right] \tag{8}$$

The behaviour of the circuit can be visualised through Bode plots, figure 7, so thus the magnitude and angle of $H(j\omega)$ determine the voltage gain and phase shift of a sine wave through the circuit for any measured frequency. So, $H(j\omega)$ is then considered a fundamental circuit descriptor for linear circuits [28].

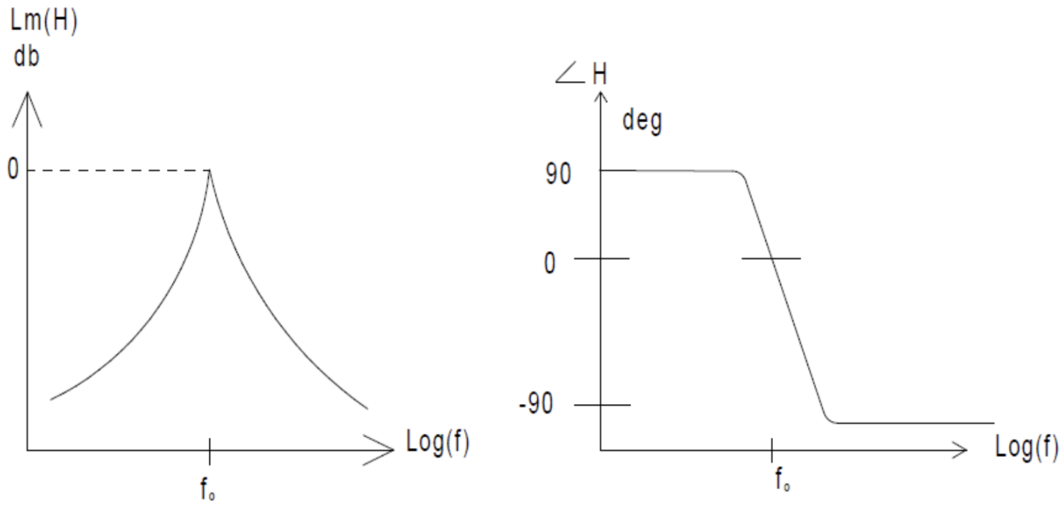

Figure 7: Typical Bode Plot Representations for the Series LCR Circuit (*Magnitude: Left, Angle of $H(j\omega)$: Right*) [28]

The magnitude, $Lm(H) = 20\log_{10}(|H|)$, given in decibels (dB), and the angle of H, $\angle H$ are shown as functions of the logatrithm of the frequency, $\log(f)$, which is what characterises the Bode plot for the system. The magnitude plot shows the gain change across varying frequencies and the phase plot shows the phase shift of the circuit for varying frequencies. The resonant frequency, f_0, is clearly labelled on the x-axis ($\log(f)$) for each plot, which is determined experimentally for the series LCR circuit and then compared to the theoretical predictions [28].

For the LCR, the gain and phase (response) curves depend greatly on the values of L, C, and R. From equation 7 it can be found that the gain reaches 0 dB, a maximum of unity, when $1 - \omega^2 LC = 0$. This observation can be used to derive the resonant frequency, ω_0 or f_0 [28]:

$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad or \qquad f_0 = \frac{1}{2\pi\sqrt{LC}} \tag{9}$$

Additionally, from equation 8, the phase shift (ϕ) is seen to vary between the input and output signals with frequency such that:

- At low frequencies ($\ll f_0$), the circuit has a 90° phase **lead**.
- At high frequency ($\gg f_0$), the circuit has a 90° phase **lag**.
- At the resonant frequency (f_0), the circuit phase shift passes through the 0° value.

These are the typical plot behaviours illustrated in figure 7 [28].

The "sharpness" of the resonance is indicated by the narrowness of the peak in the $Lm(H)$ (gain) curve and the steepness of the phase transition through the resonant frequency. This "sharpness" is quantified by the Q-factor, the wuality factor [28]:

$$Q \equiv \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC} \tag{10}$$

Greater selectivity and sharpness are a result of a higher Q-factor, shown in figure 8. The relationship between the gain (dB) and the phase angle ($\angle H$) is clearly demonstrated in this graph across a range of frequencies.

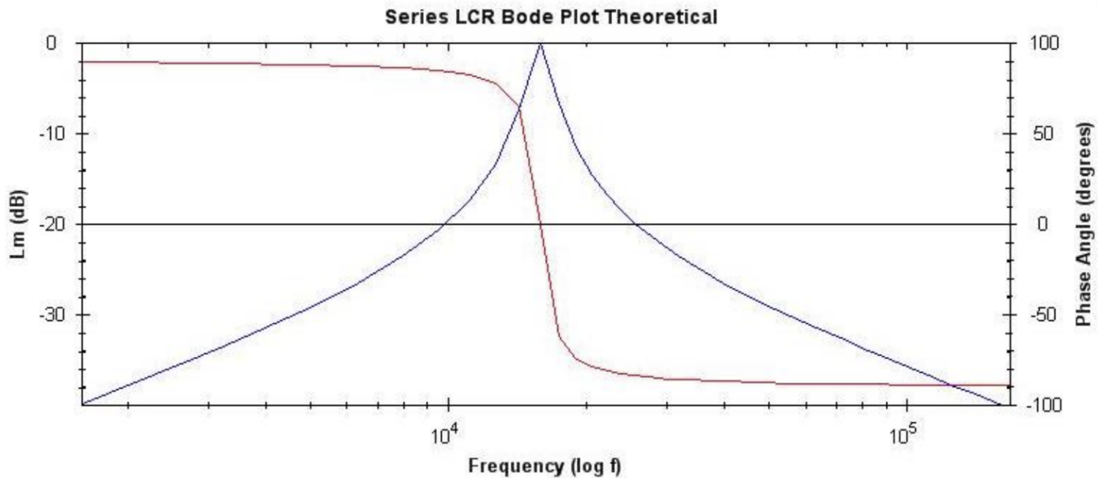

Figure 8: Example of the Computer Printout for Given Values of L, C, R, and Q. The Log of the Frequency is Plotted Against the Phase Angle ($\angle H$) and the Magnitude ($Lm(H)$) [28]

For a higher Q-factor (Q $>$ 10), the selectivity for frequencies close to f_0 is high, which makes these circuits ideal for tuning applications, such as radios receivers, to isolate different transmission frequencies. These types of circuits are often referred to as a tuned circuit [28].

For this section, the circuit is used in a similar way to a radio to tune into a specific input frequency by adjusting the resonant frequency (f_0). The resonant frequency for the LCR circuit is given by [28], similarly to equation 9:

$$f_0 = \frac{1}{2\pi\sqrt{L_0 C}} \tag{11}$$

in which L_0 is the fixed inductance of the inductor, measured ealier. By varying the capacitance (C) the resonant frequency (f_0) can be varied and adjusted as well such that [28]:

$$f_0 \propto \frac{1}{\sqrt{C}} \tag{12}$$

This shows that decreasing the capacitance will increase the resonant frequency in an inverse square-root relationship. The inverse (increasing capactitance will decrease the resonant frequency) is also true.

It has been previously observed that the maximum gain occurs when the input frequency mathces the resonant frequency of the circuit [28]. Conversly, at a fixed input frequency, the input signal will be amplified to its greatest extent when the LCR circuit is tuned (by varying the capacitance C) to match and resonate at that input frequency.

2 Methodology

This experiment is composed of three sections, each with their own set of instructions:

- The theoretical predictions for the LCR circuit method using plotting software, §2.1.
- Measurements for the Bode plot, §2.2.
- Measurement of the circuit response curves, §2.3.

The setup of the apparatus for the two experimental sections of this Methodology is shown in figure 9.

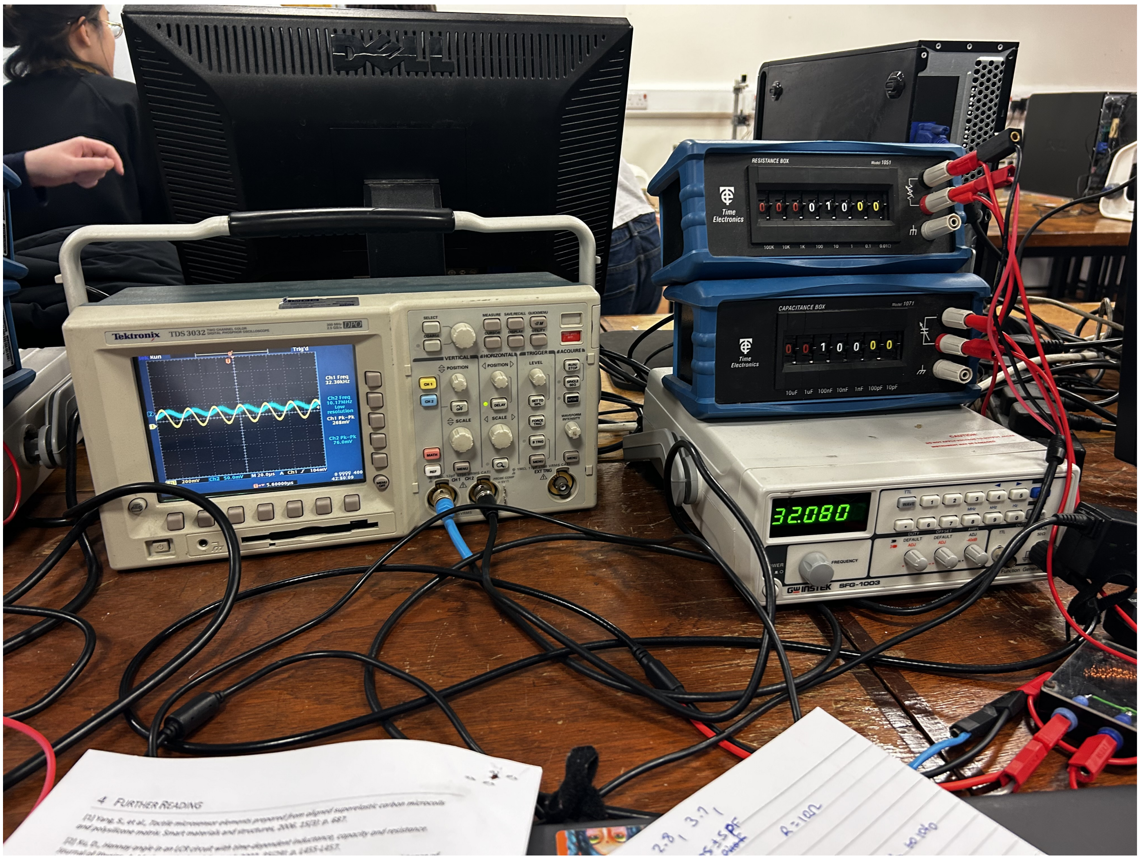

Figure 9: Apparatus setup for the experimental sections.

2.1 Theoretical Predictions for the LCR Circuit Method

The "LCR Plot" program shown in figure 10 is opened and the values for **L**, **C**, and **R** can be set to $L = 1$ mH and $C = 0.1\ \mu$F, leaving R open and available for varying values to see its effects on the plot and Q-factor.

This program calculates the resonance frequency f_0 and the Q-factor, plotting the predicted behaviour of the frequency response and the phase angle. The effect of the varying **R** values on the Q-factor for a given capacitance, **C**, can be seen in figures

The apparatus in figure 9 is then assembled with **C** set to 0.1 μF, as was done with the theory plot, and wiht **R** set to 10Ω. By observing the oscilloscope's (*right*) sinewave outputs in channel 1, ν_1, kept constant, and the varying amplitude via frequency changes in channel 2, ν_2, the resonant frequency f_0 can then be found by visually comparing the sinewaves until the phase shift is close to zero. With this value of f_0 **L** can be calculated using equation 9.

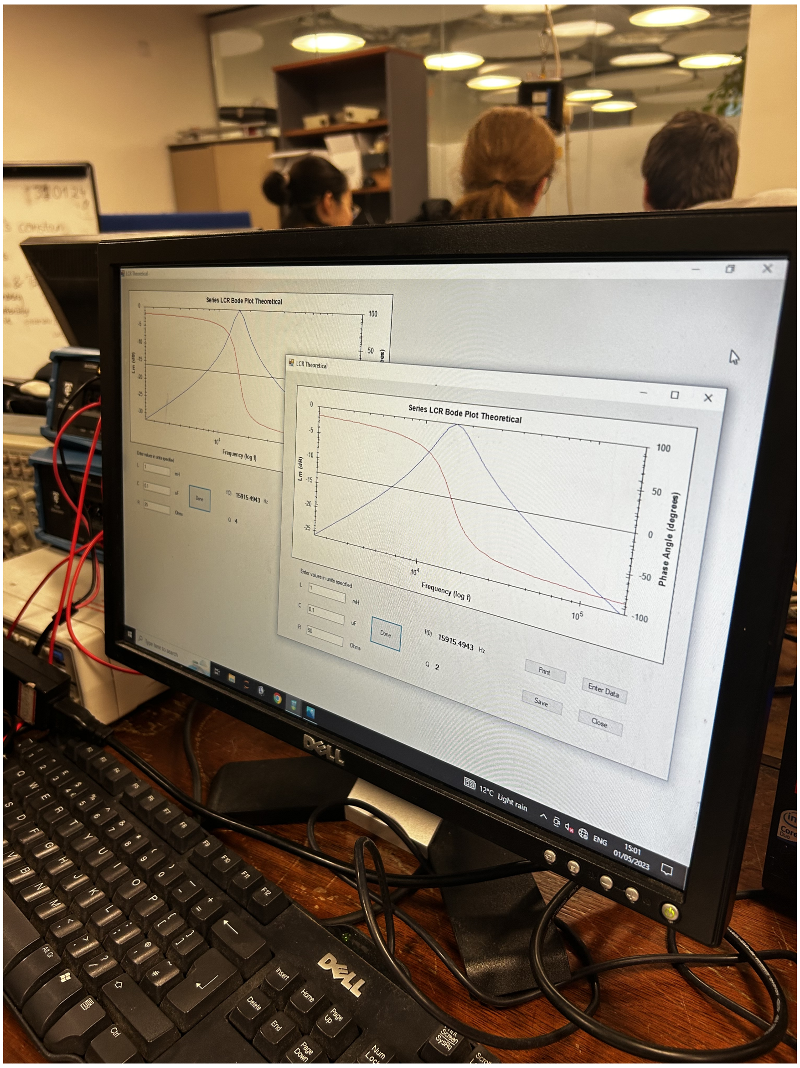

Figure 10: Theoretical software for the prediction of the LCR circuit.

2.2 Measurements for the Bode Plot Method

The behaviour of the LCR circuit is studied with varying input frequency while **L**, **C**, and **R** remain fixed. Measurements for this behaviour are taken in logatrithmic increments over two decades are selected (*ie,* $0.1f_0$ *to* $10f_0$) and values for the input and output peak-to-peak voltages are noted. The phase shift is achieved by measuring the time difference, ΔT, between the peaks and noting whether the phase shift is **lag** or **lead**. More readings should be taken closer to resonance, in which the set frequencies ideally provide evenly distributed data points to either side of the resonant frequency on a log-axis scale.

The gain (in dB) can be computed using the formula:

$$(Lm(|H|))(dB) \;=\; 20\log_{10}\frac{\nu_2}{\nu_1} \tag{13}$$

The phase shift (in degrees) can be determined by the formula:

$$\angle H(^\circ) \;=\; 360 \;\times\; f \;\times\; \Delta T \tag{14}$$

2.3 Measurement of the Circuit Response Curves Method

The circuit's response to varying capacitance, **C**, is investigated by setting the resistance to $R = 10\Omega$ and the resonant frequency to what was measured in §2.1 and keeping these values constant. The values of C are varied around the resonance with as fine of a scale as possible that allows a symmetrical and smooth graph to be produced. For each C value taken, the corresponding output frequecy is recorded from the oscilloscope. A graph like what is illustrated in figure 11 below should be the aim of what is produced with these measurements.

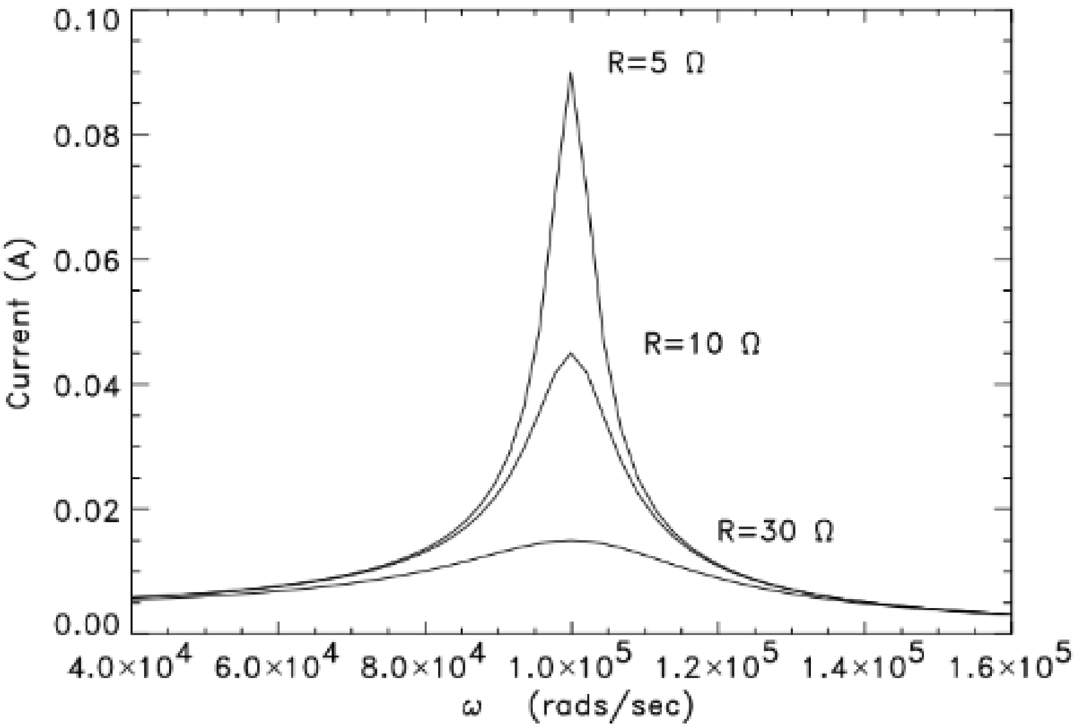

Figure 11: Theoretical response curves for the LCR series circuit assuming an input voltage amplitude of 0.45V. [28]

The above steps are repeated for a constant $R = 30\Omega$ and, if time were to permit, would have also been recreated for a constant $R = 5\Omega$. The input frequency is kept constant throughout.

3 Results and Calculations

3.1 Theoretical Predictions for the Bode Plot

The results for this section were obtained by following the method procedure in from §2.1.

The figures for the theoretical Bode plots simulated for specificied values of **R** can be found in the Appendix, §5.3.

- For $R = 5\Omega$, figure 15 shows the theoretical Bode plot.
- For $R = 10\Omega$, figure 18 shows the theoretical Bode plot.
- For $R = 20\Omega$, figure 16 shows the theoretical Bode plot.
- For $R = 30\Omega$, figure 19 shows the theoretical Bode plot.

- For $R = 40\Omega$, figure 17 shows the theoretical Bode plot.
- For $R = 50\Omega$, figure 20 shows the theoretical Bode plot.

After simulating these plots, the resonant frequency for the LCR circuit was calculated and found. At the resistance of $R = 10\Omega$ and capacitance of $C = 0.1\mu$F, as was specified in §2.1, the resonant frequency was discovered to approximately be:

$$f_0 \;=\; 16040\,\text{Hz} \,\pm\, 0.5\,\text{Hz} \tag{15}$$

With the use of this found resonant frequency value, the inductance, **L**, was calculated via a manipulation of equation 9:

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \implies \left(\frac{1}{2\pi f_0}\right)^2 = LC \implies L = \frac{1}{4\pi^2 f_0^2 C} \tag{16}$$

With the substitution for the values of $f_0 = 16040$Hz and $C = 0.1\mu$F:

$$L \;=\; \frac{1}{4\pi^2(16040)^2(0.1\times10^{-6})} \;=\; 9.845\times10^{-4}\,\text{H} \,\pm\, 7.86\times10^{-7}\text{H} \tag{17}$$

Where the unit for inductance is the *Henry (H)*.

3.2 Measurements for the Bode Plot

By making a table, table 1 as below, for the frequencies around the resonant frequency over 2 decades (a decade each side) based on a logatrithmic scale, a Bode plot can be created.

Log f_0	Frequency (Hz)	Ch1, ν_1 (V)	Ch1, $\Delta\nu_1$ (V)	Ch2, ν_2 (V)	Ch2, $\Delta\nu_2$ (V)	ΔT (s) $\times10^{-6}$	$\Delta(\Delta T)$ (s) $\times10^{-6}$
$0.1f_0$	1 604	0.28	0.004	0.043	0.01125	240	80
$0.3f_0$	4 812	0.28	0.006	0.06	0.019	50	5
$0.5f_0$	8 020	0.272	0.004	0.0732	0.038	32	8
$0.8f_0$	12 832	0.2	0.004	0.089	0.0145	16	2
f_0	16 040	0.084	0.006	0.084	0.006	0	4
$3f_0$	48 120	0.284	0.004	0.0551	0.0156	6	1
$5f_0$	80 200	0.284	0.006	0.062	0.0076	3.2	2
$8f_0$	128 320	0.284	0.006	0.055	0.012	2	1
$10f_0$	160 400	0.27	0.004	0.03	0.01	1.6	0.2

Table 1: Table of the data collected to generate a Bode plot with respective uncertainties.

With the values gathered, the gain $Lm(|H|)$ and phase angle $\angle H$ can be calculated with equations 13 and 14. These new values can be inserted into a table similar to before, table 2, to make plotting corresponding Bode plots easier.

Frequency (Hz)	$Lm(\|H\|)$ (dB)	$\Delta Lm(\|H\|)$ (dB)	$\angle H$ (°)	$\Delta\angle H$ (°)
1 604	-16.27	$\pm$ 2.276	138.59	$\pm$ 46.195
4 812	-13.38	$\pm$ 2.757	86.62	$\pm$ 8.662
8 020	-11.40	$\pm$ 4.511	92.39	$\pm$ 23.098
12 832	-7.03	$\pm$ 1.426	73.91	$\pm$ 9.239
16 040	0	$\pm$ 0.877	0	$\pm$ 23.098
48 120	-14.24	$\pm$ 2.462	103.94	$\pm$ 17.323
80 200	-13.22	$\pm$ 1.080	92.39	$\pm$ 57.744
128 320	-14.26	$\pm$ 1.904	92.39	$\pm$ 46.195
160 400	-19.08	$\pm$ 2.898	92.39	$\pm$ 11.549

Table 2: Table of the calculated gain and phase angle and respective uncertainties.

The plot of the gain against the log of the frequency is illustrated below in figure 12 using the values calculated in table 2.

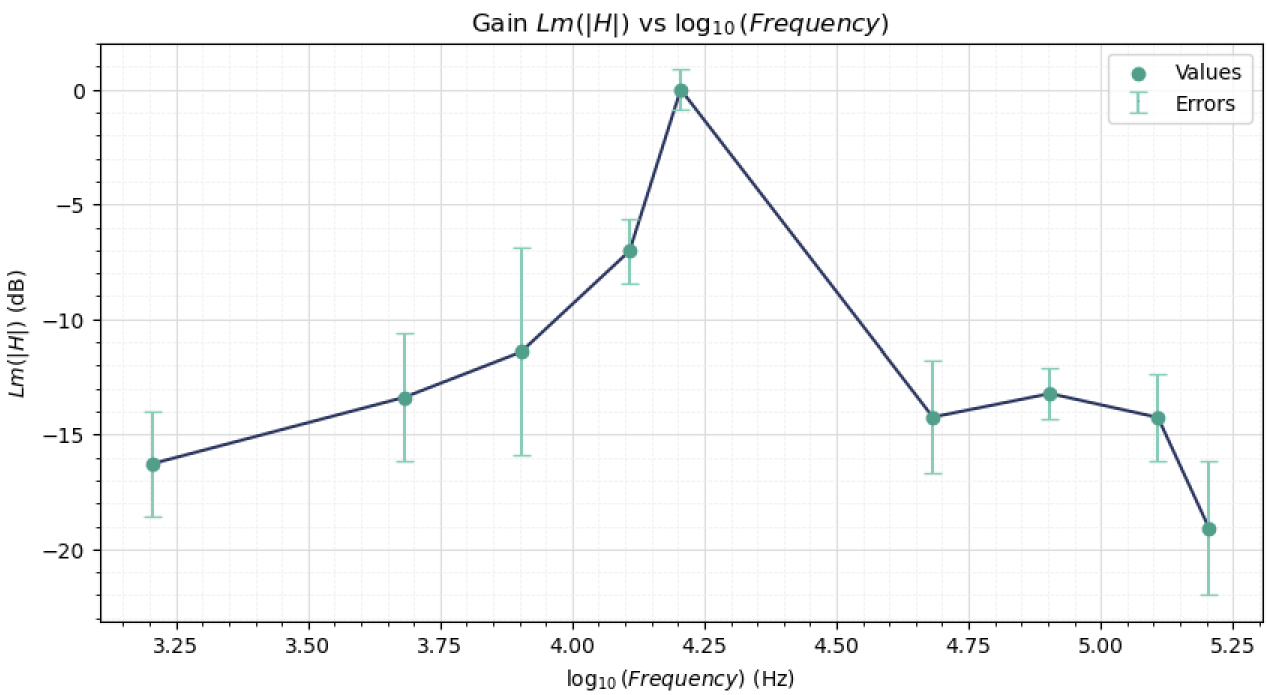

Figure 12: Gain vs. log of frequency plot for the LCR circuit.

The plot of the phase angle against the log of the frequency is illustrated below in figure 13 using the values calculated in table 2.

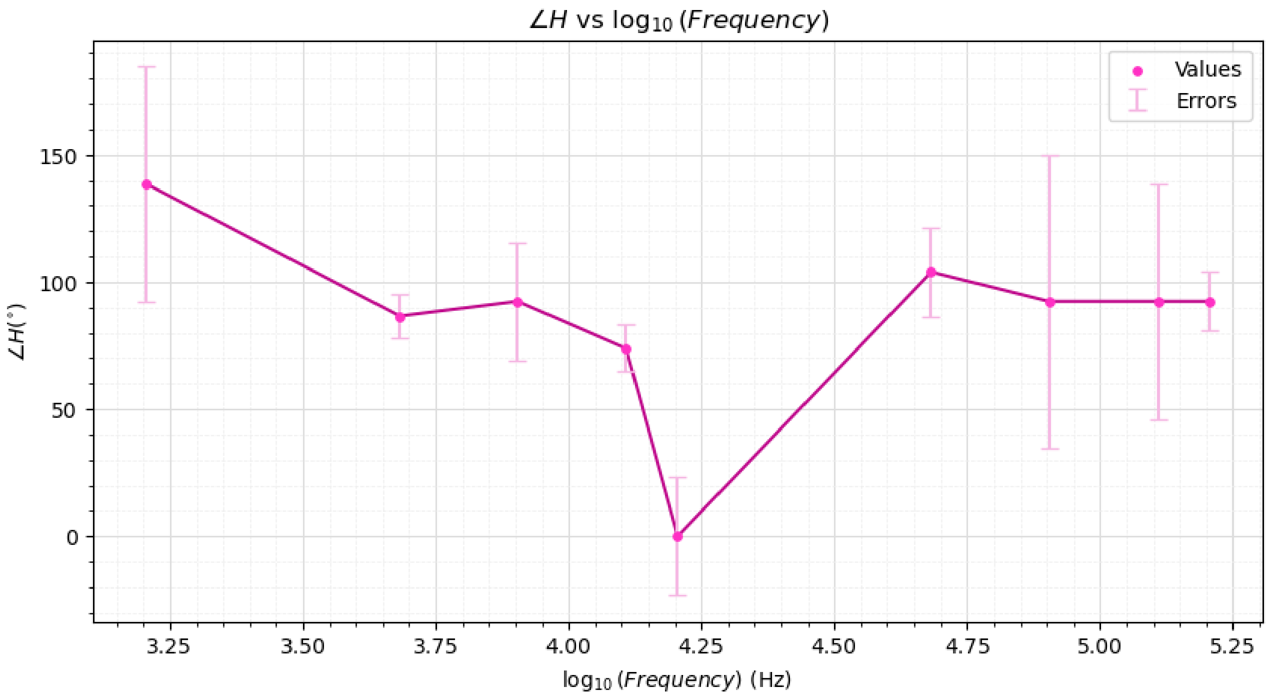

Figure 13: Phase angle vs. log of frequency plot for the LCR circuit.

3.3 Measurement of the Circuit Response Curves

The circuit was set to resistance of $R = 10\Omega$ and the resonant frequecy found in section §3.1, 16 040 Hz. Below is the table of results gathered, table 3. Values at $R = 5\Omega$ and $R = 30\Omega$ were not gathered due to shortage of time.

C (μF) $\times 10^{-6}$	V (in), ν_2 (mV) $\times 10^{-3}$	f_0 (Hz)
20	240 ± 1	1134.22
40	224 ± 1	802.01
60	188 ± 1	654.84
80	164 ± 1	567.11
100	84 ± 1	507.24
120	104 ± 1	463.04
140	144 ± 1	428.69
160	170 ± 1	401.01
180	188 ± 1	378.07
200	200 ± 1	358.67
220	208 ± 1	341.98

Table 3: Table of the circuit response at $R = 10\Omega$

Figure 14 shows the expected graph plot. closely relating to the example in figure 11. However, to achieve this result the values of the current (I) had to be inverted, as before it would illustrate a dip rather than the desired peak.

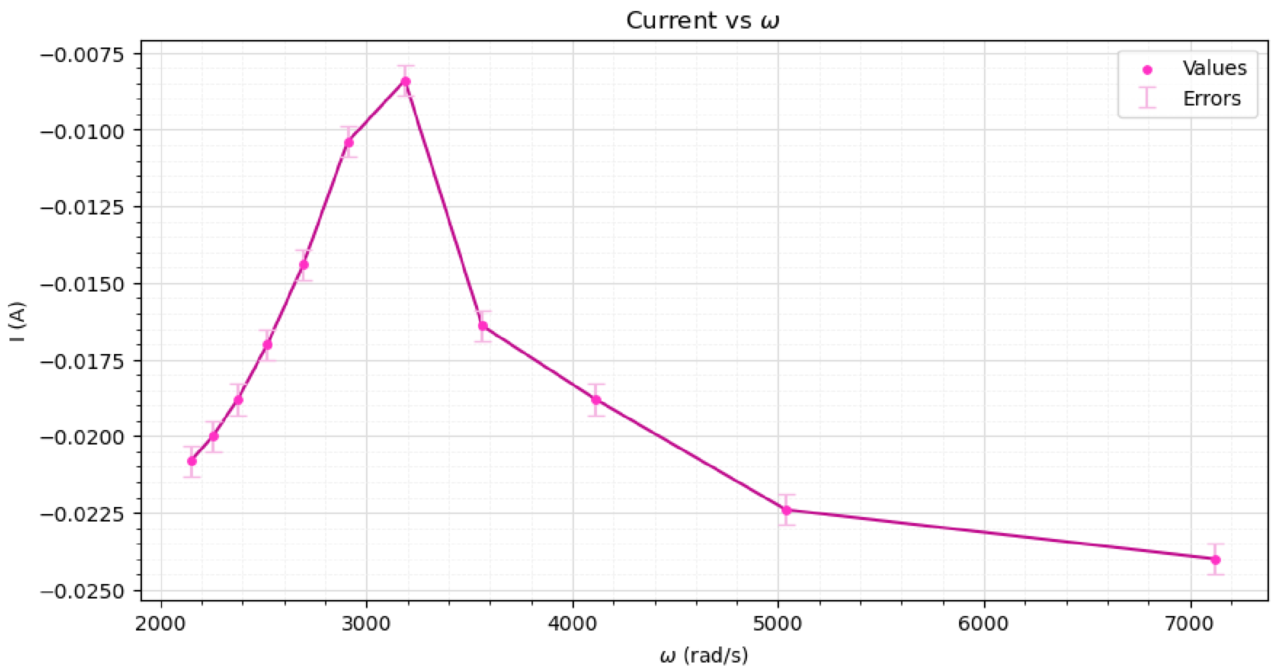

Figure 14: Plot of current vs. angluar frequency for the LCR circuit with $R = 10\Omega$ and inverted I for peak output.

By using equation 11, the Q-factor value can be calculated:

$$Q = \frac{1}{\omega_0 RC} = 0.339 \pm 0.141 \tag{18}$$

The value for the Q-factor had to be taken over the range of the angular frequencies and capacitances. With this value we can understand, and observe, that the peak is indeed not very sharp. In comparison to the theoretical graph, only the right-side of the plot agrees with it, while the left-side has a much less prominent curve.

4 Conclusion

This experiment verified and showcased the effectiveness of the LCR circuit's response curve with a reasonable resonant frequency at which the current flow and voltage was amplified and enhanced.

Through observation of the sinewave output of the oscilloscope, the resonant frequency was found to be $f_0 = 16\,040$Hz and the inductance was then calculated to be $L = 9.845 \times 10^{-4} \pm 7.86 \times 10^{-7}$H. It is difficult to confirm the effectiveness of this value for the inductance as the equipment did not readily provide the true measurement.

The Bode plot for the gain, figure 12, mostly agreed with the theoretical plots simulated in §3.1, figures 15-20, a peak clearly visible on the plot. For the phase angle, however, figure 13 in §3.2, appears as more of the inversion of the gain plot when it should follow the red line plot in figures 15-20. There's a possibility the time difference between peaks was incorrectly read on the oscilloscope, leading to issues during the calculations of the phase angle. This could be improved by studying the oscilloscope more beforehand as to know what and where exactly the values should be read from.

While the values of the current had to be inverted in order to provide the desired plot behaviour in figure 14, there is a peak that can be observed, albeit it not very sharp as the Q-factor of 0.339 ± 0.141 indicates. I was not able to be present for this section of this experiment so I am unable to directly comment on what could be done to imprive these results. From discussions with other students, it appears that the issue often lies in the unstable channel 1 voltage, so the channel 2 voltage had to be taken instead. Issues in understanding what exactly was being asked to be measured in the lab manual caused confusion and many measurements could have been taken with trepidation and uncertainty, causing errors in readings or recordings.

For future renditions of this experiment the lab manual available should be revised so that the data analysis and data-gathering for each section is clearly understandable. Reference values for the inductance could also be provided to aid guide the experimental outcome.

5 Applications of Electrical Resonance

5.1 Radios Transmissions and TV Broadcasts

Radios make use of frequency resonance to tune into broadcasts operating at specific frequencies on analogue radios, which can be heard once the the elements of the resonant circuit are in equilibrium [29]. If the peak of the frequency band is too sharp some information, like high frequencies, may be lost [28].

5.2 Voltage Multiplier/Magnification

When the capacitive and/or inductive reactances are equal with the resonant frequency, the total impedance is minimised and so there is an increase in the flow of the current as the supply voltage is less than the opposing reactive voltages. The voltage drop across the inductor and capacitor such that the current is increased is what is known as the voltage multiplier. Voltage magnification can be done passively without an external power source, which is beneficial in radio receivers in order to amplify weaker signals. [26, 29]

5.3 Environmental Monitoring

Resonant circuits can be used in **temperature monitoring**, in which variations in temperature effect changes in the inductance and capacitance sensor, thereby changing the resonant frequency. The same principle is used in **humidity monitoring**, where increases in permittivity and conductivity (proportional to humidity) leads to decreases in sensor resonant frequency. This can then be expanded upon to determine **complex permittivity**, and in turn **biological growth monitoring** by measuring the changes in complex permittivity. As humidity and temperature are bases for bacterial culture growth, changes in the Q-factor and resonant frequency can be used to monitor these developments. Additionally, changes in the resonant frequency can also be used for **pressure monitoring** as increases in pressure are directly proportional to changes in temperature ($P \propto T$) (and, in turn, humidity). More information with graphical and mathematical justification can be found at source [30].

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Appendix

Raw Data

Graphs for the Simulated Theoretical Predictions for the Bode Plot

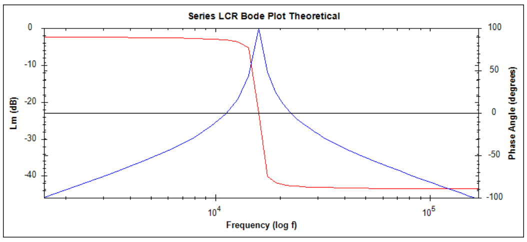

Figure 15: Theoretical Bode Plot, $R = 5\Omega$

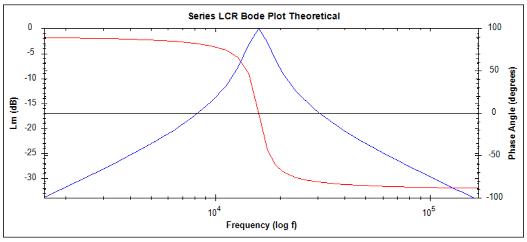

Figure 16: Theoretical Bode Plot, $R = 20\Omega$

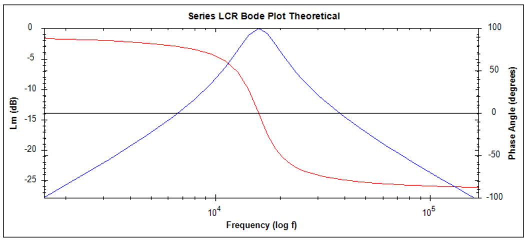

Figure 17: Theoretical Bode Plot, $R = 40\Omega$

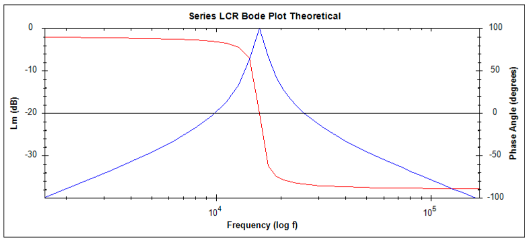

Figure 18: Theoretical Bode Plot, $R = 10\Omega$

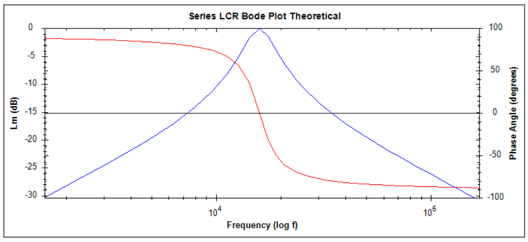

Figure 19: Theoretical Bode Plot, $R = 30\Omega$

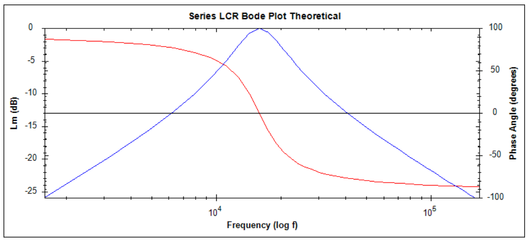

Figure 20: Theoretical Bode Plot, $R = 50\Omega$

Code

```
#log increments over 2 decades
frequency = np.array([1604, 4812, 8020, 12832, 16040, 48120, 80200, 128320, 160400])

# voltages ch1 + uncertainties
V1 = np.array([0.28, 0.28, 0.272, 0.2, 0.084, 0.284, 0.284, 0.284, 0.27])
dV1 = np.array([0.004, 0.006, 0.004, 0.004, 0.006, 0.004, 0.006, 0.006, 0.004])

# voltages ch2 + uncertainties
V2 = np.array([0.043, 0.06, 0.0732, 0.089, 0.084, 0.0551, 0.062, 0.055, 0.03])
dV2 = np.array([0.01125, 0.019, 0.038, 0.0145, 0.006, 0.0156, 0.0076, 0.012, 0.01])

# time delay + uncertainties
delta_T = np.array([240, 50, 32, 16, 0, 6, 3.2, 2, 1.6]) * 1e-6
d_delta_T = np.array([80, 5, 8, 2, 4, 1, 2, 1, 0.2]) * 1e-6

# lmh
Lm_H = 20 * np.log10(V2 / V1)
print(Lm_H)

# lmh uncertainty propagation
dR_R = np.sqrt((dV1 / V1)**2 + (dV2 / V2)**2)
print(dR_R)
dLm_H = 20 / np.log(10) * dR_R
print(dLm_H)

# phase angle
angle_H = (delta_T * frequency) * 360
print(angle_H)

# phase angle uncertainty propagation
d_angle_H = (d_delta_T * frequency) * 360
print(d_angle_H)
```

✓ 0.0s

```
[-16.27379152 -13.38013562 -11.40115646  -7.03279978   0.
 -14.24333482 -13.21853301 -14.25911301 -19.08485019]
[0.26201764 0.31739087 0.51933394 0.16414434 0.10101525 0.28347171
 0.12438792 0.21920229 0.33366239]
[2.2758563  2.75682203 4.51087726 1.42573966 0.87740735 2.46220401
 1.08041974 1.90396694 2.89815469]
[138.5856    86.616     92.3904    73.91232    0.       103.9392    92.3904
  92.3904    92.3904  ]
[46.1952    8.6616   23.0976    9.23904  23.0976   17.3232   57.744    46.1952
 11.5488  ]
```

Figure 21: Code for table 2 in §3.2

```
plt.figure(figsize=(10,5))

# Plot
plt.scatter(np.log10(frequency), Lm_H, color="#519E8A", zorder=5, label="Values")
plt.plot(np.log10(frequency), Lm_H, color="#29335C", zorder=3)

#Line of best fit
# plt.plot(np.unique(frequency), np.poly1d(np.polyfit(frequency, Lm_H, 1))(np.unique(frequency)), color = "#29335C")

# Error Bars
plt.errorbar(np.log10(frequency), Lm_H, yerr=dLm_H, fmt=",", ecolor="#89ccba", color="#519E8A", capsize=4, zorder=4, label="Errors")
# fmt=line marker, "," is a pixel.
# ecolor is for the error bars, color is for values.
#capsize is the length of the error bar cap.

# Axes, Legends, Title, etc.
plt.title(r"Gain $Lm(|H|)$ vs $\log_{10}(Frequency)$")
plt.xlabel(r"$\log_{10}(Frequency)$ (Hz)")
plt.ylabel(r"$Lm(|H|)$ (dB)")
plt.legend()

# Grids, etc
plt.minorticks_on()
plt.grid(True, which="major", linewidth=0.8, color="#DDDDDD", zorder=2)
plt.grid(True, which="minor", linewidth=0.5, color="#EEEEEE", linestyle="--", zorder=1)

plt.show()
```

✓ 0.2s

Figure 22: Code for figure 12 in §3.2

```
plt.figure(figsize=(10,5))

# Plot
plt.scatter(np.log10(frequency), angle_H, color="#ff30c4", s=15, zorder=5, label="Values")
plt.plot(np.log10(frequency), angle_H, color="#bf088b", zorder=3)

#Line of best fit
# plt.plot(np.unique(frequency), np.poly1d(np.polyfit(frequency, Lm_H, 1))(np.unique(frequency)), color = "#29335C")

# Error Bars
plt.errorbar(np.log10(frequency), angle_H, yerr=d_angle_H, fmt=",", ecolor="#f5b8e3", color="#f5b8e3", capsize=4, zorder=4, label="Errors")
# fmt=line marker, "," is a pixel.
# ecolor is for the error bars, color is for values.
#capsize is the length of the error bar cap.

# Axes, Legends, Title, etc.
plt.title(r"$\angle H$ vs $\log_{10}(Frequency)$")
plt.xlabel(r"$\log_{10}(Frequency)$ (Hz)")
plt.ylabel(r"$\angle H (^{\circ})$")
plt.legend()

# Grids, etc
plt.minorticks_on()
plt.grid(True, which="major", linewidth=0.8, color="#DDDDDD", zorder=2)
plt.grid(True, which="minor", linewidth=0.5, color="#EEEEEE", linestyle="--", zorder=1)

plt.show()
```

✓ 0.1s

Figure 23: Code for figure 13 in §3.2

```python
L = 9.845e-4
C = np.array([20, 40, 60, 80, 100, 120, 140, 160, 180, 200, 220]) * 1e-6
V2 = np.array([240, 224, 188, 164, 84, 104, 144, 170, 188, 200, 208]) * 1e-3
R = 10

f_0 = 1 / (2 * np.pi * np.sqrt(L * C))
#print(f_0)

angfreq = 2 * np.pi * f_0
print(angfreq)

I = V2 / (np.sqrt(R**2 + ((angfreq * L) - (1 / (angfreq * C)))**2))
print(I)

Q = 1 / (angfreq * R * C)
print(Q)
avgQ = np.average(Q)
print(avgQ, np.std(Q))
```

✓ 0.0s

```
[7126.51398828 5039.20636734 4114.49476952 3563.25699414 3187.07394408
 2909.38715268 2693.56910397 2519.60318367 2375.50466276 2253.601598
 2148.72481476]
[0.024  0.0224 0.0188 0.0164 0.0084 0.0104 0.0144 0.017  0.0188 0.02
 0.0208]
[0.7016053  0.49610987 0.40507201 0.35080265 0.31376743 0.28642917
 0.26518188 0.24805493 0.23386843 0.22186708 0.21154196]
0.3394818823839039 0.14103745146712773
```

Figure 24: Code for table 3 in §3.3

```python
plt.figure(figsize=(10,5))

# Plot
plt.scatter(angfreq, -I, color="#ff30c4", s=15, zorder=5, label="Values")
plt.plot(angfreq, -I, color="#bf088b", zorder=3)

#Line of best fit
# plt.plot(np.unique(frequency), np.poly1d(np.polyfit(frequency, Lm_H, 1))(np.unique(frequency)), color = "#29335C")

# Error Bars
plt.errorbar(angfreq, -I, yerr=0.0005, fmt=",", ecolor="#f5b8e3", color="#f5b8e3", capsize=4, zorder=4, label="Errors")
# fmt=line marker, "," is a pixel.
# ecolor is for the error bars, color is for values.
#capsize is the length of the error bar cap.

# Axes, Legends, Title, etc.
plt.title(r"Current vs $\omega$")
plt.xlabel(r"$\omega$ (rad/s)")
plt.ylabel(r"I (A)")
plt.legend()

# Grids, etc
plt.minorticks_on()
plt.grid(True, which="major", linewidth=0.8, color="#DDDDDD", zorder=2)
plt.grid(True, which="minor", linewidth=0.5, color="#EEEEEE", linestyle="--", zorder=1)

plt.show()
```

✓ 0.1s

Figure 25: Code for figure 14 in §3.3

List of Figures

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