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## **PHYC20080 Fields, Waves and Light**

Experiment No.1 Measurement of the Focal Lengths of Lenses and a  
Determination of Brewster's Angle

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## Abstract

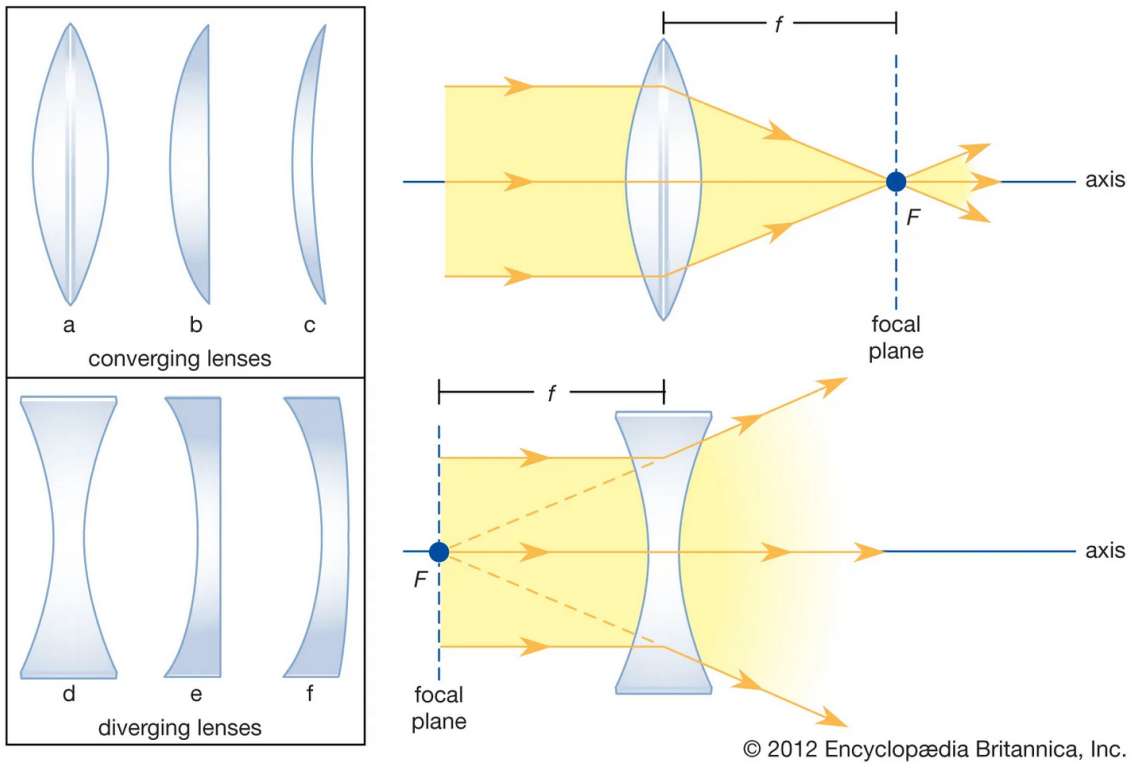
The aim of this experiment was to find the focal length of a convex and concave lens, which was calculated to be  $10.17 \pm 0.464 \text{ cm}$  and  $-40.85 \pm 7.22 \text{ cm}$  respectively.

The aim of this experiment was also to find the Brewster's angle and refractive index of a material when the light is polarised both perpendicular and parallel to the plane of incidence. For the perpendicular polarisation Brewster's angle was found to be  $\theta_B = 42.44^\circ$  and the refractive index to be  $n = 0.914$ . For the parallel polarisation the Brewster's angle was found to be  $\theta_B = 47.30^\circ$  and the refractive index to be  $n = 1.084$ .

# 1 Theory

## 1.1 Lenses

Lenses are a piece of transparent materials, typically glass, that are used to bend and focus rays of light onto one spot to form images [1, 2]. Lenses can either be shaped to be concave (depressed, caved) or convex (bulging, rounded), but have to have at least one curved surface [1, 2] The properties that make up a lens are: *focal length* (§1.1.1), *focal point*, *optical axis*, *focus*, *principal axis*, *centre of curvature* [3, 4]



**Figure 1:** Diagram of converging (convex) and diverging (concave) lenses [1].  
(a): *biconvex*, (b): *plano-convex*, (c): *positive meniscus*; (d): *biconcave*, (e): *plano-concave*, (f): *negative meniscus*

Convex lenses make the parallel travelling rays of light passing through it **converge** to a certain point known as the *focal point* (1.1.1) [5]. The degree at which light is bent depends on the absolute curvature of the lens [6]. The convex lens has two extra properties that do not apply to a concave lens: *radius of curvature*, and *aperture* [4]. The behaviour of light as it passes through a convex lens can be simulated via the interactive diagram link in *References*, §4 [7].

Concave lenses make the parallel travelling rays of light passing through it **refract** away from the focal point of the lens [5, 8]. The behaviour of light as it passes through a concave lens can be simulated via the interactive diagram link in *References*, §4 [9].

### 1.1.1 Focal Length

The focal length is the distance, typically in millimetres, of the *optical axis* (the centre of the lens) to the focal plane shown in figure 1 [10, 11]. The focal point, indicated by the point at which the focal plane and principal axis intersect, is the point at which the rays of light coincide [12]. For a concave lens, where the light diverges, there only exists an *apparent* focal point behind the lens, where a virtual image would be formed (§1.1.2) [12].

The formula below, equation 1, is used to calculate the focal length of a thin converging lens [13, 12], where  $u$  is the distance between the *object* and the centre of the lens, and  $v$  is the distance between the *image* and the centre of the lens:

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \quad (1)$$

Since a concave lens does not form a real image (§1.1.2) one must add a convex lens to overcome this issue. Therefore equation 1 becomes invalid (individually) for calculating the focal length. Instead, equation 2 below can be used to find the focal length of the combined lenses, where  $f_{convex}$  (focal length of the convex lens) and  $f_{concave}$  (focal length of the concave lens) can be calculated individually using equation 1 [13]:

$$\frac{1}{f} = \frac{1}{f_{convex}} + \frac{1}{f_{concave}} \quad (2)$$

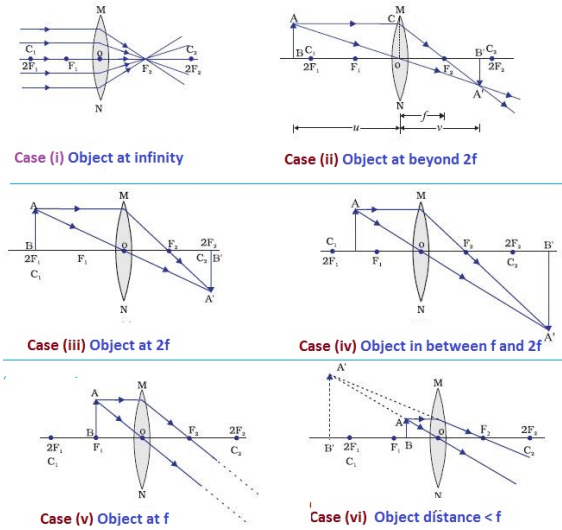
Concave lenses have *negative* focal length, as opposed to convex lenses, since the focal point is on the same side as the incident (incoming) light [3].

### 1.1.2 Image Formation

Two types of images can be formed when looking at a mirror or through a lens:

- **Real image:** an image formed when light converges at a point; inverted in nature [14].
- **Virtual image:** an image formed when light diverges away from a point; appears to be produced but is not actually present [14].

Standard image properties of convex lenses is that what is produced is a real image that is inverted and enlarged [4], but virtual images can also be formed. The image produced depends on the object position at the time of measurement. The types of images convex lenses can form are illustrated in figure 2 and described in table 1 (*sourced directly from Geeksforgeeks [4]*).

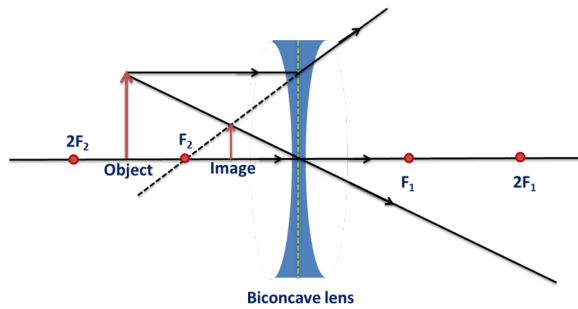


**Figure 2:** Ray diagram of the cases of convex lens image formation [15].

Object Position	Image Position	Image Size	Image Nature
Beyond 2F	Between F and 2F	Smaller	Real, Inverted
At 2F	At 2F	Same size	Real, Inverted
Between F and 2F	Beyond 2F	Larger	Real, Inverted
At F	Infinity	Infinite	Real, Inverted (Highly Diminished)
Between F and lens	Beyond 2F	Larger	Virtual, Upright
At lens	At lens	Larger	Virtual, Upright
Object inside lens	Between lens and F	Larger	Virtual, Upright

**Table 1:** Table of the different cases of convex lens image formation [4, 15].

Standard image properties of concave lenses is that what is produced is a virtual image that is upright and diminished [3]. Concave lenses cannot produce real images. The only thing that changes between the virtual images that are produced is the magnitude at which they're diminished [15, 3].



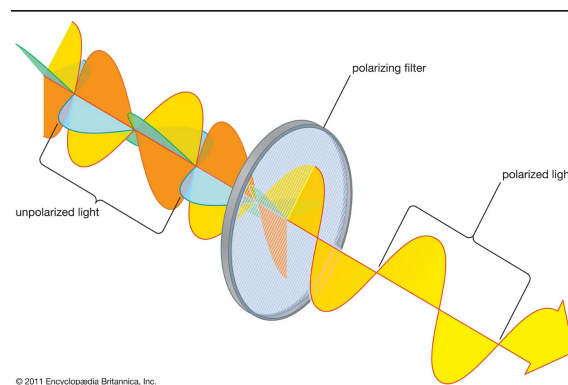
**Figure 3:** Ray diagram of concave lens image formation [15].

Object Position	Image Position	Image Size	Image Nature
At infinity	At $F_2$	Highly Diminished, Point-Sized	Virtual, Upright
Object inside lens	Between lens and $F_2$	Diminished	Virtual, Upright

**Table 2:** Table of the different cases of concave lens image formation [3, 15]

## 1.2 Polarisation

Since light, electromagnetic radiation, has both particle and (transverse) wave properties, it can be restricted to propagating in one direction, shown in figure 4 [16, 17]. Light can be polarised by passing through certain materials, like crystals, that act as a filter [17].



**Figure 4:** Visual representation of polarisation of light [17].

### 1.2.1 Reflection and Refraction

**Reflection** is when the light rays change direction as they hit a surface, or travel through a medium that has a different chemical composition to the original medium [18, 19]. For reflection to take place, the angle of incidence ( $\theta_i$ ) must equal the angle of reflection ( $\theta_r$ ), of which are measured with respect to the normal (perpendicular) to the surface. This is known as the law of reflection [18, 19]. Diffusion is a type of reflection on a rough surface, so the light rays reflect in many different directions [18].

**Refraction** occurs when light travelling through a transparent medium encounters another transparent medium [19]. The variations in the matter of the medium causes the light to "bend" (change directions) as it is transmitted through the new medium [18]. The law of refraction is more commonly known as *Snell's Law*, §1.2.2.

### 1.2.2 Snell's Law

A ray of light changes directions when it passes from a medium of specified refractive index (number indicating the magnitude of refraction of a medium) through another medium of different refractive index [18].

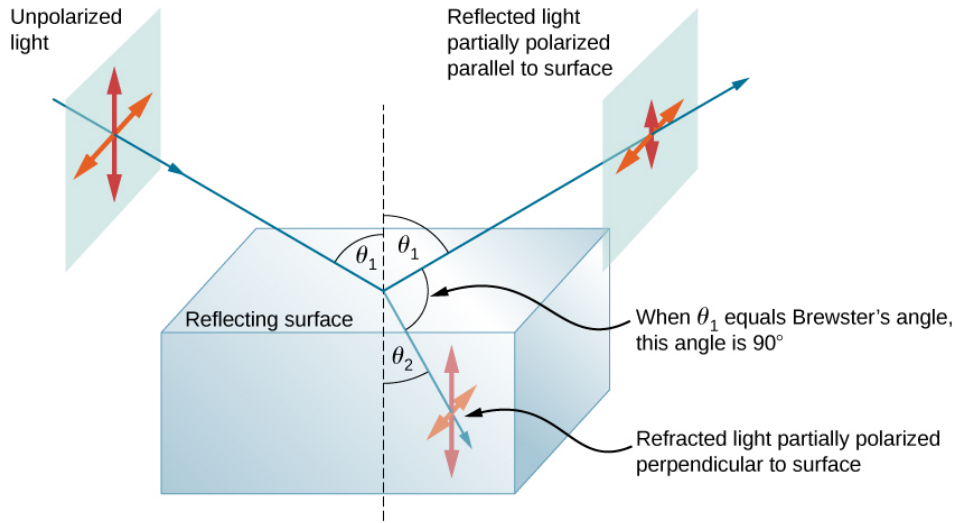
The below equation, equation 3, is the mathematical description of Snell's Law, with  $n_{1,2}$  as the refractive index of the two mediums,  $\theta_i$  as the incident angle, and  $\theta_r$  as the refractive angle:

$$n_1 \sin \theta_i = n_2 \sin \theta_r \quad \implies \quad \frac{n_2}{n_1} = \frac{\sin \theta_i}{\sin \theta_r} \quad (3)$$

The angle at which the light is refracted is dependent on the refractive index of the medium,  $\sin \theta \propto n$ , so if the refractive index of the second medium is greater than the first medium the angle of refraction will be smaller [20].

### 1.2.3 Brewster's Angle

Light can be polarised by reflection and refraction, as seen in figure 5, where majority of the light will be reflected off the medium and the rest of the light will be refracted through the medium. In doing so, the reflected ray gets partially polarised parallel to the medium's surface and the refracted ray get partially polarised perpendicular to the surface of the medium [21].



**Figure 5:** Diagram of light polarisation with Brewster's angle [22].

Brewster's angle is also known as the *polarisation angle*, which is the incident angle at which unpolarised light is separated into its vertical (perpendicular) and horizontal (parallel) components when that light is transmitted through a medium such that the angle between the reflected and refracted ray is  $90^\circ$ , Brewster's angle ( $\theta_B$ ) [23, 24].

Brewster's angle can be derived from Snell's law [13, 23]:

$$n_1 \sin \theta_i = n_2 \sin \theta_t \quad \theta_i = \theta_B, \theta_i + \theta_t \quad (4)$$

$$\implies n_1 \sin \theta_B = n_2 \sin(90^\circ - \theta_B) = n_2 \cos(\theta_B) \quad (5)$$

$$\implies \frac{\sin \theta_B}{\cos \theta_B} = \tan \theta_B = \frac{n_2}{n_1} = n \quad \therefore n = \tan \theta_B \quad (6)$$

This equation (6) can be used to both calculate the polarisation angle ( $\theta_B$ ) or the refractive index ( $n$ ) of a material, depending on the values previously known.

#### 1.2.4 Ratios of Intensity

By manipulation of Snell's law, equation 3, ratios of the intensities of the reflected to the incident light of the polarised light can be found [13]:

$$R_{parallel} = \frac{\tan^2(\theta_i - \theta_t)}{\tan^2(\theta_i + \theta_t)} \quad (7) \quad R_{perpendicular} = \frac{\sin^2(\theta_i - \theta_t)}{\sin^2(\theta_i + \theta_t)} \quad (8)$$

This is such that if  $R = 0$ , light is fully transmitted through the medium (no reflection), and if  $R = 1$ , all the light is reflected (no polarisation) [13].

## 2 Methodology

### 2.1 Determining the Focal Length

A similar equipment setup to the one used for this section of the experiment is shown in figure 6 below:



**Figure 6:** Experiment setup to determine the focal point of a convex lens [25].

### 2.1.1 Convex Lens Method

The apparatus was setup as in §2.1 with a convex lens in the holder. The image drawn in front of the light was a cross rather than the car shown in the figure. The light source was placed first 50cm away from the lens and the screen was adjusted until the image shown was at its sharpest. Five measurements at different  $u$  and  $v$  were taken and inserted into the table in §3.1.1.

For each pair of measurements the focal length was then calculated using equation 1 in §1.1.1 and added to the table. The mean of the focal lengths provided the rough estimate of the true focal length for the convex lens and the uncertainty on it.

### 2.1.2 Concave Lens Method

A similar method is used as for the convex lens (§2.1.1) with the addition of a concave lens placed behind the convex lens. Since light diverges when travelling through a concave lens and only forms a virtual image, a convex lens is needed in front of it to correct the light direction for a real image to be formed, which is measurable, as discussed in §1.1.2.

The procedure then follows the same as before, first the light source is placed 50cm away from the source and the screen is adjusted until the sharpest image forms. Five measurements are taken and a table similar to the one in §3.1.1 is made.

To calculate the focal length of the *concave* lens, equation 2 must be adjusted:

$$\frac{1}{f_{concave}} = \frac{1}{f} - \frac{1}{f_{convex}} \quad (9)$$

Since  $f_{convex}$  was calculated in §2.1.1 and  $f$  is what is found (through equation 1) by doing the above procedure, the individual focal length of the concave lens can be found through subtraction.

## 2.2 Determining Brewster's Angle Method

The apparatus is first setup in a similar fashion as to what is shown below in figure 7:



**Figure 7:** Experiment setup to determine Brewster's angle [26].



The two arms, one holding the light source (laser) and the other holding the photoresistor, are free to move around the table which holds the lens with the polaroid filter set in front of the laser light source. A multimeter was connected to the photoresistor and set to read up to 200mV. The arm holding the photoresistor is adjusted such that it is in the position to be able to read the **reflected** light, not the transmitted light.

Once the plane of incidence is found the polaroid, and thus the plane of polarisation, is adjusted such that it is perpendicular to the plane of incidence. The angle of incidence is varied and read from the table, and the intensity of the reflected light is measured, with each value being recorded in a table. This procedure is repeated for the polaroid perpendicular and parallel to the plane of incidence.

Equation 6 can then be used to calculate the refractive index of the lens material when the value for Brewster's angle is found.

## 3 Results and Calculations

### 3.1 Determining the Focal Length

For this section the aim was to find the focal length (§1.1.1) of a convex lens and concave lens.

#### 3.1.1 Convex Lens

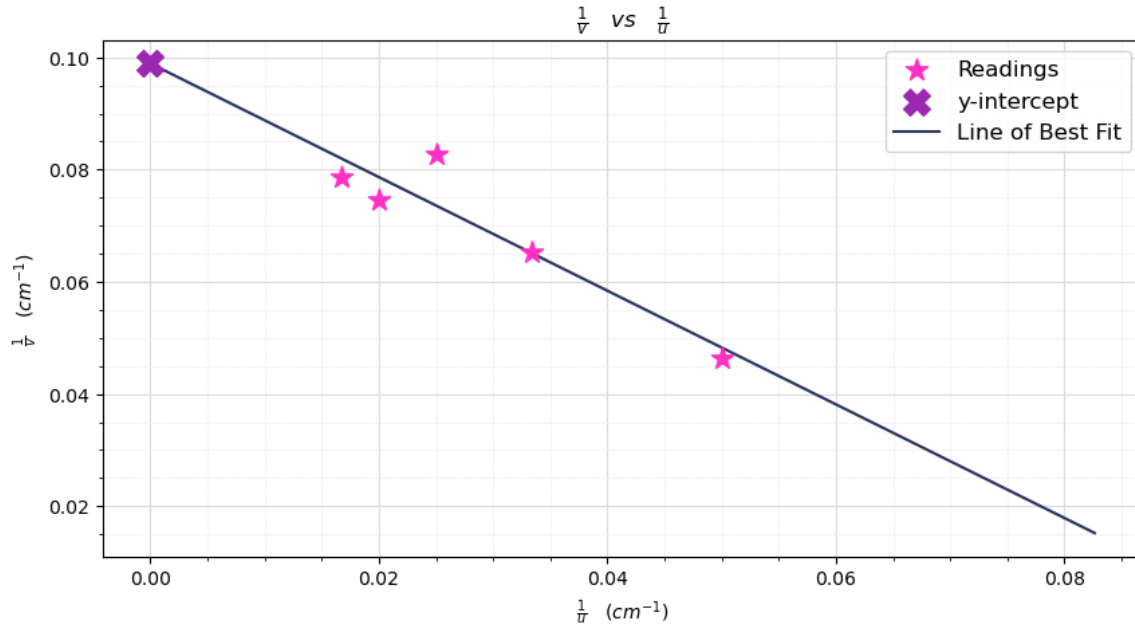
The values gathered for the convex lens are represented in the table, table 4, below:

<b>u (cm)</b>	$\Delta u$ (cm)	<b>v (cm)</b>	$\Delta v$ (cm)	f (cm)
<b>60</b>	$\pm 0.05$	<b>12.7</b>	$\pm 0.05$	10.48
<b>50</b>	$\pm 0.05$	<b>13.4</b>	$\pm 0.05$	10.57
<b>40</b>	$\pm 0.05$	<b>12.1</b>	$\pm 0.05$	9.29
<b>30</b>	$\pm 0.05$	<b>15.3</b>	$\pm 0.05$	10.13
<b>20</b>	$\pm 0.05$	<b>21.6</b>	$\pm 0.05$	10.38

**Table 3:** Table of the values gathered for the convex lens, with the focal length calculated using equation 1.

The average of the focal lengths for the convex lens with the associated standard deviation uncertainty from the calculated values is found to be  **$10.17 \pm 0.464$  cm**.

The inverse of **u**, the object-lens distance, and the inverse of **v**, the screen-lens distance, were then plotted against each other in figure 8 where the relationship between both values can be seen.



**Figure 8:** Graph of  $\frac{1}{v}$  vs  $\frac{1}{u}$  with y-intercept marked.

The value of the slope of the line of best fit is calculated to be  $-1.013$  and the y-intercept to be  $0.0989$ . Therefore to be able to compute best value of the focal length from the graph the mean of the x- and y-intercepts are found:

$$0 = mx + c \implies x = \frac{-b}{m} = \frac{-0.0989}{-1.013} = 0.0976$$

Thus the mean focal length from the graph can be calculated,

$$\left\langle \frac{1}{f} \right\rangle = \frac{0.0976 + 0.0989}{2} = 0.09825 \text{ cm}^{-1}$$

So therefore the average value of the focal length and its uncertainty as per the graph would be  $10.178 \pm 3.24 \times 10^{-4} \text{ cm}$ .

### 3.1.2 Concave Lense

The values gathered for the combined lenses are represented in the table, table 4, below:

u (cm)	$\Delta u$ (cm)	v (cm)	$\Delta v$ (cm)	f (cm)
60	$\pm 0.05$	17.2	$\pm 0.05$	13.37
50	$\pm 0.05$	18.4	$\pm 0.05$	13.45
40	$\pm 0.05$	20.2	$\pm 0.05$	13.42
30	$\pm 0.05$	25.9	$\pm 0.05$	13.90
20	$\pm 0.05$	48.7	$\pm 0.05$	14.18

**Table 4:** Table of the values gathered for the combined concave and convex lens, with the focal length calculated using equation 1.

The table below, table 5, includes all the values needed to carry out the necessary equation 9 to compute the focal length of the individual concave lens, of which the calculated values is shown in the right-most column.

$\frac{1}{f} \text{ (cm}^{-1}\text{)}$	$\frac{1}{f_{\text{convex}}} \text{ (cm}^{-1}\text{)}$	$\frac{1}{f} - \frac{1}{f_{\text{convex}}} \text{ (cm}^{-1}\text{)}$	$f_{\text{concave}} \text{ (cm)}$
0.0581	0.0787	-0.0206	-48.54
0.0543	0.0746	-0.0203	-49.31
0.0495	0.0826	-0.0331	-30.175
0.0386	0.0654	-0.0267	-37.38
0.0205	0.0463	-0.0258	-38.82

**Table 5:** Table of the calculated focal length of the concave lens by use of equation 9.

The average and associated uncertainty on the concave lens' focal length can then be surmised to be  $-40.85 \pm 7.22 \text{ cm}$ .

## 3.2 Determining Brewster's Angle

For this section the value for Brewster's angle on the material is calculated and thus the refractive index of the material can be found.

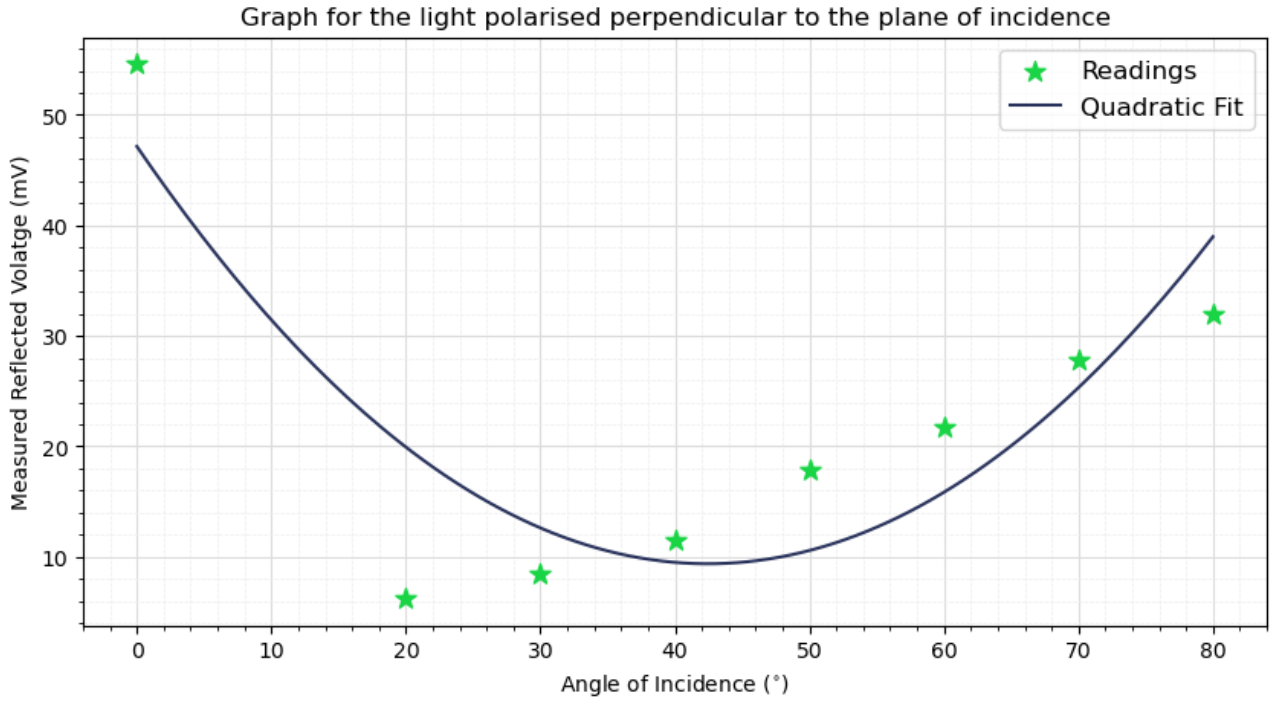
### 3.2.1 Perpendicular Polarisation

The values for the reflected light at different angles of incidence for perpendicular polarisation were found and recorded below:

Angle of Incidence ( $^{\circ}$ )	Measured Reflected Voltage (mV)
0	54.6
20	6.2
30	8.38
40	11.5
50	17.8
60	21.7
70	27.8
80	31.9

**Table 6:** Value recorded for the laser light source polarised perpendicular to the plane of incidence.

These values from table 6 are then plotted on a graph with the quadratic fitted to the values, figure 9.



**Figure 9:** Graph for the light polarised perpendicular to the plane of incidence.

The value of the minimum (lowest) point on the curve can be taken to be the Brewster's angle. The derivative of the quadratic form find the minimum,

$$y = ax^2 + bx + c \implies x_{min} = -\frac{b}{2a} \quad (10)$$

The value found can then be substituted into the original form to find the corresponding y-value for the minimum point:

$$y_{min} = a(x_{min})^2 + b(x_{min}) + c \quad (11)$$

The minimum point for the quadratic curve fitted to the values for perpendicular polarisation is then found to be  $(x, y) = (42.44, 9.37)$ , so Brewster's angle must therefore be  $\theta_B = 42.44^\circ$ .

This value can then be used in equation 6 to find the refractive index of the material used as a lens:

$$n = \tan \theta_B \implies n = \tan(42.44^\circ) = 0.914$$

So the calculated value for the refractive index of the material of the lens used is **n = 0.914**.

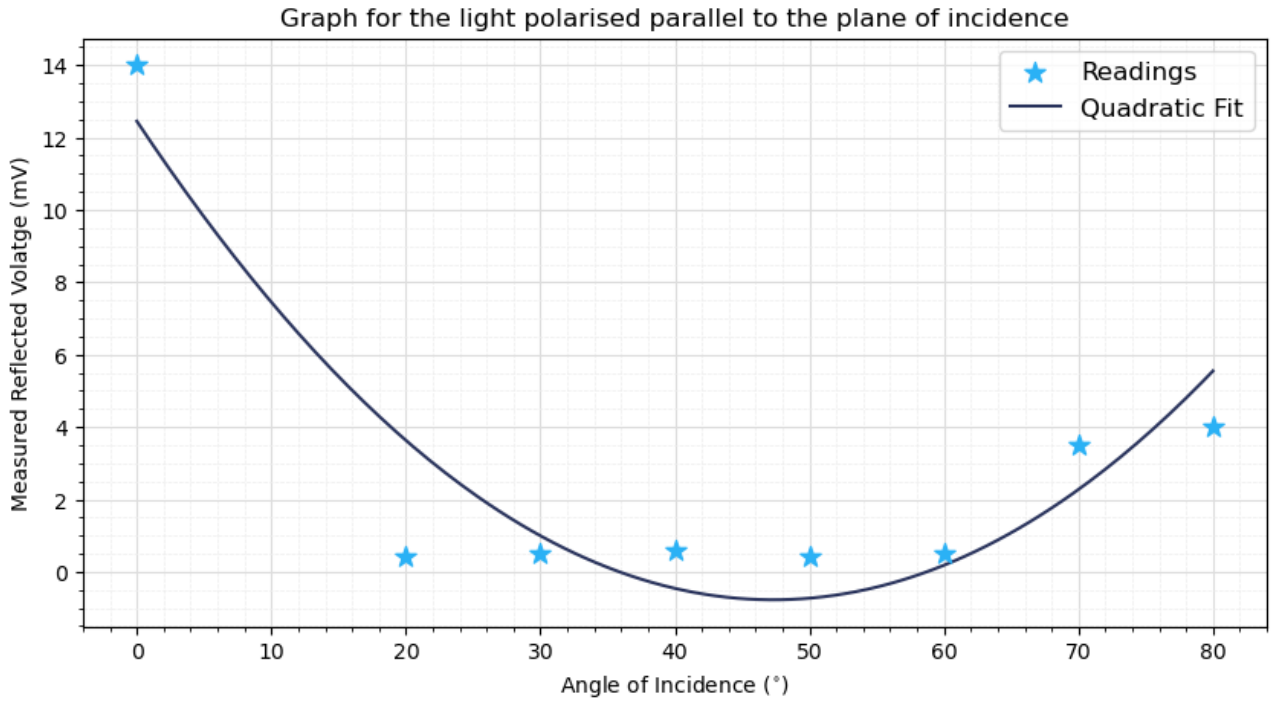
### 3.2.2 Parallel Polarisation

The values for the reflected light at different angles of incidence for parallel polarisation were found and recorded below:

Angle of Incidence ( $^{\circ}$ )	Measured Reflected Voltage (mV)
0	14
20	0.4
30	0.5
40	0.6
50	0.4
60	0.5
70	3.5
80	4

**Table 7:** Value recorded for the laser light source polarised parallel to the plane of incidence.

These values from table 7 are then plotted on a graph with the quadratic fitted to the values, figure 10.



**Figure 10:** Graph for the light polarised parallel to the plane of incidence.

Brewster's angle can be calculated the same way as was done in §3.2.1 for perpendicular polarisation.

The minimum point for the quadratic curve fitted to the values for perpendicular polarisation is then found to be  $(x, y) = (47.30, -0.77)$ , so Brewster's angle must therefore be  $\theta_B = 47.30^{\circ}$ .

This value can then be used in equation 6 to find the refractive index of the material used as a lens:

$$n = \tan \theta_B \implies n = \tan(47.30^{\circ}) = 1.084$$

So the calculated value for the refractive index of the material of the lens used is  $n = 1.084$ .

## 4 Conclusion

It can be said that the first part of the experiment, which aimed to find the focal length of both a convex and concave lens (see §3.1) achieved reasonable results. The focal length for the convex lens was calculated both explicitly and by the slope of the best line fit of the representative graph. The values calculated were  $10.17 \pm 0.464 \text{ cm}$  and  $10.178 \pm 3.24 \times 10^{-4} \text{ cm}$  respectively, and since both values lie within the other's uncertainty, it can be confidently concluded that the focal length of the convex lens used in the experiment is around  $10.178 \text{ cm}$ . With this value the focal length of the concave lens was then calculated to be  $-40.85 \pm 7.22 \text{ cm}$ , which suggests the concave lens was not very inwardly curved (flatter).

For the second part of the experiment, determining Brewster's angle (see §3.2), the results were a lot less reasonable. For the perpendicular polarisation the refractive index found with the calculated  $\theta_B$  ( $42.44^\circ$ ) was  $n = 0.914$ . This is unphysical as the refractive index of any medium is  $n \geq 1$  (*wavelength dependent*). Similarly, for the parallel polarisation the refractive index found with the calculated  $\theta_B$  ( $47.30^\circ$ ) was  $n = 1.084$ . This is most likely incorrect as the closest match in refractive index is *liquid helium* at  $n = 1.025$  [27] and the material that was used for the experiment was solid and transparent.

The likely issues during the Brewster's angle experiment could be in lack of time to gather accurate angle and voltage readings after accidentally measuring the *transmitted* voltage instead. If the smaller increments in angle (ie.,  $5^\circ$  instead of  $10^\circ$ ) could also lead to more accurate results. During the experiment the overhead light was also turned on, which likely lead to inaccuracies in readings of the photoresistor. The digital multimeter could also have been faulty as there were exposed wires visible, this could have led to inaccuracies.

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# Appendix

## Code

```
u = np.array([50, 50, 40, 30, 20])
v_convex = np.array([12.7, 13.4, 12.1, 15.3, 21.6])
v_concave = np.array([17.2, 16.4, 20.2, 25.9, 48.7])

invu = 1/u
invv_convex = 1/v_convex
invv_concave = 1/v_concave

print("1/u, invu")
print("1/v_convex, invv_convex")
print("1/v_concave, invv_concave")

f_convex = invu - invv_convex
f_concave = invu - invv_concave

print("\n", f_convex, 1/f_convex)
avgf_convex = np.average(1/f_convex)
print(avgf_convex)
avgf_concave = np.average(1/f_concave)
print(avgf_concave)

f_cconc = - f_convex - f_concave
print("\n", f_cconc)
print("\n", 1/f_cconc, np.average(1/f_cconc), np.std(1/f_cconc))
print("\n", avgf_convex, np.std(1/f_convex))

✓ 0.0s

1/u [0.01666667 0.02 0.025 0.03333333 0.05 ]
1/v_convex [0.07692308 0.07462687 0.08264463 0.06535948 0.0462963 ]
1/v_concave [0.05813953 0.05434783 0.04950405 0.03861004 0.02053388]

[13.36787555 13.4582924 13.42192691 13.89982111 14.1775837 ] [10.48143854 10.56782334 9.28982726 10.13245833 10.38461538]
10.171229370263855
13.6634999523972

[-0.02660862 -0.02827984 -0.03313968 -0.02674944 -0.02576242]

[-48.54222222 -49.312 -30.17538064 -37.38396226 -30.81623616] -40.84594585814283 7.223216638844789

10.171229370263855 0.4642822483112251
```

Figure 11: Code used for §3.1 of this report.

```
x = np.array([0.0976, 0.09825])
print(np.std(x))

✓ 0.1s

0.00032499999999999989
```

Figure 12: Code used for §3.1.2 of this report.

```
incidence = np.array([0, 20, 30, 40, 50, 60, 70, 80])
perpendicular_mv = np.array([54.6, 6.2, 8.38, 11.5, 17.8, 21.7, 27.8, 31.9])
parallel_mv = np.array([14, 0.4, 0.5, 0.6, 0.4, 0.5, 3.5, 4])

✓ 0.0s

coeff_perp = np.polyfit(incidence, perpendicular_mv, 2)

inc_smooth = np.linspace(0, 80, 200)
perp_smooth = np.polyval(coeff_perp, inc_smooth)

plt.figure(figsize=(10, 5))

plt.ylabel("Measured Reflected Volatge (mV)")
plt.xlabel("Angle of Incidence  $\theta$  (°)")

plt.scatter(incidence, perpendicular_mv, color="#194445", s=100, marker="x", zorder=6, label="Readings")

plt.plot(inc_smooth, perp_smooth, linestyle="—", color="#29335C", zorder=5, label="Quadratic Fit")

plt.minorticks_on()
plt.grid(True, which="major", linewidth=0.8, color="#000000", zorder=2)
plt.grid(True, which="minor", linewidth=0.5, color="#EEEEEE", linestyle="—", zorder=1)

plt.title("Graph for the light polarised perpendicular to the plane of incidence")
plt.legend(fontsize="large")

plt.show()
```

Figure 13: Code used for §3.2.1 of this report.

```
extended_y = np.linspace(0, max(invv_convex), 300)

slope, intercept = np.polyfit(invu, invv_convex, 1)

plt.figure(figsize=(10, 5))

plt.ylabel(r" $\frac{1}{v}$  \quad (cm-1)")
plt.xlabel(r" $\frac{1}{u}$  \quad (cm-1)")

plt.scatter(invu, invv_convex, color="#ff30c4", s=150, marker="x", zorder=6, label="Readings")
plt.scatter(0, intercept, color="#9C27B0", s=200, marker="X", zorder=6, label="y-intercept")

plt.plot(
    extended_y,
    np.polyval([slope, intercept], extended_y),
    label="Line of Best Fit",
    color="#29335C",
    zorder=5
)

plt.minorticks_on()
plt.grid(True, which="major", linewidth=0.8, color="#000000", zorder=2)
plt.grid(True, which="minor", linewidth=0.5, color="#EEEEEE", linestyle="—", zorder=1)

plt.legend(fontsize="large")
plt.title(r" $\frac{1}{v}$  \quad vs \quad  $\frac{1}{u}$ ")

print(slope)
print(intercept)

plt.show()

✓ 0.1s

-1.0127148281261837
0.09898221494998122
```

Figure 14: Code used for §3.1.1 of this report.

```
coeff_para = np.polyfit(incidence, parallel_mv, 2)

inc_smooth = np.linspace(0, 80, 200)
para_smooth = np.polyval(coeff_para, inc_smooth)

plt.figure(figsize=(10, 5))

plt.ylabel("Measured Reflected Volatge (mV)")
plt.xlabel("Angle of Incidence  $\theta$  (°)")

plt.scatter(incidence, parallel_mv, color="#2ab1f5", s=100, marker="x", zorder=6, label="Readings")

plt.plot(inc_smooth, para_smooth, linestyle="—", color="#29335C", zorder=5, label="Quadratic Fit")

plt.minorticks_on()
plt.grid(True, which="major", linewidth=0.8, color="#000000", zorder=2)
plt.grid(True, which="minor", linewidth=0.5, color="#EEEEEE", linestyle="—", zorder=1)

plt.title("Graph for the light polarised parallel to the plane of incidence")
plt.legend(fontsize="large")

plt.show()
```

Figure 15: Code used for §3.2.2 of this report.

```
a, b, c = coeff_perp
xmin_perp = -b / (2 * a)
ymin_perp = np.polyval(coeff_perp, xmin_perp)

a_p, b_p, c_p = coeff_para
xmin_para = -b_p / (2 * a_p)
ymin_para = np.polyval(coeff_para, xmin_para)

print(f"perp min: x = {xmin_perp:.2f}, y = {ymin_perp:.2f}")
print(f"para min: x = {xmin_para:.2f}, y = {ymin_para:.2f}")

xmin_perp_rad = np.pi / (180) * xmin_perp
xmin_para_rad = np.pi / (180) * xmin_para

print("\n perp:", np.tan(xmin_perp_rad))
print("\n para:", np.tan(xmin_para_rad))

✓ 0.0s

perp min: x = 42.44, y = 9.37
para min: x = 47.30, y = -0.77
n perp: 0.9142470910225726
n para: 1.0835405824710673
```

Figure 16: Code used for §3.2 of this report.

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