Coordinate systems

Contents

§1. Coordinate systems

Op. 1.1. The coordinate method is an approach that allows you to establish the correspondence between geometric objects (points) and algebraic objects (numbers) , as well as describe their properties and relations between them using analytical relations.

Op. 1.2. Coordinate line - continuous line without self-intersections, each point of which corresponds to a real number.

Opr. 1.3. The coordinate axis α is an oriented straight line that has the origin O and is equipped with a scale E. In this case, any point P of the coordinate axis corresponds to a real number x, called the coordinate of the point:

$$P \in \alpha \quad \leftrightarrow \quad x \in \mathbb{R}$$

NtB. Coordinate axes on a plane (in space) together form a coordinate system.

Opr. 1.4. A rectilinear coordinate system on a plane (in space) is a system of two (three) multidirectional coordinate axes that have a common origin.

NtB. On the plane, each point is assigned a pair of real numbers.

$$\forall P \quad \leftrightarrow \quad (x_P, y_P) \in \mathbb{R} \times \mathbb{R} = \mathbb{R}^2$$

In the space of each point, a triple of real numbers is assigned.

$$\forall P \quad \leftrightarrow \quad (x_P, y_P, z_P) \in \mathbb{R}^3$$

Opr. 1.5. A level line on a plane is any straight line parallel to one of the coordinate axes.

Opr. 1.6. A level surface in space is any plane parallel to one of the coordinate planes.

Opr. 1.7. A rectangular coordinate system is a system in which the angle between each pair of coordinate axes is a straight line. If the same scale is chosen on the coordinate axes, then such a system is called a Cartesian rectangular coordinate system.

Op. 1.8. A polar coordinate system is a coordinate system in which each point corresponds to a polar radius r - the distance from the origin (pole), and a polar angle ϕ, which is counted from the ray, going directly from the origin (polar axis), counterclockwise.

NtB. A pair of polar coordinates (r, ϕ) can be converted to Cartesian coordinates (x, y) using trigonometric functions, assuming that the polar axis coincides with the positive direction of the axis Ox

$$x = r \cos \phi$$
$$y = r \sin \phi$$

§2.

2.1. A directed segment, or connected vector, is a segment that is unambiguously defined by points, which we call the beginning and end of the directed segment.

Example 2.1. The radius vector of a point A is a directed segment drawn from the origin to the point A.

Op. 2.2. Directed segments will be called collinear if they lie on parallel lines.

Op. 2.3. Directed segments will be called coplanar if they lie on parallel planes.

NtB. Any two directed segments are coplanar.

Opr. 2.4. The module of a directed segment AB is the length of the segment AB.

Opr. 2.5. An equivalence relation ~ on a set M is a relation that has the properties of reflexivity, symmetry, and transitivity.

Opr. 2.6. The equivalence class of an element $a \in M$ is a subset of the set M in which all elements are equivalent a.

Opr. 2.7. Directed segments are called equivalent if they are co-directed and their modules are equal.

Opr. 2.8. A free vector, or simply a vector, is the equivalence class of directed segments.

§3. Actions with vectors

Consider two free vectors a and b. From an arbitrary point A, we set aside the directed segment AB, which is the image of the free vector a, and from the point B, we set aside the vector BC, which belongs

to b. Op. 3.1. The sum of vectors a and b is called the vector c, which is the equivalence class of the directed segment AC, the beginning of which coincides with the beginning of the vector AB, and the end-with end of the BC vector.

Properties of the sum of vectors

(a) Commutativity of addition: $a + b = b + a$;

(b) Associativity of addition: $a + (b + c) = (a + b) + c$;

(c) The presence of a zero vector: $\exists 0 : a + 0 = 0 + a = a$;

(d) The presence of the opposite vector: $\forall a\ \exists(-a): a + (-a) = 0$.

Opr. 3.2. The product of the vector a by the scalar λ is called the vector $b = \lambda a$ such that

(a) $|b| = / \lambda||a|$

(b) $\lambda > 0 \quad \Rightarrow \quad a \upuparrows b$

(c) $\lambda < 0 \quad \Rightarrow \quad a \uparrow\downarrow b$

(d) $\lambda = 0 \quad \Rightarrow \quad b = 0$

Properties of multiplication by a scalar

(a) Associativity of multiplication by a scalar: $\lambda(\mu a) = (\lambda\mu) a$;

(b) The presence of a unit: $1 \cdot a = a$;

(c) Distributivity: $\lambda (a + b) = \lambda a + \lambda b$.

Op. 3.3. An ort of a vector a 0 is a vector a_0 such that

$$a_0 \upuparrows a \qquad |a_0| = 1$$

4.1.

Let $\{a_1, a_2, \ldots, a_n\}$ be a set of vectors and $\lambda_1, \lambda_2, \ldots, \lambda_n \in R$ be a set of real numbers. An expression of the form

$$\lambda_1 a_1 + \lambda_2 a_2 + \ldots + \lambda_n a_n = \sum_{i=1}^{n} \lambda_i a_i$$

is called a linear combination of vectors $\{a_1, a_2, \ldots, a_n\}$ with coefficients $\lambda_1, \lambda_2, \ldots, \lambda_n$

Opr. 4.2. A basis on a plane (in space) is an ordered set of vectors such that any vector of the plane (space) can be uniquely represented as a linear combination of these vectors.

NtB. We define a basis in various spaces:

(a) On a straight line, the basis is any nonzero vector;

(b) On the plane, the basis is any ordered pair of noncollinear vectors;

(c) In three-dimensional space, a basis is an ordered triple of any non-planar vectors;

NtB. The orts of the i, j, and k axes of a Cartesian rectangular coordinate system form the basis of the space. Therefore

, the radius vector of any point A can be decomposed in terms of the basis

where X_A, Y_A, and Z_A coordinates of the point in the given coordinate system.