6.819 PSET 5
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1. Wiener Filter In Frequency Domain

$Y = H \star X + N$

$P_{XX}[u,v] = |X_F[u,v]|^2|$, $P_{HH}[u,v] = |H_F[u,v]|^2|$

$$\epsilon^2 = E(|\hat{X}_F[u,v] - X_F[u,v]^2|) = \sum_{u,v} |\hat{X}_F[u,v] - X_F[u,v]|^2$$

$$= \sum_{u,v} |X_F[u,v] - G.Y|^2$$

$$= \sum_{u,v} |X_F - G.Y|^2$$

, but $Y = HX_F + N$

$$= \sum_{u,v} |X_F - G.(HX_F + N)|^2$$

$$= \sum_{u,v} [X_F(1 - GH) \mp GN]^2$$

$$= \sum_{u,v} [X_F(1 - GH) - GN]^2$$

$$= \sum_{u,v} [X(1 - GH) - GN][X^\star(1 - GH)^\star - G^\star N^\star]$$

$$= \sum_{u,v} [XX^\star(1 - GH)(1 - GH^\star) - X(1 - GH)G^\star N^\star - GNX^\star(1 - GH)^\star + GNN^\star G^\star]$$

$$= \sum_{u,v} [X_F[u,v]^2 - X_F[u,v]^2 GH - X_F[u,v]^2 G^\star H^\star + X_F[u,v]^2 H^\star_{XX} GG^\star + GNN^\star G^\star]$$

but $P_{XX}[u,v] = |X_F[u,v]|^2|$ and $P_{HH}[u,v] = |H_F[u,v]|^2|$

$$= \sum_{u,v} [P_{XX} - P_{XX}GH - P_{XX}G^\star H^\star + P_{XX}H^\star_{XX}GG^\star + GNN^\star G^\star]$$

$$= \sum_{u,v}[P_{XX} - P_{XX}GH - P_{XX}G^{\star}H^{\star} + P_{XX}H_{XX}G]^2$$

$$= \sum_{u,v}[P_{XX} - P_{XX}GH - P_{XX}G^{\star}H^{\star} + P_{XX}H_{XX}|G|^2 + |G|^2\sigma^2]$$

but $|G|^2 = G_R^2 + G_{im}^2$

Hence ,

$$\epsilon^2 = \sum_{u,v}[P_{XX} - P_{GG}(G_R + jG_{im})(H_R + H_{im}) - P_{XX}G^{\star}(H_R - jH_{im})^{\star} + P_{XX}H_{XX}(G_R^2 + G_{im}^2)$$

$$+G^2(G_R^2 + G_{im}^2)]$$

$\frac{\partial \epsilon^2}{\partial G_R} = -P_{GG}H - P_{XX}H^{\star} + P_{XX}H_{XX} - 2GR + 2\sigma^2 G_R^2$

$G_R = \frac{H_F^{real} P_{XX}}{P_{XX}P_{HH} + \sigma^2}$

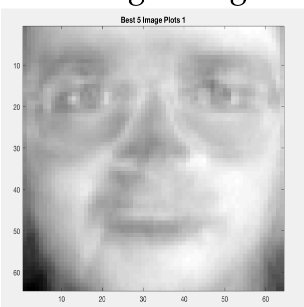
$G_{im} = \frac{H_F^{im} P_{XX}}{P_{XX}P_{HH} + |N_F[u,v]|^2}$

$G_F^{Real}[u,v] = \frac{H_F^{real} P_{XX}}{P_{XX}P_{HH} + |N_F[u,v]|^2}$

$G_F^{Complex}[u,v] = \frac{H_F^{im} P_{XX}}{P_{XX}P_{HH} + |N_F[u,v]|^2}$

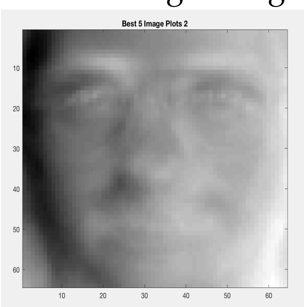
2. EigenFaces

First Image using 1st eigenvector

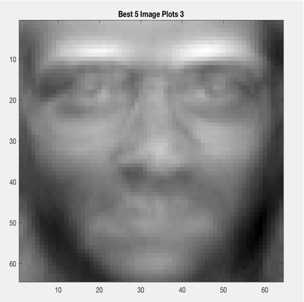


First Image

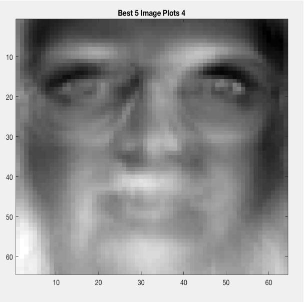
Second Image using second eigenvector



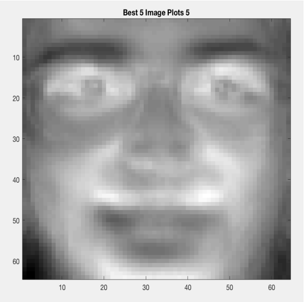
Second Image
Third Image using third eigenvector



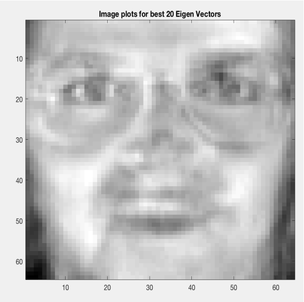
Third Image
4th Image using 4th eigenvector



4th Image
5th Image using 5th eigenvector



5th Image
Reconstruction Image using best 20 eigenvectors



Reconstruction Image

Choosing Best 5 Images

$PCA\ Reconstruction = PCScores.EigenVectors^T + Mean$

From this equation , it can be seen that the quality of the reconstruction image is directly proportional to the quality of the eigen vectors , hence the higher eigen vector values will produce better images , so I chose the *k* largest eigenvector values as the basis.