MIT CSAIL

6.869 Advances in Computer Vision

Fall 2017

Problem Set 5

Posted: Thursday, October 12, 2017 **Due:** Thursday 23:59, October 19, 2017

Please submit **two files:** 1) a **PDF** report named ⟨your_kerberos⟩.pdf, including your answers to all required questions with images and/or plots showing your results, and 2) a file named ⟨your_kerberos⟩.zip, containing relevant source code.

Late Submission Policy: We do not accept late submissions. The submission deadline has a 50-minute soft cut-off; after midnight Thursday, submissions are penalized 2% per minute late.

Problem 1 *Wiener Filter in Frequency Domain*

In this problem you will derive the Wiener Filter in frequency domain.

Assume we have a very simple image formation model: A perfect image $X(k, l)$ was taken with a imperfect approach, which may introduce degradation of blurring and noise. We aim to find a filter G so that given the non-perfect measurement $Y(k, l)$, convolving G and Y will give you the best estimate of X in a least square sense.

To be specific, we assume $Y = H \star X + N$($\star$ as convolution), where N is additive Gaussian noise with variance σ^2 and H is known. Suppose our estimate is $\hat{X}$ given by $G \star Y$. A Wiener filter is given by minimizing the reconstruction error:

$$\epsilon^2 = E(|\hat{X}_F - X_F|^2)$$

where $\hat{X}_F$ and X_F are the Fourier transform of $\hat{X}$ and X; $E()$ denotes for taking expectation, since X and N are considered random signals.(Note that since we do not assume the content of X, we model X as a track of some 2-D random process.)

Assuming the noise is completely uncorrelated with the input image, please derive the frequency representation G_F of G that minimizes ϵ^2. In your final expression, you can use the power spectrum of X, P_{XX}, and H, P_{HH}.

Hint: the best estimate G_F is given by $\frac{\partial \epsilon^2}{\partial G_F} = 0$. Since $\frac{\partial}{\partial z}(|z|^2)$ is not well defined($|z|^2$ not differentiable in complex domain), such minimization should be done for the real and complex part of G_F respectively. That is, your final result should give the real and complex part of G_F separately.

Problem 2 *Eigenfaces*

In this problem, your goal is to compute a "face basis" for the Olivetti face dataset. Load the included face database given by `faces.mat`; the resulting Matlab variable is called `faces`. Compute the eigenvectors of the covariance matrix of these faces. These so-called "eigenfaces" form a basis for the set of faces. Show the top five basis images in your report (what do we mean by 'top'? Hint: PCA). Describe in your report how you chose the top basis images, and why. Give the coefficients of the first image in the dataset in terms of the top 20 basis images. Try to reconstruct the image from these coefficients, and include the result in your report.

References