6.819 PSET 8
11/17/2017

ISAAC KONTOMAH

Collaborators: Kamoya Ikhofua,Benjamin Waar,Afika Nyati ,Andrew Zhang, Abishkar Chhetri,Suman Nepal

1 *Markov Network*

a.

$$\phi(a,d) = \phi(c,e) = \begin{bmatrix} 0.9 & 0.1 \\ 0.9 & 0.9 \end{bmatrix}$$

$$\psi(a,b) = \psi(b,c) = \begin{bmatrix} \alpha & 1-\alpha \\ 1-\alpha & \alpha \end{bmatrix}$$

n_p denote node potential

$n_p(d) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$n_p(a) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$P(a) = M_{d\to a}M_{b\to a}$

$M_{d\to a} = \phi(d,a)n_p(d) = \begin{bmatrix} 0.9 & 0.1 \\ 0.9 & 0.9 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0.9 \end{bmatrix}$,

hence ,$M_{d\to a} = = \begin{bmatrix} 0.1 \\ 0.9 \end{bmatrix}$

$M_{e\to c} = \phi(e,c) = \phi(c,e)n_p(e) = \begin{bmatrix} 0.9 & 0.1 \\ 0.9 & 0.9 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.9 \\ 0.1 \end{bmatrix}$

$M_{c\to b} = \psi(c,b)n_p(c)$,

$n_p(c) = M_{e\to c} \times \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.9 \\ 0.1 \end{bmatrix}$

$M_{c\to b} = \begin{bmatrix} 0.99 & 0.01 \\ 0.01 & 0.99 \end{bmatrix}\begin{bmatrix} 0.9 \\ 0.1 \end{bmatrix} = \begin{bmatrix} 0.892 \\ 0.108 \end{bmatrix}$

$M_{b\to a} = \psi(b,a)n_p(b)$,

$n_p(b) = M_{c\to b} \times \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.892 \\ 0.108 \end{bmatrix}$

$M_{b\to a} = \begin{bmatrix} 0.99 & 0.01 \\ 0.01 & 0.99 \end{bmatrix}\begin{bmatrix} 0.892 \\ 0.108 \end{bmatrix} = \begin{bmatrix} 0.88416 \\ 0.11584 \end{bmatrix}$

$P(a) = M_{d\to a}M_{b\to a} = \begin{bmatrix} 0.1 \\ 0.9 \end{bmatrix}\begin{bmatrix} 0.88416 \\ 0.11584 \end{bmatrix} = \begin{bmatrix} 0.088416 \\ 0.104256 \end{bmatrix}$

Normalizing the values , $P(a) = \begin{bmatrix} \frac{0.088416}{0.192672} \\ \frac{0.104256}{0.192672} \end{bmatrix} = \begin{bmatrix} 0.459 \\ 0.541 \end{bmatrix}$

b.

$$\phi(a,d) = \phi(c,e) = \begin{bmatrix} 0.9 & 0.1 \\ 0.9 & 0.9 \end{bmatrix}$$

$$\psi(a,b) = \psi(b,c) = \begin{bmatrix} \alpha & 1-\alpha \\ 1-\alpha & \alpha \end{bmatrix}$$

n_p denote node potential

$n_p(d) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$n_p(a) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$P(a) = M_{d\to a} M_{b\to a}$

$M_{d\to a} = \phi(d,a) n_p(d) = \begin{bmatrix} 0.9 & 0.1 \\ 0.9 & 0.9 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0.9 \end{bmatrix},$

hence ,$M_{d\to a} = = \begin{bmatrix} 0.1 \\ 0.9 \end{bmatrix}$

$M_{e\to c} = \phi(e,c) = \phi(c,e) n_p(e) = \begin{bmatrix} 0.9 & 0.1 \\ 0.9 & 0.9 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.9 \\ 0.1 \end{bmatrix}$

$M_{c\to b} = \psi(c,b) n_p(c),$

$n_p(c) = M_{e\to c} \times \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.9 \\ 0.1 \end{bmatrix}$

$M_{c\to b} = \begin{bmatrix} 0.6 & 0.4 \\ 0.4 & 0.6 \end{bmatrix} \begin{bmatrix} 0.9 \\ 0.1 \end{bmatrix} = \begin{bmatrix} 0.58 \\ 0.42 \end{bmatrix}$

$M_{b\to a} = \psi(b,a) n_p(b),$

$n_p(b) = M_{b\to a} \times \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.58 \\ 0.42 \end{bmatrix}$

$M_{b\to a} = \begin{bmatrix} 0.6 & 0.4 \\ 0.4 & 0.6 \end{bmatrix} \begin{bmatrix} 0.58 \\ 0.42 \end{bmatrix} = \begin{bmatrix} 0.516 \\ 0.484 \end{bmatrix}$

$P(a) = M_{d\to a} M_{b\to a} = \begin{bmatrix} 0.1 \\ 0.9 \end{bmatrix} \begin{bmatrix} 0.516 \\ 0.484 \end{bmatrix} = \begin{bmatrix} 0.0516 \\ 0.4356 \end{bmatrix}$

Normalizing the values , $P(a) = \begin{bmatrix} \frac{0.0516}{0.4872} \\ \frac{0.484}{0.4872} \end{bmatrix} = \begin{bmatrix} 0.1059 \\ 0.8941 \end{bmatrix}$

Explanation of difference in results

The marginal probabilities of $P(a)$ being in its different states , state 0 and 1 are almost quite likely when $\alpha = 0.99$ but very far apart when $\alpha = 0.6$.This is because the transition matrix multiplies the node potential at each node to update the node potential before a message is then passed among the edge connecting the two nodes , hence a higher α value means there is higher compatibility between the states and the node will more on less prefer to be in any of the two , but a lesser value of α will mean it has lesser compatibility hence will prefer one state over the other.

2 *Belief Propagation*

a.

Flower Contours

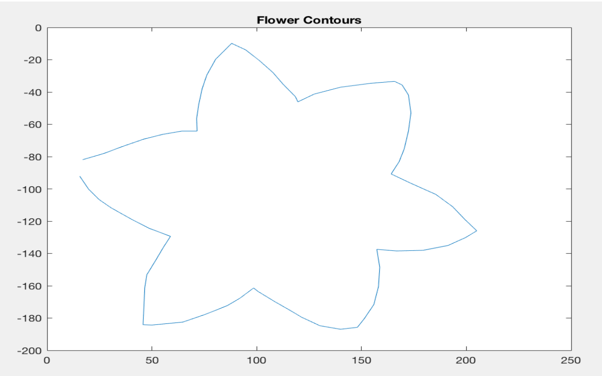

Image showing contours of flower image

Hand Contours

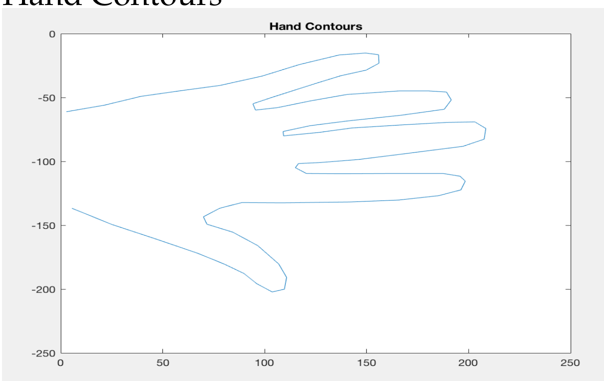

Image showing contours of hand

Pedestrian contours

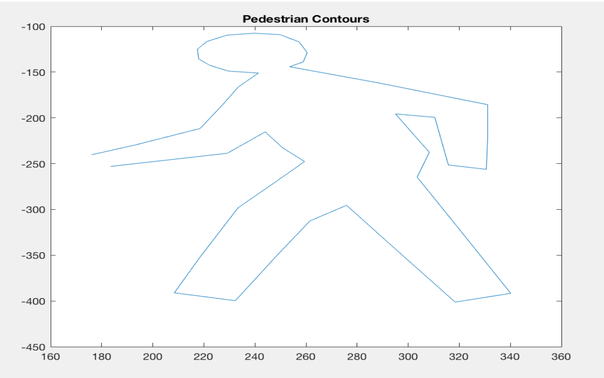

Image showing contours of pedestrian image

b.

Local Direction of Each Node Before Running Belief Propagation

Flower Contours with Local evidence

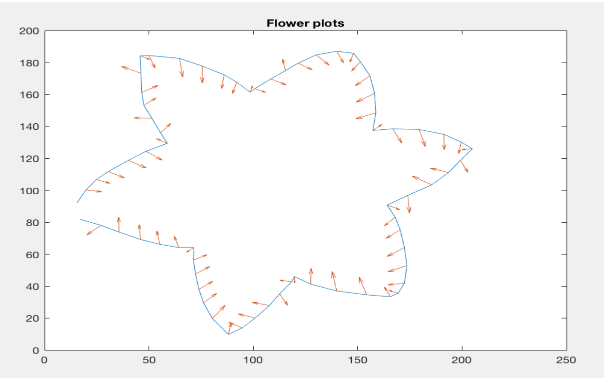

Image showing flower contours with local evidence before belief propagation

Hand Contours with Local Evidence

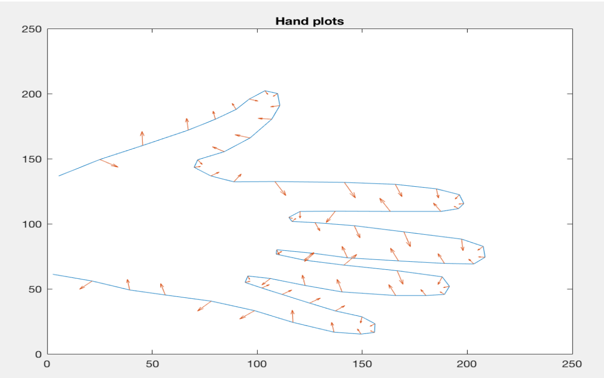

Image showing hand contours with local evidence before belief propagation

Pedestrian contours with Local Evidence

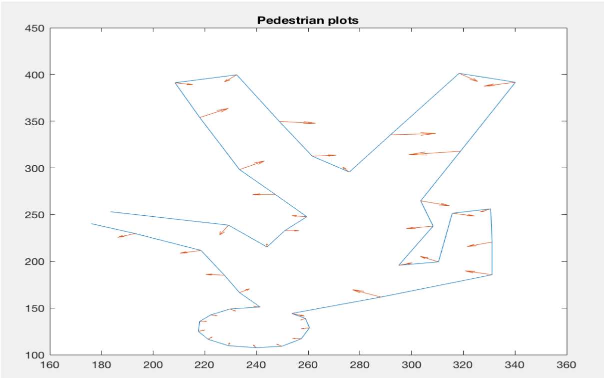

Image showing pedestrian contours with local evidence before belief propagation

c.***Final Estimated Direction after Running Belief Propagation***

Direction of Each Node After Belief Propagation

Flower Contours with belief propagation

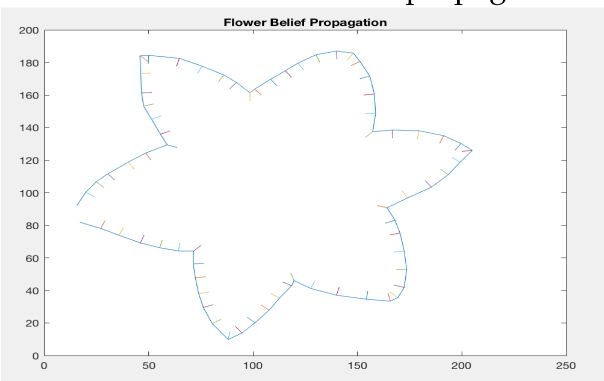

Image showing flower contours with belief propagation

Hand Contours with belief propagation

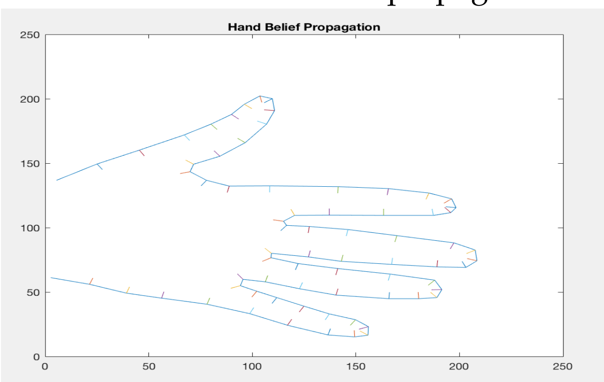

Image showing hand contours with belief propagation

Pedestrian contours with Belief Propagation

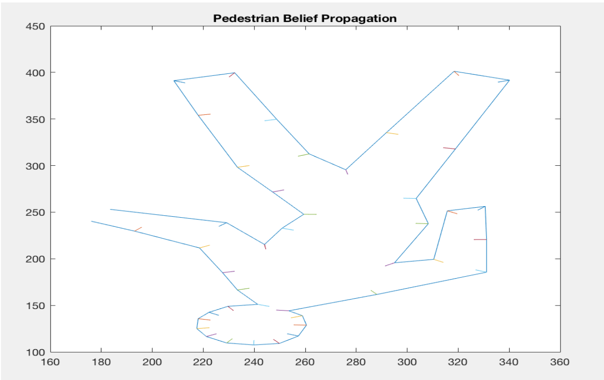

Image showing pedestrian contours with belief propagation