

Algebraic Complexity

Markus Bläser, Alexander Ikonomou and Shreyas Srinivas

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Lemma 1. *Every algebraic formula where the root is an addition, subtraction or negation has at least size 1.*

Proof. This follows by the definition of size. □

Lemma 2. *Every algebraic formula with size 0 is a variable or a constant.*

Proof. Proof by contraposition. Assume that f is an algebraic formula that is neither a variable nor a constant. Then, the root of f must be an addition, a subtraction or a negation. With 1 it follows that f has at least size 1. □