STRUCTURED SCALE FREE NETWORKS

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EXCELENCIA
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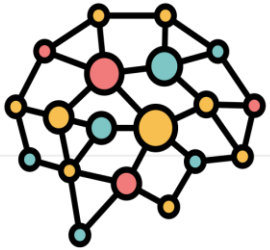
OUTLINE

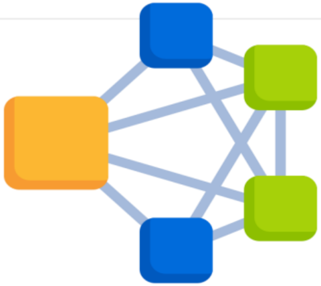
Introduction

Generation of the Networks



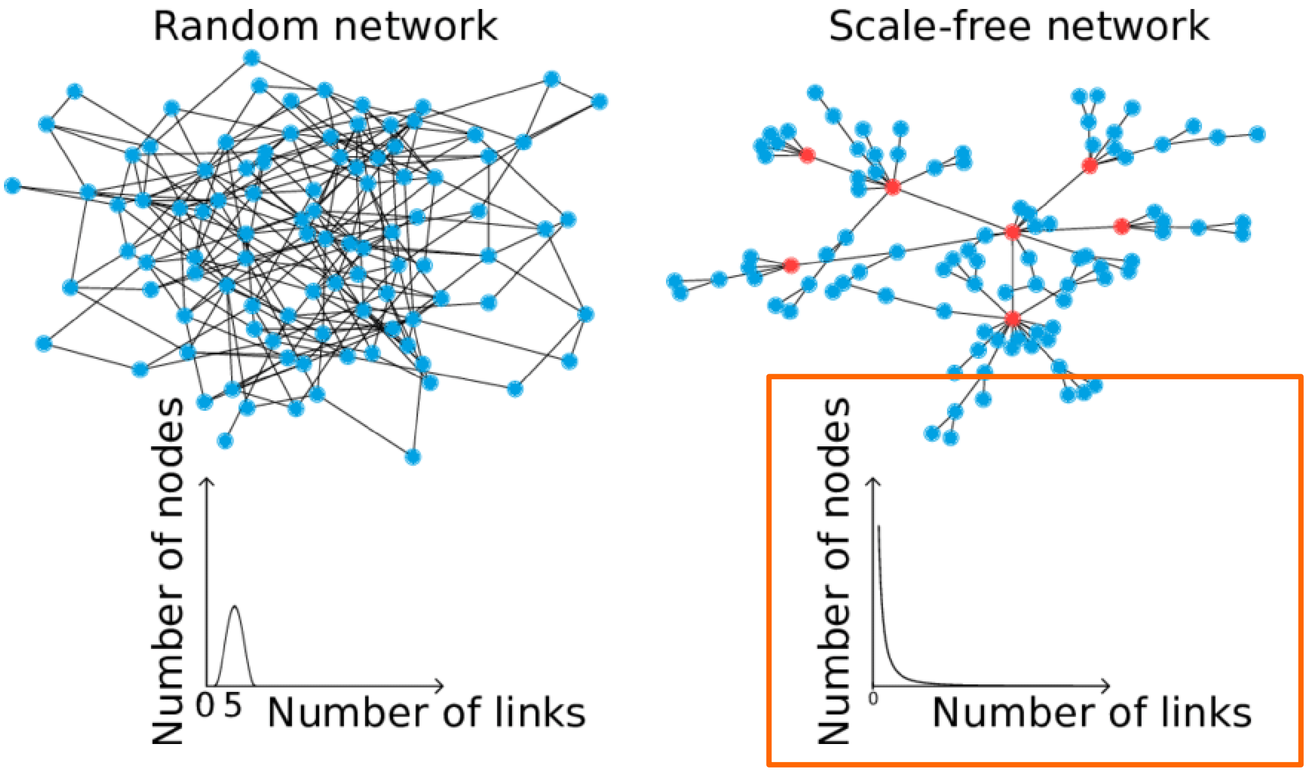
Degree Distribution

Clustering Coefficient



Dimension

Conclusion and questions



Nat Commun. 2019; 10(1):1017.

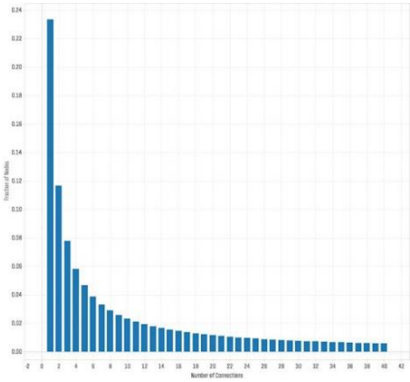
Scale-free networks are a type of complex network where the degree distribution (connections per node) follows a power law instead of a Poisson distribution (as in random networks).

Real-world networks

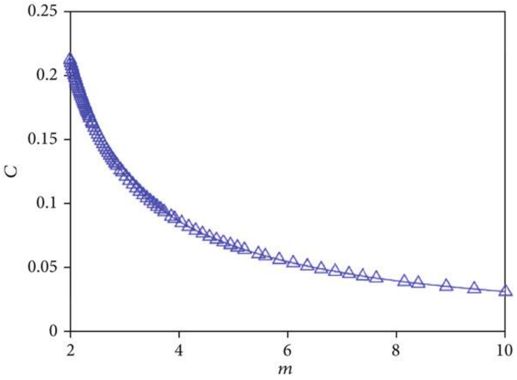
Structure and dynamics of complex systems.

INTRODUCTION

Scale free networks (Barabasi –Albert)



Scale-free degree distribution



Clustering decreases with N, but more slowly than in a random network

Random networks

Scales with N

$$C_{rand} \sim \frac{\langle k \rangle}{N}$$

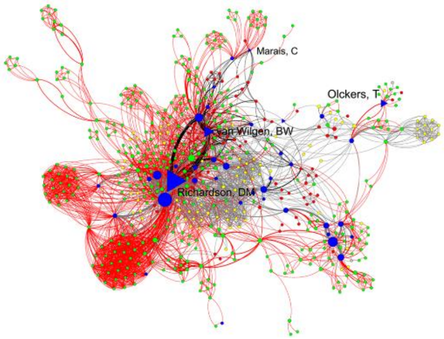
Small world networks (Watts-Strogatz)

$$C_{WS} \sim constant$$

Scale free networks (Barabasi –Albert)

$$C_{BA} \sim \frac{(\ln N)^2}{N}$$

Dynamic Games and Applications, 2024, 15(5):1564-1586
Proc Natl Acad Sci U S A. 2004;101(Suppl 1):5200-5.



In real networks (such as the Internet, social networks, scientific collaboration networks), the clustering coefficient is observed to be high and almost independent of N (**as in the Small World model**), while the network has a scale-free degree distribution (**as in the BA model**).

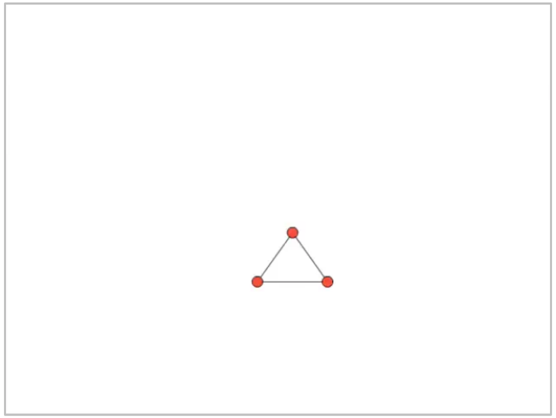
INTRODUCTION

Network assembly mechanisms:

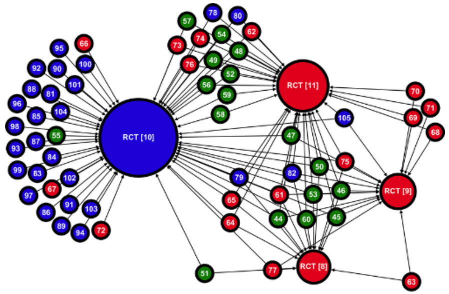
Barabási-Albert (BA)

Preferential attachment: The probability that a node gains a connection is proportional to its current degree *k*.

Strongly scale-free structure is empirically rare



Scientific citations networks



There is an age dependence of the growth dynamics of the network.

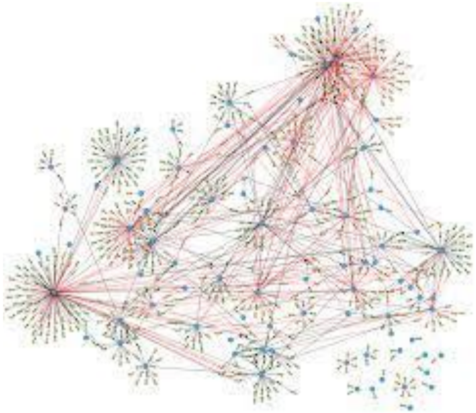
Modeling **highly clustered scale-free networks**

- Networks that retain the power-law distribution of the degree.
- High clustering independent of N
- Linear preferential attachment of new links
- Negative correlation between the age of a node and its link attachment rate (in BA older nodes tend to have more connections simply because they have been in the network longer).

A highly clustered scale-free network (HCSF) is a scale-free network with aging, which differs from the pure Barabási-Albert model due to the negative correlation between age and attachment rate.

BA the advantage is purely cumulative ("the rich get richer")

In HCSF there is a balance between cumulative advantage and novelty—new nodes have a "boost" that counteracts the advantage of established nodes.



This modification makes the model **more realistic** for many systems where novelty or recent activity are important factors in attracting connections.

INTRODUCTION

PHYSICAL REVIEW E, VOLUME 65, 036123

Highly clustered scale-free networks

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[2]*Instituto Mediterráneo de Estudios Avanzados IMEDEA (CSIC-UIB), E07071 Palma de Mallorca, Spain*

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(Received 28 August 2001; published 21 February 2002)

We propose a model for growing networks based on a finite memory of the nodes. The model shows stylized features of real-world networks: power-law distribution of degree, linear preferential attachment of new links, and a negative correlation between the age of a node and its link attachment rate. Notably, the degree distribution is conserved even though only the most recently grown part of the network is considered. As the network grows, the clustering reaches an asymptotic value larger than that for regular lattices of the same average connectivity and similar to the one observed in the networks of movie actors, coauthorship in science, and word synonyms. These highly clustered scale-free networks indicate that memory effects are crucial for a correct description of the dynamics of growing networks.

DOI: 10.1103/PhysRevE.65.036123 PACS number(s): 89.75.Hc, 87.23.Ge, 89.65.−s

OBJECTIVES

- Produce the code for generating networks using the structured scale-free network model of different sizes.
- Generate code for calculate degree distribution and the clustering coefficient to the generated networks.
- Evaluate the dimensionality of generated networks.

Properties

- Average connectivity of the network:

  Each new node → m new edges. For $N \gg m$ (neglecting the initial condition):

  Total edges are

  $$L \approx N \cdot m \implies \langle \kappa \rangle = \frac{N}{L} = m$$

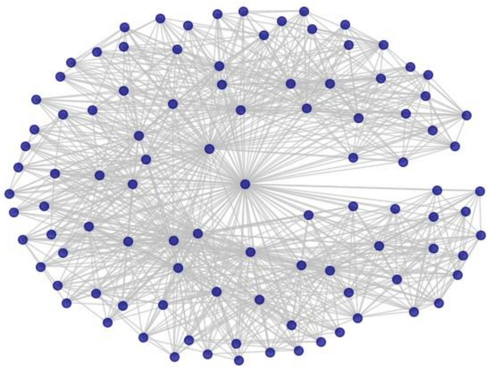
- Degree of node i:
  Node receives incoming links during its active lifetime T:

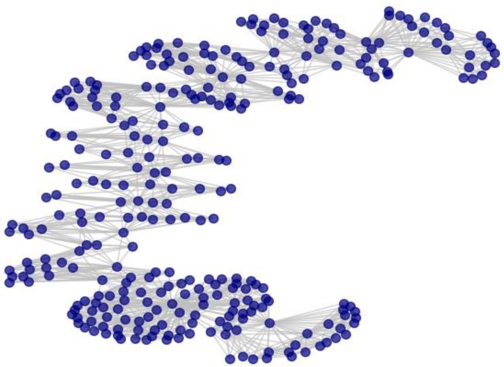
  $$k_i = T_i$$

- Strong deactivation mechanism:

  Forgotten nodes, no rediscovery.

- Reproduce well several real growing networks.





Model: Growth and deactivation

- Initial condition:
  - Directed network,
  - m initial *active* nodes,
  - Fully connected (complete digraph).
- Parameters:
  - N, number of nodes,
  - $a > 0$, the constant bias,
  - m, number of active nodes at any step.

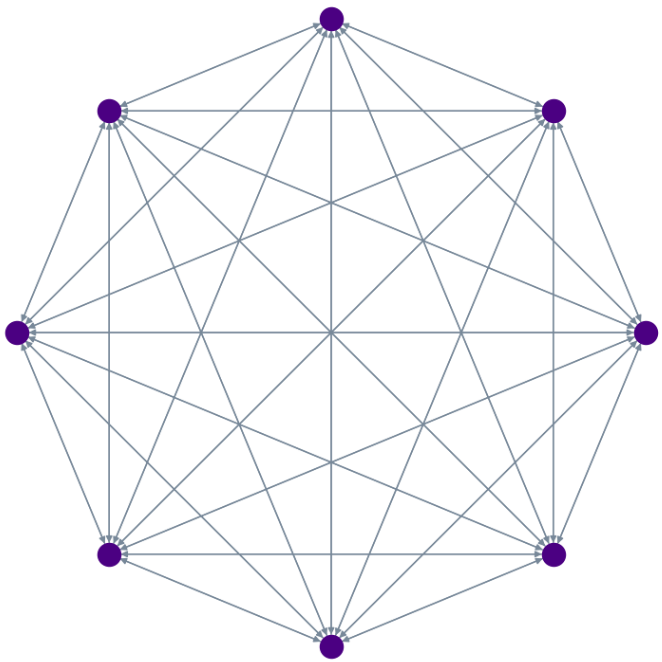


Fig: Complete digraph – initial configuration.

Model: Growth and deactivation

- Dynamics:

  (1) **Add** new node i to network ($k_i = 0$)

  (2) **Attach** m links from node i to active nodes j ($k_j \rightarrow k_j + 1$)

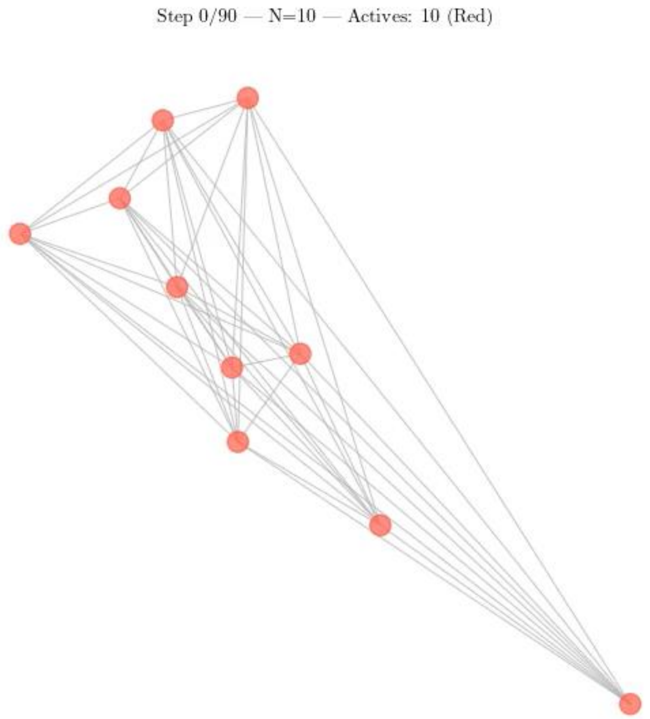
  (3) **Activate** the node i

  (4) **Deactivate** one active node, with probability:

  $$P(k_j) = \frac{c}{a + k_j}$$

  (c is a normalization constant)

  (5) **Resume** at 1



Animation of assembling (N = 100)

Model: Growth and deactivation

- Algorithm

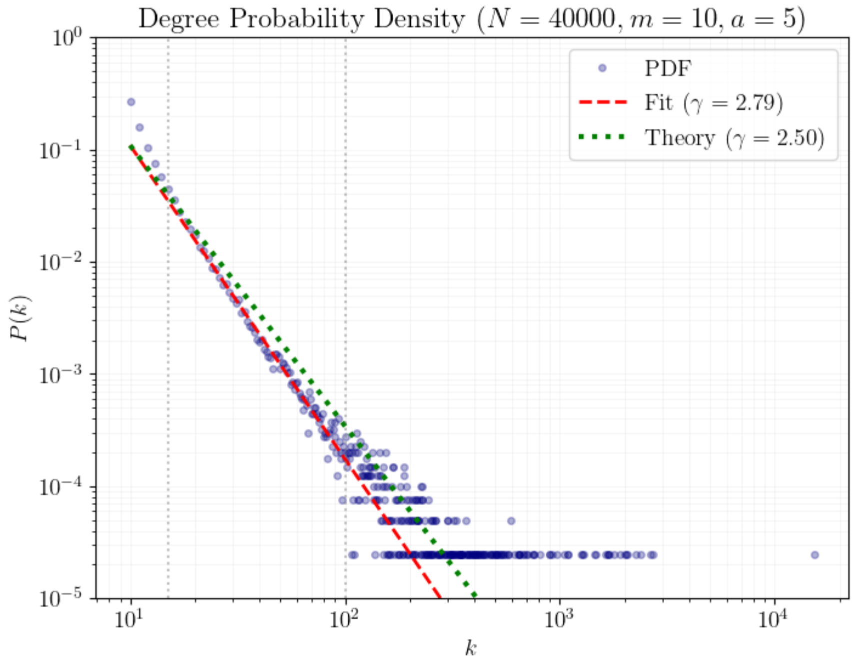
```
initialize m, a, N
V = {1,…,m}
E = {(1,2),(2,1),…,(m,m-1),(m-1,m)}

k_in = {1:m-1, 2:m-1,…, m:m-1}
V_act = V

for i in [m+1,N]:
  add i to V
  add i:0 to k_in
  for j in V_act:
    add (i,j) to E
    k_in[j] = k_in[j] + 1
  end for
  add i to V_act

  probs = {j in V_act: 1/(a + k_in[j])}
  s = sum(probs)
  r = real_random(0,s)
  acc = 0.0
  for p in probs:
    acc = acc + p
    if r <= acc:
      remove j from V_act
      end for
    end if
end for

Graph = (V, E)
```

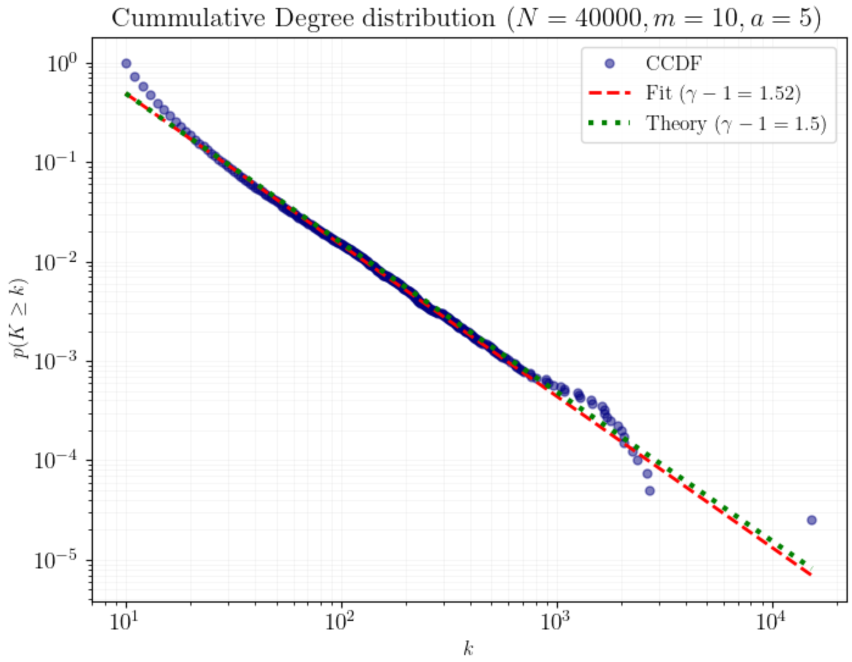
Scale-free network : Power-law degree distribution

In-degree: $N(k) = b(a+k)^{-\gamma}$

Total: $P(k) \sim k^{-\gamma}$

$$m = c\int_0^{\infty} \frac{k}{(a+k)^{\gamma}}\,dk \;\longrightarrow\; \gamma = 2 + \frac{a}{m}$$

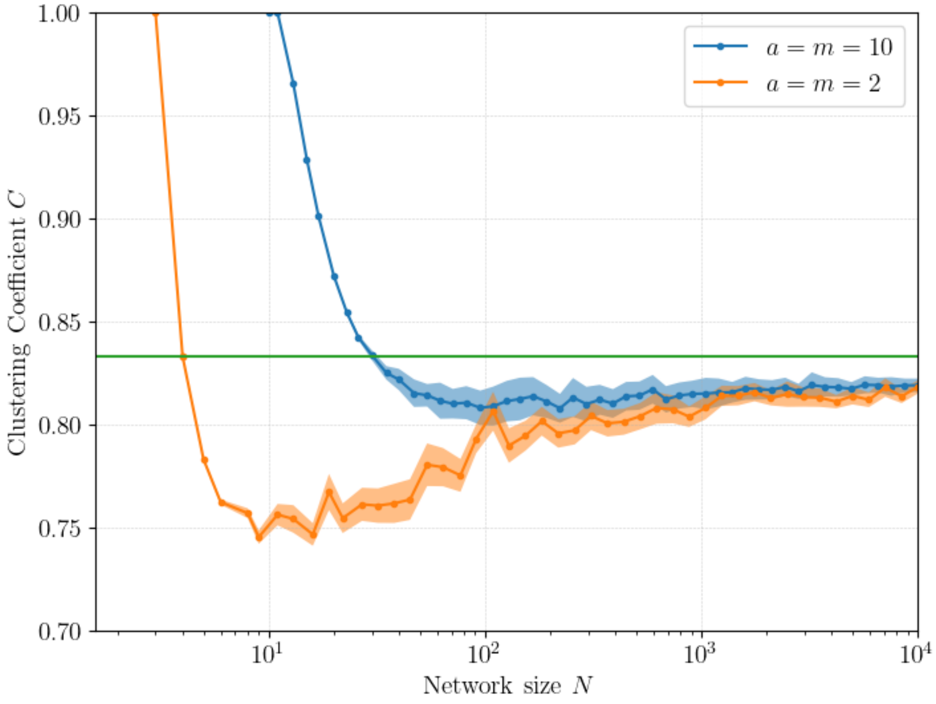




Some issues with the clustering coefficient. Initially, we didn't consider leaf nodes or isolated nodes, and our function diverged from the NetworkX function. We eventually realized the problem and corrected it.

The Klemm-Eguíluz model predicts that C approach to a non-zero constant value as $N \to \infty$, similar to regular lattices.
When $a = m$, the clustering coefficient gives an asymptotic value:

$$C_{a=m} = \frac{5}{6}$$



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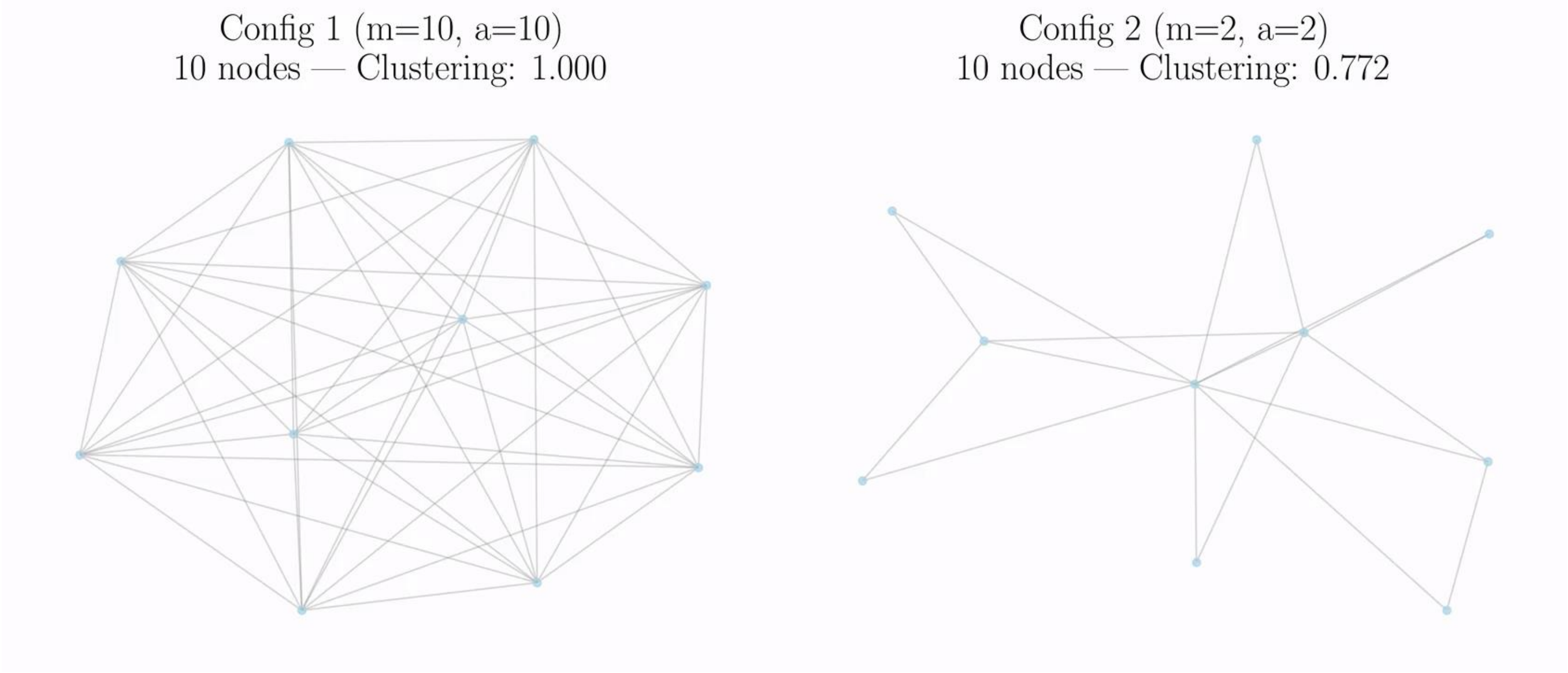

Config 1 (m=10, a=10)
10 nodes — Clustering: 1.000
Config 2 (m=2, a=2)
10 nodes — Clustering: 0.772

How can we characterize the dimension of a Complex Network?

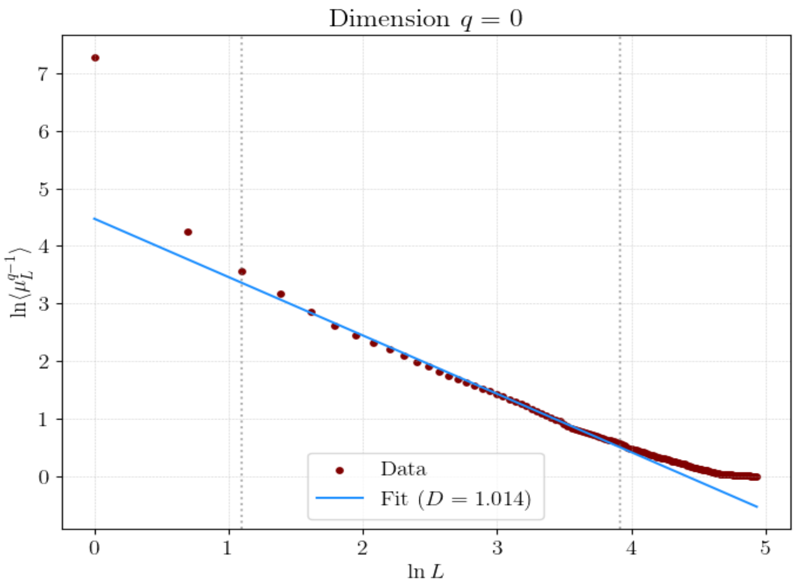
- Not explored topic.
- Clasically three dimensional Euclidean Dimension.
- 3 Types of Dimension.
- Good results for regular lattices.
- Infinite Dimension for nonregular Complex Networks.
- But for Structured Scale Free Networks?

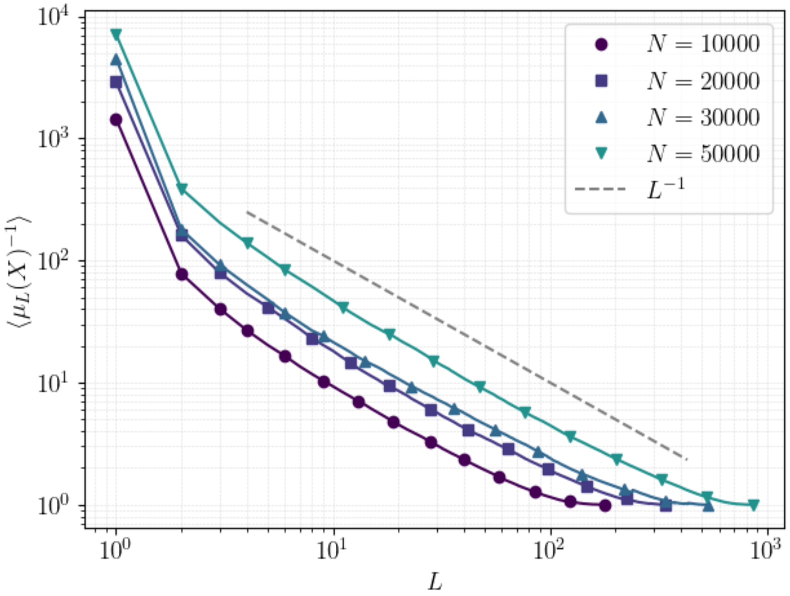
$$D_q = \lim_{L \to \infty} \frac{1}{q-1} \frac{\ln \langle \mu_L(X)^{q-1} \rangle_X}{\ln L}$$

$$\mu_L(X_i) = (N-1)^{-1} \sum_{j \neq i} \Xi(L - dist(X_i, X_j))$$

Eguiluz et al (2003)

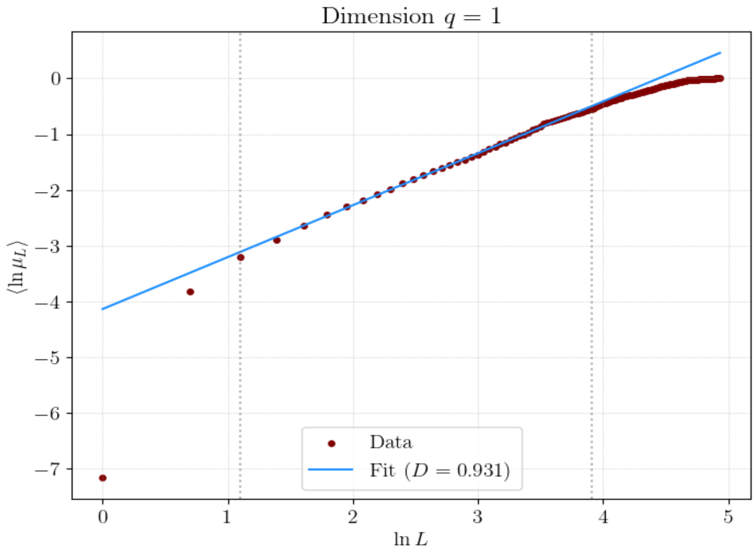
Capacity Dimension (q = 0)

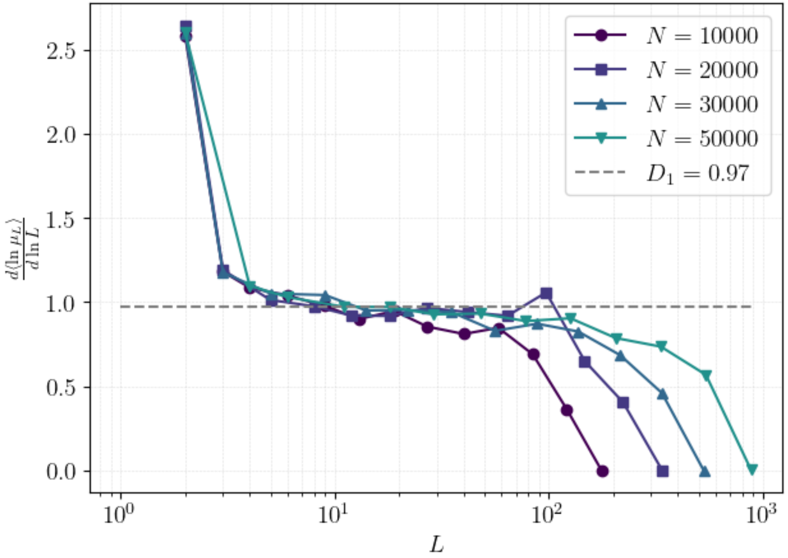




- Measures the overall scaling behaviour of the Network.
- Box covering approach.

Information Dimension (q = 1)

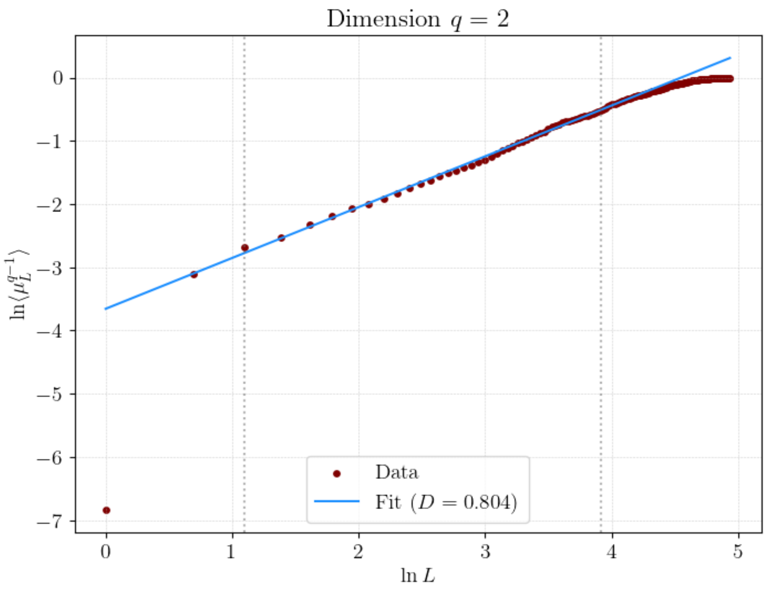


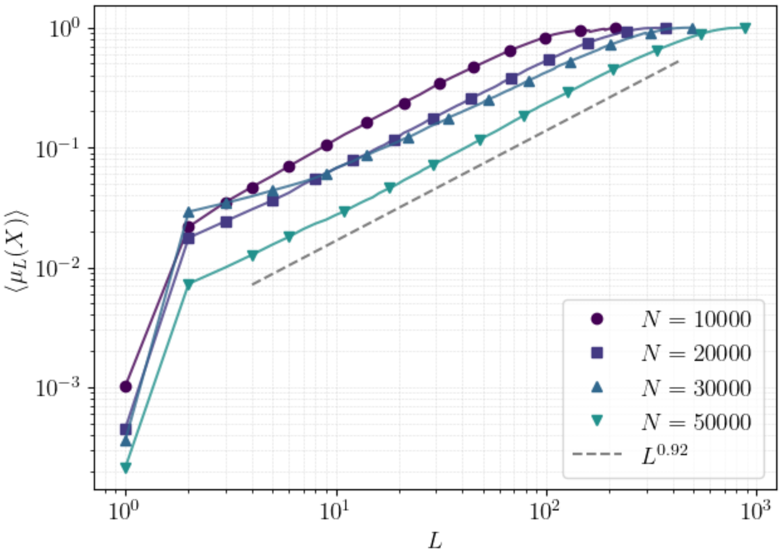


- Information content of the Network structure.
- Average local slope of the correlation function.

$$D_1 = \lim_{L \to \infty} \left\langle \frac{\ln \mu_L(X)}{\ln L} \right\rangle_X$$

Correlation Dimension (q = 2)





- Measures how pair of nodes are distributed.
- Most commonly used in Network Analysis.

Final Analysis

- Always close to 1.
- Significant difference with random Scale Networks.
- Information propagation.
- Percolation Treshold.
- Dimensionality as a descriptor of Network behavior.
- Not a complete analysis.

Conclusions

1. It was successfully implemented Python code to generate Highly Clustered Scale-Free networks of varying sizes using the Klemm–Eguiluz growth and deactivation model.

2. The generated code calculates and confirms a power-law degree distribution and a high, size-independent clustering coefficient, aligning with the model's theoretical predictions.

3. Analysis shows low effective dimensionality and hierarchical structure, consistent with the highly clustered, scale-free nature of the generated networks.

GitHub Project

IFISC

THANK YOU

for your attention

Questions?

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