A Study of Volatility in the Japanese Stock Market through the Nikkei 225 Index

Final Project

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In this project, we analyze and model the Nikkei 225 index time series, the most important index in the Japanese stock market. An analysis of its series is done in order to estimate models for the mean and variance of log-returns, and some results regarding volatility prediction and news impact are used to select the volatility model. We also present risk measures and comparisons with traditional methods with our chosen model. Then, we consider the FTSE100 time series in order to analyze possible relations and the impact of news on the volatility of both markets using a GARCH-DCC model. Finally, we expose the concluding remarks of the analysis.

1 Introduction

Most of the literature studying volatility (and other finance topics) shown in universities and academic contexts concentrates on and uses data about the US stock markets, as they are the most developed and have the most influence worldwide. However, less emphasis has been given to other kinds of stock markets such as eastern stock markets and emerging economies stock markets. With the purpose of leaving this idea behind and adapting to the current financial literature trend about studying different kinds of markets, we base this project on studying the volatility of a concrete market that has been highly influential and is one of the most developed in the globe: the Japanese stock market.

In this project, we analyze the distribution of returns of the Nikkei225 in order to obtain a model for both the mean and the volatility of these returns. The main goal is to compare different models for the volatility to illustrate the different aspects to take into account when modeling with a GARCH framework and to use the results obtained from these models to compare them with alternative traditional methods, analyze the different impacts on volatility and predictions of volatility, and apply them to risk management purposes such as the computation of risk measures. We also try to characterize how is the index related to other markets worldwide, so we consider the London Stock Exchange by using data about the FTSE100 index in order to estimate a multivariate DCC-GARCH model and study the correlation and the impact of shocks in both markets.

To study the volatility in the Japanese market, we make use of data about the Nikkei225 stock index, one of the most influential worldwide. We use the daily adjusted closing prices of the index from 2000 until the end of 2022, extracted from Yahoo Finance. The data about FTSE100 is also daily, adjusted and is extracted from the same source as the Nikkei225 data.

We structure this project as follows. First, we present a brief analysis of the time series for the prices and the returns of the Nikkei225 index. Then, we start the modeling task for the mean and the volatility and we present different estimation results. We proceed with a section discussing the different results from the models and different applications such as prediction, comparison with traditional methods, and the computation of risk measures. At last, we present the results for the DCC-GARCH, including data from the FTSE100, and analyze news impact and the correlation between indices. We end the project with a brief recap of the different results in form of conclusions.

2 Analysis of the Time Series

As a first step, we analyze the time series of the index to emphasize important events that had an effect on volatility, analyze the stationarity, and give some informative statistics.

In Figure 1, we plot the time series of the index prices and the log-returns so that one can visually localize important events, understood as events where there are large declines or increases in prices caused by external events, which translate into very high or low log-returns in different points in time.

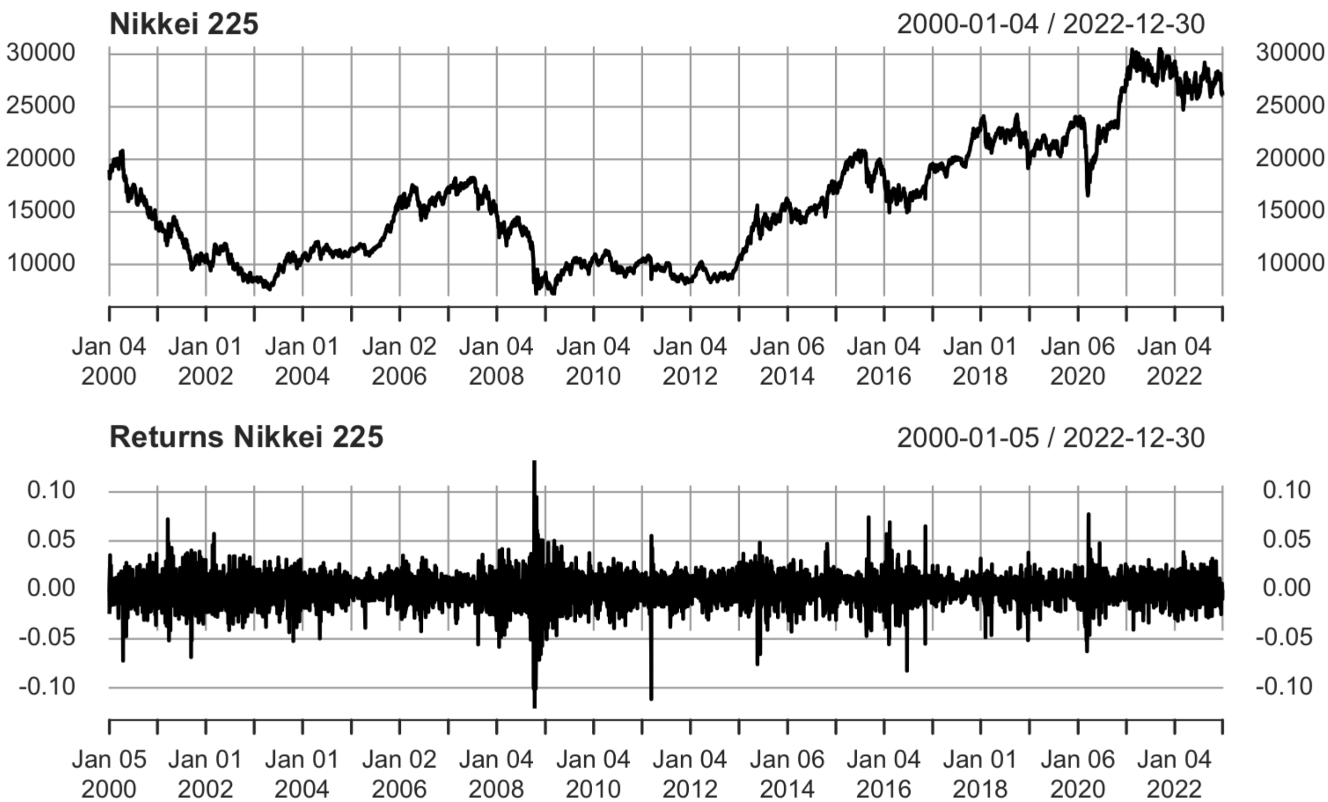

Figure 1: Nikkei225 time series

As we can see, there are various times when important events had happened. We summarize them in the following timeline:

- **2000**: During this year, the dot-com bubble surged and made the index increase until March, when it began to fall and the bubble burst, causing an extended fall in tech stock prices until August 2001.
- **September 2001**: The 9/11 terrorist attacks happened, shocking the world.
- **June 2002**: Fears of a US worsening economy make downward pressure on the index price and the incoming Iraq war augmented this negative effect until March 2003.
- **2008**: The subprime loan bubble burst and the 2008's financial crisis began, causing a steep decrease in indexes' prices worldwide.
- **March 2011**: The Great East Japan earthquake and the Fukushima incident made the index decrease steeply in a very few days.
- **2013**: Japan's Central Bank tries to fight deflation through policies that led to investors sped up investing in Japanese stock, increasing the index price.
- **2016**: Brexit and other unexpected events occurred, which led to a huge decrease.

- **2020**: The COVID-19 outbreak caused stock indexes around the world to fall, recovering steeply after governments around the world announced policies to fight the incoming crisis.

After a qualitative explanation of the series, we can now begin with a more empirical analysis regarding stationarity and statistical properties. To assess the stationarity of our time series, we will look at the autocorrelation (ACF) and the partial autocorrelation (PACF) functions, we will compute unit root tests and we will also carry on independence tests such as the Ljung-Box test.

By looking at the ACF and the PACF of the index prices series, non-stationary is evident given the persistent behavior of the lags in the ACF.

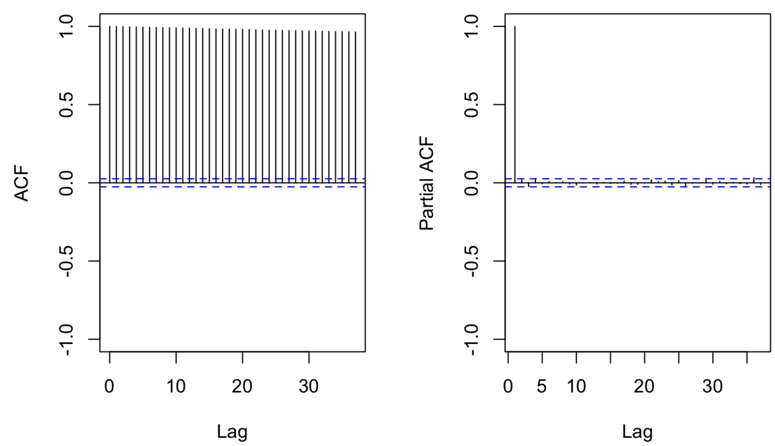

Figure 2: Nikkei225 Index Prices' ACF & PACF

However, when considering the index log-returns, the ACF and the PACF indicate that the log-return series might possibly behave as white noise due to the insignificance of the majority of lags on both functions.

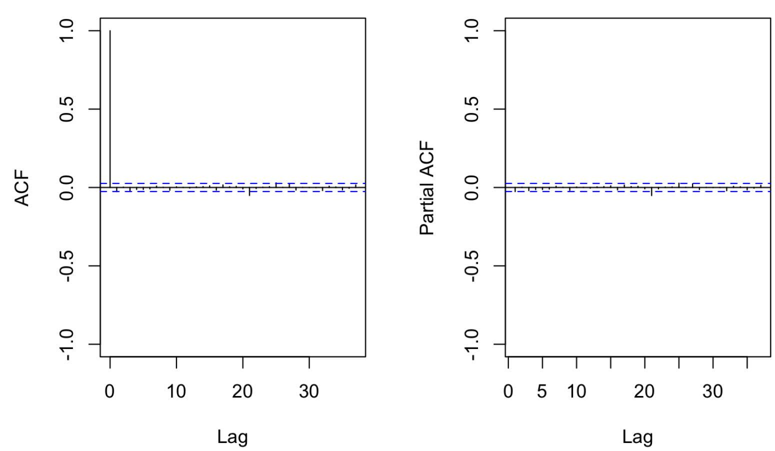

Figure 3: Nikkei225 Index log-returns' ACF & PACF

Because of this primary analysis, we consider both time series when computing unit root and independence tests. Both kinds of tests are shown in Tables 1 to 4, where we show results of Augmented Dickey-Fuller (ADF) tests, Phillips-Perron (PP) tests, KPSS tests, and Ljung-Box tests for both the prices and the log-return series. We consider 20 lags (approximately three weeks) when running all the unit root tests, and we choose to use BIC over AIC because of the bias that the AIC has.

Test type	Type	Model	Lag Selection	Statistic	p-value	Conclusion
ADF	None	None	BIC	0.0741	> 0.1	Not Reject H_0
ADF	With Constant	None	BIC	-0.8928	> 0.1	Not Reject H_0
ADF	With Trend	None	BIC	-2.8561	> 0.1	Not Reject H_0
PP	With Constant	Z-Tau	None	-0.8371	> 0.1	Not Reject H_0
PP	With Trend	Z-Tau	None	-2.714	> 0.1	Not Reject H_0
KPSS	With Constant	None	None	16.0308	< 0.01	Reject H_0
KPSS	With Trend	None	None	4.084	< 0.01	Reject H_0

Table 1: Unit Root Tests for Nikkei225 Index Prices

Test type	Type	Model	Lag Selection	Statistic	p-value	Conclusion
ADF	None	None	BIC	-54.0899	< 0.01	Reject H_0
ADF	With Constant	None	BIC	-54.0862	< 0.01	Reject H_0
ADF	With Trend	None	BIC	-54.1081	< 0.01	Reject H_0
PP	With Constant	Z-Tau	None	-78.0254	< 0.01	Reject H_0
PP	With Trend	Z-Tau	None	-78.0575	< 0.01	Reject H_0
KPSS	With Constant	None	None	0.2505	> 0.1	Not Reject H_0
KPSS	With Trend	None	None	0.0638	> 0.1	Not Reject H_0

Table 2: Unit Root Tests for Nikkei225 Index Log-returns

The results in Tables 1 & 2 clearly show the non-stationarity of the prices' time series and the stationarity of the log-returns' series, as the hypothesis of stationarity is not supported by the evidence for the first series while it is supported in all instances in the second case. The following tables illustrate how the prices' series do not respect independence between lags (we reject the independence hypothesis), while the tests on log-returns do not reject the independence hypothesis. We consider different numbers of lags for robustness, but the results are the same.

Test type	Nº of lags	Statistic	p-value	Conclusion
Ljung-Box	1	5748.5	$< 2.2e\text{-}16$	Reject H_0
Ljung-Box	5	28640	$< 2.2e\text{-}16$	Reject H_0
Ljung-Box	10	57027	$< 2.2e\text{-}16$	Reject H_0
Ljung-Box	15	85157	$< 2.2e\text{-}16$	Reject H_0
Ljung-Box	20	113029	$< 2.2e\text{-}16$	Reject H_0

Table 3: Ljung-Box Tests for Nikkei225 Index Prices

Test type	Nº of lags	Statistic	p-value	Conclusion
Ljung-Box	1	3.9655	0.04644	Not Reject H_0 at 0.01
Ljung-Box	5	8.0446	0.1538	Not Reject H_0
Ljung-Box	10	10.809	0.3726	Not Reject H_0
Ljung-Box	15	11.851	0.6903	Not Reject H_0
Ljung-Box	20	15.911	0.7221	Not Reject H_0

Table 4: Ljung-Box Tests for Nikkei225 Index Log-returns

Both the tests and the ACF and PACF seem to indicate that the index price series is not stationary, so we will need to consider the log-return series for further analysis. Some basic statistics and a Jarque-Bera normality test for both series are shown in Table 5.

Statistic	Price Series	Log-returns series
Mean	15764.406	0.000055
Median	15069.480	0.000096
Variance	34246671.105	0.000211
Std. Dev.	5852.066	0.014534
Skewness	0.645	-0.362362
Kurtosis	-0.517	6.355476
Minimum	7054.980	-0.121110
Maximum	30670.100	0.132346
JB test	Not normal	Not normal

Table 5: Basic Statistics for both series of the Nikkei225

All in all, we can see that the index prices seem to have the stylized properties of financial time series, such as non-stationarity or a non-zero correlation between lags. The return series, however, seems not to have this problem, resembling a stationary series, and having stylized facts about financial time series for returns (leptokurtosis, asymmetry, etc.).

3 Time Series Modeling

Now, we proceed to identify and estimate a possible model for the mean of the Nikkei225 time series by considering the stationary time series of log-returns. We first start by modeling the mean of the process and we then begin with the variance modeling.

3.1 A Model for the Mean

In order to identify a model for the mean, we first look at the ACF and the PACF functions plotted in Figure 3 above. As mentioned, most of the lags seem to be non-significant (they do not surpass the confidence bands) in both the ACF and the PACF.

Hence, the most sensible model to consider is an ARMA(0,0) or, equivalently, a white noise process. However, we also compare the estimations with other alternative models, such as an AR(1) and a MA(1). Table 6 shows the estimation results for all models.

Model	**Mean**	**ar1**	**ma1**	σ^2	**AIC**	**BIC**
ARMA(0,0)	1e-04	-	-	0.0002112	-32372.35	-32359.03
s.e.	2e-04	-	-	-	-	-
AR(1)	1e-04	-0.0262	-	0.0002111	-32374.31	-32354.34
s.e.	2e-04	0.0132	-	-	-	-
MA(1)	1e-04	-	-0.0262	0.0002111	-32374.3	-32354.32
s.e.	2e-04	-	0.0132	-	-	-

Table 6: Estimation of mean models for the Nikkei225

The results show that, for each model considered, the constant parameter is not significant, so the mean is not significantly different from 0. However, the AR(1) and the MA(1) parameters are significant at 0.1 and 0.05 significance levels, so we need to use the goodness of fit and other criteria to select the model.

The AIC and the BIC give contrary results, as the AIC favors the AR(1) model but the BIC favors the white noise one. Moreover, σ^2 is very similar between models, so we make use of specific knowledge about return modeling to decide. Because the AR(1) model has an intuitive interpretation (the return in a given moment depends partially on the return from the previous moment), it has been used in the finance literature and the goodness of fit does not differ significantly, we decide to keep the AR(1) model as our selected model for the mean of the Nikkei225 return series.

We run some diagnostics for the selected model to check the validity, which can be represented in the graphs from Figure 4.

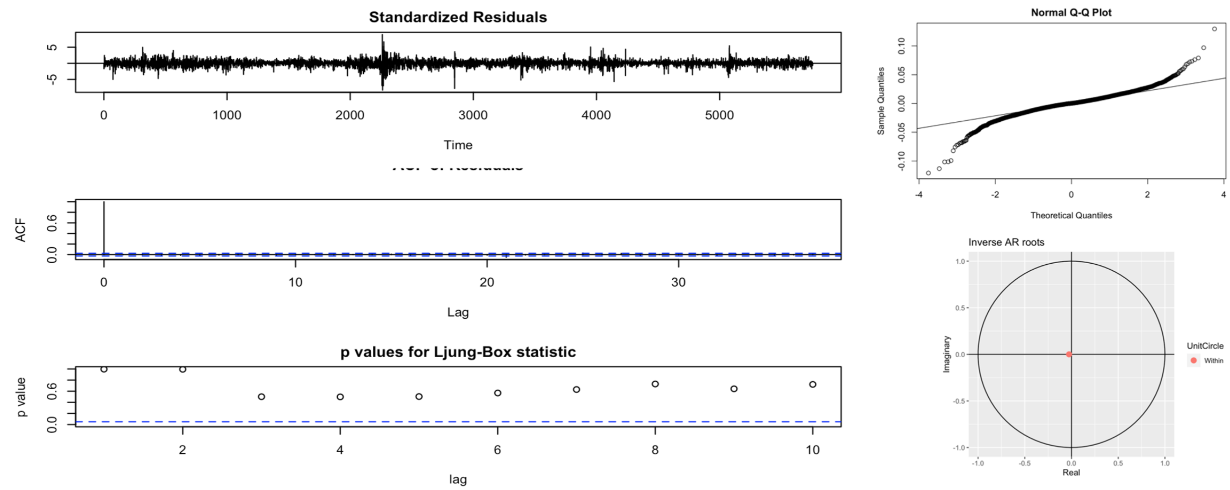

Figure 4: AR(1) graphs for diagnostics

The diagnostics show that the model's residuals behave like stationary white noise, even if it is not a Gaussian white noise (normality does not hold), so we can validate our model and use it for the estimation of the variance model.

3.2 A Model for the Variance

Once a model for the mean has been estimated, we can build a model for the variance, where our interest resides. For doing so, we extract the residuals from the previous AR(1) model and use them to identify the model for the variance by using the squared residuals. The first step is to plot the ACF and the PACF of the squared residuals of the mean model so that we can identify the possible presence of volatility through the lags, and also to use the Ljung-Box tests to know whether there might be dependence between lags.

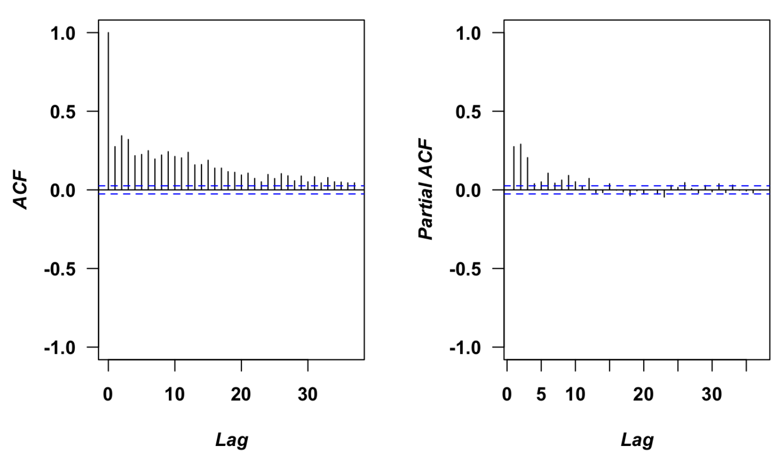

Figure 5: ACF & PACF Squared Residuals of the AR(1) model

Test type	Nº of lags	Statistic	p-value	Conclusion
Ljung-Box	1	434.8	$<$ 2.2e-16	Reject H_0
Ljung-Box	5	2271.5	$<$ 2.2e-16	Reject H_0
Ljung-Box	15	4802.3	$<$ 2.2e-16	Reject H_0
Ljung-Box	20	5223.5	$<$ 2.2e-16	Reject H_0

Table 7: Ljung-Box Tests for AR(1) model

The results of Figure 5 and Table 7 indicate that volatility is present, so we need, indeed, to model variance through a model. For this purpose, we will model variance through three different versions of the GARCH model: the standard version, the exponential version, and the GJR version. We chose these models because they are the most used in the literature, even though other models could also be fitted to compare and analyze their implications. We restrict ourselves to the use of just these three options not to expand the analysis more than needed for our aims.

For the estimation of all models, we have used the AR(1) model previously estimated and we have also used the t-student distribution, as we argued how the normality assumption does not hold, and given the kurtosis and the fat tails, it should be more adequate to use the t-student distribution.

Additionally, we just consider the (1,1) version of the models (with just two parameters apart from the constant term), as we have tried other configurations which yield an equivalent or worse fit and we opt to restrain our modeling to models which have the fewest parameters so that their interpretation and the results the models yield are easier and more comprehensible. We present the estimation results for the different models in Table 8.

Model	ω	α_1	β_1	γ_1	**AIC**	**BIC**
GARCH(1,1)	0.000003	0.085970	0.902638	-	-5.9016	-5.8946
p-values		0.219676	0.000000	-	-	-
EGARCH(1,1)	-0.281791	-0.111960	0.967869	0.166606	-5.9271	-5.9190
p-values	0.000000	0.000000	0.000000	0.000000	-	-
GJR-GARCH(1,1)	0.000005	0.019386	0.884412	0.141981	-5.9199	-5.9118
p-values	0.000013	0.000003	0.000000	0.000000	-	-

Table 8: Estimation of mean models for the Nikkei225

In this case, all parameters are significant for all models except for the intercept in the GARCH(1,1). Regarding the information measures, the asymmetric models have a better fit than the standard GARCH(1,1) and the model with the best fit would be the EGARCH(1,1). However, despite the negative parameters do not matter for an EGARCH (because there is no positivity restriction on the value of the parameters), the interpretation becomes more complex and depends on the parametrization. Hence, because there is not a very significant difference in the goodness of fit between both models, we choose to stay with the third, the GJR-GARCH(1,1) model.

For these models, the α_1 parameter indicates the effect of market shocks on volatility, while β_1 indicates the persistence of volatility and ω is the constant parameter. In the asymmetric models, the γ_1 parameter is related to the leverage effect that creates asymmetry (negative news affects volatility more than positive ones), which should be negative for the EGARCH but positive for the GJR-GARCH. The election of the latter model comes from the fact that γ_1 can be directly interpreted as the extra effect on volatility that comes from negative shocks (negative residuals). This is easier to understand for the reader and allows to work more comfortably when explaining how shocks impact volatility.

Considering the parameters, we see that the gamma parameters in both models that introduce asymmetry (the EGARCH and the GJR-GARCH) are significant, so we might expect the presence of asymmetric effects in volatility. Moreover, the persistence of volatility is high compared to the effect of market shocks on volatility in all the models, so we can conclude that volatility in the Nikkei225 is highly persistent.

We discuss the different results obtained from the models in order to discuss them and select the one which has the best properties considering diverse aspects.

4 Comparing the Volatility Models

After having a model for the mean and different models for the variance to study volatility, we can analyze the results that the selected models yield when analyzing the estimated volatility, the news impact, the prediction capability and hence conclude which model would be the most adequate.

4.1 Estimated Volatility

The selected models allow obtaining the estimation of annualized volatility for the whole time series observation window. In Figure 7 we have plotted the volatility estimated with each of the models, where the GARCH(1,1) model is blue, the EGARCH(1,1) model is green and the GJR-GARCH(1,1) model is red. The observed volatility (computed with the absolute values of residuals from the mean model) are also plotted so that one can see the contrast between different estimations.

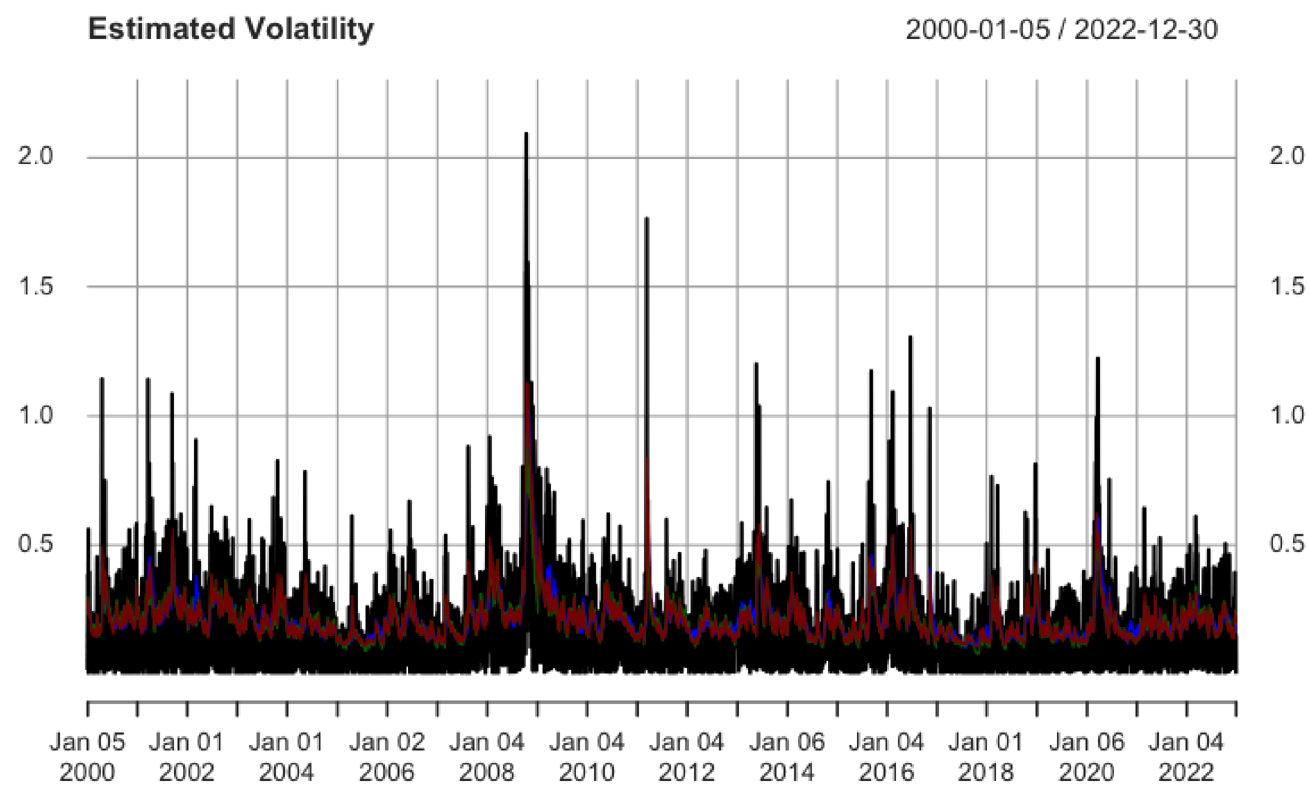

Figure 6: Volatility estimation for the different volatility models

As we can see here, the estimated volatility of the different models is very similar, but we can spot that the model which gives a more similar volatility to the observed one would be the GJR-GARCH(1,1), as it fits better the volatility peaks. Hence, the estimation of volatility would favor this model.

4.2 News Impact Curve

Now, we can also analyze which is the impact of news or innovations (new information that arrives in the market in form of shocks) on the index volatility, as the estimation results indicate that asymmetry might be present regarding negative shocks. Figure 7 shows the news impact curve, which is a visual representation of the volatility impact of different shocks. In this case, the colors that represent each model are the same as before, so we refer to Figure 6 to remember the color allocation of each model.

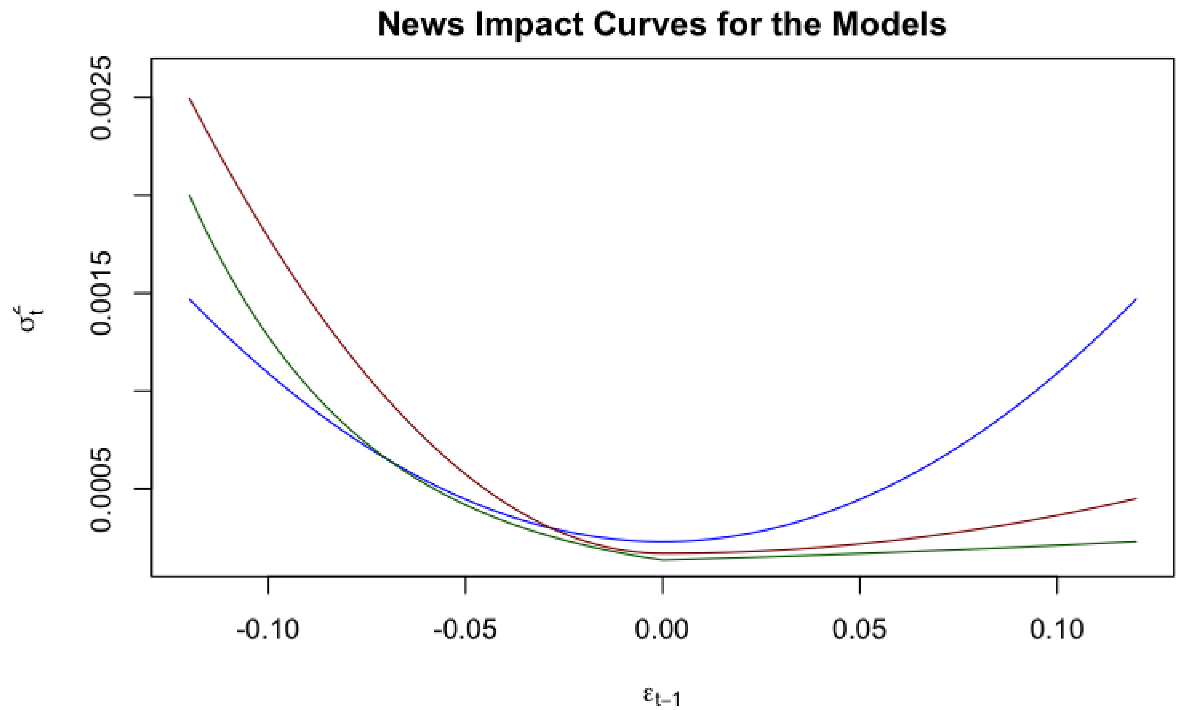

Figure 7: News impact curves for the different models

The illustrated results show the asymmetry that is introduced in the EGARCH(1,1) and the GJR-GARCH(1,1), while the GARCH(1,1) model treats the shocks symmetrically. In this graph, we can see how large negative shocks have a higher impact on volatility when considering the GJR-GARCH (the red line), while for low-level negative shocks, other models have a higher impact. When looking at the EGARCH(1,1) (the green line), we can see that the impact on volatility is less than with any other model for medium and low-level negative shocks (take -0.075 as a reference), but it is higher than the standard GARCH(1,1) for large negative shocks.

When looking at the right side, illustrating the effect of positive shocks on volatility, the asymmetric models allocate a lower impact than the symmetric model (the blue line). However, the impact of positive news is greater in the GJR-GARCH than in the EGARCH. All of these facts jointly with the significance of the asymmetry parameter γ_1 support the idea of choosing the GJR-GARCH(1,1) as the volatility model for the Nikkei225.

4.3 Volatility Prediction

The estimated models also allow us to compare the predictive capability and compare them. Therefore, we will first provide static and dynamic predictions for a 90 days horizon and then we will compare the predictions of the different models and interpret the results obtained.

The forecasting is done in two different ways: a static way, in which the forecasts are done using only the previously estimated model, and a dynamic way, in which we re-estimate the model whenever a new forecast is done (it is included, so the window expands). Another option for dynamic forecasting would be o consider a moving window, but we prefer to expand the window not to lose past observations.

In Figures 8 to 10 we present the static forecasts in the left panel and the dynamic forecasts in the right panel for each model.

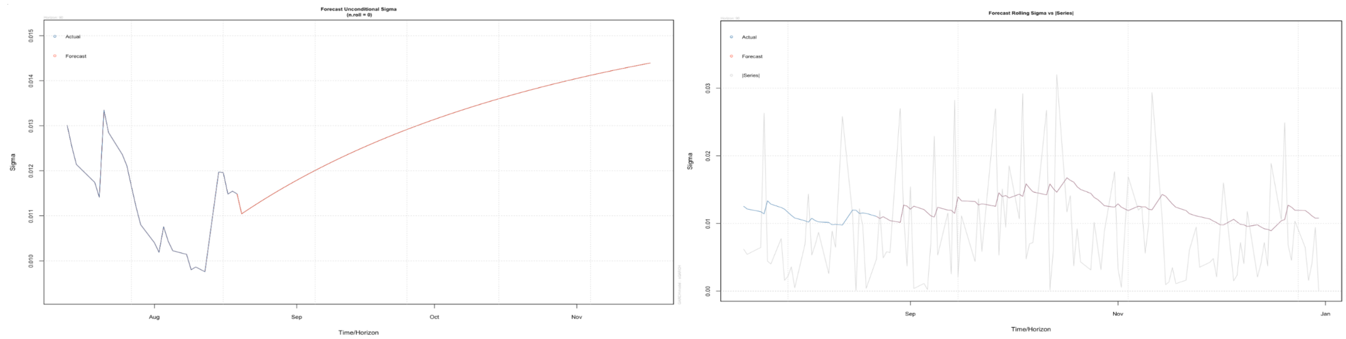

Figure 8: Static and dynamic (respectively) prediction for volatility in the GARCH(1,1) model

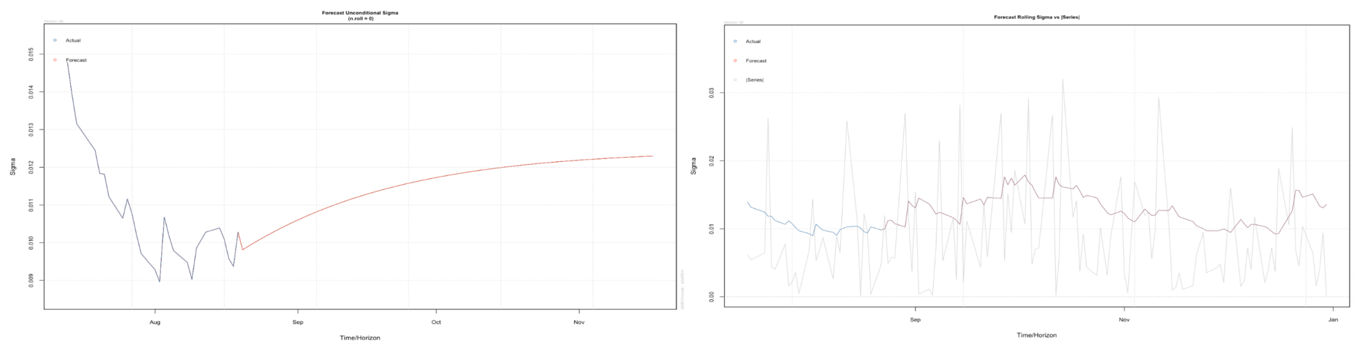

Figure 9: Static and dynamic (respectively) prediction for volatility in the EGARCH(1,1) model

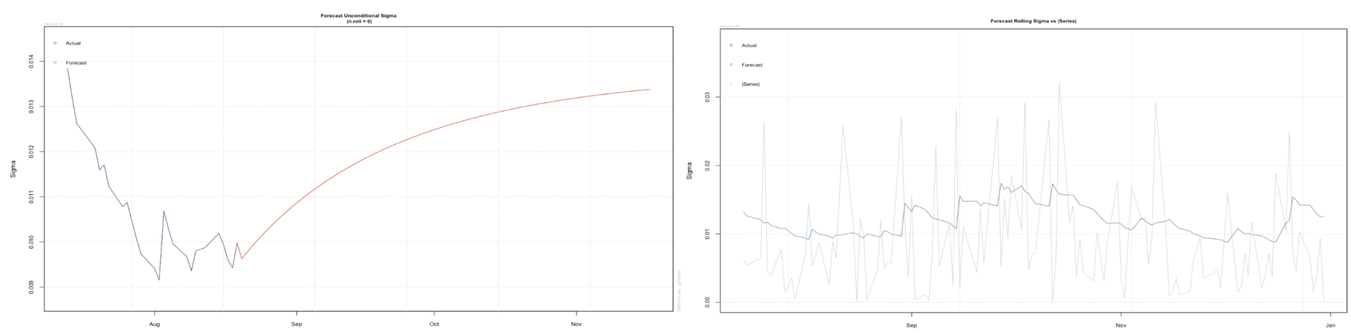

Figure 10: Static and dynamic (respectively) prediction for volatility in the GJR-GARCH(1,1) model

As we can see here, the static predictions for the Nikkei225 log-returns seem similar, but there is a difference regarding the levels to which they tend. The GARCH(1,1) tends to a higher level than the other models, while the EGARCH(1,1) and the GJR-GARCH(1,1) tend to a more moderate level, which will be the long-term average level. When it comes to dynamic forecasts, we can see that they are pretty similar, with little differences. In all instances, volatility is predicted to increase during the first half of the forecasting horizon, while it slightly decreases at the end.

We can present some error measures for the different forecasting methods and models so that we can compare the predictive capability numerically. We present these results in Tables 9 and 10.

Measure	GARCH(1,1)	EGARCH(1,1)	GJR-GARCH(1,1)
MSE	0.00014	0.00014	0.00014
MAE	0.00900	0.00897	0.00898

Table 9: Error measures for static forecasts by model

Measure	GARCH(1,1)	EGARCH(1,1)	GJR-GARCH(1,1)
MSE	0.00014	0.00014	0.00014
MAE	0.00899	0.00896	0.00897

Table 10: Error measures for dynamic forecasts by model

The measures support the claims we made visually: the error measures are merely the same between different GARCH models. Consequently, we can conclude that the forecasts are very similar among the models, so we should base our model selection for the variance based on the combination of the different aspects seen because forecasting capability seems not to differ significantly.

4.4 Model Selection

All in all, we would like to keep the GJR-GARCH(1,1) because is a model which has better properties than the other two models considered in some aspects such as the asymmetric impact on volatility or the estimations obtained for the parameters and the volatility, while it is very similar in other aspects such as forecasting capability.

Hence, we will work especially with the GJR-GARCH model to compare it to other methods of estimating volatility and to compute risk measures, which are shown in the next section.

5 Classical Methods and Risk Measures

As previously mentioned, we now use the selected model in order to do a comparison among traditional or alternative methods to GARCH models, such as the historical volatility or the exponentially weighted moving average, and we use the model and the others proposed to compute the Value-at-Risk in an index portfolio context (a replicating portfolio consisting on the stocks included in the Nikkei225 index).

5.1 Historical Volatility & EWMA

Two alternatives to the GARCH modeling framework used in the previous development are the historical volatility and the exponentially weighted moving average (EMWA), which are methods based on averages. In this section, we compare the GJR-GARCH(1,1) results to the ones we compute for both alternative methods so that we can get some interesting insights about the selected model. We do not consider the other models for this section to ease the comparison between estimated volatilities.

First, we compare the historical volatility results with our volatility model. The historical volatility estimator is based on a moving average of the squared residuals of the mean model (our AR(1) model), so we compare the estimation of the annualized volatility of the GJR-GARCH(1,1) model to the obtained with the historical volatility estimator using a different number of observations. For this purpose, we consider the historical volatility with a window of a week, a month, a quarter, and half a year. The comparisons are shown in Figure 11.

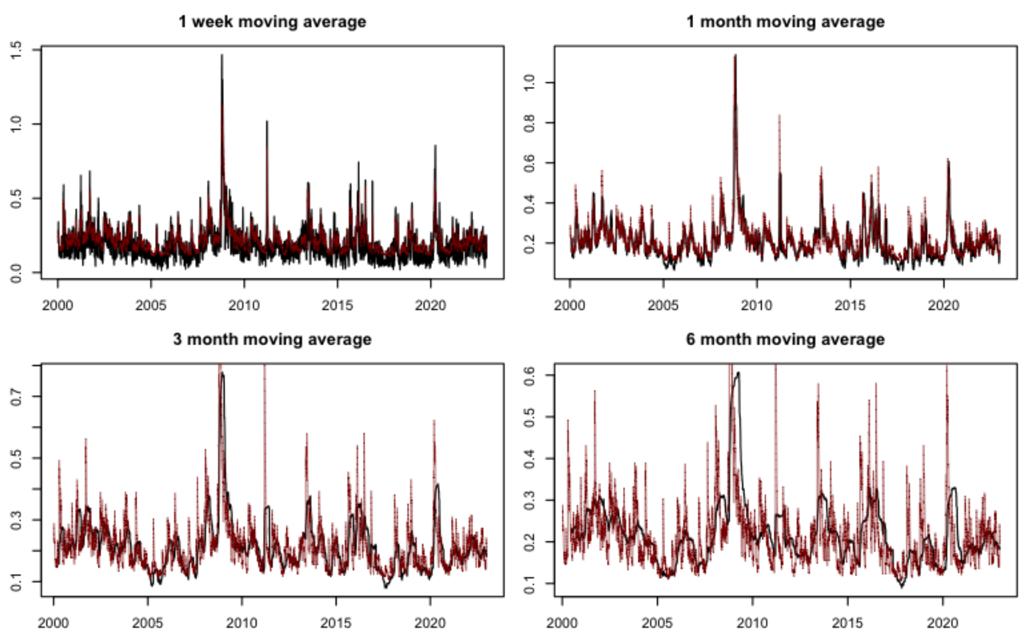

Figure 11: Historical volatility compared to estimated volatility of a GJR-GARCH(1,1)

Here, one can see that volatility is rougher than with our volatility model, where the "valleys" are more pronounced than the peaks compared to the volatility estimated with our model. The one-month historical volatility, in contrast, is quite similar to the results obtained with our model, even though our model has higher volatility peaks than the historical volatility. Once we pass from the month, it seems like the estimated volatilities start to clearly differ, having lower values for different moments and also being smoother (because of the averaging effect).

Now, we instead look at the EWMA estimations, which apply exponential weighting through a parameter λ, which controls the intensity of the reaction to market shocks and the persistence of volatility. The higher the λ, the more persistent volatility is. Hence, we consider four different values of λ: 0.95, 0.75, 0.5, and 0.25. We present a comparison with the results of our volatility model in Figure 12.

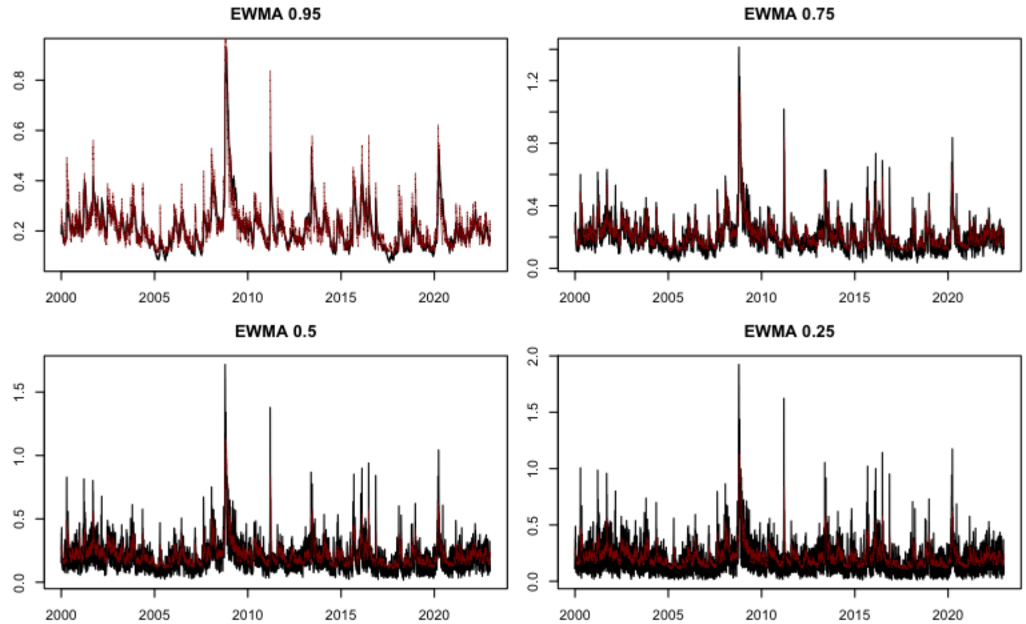

Figure 12: EWMA volatility compared to estimated volatility of a GJR-GARCH(1,1)

We can see that, for $\lambda = 0.95$, both estimated volatilities seem to be very similar, with little differences in the peaks, but as we decrease the value of λ, we can see that the estimation of volatility becomes rougher and rougher, as the impact of new information or shocks in the market are taken into account with higher weight and creates more jumps. For these cases, the volatility estimated with our model does not seem to be similar, as the EWMA estimation has higher peaks and lower "valleys" than the GJR-GARCH(1,1).

All in all, we can see that there are some instances where these traditional volatility estimation methods reproduce volatility more or less like our selected GARCH model, but they are very sensitive to the parameters chosen, as different elections lead to very different results.

5.2 VaR Computation

A very interesting application of volatility modeling is to predict risk measures that help manage risk in an investment portfolio taking into account the volatility of the assets they are holding. In this context, we will consider a replicating portfolio of the Nikkei225: a portfolio that consists of the same stocks the Nikkei225 holds and applies the same weighting to each stock. Even though there are more than a couple of risk measures, for this exercise we just consider the Value-at-Risk (VaR) measure, which would allow an investor to know what is the maximum log-return loss with a given probability level.

To compute the VaR predictions, we use 0.95 and 0.99 probability levels, as these are the most common, and we always assume a t-student distribution, so that the fit of the estimation is according to the empirical evidence of the log-returns of the Nikkei225. Even though we have defended the use of the GJR-GARCH(1,1) model, we will compute the VaR for all the models proposed in order to compare the different predictions for the VaR at different levels and the implications of using our selected model among the proposed.

We predict the VaR at 95% and 99% probability levels for the 2023 year (the following year) using a rolling window method, so that every 21 observations (the trading days of a single month) we re-estimate the volatility model to forecast the following observations. The results for the different models and probability levels are shown in Figures 12 & 13, and we maintain the colors allocated to each model in previous graphs.

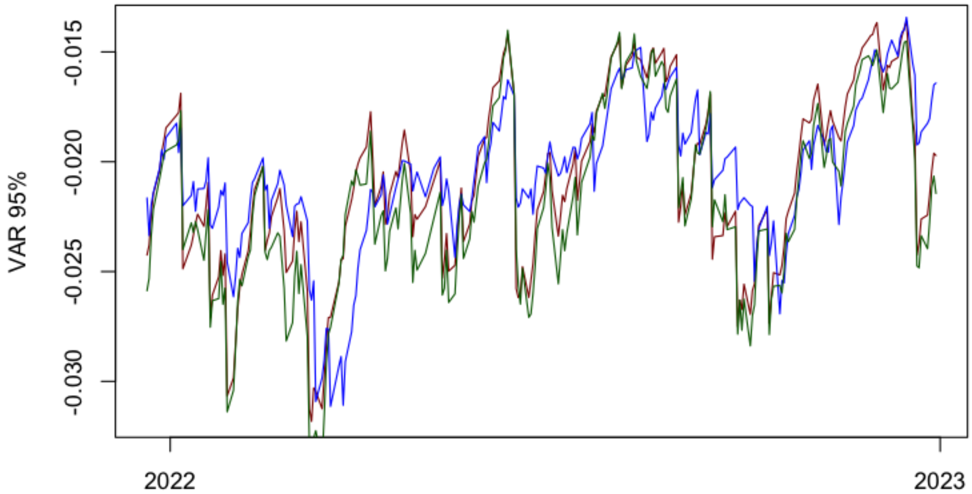

Figure 13: VaR at 95% for the proposed models for the NIkkei225

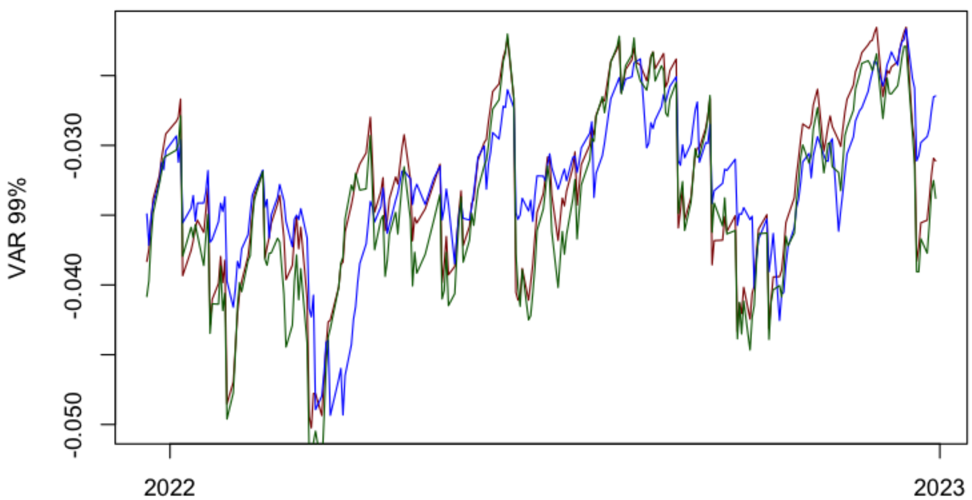

Figure 14: VaR at 99% for the proposed models for the NIkkei225

From the figures, one can observe that the VaR at 95% fluctuates between -1.5% and -3% approximately, while the VaR at 99% fluctuates between -2.5% and -5% approximately. There are also observable differences between models. The GARCH(1,1) model (the blue line) seems to have less width than the other models, so it does a more moderate VaR prediction (high values are lower and low values are higher than with the other models). In contrast, the EGARCH(1,1) model gives more extreme predictions for the VaR, yielding generally more negative values than the other models. The GJR-GARCH, our selected models, seems to be in the middle between both models commented.

The graphs show that we predict a high VaR value at the start of the year, meaning that, with 95% probability, losses would be no greater than -3.5% more or less for a 95% VaR and -5.5% for a 99% VaR. This would be bad for investors, as higher extreme losses would mean that other negative losses would be more common or will occur with a higher probability at the start of the year (as the extremes are more negative).

From that period onwards, however, it seems that the VaR will return to more stable levels, fluctuating between -1.5% and -2.5% for a 95% VaR and -2.5% and -4.5% for a 99% VaR, meaning that the negative losses would be less common or will occur with a lower probability (as the extremes are less negative).

6 Multivariant Modeling with FTSE100

The last goal of our project is to establish relations between the volatility of the Nikkei225 and the one from other indices around the world, as this would give some insights into (from the volatility point of view) the relations between the Japanese and other stock markets and how are they affected (how do shocks affect one or another).

In order to illustrate these relations, we consider a multivariate volatility model for explaining the volatility and the relations among different indices. We consider the Nikkei225 and the FTSE100, which is the most important stock index on the London Stock Exchange and represents both the English stock market and a part of the whole Western stock market. Other famous indices might have been considered, such as the S&P500 or the NASDAQ100, but we prefer to use non-American indices to compare ours with a "less common" index.

We will first discuss the multivariate model we use and then we also discuss the estimation results for both the variance and the correlations.

6.1 Estimation Results

The model that we use for modeling the volatility of both indices is the DCC-GARCH model, as it allows us to model both the variances and the correlations between indices. To use this model, we first have to specify a volatility model for the Nikkei225 and another for the FTSE100. For the Nikkei225, we use the GJR-GARCH(1,1) we discussed earlier, as it works better to model asymmetric impact and has a better fit, but for the FTSE100 log-returns we consider again the three models proposed before.

We assume that the AR(1) is the model for the mean in this index and that the distribution will be a t-student, assuming that the stylized facts of log-returns hold. The estimation results for the three models under this assumptions are shown in Table 11.

Model	ω	α_1	β_1	γ_1	**AIC**	**BIC**
GARCH(1,1)	0.000002	0.114741	0.873877	-	-6.4876	-6.4808
p-values	0.556708	0.024075	0.000000	-	-	-
EGARCH(1,1)	-0.162315	-0.138017	0.982920	0.122505	-6.5266	-6.5186
p-values	0.000000	0.000000	0.000000	0.000000	-	-
GJR-GARCH(1,1)	0.000002	0.000001	0.890704	0.183501	-6.5213	-6.5133
p-values	0.052923	0.999935	0.000000	0.000000	-	-

Table 11: Estimation of mean models for the FTSE100

Results demonstrate how the EGARCH(1,1) has significant parameters and has the best fit, in contrast to the other models. Because we do not need to interpret anything, but just use it for estimation of the DCC-GARCH, we keep the EGARCH(1,1) model as a model for the volatility of the FTSE100 log-returns.

The estimation results for the parameters and other measures are exposed in Table 12.

Model	$\alpha_{1,DCC}$	$\alpha_{2,DCC}$	$\beta_{1,DCC}$	$\beta_{2,DCC}$	**AIC**	**BIC**
DCC-GARCH(1,1)	0.009694	-	0.895505	-	-12.532	-12.510
p-values	0.321384	-	0.000000	-	-	-
DCC-GARCH(2,1)	0.009694	0.000000	0.895460	-	-12.531	-12.509
p-values	0.437527	0.999974	0.000000	-	-	-
DCC-GARCH(1,2)	0.009694	-	0.155826	0.639752	-12.532	-12.509
p-values	0.119960	-	0.241180	0.000148	-	-

Table 12: Estimation results for the DCC-GARCH

We can appreciate how all the $\alpha_D CC$ parameters seem to be insignificant at common significance levels so that the impact of shocks on the markets seems to not affect volatility significantly. However, $\beta_{1,DCC}$ is significant if another $\beta_{2,DCC}$ is not included (this latter becomes significant) so one can say that persistence of volatility is high. Given the information criteria and the significance of parameters, we choose to keep the DCC-GARCH(1,1) model from now on.

6.2 Correlation and News Impact

We now can discuss results about the correlation between both indices and the news impact on the correlation. The GARCH-DCC model allows to estimate the correlations between the volatility of both indices, so we can show and interpret the correlation series for the indices, shown in Figure 15.

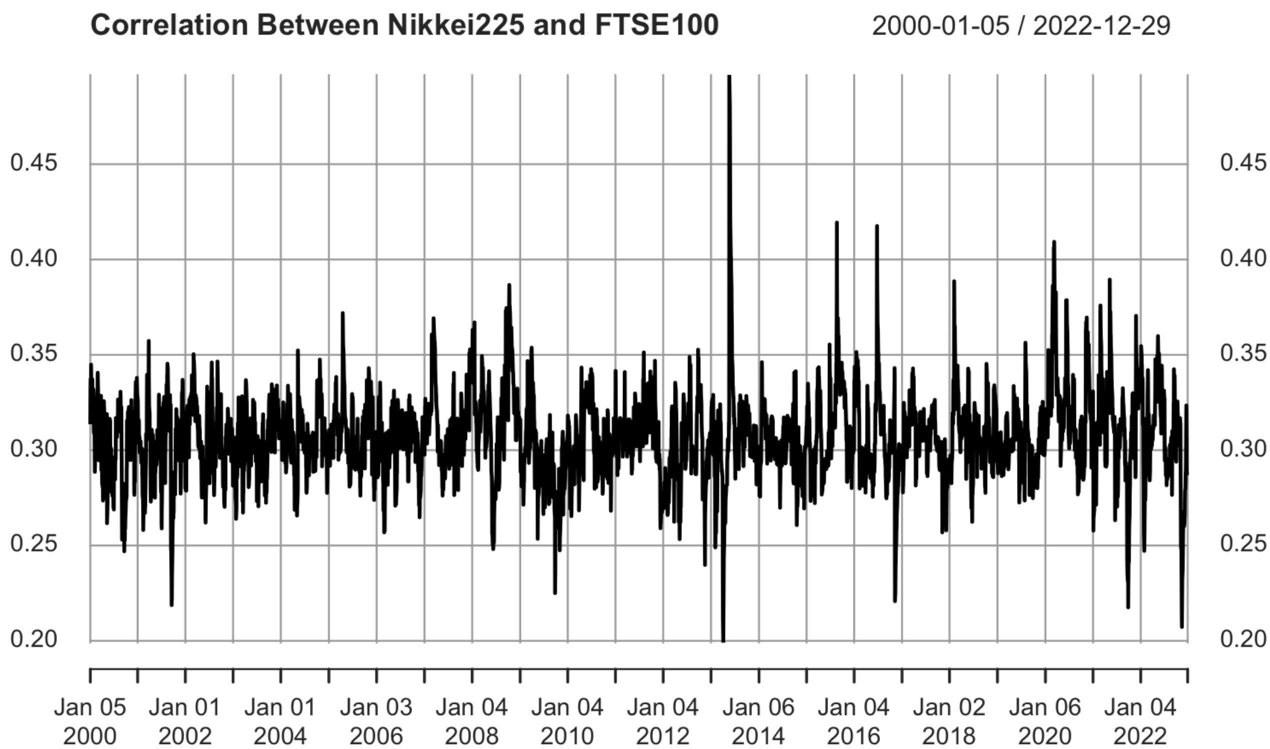

Figure 15: Correlation time series between the NIkkei225 and the FTSE100

The series makes clear that the correlation between the Nikkei225 and the FTSE100 changes depending on the moment. We can identify that from 2000 to 2013, the correlation between both indices fluctuates in an interval between 0.25 and 0.35, despite different events (presented previously) that caused a higher or a lower correlation compared to the values of this period (for example, 9/11 attacks or 2008 crisis). But after the Great East Japan Earthquakes of 2013, the variability of the correlations increases and there seem to be more peaks (where the correlation is higher than normal) and "valleys" (where the correlation decreases more than normal). This is relevant from the point of view of an investor, as portfolios composed of these indices would be affected by the change in correlations.

Now, when considering the news impact, we want to study how shocks in both indices' markets affect the correlation. The results for this both indices are quite interesting and are plotted in Figure 16. Before interpreting the results, we highlight that $shock[z_1]$ refers to shocks related to the Nikkei225, while $shock[z_2]$ refers to shocks related to the FTSE100.

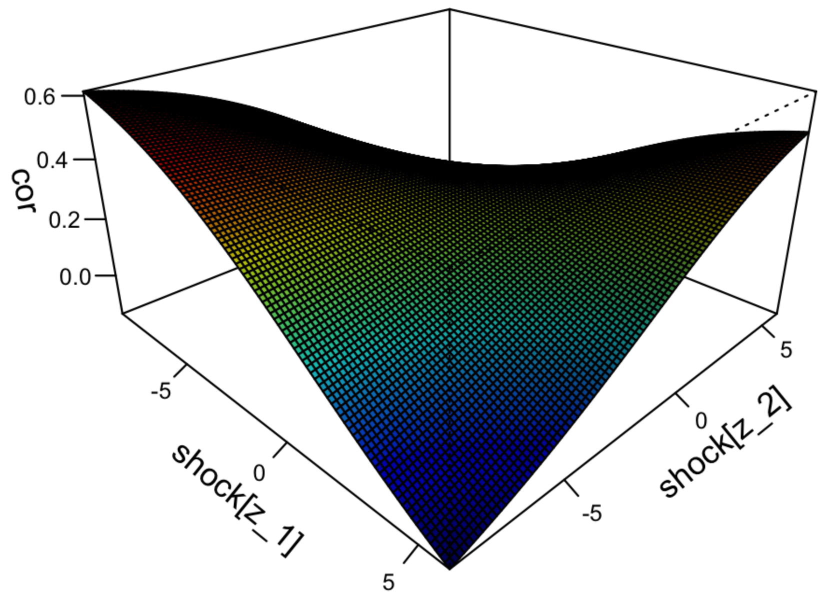

Figure 16: News impact surface for the DCC-GARCH model

The surface plot shows that when there are negative shocks in one market but positive on the other, the DCC-GARCH estimates that correlation would decrease. However, when both markets have shocks with the same signs, the correlation increases. However, we can look that the correlation is not as high when both shocks are positive as when both shocks are negative. This can be interpreted as an indication of the present asymmetry on the volatility impact of both indices: when news is bad, the correlation is higher than when is positive, which coincides with the idea that volatility is higher when news is bad.

7 Concluding Remarks

We finish this project by summarizing the most important conclusions from the analyses of the various topics we have covered in the previous section. To ease the reading, we present the conclusions in a brief and concise way in the following list:

- When looking at the Nikkei225 index prices, we can see that the series is, as expected, non-stationary, while the log-returns series is stationary. This latter series has the stylized properties of returns series, such as leptokurtosis or asymmetry.
- By considering the log-returns, we fitted various models for the mean and for the variance (to model volatility). We selected an AR(1) model due to its goodness of fit and intuitive interpretation, and through the squared residuals of the model, we identified a standard GARCH, an EGARCH, and a GJR-GARCH.
- The estimation results show that volatility is very persistent, while the effect of news is not too large. Moreover, the asymmetric models considered have significant asymmetry-related parameters, which allows us to assume there is an asymmetry in the impact of negative and positive shocks on volatility. Because of the goodness of fit and the easier interpretation of the asymmetry-related parameter, we favor the use of the GJR-GARCH(1,1) model.
- Looking at the volatility estimation of the different methods, we can see how the GJR-GARCH fits better the observed volatility peaks caused by some shocks. Moreover, when looking at news impact, we can see how the asymmetric curve has a higher impact than the other models for negative shocks but has a higher impact than the EGARCH for positive shocks.
- When considering the forecasts made by the different models, we can see that the results are very similar and this is also reflected in the error measures, which are mostly equivalent. Hence, given all the previous facts, we prefer to work with a GJR-GARCH.
- Comparing the GJR-GARCH volatility estimates with the estimates of the historical volatility estimators and the exponentially weighted moving average, we can see that there are some instances in which volatility is similar, but the similarity between the model and the traditional methods varies highly with the election of the window length and the λ parameter.
- The computation of VaR forecasts for the 2023 year with the different models shows that current values for a 95% VaR fluctuate between -1.5% and -3% approximately, while for a 99% VaR fluctuate between -2.5% and -5%. These forecasts show that at the start of the year, negative returns would be more likely than the returns from half to the end of the year, which would mean that investors should protect with capital possible losses (which would be bigger) in this initial period.
- The estimation of different DCC-GARCH models shows that the best parametrization is a DCC-GARCH(1,1) model when we consider a GRJ-GARCH(1,1) for the Nikkei225 and an EGARCH(1,1) for the FTSE100.
- The analysis of the correlation series indicates that the correlation fluctuates between 0.25 and 0.35 from 2000 to 2013, but after the Great East Japan Earthquake shock, the correlations start being more volatile and reached higher and lower values than before.
- Looking at the news impact surface, one can see how the correlation is negative when the shocks have negative signs and positive when they have the same signs, but we detect an asymmetric effect depending on the positivity or negativity of the shocks. If both shocks are negative, the impact on correlation is higher than when both are positive.

Through all the analyses seen, we expect the reader to have obtained useful insights into the volatility of the Japanese stock market, the modeling of volatility using a GARCH framework, and the different applications that these models have in forecasting and risk management. Moreover, some conclusions can be made when considering the FTSE100 as a complementary series for the estimation and analysis of a multivariate GARCH model. All of the results obtained are interesting for passive investors seeking to invest in Asian markets such as the Japanese and to other kinds of investors that seek a theoretical but practical analysis that allows working with a volatility model to help manage risk and exposure through risk measurement or forecasts of future volatility, also being possible taking into account other series.

Appendix

Iker Caballero Bragagnini

2023-04-05

Package Installing

```
library(tseries)
```

```
## Registered S3 method overwritten by 'quantmod':
##   method            from
##   as.zoo.data.frame zoo
```

```
library(fBasics)
library(car)
```

```
## Loading required package: carData
```

```
##
## Attaching package: 'car'
```

```
## The following object is masked from 'package:fBasics':
##
##     densityPlot
```

```
library(tseries)
library(urca)
library(forecast)
library(rugarch)
```

```
## Loading required package: parallel
```

```
##
## Attaching package: 'rugarch'
```

```
## The following objects are masked from 'package:fBasics':
##
##     qgh, qnig
```

```
## The following object is masked from 'package:stats':
##
##     sigma
```

```
library(quantmod)
```

```
## Loading required package: xts

## Loading required package: zoo

##
## Attaching package: 'zoo'

## The following objects are masked from 'package:base':
##
##     as.Date, as.Date.numeric

## Loading required package: TTR

##
## Attaching package: 'TTR'

## The following object is masked from 'package:fBasics':
##
##     volatility
```

```
library(ggplot2)
library(fTrading)
```

```
## Loading required package: timeDate

## Loading required package: timeSeries

##
## Attaching package: 'timeSeries'

## The following object is masked from 'package:zoo':
##
##     time<-

##
## Attaching package: 'fTrading'

## The following object is masked from 'package:TTR':
##
##     SMA
```

```
library(rmgarch)
```

```
##
## Attaching package: 'rmgarch'

## The following objects are masked from 'package:xts':
##
##     first, last
```

Exploratory Analysis

Data Import

```
getSymbols("^N225",from="2000-01-01", to="2022-12-30")
```

```
## Warning: ^N225 contains missing values. Some functions will not work if objects
## contain missing values in the middle of the series. Consider using na.omit(),
## na.approx(), na.fill(), etc to remove or replace them.
```

```
dim(N225)
head(N225)
tail(N225)
nik=N225[,6]
```

Graphs

```
plot(nik,main="Nikkei 225")
```

```
nik<-na.locf(nik, fromLast = TRUE)
length(nik)
plot(nik,main="Nikkei 225")
```

```
rendnik=diff(log(nik))
rendnik<-rendnik[-1]

par(mfrow=c(2,1))
plot(nik,main="Nikkei 225")
plot(rendnik, main="Returns Nikkei 225")
```

Stationarity

ACF & PACF

```
par(mfrow=c(1,2))
acf(nik,ylim=c(-1,1),main="Nikkei 225")
pacf(nik,ylim=c(-1,1),main="Nikkei 225")
```

```
par(mfrow=c(1,2))
acf(rendnik,ylim=c(-1,1),main="Returns Nikkei 225")
pacf(rendnik,ylim=c(-1,1),main="Returns Nikkei 225")
```

Independence of lags

```r
Box.test(nik, lag = 1, type = c("Ljung-Box"))
Box.test(nik, lag = 5, type = c("Ljung-Box"))
Box.test(nik, lag = 10, type = c("Ljung-Box"))
Box.test(nik, lag = 15, type = c("Ljung-Box"))
Box.test(nik, lag = 20, type = c("Ljung-Box"))

Box.test(rendnik, lag = 1, type = c("Ljung-Box"))
Box.test(rendnik, lag = 5, type = c("Ljung-Box"))
Box.test(rendnik, lag = 10, type = c("Ljung-Box"))
Box.test(rendnik, lag = 15, type = c("Ljung-Box"))
Box.test(rendnik, lag = 20, type = c("Ljung-Box"))
```

Unit root tests for prices

```r
# Tests
nik.df.b<-ur.df(nik, lags=20, selectlags = c("BIC"))
nik.df.db<-ur.df(nik, type = c("drift"), lags=20, selectlags = c("BIC"))
nik.df.tb<-ur.df(nik, type = c("trend"), lags=20, selectlags = c("BIC"))
nik.pp.c<-ur.pp(nik, type = c("Z-tau"), model = c("constant"), use.lag = 20)
nik.pp.t<-ur.pp(nik, type = c("Z-tau"), model = c("trend"), use.lag = 20)
nik.kpss.c<-ur.kpss(nik, type = c("mu"), use.lag = 20)
nik.kpss.t<-ur.kpss(nik, type = c("tau"), use.lag = 20)

# Summary
summary(nik.df.b)
summary(nik.df.db)
summary(nik.df.tb)
summary(nik.pp.c)
summary(nik.pp.t)
summary(nik.kpss.c)
summary(nik.kpss.t)

# Plot
plot(nik.df.b)
```

```r
plot(nik.df.db)
```

```r
plot(nik.df.tb)
```

```r
plot(nik.pp.c)
```

```r
plot(nik.pp.t)
```

```r
plot(nik.kpss.t)
```

Unit root tests for returns

```r
# Tests
rendnik.df.b<-ur.df(rendnik, lags=20, selectlags = c("BIC"))
rendnik.df.db<-ur.df(rendnik, type = c("drift"), lags=20, selectlags = c("BIC"))
rendnik.df.tb<-ur.df(rendnik, type = c("trend"), lags=20, selectlags = c("BIC"))
```

```r
rendnik.pp.c<-ur.pp(rendnik, type = c("Z-tau"), model = c("constant"), use.lag = 20)
rendnik.pp.t<-ur.pp(rendnik, type = c("Z-tau"), model = c("trend"), use.lag = 20)
rendnik.kpss.c<-ur.kpss(rendnik, type = c("mu"), use.lag = 20)
rendnik.kpss.t<-ur.kpss(rendnik, type = c("tau"), use.lag = 20)

# Summary
summary(rendnik.df.b)
summary(rendnik.df.db)
summary(rendnik.df.tb)
summary(rendnik.pp.c)
summary(rendnik.pp.t)
summary(rendnik.kpss.c)
summary(rendnik.kpss.t)
```

Basic Statistics

```r
normalTest(rendnik,method="jb")
normalTest(nik,method="jb")
basicStats(rendnik)
round(basicStats(nik),3)
```

```r
hist(rendnik,breaks=20,freq=F, main = "Nikkei225 Log-returns")
curve(dnorm(x, mean=mean(rendnik), sd=sd(rendnik)), col=2, add=T)
```

ARMA Model Identification, Estimation & Diagnosis

Model Identification

```r
par(mfrow=c(1,2),font=2,font.lab=4,font.axis=2,las=1)
acf(rendnik,ylim=c(-1,1),main="Returns Nikkei 225")
pacf(rendnik,ylim=c(-1,1),main="Returns Nikkei 225")
```

Model Estimation

```r
# White noise
model_w = Arima(rendnik, order = c(0,0,0),include.mean = TRUE)
model_w

# AR(1)
model_ar1 = Arima(rendnik, order = c(1,0,0),include.mean = TRUE)
model_ar1

# MA(1)
model_ma1 = Arima(rendnik, order = c(0,0,1),include.mean = TRUE)
model_ma1
```

Diagnosis

```
autoplot(model_ar1)
```

```
tsdiag(model_ar1)
```

```
par(mfrow=c(1,2),font=2,font.lab=4,font.axis=2,las=1)
acf(model_ar1$residuals,ylim=c(-1,1),main="residuosar2")
pacf(model_ar1$residuals,ylim=c(-1,1),main="residuosar2")
```

```
qqnorm(model_ar1$residuals)
qqline(model_ar1$residuals, datax = FALSE)
normalTest(model_ar1$residuals,method="jb")
```

GARCH Identification, Estimation & Diagnosis

Model Identification

```
residuos=model_ar1$residuals
residuos2=residuos^2

par(mfrow=c(1,2),font=2,font.lab=4,font.axis=2,las=1)
acf(residuos2,ylim=c(-1,1))
pacf(residuos2,ylim=c(-1,1))
```

```
Box.test(residuos2,lag=1,type='Ljung')
Box.test(residuos2,lag=5,type='Ljung')
Box.test(residuos2,lag=15,type='Ljung')
Box.test(residuos2,lag=20,type='Ljung')
```

Model Estimation GARCH

```
# Model_ar1
spec_ar1=ugarchspec(variance.model = list(model = "sGARCH",
garchOrder = c(1,1)), mean.model=list(armaOrder=c(1,0)),
distribution.model = "std")
m_ar1 = ugarchfit(spec=spec_ar1,data=rendnik)
m_ar1

v_GARCH_ar1 = sigma(m_ar1)
v_anualizada_GARCH_ar1=(250)^0.5*v_GARCH_ar1
```

Model Estimation EGARCH

```
spec2=ugarchspec(variance.model=list(model="eGARCH", garchOrder = c(1,1)),mean.model=list(armaOrder=c(1
m2_ar1=ugarchfit(spec=spec2,data=rendnik)
m2_ar1

v_GARCH_e = sigma(m2_ar1)
v_anualizada_GARCH_e=(250)^0.5*v_GARCH_e
```

Model Estimation GJR-GARCH

```
spec3=ugarchspec(variance.model=list(model="gjrGARCH", garchOrder = c(1,1)), mean.model=list(armaOrder=c
m3_ar1=ugarchfit(spec=spec3,data=rendnik)
m3_ar1

v_GJR_GARCH = sigma(m3_ar1)
v_anualizada_GJR_GARCH=(250)^0.5*v_GJR_GARCH

vol.obs <- (250)^0.5*abs(rendnik)
```

Graph

```
plot(vol.obs,col="black", main="Estimated Volatility",
     cex.main=0.5,
     ylim=c(0,2.3))
```

```
lines(v_anualizada_GARCH_ar1,ylab="1 = 100%",col="blue")
```

```
lines(v_anualizada_GARCH_e,ylab="1 = 100%",col="darkgreen")
```

```
lines(v_anualizada_GJR_GARCH,ylab="1 = 100%",col="darkred")
```

News Impact

```
ni1=newsimpact(z = NULL, m_ar1)
ni2=newsimpact(z = NULL, m2_ar1)
ni3=newsimpact(z = NULL, m3_ar1)
plot(ni1$zx, ni1$zy, ylab=ni1$yexpr, xlab=ni1$xexpr, type="l", col="blue",
     ylim=c(0.00015,0.0026),main = "News Impact Curves for the Models")
lines(ni2$zx, ni2$zy, ylab=ni2$yexpr, xlab=ni2$xexpr, type="l",
      col="darkgreen")
lines(ni3$zx, ni3$zy, ylab=ni3$yexpr, xlab=ni3$xexpr, type="l",
      col="darkred")
```

Prediction

Static

```
mpred_ar1=ugarchfit(spec=spec_ar1,data=rendnik,out.sample = 90)
forc = ugarchforecast(mpred_ar1, n.ahead=90, n.roll= 0)
show(forc)
mpred_ar1@model$modeldata$T

round(fpm(forc),6)

uncvariance(m_ar1)^0.5

mpred_e=ugarchfit(spec=spec2,data=rendnik,out.sample = 90)
forc2 = ugarchforecast(mpred_e, n.ahead=90, n.roll= 0)
show(forc2)
mpred_e@model$modeldata$T

round(fpm(forc2),6)

uncvariance(m2_ar1)^0.5

mpred_gjr=ugarchfit(spec=spec3,data=rendnik,out.sample = 90)
forc3 = ugarchforecast(mpred_gjr, n.ahead=90, n.roll= 0)
show(forc3)
mpred_gjr@model$modeldata$T

round(fpm(forc3),6)
uncvariance(m3_ar1)^0.5
```

Dynamic

```
forc_1 = ugarchforecast(mpred_ar1, n.ahead=1, n.roll= 90)
show(forc_1)
mpred_ar1@model$modeldata$T

round(fpm(forc_1),5)
uncvariance(m_ar1)^0.5

forc2_1 = ugarchforecast(mpred_e, n.ahead=1, n.roll= 90)
mpred_e@model$modeldata$T

round(fpm(forc2_1),5)
uncvariance(m2_ar1)^0.5

mpred_gjr_1=ugarchfit(spec=spec3,data=rendnik,out.sample = 90)
forc3_1 = ugarchforecast(mpred_gjr, n.ahead=1, n.roll= 90)
show(forc3_1)
mpred_gjr@model$modeldata$T
```

```
round(fpm(forc3_1),5)
uncvariance(m3_ar1)^0.5
```

Historical & EMWA volatility

Historical Volatility

```
Fechas<-as.Date(rownames(zoo(nik)))
Fechas<-Fechas[-1] #eliminamos la primera observación de Fechas, la hemos perdido al calcular los rendi
Tf=length(Fechas)

resid.mod<- m_ar1@fit$residuals

n_vol=5 # periodos de media móvil
vol.hist20 <- sqrt(SMA(resid.mod^2, n=n_vol) * 252)
Fechas2<-Fechas[(n_vol+1):(Tf+1)]#hemos perdido las primeras 20 observaciones para calcular la primera

n_vol=21
vol.hist80 <- sqrt(SMA(resid.mod^2, n=n_vol) * 252)
Fechas3<-Fechas[(n_vol+1):(Tf+1)]

n_vol=63
vol.hist160 <- sqrt(SMA(resid.mod^2, n=n_vol) * 252)
Fechas4<-Fechas[(n_vol+1):(Tf+1)]

n_vol=126
vol.hist240 <- sqrt(SMA(resid.mod^2, n=n_vol) * 252)
Fechas5<-Fechas[(n_vol+1):(Tf+1)]

par(mfrow=c(2,2), cex=0.6, mar=c(2,2,3,1))
plot(Fechas2, vol.hist20, type="l", ylab='variance', main='1 week moving average')
lines(Fechas, v_anualizada_GJR_GARCH, type="l", ylab='variance',col="darkred",
      lwd=0.5)
plot(Fechas3, vol.hist80, type="l", ylab='variance', main='1 month moving average')
lines(Fechas, v_anualizada_GJR_GARCH, type="l", ylab='variance',col="darkred",
      lwd=0.5)
plot(Fechas4, vol.hist160, type="l", ylab='variance', main='3 month moving average')
lines(Fechas, v_anualizada_GJR_GARCH, type="l", ylab='variance',col="darkred",
      lwd=0.5)
plot(Fechas5, vol.hist240, type="l", ylab='variance', main='6 month moving average')
lines(Fechas, v_anualizada_GJR_GARCH, type="l", ylab='variance',col="darkred",
      lwd=0.5)
```

Exponential Weighted Moving Average

```
vol.ewma0.95 <- sqrt(EWMA(resid.mod^2, lambda = 0.05)*252)# note: in EWMA lambda is actually 1-lambda
vol.ewma0.75 <-  sqrt(EWMA(resid.mod^2, lambda = 0.25)*252) # note: in EWMA lambda is actually 1-lambda
vol.ewma0.5 <-  sqrt(EWMA(resid.mod^2, lambda = 0.5)*252) # note: in EWMA lambda is actually 1-lambda
vol.ewma0.25 <-  sqrt(EWMA(resid.mod^2, lambda = 0.75)*252) # note: in EWMA lambda is actually 1-lambda
```

```
par(mfrow=c(2,2), cex=0.6, mar=c(2,2,3,1))
plot(Fechas, vol.ewma0.95, type="l", ylab='variance', main='EWMA 0.95')
lines(Fechas, v_anualizada_GJR_GARCH, type="l", ylab='variance',col="darkred",
      lwd=0.5)
plot(Fechas, vol.ewma0.75, type="l", ylab='variance', main='EWMA 0.75')
lines(Fechas, v_anualizada_GJR_GARCH, type="l", ylab='variance',col="darkred",
      lwd=0.5)
plot(Fechas, vol.ewma0.5, type="l", ylab='variance', main='EWMA 0.5')
lines(Fechas, v_anualizada_GJR_GARCH, type="l", ylab='variance',col="darkred",
      lwd=0.5)
plot(Fechas, vol.ewma0.25, type="l", ylab='variance', main='EWMA 0.25')
lines(Fechas, v_anualizada_GJR_GARCH, type="l", ylab='variance',col="darkred",
      lwd=0.5)
```

VAR estimation

```
spec1=ugarchspec(variance.model=list(model="gjrGARCH", garchOrder = c(1,1)), mean.model=list(armaOrder=
var.t=ugarchroll(spec1, data = rendnik, n.ahead = 1, forecast.length = 252,
refit.every = 21, calculate.VaR = TRUE, VaR.alpha = 0.05,refit.window = "rolling")

spec2=ugarchspec(variance.model=list(model="sGARCH", garchOrder = c(1,1)), mean.model=list(armaOrder=c(
var.t2=ugarchroll(spec2, data = rendnik, n.ahead = 1, forecast.length = 252,
refit.every = 21, calculate.VaR = TRUE, VaR.alpha = 0.05,refit.window = "rolling")

spec3=ugarchspec(variance.model=list(model="eGARCH", garchOrder = c(1,1)), mean.model=list(armaOrder=c(
var.t3=ugarchroll(spec3, data = rendnik, n.ahead = 1, forecast.length = 252,
refit.every = 21, calculate.VaR = TRUE, VaR.alpha = 0.05,refit.window = "rolling")


plot(Fechas[(length(Fechas)-252+1):length(Fechas)],
     var.t@forecast$VaR$`alpha(5%)`,type="l",ylab="VAR 95%",xlab="",col="darkred")
lines(Fechas[(length(Fechas)-252+1):length(Fechas)],
     var.t2@forecast$VaR$`alpha(5%)`,type="l",ylab="VAR 95%",xlab="",col="blue")
lines(Fechas[(length(Fechas)-252+1):length(Fechas)],
     var.t3@forecast$VaR$`alpha(5%)`,type="l",ylab="VAR 95%",xlab="",col="darkgreen")
```

```
spec199=ugarchspec(variance.model=list(model="gjrGARCH", garchOrder = c(1,1)), mean.model=list(armaOrde
var.t99=ugarchroll(spec1, data = rendnik, n.ahead = 1, forecast.length = 252,
refit.every = 21, calculate.VaR = TRUE, VaR.alpha = 0.01,refit.window = "rolling")

spec299=ugarchspec(variance.model=list(model="sGARCH", garchOrder = c(1,1)), mean.model=list(armaOrder=
var.t299=ugarchroll(spec2, data = rendnik, n.ahead = 1, forecast.length = 252,
refit.every = 21, calculate.VaR = TRUE, VaR.alpha = 0.01,refit.window = "rolling")

spec399=ugarchspec(variance.model=list(model="eGARCH", garchOrder = c(1,1)), mean.model=list(armaOrder=
var.t399=ugarchroll(spec3, data = rendnik, n.ahead = 1, forecast.length = 252,
refit.every = 21, calculate.VaR = TRUE, VaR.alpha = 0.01,refit.window = "rolling")


plot(Fechas[(length(Fechas)-252+1):length(Fechas)],
```

```
     var.t99@forecast$VaR$`alpha(1%)`,type="l",ylab="VAR 99%",xlab="",col="darkred")
lines(Fechas[(length(Fechas)-252+1):length(Fechas)],
     var.t299@forecast$VaR$`alpha(1%)`,type="l",ylab="VAR 99%",xlab="",col="blue")
lines(Fechas[(length(Fechas)-252+1):length(Fechas)],
     var.t399@forecast$VaR$`alpha(1%)`,type="l",ylab="VAR 99%",xlab="",col="darkgreen")
```

Multivariant GARCH Models

```
# Mean and Volatility for FTSE100

getSymbols("^FTSE",src="yahoo",from="2000-01-01", to="2022-12-30")
```

```
## Warning: ^FTSE contains missing values. Some functions will not work if objects
## contain missing values in the middle of the series. Consider using na.omit(),
## na.approx(), na.fill(), etc to remove or replace them.
```

```
fts=FTSE[,6]
head(fts)
plot(fts)
```

```
dim(fts)
fts<-na.locf(fts, fromLast = TRUE)
rendfts <- dailyReturn(fts)

rX = cbind(rendnik, rendfts)

spec_fts = ugarchspec(mean.model = list(armaOrder = c(1,0)), variance.model =
list(garchOrder = c(1,1), model = "sGARCH"), distribution.model = "std")
mod_fts1=ugarchfit(spec=spec_fts,data=rendfts)
mod_fts1

spec_fts2 = ugarchspec(mean.model = list(armaOrder = c(1,0)), variance.model =
list(garchOrder = c(1,1), model = "eGARCH"), distribution.model = "std")
mod_fts2=ugarchfit(spec=spec_fts2,data=rendfts) # -6.5266
mod_fts2

spec_fts3 = ugarchspec(mean.model = list(armaOrder = c(1,0)), variance.model =
list(garchOrder = c(1,1), model = "gjrGARCH"), distribution.model = "std")
mod_fts3=ugarchfit(spec=spec_fts3,data=rendfts) # -6.5213
mod_fts3

# DCC-GARCH

mod_1 = ugarchspec(mean.model = list(armaOrder = c(1,0)), variance.model =
list(garchOrder = c(1,1), model = "gjrGARCH"), distribution.model = "std")
mod_2 = ugarchspec(mean.model = list(armaOrder = c(1,0)), variance.model =
list(garchOrder = c(1,1), model = "eGARCH"), distribution.model = "std")
dcc.garch11 = dccspec(uspec = multispec(c(mod_1,mod_2)), dccOrder = c(1,1),
distribution = "mvt")
```

```r
dcc.fit = dccfit(dcc.garch11, data = na.omit(rX))
dcc.fit

# Correlation and News Impact

cor1 = rcor(dcc.fit)
dim(cor1)
cor1
cor_AM <- cor1[1,2,]
cor_AM <- as.xts(cor_AM)
plot(cor_AM,main="Correlation Between Nikkei225 and FTSE100")
```

```r
write.table(cor_AM,file="correlaciones.csv")
nisurface(dcc.fit,type="cor")
```