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Final Exam Part 2

5.2 Derive an algorithm that, given a symmetric indefinite tridiagonal matrix A, computes $A = UDU^T$. Overwrite only the upper triangular part of A. (Fill in figure 9.) Show how you came up with the algorithm, similar to how we derived the algorithm for LDL^T.

Algorithm: $A := \text{UDUT\_Tri}(A)$

Partition $A \rightarrow \left(\begin{array}{c|c|c} A_{FF} & \alpha_{FM} e_L^T & 0 \\ * & \alpha_{MM} & \alpha_{ML} e_F^T \\ * & * & A_{LL} \end{array}\right)$ where A_{LL} is 0×0

While $m(A_{LL}) < m(A)$ do

Repartition $\left(\begin{array}{c|c|c} A_{FF} & \alpha_{FM} e_L & 0 \\ * & \alpha_{MM} & \alpha_{FM} e_F^T \\ * & * & A \end{array}\right) \rightarrow \left(\begin{array}{c|c|c|c} A_{00} & \alpha_{01} e_L & 0 & 0 \\ * & \alpha_{11} & \alpha_{12} & 0 \\ * & * & \alpha_{22} & \alpha_{23} \\ * & * & * & A_{33} \end{array}\right)$

where

There are 3 possible algorithms

1 $\lambda_{12} := \dfrac{\alpha_{12}}{\alpha_{11}}$	$\alpha_{12} = \dfrac{\alpha_{12}}{\alpha_{11}}$ 2	3 $\alpha_{22} = \alpha_{22} - \dfrac{1}{\alpha_{11}} (\alpha_{12})^2$
$\alpha_{22} := \alpha_{22} - \lambda_{12} \alpha_{12}$ (updating upper triangle)	$\alpha_{22} = \alpha_{22} - \alpha_{11} \alpha_{12}^2$ (updating upper triangle)	(updating upper triangle)
$\alpha_{12} = \lambda_{12}$		$\alpha_{12} = \alpha_{12} / \alpha_{11}$

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continue with $\left(\begin{array}{c|c|c} A_{FF} & * & * \\ \alpha_{MF} e_L & \alpha_{MM} & * \\ 0 & \alpha_{LM} e_F & A_{LL} \end{array}\right) \leftarrow \left(\begin{array}{c|c|c|c} A_{00} & * & * & \\ \alpha_{10} e_L & \alpha_{11} & * & * \\ 0 & \alpha_{21} & \alpha_{22} & * \\ 0 & 0 & \alpha_{32} e_F & A_{33} \end{array}\right)$

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endwhile

5.2 Using figure 6 from the twisted factorization document as a reference, the algorithm for 5.2 is similar.

using (1) requires a ratio between α_{12} and α_{11} to do a proper update of the upper triangle. This ratio, λ_{12}, allows for removal of any non-orthonormal element from α_{22} (i.e. $\alpha_{22} = \alpha_{22} - \lambda_{12}\alpha_{12}$). We can then replace α_{12} with λ_{12}, as we will repeat this process

(2) and (3) effectively achieve the same goal as (1), with the main difference being that we are no longer calculating λ_{12} directly. Instead, we are implicitly calculating it in the update of α_{22}:

(2) $\alpha_{22} = \alpha_{22} - \alpha_{11}(a_{12})^2 = \frac{\alpha_{12}}{\alpha_{11}}$

(3) $\alpha_{22} = \alpha_{22} - \left(\frac{1}{\alpha_{11}}\right)\left(\alpha_{12}\right)^2$ will be updated to $\frac{\alpha_{12}}{\alpha_{11}}$