(c) Provided ϕ_1 is chosen appropriately

$$A = \begin{pmatrix} A_{00} & \alpha_{10} e_L & 0 \\ \alpha_{10} e_L^T & \alpha_{11} & \alpha_{21} e_F^T \\ 0 & \alpha_{21} e_F & A_{22} \end{pmatrix}, \quad L = \begin{pmatrix} L_{00} & 0 & 0 \\ \lambda_{10} e_L^T & 1 & 0 \\ 0 & \lambda_{21} e_F & L_{22} \end{pmatrix}, \quad U = \begin{pmatrix} U_{00} & u_{01} e_L & 0 \\ 0 & 1 & v_{12} e_F^T \\ 0 & 0 & U_{22} \end{pmatrix},$$

$$D = \begin{pmatrix} D_{00} & 0 & 0 \\ 0 & \delta_1 & 0 \\ 0 & 0 & D_{22} \end{pmatrix}, \quad E = \begin{pmatrix} E_{00} & 0 & 0 \\ 0 & \varepsilon_1 & 0 \\ 0 & 0 & E_{22} \end{pmatrix}$$

$A = LDL^T$, $A = UEU^T$ Show that $\phi_1 = \delta_1 + \varepsilon_1 - \alpha_{11}$

$$A = \begin{pmatrix} L_{00} & 0 & 0 \\ \lambda_{10} e_L^T & 1 & v_{12} e_F^T \\ 0 & 0_F & U_{22} \end{pmatrix} \begin{pmatrix} D_{00} & 0 & 0 \\ 0 & \phi_1 & 0 \\ 0 & 0 & E_{22} \end{pmatrix} \begin{pmatrix} L_{00} & \lambda_{10} e_L^T & 0 \\ 0 & 1 & 0_F \\ 0 & \lambda_{21} e_F & U_{22} \end{pmatrix}$$

$$A = \begin{pmatrix} L_{00} D_{00} & 0 & 0 \\ \lambda_{10} e_L^T D_{00} & \phi_1 & v_{12} e_F^T E_{22} \\ 0 & 0 & U_{22} E_{22} \end{pmatrix} \begin{pmatrix} L_{00} & \lambda_{10} e_L^T & 0 \\ 0 & 1 & \lambda_{21} e_F \\ 0 & 0 & L_{22} \end{pmatrix}$$

$$A = \begin{bmatrix} L_{00} D_{00} L_{00} & L_{00} D_{00} \lambda_{10} e_L^T & 0 \\ \lambda_{10} e_L^T D_{00} L_{00} & (\lambda_{10} e_L^T) D_{00} (\lambda_{10} e_L^T) + \phi_1 + E_{22} (\lambda_{21} e_F)(\lambda_{21} e_F) & E_{22} U_{22} v_{12} e_F^T \\ 0 & U_{22} E_{22} \lambda_{21} e_F & E_{22} U_{22} U_{22} \end{bmatrix}$$

$$\alpha_{11} = D_{22} (\lambda_{10} e_L^T)(\lambda_{10} e_L^T) + \phi_1 + E_{22} (\lambda_{21} e_F)(\lambda_{21} e_F)$$

$$-\alpha_{11} = d_1 + D_{22} (\lambda_{10} e_L^T)(\lambda_{10} e_L^T)$$

$$0 = -d_1 + \phi + E_{22} (\lambda_{21} e_F)(\lambda_2 e_F)$$

$$-\alpha_{11} = \varepsilon_1 + E_{22} (\lambda_{21} e_F)(\lambda_{21} e_F)$$

$$-\alpha_{11} = -\delta_1 - \varepsilon_1 + \phi \implies \boxed{\phi_1 = \delta_1 + \varepsilon_1 - \alpha_{11}}$$

- What is the cost of computing one twisted factorization given that you have already computed the LDL^T and UEU^T factorizations?

  $\boxed{O(n^2)}$ should be the cost, as the diagonal matrices D and E remove the need to multiply them out as whole matrices. This leaves the bulk of the compution cost to be determined by L and U

- What is the cost of computing all twisted factorization given that you have already computed the LDL^T and UEU^T factorizations?

  $\boxed{O(n^3)}$, as the placement of Φ_1 can increase the number of computations needed to finish a twisted factorization

$$\begin{pmatrix} D_{00} & 0 & 0 \\ 0 & \phi_1 & 0 \\ 0 & 0 & E_{22} \end{pmatrix}$$

ϕ_1 shifts depending on the size of D_{00} and E_{22}.