$$LDL^T\begin{pmatrix} x_0 \\ \chi_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Hint choose x_0, χ_1, x_2 so that equals $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$$L^T\begin{pmatrix} x_0 \\ \chi_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$L_{00}x_0 + \lambda_{10}e_L^T\chi_1 = 0$$

$$\boxed{\chi_1 = 1}$$

$$V_{12}e_F\chi_1 + U_{22}x_2 = 0$$

$$(L_{00})(x_0) + (\lambda_{10}e_L^T) = 0 \Rightarrow \boxed{x_0 = \frac{-\lambda_{10}e_L^T}{L_{00}}}$$

$$(V_{12}e_F) + (U_{22})(x_2) = 0 \Rightarrow \boxed{x_2 = \frac{-V_{12}e_F^T}{U_{22}}}$$

$$\begin{pmatrix} L_{00} & \lambda_{10}e_L^T & 0 \\ 0 & 1 & 0 \\ 0 & V_{12}e_F^T & U_{22} \end{pmatrix}\begin{pmatrix} x_0 \\ \chi_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} (L_{00})(x_0) + (\lambda_{10}e_L^T)(\chi_1) \\ \chi_1 \\ (V_{12}e_F^T)(\chi_1) + (U_{22})(x_2) \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\boxed{\begin{pmatrix} x_0 \\ \chi_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -\lambda_{10}e_L^T / L_{00} \\ 1 \\ -V_{12}e_F^T / U_{22} \end{pmatrix}}$$

$$LD\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = L\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

What is the cost of this computation, given that L_{00} and U_{22} have special (bidiagonal) structure?

As determined during part 1 of this exam, operating on a bidiagonal matrix (e.g. L_{00} and U_{22}) is $O(n)$. Given we are multiplying both, the computation increases to $\boxed{O(n^2)}$