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# Final Exam Part 2

5.2 Derive an algorithm that, given a symmetric indefinite tridiagonal matrix A, computes $A = UDU^T$. Overwrite only the upper triangular part of A. (Fill in figure 9.) Show how you came up with the algorithm, similar to how we derived the algorithm for $LDL^T$.

Algorithm: $A := UDU^T\_Tri(A)$

Partition $A \rightarrow \left(\begin{array}{c|c|c} A_{FF} & \alpha_{FM} e_L^T & 0 \\ \hline * & \alpha_{MM} & \alpha_{ML} e_F^T \\ \hline * & * & A_{LL} \end{array}\right)$ where $A_{LL}$ is $0 \times 0$

While $m(A_{LL}) < m(A)$ do

Repartition $\left(\begin{array}{c|c|c} A_{FF} & \alpha_{FM} e_L & 0 \\ \hline * & \alpha_{MM} & \alpha_{FM} e_F^T \\ \hline * & * & A \end{array}\right) \rightarrow \left(\begin{array}{c|c|c|c} A_{00} & \alpha_{01} e_L & 0 & 0 \\ \hline * & \alpha_{11} & \alpha_{12} & 0 \\ \hline * & * & \alpha_{22} & \alpha_{23} \\ \hline * & * & * & A_{33} \end{array}\right)$

where

There are 3 possible algorithms

$u_{01} := \frac{\alpha_{01} e_L}{\alpha_{11}}$

$A_{00} := A_{00} - u_{01} \alpha_{01} e_L^T$ (updating upper triangle)

$\alpha_{01} e_L = u_{01}$

$\alpha_{01} e_L = \frac{\alpha_{01} e_L}{\alpha_{11}}$

$A_{00} = A_{00} - \alpha_{11} \alpha_{01} e_L \alpha_{01} e_L^T$ (updating upper triangle)

$A_{00} = A_{00} - \frac{1}{\alpha_{11}} (\alpha_{01} e_L)(\alpha_{01} e_L^T)$ (updating upper triangle)

$\alpha_{01} e_L = \alpha_{01} e_L / \alpha_{11}$

continue with $\left(\begin{array}{c|c|c} A_{FF} & * & * \\ \hline \alpha_{MF} e_L & \alpha_{MM} & * \\ \hline 0 & \alpha_{LM} e_F & A_{LL} \end{array}\right) \leftarrow \left(\begin{array}{c|c|c|c} A_{00} & * & * & \\ \hline \alpha_{10} e_L & \alpha_{11} & * & * \\ \hline 0 & \alpha_{21} & \alpha_{22} & * \\ \hline 0 & 0 & \alpha_{32} e_F & A_{33} \end{array}\right)$

continue next pass | endwhile